IET Generation, Transmission & Distribution

Research Article

Loss and emission reduction allocation in distribution networks using MCRS method and Aumann–Shapley value method

ISSN 1751-8687
Received on 7th February 2018
Revised 5th July 2018
Accepted on 11th October 2018
E-First on 20th November 2018
doi: 10.1049/iet-gtd.2018.6086
www.ietdl.org

Qingyun Yu[1], Jun Xie[1] ✉, Xingying Chen[1], Kun Yu[1], Lei Gan[1]
[1]*College of Energy and Electrical Engineering, Hohai University, Nanjing 211100, Jiangsu, People's Republic of China*
✉ *E-mail: eejxie@163.com*

Abstract: In order to provide reasonable economic signals for distribution companies (DISCOs) and further compensate for distributed generators (DGs) integrated in distribution networks equitably, the contributions of DGs to loss and emission reduction in distribution networks should be allocated according to their own responsibilities. Generally, the loss and emission reduction of the network can be allocated based on traditional cooperative-game-based allocation methods such as nucleolus method and Shapley value method. However, traditional cooperative-game-based allocation methods will result in the combinational explosion problem with the integration of a large number of DGs. In order to tackle this problem, minimum costs-remaining savings (MCRS) method and Aumann–Shapley value method are employed for loss and emission reduction allocation. Simulation results of two cases show that compared with the allocation results of traditional cooperative-game-based allocation methods, the proposed MCRS method and Aumann–Shapley value method both have the characteristics of individual rationality, coalition rationality, and global rationality. Furthermore, neither MCRS method nor Aumann–Shapley value method has the problem of combinational explosion, which can reduce computational burden with regard to the integration of a large number of DGs.

Nomenclature

Indices

br	branch index
i	DG unit index
grid	main grid index
k	objective index
l	loss index
e	emission index

Constants

N_{br}	number of branches
R_{b}	resistance of branch b, Ω
N_{DG}	number of DGs
EF_{grid}	emission coefficients of the main grid, kg/kW
$EF_{\text{DG}i}$	emission coefficients of DG_i, kg/kW
ε^k	an arbitrary small real number for loss/emission
S_1	coalition of all DGs
S_2	all non-empty sub-coalitions of DGs
N	grand coalition of all DGs
$\hat{S}$	set of all combinations of players containing DG_i
$\lvert S\rvert$	number of DGs in particular coalition S
λ	integration parameter

Variables

P_{loss}	active power losses, kW
I_{b}	current of branch b, A
E	total emission produced by the main grid and DGs, kg
E_{grid}	emission produced by the main grid, kg
$E_{\text{DG}i}$	emission produced by DG_i, kg
P_{grid}	active power of the main grid, kW
P_i	active power of DG_i, kW
X^k	set of loss/emission reduction allocation
Y^k	set of loss/emission reduction allocation imputation of each DG unit
$e^k(S, y^k)$	excess value for loss/emission of imputation $y^k \in Y^k$ of coalition S
$\varphi^k(y^k)$	maximum excess value for loss/emission of coalition S
$v^k(S)$	loss/emission reduction caused by coalition S
$V^k(S)$	loss/emission reduction caused by the alternation of DGs of coalition S
x_i^k	loss/emission reduction allocation of DG_i
$x_{i,\min}^k$	minimum loss/emission reduction allocation of DG_i
$x_{i,\max}^k$	maximum loss/emission reduction allocation of DG_i
I_c^k	remainder of the loss/emission reduction sharing
β_i^k	coefficient of the ratio
$W(\lvert\hat{S}\rvert)$	weight factor
b_i	output of DG_i
$f(\cdot)$	the function that represents the marginal contributions to loss and emission reduction for a given λ

1 Introduction

With the integration of distributed generators (DGs) in distribution networks, the radial distribution networks have been shifted from passive unidirectional networks to active bidirectional networks, and DGs can affect power losses and the emission produced by the substation bus [1]. Therefore, DGs should be rewarded according to their responsibilities to the losses and emission reduction. Consequently, the deviation of active power losses and emission from those of the base case (the network with 'no DGs') should fairly be allocated to all integrated DGs, which can provide correct economic signals for distribution companies (DISCOs) and further prompt efficient operation of the networks [2].

The problem of loss reduction allocation in distribution networks due to the integration of DGs is essentially the same as loss allocation, which is highly challenging due to the non-linear and non-separable characteristics between branch losses and the network power flow [3]. There have been various loss allocation methods developed for transmission and distribution networks, which is summarised in Table 1.

Table 1 Loss allocation methods for transmission and distribution networks

Method	Description	Note
Pro-rata method [1]	In proportion to their corresponding generation and consumption without considering the network topology and their relative location within the network	• Easy to compute but cannot provide correct economic signals
Marginal loss coefficient (MLC) method [4]	In accordance with the MLC based on the results of power flow	• Consider the network topology and is superior to pro-rata method • The sum of loss allocation results of all network users is not equal to the total losses and therefore a reconciliation procedure is required to compensate for the over-recovery of losses
Proportional sharing method [5, 6]	The power flow reaching a bus from any power line splits among the lines evacuating power from the bus in proportion to their corresponding power flows	• All the information that needed is power flow calculation results, losses in each line, and the power level in each bus • The allocation method is based on the solved power flow on the premise of the linear proportionality assumption and neglects the cross-terms of losses
Z-bus method [7]	Based on the network impedance matrix and a solved power flow without any special approximations	• Reflect the network topology and provide economic incentives for market participants • Applicable only to networks with non-singular admittance matrix
Costa's method [3]	Network losses are distributed to loads using downstream algorithm in the first phase and then to DGs using upstream algorithm in the second phase. In the third phase, the remaining losses are allocated to DGs in proportion to their apparent power	• Based on the downstream-looking and upstream-looking tracing algorithm and operates in three phases
Branch current decomposition method [8]	Based on the real and imaginary components of the currents with respect to branches connecting the node to the root node	• Establish the relation between branch components (branch current, power, and energy) and nodal injections • Distribute the loss variations to loads and DGs simultaneously, which may cause spatial and temporal cross-subsidies and thus draw unfair allocation results
Power/energy summation method [9, 10]	Both based on the decomposition of branch power losses and average branch power losses (energy) into injections related to nodes, both based on branch-oriented approach	

In recent years, traditional cooperative-game-based allocation methods such as nucleolus method and Shapley value method have attracted attention because these methods satisfy the axioms of fairness and have been confirmed to have the characteristics of individual rationality, coalition rationality, and global rationality [11]. Therefore, in the past decades, several studies have been conducted for allocation methods based on traditional cooperative game theory [11] for the allocation of common costs in power systems, for example transmission cost allocation [12–15], transmission loss allocation [16–18], thermal unit start-up costs allocation [19], electricity purchasing cost decrement allocation [20].

As a result, traditional cooperative-game-based allocation methods can be used to specify the share of each DG's contributions to loss and emission reduction in a fair way. Refs. [21, 22] employs nucleolus method and Shapley value method, respectively, to allocate loss and emission reduction due to the integration of DGs in distribution networks. However, with a large number of DGs integrated in distribution networks, the combinational explosion problem will occur and traditional cooperative-game-based allocation methods will become computationally unfeasible due to the combinational nature (e.g. there are totally $2^n - 1$ possible permutations for a case with n DGs).

In order to overcome the combinational explosion problem and reduce the computational burden with a large number of DGs in distribution networks, the minimum costs-remaining savings (MCRS) method, which is an extension of nucleolus method, and Aumann–Shapley value method, which is an extension of Shapley value method, are introduced in this paper to allocate the loss and emission reduction in distribution networks due to the integration of DGs.

The MCRS method is applied in [23] to allocate the treatment costs. The MCRS model considering risk factors is proposed in [24] to solve the profit allocation problem in apparel supply chain. However, to the best knowledge of the authors, the MCRS method has not been employed to solve common cost allocation problems in distribution networks.

Meanwhile, Aumann–Shapley value method has been applied to solve a variety of common cost allocation problems in power systems, for example transmission network cost allocation among generators and loads [25, 26], complex losses allocation among generators and loads [27], congestion cost allocation among transmission users [28], and firm-energy rights allocation among hydro plants [29]. In [25], a method based on circuit theory and the Aumann–Shapley value method is proposed to allocate the costs of the transmission network among generators and loads, which can ensure equitable allocation and recovery of the total costs. In [26], a methodology based on Aumann–Shapley value method is presented to allocate transmission service cost among network users in energy markets and is shown to be computational feasible and can ensure economic coherence and isonomy. A method coupling with circuit theory and the Aumann–Shapley value method is employed in [27] to allocate active and reactive losses simultaneously to generators and loads considering counterflow, cross-subsidy, and negative allocation issues. A methodology for congestion cost allocation based on marginal costs and Aumann–Shapley is presented in [28], which can not only provide fair and efficient prices, but also avoid the merchandising surplus. However, to the best knowledge of the authors, the Aumann–Shapley value method has not been employed to solve the problem of loss and emission reduction allocation due to the integration of DGs in distribution networks.

With the above background, we explore MCRS method and Aumann–Shapley value method to allocate loss and emission reduction for distribution networks with DGs. After comparing with the allocation results of traditional cooperative-game-based allocation methods on the modified 33-bus distribution network, the rationality and validity of the proposed two methods are verified. Simulation studies on the modified 69-bus distribution networks are used to demonstrate the superiority of MCRS method and Aumann–Shapley value method over traditional cooperative-game-based allocation methods due to their computational feasibility.

The main contributions of this paper are as follows:

(i) Employing MCRS method and Aumann–Shapley value method for loss and emission reduction allocation in distribution networks due to the integration of DGs;
(ii) Providing appropriate economic signals to DISCOs and further imposing financial compensation (punishment) to all integrated DGs when employing the proposed MCRS method and Aumann–Shapley value method;
(iii) Having the characteristics of individual rationality, coalition rationality, and global rationality when employing the proposed MCRS method and Aumann–Shapley value method;
(iv) Overcoming the combinational explosion problem with a large number of DGs integrated in distribution networks when comparing with traditional cooperative-game-based allocation methods (nucleolus method and Shapley value method).

The remainder of this paper is organised as follows. In Section 2, the loss and emission reduction due to the integration of DGs is identified, and the nucleolus method as well as its extension (MCRS method) is presented for loss and emission reduction allocation. In Section 3, Shapley method and its extension (Aumann–Shapley value method) are presented for loss and emission reduction allocation. In Section 4, numerical simulation results with a modified 33-bus test network and a modified 69-bus test network are reported. Finally, the main conclusions are summarised in Section 5.

2 Nucleolus method and MCRS method

2.1 Loss and emission calculation

The penetration of DGs in distribution networks will affect active power losses and emission produced by DGs and the main grid. The formulations of active power losses P_{loss} and total emission E produced by the main grid and DGs in distribution networks are as follows [21]:

$$P_{\text{loss}} = \sum_{b=1}^{N_{\text{br}}} R_b |I_b|^2, \tag{1}$$

$$\left.\begin{aligned} E &= \sum_{i=1}^{N_{\text{DG}}} E_{\text{DG}_i} + E_{\text{grid}} \\ E_{\text{DG}_i} &= \text{CO}_2^{\text{DG}_i} + \text{NO}_x^{\text{DG}_i} + \text{SO}_2^{\text{DG}_i} = EF_{\text{DG}_i} P_i \\ E_{\text{grid}} &= \text{CO}_2^{\text{grid}} + \text{NO}_x^{\text{grid}} + \text{SO}_2^{\text{grid}} = EF_{\text{grid}} P_{\text{grid}} \end{aligned}\right\} \tag{2}$$

2.2 Nucleolus method

Suppose:

$$X^k = \{x_1^k, x_2^k, \ldots, x_n^k\}, \quad k = l, e, \tag{3}$$

$$Y^k = \{y_1^k, y_2^k, \ldots, y_n^k\}, \quad k = l, e \tag{4}$$

Nucleolus is based on the minimum core and is determined by the following (5)–(7):

$$\left.\begin{aligned} C^{+k}(\varepsilon^k) &= \{y^k \in Y^k | \varphi^k(y^k) \le \varepsilon^k\} \\ \varphi^k(y^k) &= \max_{S \in N} e^k(S, y^k), \quad k = l, e \end{aligned}\right\}, \tag{5}$$

$$e^k(S, y^k) = V^k(S) - \sum_{i \in S} y_i^k, \quad k = l, e, \tag{6}$$

$$V^k(S) = v^k(S) - \sum_{i \in S} v^k(i), \quad k = l, e. \tag{7}$$

The problem of formula (5) can be solved by linear programming (LP) as follows [17, 22]:

$$\left.\begin{aligned} &\min \quad \varepsilon^k \\ &\text{s.t.} \quad V^k(S) = \sum_{i \in S_1} y_i^k \\ &\qquad V^k(S) - \sum_{i \in S_2} y_i^k \le \varepsilon^k, \quad k = l, e \end{aligned}\right\} \tag{8}$$

It is known that the loss/emission reduction allocation of DG_i is the summation of the loss/emission reduction caused by all DGs and the one caused by the individual DG player as (9)–(10) show the following:

$$x_i^l = y_i^l + v^l(i), \quad i = 1, 2, \ldots, N_{\text{DG}}, \tag{9}$$

$$x_i^e = y_i^e + v^e(i), \quad i = 1, 2, \ldots, N_{\text{DG}}. \tag{10}$$

2.3 MCRS method

With the integration of a large number of DGs in distribution networks, the combinations of DGs for calculating coalition S will increase in an exponential form. Therefore, for conquering combinational explosion, MCRS method, as an effective method when it comes to a game with infinite players [23], is employed in this paper. The basic idea of MCRS method is to draw out the nucleolus outline and to allocate the loss and emission reduction in accordance with the proportion between the maximum and the minimum loss and emission reduction allocation.

The MCRS method for loss and emission reduction allocation is represented as follows:

$$\left.\begin{aligned} &\max \text{ or } \min x_i^k \\ &\sum_{i=1}^{N_{\text{DG}}} x_i^k = v^k(N) \\ &\sum_{j \in \{N - \{i\}\}} x_j^k \le v^k(N - \{i\}), \quad k = l, e \end{aligned}\right\} \tag{11}$$

The equality constraint describes the global rationality, where the sum of the loss/emission reduction allocation of all DGs is equal to the loss/emission reduction caused by the grand coalition N with all DGs in operation. The inequality constraint indicates that the sum of the loss/emission reduction allocation to DGs except DG_i is no more than the loss/emission reduction of the grand coalition N without DG_i.

The minimum loss/emission reduction allocation of DG_i can be defined as the incremental loss/emission reduction with DG_i participating in the grand coalition N, known as the marginal contributions of DG_i:

$$x_{i,\min}^k = v^k(N) - v^k(N - \{i\}) \le x_i^k. \tag{12}$$

The maximum loss/emission reduction allocation of DG_i can be defined as the loss/emission reduction with only DG_i in operation:

$$x_i^k \le x_{i,\max}^k = v^k(i). \tag{13}$$

As a result, the loss/emission reduction allocated to DG_i is calculated by:

$$x_i^k = x_{i,\min}^k + \beta_i^k I_c^k \tag{14}$$

$$I_c^k = v^k(N) - \sum_{i \in N} x_{i,\min}^k \tag{15}$$

$$\beta_i^k = \frac{x_{i,\max}^k - x_{i,\min}^k}{\sum_{j=1}^{N_{\rm DG}}\left(x_{j,\max}^k - x_{j,\min}^k\right)} \tag{16}$$

3 Shapley value method and Aumann–Shapley value method

3.1 Shapley value method

Shapley value method is a typical solution concept to allocation problems in cooperative game theory and behaves well in terms of fairness and effectiveness. In order to allocate loss and emission reduction, each DG can be regarded as a player in a cooperative game. Since the entrance order of each DG in coalition S affects its incremental contributions to loss and emission reduction, the Shapley value considers all sub-coalitions without the particular DG and calculates the average value of incremental contributions to loss/emission reduction of including the particular DG. In Shapley value method, the marginal contribution to loss and emission reduction of a DG unit is the only factor that determines its allocation results [13, 14, 19].

Therefore, the loss/emission allocation of DG_i based on Shapley value is defined as [15, 20]:

$$x_i^k = \sum_{i \in S}\left\{W\left(\left|\hat{S}\right|\right) \times \left[v^k\left(\hat{S}\right) - v^k\left(\hat{S} - \{i\}\right)\right]\right\}, \quad k = l, e, \tag{17}$$

$$W\left(\left|\hat{S}\right|\right) = \frac{\left(N_{\rm DG} - \left|\hat{S}\right|\right)!\left(\left|\hat{S}\right| - 1\right)!}{N_{\rm DG}!} \tag{18}$$

where the term $\left[v^k(S) - v^k(S - \{i\})\right]$ refers to the incremental contribution that the player DG_i makes to coalition S.

Since the Shapley value considers all orderings equally, that is all DG players have the same opportunity to be in the first and last order positions, the allocation value to each DG player can be considered fair and desirable. However, the combination number as well as coalition permutations will increase which will result in the combinational explosion with the number of DG players due to the combinational nature of Shapley value. Therefore, the Shapley value method is limited to applying in networks with small number of DGs.

3.2 Aumann–Shapley value method

Aumann–Shapley value method, an extension of the Shapley value method, is an analytical solution to the allocation problem with large number of players. It can be explained as dividing each player (DG) power output into infinitesimal segments and then applying Shapley value method to each one as if each sub-player was an individual. When the output of each DG player connected to the distribution networks grows from zero to its maximum power output, Aumann–Shapley value method calculates the average of the incremental contributions of sub-players to loss and emission reduction which makes the allocation problem insensitive to the entrance order of DG players. Therefore, Aumann–Shapley value method provides fair, desirable, and efficient allocation results [25].

The computational burden seems to be greater due to considerable increase in the number of players as well as their permutations in the Aumann–Shapley value method. However, as mentioned before, Aumann–Shapley value method does not depend on the order of entry since power output of each DG player is divided into infinitesimal parts.

Assuming that the output of a player DG_i at a specific time in the game is b^*, and its contribution to loss and emission reduction denotes as $f(b^*)$. If its output grows up to $(b^* + \Delta b_i)$ at another point, where Δb_i is an infinitesimal value $(\Delta b_i \to 0)$, thus the incremental contribution of the new sub-player Δb_i in the loss and emission reduction is [25]:

$$\frac{f(b^* + \Delta b_i) - f(b^*)}{\Delta b_i} \cong \left.\frac{\partial f(b)}{\partial b_i}\right|_{b = b^*}. \tag{19}$$

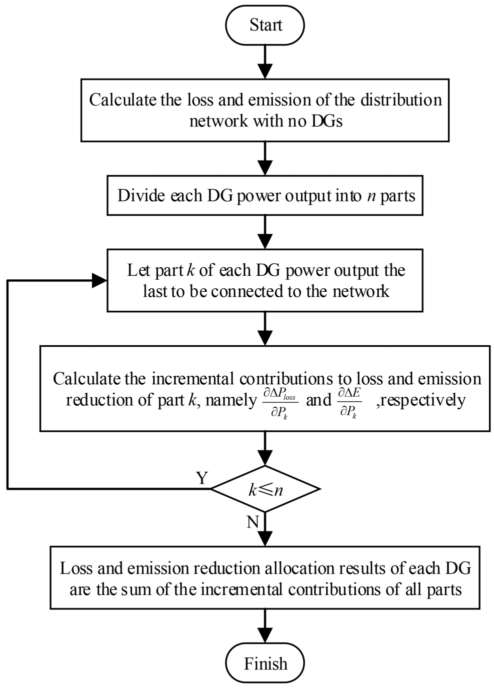

Fig. 1 *Flowchart of Aumann–Shapley value method*

When the output of DG_i increases from zero to its maximum value, we can determine the loss and emission allocation results of DG_i as follows:

$$x_i^k = b_i \int_{\lambda=0}^{1} \frac{\partial f^k(\lambda b)}{\partial b_i} \mathrm{d}\lambda \tag{20}$$

Since the complexity of loss and emission calculation in distribution network, it is difficult to calculate the allocation of loss and emission reduction in a regular Aumann–Shapley formulation as (20) shows. Then, we consider the calculation of Aumann–Shapley value based on the premise that the power output of each DG unit is divided into n parts and sum up the incremental contributions to loss and emission reduction when each part of the DG power output is the last to be connected to the distribution network. Therefore, the loss and emission allocation results are formulated as:

$$x_i^l = \sum_{k=1}^{n} \frac{\partial \Delta P_{\rm loss}}{\partial P_k}, \tag{21}$$

$$x_i^e = \sum_{k=1}^{n} \frac{\partial \Delta E}{\partial P_k} \tag{22}$$

where $\partial \Delta P_{\rm loss}/\partial P_k$ and $\partial \Delta E/\partial P_k$ represent the incremental contributions to loss and emission reduction of the k part of the power output of DG_i connected to distribution network, respectively.

Fig. 1 gives the flowchart of Aumann–Shapley value method to allocate the loss and emission reduction.

4 Simulation results

In this section, the proposed MCRS method and Aumann–Shapley value method are evaluated with a modified 33-bus distribution network and a modified 69-bus distribution network.

4.1 Modified 33-bus distribution network

Fig. 2 shows the topology of the modified 33-bus distribution network. The network is composed of 33 buses and 4 feeders. Three DG units with different types, namely gas internal combustion engine, combined cycle gas turbine, and diesel internal

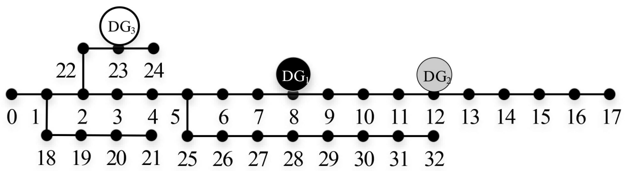

Fig. 2 *Diagram of the modified 33-bus test network*

Table 2 Emission factors of DGs and the main grid

DG unit	Emission factors, g/kW			
	CO_2	SO_2	NO_x	CO
DG_1	430	0.022	0.014	0
DG_2	626	1.13	1.92	2.52
DG_3	563	0.029	0.26	0.38
Main grid	869	5.08	1.5	0.063

Table 3 Loss and emission reduction under different DG unit combinations

DG unit combination	Loss reduction, kW	Emission reduction, kg
{1}	75.23	288.66
{2}	85.70	197.07
{3}	30.67	182.81
{1,2}	132.99	461.30
{1,3}	101.88	467.99
{2,3}	112.21	376.27
{1,2,3}	155.92	637.36

Table 4 Allocation results under nucleolus and MCRS

DG unit	Nucleolus method		MCRS method	
	Loss reduction, kW	Emission reduction, kg	Loss reduction, kW	Emission reduction, kg
DG_1	59.4	274.8	59.4	274.8
DG_2	69.9	182.3	69.8	183.2
DG_3	26.6	179.3	26.7	179.4
Total	155.9	637.4	155.9	637.4

combustion engine [22], are located at buses 8, 12, and 23, respectively. The emission factor of these DGs and the main grid is shown in Table 2, which is cited from Ref. [22] with a small modification. These DGs are modelled with constant power factor of 0.9 lagging [30]. The base case is the situation where there are no DGs connected to the network and the total active power loss and emission in the base case are 259.6 kW and 3480.4 kg, respectively.

This section aims to show the effectiveness of MCRS method and Aumann–Shapley value method. For this purpose, the loss and emission reduction allocation results of MCRS method and Aumann–Shapley value method will be compared with those of nucleolus method and Shapley value method. The active power output of each DG unit is 500 kW. Each DG unit is considered as a player in the allocation problem and the set of all DG units and every non-empty subset form a coalition. Table 3 shows the loss and emission reduction of all combinations.

Table 4 shows the allocation results of loss and emission reduction in nucleolus method and MCRS method.

When calculating the loss and emission reduction allocation using MCRS method, we should obtain the incremental loss/emission reduction with DG_i participating in the grand coalition N and the loss/emission reduction with only DG_i in service. The former represents the minimum loss/emission reduction allocation of DG_i and the latter represents the maximum one.

Table 5 Loss reduction allocation results of each step when using Aumann–Shapley value method

Step	DG_1, kW	DG_2, kW	DG_3, kW
1	8.86	10.68	3.47
2	8.18	9.74	3.27
3	7.52	8.84	3.08
4	6.89	7.97	2.89
5	6.29	7.15	2.70
6	5.70	6.36	2.52
7	5.14	5.60	2.34
8	4.59	4.86	2.17
9	4.07	4.16	2.00
10	3.56	3.48	1.83
Total	60.8	68.8	26.3

Table 6 Allocation results under Shapley value and Aumann–Shapley value

DG unit	Shapley value		Aumann–Shapley value (n = 10)	
	Loss reduction, kW	Emission reduction, kg	Loss reduction, kW	Emission reduction, kg
DG_1	59.4	274.8	60.8	276.1
DG_2	69.8	183.2	68.8	182.3
DG_3	26.7	179.4	26.3	179.0
Total	155.9	637.4	155.9	637.4

From Table 4, we can see that compared with the loss and emission reduction allocation results in nucleolus method, MCRS method has little difference and performs well in terms of individual rationality, coalition rationality, and global rationality (take loss reduction allocation results as example, and the conclusion is the same with emission reduction allocation):

(i) Individual rationality:

$$x^l_{\mathrm{DG}_1} = 59.4 \le v\{1\} = 75.2,$$

$$x^l_{\mathrm{DG}_2} = 69.8 \le v\{2\} = 85.7,$$

$$x^l_{\mathrm{DG}_3} = 26.7 \le v\{3\} = 30.7.$$

(ii) Coalition rationality:

$$x^l_{\mathrm{DG}_1} + x^l_{\mathrm{DG}_2} = 129.2 \le v\{1,2\} = 133.0,$$

$$x^l_{\mathrm{DG}_1} + x^l_{\mathrm{DG}_3} = 86.1 \le v\{1,3\} = 101.9,$$

$$x^l_{\mathrm{DG}_2} + x^l_{\mathrm{DG}_3} = 96.5 \le v\{2,3\} = 112.2.$$

(iii) Global rationality:

$$x^l_{\mathrm{DG}_1} + x^l_{\mathrm{DG}_2} + x^l_{\mathrm{DG}_3} = 155.9 = v\{1,2,3\}.$$

In Aumann–Shapley value method, we divide each DG unit power output into n parts and sum up the incremental contributions to loss and emission reduction when each part of the DG unit power output is the last to be connected to the test network. To illustrate the calculation, we take $n = 10$, for example. Table 5 shows the loss reduction allocation results of each step when using Aumann–Shapley value method.

Table 6 presents the allocation results obtained by Shapley value method and Aumann–Shapley value method ($n = 10$).

As can be seen from Tables 3 and 6, compared with the loss and emission allocation results in Shapley value method, the Aumann–Shapley value method performs well in terms of individual rationality, coalition rationality, and global rationality (take loss reduction allocation results as example, and the conclusion is the same with emission reduction allocation):

(i) Individual rationality:

$$x^l_{\mathrm{DG}_1} = 60.8 \le v\{1\} = 75.2,$$

$$x^l_{\mathrm{DG}_2} = 68.8 \le v\{2\} = 85.7,$$

$$x^l_{\mathrm{DG}_3} = 26.3 \le v\{3\} = 30.7.$$

(ii) Coalition rationality:

$$x^l_{\mathrm{DG}_1} + x^l_{\mathrm{DG}_2} = 129.6 \le v\{1,2\} = 133.0,$$

$$x^l_{\mathrm{DG}_1} + x^l_{\mathrm{DG}_3} = 87.1 \le v\{1,3\} = 101.9,$$

$$x^l_{\mathrm{DG}_2} + x^l_{\mathrm{DG}_3} = 95.1 \le v\{2,3\} = 112.2.$$

(iii) Global rationality:

$$x^l_{\mathrm{DG}_1} + x^l_{\mathrm{DG}_2} + x^l_{\mathrm{DG}_3} = 155.9 = v\{1,2,3\}.$$

Fig. 3 illustrates the relationship between the number of division parts of DGs' power output and the error rate of allocation results compared with Shapley value method. We can observe from Fig. 3 that when the number of division parts of each DG's power output increases, the allocation results in Aumann–Shapley value method are more accurate.

To sum up, as we can see from the loss and emission allocation results obtained by MCRS method, Aumann–Shapley value method, nucleolus method, and Shapley value method, all the four methods have the characteristics of individual rationality, coalition rationality, and global rationality. However, when it comes to the distribution networks with a large number of integration of DGs, traditional cooperative-game-based allocation methods (nucleolus method and Shapley value method), will become computationally unfeasible due to their combinational nature, while MCRS method and Aumann–Shapley value method can overcome the above combinational explosion problem and can dramatically reduce computational burden.

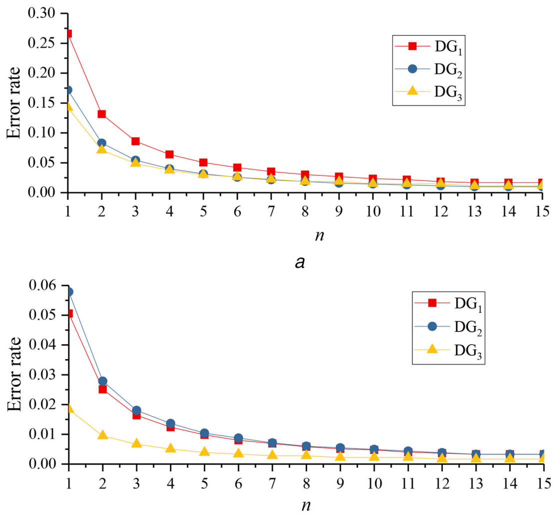

Fig. 3 *Error rate of allocation results with variation of n in Aumann–Shapley value method*

(a) Error rate of loss reduction allocation result, ***(b)*** Error rate of emission reduction allocation result

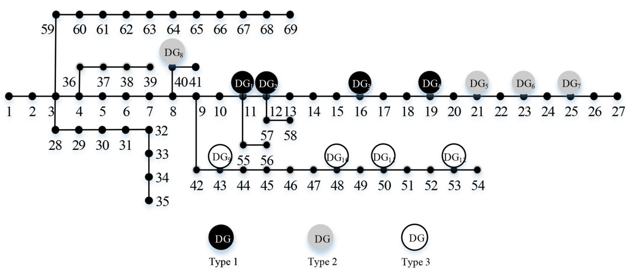

Fig. 4 *Diagram of the modified 69-bus test network*

4.2 Modified 69-bus distribution network

The network shown in Fig. 4 is the topology of the modified 69-bus distribution network. There are totally 12 DG units connected to the test network, which can be classified into three types and the emission factors of them as well as the main grid are listed in Table 7, which is directly cited from Ref. [22]. All the DGs are modelled with constant power factor of 0.9 lagging [30]. The base case is the situation where there are no DGs connected to the network and the total active power loss and emission in the base case are 570.2 kW and 5463.3 kg, respectively.

The simulations have been implemented on a laptop computer with 2.60 GHz processors and 6.0 GB of RAM using MATLAB 8.3. For the case of the test network with 12 DGs, there are $(2^{12} - 1)$ possible permutations to compute in traditional cooperative-game-based allocation methods, which will considerably increase computational burden. Table 8 lists the time taken to allocate loss and emission reduction under traditional cooperative-game-based allocation methods (nucleolus method and Shapley value method), MCRS method and Aumann–Shapley value method (take $n = 10$ as example), and Fig. 5 plots the computational time in Aumann–Shapley value method with the variation of number of division parts of DGs' power output.

We can see from Table 8 and Fig. 5 that compared with traditional cooperative-game-based allocation methods, MCRS method and Aumann–Shapley value method can allocate loss and emission reduction in considerably less time and become more feasible when it comes to the distribution networks with a large number of integration of DGs.

Fig. 6 presents the allocation results obtained by MCRS method and Aumann–Shapley value method (take $n = 10$ as example) in the test network. As shown in Fig. 6, the emission reduction allocations of DG_1, DG_2, DG_3, and DG_4 are smaller than other DG units due to their greater emission factors, which are in accordance with the principle of fairness. The proposed MCRS method and Aumann–Shapley value method can determine the allocation of DG players according to their contributions to loss and emission reduction, provide reasonable economic signals for DISCOs, as well as overcome the combinational explosion problem in terms of large numbers of DGs integrated to distribution networks.

5 Conclusion

With the integration of DGs to distribution networks, the active power losses and the produced emission will be affected. In order to clarify the responsibility of DGs and compensate for DG units economically, the MCRS method and Aumann–Shapley value method are employed to allocate the loss and emission reduction due to the integration of DGs.

The simulation results show that the allocation results of MCRS method and Aumann–Shapley value method are close to those of traditional cooperative-game-based allocation methods and have been confirmed to have the characteristics of individual rationality, coalition rationality, and global rationality. The other feature of MCRS method and Aumann–Shapley value method is that they can effectively overcome the combinational explosion problem and dramatically reduce computational burden with regard to the integration of a large number of DGs.

6 Acknowledgments

This work is supported by the National Key R&D Program of China under Grant No. 2016YFB0901100, the China State Grid Technology Research Program "Research on the system for friendly supply-demand interaction between urban electric power

Table 7 Emission factors of DGs and the main grid

Type no.	Capacity of DG, kW	Emission factors, g/kW			
		CO_2	SO_2	NO_x	CO
Type 1	500	695	1.25	2.13	2.8
Type 2	500	477	0.024	0.015	0
Type 3	500	625	0.032	0.29	0.42
main grid		965	5.64	1.5	0.063

Table 8 Total computational time of the four methods

Method	Total time taken, s
Traditional cooperative-game-based method	≥1590.09
MCRS method	0.26
Aumann–Shapley value method (n = 10)	0.075

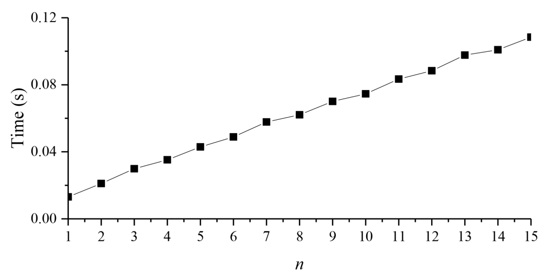

Fig. 5 *Total time taken in Aumann–Shapley value method under different n*

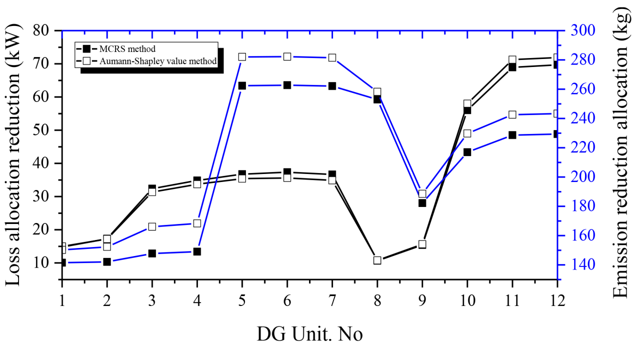

Fig. 6 *Allocation results in MCRS method and Aumann–Shapley value method*

customers and power grid" and the Fundamental Research Funds for the Central Universities of China under Grant No. 2018B01814.

7 References

[1] Kashyap, S.S., De, M.: 'Loss allocation and loss minimisation for radial distribution system including DGs', *IET Renew. Power Gener.*, 2017, **11**, (6), pp. 806–818

[2] Ghofrani-Jahromi, Z., Mahmoodzadch, Z., Ehsan, M.: 'Distribution loss allocation for radial systems including DGs', *IEEE Trans. Power Deliv.*, 2014, **29**, (1), pp. 72–80

[3] Costa, P.M., Matos, M.A.: 'Loss allocation in distribution networks with embedded generation', *IEEE Trans. Power Syst.*, 2004, **19**, (1), pp. 384–389

[4] Mutale, J., Strbac, G., Curcic, S.: 'Allocation of losses in distribution systems with embedded generation', *IEE Proc., Gener. Transm. Distrib.*, 2000, **147**, (1), pp. 7–14

[5] Bialekm, J.W.: 'Tracing the flow of electricity', *IEE Proc., Gener. Transm. Distrib.*, 1996, **143**, (4), pp. 313–320

[6] Bialekm, J.W., Kattuman, P.A.: 'Proportional sharing assumption in tracing methodology', *IEE Proc., Gener. Transm. Distrib.*, 2004, **151**, (4), pp. 526–532

[7] Conejo, A.J., Galiana, F.D., Kockar, I.: 'Z-bus loss allocation', *IEEE Trans. Power Syst.*, 2000, **16**, (1), pp. 105–109

[8] Carpaneto, E., Chicco, G., Akilimali, J.S.: 'Branch current decomposition method for loss allocation in radial distribution systems with distributed generation', *IEEE Trans. Power Syst.*, 2006, **21**, (3), pp. 1170–1179

[9] Atanasovski, M., Taleski, R.: 'Power summation method for loss allocation in radial distribution networks with DG', *IEEE Trans. Power Syst.*, 2011, **26**, (4), pp. 2491–2499

[10] Atanasovski, M., Taleski, R.: 'Energy summation method for loss allocation in radial distribution networks with DG', *IEEE Trans. Power Syst.*, 2012, **27**, (3), pp. 1433–1440

[11] Young, H.P.: '*Equity: in theory and practice*' (Princeton University Press, Princeton, USA, 1994)

[12] Tsukamoto, Y., Iyoda, I., Corp, M.E.: 'Allocation of fixed transmission cost to wheeling transactions by cooperative game theory', *IEEE Trans. Power Syst.*, 1996, **11**, (2), pp. 620–629

[13] Tan, X., Lie, T.T.: 'Application of the Shapley value on transmission cost allocation in the competitive power market environment', *IEE Proc., Gener. Transm. Distrib.*, 2002, **149**, (1), pp. 15–20

[14] Stamtsis, G.C., Erlich, I.: 'Use of cooperative game theory in power system fixed-cost allocation', *IEE Proc., Gener. Transm. Distrib.*, 2004, **151**, (3), pp. 401–406

[15] Zolezzi, J.M., Rudnick, H.: 'Transmission cost allocation by cooperative games and coalition formation', *IEEE Trans. Power Syst.*, 2002, **17**, (4), pp. 1008–1015

[16] Galiana, F.D., Phelan, M.: 'Allocation of transmission losses to bilateral contracts in a competitive environment', *IEEE Trans. Power Syst.*, 2000, **15**, (1), pp. 143–150

[17] Du, S., Zhou, X., Mo, L., *et al.*: 'A novel nucleolus-based loss allocation method in bilateral electricity markets', *IEEE Trans. Power Syst.*, 2006, **21**, (1), pp. 28–33

[18] Lima, D.A., Contreras, J., Padilha-Feltrin, A.: 'A cooperative game theory analysis for transmission loss allocation', *Electr. Power Syst. Res.*, 2008, **78**, (2), pp. 264–275

[19] Hu, Z.Y., Chen, L., Gan, D.Q., *et al.*: 'Allocation of unit start-up costs using cooperative game theory', *IEEE Trans. Power Syst.*, 2006, **21**, (2), pp. 653–662

[20] Cui, Q., Wang, X.L., Wang, X.F., *et al.*: 'Residential appliances direct load control in real-time using cooperative game', *IEEE Trans. Power Syst.*, 2016, **31**, (1), pp. 226–233

[21] Shaloudegi, K., Madinehi, N., Hosseinian, S.H., *et al.*: 'A novel policy for locational marginal price calculation in distribution systems based on loss reduction allocation using game theory', *IEEE Trans. Power Syst.*, 2012, **27**, (2), pp. 811–820

[22] Farsani, E.A., Abyaneh, H.A., Abedi, M., *et al.*: 'A novel policy for LMP calculation in distribution networks based on loss and emission reduction allocation using nucleolus theory', *IEEE Trans. Power Syst.*, 2016, **31**, (1), pp. 143–152

[23] Chen, W.Y., Fang, D., Xue, D.Z.: 'Optimal treatment cost allocation methods in pollution control', *Tsinghua Sci. Technol.*, 1999, **4**, (4), pp. 1642–1646

[24] Jiu, X.T., Xu, Q.: 'The MCRS model of profit distribution considering risk factors in apparel supply chain'. 2009 Second Int. Conf. on Intelligent Computation Technology and Automation, Changsha, Hunan, China, October 2009, vol. 3, pp. 1017–1020

[25] Molina, Y.P., Saavedra, O.R., Amaris, H.: 'Transmission network cost allocation based on circuit theory and the Aumann-Shapley method', *IEEE Trans. Power Syst.*, 2013, **28**, (4), pp. 4568–4577

[26] Junqueira, M., Costa, L.C.D., Barroso, L.A., *et al.*: 'An Aumann-Shapley approach to allocate transmission service cost among network users in electricity markets', *IEEE Trans. Power Syst.*, 2007, **22**, (4), pp. 1532–1546

[27] Molina, Y.P., Prada, R.B., Saavedra, O.R.: 'Complex losses allocation to generators and loads based on circuit theory and Aumann-Shapley method', *IEEE Trans. Power Syst.*, 2010, **25**, (4), pp. 1928–1936

[28] Bakirtzis, A.G.: 'Aumann-Shapley transmission congestion pricing', *IEEE Power Eng. Rev.*, 2000, **21**, (3), pp. 67–69

[29] Faria, E., Barroso, L.A., Kelman, R., *et al.*: 'Allocation of firm-energy rights among hydro plants: an Aumann-Shapley approach', *IEEE Trans. Power Syst.*, 2009, **24**, (2), pp. 541–551

[30] Walling, R.A., Saint, R., Dugan, R.C., *et al.*: 'Summary of distributed resources impact on power delivery systems', *IEEE Trans. Power Deliv.*, 2008, **23**, (3), pp. 1636–1644