Income Elasticity of Demand, a Micro-Economic and a Macro-Economic Interpretation

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Source: *The Economic Journal*, Vol. 51, No. 201 (Apr., 1941), pp. 140-145
Published by: Wiley on behalf of the Royal Economic Society
Stable URL: http://www.jstor.org/stable/2225666
Accessed: 08-03-2015 06:20 UTC

NOTES AND MEMORANDA

INCOME ELASTICITY OF DEMAND, A MICRO-ECONOMIC AND A MACRO-ECONOMIC INTERPRETATION

THE concept of income elasticity of demand has been used with two entirely different meanings: a micro- and a macro-economic one.

The micro-economic interpretation refers to the relation between income and outlay on a certain commodity *for a single person or family.*

The macro-economic interpretation is derived from the corresponding relation between total income and total outlay *for a large group of persons or families* (social strata, nations, etc.).

If we make the assumption that an individual family's outlay c on a certain commodity is wholly determined by its income i and a number of other factors (size of family, tastes, etc.), it is possible to ascribe an unambiguous sense to the (micro-economic) income elasticity ε, viz.—

$$\varepsilon = \frac{i}{c} \cdot \frac{\partial c}{\partial i} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Our empirical knowledge of the micro-economic relation is very limited. This relation might be determined statistically by comparing the simultaneous fluctuations of income and outlay in time. As, however, other factors also may be expected to play a rôle, it would be necessary to apply the method of multiple correlation. As far as I know, this method has not yet been used for individual families. Our statistical knowledge of the micro-economic relation is up to now wholly based on the results of family budget inquiries.[1] From these data we may, at best, derive an average micro-economic relation, because we generally lack data on a number of determining factors (tastes, etc.). Moreover, we must assume that outlay is not dependent on the rate of increase of income, and that the reaction of income on outlay is not lagged (it would be impossible to derive such dynamic relations from the data of budget inquiries, unless these data refer to the expenditures of the same families during at least two different periods).

[1] Cf. *Family Expenditure*, by R. D. G. Allen and A. L. Bowley, London, 1935.

Although the statistical determination of the micro-economic relation is thus not free from difficulties, the relation itself, as has been shown, has a perfectly clear meaning, and the same holds true of the income elasticity derived from it. It is not so simple, however, to give an unassailable definition of the macro-economic income elasticity E. If we know the micro-economic relation between outlay and income, total outlay will be determined as soon as we know the income distribution. This may be described by a function $F(i)$ of i, telling us the number of families $F(i)di$ with incomes between i and $i + di$. If the total number of families is denoted by N, total income by I and total outlay on the considered commodity by C we have the following relations :

$$N = \int_0^\infty F(i)di \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$I = \int_0^\infty iF(i)di \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$C = \int_0^\infty c(i).F(i)di \quad . \quad . \quad . \quad . \quad . \quad (4)$$

(If $F(i)$ proves to vanish outside a certain range $\alpha-\beta$, the limits of integration 0 and ∞ may be replaced by α and β without altering the meaning of the integrals.) In general it is not possible to study the relation between C and I given by (3) and (4), without making some assumption about the character of $F(i)$. There is, however, one exception : viz. *when $c(i)$ is a linear function of i :*

$$c(i) = a + bi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

a and b being constants.

In this case (4) immediately reduces to :

$$C = aN + bI \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Now, generally, N and I will be independent variables (in most cases N will practically be constant or show a slight trend), and therefore E may be put equal to :

$$E = \frac{I}{C} \cdot \frac{\partial C}{\partial I} = b\,\frac{I}{C} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

In all other cases we must know the properties of $F(i)$, in order to be able to perform the transition from ε to E.

Now there are practically no theories about the shape of the income distribution function. But nevertheless we may assume that such a theory would lead to a mathematical expression for the distribution function, containing a number of constants or parameters λ, μ, ν . . ., depending on the economic structure of the group of families. We shall denote this general expression by $F(i;\ \lambda, \mu, \nu\ .\ .\ .)$. We have already observed that the number of families may be considered as practically a constant. If we assume that this holds rigorously, we may eliminate one of the parameters, say λ. Instead of F we will then obtain a function $f(i;\ \mu, \nu\ .\ .\ .)$.

Now we are able to prove that, if f contains only one parameter, it is possible to generalise the definition of ε to the macro-economic case.

Equations (3) and (4) will then take on the following forms :

$$I = \int_0^\infty i f(i;\ \mu) di \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

$$C = \int_0^\infty c(i) . f(i;\ \mu) di \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

I and C both become functions of μ. If μ is kept constant, I and C are constant too. (The variable i has disappeared after the process of integration.) If μ changes, I and C will change at the same time, and their fluctuations are uniquely connected with the fluctuations of μ.

Consequently C may be considered as a function of I, and the derivative $\frac{\partial C}{\partial I}$ may be calculated with the help of the ordinary function of a function rule :

$$\frac{\partial C}{\partial I} = \frac{\partial C}{\partial \mu} : \frac{\partial I}{\partial \mu} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

and E becomes :

$$E = \frac{I}{C} \cdot \frac{\partial C}{\partial \mu} : \frac{\partial I}{\partial \mu} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

As soon as, however, the number of parameters of f becomes greater than one, it is no longer possible to reason in the same way. Suppose that f contains two parameters μ and ν, consequently I and C will also become functions of these parameters. The variations of C are now no longer uniquely connected with the variations of I. We may, for instance, compare the simultaneous changes ΔC of C and ΔI of I brought about by changes of μ, keeping ν constant, or, on the contary, caused by variations of ν, treating μ as a constant. More generally we may vary μ and ν together, keeping constant a certain relation between them : $\phi(\mu, \nu) =$ constant. The quotient $\frac{\Delta C}{\Delta I}$ will be different for the different choices, and the derivative $\frac{\partial C}{\partial I}$ as well as E becomes wholly arbitrary. The same holds true *a fortiori*, if f contains three or more parameters. In such cases we have still more freedom to choose the variations of the parameters (we have to introduce more than one constant function ϕ between the parameters in order to obtain a definite value for $\frac{\partial C}{\partial I}$). We may still arrive at an unambiguous definition, by restricting ourselves to a given ϕ (or generally a number of ϕ's equal to the number of parameters of f minus one), *it will, however, always be necessary clearly to state which quantities ϕ are assumed to be constant.* It will perhaps be best to illustrate this by some examples.

Pareto's well-known distribution function is characterised by three constants. If we state Pareto's law in the following form : the number of persons (families) with incomes exceeding a given number i may be represented by Ki^{-a}, these constants are K, a and i_0, the minimum income (i_0 cannot be put equal to 0, as in this case the total number of persons would become infinite). The function f would consequently contain two parameters—say K and α (i_0 being eliminated with the help of (2)). Now we may consider one (macro-economic) income elasticity with the additional condition α is constant, another with the condition K is constant, etc. The value of these concepts is, however, very much reduced, as we lack a theory telling us the economic meaning of changes of the income distribution for which a (or K) is constant.

The second example refers to a paper of Staehle,[1] in which he tries to improve the relation between total consumption outlay (on all commodities) and total income of German workers by introducing a coefficient β connected with the concentration of incomes (due to Mendershausen). His result reads as follows :

$$C = aI + b\beta + c \quad . \quad . \quad . \quad . \quad . \quad (12)$$

a, b and c being constants. Here again we can derive a formula for E :

$$E = a\frac{I}{C} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

If we are allowed to assume that the distribution function F in this case contains only two parameters (of which one can be eliminated, whereas the other is β), the condition which has to be added in order to ascribe a definite sense to (13) is : β is constant. But there is no reason at all to make this assumption; on the contrary, Staehle introduces the coefficient β, instead of Pareto's a, to enable him to apply his methods to distributions for

[1] Hans Staehle : " Short-Period Variations in the Distribution of Incomes," *Review of Economic Statistics*, August 1937.

which Pareto's law does not hold and which will generally be characterised by a greater number of parameters. *In this case the theoretical meaning of the income elasticity derived from* (12) *becomes rather vague.*

From the preceding we learn that, with two exceptions, it is not possible to define the macro-economic income elasticity in an unambiguous way. In the general case the definition refers to a special type of variations of the distribution function, and this type must be carefully specified. As a general theory of income distribution is still lacking, there are no theoretical grounds to prefer a particular definition, and it seems hardly possible to avoid ambiguity.

Sometimes, however, a particular definition intrudes itself upon us on practical grounds. Suppose we know that the income distribution for a certain country in different years can be described by a function $f(i;\ \mu, \nu)$ with two parameters (we presume that this function has already been chosen so as to satisfy the condition: N is constant). If we now observe that the time series of μ and ν (and consequently also the series of μ and I and of ν and I) are highly correlated, it is allowed to consider μ as a function $\phi(\nu)$ of ν (at least for the variations which occurred in the past). Strictly speaking, *there is now only one parameter*, and we already know that in this case an unambiguous definition can be given. It will be obvious that the additional definition in this case must be: ϕ is constant. Moreover, the income elasticity, corresponding to this definition, is the only one which can statistically be determined, as, according to our supposition, all our observations refer only to variations for which ϕ has been constant.

A high degree of interrelatedness of the parameters seems to occur rather frequently. For the Netherlands 1921–38 Pareto's α is highly correlated with I; the same is true for the United States 1919–32.[1] If the other parameters show corresponding relations (this has not yet been verified), we are justified in applying the last-mentioned definition in these two cases. When using this definition we should, however, remember the following two points:

(1) It is meaningless to introduce a parameter of the distribution as a separate factor in order to improve the result of a correlation calculation between C and I;

(2) the income elasticity calculated according to it should

[1] J. Tinbergen, *Business Cycles in the United States of America, 1919–1932* League of Nations, 1939.

not be applied to cases in which the condition of a constant volume for ϕ is probably not fulfilled—*e.g.*, to study the effect of a sudden and heavy change of the progressivity of income tax, etc.

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OBITUARY

JAMES BONAR (1852–1941)

FELLOWS of the Society throughout the world will have read with deep regret of the death of Dr. James Bonar, in his 89th year, on January 18, 1941. With his passing one of the last of the older Economists has gone. He belonged to the generation of Marshall, Edgeworth and Foxwell and of John Neville Keynes and Bastable who are still with us. After Scott's death he was without any shadow of doubt the leading authority in the world on Adam Smith, and his work on Malthus and population had brought him much renown. Like other Scotsmen, such as James Mill and his distinguished son, John Stuart Mill, he was a Civil Servant. He entered the Civil Service at the age of twenty-nine, and retired at sixty-seven.

The name " Bonar " is said to have been originally " Bonare." Guillaume de Bonare came from the province of Anjou to Perthshire in the thirteenth century. The spelling Bonar dates from the seventeenth century, and the family crest (a crusader's sword placed pale-wise, and the motto " Denique Coelum ") dates from Guillaume Roger Sire de Bonare, who fought in the Holy Land under St. Louis. James Bonar's father was the Rev. Andrew Bonar (1810–92), after whom Andrew Bonar Law, the former Prime Minister, was named, and his uncles were also well-known divines, Horatius Bonar, the hymn-writer, and John James Bonar. It is interesting to note that of the seven Bonars in the *Dictionary of National Biography*, five were Scottish divines, and the divinity strain was indeed strong in Bonar himself. He was born at Collace, a few miles from Perth, on September 27, 1852, when his father was the Free Church Minister of that parish. When his father became Minister of Finnieston, Glasgow, in 1856, Bonar was four years old. His father remained