

# Application of chaotic particle swarm optimization (CPSO) algorithm in synchronization of modified modeling of the heart in presence of unknown parameters

1.Siroos Nazari 2.Aghileh Heydari 3.Reza ghanbari 4.Mabonhe Tavakoli 5.Javad Khaligh

Department of Mathematics, (PNU), Tehran, Iran

Email: [siroos\\_nazari@yahoo.com](mailto:siroos_nazari@yahoo.com); [a\\_heidari@pnu.ac.ir](mailto:a_heidari@pnu.ac.ir); [zj.khaligh6270@yahoo.com](mailto:zj.khaligh6270@yahoo.com)

## Abstract

In this paper, chaotic particle swarm optimization (CPSO) is applied to synchronization of modified modeling of the heart in presence of parameters uncertainties. Particle swarm optimization is an evolutionary algorithm which is introduced by Kennedy and Eberhart. This algorithm is inspired by birds flocking. Optimization algorithms can be applied to control by defining an appropriate cost function that guarantees stability of system. In presence of environment noise and parameter uncertainty, robustness plays a crucial role in succeed of controller. Since PSO needs only rudimentary information about the system, it can be a suitable algorithm for this case. Simulation results confirm that the proposed controller can handle the uncertainty and environment noise without any extra information about them. Also, in this paper we show that conductive system of the heart can be stimulated to action as a network of elements and these elements show the oscillatory behavior then can be modeled as nonlinear oscillators, and provides the mathematical model of the heart rhythm by considering different states of Vanderpol nonlinear oscillators.

Proposed oscillator model is designed in order to reproduce time series of action potential of natural pacemakers cardiac, such as SA or AV nodes. So model of heart is presented by a system of differential equations. Finally, synchronized the model is by applying CPSO and proposed appropriate control signal.

**Keywords:** Chaotic particle swarm optimization, Model of heart, Nonlinear oscillators, Vander pol equations, Action potential, Control signal, Synchronization.

## 1. Introduction

Synchronization in chaotic dynamic systems attains a great deal of interest among scientists from various fields [1–7]. Applications of chaotic systems synchronization are very important in nonlinear fields such as secret communication and chemical reaction synchronization. Several methods have been applied to synchronize chaotic systems. A number of methods based on master–slave pattern have been proposed [5,6]. The synchronization of near-identical chaotic systems with unknown parameters is investigated by Mu and Pei [8]. In presence of environmental disturbance, measurement noise, or parameter uncertainties the problem becomes more challenging. In [9] integral sliding-mode technique is applied to design a controller to synchronize the behavior of two different chaotic systems, in which some random uncertainties with determined bounds are considered in the parameters. The method presented in that paper is based on minimizing the summation of the norm of synchronization error in discrete model of the system via CPSO.

Normal rhythm can be regular or irregular through time and place. Each type of dynamic behaviors related to

medical systems may be related to the performance of both normal physiological and pathological groups.

Rhythmic variations in blood pressure, heart pulse and other cardiovascular measures, indicate importance of understanding the dynamic aspects of cardiovascular rhythms. Cardiac conduction system can be considered as a network of elements self stimulates, such as: SA node (the first pacemaker), AV node and His-Purkinje system. Because these elements show oscillation behavior, they can be modeled as a nonlinear oscillators.

There are different methods for evaluation of cardiac function by measuring certain signals. Strip (ECG) measures the electrical activity of the heart. Electrical signals associated with cardiac function, which represents electric current in different regions of the heart are recorded waveform [10].

Mathematical modeling of heart rhythm is the goal of many research efforts. Since the qualitative features of the excitation potential of heart is very close to the dynamic behavior of the classical oscillator Vanderpol [11], so this oscillator can be considered as starting point for this modeling.

For modeling, cardiac pacemaker the Vander pol oscillator, is offered to be considered as followed

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -d(x_1^2 - 1)x_2 - x_1 + a \cos \omega t \end{cases} \quad (1)$$

Where  $a, d, \omega$  are system parameters, that depends on their amounts, the heart dynamics may be chaotic or nonchaotic.

In this paper, we propose a CPSO based controller to synchronize two different chaotic systems, in presence of completely unknown parameter uncertainty. To achieve acceptable robustness, the parameters of controller adapts according to PSO rule to minimize an appropriate cost function which guarantees the stability of system. Since PSO algorithm is not significantly sensitive to the dynamics of model, it can handle unknown parameter uncertainties by online adapting the controller's gains to pursue its goal to minimize the cost function. In this paper, the robustness of controller is confirmed by simulation. As a case study the method presented has been applied to a nonlinear oscillators of heart system as the master and the slave systems, respectively.

In section 2, we describe the chaotic particle swarm optimization (CPSO) algorithm and synchronization method, In section 3, we present modelling of the heart. In section 4, designing proper control signal, Section 5, contains implementation of the method and simulation and Section 6 is the conclusions.

## 2. The chaotic particle swarm optimization (CPSO) algorithm and synchronization method

### 2.1. Particle swarm optimization algorithm

For non linear system optimisation, this technique involves simulating social behaviour among particles that fly through a multidimensional search space, where each particle represents a single intersection of all search dimensions. Particles would evaluate their positions or fitness levels with respect to the objective function in each of the iterations. In addition, particles in local neighbourhoods share memories of their "best" positions, and then use those memories to adjust their own velocities for subsequent positions. In the PSO algorithm the  $i$ th particle  $X_i$  is defined as a potential solution in  $D$ -dimensional space, where

$$X_i = (x_{i_1}, x_{i_2}, \dots, x_{i_D})$$

Each particle also maintains a memory of its previous best position

$$P_i = (p_{i_1}, p_{i_2}, \dots, p_{i_D})$$

and velocity

$$V_i = (v_{i_1}, v_{i_2}, \dots, v_{i_D})$$

along each dimension. Following each iteration, the particle vector

$$P = [P_1, P_2, \dots, P_n]$$

is adjusted with regard to the best fitness in the local neighbourhood. This adjustment will be implemented using the "gbest" and the "pbest" factors leading to the best fitness for the population. Velocity adjustment along each dimension is described by Equation (2), where a new position for the particle can be determined [12]:

$$\begin{aligned} v_i &= w \cdot v_{i-1} + c_1 \times \text{rand}(0,1) \times (x_{i_{gbest}} - x_i) \\ &+ c_2 \times \text{rand}(0,1) \times (x_{i_{pbest}} - x_i) \end{aligned} \quad (2)$$

$$x_{i+1} = x_i + v_i$$

where  $w$  is the inertia weight factor;  $c_1, c_2$  are the acceleration constants,  $\text{rand}(0,1)$  and  $\text{rand}(0,1)$  are random numbers;  $x_{i_{gbest}}$  is the best particle among all particles in the population and  $x_{i_{pbest}}$  is the best historical position for particle  $x_i$ . The constants  $c_1$  and  $c_2$  represent the weighting of the stochastic acceleration terms that pull each particle  $x_i$  towards  $x_{i_{gbest}}$  and  $x_{i_{pbest}}$  positions.

According to existing literature  $c_1$  and  $c_2$  are often set to be 2.05 [13,14]. In order to reduce the number of iterations required to reach the optimal solution, a suitable selection of inertia weight ( $w$ ) is introduced to provide a balance between global and local explorations. The inertia weight normally decreases linearly from 0.9 to 0.4 during the optimization process. The inertia weight can be set according to the Equation (3) [12].

$$w = w_{\max} - \frac{w_{\max} - w_{\min}}{\text{iter}_{\max}} \times \text{iter} \quad (3)$$

where  $\text{iter}_{\max}$  is the maximum number of iterations (generations), and  $\text{iter}$  is the current number of iterations. In this study, the population size is considered 250 and  $\text{iter}_{\max} = 60$ .

## 2.2. Chaotic PSO algorithm

One of the major drawbacks of the PSO algorithm is its premature convergence, especially for search spaces with several local optima. In order to overcome this problem, some researchers proposed to introduce chaotic maps with certainty, ergodicity and pseudo-randomness property into PSO algorithm so as to improve the global convergence [15]. Moreover, in [16] and [17] is stated that ( due to the non-repetition of chaos ) chaos-based optimization algorithms can carry out overall searches at higher speeds than stochastic ergodic searches that depend on probabilities. The ways in which the chaos entries into PSO algorithms are different but in this study we consider the most diffused one: in detail, we assume that chaotic maps are adopted to select the numerical values for the parameters of the particle's velocity (2). Different numerical studies have been conducted for choosing the better chaotic maps. In order to enrich the searching behavior and to avoid being trapped into local optimum, A well-known lorenz equations is employed for the hybrid PSO. The lorenz equations is defined as follows.

$$\begin{aligned}\dot{x} &= \delta(x - y) \\ \dot{y} &= \rho x - y - xz \\ \dot{z} &= -\beta z + xy\end{aligned}\quad (4)$$

Where

$$[x, y, z] = [0, 1, 0] \quad , \quad (\delta, \rho, \beta) = (10, 28, 8/3) .$$

## 2.3. Description of the synchronization method

The synchronization of chaos is process in which two or more identical or nonidentical chaotic systems, a distinct feature of motion set through a foreign force to achieve a set of common behaviors.

The most common configuration for synchronous systems is considered as two subsystems are coupled, the one as slave system and the other as the master system.

Purpose of synchronous this is that slave system follow dynamics of master system. Chaotic systems, according to the type and intensity of the coupling between the two systems is established, with different methods are synchronous with each other.

For clarification, the following relationship between slave and master systems, respectively, as we think

$$\begin{aligned}\dot{u}(t) &= f_u(u, t) \\ \dot{v}(t) &= f_v(v, t)\end{aligned}$$

where  $u \in R^n$  ,  $v \in R^n$  the system state variables  $f_u$  , and  $f_v$  are chaotic nonlinear functions. These two systems can, by applying an appropriate control signal, and the special relationship between their trajectories, be synchronous.

In this case we have

$$\lim_{t \rightarrow \infty} \|D_1(u(t)) - D_2(u(t))\| \rightarrow 0 \quad (5)$$

Functions  $D_1$  and  $D_2$  determine the optimum relationship between state variables systems. In fact, they specify the type of synchronization.

## 3. Modelling of the heart

### 3.1. The heart and the record of its electrical activity

The walls of the heart muscle called myocardium, which consists of four cavities, such as: right and left atrium (Upper part), and right and left ventricles (lower part). Blood returning from the systemic circulation goes into the right atrium and then right ventricle, and finally into the lungs. After being oxygenated, oxygenated blood from the lungs goes to the left atrium and left ventricle then blood through the aortic valve goes into the aorta and then is pumped in systemic circulation. Sinus node (SA node) is located in the right atrium the upper vnaakavay. SA node cells, are self stimulate pacemaker cells, which produce an active potential. Activation of the SA node through the atria will release, but can not be spread of the wall between the atria and ventricles. Atrio ventricular node (AV node) is located at boundary between the atria and ventricles. In a normal heart, node AV provides the only guidance signals of atrial to ventricular. So, in unusual conditions, the ventricles can stimulate only through pulses that will be spread among them.

There are different methods for the assessment of cardiac function is done by measuring certain signals, including tape (ECG) is the recording of the waves of electrical activity of the heart record by putting electrodes on the chest and around the heart. Each of the components on tape indicates, the electrical activity of the heart cells. Stimulates sinus node cause electrical stimulation of the atria and creates a P wave, that is the wave of atrial depolarization. After the wave of ventricular depolarization, the QRS complex series arise, and the subsequent wave creates T wave by ventricular repolarization. Sometimes another small wave after wave of T is seen that is called U wave. The reason of creation

of the wave is repolarization papillary muscles in the heart. In distance of this waves There is not appropriate electrical activity and electrogram draws a horizontal straight line that is called the isoelectric line, **Figure 1**.

In general, signal waves are as follow [18].

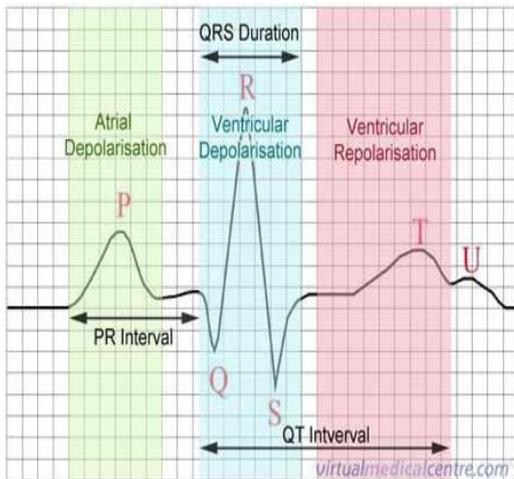
**Wave P:** The first wave was recorded in the ECG, and indicates right atrial activation after stimulation of the sinus node. Typically lasts between 60 and 90 milliseconds in the adults, its shape is round and has a maximum range between 0.25 and 0.30 volts.

**Interval PR:** From beginning of P wave to beginning of the QRS complex and lasts 90 ms.

**Complex QRS:** Is composed of three waves and sum of total of three waves is indicated depolarization. The first negative wave after P wave is Q. The first positive wave after P wave is R, and The first negative wave after of R, is called S. Because of all three waves may be not seen, the sum of these three waves together is called a QRS complex.

**Interval ST:** Starts from the end of QRS complex and lasts to the beginning of the T wave and is part of process of the repolarization.

**Wave T:** Indicates ventricular activation and has a circular shape with amplitude is 0.60 volts.



**Figure 1. ECG description.**

### 3.2. Mathematical model of the heart

The idea modeling of the heartbeat system with a coupled nonlinear oscillators, first was explained in 1928 by Vander pol and Vander mark. Cardiac conduction system may be assumed as a self stimulate pacemaker that is composed of two oscillator subsystems. The first subsystem is atrial sinus node (SA) that between other oscillators of heart has the highest pulse (60-100 impulse per minute). The second subsystem consists of AV node.

(40-60 impulse per minute). Fluctuation of sinus node and atrioventricular node (AV) can be nonlinear modeled equations of two coupled oscillator Vander pol.

Modeling the interaction between the oscillators heart model is considered the following equation[19,20,21],

$$SA : \begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -d_1(x_1^2 - 1)x_2 - c_1x_1 + a_1 \cos \omega t \\ \quad + R_1(x_1 - x_3) \end{cases} \quad (6)$$

$$AV : \begin{cases} \dot{x}_3 = x_4 \\ \dot{x}_4 = -d_2(x_3^2 - 1)x_4 - c_2x_3 + a_2 \cos \omega t \\ \quad + R_2(x_2 - x_4) \end{cases}$$

Where pairs  $(x_1, x_2)$  and  $(x_3, x_4)$ , respectively, show SA and AV oscillators. Frequency of the SA node is denoted by  $c_1$  and frequency of the AV node, is denoted by  $c_2$ . and  $R_1, R_2$  express coupling coefficients between two nodes. If  $R_1 = 0, R_2 > 0$  indicates that only oscillator SA affects on oscillator AV in a physiological way. The condition  $R_1, R_2 > 0, R_1 < R_2$ , indicates this fact that node AV has little effect on the SA node. coefficients  $d_1$  and  $d_2$  affects on term of nonlinear equation, and cause stability of limit cycle in the phase plate  $(x_1, x_2)$  that a limit cycle is adapted with the behavior of the heart in a physiological way.

As was mentioned earlier, first normal cardiac rhythm is produced by SA node (Pacemaker normal) and causes stimulate the AV node. However, it was observed that the two oscillators for producing ECG signals are not very accurate. This is because, the signal of first oscillator is related to the activation of SA node and right atrium, the signal of second oscillator is of only related to the left ventricular depolarization. According to this hypothesis, it is possible to produce P curve, but complex QRS may not be produced, because this distance is mainly due to ventricular repolarization. These observations makes us incorporate a third oscillator, which represents the spread of a pulse through the heart, that indicate His-Purkinje complex in a Physiological way.

In order to create a general model, we assume that all oscillators should be coupled asymmetry. In addition, external stimulation is entered into the system with regard to the oscillator frequency, **Figs 2,3..**

This developed model can be shown with a set of differential equations as follows

$$SA : \begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -d_1(x_1^2 - 1)x_2 - c_1x_1 + a_1 \cos \omega t \\ \quad + R_{13}(x_1 - x_3) + R_{15}(x_1 - x_5) \end{cases} \quad \begin{aligned} \dot{X} &= f(X) \\ \dot{Y} &= g(Y) + U \end{aligned} \quad (9)$$

$$AV : \begin{cases} \dot{x}_3 = x_4 \\ \dot{x}_4 = -d_2(x_3^2 - 1)x_4 - c_2x_3 + a_2 \cos \omega t \\ \quad + R_{31}(x_3 - x_1) + R_{35}(x_3 - x_5) \end{cases} \quad (7)$$

$$HP : \begin{cases} \dot{x}_5 = x_6 \\ \dot{x}_6 = -d_3(x_5^2 - 1)x_6 - c_3x_5 + a_3 \cos \omega t \\ \quad + R_{51}(x_5 - x_1) + R_{53}(x_5 - x_3) \end{cases}$$

Where coefficients  $d_i$  and  $c_i$  expressed in the previous formula and the coupling coefficients of  $R_{ij}$  are between pair oscillators. Because even small delays may alter the dynamics of the system, differential equations incorporating time delay can cause drastic changes and creation of chaos in the system that described by the regular behavior.

Accordingly, proposed mathematical model can change to consideration of aspects of delay in coupling terms. Thus, the governing equations is changed as following, where  $x_i^\tau = x_i(t - \tau)$  and  $\tau$ , are time delay.

$$SA : \begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -d_1(x_1^2 - 1)x_2 - c_1x_1 + a_1 \cos \omega t \\ \quad + R_{13}(x_1 - x_3^{\tau_{13}}) + R_{15}(x_1 - x_5^{\tau_{15}}) \end{cases} \quad (8)$$

$$AV : \begin{cases} \dot{x}_3 = x_4 \\ \dot{x}_4 = -d_2(x_3^2 - 1)x_4 - c_2x_3 + a_2 \cos \omega t \\ \quad + R_{31}(x_3 - x_1^{\tau_{31}}) + R_{35}(x_3 - x_5^{\tau_{35}}) \end{cases}$$

$$HP : \begin{cases} \dot{x}_5 = x_6 \\ \dot{x}_6 = -d_3(x_5^2 - 1)x_6 - c_3x_5 + a_3 \cos \omega t \\ \quad + R_{51}(x_5 - x_1^{\tau_{51}}) + R_{53}(x_5 - x_3^{\tau_{53}}) \end{cases}$$

## 4. Designing proper control signal

### 4.1. Problem definition

Consider the following two systems defined by

where  $f(X)$  and  $g(Y)$  are nonlinear chaotic functions in  $R^n$ . Here the first system is considered as master (drive) system and the other one as the slave (response) system. Let us suppose that the functions  $f(X)$  and  $g(Y)$  are totally different nonlinear functions. Our goal is to design a control law for the slave system in such a way that both master and slave systems are synchronized. This means that the error state vector  $e = Y - X$  converges to zero as time converges to infinity.

In problem of synchronization of system explained in Equations (6), First system is considered as master system and second system as a slave system. However, by applying a suitable control signal on it, which has the form

$$SA : \begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -d_1(x_1^2 - 1)x_2 - c_1x_1 + a_1 \cos \omega t \\ \quad + R_1(x_1 - x_3) \end{cases} \quad (10)$$

$$AV : \begin{cases} \dot{x}_3 = x_4 \\ \dot{x}_4 = -d_2(x_3^2 - 1)x_4 - c_2x_3 + a_2 \cos \omega t \\ \quad + R_2(x_2 - x_4) + u \end{cases}$$

A state variables of system slave will converget to state variables of system master after a transient time, **Fig 6**. In fact, the second oscillator has to follow the behavior of the first oscillator, which has a dominant frequency.

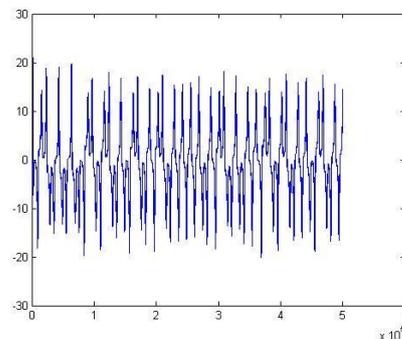
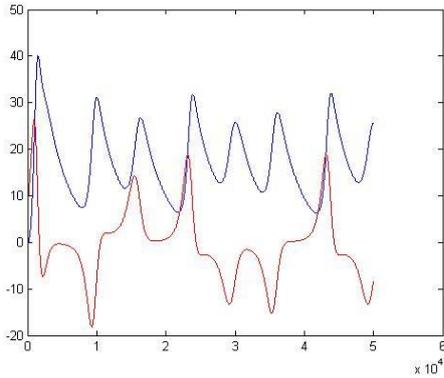
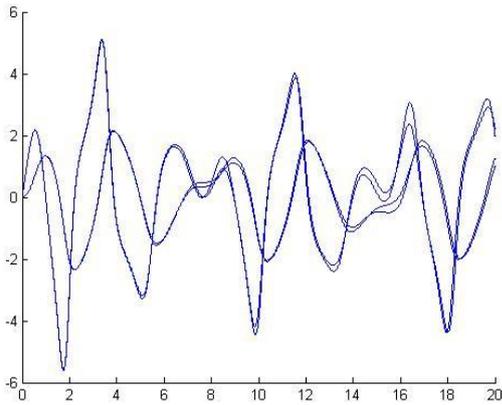


Figure 2. curve of  $x_1, x_3$  before of synchronization.



**Figure 3.** curve of  $x_2, x_4$  before of synchronization



**Figure 4.** curve of  $x_1, x_3$  before of synchronization in presence of noise.

Since we want the slave system state variables to converge to the master system state variables after a transient time, the error of synchronization should be considered as follow [22,23,24,25].

$$X_m : \begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -d_1(x_1^2 - 1)x_2 - c_1x_1 + a_1 \cos \omega t \\ \quad + R_1(x_1 - x_3) \end{cases} \quad (11)$$

$$X_s : \begin{cases} \dot{x}_3 = x_4 \\ \dot{x}_4 = -d_2(x_3^2 - 1)x_4 - c_2x_3 + a_2 \cos \omega t \\ \quad + R_2(x_2 - x_4) + u \end{cases} \quad (12)$$

$$E = X_m - X_s$$

Therefore have

$$\begin{aligned} e_1 &= x_1 - x_3 \\ e_2 &= x_2 - x_4 \end{aligned} \quad (13)$$

The purpose of the synchronization is to vanish the error, **Figs.9,10**. Therefore, control signal is calculated as follow

$$\begin{aligned} \dot{e}_1 &= \dot{x}_1 - \dot{x}_3 \\ \dot{e}_2 &= \dot{x}_2 - \dot{x}_4 \end{aligned} \quad (14)$$

With placement, have

$$\begin{aligned} \dot{e}_1 &= x_2 - x_4 \\ \dot{e}_2 &= \dot{x}_2 - \dot{x}_4 = -d_1(x_1^2 - 1)x_2 - c_1x_1 + a_1 \cos \omega t \\ &+ R_1(x_1 - x_3) + d_2(x_3^2 - 1)x_4 + c_2x_3 \\ &- a_2 \cos \omega t - R_2(x_2 - x_4) - u \end{aligned} \quad (15)$$

In this case

$$\begin{aligned} \dot{e}_1 &= e_2 \\ \dot{e}_2 &= d(x_4(x_3^2 - 1) - x_2(x_1^2 - 1)) \\ &- c e_1 + (x_1 - x_3 - x_2 + x_4)R - u \end{aligned} \quad (16)$$

Have

$$\begin{aligned} \dot{e}_1 &= e_2 \\ \dot{e}_2 &= d(x_4(x_3^2 - 1) - x_2(x_1^2 - 1)) \\ &- c e_1 + (e_1 - e_2)R - u \end{aligned} \quad (17)$$

#### 4.2. Adaptive PSO based robust control

First, suppose that there are no parameter uncertainties in the system models. In order to solve synchronization problem, we can apply the idea of sliding-mode controller stated in [26,27]. Let us define the synchronization error as  $e = [\int e_1, e_1, \dot{e}_1]$ . Also, let us define a time-varying surface  $S(t)$  in the state-space  $R^3$  by the scalar equation  $s(X, t)$ . Consider the variable of interest as follows [39]:

$$s(t) = (\lambda + \frac{d}{dt})^2 (\int e_1 dt) \quad (18)$$

Or

$$s(t) = \lambda^2 \int e_1 dt + 2\lambda e_1 + \dot{e}_1 \quad (19)$$

The assumptions

$$\dot{e}_1 = e_2, \lambda^2 = \xi_1, 2\lambda = \xi_2$$

Then

$$s(t) = \xi_1 \int e_1 dt + \xi_2 e_1 + e_2 \quad (20)$$

where  $\xi_1$  and  $\xi_2$  are parameters of sliding surface.

Using the Filippov's construction, error dynamics in sliding mode can be written as  $\dot{s} = 0$ . Solving this equation for the control input, we obtain an expression for  $u$  as:

$$\begin{aligned} \dot{s}(t) = \xi_1 e_1 + \xi_2 \dot{e}_1 + \dot{e}_2 = 0 \\ \xi_1 e_1 + \xi_2 e_2 + d(x_4(x_3^2 - 1) - x_2(x_1^2 - 1)) \\ - c e_1 + (e_1 - e_2)R - u = 0 \end{aligned} \quad (21)$$

Therefore

$$u = \xi_1 e_1 + \xi_2 e_2 + d(x_4(x_3^2 - 1) - x_2(x_1^2 - 1)) \\ - c e_1 + (e_1 - e_2)R \quad (22)$$

When there are no parameter uncertainties in the system dynamics, control law (22) with sliding surface defined in (13) with  $\lambda > 0$ , can made the system asymptotically stable. In presence of uncertainty, control law (22) cannot be applied. In [26], a nonlinear term is added to overcome uncertainty. When the uncertainty bounds are known, it can be proved that the error signal will converge to zero [26]. In the control approach we state here, determination of uncertainty bounds is not necessary. Our method is based on adapting the controller parameters denoted by  $(\xi_1, \xi_2)$ , in a way that minimizes an appropriate cost function. The cost function must be determined such that take its global minimum when the error of synchronization vanishes. To this aim, these parameters are treated as positions of the particles in PSO algorithm. So, the PSO algorithm adapts the controller in presence of noises and uncertainties which may exist in the system.

### 4.3. Cost function

As mentioned before, the cost function must be defined carefully, because it strongly affects the controller performance. We started with the cost function:

$$J = \int_0^T ((e_1)^2 + (e_2)^2) dt \quad (23)$$

which  $T$  is the simulation time and it is the continuous form of the cost function proposed in [28]. But, with this cost function, one must run the system several times and from exactly the same initial condition which may not be appropriate in many practical cases. The position vector of the PSO parameters must be updated online while the system is running. Before introducing our cost function, let us define the following sequence

$$t_{i+1} = t_i + n_i h, \quad t_1 = 0 \quad (24)$$

where  $h$  is a small positive constant and  $n_i$  denotes a non-increasing sequence of natural numbers that finally tends to 1, that is  $n_i \geq n_{i+1}$  and  $n_p = 1$  for some  $p$ . We proposed the cost function as follows

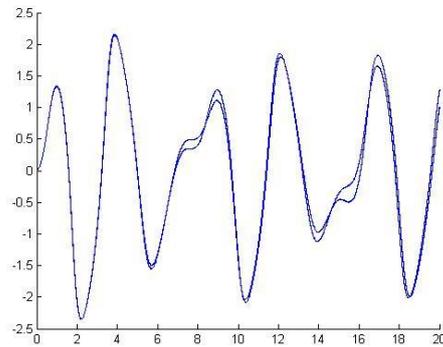
$$J(i) = \frac{1}{n_i} \int_{t_i}^{t_i + n_i h} ((e_1)^2 + (e_2)^2) dt \quad (25)$$

where  $h$  is a constant time step size and  $t_i$  and  $n_i$  are introduced in (24) with additional condition that for every  $i$ ,  $t_i \leq T$  where  $T$ , where  $T$  is the simulation time. In fact, the introduced index  $i$  indicates how many times the PSO algorithm is updated. In other words, it reflects how many times the parameters of controller are updated. In addition, updating times in which optimization algorithm updates, are defined by  $t_i$  in (24). Therefore, the cost function of PSO defined in (25) demonstrates the integral of square of synchronization error, in an interval between two sequential updating times. The parameter  $n_i$ , which represents the period of updating, reduced when the cost function does not change for a while. The most crucial part of this algorithm is the way that the cost function is defined. The cost function must be defined in a way that its minimization guarantees the control of the system. As a solution, it can be a positive semi definite function of synchronization error. We choose the square of the norm of error vector. In addition, it must be calculated online while the system is running. Therefore, this cost function is also a function of time and it may lead to harsh changes in control signal and cause chattering in it especially in the beginning when the error signal usually

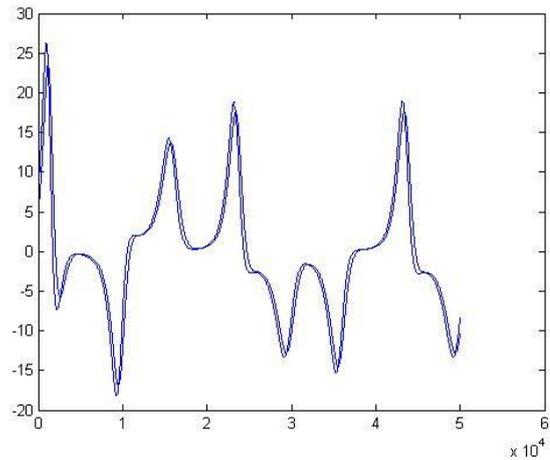
changes rapidly. To overcome this problem, we define the cost function as the average of square of error vector norm in an appropriate interval. This averaging, if the time interval of integration does not change appropriately, can make the adaptation more reluctant and unfortunately, in contrast to beginning time, it can have inappropriate effects like chattering when the error signals approach zero, especially in presence of time varying parameter uncertainties. In other way, this averaging fools the PSO algorithm by reporting a smaller cost function. To reduce this effect and getting a better response, we reduce the length of averaging intervals while the system is running. As a summary, the proposed cost function has the following properties

1. It can be calculated online while the system is running.
2. The cost function calculates the average of square of error norm in interval  $[t_i, t_{i+1}]$ . This can strongly prevents from chattering of the response of system. Also, it can protect the algorithm from failure near the crossover points where the error signal value is small.
3. As the system tends to converge, the integration interval become smaller until it reaches to some minimum length. This prevents chattering of response of system while its average remains constant near zero. The minimum interval length must be considered to allow the controller to influence the system behavior. As a result, PSO algorithm will be able to analyze the effectiveness of controller.

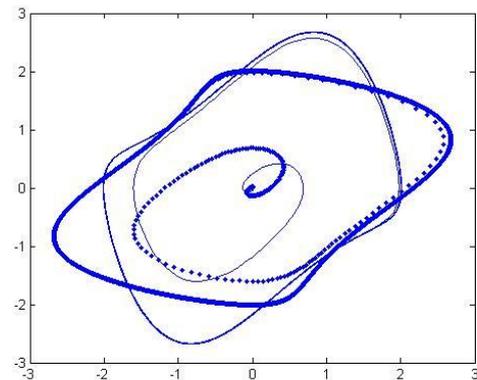
So the strategy is to determine the parameters of controller such that applying control law (22), minimizes the cost function defined in (25). Since the integrand of  $J$  defined in (25) is a positive semi-definite function, it would be tend to zero in order to minimize the integral and it guarantee the stability of system. To minimize the cost function, PSO algorithm is applied. The PSO parameters update when cost function is calculated while the system is running. Therefore, the algorithm updates rarely at the beginning of simulation. Although the dynamic of control signal defined in (22) is important, uncertainties and noises can not perturb the controller performance so much. The PSO algorithm tends to minimize the cost function defined in (25) and it is not depends significantly on the parameters of (22). Thus, PSO algorithm can defeat the uncertainty, without any additional information about it.



**Figure 5.** curve of  $x_1, x_3$  after of synchronization in presence of noise



**Figure 6.** synchronization after applying CPSO



**Figure 7.** curve of  $x_1, x_3$  before of synchronization in presence of certain parameters

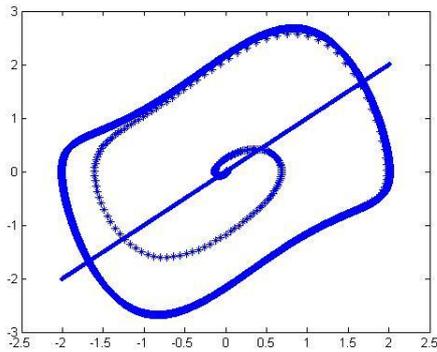


Figure 8. curve of  $x_1, x_3$  before of synchronization in presence of uncertainty parameters .

## 5. Implementation of the method and Simulation

### 5.1. Implementation of the method

As mentioned above, the controller consists of a Fillippov's structure. The parameters of the controller are considered as the positions of particle in particle swarm optimization algorithm. The adaptive method proposed so far can be applied to

**Step 1:** The initial positions and velocities of particles are determined randomly. In other words, the controller launches with randomly defined parameters and parameter changes.

**Step 2:** The control signal is calculated by Equation. (22) and applied to system while system is running.

**Step 3:** The cost function defined in (25) is calculated in the time period between the two last subsequent updating times for each particle.

**Step 4:** When the time reaches the optimization point defined by (24), then the best solution of each particle ( $P_i^k$ ) and the swarm ( $X_g^k$ ) are found and saved.

Otherwise, the algorithm is followed by step 8.

**Step 5:** Convergence of algorithm is checked.

**Step 6:** The PSO algorithm updates. It means that the new positions and velocities of particles are calculated by (2).

**Step 7:** When the value of cost function does not change, the interval of integration in cost function, i.e. the period between updating points, is reduced.

**Step 8:** Going back to step 2.

### 5.2. Simulation

In order to verify the performance of the offered control algorithm, numerical simulations are performed. The equations are solved with Runge–Kutta method of order four with maximum step size 0.005. We perform some simulations. All parameters are set as stated in sub

section 4.1. Since in real systems, it is not possible to apply an unbounded control law, the control signal is assumed to be saturated at  $u = \pm 60$ .

#### Case study 1: system without uncertainty

In this case, no uncertainty presents in the system and the values of  $pop = 20$ ,  $c_1 = c_2 = 2.2$   $w = 0.99$  are set for the parameters of PSO. where,  $pop$  represents the population size of swarm under consideration,  $c_1$  and  $c_2$  correspond to the cognitive and the social parameters respectively.  $w$  is the inertia weight. Simulation results are shown in **Figs. 7**. These results contain the variation of cost function versus time, three error signals and the controller output signal.

#### Case study 2: system with uncertainty

In this simulation, the parameters of PSO are just the same as case 1. But this time the model contains parameter uncertainty. In order to show the ability of controller, control law (22) is applied to a system which its real dynamic follow (10) with following parameters:

$$\begin{aligned} R_1 &= 5 + 0.1r_1(t) & R_2 &= 5 + 0.1r_2(t) \\ d_1 &= 3 + 0.1r_3(t) & d_2 &= 3 + 0.1r_4(t) \\ a_1 &= 5 + 0.1r_5(t) & a_2 &= 5 + 0.1r_6(t) \end{aligned} \quad (26)$$

Where  $r_i(t)$ 's are normally distributed random functions. Simulation results are shown in **Fig. 8**. These results contain the variation of cost function versus time, three error signals and the controller output signal.

#### Case study 3 system with uncertainty and measurement noise

To consider the effect of measurement noise on the performance of the controller, a sinusoidal noise with random frequency is added to all the states of both master and slave systems. The amplitude of noise is set equal to 0.02. Simulation results are shown in **Fig. 4** and **Fig. 5**.

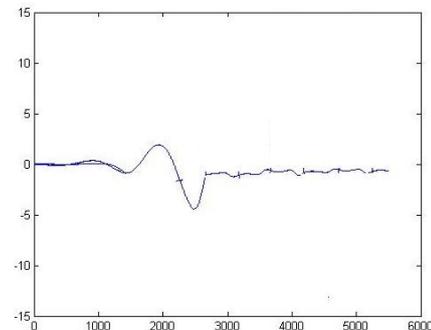
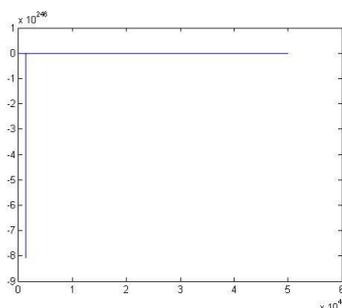


Figure 9. error of synchronization before of applying CPSO algorithm



**Figure 10. error of synchronization by applying CPSO algorithm**

## 6. Conclusion

An adaptive robust control algorithm was proposed to synchronize two different chaotic systems with parameter uncertainty in presence of measurement noise. The controller was based on Filippov's construction and its parameters were adapted with PSO algorithm to minimize tracking error. The ability to eliminate tracking error without any information about uncertainty bounds is the advantage of this controller. Besides, this controller is robust and it needs only a rough model of system and the PSO algorithm can defeat parameter uncertainties of the system and the controller endures even in presence of measurement noise in both master and slave systems. also, we have shown that nonlinear oscillators can be used to model the heartbeat activity and mathematical modeling of dynamic rhythm was provided in different states of single oscillator, two oscillator and three oscillators modified. Moreover, by applying CPSO algorithm and Synchronization, proposed an proper control signal for the model of heart. In this model, each oscillator is represented one of the heart natural important pacemaker: atrial sinus node (node SA ) and atrial ventricular (node AV ) and His-Purkinje complex.

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