Chapter 2: Motion Along a Straight Line

- ✓ Position, Displacement and Distance
- ✓ Velocity and Speed
- ✓ Acceleration
- ✓ Finding Displacement and Velocity from Acceleration
- ✓ Motion with Constant Acceleration
- ✓ Free Fall

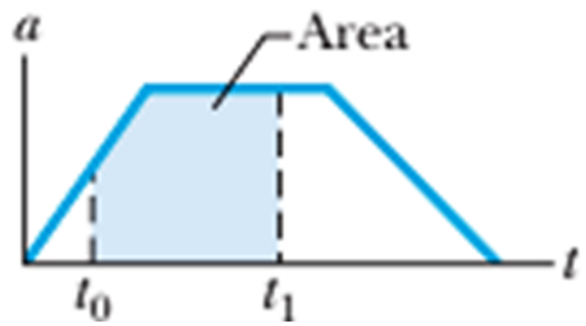
Chapter 2: Motion Along a Straight Line

Session 3:

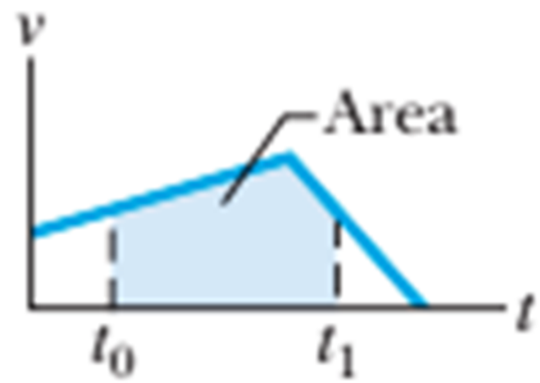
- ✓ **Finding Displacement and Velocity from Acceleration**
- ✓ **Motion with Constant Acceleration**
- ✓ **Examples**

Finding Displacement and Velocity from Acceleration

$$a = \frac{dv}{dt} \Rightarrow dv = a\,dt \Rightarrow \int_{v_i}^{v_f} dv = \int_{t_i}^{t_f} a\,dt \Rightarrow \boxed{v_f - v_i = \int_{t_i}^{t_f} a\,dt}$$



$$v = \frac{dx}{dt} \Rightarrow dx = v\,dt \Rightarrow \int_{x_i}^{x_f} dx = \int_{t_i}^{t_f} v\,dt \Rightarrow \boxed{x_f - x_i = \int_{t_i}^{t_f} v\,dt}$$



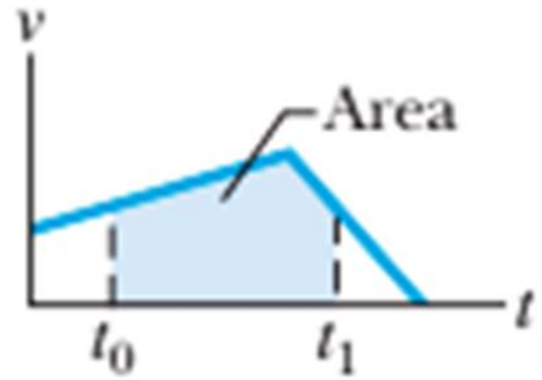
❖ **The acceleration of an object is related to the total force exerted on the object.**

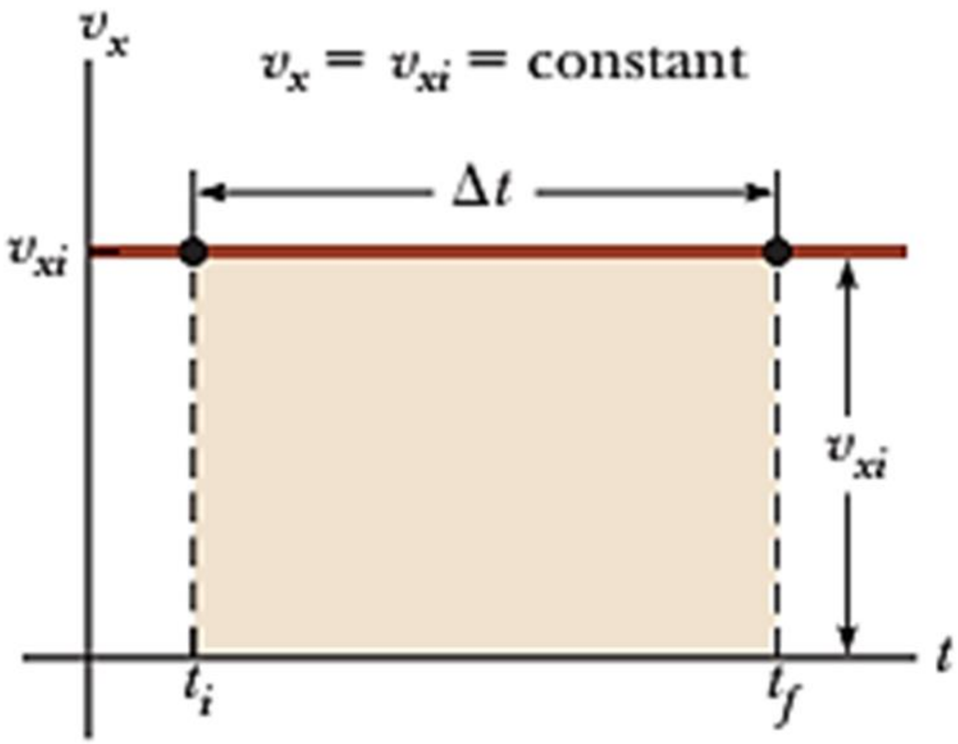
The force is proportional to the acceleration, $\mathbf{F} \propto \mathbf{a}$.

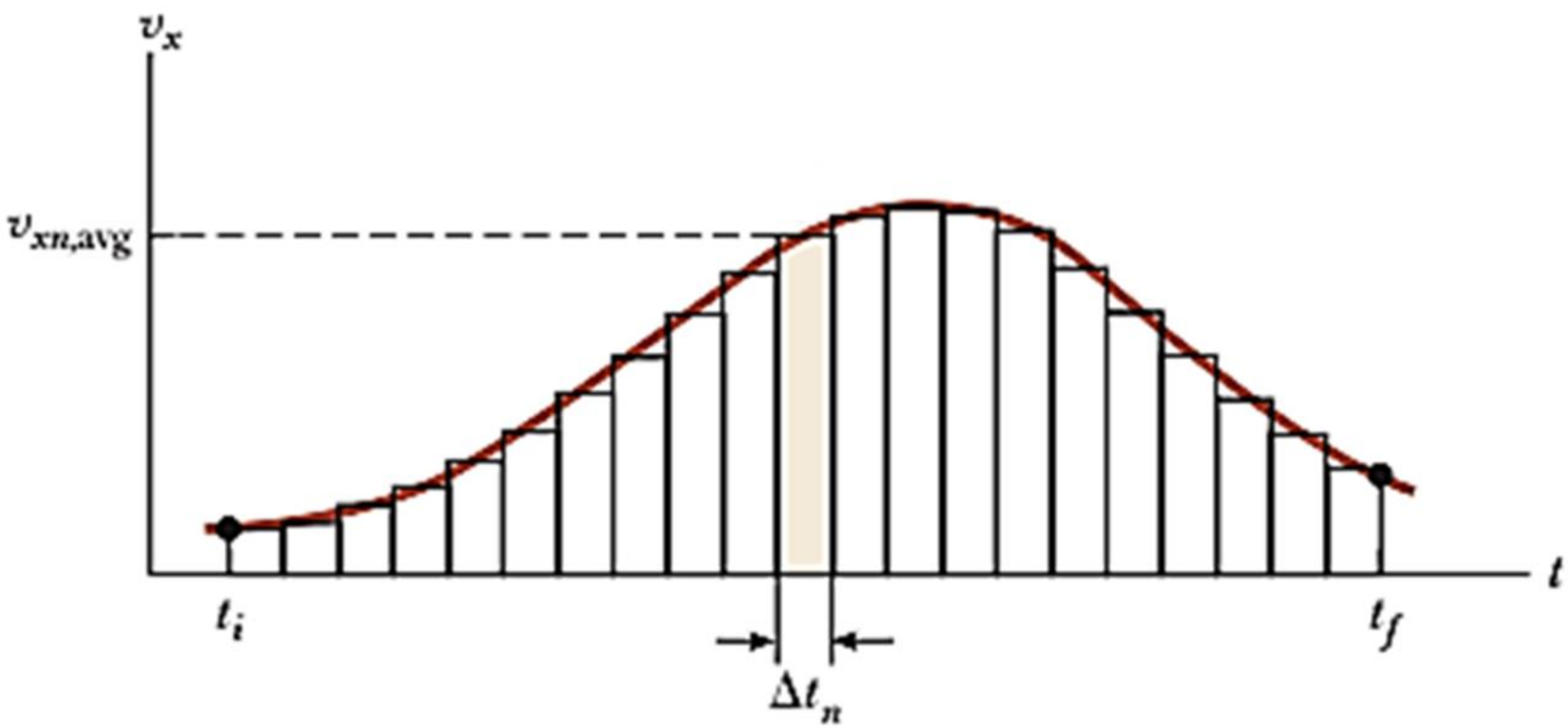
F ➡ **a** ➡ $\boldsymbol{v}$ ➡ $\boldsymbol{x}$

Integral

$$x_f - x_i = \int_{t_i}^{t_f} v\,dt$$







$$\Delta x = v_x\,\Delta t$$

$$\Delta x = \sum_n v_{xn,avg}\,\Delta t_n$$

$$\lim_{\Delta t_n \to 0} \sum_n v_{xn} \Delta t_n = \int_{t_i}^{t_f} v_x(t)\,dt$$

Ex 4: (Problem 2.70 Halliday)

Two particles move along an x axis. The position of **particle 1** is given by **x= 6t^2 + 3t+ 2** (in meters and seconds); the acceleration of **particle 2** is given by **a=-8t** (in meters per second squared and seconds) and, at **t=0**, its velocity is **20 m/s**. When the **velocities of the particles match**, what is their velocity?

$$x = 6t^2 + 3t + 2 \Rightarrow \boxed{v_1 = \frac{dx}{dt} = 12t + 3}$$

$$a = -8t \Rightarrow v_2 - v_{02} = \int_0^t a\,dt \Rightarrow v_2 - 20 = \int_0^t (-8t)\,dt = -4t^2 \Rightarrow \boxed{v_2 = 20 - 4t^2}$$

$$\int t^n dt = \frac{t^{n+1}}{n+1}, \; n \neq -1$$

$$v_1 = v_2 \Rightarrow 12t + 3 = 20 - 4t^2 \Rightarrow 4t^2 + 12t - 17 = 0 \Rightarrow \boxed{t = 1.05\ s}$$

$$\boxed{v_1 = v_2 = 15.6\ (m/s)}$$

Motion with Constant Acceleration

$$\text{if } a = \text{constant} \xrightarrow{t_i = 0, t_f = t} v - v_0 = \int_0^t a\,dt = a\int_0^t dt = at \Rightarrow \boxed{v(t) = v_0 + at}$$

$$x - x_0 = \int_0^t v\,dt = \int_0^t (v_0 + at)\,dt = \int_0^t v_0\,dt + \int_0^t at\,dt = v_0 t + \frac{1}{2}at^2 \Rightarrow \boxed{x = x_0 + v_0 t + \frac{1}{2}at^2}$$

$$v_{avg} \equiv \frac{\Delta x}{\Delta t} = \frac{x - x_0}{t} \Rightarrow \boxed{x = x_0 + v_{avg} t} \Rightarrow v_{avg} = v_0 + \frac{1}{2}at$$

$$v_{avg} = v_0 + \frac{1}{2}at = \frac{v_0}{2} + \frac{v_0}{2} + \frac{1}{2}at = \frac{v_0}{2} + \frac{1}{2}(v_0 + at) \Rightarrow \boxed{v_{avg} = \frac{v_0 + v(t)}{2}}$$

$$t = \frac{v - v_0}{a}$$

$$x = x_0 + v_0 t + \frac{1}{2}at^2 \Rightarrow \boxed{v^2 - v_0^{\,2} = 2a(x - x_0)}$$

Motion with Constant Acceleration

if $a = \text{constant}$

$$v(t) = v_0 + at$$

$$x = x_0 + v_0 t + \frac{1}{2} a t^2$$

$$x = x_0 + v_{avg} t$$

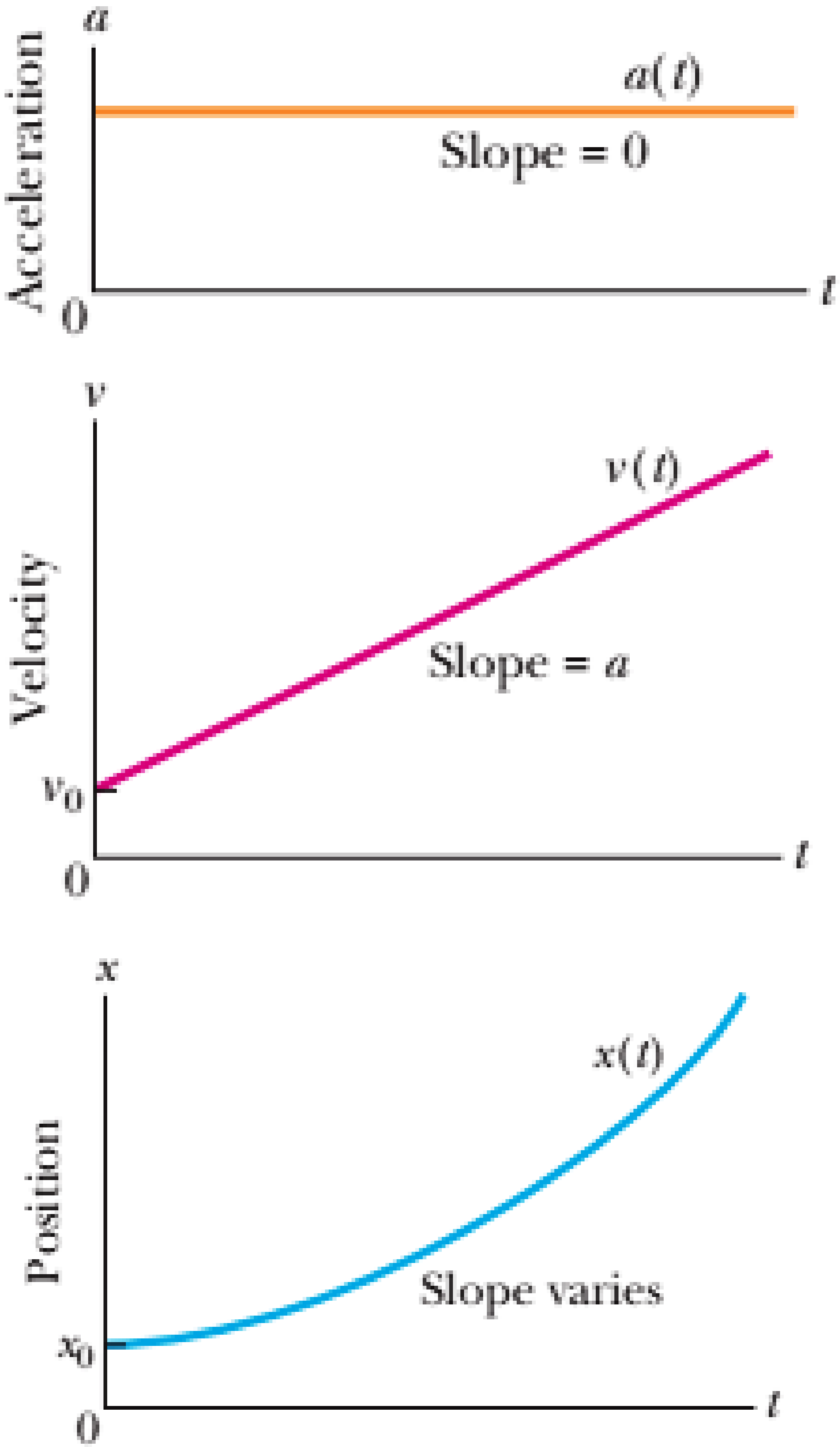
$$v_{avg} = \frac{v_0 + v(t)}{2}$$

$$v^2 - v_0^{\,2} = 2a(x - x_0)$$

if $a = 0$

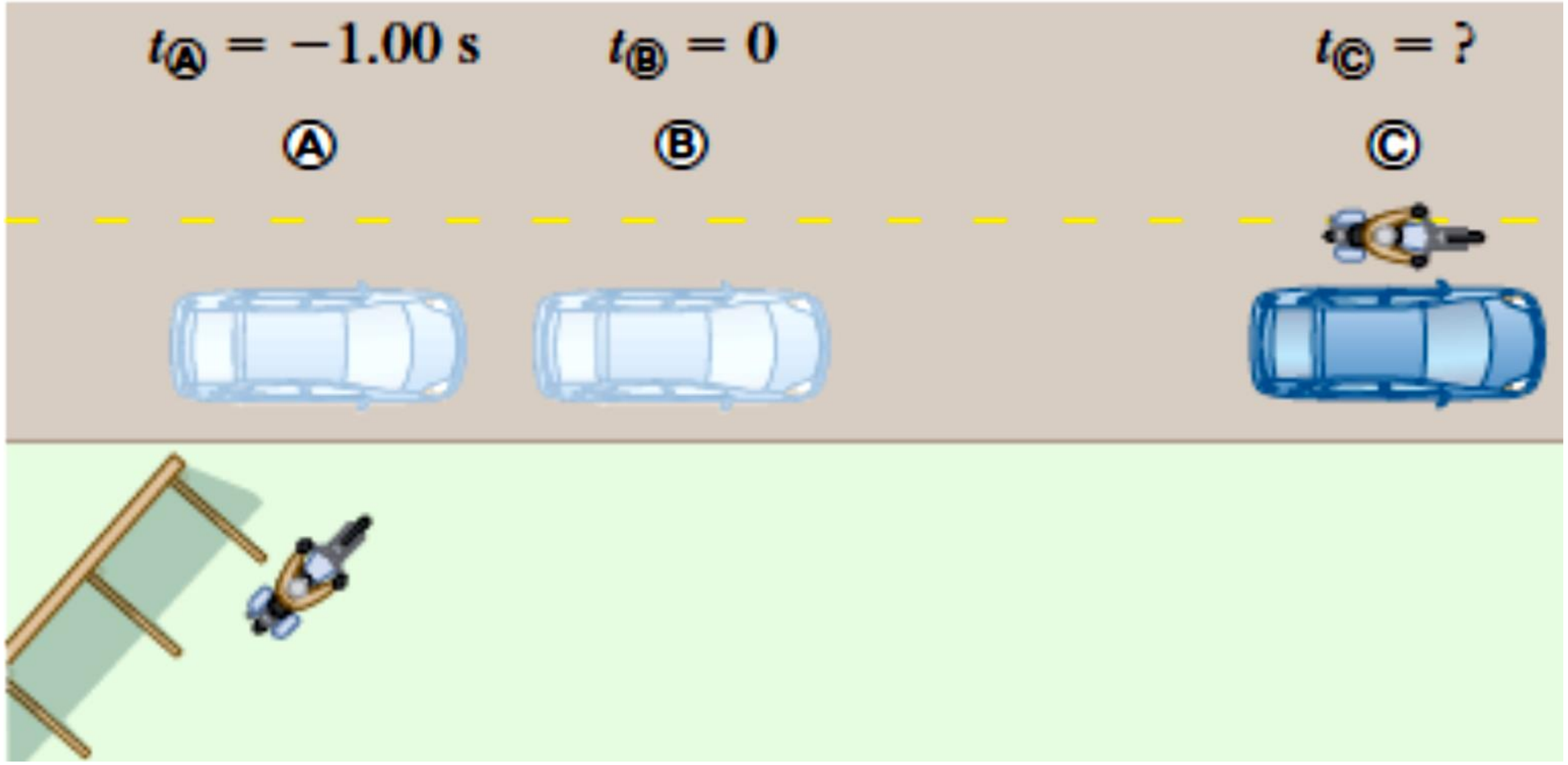
$$v(t) = v_0 = \text{constant}$$

$$x = x_0 + vt$$



Ex 5:

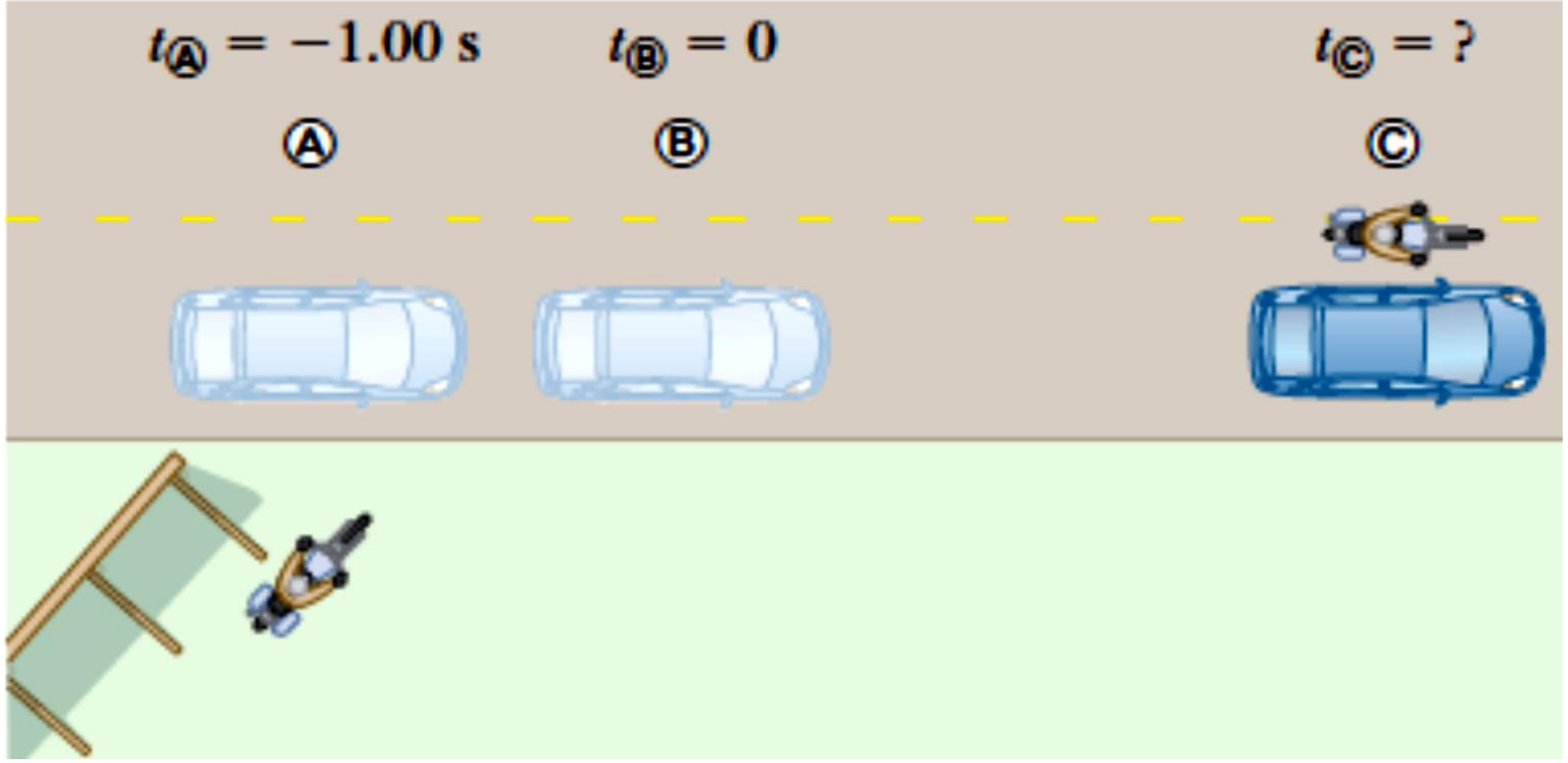
A car traveling at a constant speed of **45.0 m/s** passes a trooper on a motorcycle hidden behind a billboard. **One second** after the speeding car passes the billboard, the trooper sets out from the billboard to catch the car, accelerating at a constant rate of **3.00 m/s²**. **a)** How long does it take the trooper to overtake the car? **b)** what is the displacement of car relative to the billboard at this time?



Ex 5:

$$x_A = 0$$

$$x_B = 45\ m$$



$$\begin{cases} x_{car} = x_B + v_{car}\, t \\ x_{trooper} = \underbrace{x_A}_{0} + \underbrace{v_0}_{0} t + \frac{1}{2} a t^2 = \frac{1}{2} a t^2 \end{cases}$$

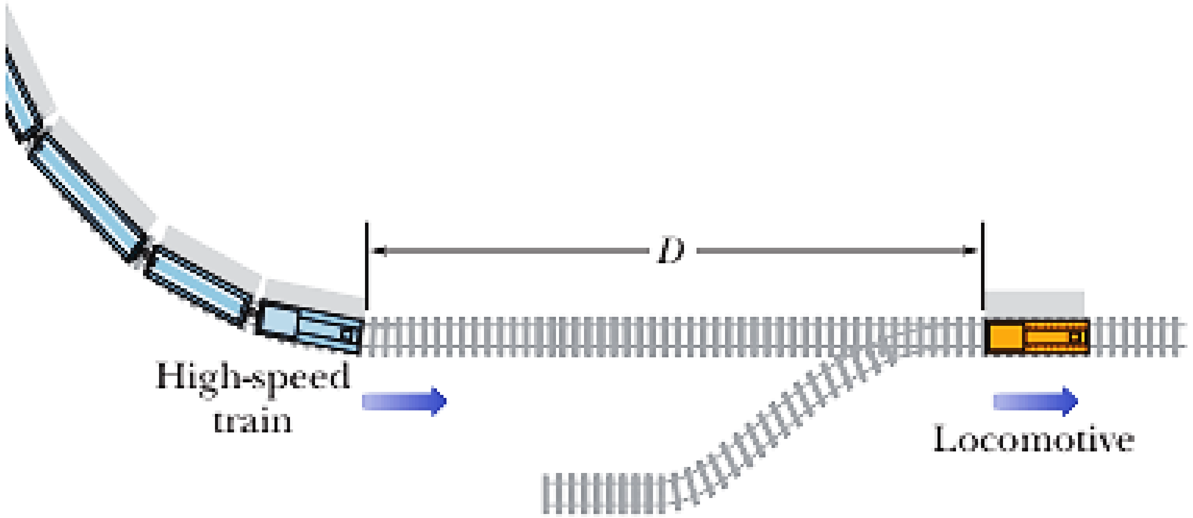
$$x_{car} = x_{trooper} \Rightarrow x_B + v_{car}\, t = \frac{1}{2} a t^2 \Rightarrow \frac{1}{2} a t^2 - v_{car}\, t - x_B = 0$$

$$\boxed{t = \frac{v_{car} \pm \sqrt{v_{car}{}^2 + 2a x_B}}{a} = \frac{45 + \sqrt{(45)^2 + 2(3)(45)}}{3} = 30.9 \approx 31\ s}$$

$$\boxed{x_{car} = 45 + 45(31) = 1440\ \ m}$$

Ex 6: (Problem 2.43 Halliday)

When a **high-speed passenger train** traveling at **161 km/h** rounds a bend, the engineer is shocked to see that a **locomotive** has improperly entered onto the track from a siding and is a distance **D=676 m** ahead. The locomotive is moving at **29.0 km/h**. The engineer of the high-speed train immediately applies the brakes. What must be the magnitude of the resulting constant deceleration if a collision is to be just avoided?



$$v^2 - v_0^2 = 2a(x - x_0) \Rightarrow v_l^2 - v_t^2 = 2a(D + v_l t)$$

$$v(t) = v_0 + at \Rightarrow t = \frac{v_l - v_t}{a} \Rightarrow v_l^2 - v_t^2 = 2a(D + v_l \frac{v_l - v_t}{a})$$

$$v_l^2 - v_t^2 = 2aD + 2v_l(v_l - v_t) \Rightarrow (v_l - v_t)(v_l + v_t) = 2aD + 2v_l(v_l - v_t)$$

$$-(v_t - v_l)^2 = 2aD \Rightarrow -(44.72 - 8.05)^2 = 2a(676) \Rightarrow \boxed{a = -0.99\ (m/s^2)}$$