Chapter 4: Motion in Two and Three Dimensions

- ✓ **Position and Displacement**
- ✓ **Velocity**
- ✓ **Acceleration**
- ✓ **Finding Displacement and Velocity from Acceleration**
- ✓ **Projectile Motion**
- ✓ **Uniform Circular Motion**
- ✓ **Relative Motion**

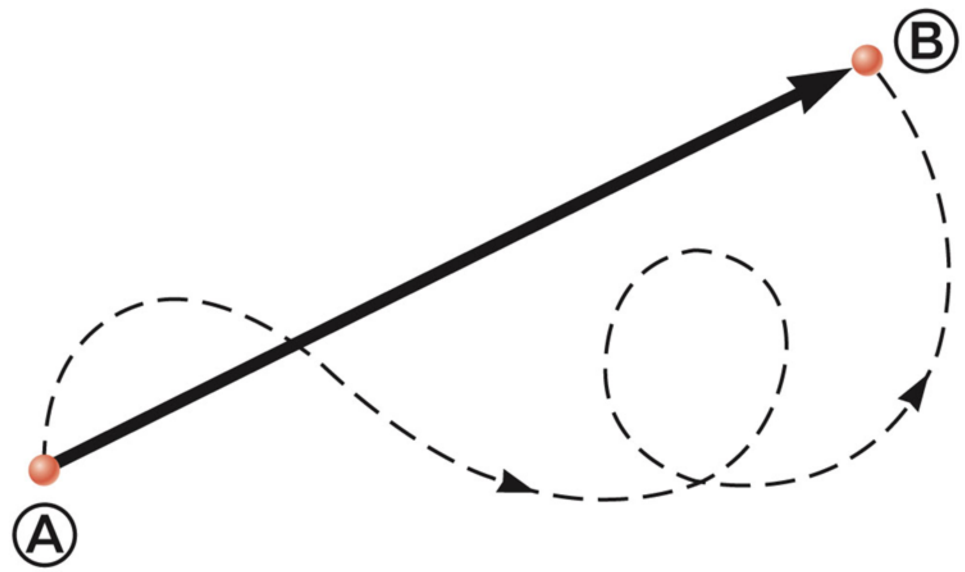
Chapter 4: Motion in Two and Three Dimensions

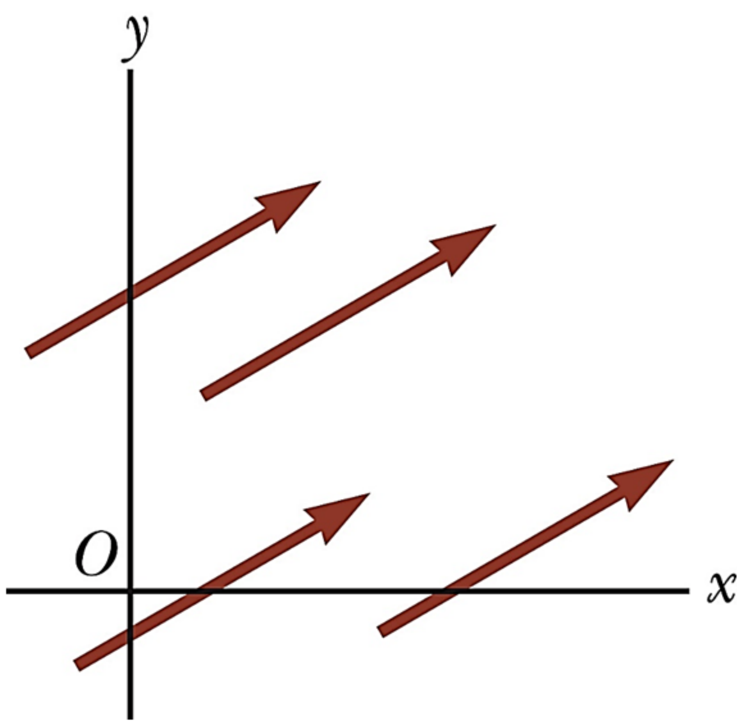
Session 5:

- ✓ Introduction to Vectors
- ✓ Position and Displacement
- ✓ Velocity
- ✓ Examples

Introduction to Vectors

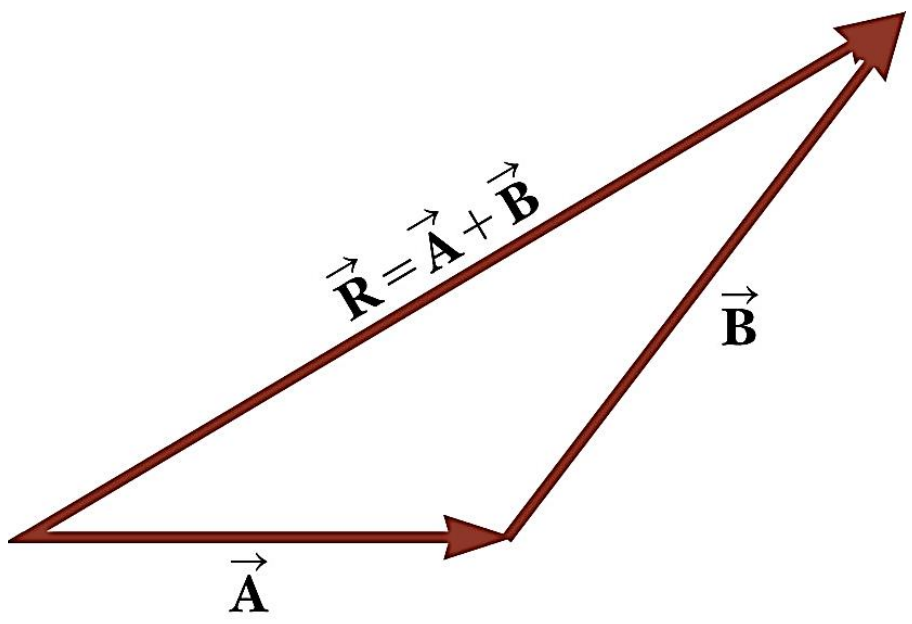
- A scalar quantity is completely specified by a single value with an appropriate unit and has no direction. (Mass, Time, Distance)
- A vector quantity is completely described by a number and appropriate units plus a direction. (Displacement, Velocity)
- Text uses arrow to denote a vector: $\vec{\mathbf{A}}$
- Two vectors are equal if they have the same magnitude ($\vec{\mathbf{A}} = \vec{\mathbf{B}}$ *if* $|\vec{\mathbf{A}}| = |\vec{\mathbf{B}}|$) and the same direction.

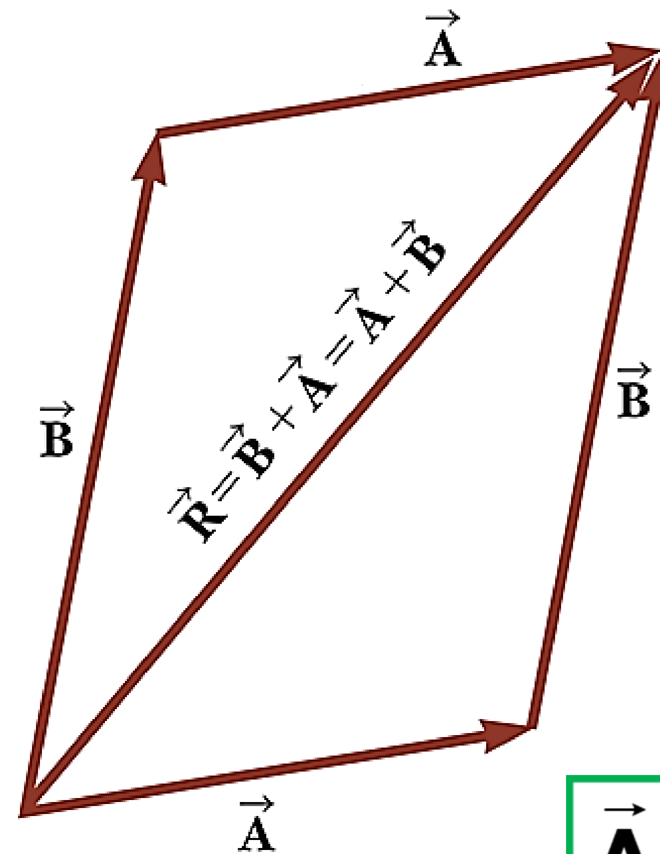




Introduction to Vectors

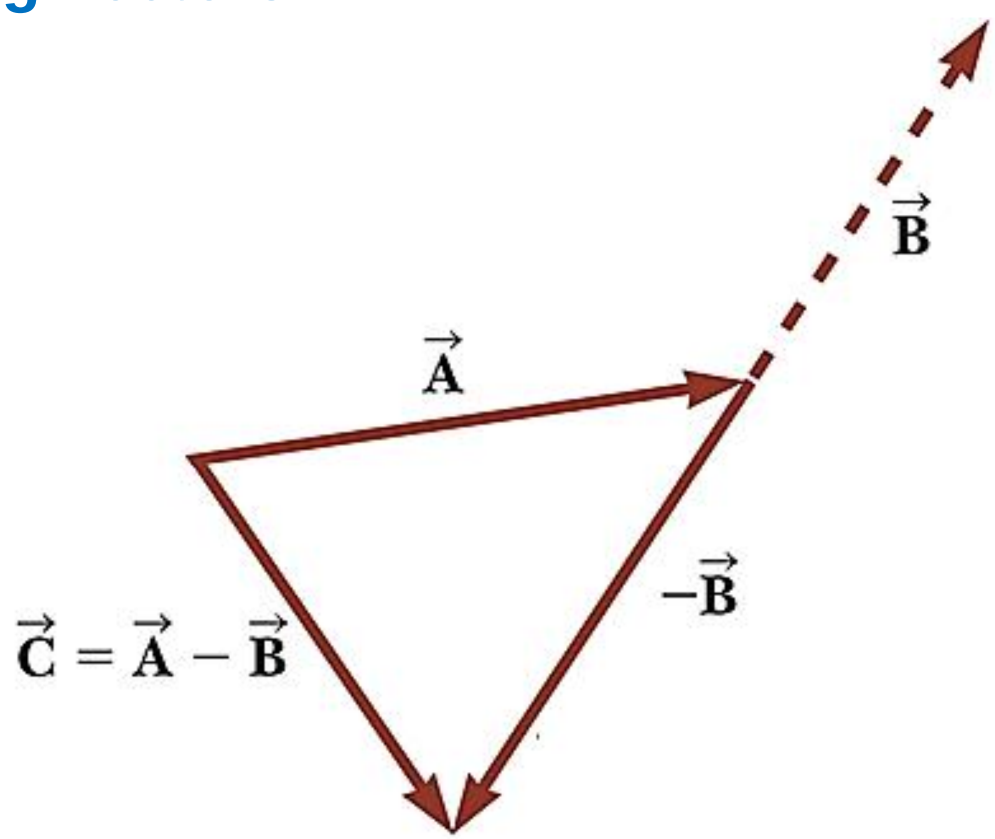
❖ Adding Vectors (Graphical Method):

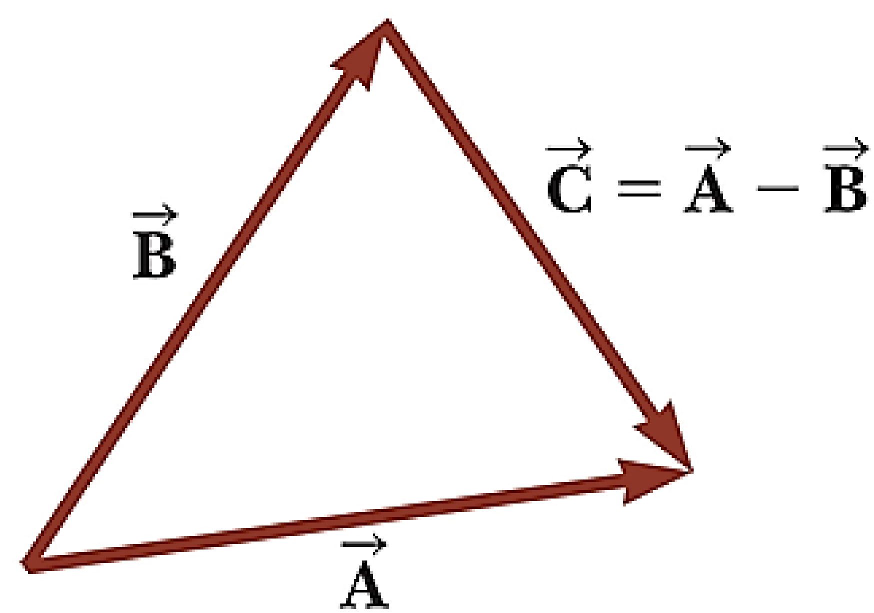




$$\boxed{\vec{A}+\vec{B}=\vec{B}+\vec{A}}$$

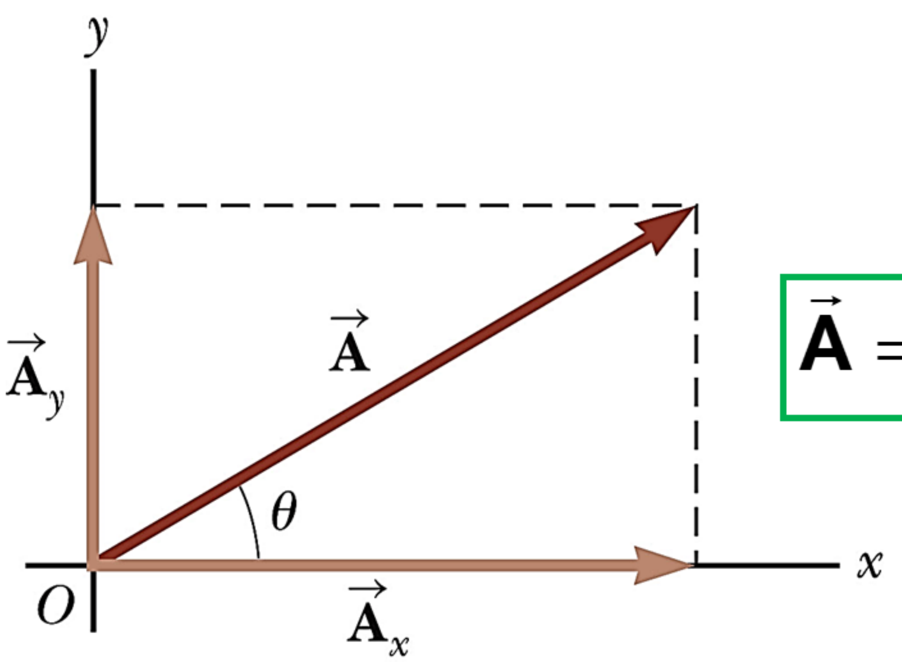
❖ Subtracting Vectors:



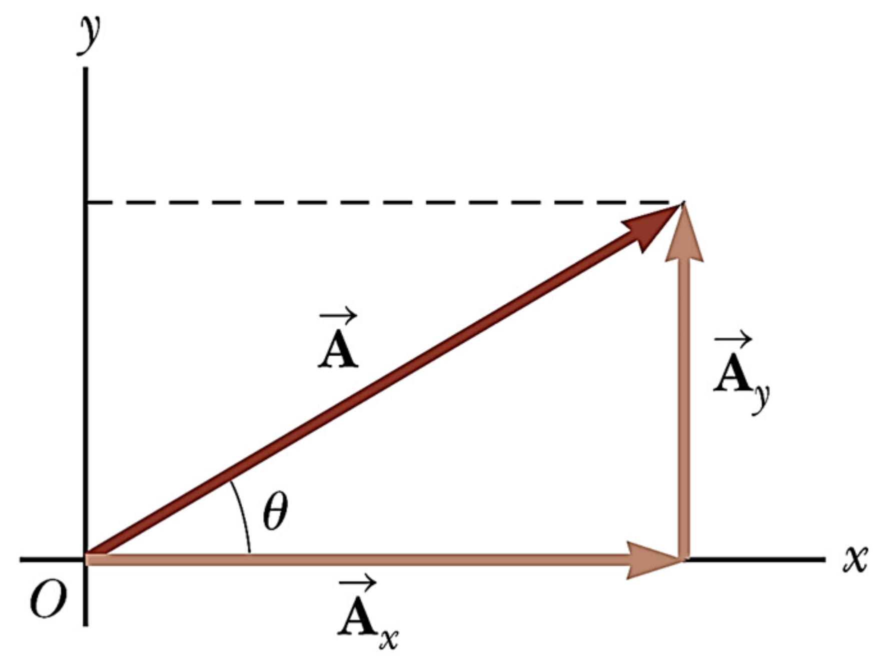


Introduction to Vectors

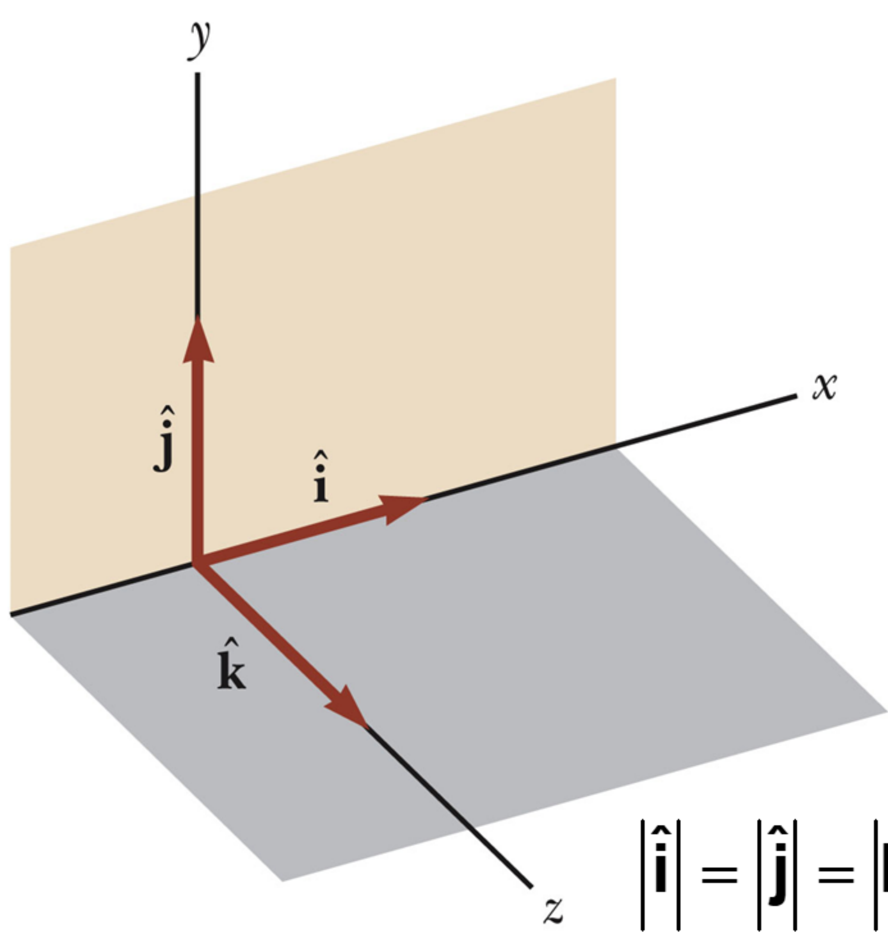
❖ Adding Vectors (Component Method):



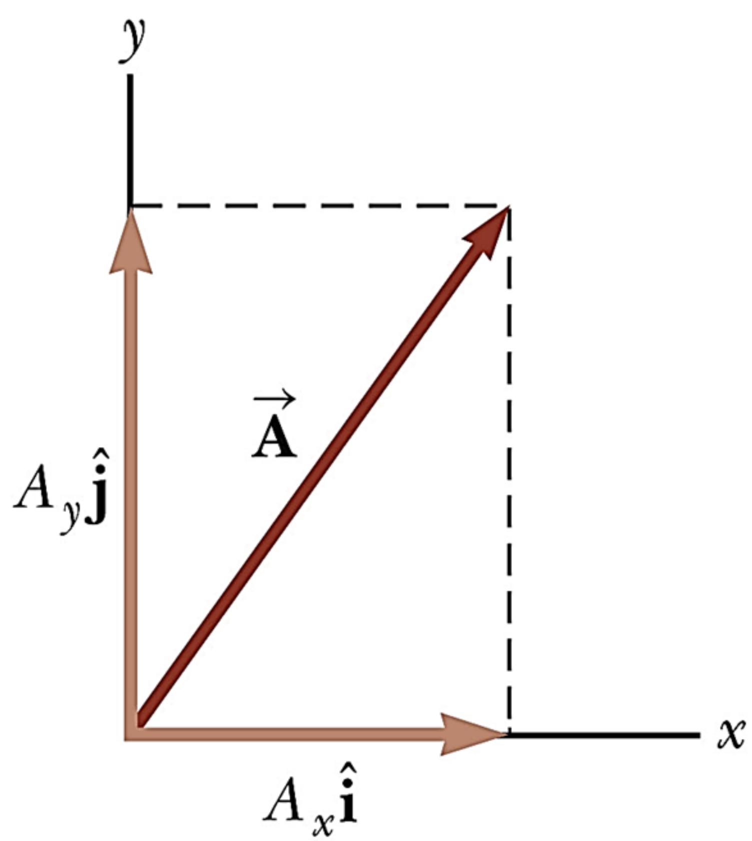
$$\boxed{\vec{\mathbf{A}} = \vec{\mathbf{A}}_x + \vec{\mathbf{A}}_y}$$



$$\left\{\begin{aligned} A_x &= A\cos\theta \\ A_y &= A\sin\theta \\ A &= \sqrt{A_x^2 + A_y^2} \\ \theta &= \tan^{-1}\frac{A_y}{A_x} \end{aligned}\right.$$



$$|\hat{\mathbf{i}}| = |\hat{\mathbf{j}}| = |\hat{\mathbf{k}}| = 1$$



$$\boxed{\vec{\mathbf{A}} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}}}$$

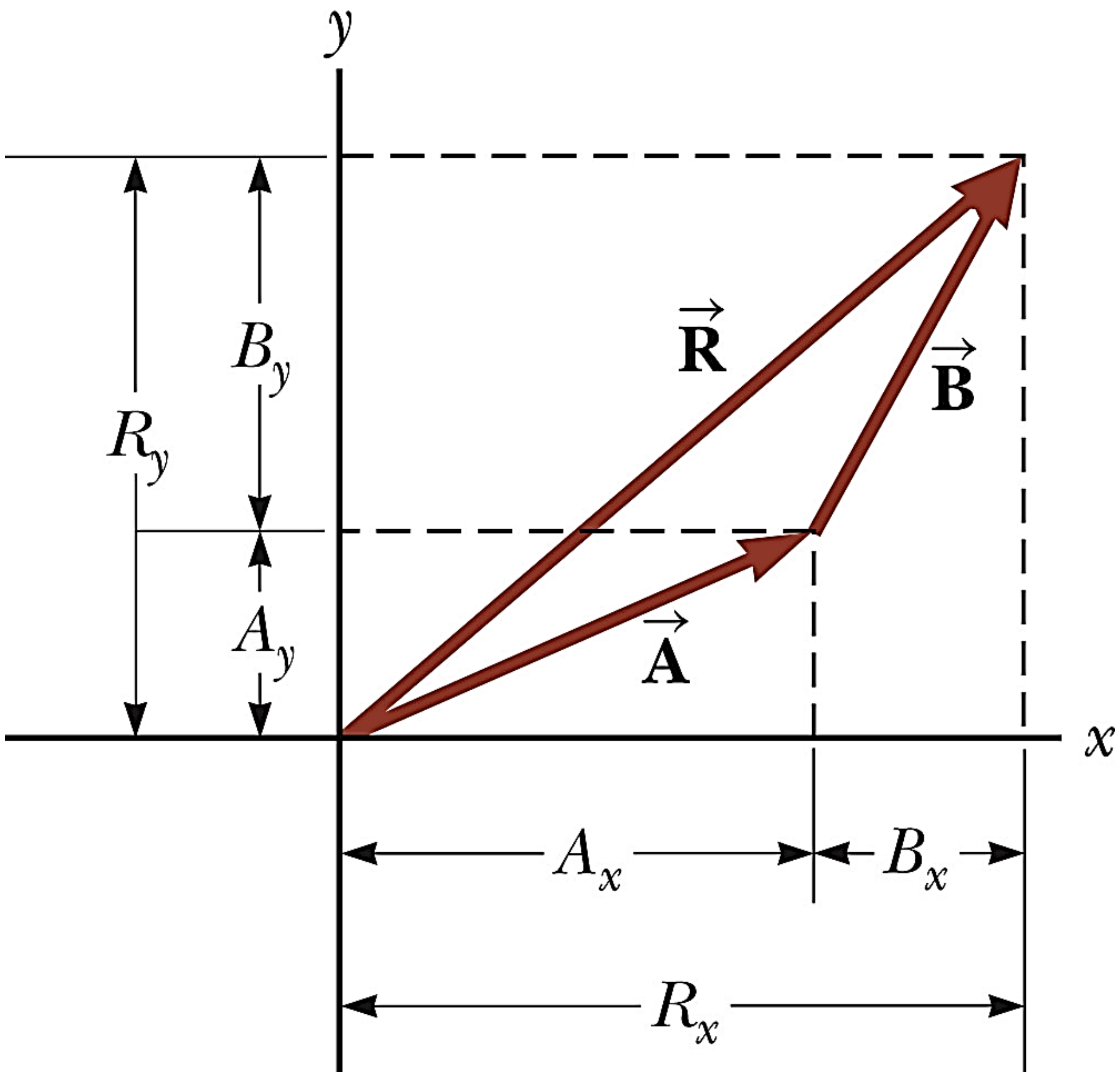
❖ Adding Vectors (Component Method):

$$\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$$

$$\vec{\mathbf{R}} = \left(A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}}\right) + \left(B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}}\right)$$

$$\vec{\mathbf{R}} = \left(A_x + B_x\right)\hat{\mathbf{i}} + \left(A_y + B_y\right)\hat{\mathbf{j}}$$

$$\vec{\mathbf{R}} = R_x\hat{\mathbf{i}} + R_y\hat{\mathbf{j}}$$



❖ Three-Dimensional Extension

$$\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$$

$$\vec{\mathbf{R}} = \left(A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}}\right) + \left(B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}}\right)$$

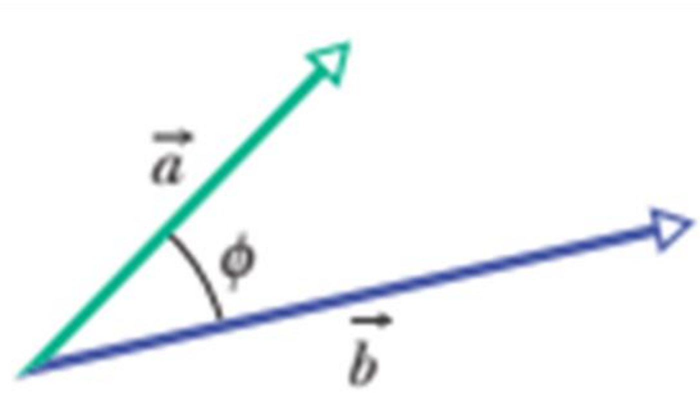
$$\vec{\mathbf{R}} = \left(A_x + B_x\right)\hat{\mathbf{i}} + \left(A_y + B_y\right)\hat{\mathbf{j}} + \left(A_z + B_z\right)\hat{\mathbf{k}}$$

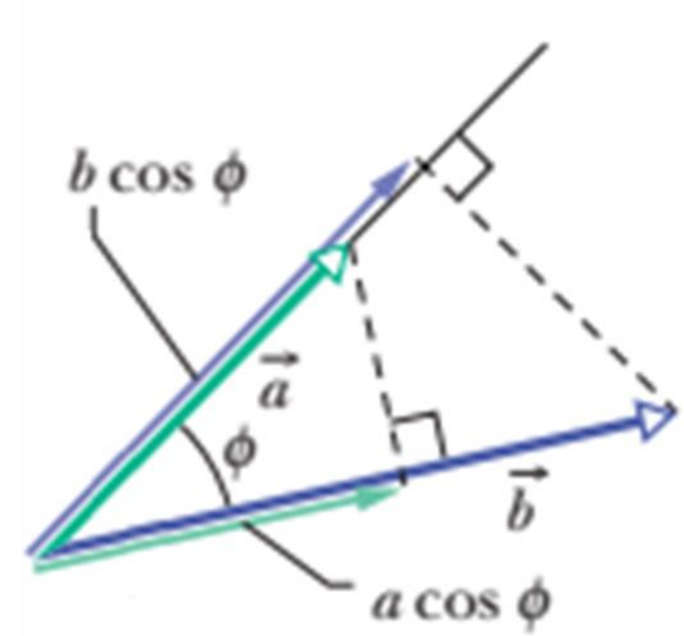
$$\vec{\mathbf{R}} = R_x\hat{\mathbf{i}} + R_y\hat{\mathbf{j}} + R_z\hat{\mathbf{k}}$$

Introduction to Vectors

❖ Multiplying Vectors:

1) The Scalar Product





$$\vec{a}.\vec{b} = (a_x\,\hat{\mathbf{i}} + a_y\,\hat{\mathbf{j}} + a_z\,\hat{\mathbf{k}}).(b_x\,\hat{\mathbf{i}} + b_y\,\hat{\mathbf{j}} + b_z\,\hat{\mathbf{k}})$$

$$\boxed{\vec{a}.\vec{b} = a_x b_x + a_y b_y + a_z b_z}$$

$$\boxed{\vec{a}.\vec{b} = (a\cos\varphi)(b) = (a)(b\cos\varphi)}$$

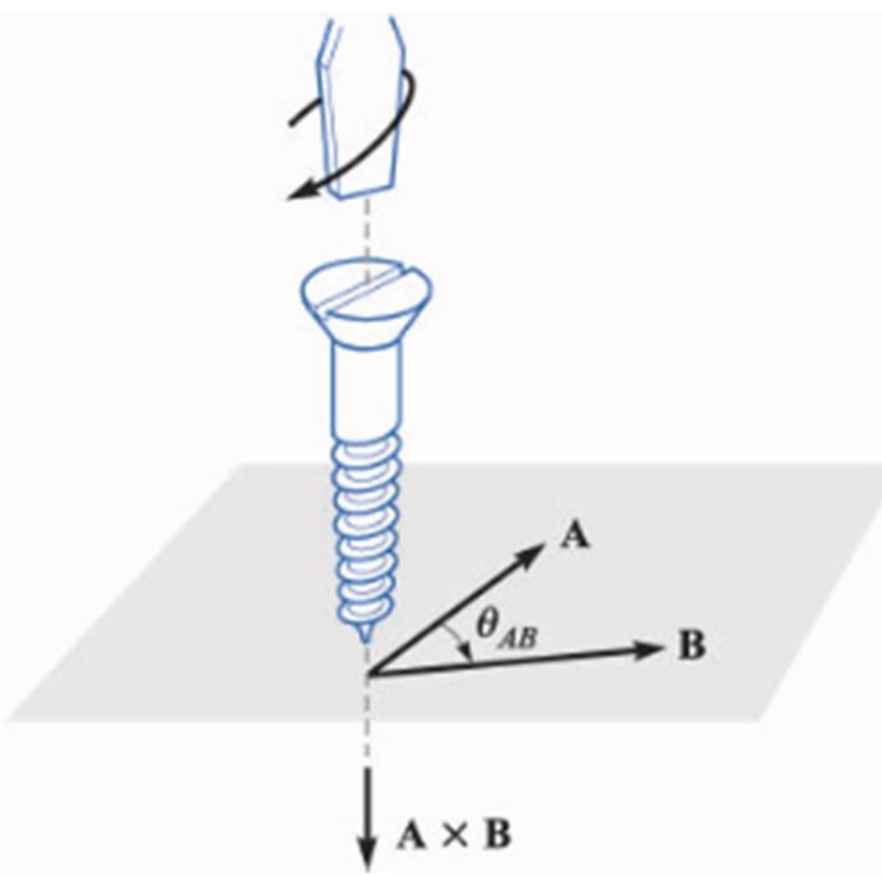
2) The Vector Product

$$\vec{A}\times\vec{B} : Vector$$

$$\vec{A}\times\vec{B} = (A_x\,\hat{\mathbf{i}} + A_y\,\hat{\mathbf{j}} + A_z\,\hat{\mathbf{k}})\times(B_x\,\hat{\mathbf{i}} + B_y\,\hat{\mathbf{j}} + B_z\,\hat{\mathbf{k}})$$

$$\boxed{\vec{A}\times\vec{B} = (A_y B_z - A_z B_y)\hat{\mathbf{i}} + (A_z B_x - A_x B_z)\hat{\mathbf{j}} + (A_x B_y - A_y B_x)\hat{\mathbf{k}}}$$

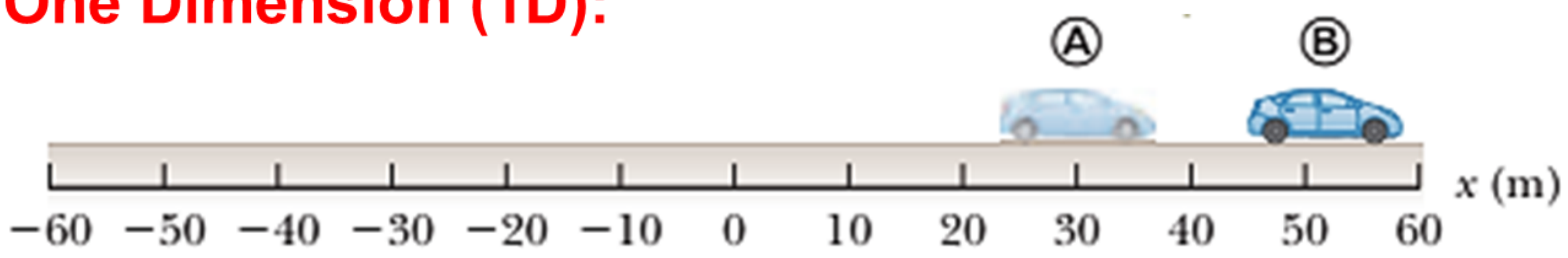
$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$



$$\boxed{\left|\vec{A}\times\vec{B}\right| = A\,B\sin\theta}$$

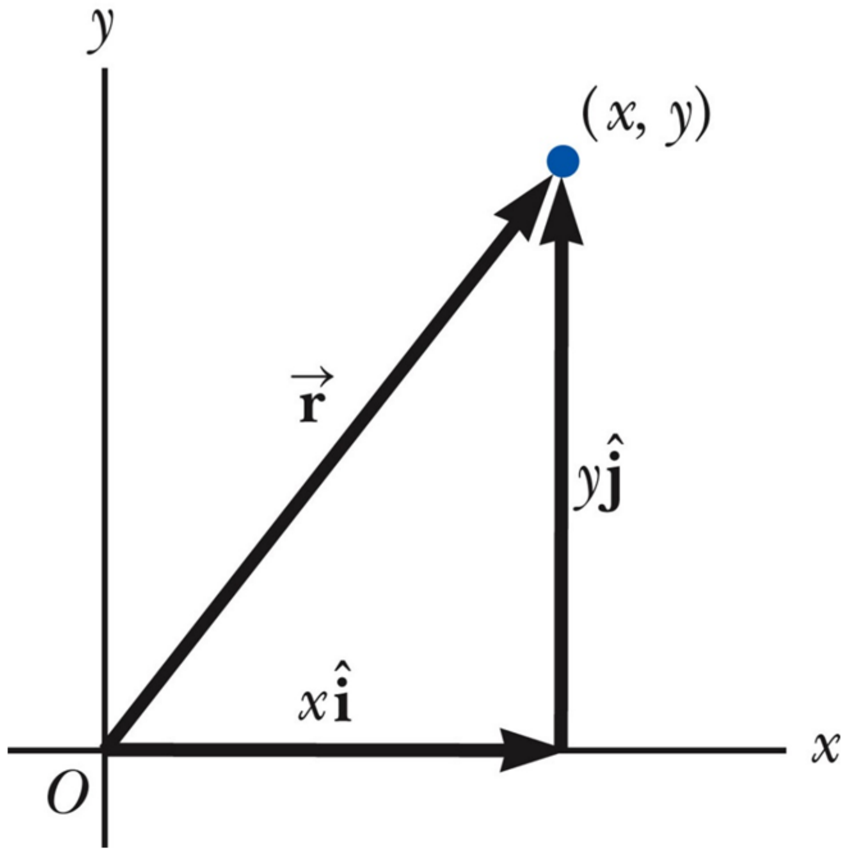
Position and Displacement

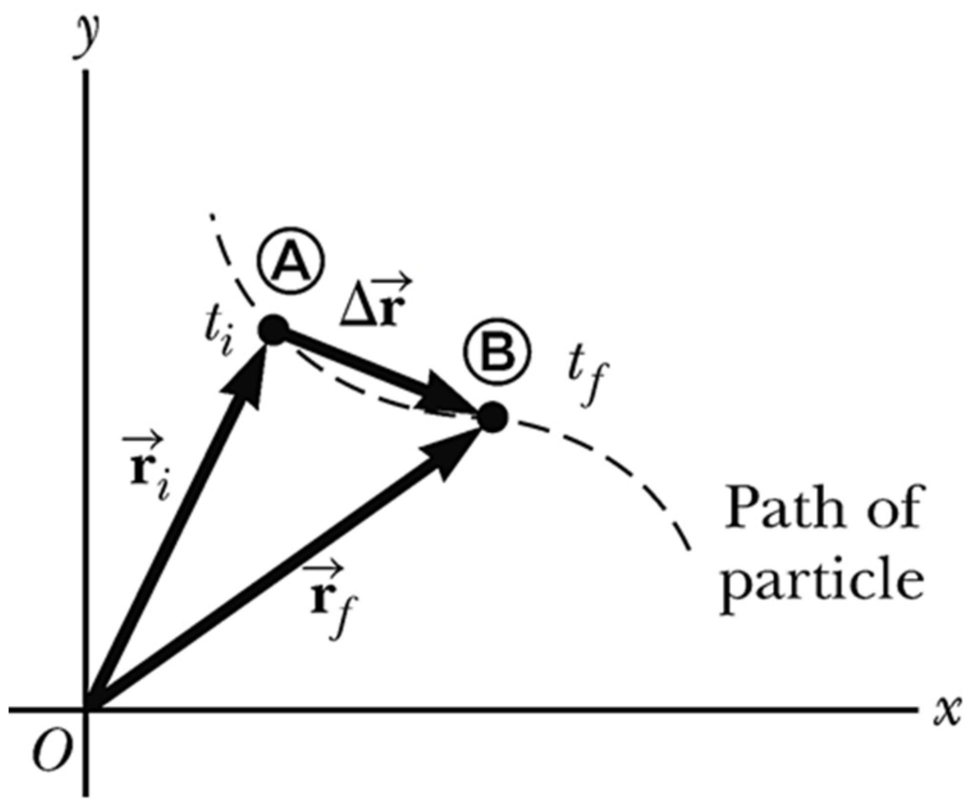
❖ **One Dimension (1D):**



$$\boxed{\Delta x = x_f - x_i}$$

❖ **Two Dimensions (2D):**





$$\boxed{\vec{r} = x\,\hat{i} + y\,\hat{j}}$$

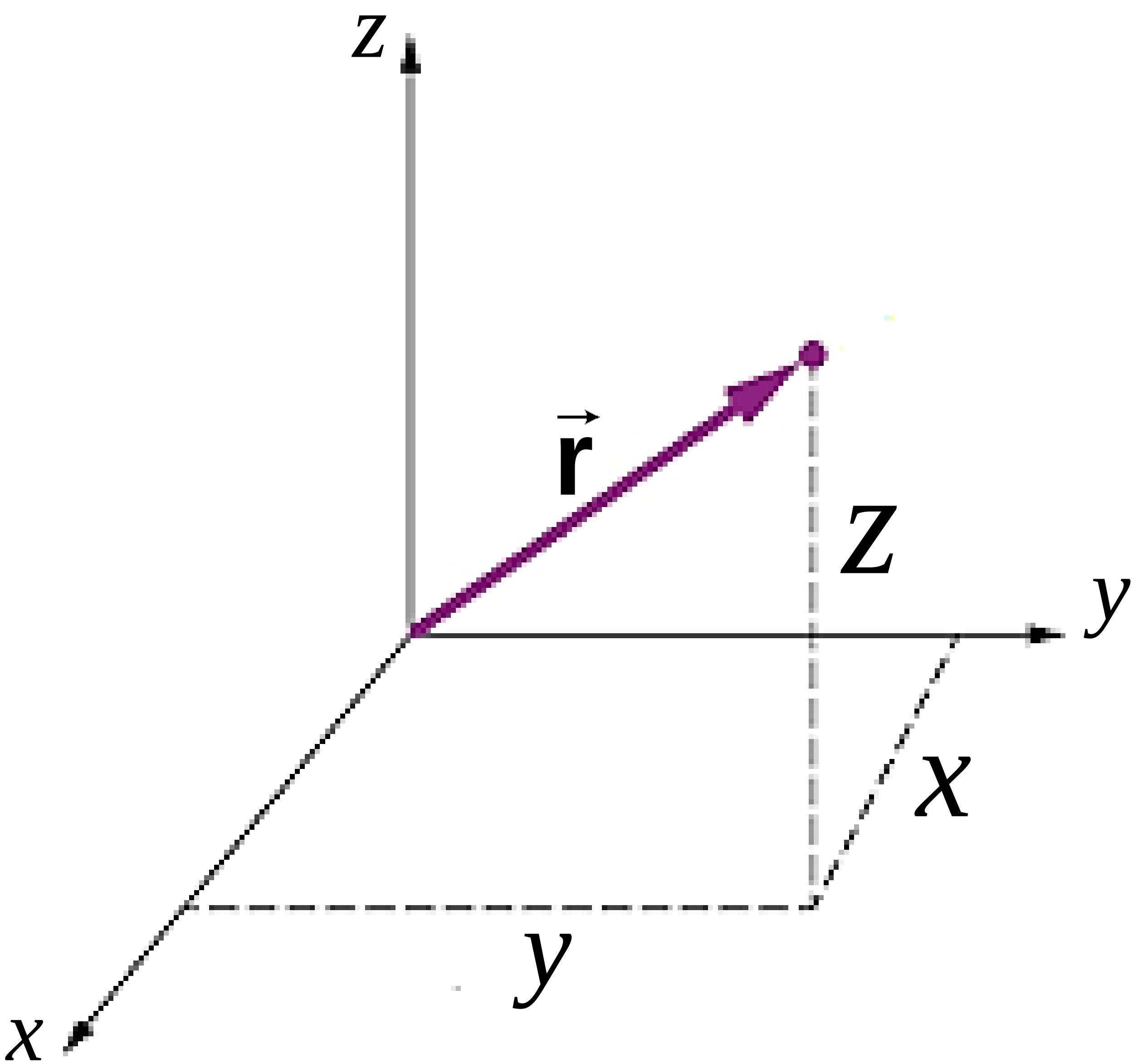
$$\Delta\vec{r} \equiv \vec{r}_f - \vec{r}_i$$

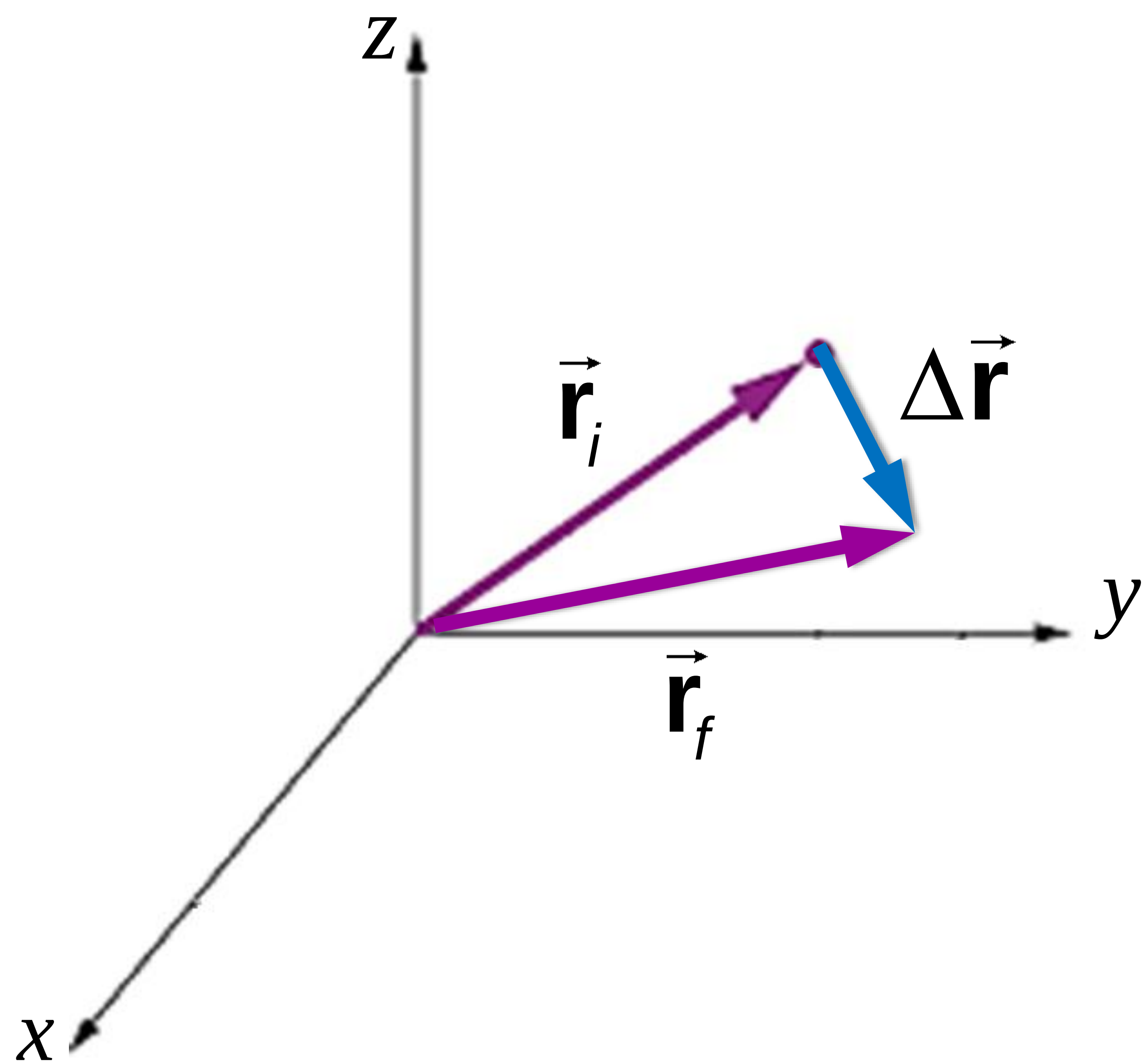
$$\Delta\vec{r} = (x_f - x_i)\,\hat{i} + (y_f - y_i)\,\hat{j}$$

$$\boxed{\Delta\vec{r} = \Delta x\,\hat{i} + \Delta y\,\hat{j}}$$

Position and Displacement

❖ **Three Dimensions (3D):**





$$\boxed{\vec{\mathbf{r}} = x\,\hat{\mathbf{i}} + y\,\hat{\mathbf{j}} + z\,\hat{\mathbf{k}}}$$

$$\Delta\vec{\mathbf{r}} \equiv \vec{\mathbf{r}}_f - \vec{\mathbf{r}}_i$$

$$\Delta\vec{\mathbf{r}} = (x_f - x_i)\,\hat{\mathbf{i}} + (y_f - y_i)\,\hat{\mathbf{j}} + (z_f - z_i)\,\hat{\mathbf{k}}$$

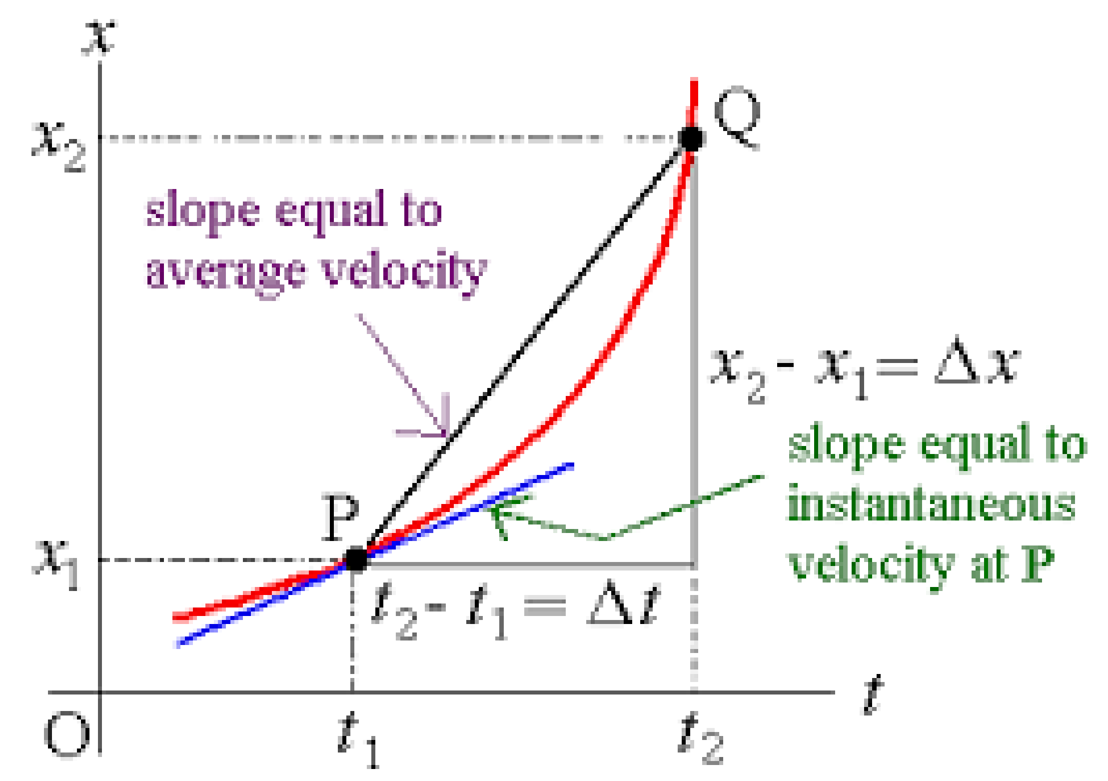
$$\boxed{\Delta\vec{\mathbf{r}} = \Delta x\,\hat{\mathbf{i}} + \Delta y\,\hat{\mathbf{j}} + \Delta z\,\hat{\mathbf{k}}}$$

Velocity

❖ One Dimension (1D):

$$v_{avg} \equiv \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{\Delta t}$$

$$v = \frac{dx}{dt}$$

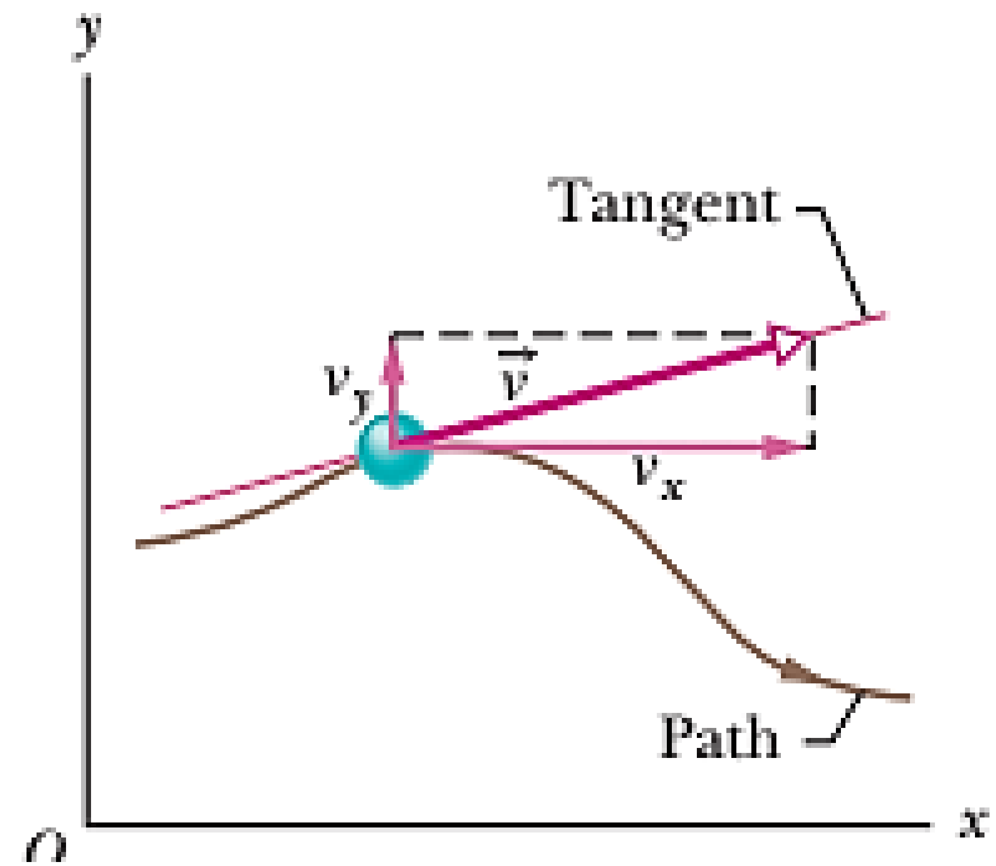


❖ Two Dimensions (2D):

$$\vec{v}_{avg} \equiv \frac{\Delta \vec{\mathbf{r}}}{\Delta t} = \frac{\Delta x}{\Delta t}\hat{\mathbf{i}} + \frac{\Delta y}{\Delta t}\hat{\mathbf{j}} = v_{avg,x}\hat{\mathbf{i}} + v_{avg,y}\hat{\mathbf{j}}$$

$$\vec{v} \equiv \frac{d\vec{\mathbf{r}}}{dt} = \frac{dx}{dt}\hat{\mathbf{i}} + \frac{dy}{dt}\hat{\mathbf{j}} = v_x\hat{\mathbf{i}} + v_y\hat{\mathbf{j}}$$

$$\left|\vec{v}\right| = \sqrt{v_x^{\ 2} + v_y^{\ 2}}$$



Direction of Motion: $\theta = \tan^{-1}\frac{v_y}{v_x}$

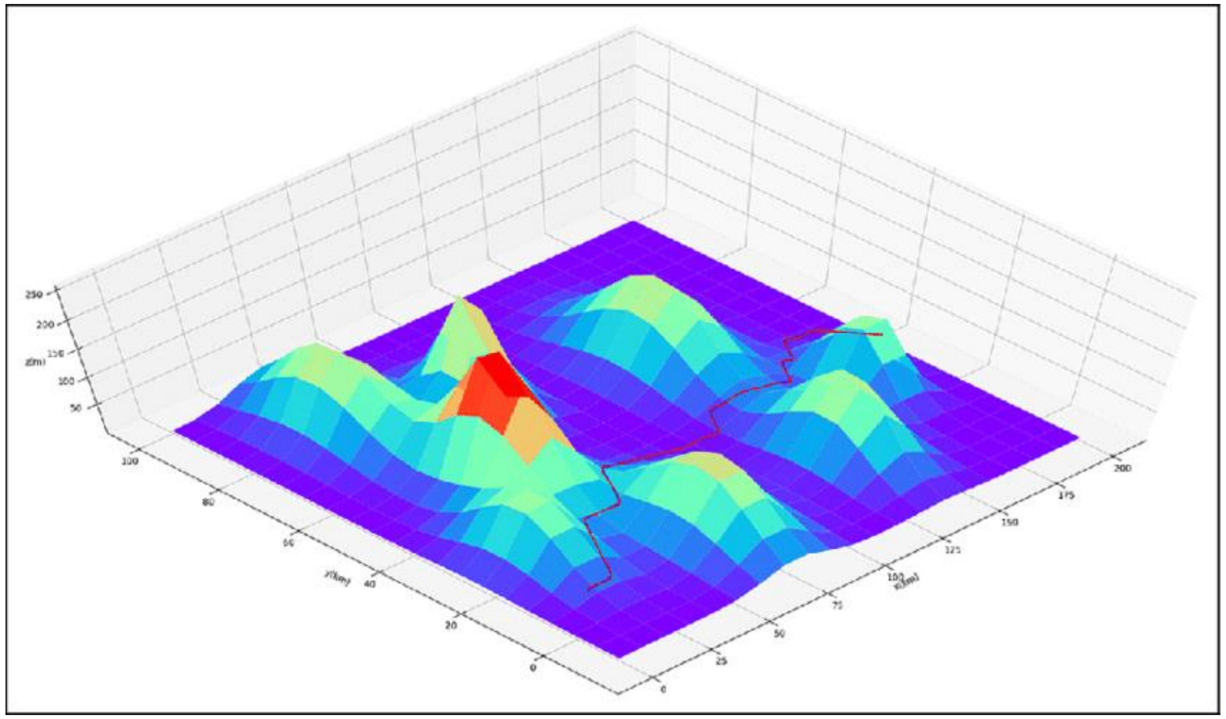
Velocity

❖ **Three Dimensions (3D):**

$$\vec{v}_{avg} \equiv \frac{\Delta\vec{\mathbf{r}}}{\Delta t} = \frac{\Delta x}{\Delta t}\hat{\mathbf{i}} + \frac{\Delta y}{\Delta t}\hat{\mathbf{j}} + \frac{\Delta z}{\Delta t}\hat{\mathbf{k}} = v_{avg,x}\hat{\mathbf{i}} + v_{avg,y}\hat{\mathbf{j}} + v_{avg,z}\hat{\mathbf{k}}$$

$$\vec{v} \equiv \frac{d\vec{\mathbf{r}}}{dt} = \frac{dx}{dt}\hat{\mathbf{i}} + \frac{dy}{dt}\hat{\mathbf{j}} + \frac{dz}{dt}\hat{\mathbf{k}} = v_x\hat{\mathbf{i}} + v_y\hat{\mathbf{j}} + v_z\hat{\mathbf{k}}$$

$$|\vec{v}| = \sqrt{v_x{}^2 + v_y{}^2 + v_z{}^2}$$



Ex 1:

An electron's position is given by $\vec{\mathbf{r}} = 3t\,\hat{i} - 4t^2\,\hat{j} + 2\hat{\mathbf{k}}$, with **t** in seconds **r** and in meters. (a) In unit-vector notation, what is the electron's **velocity**? At **t = 2 s**, what is **velocity** (b) in unit vector notation and as (c) a magnitude and (d) an angle relative to the positive direction of the x axis?

$$\vec{v} = \frac{d\vec{\mathbf{r}}}{dt} \quad \Rightarrow \quad \boxed{\vec{v} = 3\,\hat{\mathbf{i}} - 8t\,\hat{\mathbf{j}}}$$

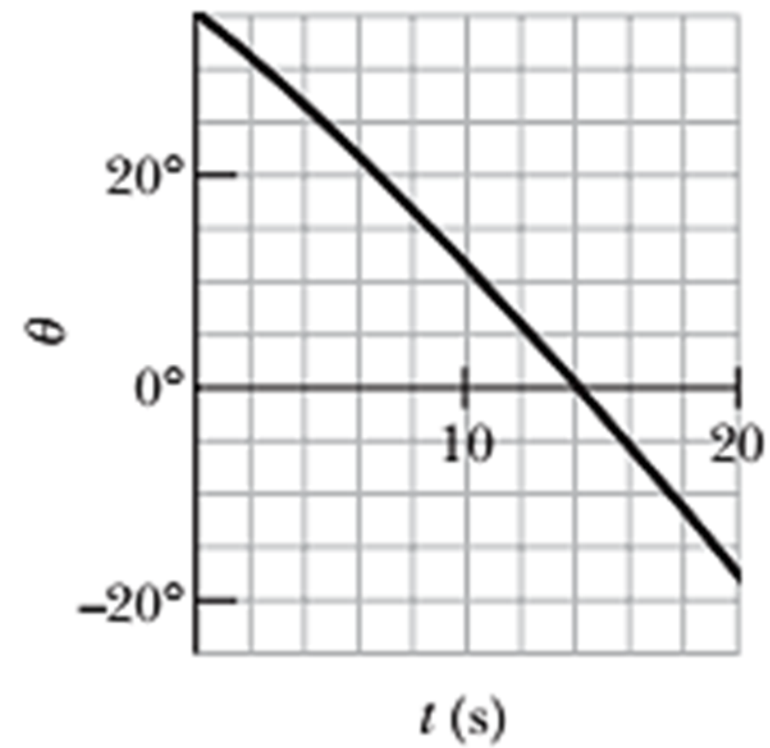
$$t = 2\ s \quad \Rightarrow \quad \boxed{\vec{v} = 3\,\hat{\mathbf{i}} - 16\,\hat{\mathbf{j}}}$$

$$|\vec{v}| = \sqrt{{v_x}^2 + {v_y}^2} \quad \Rightarrow \quad \boxed{|\vec{v}| = \sqrt{9 + 256} = 16.3\ \ (m/s)}$$

$$\theta = \tan^{-1}\frac{v_y}{v_x} \quad \Rightarrow \quad \boxed{\theta = \tan^{-1}\frac{-16}{3} = -79.4}$$

Ex 2: (Prob 4.10)

The position vector $\vec{\mathbf{r}} = 5t\,\hat{i} + (e\,t + f\,t^2)\,\hat{j}$ locates a particle as a function of time **t**. Vector **r** is in meters, **t** is in seconds, and factors **e** and **f** are **constants**. Figure 4-31 gives the angle **θ** of the particle's direction of travel as a function of **t** (**θ** is measured from the positive x direction). What are (a) **e** and (b) **f**, **including units**?



$$\vec{v} = \frac{d\vec{\mathbf{r}}}{dt} \Rightarrow \boxed{\vec{v} = 5\,\hat{i} + (e + 2f\,t)\,\hat{j}}$$

$$\theta = \tan^{-1}\frac{v_y}{v_x} \Rightarrow \boxed{\theta = \tan^{-1}\frac{e + 2f\,t}{5}}$$

$$\text{If } t = 0 \Rightarrow \theta = 35^\circ \Rightarrow 35 = \tan^{-1}\frac{e}{5} \Rightarrow \boxed{e = 5\tan(35) = 3.50 \quad (m/s)}$$

$$\text{If } t = 14 \Rightarrow \theta = 0 \Rightarrow 0 = \tan^{-1}\frac{e + 2f(14)}{5} \Rightarrow e + 28f = 5\tan(0) = 0$$

$$\boxed{f = -\frac{e}{28} = -0.125\ (m/s^2)}$$