LALR Parsing

- Canonical sets of LR(1) items
- Number of states much larger than in the SLR construction
- LR(1) = Order of thousands for a standard prog. Lang.
- SLR(1) = order of hundreds for a standard prog. Lang.
- LALR(1) (lookahead-LR)
- A tradeoff:
  - Collapse states of the LR(1) table that have the same *core* (the “LR(0)” part of each state)
  - LALR never introduces a Shift/Reduce Conflict if LR(1) doesn’t.
  - It might introduce a Reduce/Reduce Conflict (that did not exist in the LR(1))…
  - Still much better than SLR(1) (larger set of languages)
  - … but smaller than LR(1)
- What Yacc and most compilers employ.

Conflict Example

S → L=R
S → R
L→ *R
L → id
R → L

I_0: S' → .S
S → .L=R
S → .R
L → .*R
L → .id
R → .L

I_1: S' → S.

I_2: S → L.=R
R → L.

I_3: S → R.

I_4: L → *.R
R → .L
L→ .*R
L → .id

I_5: L → id.

I_6: S → L=.R
R → .L
L→ .*R
L → .id

I_7: L → *R.

I_8: R → L.

I_9: S → L=R.

Problem

FOLLOW(R)={=,$}

= shift 6

reduce by R → L

shift/reduce conflict

Conflict Example2

$S \rightarrow AaAb$

$S \rightarrow BbBa$

$A \rightarrow \varepsilon$

$B \rightarrow \varepsilon$

I_0: $S' \rightarrow .S$

$S \rightarrow .AaAb$

$S \rightarrow .BbBa$

$A \rightarrow .$

$B \rightarrow .$

Problem

FOLLOW(A)={a,b}

FOLLOW(B)={a,b}

a → reduce by $A \rightarrow \varepsilon$

a → reduce by $B \rightarrow \varepsilon$

reduce/reduce conflict

b → reduce by $A \rightarrow \varepsilon$

b → reduce by $B \rightarrow \varepsilon$

reduce/reduce conflict

Constructing Canonical LR(1) Parsing Tables

- In SLR method, the state i makes a reduction by $A \rightarrow \alpha$ when the current token is a:
  - if the $A \rightarrow \alpha\bullet$ in the I_i and a is FOLLOW(A)
- In some situations, βA cannot be followed by the terminal a in a right-sentential form when $\beta\alpha$ and the state i are on the top stack. This means that making reduction in this case is not correct.

$S \rightarrow AaAb$	$S \Rightarrow AaAb \Rightarrow Aab \Rightarrow ab$	$S \Rightarrow BbBa \Rightarrow Bba \Rightarrow ba$
$S \rightarrow BbBa$		
$A \rightarrow \varepsilon$	$Aab \Rightarrow \varepsilon ab$	$Bba \Rightarrow \varepsilon ba$
$B \rightarrow \varepsilon$	$AaAb \Rightarrow Aa \varepsilon b$	$BbBa \Rightarrow Bb \varepsilon a$

LR(1) Item

- To avoid some of invalid reductions, the states need to carry more information.
- Extra information is put into a state by including a terminal symbol as a second component in an item.
- A LR(1) item is:

  $A \rightarrow \alpha \bullet \beta, a$ where **a** is the look-head of the LR(1) item

  (**a** is a terminal or end-marker.)

LR(1) Item (cont.)

- When β (in the LR(1) item $A \rightarrow \alpha \bullet \beta, a$) is not empty, the look-head does not have any affect.
- When β is empty ($A \rightarrow \alpha \bullet, a$), we do the reduction by $A \rightarrow \alpha$ only if the next input symbol is **a** (not for any terminal in FOLLOW(A)).
- A state will contain $A \rightarrow \alpha \bullet, a_1$ where $\{a_1,...,a_n\} \subseteq$ FOLLOW(A)

  ...

  $A \rightarrow \alpha \bullet, a_n$

Canonical Collection of Sets of LR(1) Items

- The construction of the canonical collection of the sets of LR(1) items are similar to the construction of the canonical collection of the sets of LR(0) items, except that *closure* and *goto* operations work a little bit different.

closure(I) is: (where I is a set of LR(1) items)

- every LR(1) item in I is in closure(I)
- if $A \rightarrow \alpha . B\beta, a$ in closure(I) and $B \rightarrow \gamma$ is a production rule of G; then $B \rightarrow .\gamma, b$ will be in the closure(I) for each terminal b in FIRST(βa) .

goto operation

- If I is a set of LR(1) items and X is a grammar symbol (terminal or non-terminal), then goto(I,X) is defined as follows:
  - If $A \rightarrow \alpha.X\beta,a$ in I
    then every item in **closure({$A \rightarrow \alpha X.\beta,a$})** will be in goto(I,X).

Construction of The Canonical LR(1) Collection

- ***Algorithm***:

  C is { closure({S'→.S,$}) }

  repeat the followings until no more set of LR(1) items can be added to ***C***.

  for each I in ***C*** and each grammar symbol X

  if goto(I,X) is not empty and not in ***C***

  add goto(I,X) to ***C***

- goto function is a DFA on the sets in C.

A Short Notation for The Sets of LR(1) Items

- A set of LR(1) items containing the following items

$$A \rightarrow \alpha \bullet \beta, a_1$$

$$\ldots$$

$$A \rightarrow \alpha \bullet \beta, a_n$$

can be written as

$$A \rightarrow \alpha \bullet \beta, \{a_1, a_2, \ldots, a_n\}$$

Canonical LR(1) Collection -- Example

$S \rightarrow AaAb$

$S \rightarrow BbBa$

$A \rightarrow \varepsilon$

$B \rightarrow \varepsilon$

I_0: $S' \rightarrow .S$,\$
$S \rightarrow .AaAb$,\$
$S \rightarrow .BbBa$,\$
$A \rightarrow .$,a
$B \rightarrow .$,b

S → I_1: $S' \rightarrow S.$,\$

A → I_2: $S \rightarrow A.aAb$,\$ —a→ to I_4

B → I_3: $S \rightarrow B.bBa$,\$ —b→ to I_5

I_4: $S \rightarrow Aa.Ab$,\$ —A→ I_6: $S \rightarrow AaA.b$,\$ —a→ I_8: $S \rightarrow AaAb.$,\$
$A \rightarrow .$,b

I_5: $S \rightarrow Bb.Ba$,\$ —B→ I_7: $S \rightarrow BbB.a$,\$ —b→ I_9: $S \rightarrow BbBa.$,\$
$B \rightarrow .$,a

Canonical LR(1) Collection – Example2

S' → S
1) S → L=R
2) S → R
3) L→ *R
4) L → id
5) R → L

I_0: S' → .S,$
S → .L=R,$
S → .R,$
L → .*R,{$,=}
L → .id, {$,=}
R → .L,$

S → I_1
L → I_2
R → I_3
* → I_4
id → I_5

I_1: S' → S.,$

I_2: S → L.=R,$ → to I_6
R → L.,$

I_3: S → R.,$

I_4: L → *.R, {$,=}
R → .L, {$,=}
L→ .*R, {$,=}
L → .id, {$,=}

R → to I_7
L → to I_8
* → to I_4
id → to I_5

I_5: L → id., {$,=}

I_6: S → L=.R,$
R → .L,$
L → .*R,$
L → .id,$

R → to I_9
L → to I_{10}
* → to I_{11}
id → to I_{12}

I_7: L → *R., {$,=}

I_8: R → L., {$,=}

I_9: S → L=R.,$

I_{10}: R → L.,$

I_{11}: L → *.R,$
R → .L,$
L→ .*R,$
L → .id,$

R → to I_{13}
L → to I_{10}
* → to I_{11}
id → to I_{12}

I_{12}: L → id.,$

I_{13}: L → *R.,$

I_4 and I_{11}

I_5 and I_{12}

I_7 and I_{13}

I_8 and I_{10}

Construction of LR(1) Parsing Tables

1. Construct the canonical collection of sets of LR(1) items for G'.
   $C \leftarrow \{I_0,...,I_n\}$

2. Create the parsing action table as follows
   - If a is a terminal, $A \rightarrow \alpha \bullet a\beta,b$ in I_i and $goto(I_i,a)=I_j$ then action[i,a] is ***shift j.***
   - If $A \rightarrow \alpha \bullet,a$ is in I_i, then action[i,a] is ***reduce*** $A \rightarrow \alpha$ where $A \neq S'$.
   - If $S' \rightarrow S \bullet,\$$ is in I_i, then action[i,$] is ***accept***.
   - If any conflicting actions generated by these rules, the grammar is not LR(1).

3. Create the parsing goto table
   - for all non-terminals A, if $goto(I_i,A)=I_j$ then goto[i,A]=j

4. All entries not defined by (2) and (3) are errors.

5. Initial state of the parser contains $S' \rightarrow .S,\$$

LR(1) Parsing Tables – (for Example2)

	id	*	=	$	S	L	R
0	s5	s4			1	2	3
1				acc			
2			s6	r5			
3				r2			
4	s5	s4				8	7
5			r4	r4			
6	s12	s11				10	9
7			r3	r3			
8			r5	r5			
9				r1			
10				r5			
11	s12	s11				10	13
12				r4			
13				r3			

no shift/reduce or
no reduce/reduce conflict

so, it is a LR(1) grammar

LALR Parsing Tables

- **LALR** stands for **LookAhead LR.**

- LALR parsers are often used in practice because LALR parsing tables are smaller than LR(1) parsing tables.
- The number of states in SLR and LALR parsing tables for a grammar G are equal.
- But LALR parsers recognize more grammars than SLR parsers.
- ***yacc*** creates a LALR parser for the given grammar.
- A state of LALR parser will be again a set of LR(1) items.

Creating LALR Parsing Tables

Canonical LR(1) Parser ➔ LALR Parser

shrink # of states

- This shrink process may introduce a **reduce/reduce** conflict in the resulting LALR parser (so the grammar is NOT LALR)
- But, this shrik process does not produce a **shift/reduce** conflict.

The Core of A Set of LR(1) Items

- The core of a set of LR(1) items is the set of its first component.

Ex: $S \rightarrow L \bullet =R,\$$ ➔ $S \rightarrow L \bullet =R$ ← Core

$R \rightarrow L \bullet ,\$$ $R \rightarrow L \bullet$

- We will find the states (sets of LR(1) items) in a canonical LR(1) parser with same cores. Then we will merge them as a single state.

I_1: $L \rightarrow id \bullet ,=$ A new state: I_{12}: $L \rightarrow id \bullet ,=$

➔ $L \rightarrow id \bullet ,\$$

I_2: $L \rightarrow id \bullet ,\$$ have same core, merge them

- We will do this for all states of a canonical LR(1) parser to get the states of the LALR parser.
- In fact, the number of the states of the LALR parser for a grammar will be equal to the number of states of the SLR parser for that grammar.

Creation of LALR Parsing Tables

- Create the canonical LR(1) collection of the sets of LR(1) items for the given grammar.
- Find each core; find all sets having that same core; replace those sets having same cores with a single set which is their union.

  $C=\{I_0,...,I_n\} \rightarrow C'=\{J_1,...,J_m\}$ where $m \leq n$
- Create the parsing tables (action and goto tables) same as the construction of the parsing tables of LR(1) parser.
  - Note that: If $J=I_1 \cup ... \cup I_k$ since $I_1,...,I_k$ have same cores

    $\rightarrow$ cores of $goto(I_1,X),...,goto(I_2,X)$ must be same.
  - So, $goto(J,X)=K$ where K is the union of all sets of items having same cores as $goto(I_1,X)$.
- If no conflict is introduced, the grammar is LALR(1) grammar. (We may only introduce reduce/reduce conflicts; we cannot introduce a shift/reduce conflict)

Shift/Reduce Conflict

- We say that we cannot introduce a shift/reduce conflict during the shrink process for the creation of the states of a LALR parser.
- Assume that we can introduce a shift/reduce conflict. In this case, a state of LALR parser must have:

  $A \rightarrow \alpha\bullet,a$ and $B \rightarrow \beta\bullet a\gamma,b$

- This means that a state of the canonical LR(1) parser must have:

  $A \rightarrow \alpha\bullet,a$ and $B \rightarrow \beta\bullet a\gamma,c$

  But, this state has also a shift/reduce conflict. i.e. The original canonical LR(1) parser has a conflict.

  (Reason for this, the shift operation does not depend on lookaheads)

Reduce/Reduce Conflict

- But, we may introduce a reduce/reduce conflict during the shrink process for the creation of the states of a LALR parser.

$I_1 : A \rightarrow \alpha\bullet, a$
$\quad B \rightarrow \beta\bullet, b$

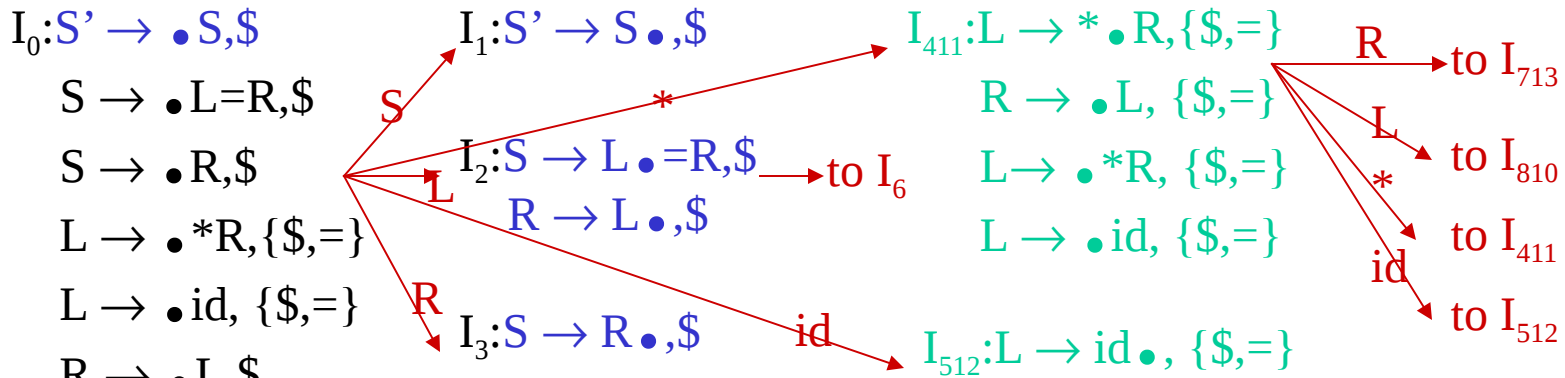
$I_2 : A \rightarrow \alpha\bullet, b$
$\quad B \rightarrow \beta\bullet, c$

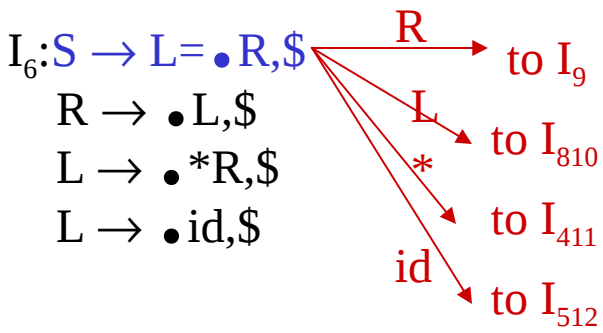
$\Downarrow$

$I_{12} : A \rightarrow \alpha\bullet, \{a,b\}$ ➔ reduce/reduce conflict
$\quad B \rightarrow \beta\bullet, \{b,c\}$

Canonical LALR(1) Collection – Example2

S' → S
1) S → L=R
2) S → R
3) L→ *R
4) L → id
5) R → L





I9:S → L=R•,$

I713:L → *R•, {$,=}

I810: R → L•, {$,=}

Same Cores

I_4 and I_{11}

I_5 and I_{12}

I_7 and I_{13}

I_8 and I_{10}

LALR(1) Parsing Tables – (for Example2)

	id	*****	**=**	**$**	**S**	**L**	**R**
0	s5	s4			1	2	3
1				acc			
2			s6	r5			
3				r2			
4	s5	s4				8	7
5			r4	r4			
6	s12	s11				10	9
7			r3	r3			
8			r5	r5			
9				r1			

no shift/reduce or
no reduce/reduce conflict

so, it is a LALR(1) grammar

Using Ambiguous Grammars

- All grammars used in the construction of LR-parsing tables must be un-ambiguous.
- Can we create LR-parsing tables for ambiguous grammars ?
  - Yes, but they will have conflicts.
  - We can resolve these conflicts in favor of one of them to disambiguate the grammar.
  - At the end, we will have again an unambiguous grammar.
- Why we want to use an ambiguous grammar?
  - Some of the ambiguous grammars are **much natural**, and a corresponding unambiguous grammar can be very complex.
  - Usage of an ambiguous grammar may **eliminate unnecessary reductions**.
- Ex.

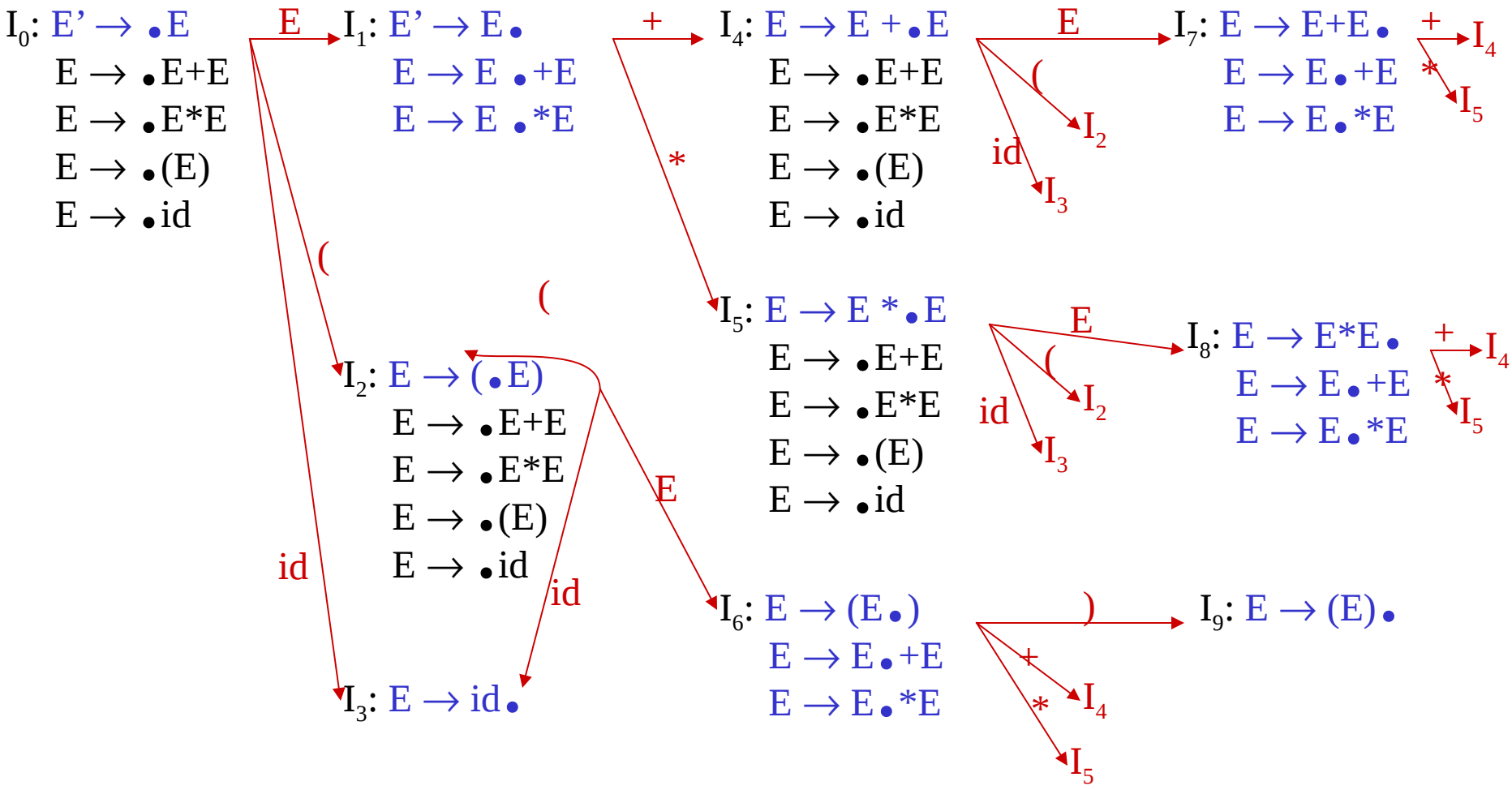
$E \rightarrow E+E \mid E*E \mid (E) \mid id$ ➔

$E \rightarrow E+T \mid T$

$T \rightarrow T*F \mid F$

$F \rightarrow (E) \mid id$

Sets of LR(0) Items for Ambiguous Grammar



SLR-Parsing Tables for Ambiguous Grammar

FOLLOW(E) = { $, +, *,) }

State I_7 has shift/reduce conflicts for symbols + and *.

$I_0 \xrightarrow{E} I_1 \xrightarrow{+} I_4 \xrightarrow{E} I_7$

when current token is +

shift ➔ + is right-associative

reduce ➔ + is left-associative

when current token is *

shift ➔ * has higher precedence than +

reduce ➔ + has higher precedence than *

SLR-Parsing Tables for Ambiguous Grammar

FOLLOW(E) = { $, +, *,) }

State I_8 has shift/reduce conflicts for symbols + and *.

$$I_0 \xrightarrow{E} I_1 \xrightarrow{*} I_5 \xrightarrow{E} I_8$$

when current token is *

shift ➔ * is right-associative

reduce ➔ * is left-associative

when current token is +

shift ➔ + has higher precedence than *

reduce ➔ * has higher precedence than +

SLR-Parsing Tables for Ambiguous Grammar

	Action							Goto
	id	**+**	*****	**(**	**)**	**$**		**E**
0	s3			s2				1
1		s4	s5			acc		
2	s3			s2				6
3		r4	r4		r4	r4		
4	s3			s2				7
5	s3			s2				8
6		s4	s5		s9			
7		r1	s5		r1	r1		
8		r2	r2		r2	r2		
9		r3	r3		r3	r3		

Error Recovery in LR Parsing

- An LR parser will detect an error when it consults the parsing action table and finds an error entry. All empty entries in the action table are error entries.
- Errors are never detected by consulting the goto table.
- An LR parser will announce error as soon as there is no valid continuation for the scanned portion of the input.
- A canonical LR parser (LR(1) parser) will never make even a single reduction before announcing an error.
- The SLR and LALR parsers may make several reductions before announcing an error.
- But, all LR parsers (LR(1), LALR and SLR parsers) will never shift an erroneous input symbol onto the stack.

Panic Mode Error Recovery in LR Parsing

- Scan down the stack until a state **s** with a goto on a particular nonterminal **A** is found. (Get rid of everything from the stack before this state s).
- Discard zero or more input symbols until a symbol **a** is found that can legitimately follow A.
  - The symbol a is simply in FOLLOW(A), but this may not work for all situations.
- The parser stacks the nonterminal **A** and the state **goto[s,A]**, and it resumes the normal parsing.
- This nonterminal A is normally is a basic programming block (there can be more than one choice for A).
  - stmt, expr, block, ...

Phrase-Level Error Recovery in LR Parsing

- Each empty entry in the action table is marked with a specific error routine.
- An error routine reflects the error that the user most likely will make in that case.
- An error routine inserts the symbols into the stack or the input (or it deletes the symbols from the stack and the input, or it can do both insertion and deletion).
  - missing operand
  - unbalanced right parenthesis