Chapter 4: Motion in Two and Three Dimensions

- ✓ **Position and Displacement**
- ✓ **Velocity**
- ✓ **Acceleration**
- ✓ **Finding Displacement and Velocity from Acceleration**
- ✓ **Projectile Motion**
- ✓ **Uniform Circular Motion**
- ✓ **Relative Motion**

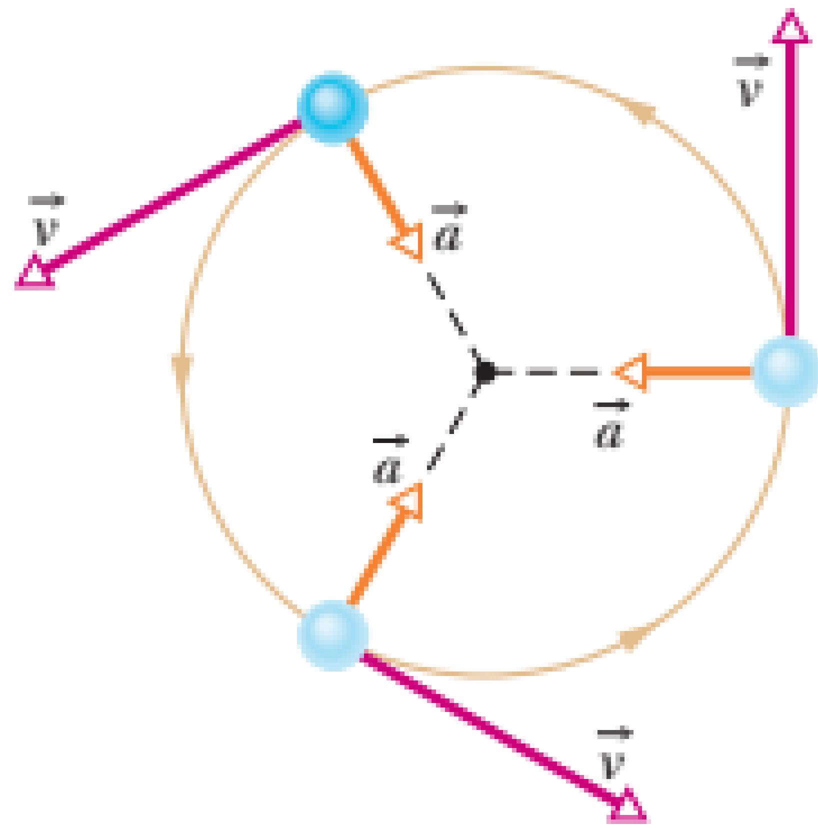
Session 8:

- ✓ Uniform Circular Motion
- ✓ Relative Motion
- ✓ Examples

Uniform Circular Motion

- A particle is in **uniform circular motion** if it travels around a circle or a circular arc at **constant (uniform) speed.**

- The particle is **accelerating** because the **velocity changes in direction**.

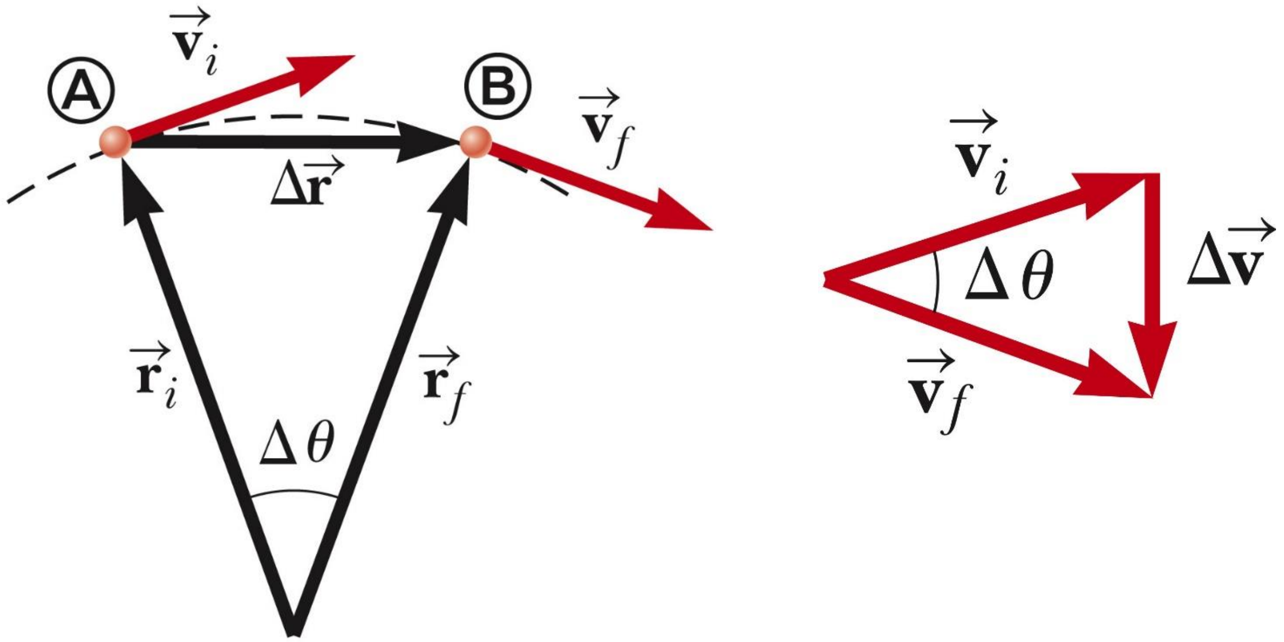


Centripetal Acceleration

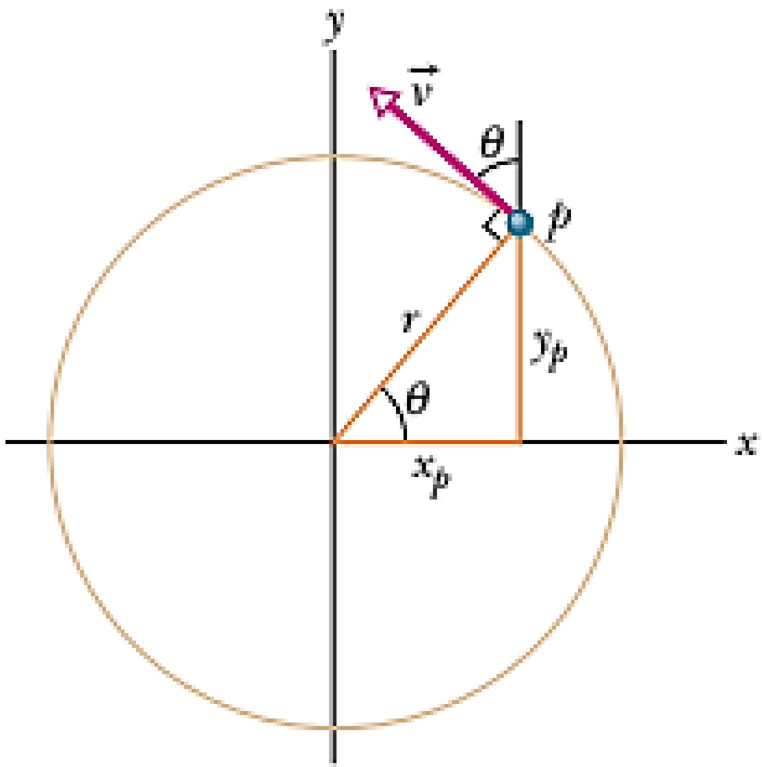
$$\boxed{a_C = \frac{v^2}{r}}$$

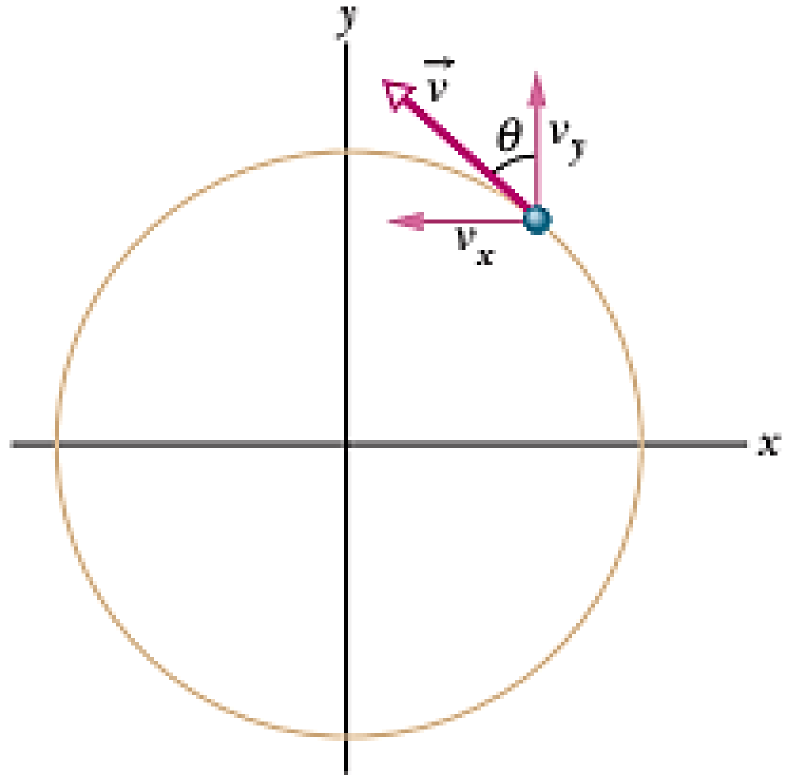
Period

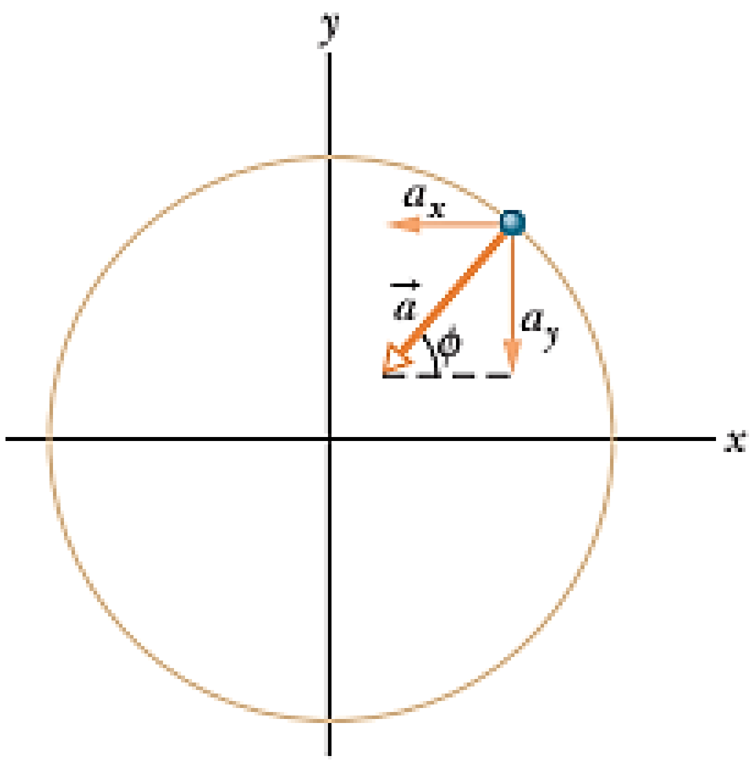
$$\boxed{T = \frac{2\pi r}{v}}$$



Uniform Circular Motion







$$\begin{cases} \vec{\mathbf{r}} = x_p\,\hat{\mathbf{i}} + y_p\,\hat{\mathbf{j}} \\ x_p = r\cos\theta \\ y_p = r\sin\theta \end{cases}$$

$$\begin{cases} \vec{v} = v_x\hat{\mathbf{i}} + v_y\hat{\mathbf{j}} = (-v\sin\theta)\hat{\mathbf{i}} + (v\cos\theta)\hat{\mathbf{j}} \\ \vec{v} = (-\dfrac{v\,y_p}{r})\hat{\mathbf{i}} + (\dfrac{v\,x_p}{r})\hat{\mathbf{j}} \end{cases}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = (-\frac{v}{r}\frac{dy_p}{dt})\hat{\mathbf{i}} + (\frac{v}{r}\frac{dx_p}{dt})\hat{\mathbf{j}} \Rightarrow \vec{a} = (-\frac{v\,v_y}{r})\hat{\mathbf{i}} + (\frac{v\,v_x}{r})\hat{\mathbf{j}} = (-\frac{v^2}{r}\cos\theta)\hat{\mathbf{i}} + (-\frac{v^2}{r}\sin\theta)\hat{\mathbf{j}}$$

$$\boxed{|\vec{a}| = \sqrt{a_x^{\;2} + a_y^{\;2}} = \frac{v^2}{r}}$$

$$\boxed{\tan\phi = \frac{a_y}{a_x} = \frac{-\dfrac{v^2}{r}\sin\theta}{-\dfrac{v^2}{r}\cos\theta} = \tan\theta}$$

Ex 9:

What is the magnitude of the centripetal acceleration of an object on Earth's equator due to the rotation of Earth?

$$\boxed{R = 6370\ km = 6.37 \times 10^{6}\ m} \qquad \boxed{T = 24\,h = 86400\ s = 8.64 \times 10^{4}\ s}$$

$$T = \frac{2\pi R}{v} \Rightarrow \boxed{v = \frac{2\pi R}{T} = \frac{2\pi(6.37 \times 10^{6})}{(8.64 \times 10^{4})} \simeq 463\ \ (m/s)}$$

$$a_C = \frac{v^2}{R} \Rightarrow \boxed{a_C = \frac{(463)^2}{6.37 \times 10^{6}} \simeq 34 \times 10^{-3}\ \ (m/s^2)}$$

Circular Motion, General

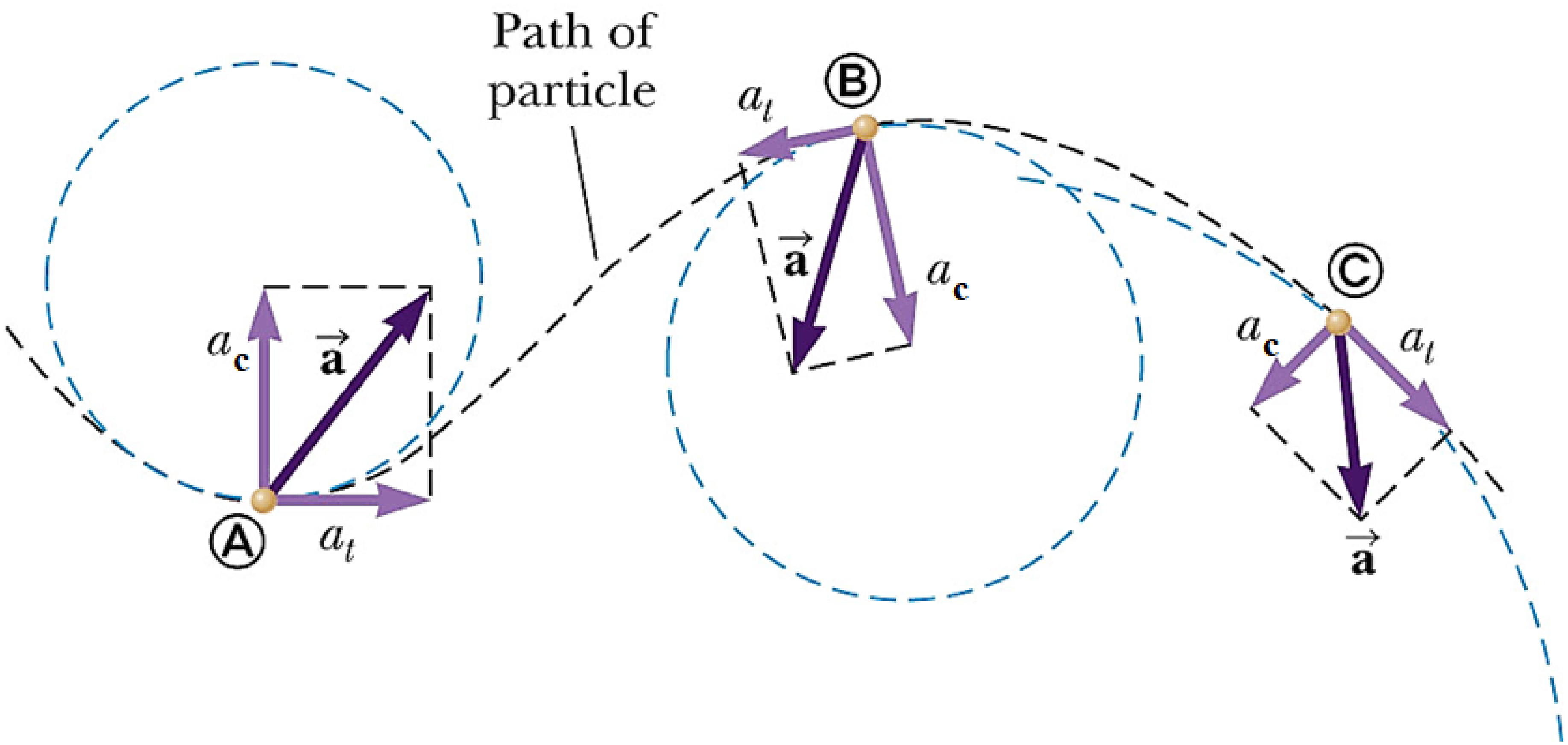
Tangential Acceleration:

- The **centripetal acceleration** comes from a **change in the direction** of the velocity vector.
- The **tangential acceleration** causes the **change in the speed** of the particle.

$$a_C = \frac{v^2}{r}$$

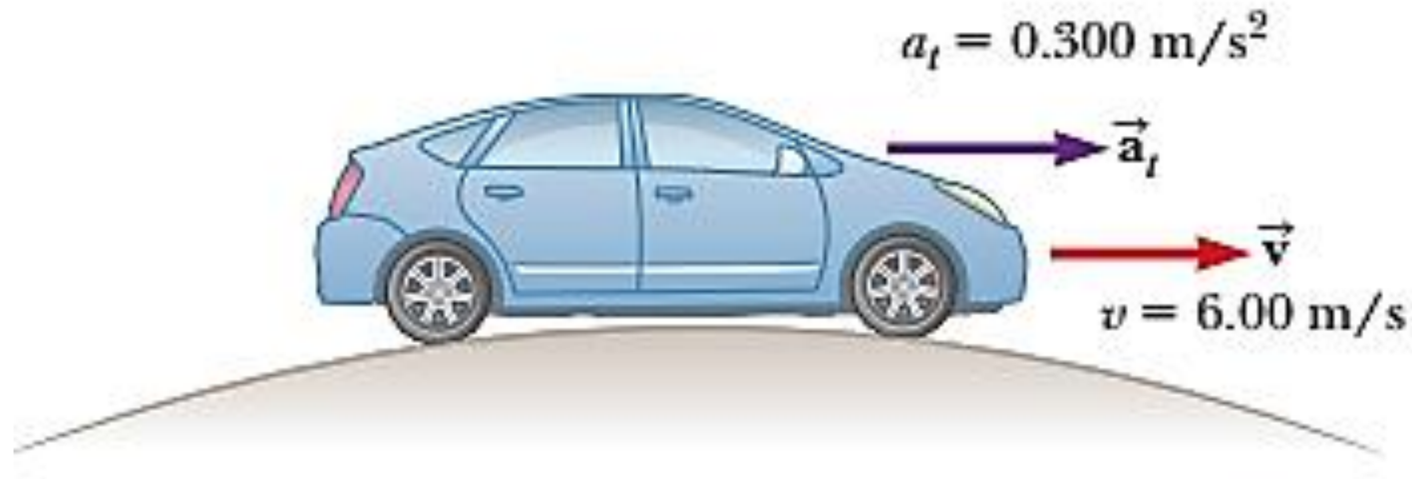
$$a_t = \left|\frac{dv}{dt}\right|$$

$$|\vec{a}| = \sqrt{a_c^{\,2} + a_t^{\,2}}$$



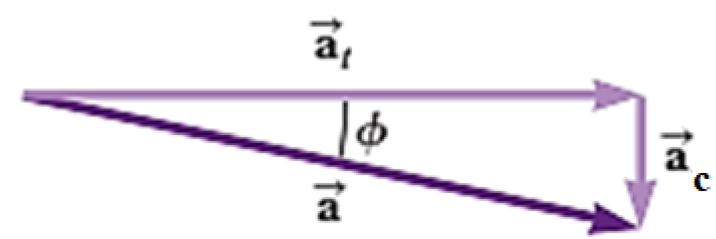
Ex 10:

A car leaves a stop sign and exhibits a constant acceleration of **0.300 m/s²** parallel to the roadway. The car passes over a rise in the roadway such that the top of the rise is shaped like an arc of a circle of radius **500 m**. At the moment the car is at the top of the rise, its velocity vector is horizontal and has a magnitude of **6.00 m/s**. What are the **magnitude and direction of the total acceleration** vector for the car at this instant?



$$a_C = \frac{v^2}{r}$$

$$a_C = \frac{(6)^2}{500} = 0.072\ \ (m/s^2)$$



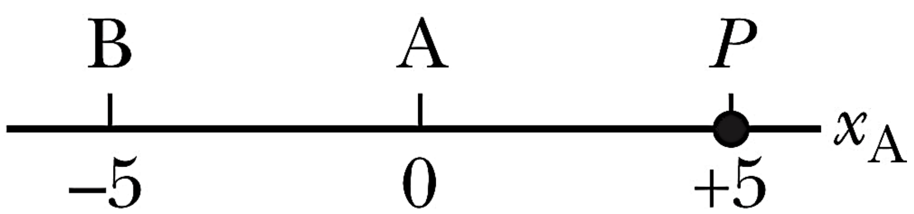
$$|\vec{a}| = \sqrt{a_c{}^2 + a_t{}^2}$$

$$\boxed{|\vec{a}| = \sqrt{(0.072)^2 + (0.3)^2} = 0.309\ \ (m/s^2)}$$

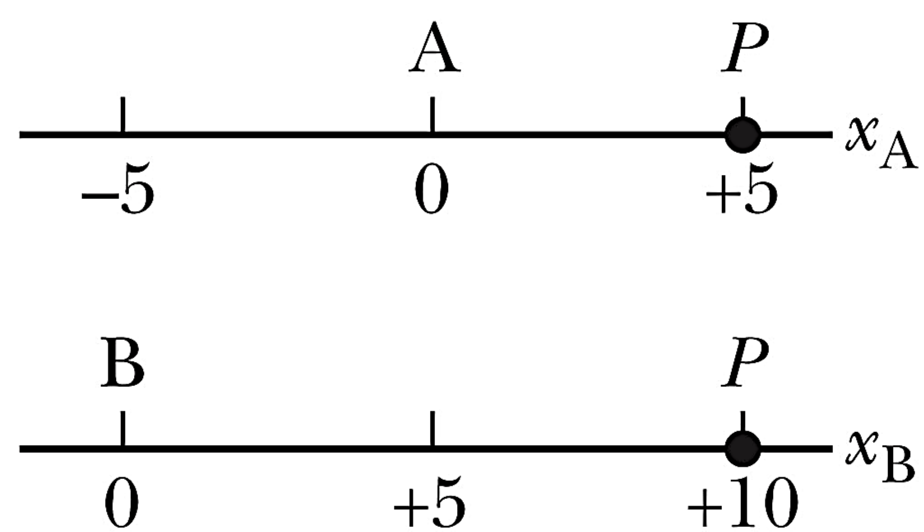
$$\boxed{\phi = \tan^{-1}\frac{a_c}{a_t} = \tan^{-1}\frac{0.072}{0.3} = 13.5^\circ}$$

Relative Motion

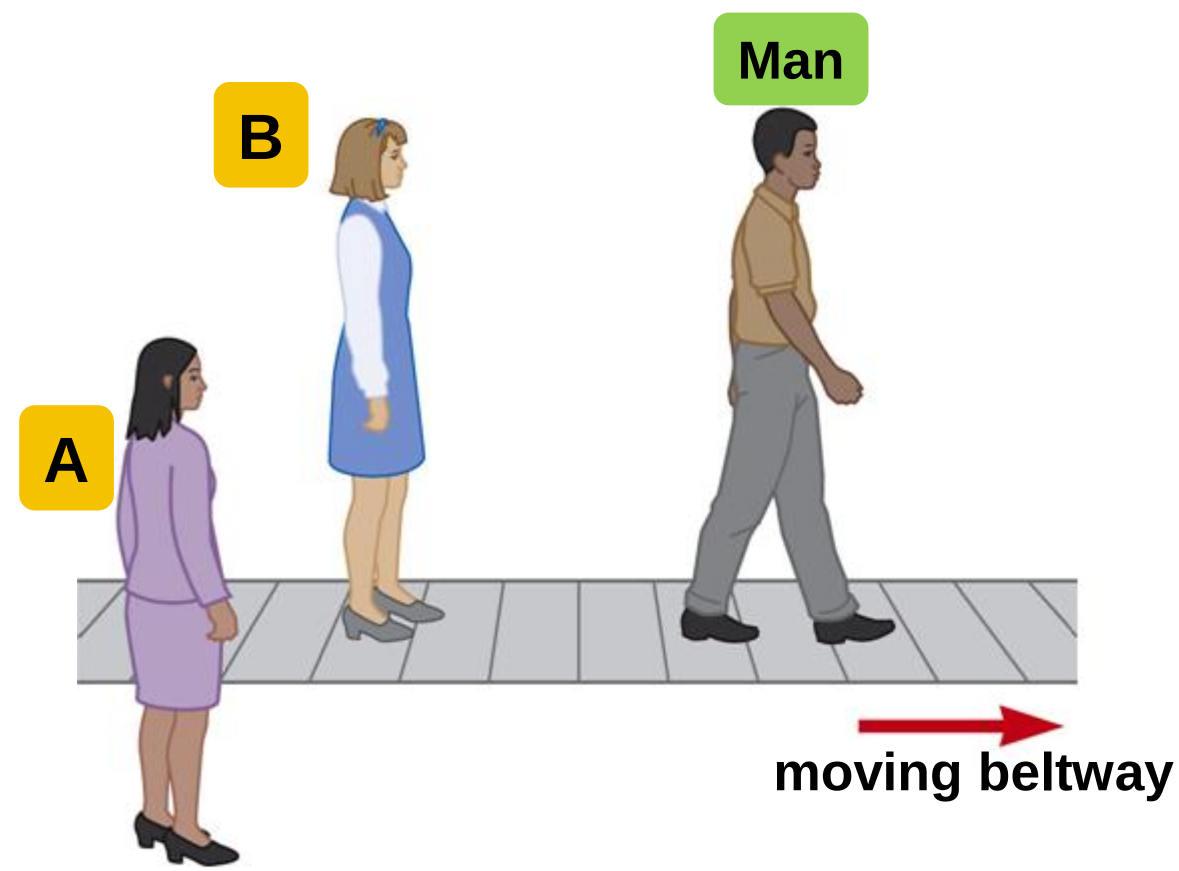
Different measurements due to the different frames of reference:



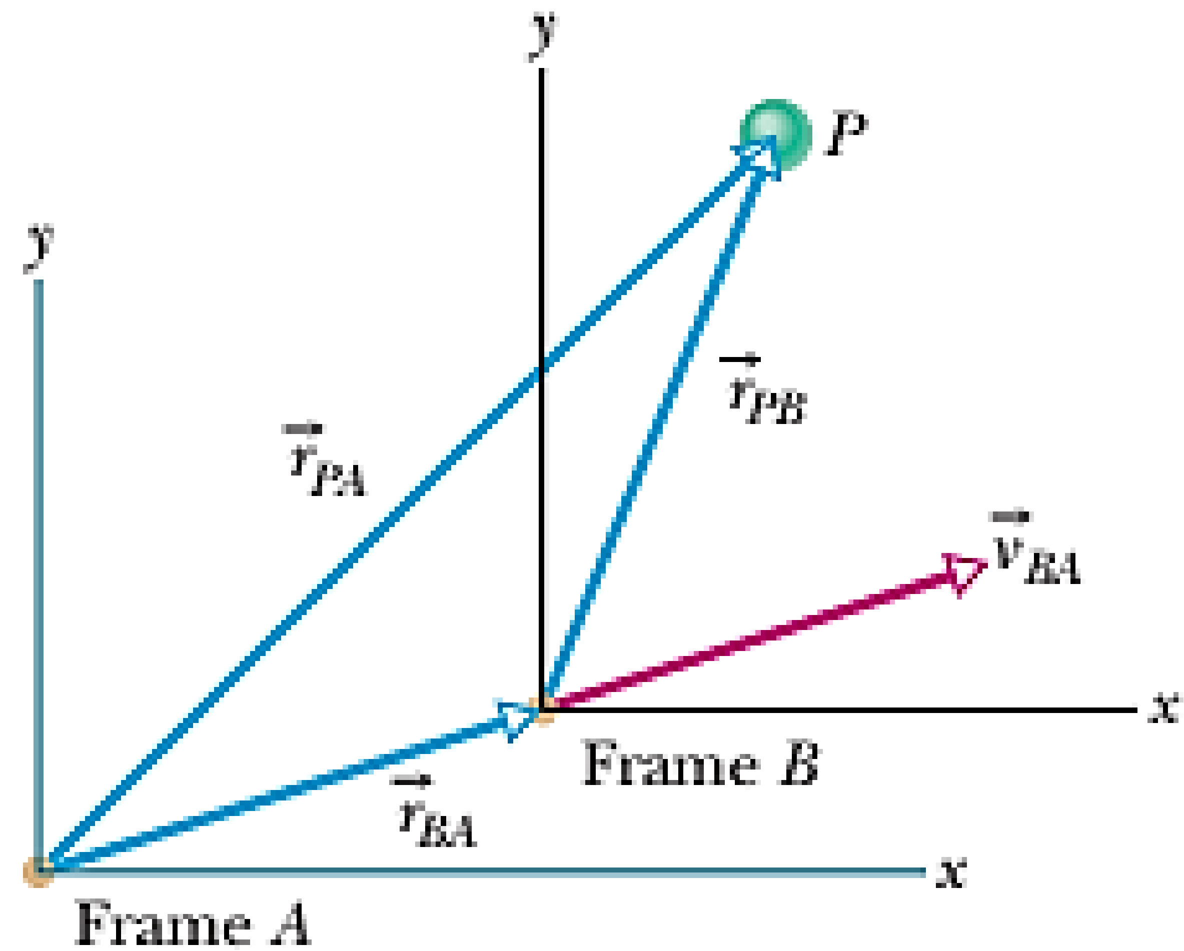
$$\left\{\begin{aligned} x_{pA} &= +5\,m \\ x_{pB} &= +10\,m \end{aligned}\right.$$



$$\boxed{v_{MA} > v_{MB}}$$



Relative Motion



$$\boxed{\vec{\mathbf{r}}_{PA} = \vec{\mathbf{r}}_{PB} + \vec{\mathbf{r}}_{BA}}$$

$$\frac{d}{dt}(\vec{\mathbf{r}}_{PA}) = \frac{d}{dt}(\vec{\mathbf{r}}_{PB}) + \frac{d}{dt}(\vec{\mathbf{r}}_{BA})$$

$$\boxed{\vec{v}_{PA} = \vec{v}_{PB} + \vec{v}_{BA}}$$

Galilean transformation equations.

$$\frac{d}{dt}(\vec{v}_{PA}) = \frac{d}{dt}(\vec{v}_{PB}) + \frac{d}{dt}(\vec{v}_{BA}) \Rightarrow \vec{a}_{PA} = \vec{a}_{PB} + \vec{a}_{BA}$$

$$\text{if } \vec{v}_{BA} = \text{constant} \Rightarrow \vec{a}_{BA} = 0 \Rightarrow \boxed{\vec{a}_{PA} = \vec{a}_{PB}}$$

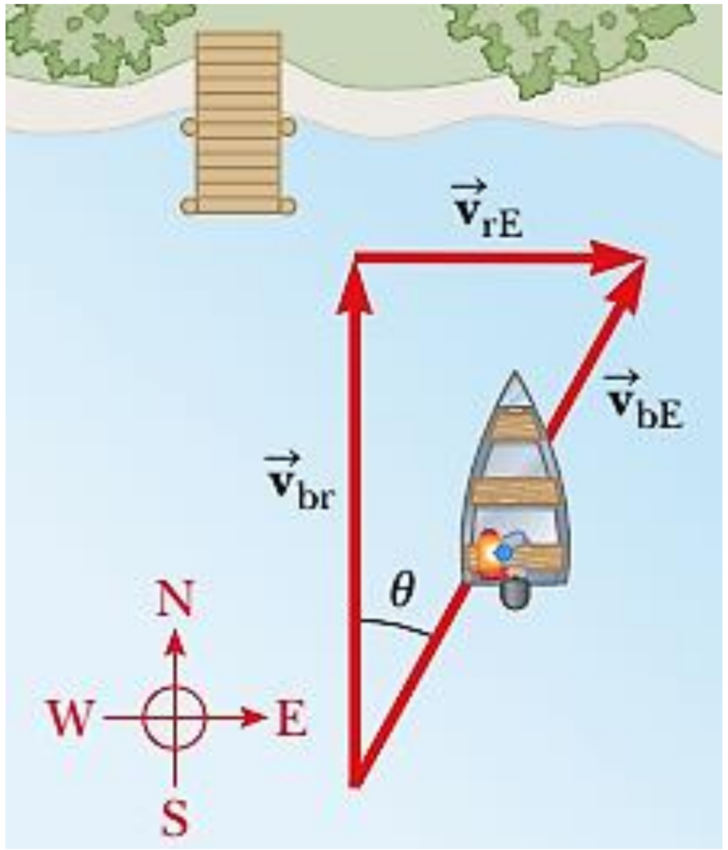
Ex 11: A boat crossing a wide river moves with a speed of **10.0 km/h** relative to the water. The water in the river has a uniform speed of **5.00 km/h** due east relative to the Earth. If the boat heads due north, determine the velocity of the boat relative to an observer standing on either bank.

$$\vec{v}_{bE} = \vec{v}_{br} + \vec{v}_{rE}$$

$$\left|\vec{v}_{bE}\right| = \sqrt{\left|\vec{v}_{br}\right|^2 + \left|\vec{v}_{rE}\right|^2}$$

$$\boxed{\left|\vec{v}_{bE}\right| = \sqrt{(10)^2 + (5)^2} \simeq 11.2 \quad (km/h)}$$

$$\boxed{\theta = \tan^{-1}\frac{v_{rE}}{v_{br}} = \tan^{-1}\frac{5}{10} \simeq 26.6^\circ}$$



$$\begin{cases} \left|\vec{v}_{bE}\right| = \sqrt{\left|\vec{v}_{br}\right|^2 - \left|\vec{v}_{rE}\right|^2} \simeq 8.66 \ (km/h) \\ \varphi = \tan^{-1}\frac{v_{rE}}{v_{bE}} = \tan^{-1}\frac{5}{8.66} = 30^\circ \end{cases}$$