## Chapter 4: Electric Potential

$\checkmark$ Electric Potential and Electrical Potential Energy
$\checkmark$ Potential of a Point Charge
$\checkmark$ Electric Potential for Multiple Charges
$\checkmark$ Electric Potential for a Continuous Charge Distribution
$\checkmark$ V Due to a Charged Conductor

## Session 9:

$\checkmark$ Electric Potential and Electrical Potential Energy
$\checkmark$ Potential of a Point Charge
$\checkmark$ Electric Potential for Multiple Charges
$\checkmark$ Examples

## Electrical Potential Energy

$$
\frac{W}{q_{0}}=-\int_{a}^{b} \vec{E} \cdot d \vec{s}=V_{b}-V_{a}=V_{b a}=\frac{\Delta U}{q_{0}} \Rightarrow \Delta U=q_{0} \Delta V=-W_{E}
$$

$$
\left(V_{a}=V_{o}=0\right)
$$

$$
V(\mathrm{r})=-\int_{0}^{r} \vec{E} \cdot d \vec{S}
$$

$$
\begin{aligned}
& \overrightarrow{F_{E}}=q_{0} \vec{E} \\
& \vec{F}_{\text {apply }}=-q_{0} \vec{E} \\
& W=\int_{a}^{b} \vec{F}_{a p p l} \cdot d \vec{s}=\int_{a}^{b}-q_{0} \vec{E} \cdot d \vec{s}=-q_{0} \int_{a}^{b} \vec{E} \cdot d \vec{s}
\end{aligned}
$$

## Electric Potential

The potential energy per unit charge is the electric potential.

- The potential is characteristic of the field only.
- The potential energy is characteristic of the charge-field system.
- The potential is independent of the value of $\boldsymbol{q}_{\mathbf{o}}$.
- The potential has a value at every point in an electric field.

The electric potential is

$$
V=\frac{U}{q_{0}}
$$

- Unit: 1 V $\equiv 1 \mathrm{~J} / \mathrm{C}$
$-1 \mathrm{eV}=1.60 \times 10^{-19} \mathrm{~J}$


## Potential Difference in a Uniform Field

$$
V_{B}-V_{A}=\Delta V=-\int_{A}^{B} \overrightarrow{\mathbf{E}} \cdot d \overrightarrow{\mathbf{s}}=-E \int_{A}^{B} d \mathbf{s}=-E d
$$

$$
\Delta V=-E d
$$

The electric field vector points from higher potential toward lower potential.

$$
|E|=\frac{\Delta V}{d}
$$

$|E|=\frac{12}{3 \times 10^{-3}}=4 \times 10^{3}(\mathrm{~V} / \mathrm{m})$


Ex 1. A proton is released from rest at point $A$ in a uniform electric field that has a magnitude of $8.0 \times 10^{4} \mathrm{~V} / \mathrm{m}$. The proton undergoes a displacement of magnitude $d=0.50 \mathrm{~m}$ to point $B$ in the direction of $E$. Find the speed of the proton after completing the displacement.

Conservation of Energy : $\Delta K+\Delta U=0$

$$
\begin{aligned}
& \left(\frac{1}{2} m v^{2}-0\right)+e \Delta V=0 \\
& \frac{1}{2} m v^{2}+e(-E d)=0
\end{aligned}
$$



$$
v=\sqrt{\frac{2 e E d}{m}}=\sqrt{\frac{2\left(1.6 \times 10^{-19}\right)\left(8 \times 10^{4}\right)(0.5)}{1.67 \times 10^{-27}}}=2.8 \times 10^{6}(\mathrm{~m} / \mathrm{s})
$$

## Equipotential Surfaces



The name equipotential surface is given to any surface consisting of a continuous distribution of points having the same electric potential.

## Potential of a Point Charge

$$
\begin{gathered}
V_{b}-V_{a}=-\int_{a}^{b} \vec{E} \cdot d \vec{s}=-\int_{a}^{b} E d s(\cos 0)=-\int_{a}^{b} E d s \\
V_{b}=0(a t \infty) \text { and } V_{a}=V(a t R) \\
E=\frac{q}{4 \pi \varepsilon_{0} r^{2}} \\
0-V=-\int_{R}^{\infty} \frac{q}{4 \pi \varepsilon_{0} r^{2}} d r=\frac{q}{4 \pi \varepsilon_{0}}\left[\frac{1}{r}\right]_{R}^{\infty}=0-\frac{q}{4 \pi \varepsilon_{0}} \frac{1}{R} \\
V(\mathrm{r})=\frac{q}{4 \pi \varepsilon_{0} r}
\end{gathered}
$$

## Electric Potential for Multiple Charges

$\nLeftarrow$ The electric potential due to several point charges is the sum of the potentials due to each individual charge.
$\square$ This is another example of the superposition principle.
$\square$ The sum is the algebraic sum
ㅁ $V=0$ at $r=\infty$


$$
V=V_{1}+V_{2}+\ldots=\frac{1}{4 \pi \varepsilon_{0}} \sum_{i=1}^{n} \frac{q_{i}}{r_{i}}
$$

Ex 2. Charges $q_{1}=+q$ and $q_{2}=-q$ are located on the $z$ axis as shown in Figure (electric dipole). Find the electric potential at the point P .

$$
\begin{aligned}
& V=V_{+}+V_{-}=\frac{1}{4 \pi \varepsilon_{0}}\left(\frac{q}{r_{(+)}}+\frac{-q}{r_{(-)}}\right) \\
& V=V_{+}+V_{-}=\frac{q}{4 \pi \varepsilon_{0}}\left(\frac{r_{(-)}-r_{(+)}}{r_{(+)} r_{(-)}}\right) \\
& r_{(-)}-r_{(+)} \approx d \cos \theta \text { and } r_{(+)} r_{(-)} \approx r^{2} \\
& V=\frac{q}{4 \pi \varepsilon_{0}} \frac{d \cos \theta}{r^{2}} \\
& V \pi \varepsilon_{0} \frac{p \cos \theta}{r^{2}}
\end{aligned}
$$

## Potential Energy of Multiple Charges

$$
\begin{gathered}
U_{f}-U_{i}=q_{2}\left(V_{f}-V_{i}\right) \\
V_{i}=0(\text { at } \infty) \text { and } V_{f}=\frac{q_{1}}{4 \pi \varepsilon_{0} r} \\
U=\frac{1}{4 \pi \varepsilon_{0}} \frac{q_{1} q_{2}}{r}
\end{gathered}
$$

## $q_{1}$





Two-particle system

If the two charges are the same sign, $U$ is positive and work must be done to bring the charges together. If the two charges have opposite signs, $U$ is negative and work is done to keep the charges apart.


$$
\begin{gathered}
W_{1}=0 \\
W_{2}=q_{2}[V_{1}\left(\mathrm{r}_{2}\right)-\overbrace{V_{1}(\infty)}^{0}]=\frac{1}{4 \pi \varepsilon_{0}} q_{2}\left(\frac{q_{1}}{r_{12}}\right) \\
W_{3}=q_{3} V_{1,2}\left(\mathrm{r}_{3}\right)=\frac{1}{4 \pi \varepsilon_{0}} q_{3}\left(\frac{q_{1}}{r_{13}}+\frac{q_{2}}{r_{23}}\right) \\
W=W_{1}+W_{2}+W_{3}=\frac{1}{4 \pi \varepsilon_{0}}\left(\frac{q_{1} q_{2}}{r_{12}}+\frac{q_{1} q_{3}}{r_{13}}+\frac{q_{2} q_{3}}{r_{23}}\right)
\end{gathered}
$$

