

Acoustic Shooter Localization with a Minimal Number of Single-Channel Wireless Sensor Nodes

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Abstract

Acoustic shooter localization systems are being rapidly deployed in the field. However, these are standalone systems—either wearable or vehicle-mounted—that do not have networking capability even though the advantages of widely distributed sensing for locating shooters have been demonstrated before. The reason for this is that certain disadvantages of wireless network-based prototypes made them impractical for the military. The system that utilized stationary single-channel sensors required many sensor nodes, while the multi-channel wearable version needed to track the absolute self-orientation of the nodes continuously, a notoriously hard task. This paper presents an approach that overcomes the shortcomings of past approaches. Specifically, the technique requires as few as five single-channel wireless sensors to provide accurate shooter localization and projectile trajectory estimation. Caliber estimation and weapon classification are also supported. In addition, a single node alone can provide reliable miss distance and range estimates based on a single shot as long as a reasonable assumption holds. The main contribution of the work and the focus of this paper is the novel sensor fusion technique that works well with a limited number of observations. The technique is thoroughly evaluated using an extensive shot library.

Categories and Subject Descriptors

C.2.4 [Computer-Communications Networks]: Distributed Systems

General Terms

Algorithms, Experimentation, Measurement, Theory

Keywords

Localization, Sensor Networks, Acoustics

1 Introduction

The US Army has recently deployed a large number of personal wearable shooter location systems by QinetiQ [9]. The British military has been using these for a number of years now. The system has two main components; each has the size of a deck of cards. One is the sensor with a small 4-microphone array mounted on the soldier's shoulder, while the other has a small monochrome LCD display and a few buttons and acts as the user interface. Being a commercial military system, its exact specification is not readily available, but its user interface—both visual and audio—indicates the bearing to the shooter at a low resolution. It provides the result in a format familiar to the soldiers: "Shooter at four o'clock, 60 yards." This indicates a 30-degree resolution and consequently, quite limited accuracy. To understand why this relatively low performance is commonplace in today's deployed systems, we need to overview how acoustic shooter localization works.

When a typical rifle is fired, there are two acoustic phenomena that can be observed. The first is the muzzle blast that is created by the explosion inside the barrel as the bullet exits the muzzle. This sound propagates from the muzzle spherically at the speed of sound. The second is a miniature sonic boom, the ballistic shockwave, that is generated by the supersonic projectile. This is a conical wavefront with its axis being the bullet trajectory and it propagates at the speed of sound also. See Figure 1. Both the muzzle blast and the shockwave can be detected by regular microphones. A typical acoustic shooter localization system relies on one or a couple of wired microphone arrays with precisely known microphone separation. This makes it possible to estimate the Angle of Arrival (AOA) of both events by measuring the Time of Arrival (TOA) at every microphone and using the known geometry. Then a simple analytical formula containing the AOAs of the two acoustic events and the Time Difference of Arrival (TDOA) between the muzzle blast and the shockwave provides the shooter location [14].

Even with tight microphone spacing and long range shots, shooter location estimates can be very accurate. Völgyesi et al. report 1-degree average accuracy with 4" microphone separation [14]. However, the results correspond to stationary sensors. When a soldier actually wears a system, such as QinetiQ's, the orientation of the microphone array needs to be precisely known. How it is aligned with the sol-

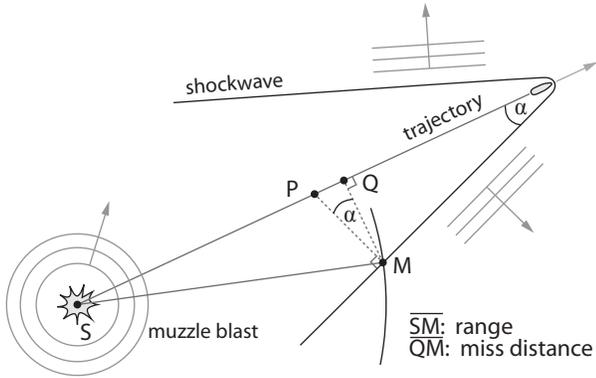


Figure 1. Acoustic phenomenon generated by a shot from location S and observed at location M .

dier’s body is one question, but the more problematic issue is that the sensor orientation can change significantly between the detection of the shockwave and the corresponding muzzle blast. For example, for a shot from 100 m that hardly misses the sensor, the difference between the two acoustic events can be 0.2 s. That is enough time for a person’s head to turn tens of degrees. While this can be measured, it does make the system more complicated and brings in additional error sources. Along with the body-sensor alignment issue, it can significantly degrade the 1-degree error in the stationary case. Note that the sensor displacement between the detection of the two events is very small for dismounted soldiers and can be disregarded.

Another interesting issue is that neither the QinetiQ system, nor any other commercially available shooter location system is networked. Clearly, the more observation one can bring to bear, the better performance one can expect. Yet, the only networked acoustic shooter location systems are research prototypes [6, 14, 3, 7]. Some of the reported results are excellent, yet none of these systems have been commercialized. Why could this be?

The PinPtr system developed in 2004 was based on a large number of single-channel sensors distributed in the environment [6]. It provided approximately 1 m 3D shooter localization accuracy for shots originating within or near the sensor network. It could eliminate echoes and resolve multiple near simultaneous shots. However, the need for a relatively large number of sensors and the fact that once they are deployed they cover a certain area and cannot be easily moved make this approach impractical for many relevant military scenarios. The wearable version relying on four-channel sensor nodes [14]—just like QinetiQ—alleviate this problem, but brings in others.

Once one needs to fuse AOA observations from distributed sensor nodes, it is not enough to track the orientation change of the microphone array between the shockwave and the muzzle blast detections, instead, one needs to know the absolute self orientation of every sensor all the time. Digital compasses are expensive and power hungry and they do not have great resolution and accuracy especially near metal objects. This makes wearable, networked, multi-channel sys-

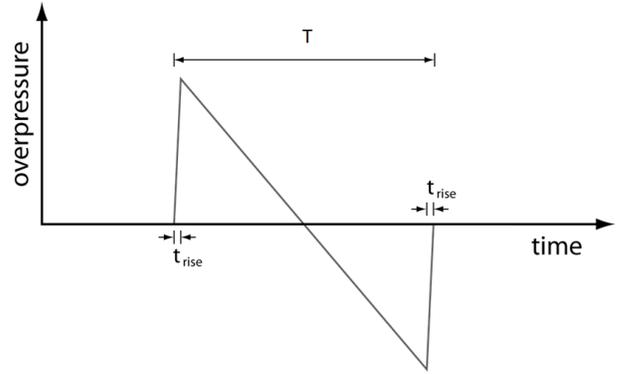


Figure 2. Typical shockwave shape

tems extremely challenging if feasible at all. In fact, all the tests in [14] were stationary.

On the other hand, the promise of networked shooter localization is great as these demonstrated results show. The question is then this: is it possible to create a networked acoustic shooter localization system from a low number of wireless, single-channel sensor nodes? It needs to be single-channel because of the self-orientation problem. It needs to work with only a handful of sensors because it must be wearable to protect the dismounted soldier wherever (s)he goes and soldiers can operate in small units. In fact, such a system should be able to work in standalone mode *to some extent* to provide utility to the warfighter who is alone or when the network is down. One of the most intriguing questions is what can be done with a single omnidirectional microphone. We have shown that making a single realistic assumption, i.e. a known weapon type, makes it possible to accurately estimate the miss distance and the range to the shooter based on a single shot using a single microphone [11]. This paper will show that utilizing a handful of wireless single-channel sensor nodes, it is possible to localize the shooter and the trajectory of the bullet with high accuracy without assuming that the weapon type is known. Moreover, it is also feasible to estimate the caliber and classify the rifle.

The rest of the paper is organized as follows. We start by reviewing related work. Then we continue by summarizing the single-sensor approach that we introduced in [11]. Section 4 is on sensor fusion and it presents several novel contributions that provide excellent results and require only a limited number of sensors. Then we evaluate the approach and present error sensitivity analysis. We conclude by describing our future plans.

2 Related Work

Acoustic gunshot detection and localization have a long history. Fansler studied the ideal muzzle blast pressure wave in the near field without contamination from echoes or propagation effects [5]. Stoughton recorded ballistic shockwaves at longer ranges using calibrated pressure transducers [12]. He measured bullet speeds and miss distances of 3–55 meters for projectiles of various calibers.

A typical shockwave signature, shown in Figure 2, has less than one-microsecond rise time and 150–500 microsec-

ond length. Whitham showed that the shockwave length T is a function of the projectile diameter d and length l , the distance b of the microphone from the trajectory of the bullet (miss distance), the Mach number M (the ratio of the speed of the projectile and that of sound), and the speed of sound c [15]:

$$T = \frac{1.82Mb^{1/4}}{c(M^2 - 1)^{3/8}} \frac{d}{l^{1/4}} \approx \frac{1.82d}{c} \left(\frac{Mb}{l}\right)^{1/4} \quad (1)$$

The shockwave length depends linearly on the diameter (caliber) and the known speed of sound. The miss distance is also significant as it can vary between 0 and 50 m beyond which the signal deteriorates too much for reliable detection. On the other hand, the reasonable range for the Mach number is between 1 and 3. (Note that it cannot be less than 1 as the bullet becomes subsonic at that point and hence, no shockwave would be generated.) The length of the projectile varies even less. For example, 7.62 mm rifle ammunition comes in 39 and 51 mm varieties. The 0.25th power makes their effect even smaller. That means that Equation 1 is dominated by the caliber and the miss distance. Consequently, Sadler et al. were able to demonstrate accurate caliber estimation using shockwave length measurements [10].

In our previous system, we demonstrated that the shockwave length can also be used for weapon classification [14]. The system estimates the shooter location and the bullet trajectory fairly accurately. The trajectory and the measured shockwave AOAs then provide the projectile speed. A simple approximation of the bullet deceleration provides a muzzle speed estimate. The caliber estimate using Equation 1 and the muzzle speed are characteristic of rifles. [14] showed good results for most weapons tested.

As we have seen, shockwave length proved to be very useful in estimating properties of the bullet and rifle. To the best of our knowledge, however, the work presented here is the first method that utilizes it for localization per se.

The standard way of locating a point source, such as the muzzle blast, is multilateration. When the microphone separation is small compared to the distance to the source, the errors can go large. Also, detection becomes harder and less accurate with increasing range. In an acoustically reverberant environment, a single non line-of-sight (LOS) measurement can render multilateration unusable. For all these reasons, the shockwave plays a very important role in shooter localization. It does not matter how far the shooter is, the projectile will pass close to the sensor if the opponent is shooting in its direction. The sharp edge of the shockwave makes its detection easy and TOA estimation precise. While the shockwave cannot provide range without a muzzle blast detection, it can be used for trajectory estimation. Danicki provides a good summary of analytical methods using shockwave detections for shooter localization [4].

The most significant issue with multilateration and analytical methods in general is the possible presence of echoes. This is where a wireless sensor system can provide unsurpassed performance. The geographically distributed sensing increases the probability that there will be a sufficient number of LOS detections. However, there is still a need for

outlier rejection. Previously, we introduced a consistency function-based sensor fusion for shooter localization [6]. The underlying idea is to search for the shooter position that is consistent with the highest number of sensor observations. Consider four muzzle blast detections with three being LOS and one non-LOS. The true shooter position will be consistent with the TOA observations of the three LOS sensors, as a simple computation using the known sensor locations will confirm, but it will be inconsistent with the non-LOS sensor. The method not only eliminates the outliers due to echoes, it can also resolve multiple simultaneous shots. The only disadvantage of this technique is the need for searching the entire space of possible shooter locations. However, we reported less than one second latency of our multiresolution search procedure. In [6] the method was generalized to use TOAs and AOAs of both shockwaves and muzzle blasts.

The original consistency function-based muzzle blast fusion approach relied on lots of observations. The revised version relaxed this requirement, but needed multiple microphones per sensor node to provide AOA estimates. In this work, we present a sensor fusion technique that relies on a consistency function also. However, it works with a limited number of single-channel sensors. This is made possible by utilizing the shockwave length measurements, in addition to shockwave and muzzle blast TOAs.

Recently, two groups proposed a novel approach to networked shooter localization independently. The method Damarla et al. and Lindgren et al. present does not require the distributed sensors to be time synchronized, but assumes a known bullet speed [3, 7]. The technique relies only on the TDOA of the shockwave and muzzle blast on the same nodes. Early results are somewhat mixed and are based on data collected on high-quality instrumentation microphones and offline processing. The disadvantage of the technique is that if a node only detects one acoustic event, either the muzzle blast or the shockwave, it cannot be included in the sensor fusion at all. Also, while it is interesting that one can do away with time synchronization, its value is questionable as time synchronization is one of the mature services available in WSNs today. Both [6] and [14] use a time synchronization approach with negligible overhead as the synchronization data is included in the messages that contain the detection data. In our current system, we plan to use GPS for sensor node localization, hence, precise absolute time will be available on each node.

3 Single Sensor Approach

The main disadvantage of a networked system is its reliance on the network. What happens if the network is down? In other words, can a single sensor using only its own detections provide useful information? As we have shown in [11], a single channel sensor can accurately estimate the miss distance of a shot and the range to the shooter based on a single shot assuming a known weapon. Here we summarize the approach presented in [11] and provide improved results.

As we have seen in the previous section, the shockwave length depends primarily on the caliber and the miss distance. This has been used to estimate the caliber successfully in the past. Notice that the opposite is also possible. Assum-

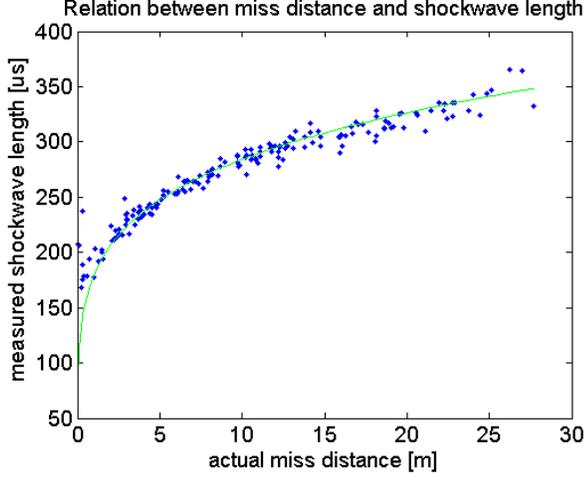


Figure 3. Miss distances vs. shockwave length.

ing a known caliber, one can estimate the miss distance. The most widely used rifle today is the AK-47, so for the rest of this section we shall assume a 7.62 mm caliber projectile. Note that the user of the system can pick any other weapon and the system will work the same using a couple of different parameters.

Let us use the shot library gathered during a field test of our previous system at Aberdeen Proving Grounds in 2006. Figure 3 shows the relationship between the measured shockwave length and the actual miss distance for 168 shockwave detections. The data represents AK-47 shots from ranges between 50 and 130 m and miss distances between 0 and 28 m. The signals were sampled at 1 MSPS. Unlike in [11], we did not downsample the signals. We have settled on our hardware platform in the meantime and it will enable 1 MSPS sampling rate as well. Figure 3 also shows the curve corresponding to equation 1 using appropriately selected AK-47-specific constraints. Miss distance estimation is carried out by fitting the observations to this function.

The resulting actual vs. estimated miss distances are

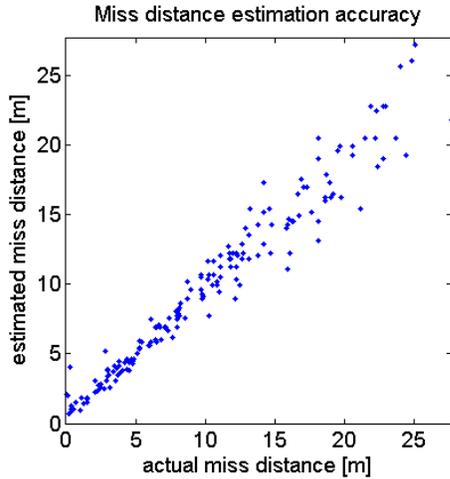


Figure 4. Actual vs. estimated miss distances.

shown in Figure 4. Note that both the ground truth and the calculations are provided in 2D. The couple of outliers with near zero miss distances correspond to shots that passed directly over the sensors. The mean absolute miss distance estimation error was 1.0 m. This is somewhat better than the results reported in [11] due to the higher sampling rate. The utility of this information to the soldier is quite high. Knowing that a bullet missed by a mere meter or 10 meters indicates whether the shooter is aiming at the given soldier or somebody else.

Moreover, making the same assumption of a known caliber and rifle, one can estimate the range of the shooter using the known muzzle velocity. For example, the muzzle speed of an AK-47 is about 700 m/s. Consider Figure 1.

Points S and M represent the locations of the shooter and the microphone detecting the acoustic events, respectively. Let us denote the range, i.e. the Euclidean distance between points S and M, with $d_{S,M}$. Then the detection time of the muzzle blast can be written as

$$t_{mb} = t_{shot} + \frac{d_{M,S}}{c} \quad (2)$$

where t_{shot} is the time of shot, and c is the speed of sound which is assumed to be known.

Since the shockwave originates at the bullet traveling along its trajectory, we need to consider the time traveled by the bullet from the muzzle to a point P on the trajectory, as well as the time traveled by the wavefront from point P to the sensor. For simplicity, let us assume for now that the bullet travels at a known constant speed v . The shockwave detection time is then

$$t_{sw} = t_{shot} + \frac{d_{S,P}}{v} + \frac{d_{P,M}}{c} \quad (3)$$

Since we assume a constant bullet speed v , the shockwave front has a conical shape, such that the angle between the axis and the surface of the cone is

$$\alpha = \sin^{-1} \frac{c}{v} \quad (4)$$

Then the following closed-form formula provides the range to the shooter, as was shown in [11]:

$$d_{S,M} = \frac{1}{2(c^4 - v^4)} (A - 2\sqrt{B}) \quad (5)$$

where A and B are defined as

$$A = -2v^3 d_{Q,M} \sqrt{v^2 + c^2} - 2(t_{mb} - t_{sw})c^3 v^2 + 2c^2 d_{Q,M} v \sqrt{v^2 + c^2} - 2(t_{mb} - t_{sw})c v^4$$

$$B = -2c^4 v^4 d_{Q,M}^2 + 2(t_{mb} - t_{sw})^2 c^6 v^4 + (t_{mb} - t_{sw})^2 c^4 v^6 - 2c^7 d_{Q,M} (t_{mb} - t_{sw}) v \sqrt{v^2 + c^2} + c^8 (t_{mb} - t_{sw})^2 v^2 + 2c^8 d_{Q,M}^2 + 2v^5 d_{Q,M} \sqrt{v^2 + c^2} (t_{mb} - t_{sw}) c^3$$

If a node successfully measures both the shockwave length and the TDOA between the shockwave and the muz-

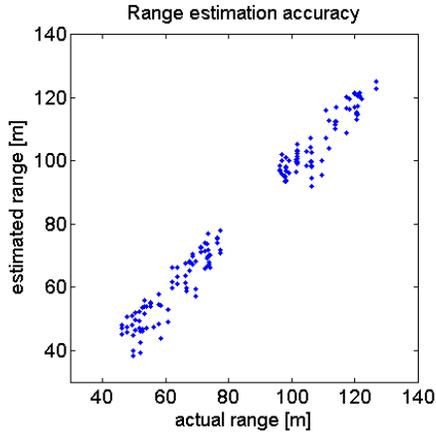


Figure 5. Actual vs. estimated ranges.

zle blast, the formula is easy to compute, since all the parameters are known or measured. For the same 168 data points we used for miss distance estimation, we computed the range, assuming a projectile speed of 620 m/s and sound propagation speed of 340 m/s. Note that we use a bullet speed that is lower than the muzzle speed because bullet decelerates after leaving the barrel. The 620 m/s average speed corresponds to approximately 100 m range for an AK-47. In fact, a simple iterative procedure could adjust the projectile speed according to the current range estimate and compute a new range accordingly. In practice, however, this does not improve the accuracy significantly. The 168 range estimates versus the ground truth are shown in Figure 5.

The mean absolute range estimation error was 3.5 m. Expressing the errors as a percentage of the range gives an average absolute range estimation error of 4.7%.

4 Networked Operation

While we have not build the hardware for this system yet, there are a few architectural assumptions we need to discuss since they have an impact on the approach we cover in this section. The soldiers are assumed to have a smartphone to perform their duties and we will utilize it for the shooter localization application as well. This phone will act as a user interface and it will perform the sensor fusion also. We will design and build a sensor node that will perform the detection and signal processing tasks. It will communicate with the phone either through USB or via Bluetooth. The wireless network will be either Wifi or GSM.

The heart of our approach is the sensor fusion technique. It is a centralized algorithm, so the individual sensor observations need to be gathered. It does not necessarily have to be at one central place, instead, it is better if the nodes broadcast their data to every other node in the vicinity. That way each soldier can run the sensor fusion and get the most accurate shooter localization as fast as possible. Since there are only a handful of sensors in any one area and the amount of data is small, broadcast is a perfectly feasible approach.

Furthermore, the sensor locations need to be known. As the system is assumed to be mobile, the accuracy needs to be high and the latency must be minimal, none of the traditional WSN localization methods would suffice. Hence, we will

utilize GPS for this purpose. We evaluate how GPS-error effects shooter localization accuracy in Section 5.

In order to fuse the distributed observations, they need to share a common time base. There are various feasible approaches to time synchronization in WSNs. We can use a continuous time synchronization service such as FTSP [8] or apply the post-facto approach utilized in previous systems [14]. However, we have GPS at our disposal, so we will have very accurate absolute time available to every sensor node with minimal overhead.

Note that there are potential problems with relying on the GPS in a phone. First, the localization accuracy is typically not good with general purpose devices because they rely on low-cost GPS chips. Second, even if it possible to access the absolute time information from the GPS, it is typically done through numerous OS layers. Hence, the timing accuracy can be subpar. Therefore, we will include a high-precision GPS chip in our initial sensor node design.

Further discussion of the networking and middleware aspects of the system is beyond the scope of this paper. Our focus here is primarily the sensor fusion.

4.1 Sensor Fusion Overview

The sensor fusion presented below is a complex multi-step procedure. Here we outline the technique and then subsequent subsections present detailed descriptions of the individual steps.

It is very important to note that the networked fusion approach does not assume a known weapon or caliber. While it was necessary to make this assumption, so that both the caliber and the muzzle velocity were known for the single sensor approach, the networked sensor fusion works without such restriction and is generally applicable to all supersonic rifles.

All or a subset of the following parameters are available for the sensor fusion from each sensor: shockwave length, shockwave TOA and muzzle blast TOA. The sensor locations are also known and all TOAs use the same time base. Note that the individual miss distance and range estimates are not used since they are based on the known weapon type assumption. The sensor fusion algorithm carries out the following steps:

- **Initial shooter localization.** Traditional multilateration based on the muzzle blast TOAs is applied. The main goal of this step is to reduce the trajectory search space for the next step. If there are not enough observations, this step is skipped.
- **Projectile trajectory estimation.** A search is performed to find the minimum of an error function that is defined on the space of possible trajectories. It has two components. The first is the least squares error of the miss distances computed using Equation 1 and the measured shockwave lengths. The second is based on the variance of the shockwave angles computed using pairwise shockwave TDOAs. Both of these incorporate outlier rejection also. The value of the error function is the product of the two components. The result of the search is the best trajectory that matches the shockwave observations.

- **Final shooter localization.** The side effect of the trajectory search is that a good estimate of the projectile speed is available because the angle of the shock wavefront and the trajectory provides it. The trajectory and the sensor locations provide the miss distances. Using the same approach as for single sensors, the range from each sensor can be estimated without the known weapon assumption. Basic trilateration then provides the shooter location. Since there are typically more than three sensors and the trajectory is also available, the system is overdetermined, so a simple optimization method is applied.
- **Caliber estimation and weapon classification.** Since the projectile speed is known at this point, the two unknowns that remain in Equation 1 are the caliber and the bullet length. As there are only a handful of discrete choices for these, the caliber and the projectile length can be determined. Since the range to the shooter is also available from the previous step, weapon classification can be performed using the technique we introduced in [14].

4.2 Initial shooter localization

The first step of the sensor fusion algorithm is computing an approximate shooter position. Finding the shooter location given the muzzle blast TOAs and assuming a known speed of sound is an instance of the classic multilateration problem. Pairwise muzzle blast TDOAs constrain the shooter location to hyperbolas (or hyperboloid surfaces in three dimensions), the intersection of which is the shooter location itself. There is extensive literature on multilateration, with methods ranging from direct linearization [2] to nonlinear optimization approaches. Unfortunately, such techniques perform poorly when unbounded errors may be present in the inputs. Non-line of sight conditions and echoes often result in TOAs that are not consistent with the geometric configuration of the shooter and the microphones. Also, diffraction may alter the shockwave signature to a degree such that it is classified as a muzzle blast. The corresponding erroneous TOAs manifest as large outliers, and cause the overall error distribution to be non-Gaussian, violating the assumptions that are required for traditional multilateration techniques to work correctly.

Clearly, this problem mandates a multilateration approach that is robust against unbounded non-Gaussian TOA errors. We use a generalization of the Hough transformation, a well known image processing algorithm to find imperfect instances of objects (i.e. lines) using a voting procedure carried out in a parameter space. The underlying idea is similar to the sensor fusion in [6], but the details differ significantly.

For a given location S and for each microphone, we compute when a shot must have been fired from S to give the measured TOA. We expect that the shot times will not align for any location other than the true shooter position. Since errors are assumed to be present in the TOA measurements, the computed shot times for the true shooter location will not be identical, but will exhibit some scattering, as well as some (possibly large) outliers.

Our algorithm works as follows. We subdivide the search

space (the area where the shooter can possibly be, based on the sensing range of the microphones) into a rectangular grid of a predefined resolution. For each point S on the grid, we compute the shot times that correspond to the muzzle blast TOAs supposing that the shooter is at S , effectively mapping the TOA measurements to a parameter space of shot times. For a given point S , clustering of points in the parameter space indicate that a shot is likely to have been fired from S . The search algorithm, therefore, is looking for the most *consistent* cluster, and the corresponding grid point is returned as the shooter position estimate.

We use the variance of the shot times within a cluster as the metric of consistency. Intuitively, the scattering of shot times within a consistent cluster should be no more than the measurement error of the muzzle blast TOAs plus the error introduced by the discretization of the search space. Therefore, we can empirically find a threshold for the variance of shot times above which the clusters are rejected as invalid. Also, the cardinality of the cluster must be at least 3 (4 in three dimensions), since this is the minimum number of TOAs required to unambiguously find a point source. Since non-Gaussian TOA errors are assumed, we find it reasonable to increase the minimum required cardinality of the clusters from the theoretical minimum to $N > 3$. This will decrease the probability of finding false consistent clusters, thereby improving the robustness of the algorithm.

An inherent property of multilateration is that the accuracy of the result is highly dependent on the geometry. For instance, when the microphones are located in close proximity of each other, but the shooter is relatively far from the sensors, which is the most common case in this problem domain, the direction estimate to the source will be accurate, but the range can exhibit relatively large errors. See Section 4.4 and Figure 8a. Since the initial shooter position estimate will be used exclusively to reduce the search space for trajectory estimation, we are hardly affected by this geometric dilution of precision of multilateration. Since we are expecting that the bullet passes over the sensor field or close by (otherwise no shockwave signatures are picked up by the microphones), the direction component of the shooter position estimate provides the information we need.

4.3 Projectile trajectory estimation

Trajectory estimation is a search procedure in the space of possible trajectories for the minimum of an error function. The error function has two components. The first one is based on the measured shockwave lengths.

Let us rearrange Whitham's equation (Equation 1):

$$b \approx kT^4 \quad (6)$$

That is, the miss distance is proportional to the fourth power of the shockwave length, with a scaling factor of k . k is bounded and the bounds can be obtained by substituting the minimum and maximum parameter values (projectile speed, length and caliber, speed of sound) into Equation 1. We need to define the error function to jointly optimize the trajectory angle and k . Formally, the problem formulation to find the trajectory estimate is the following. Given a set of

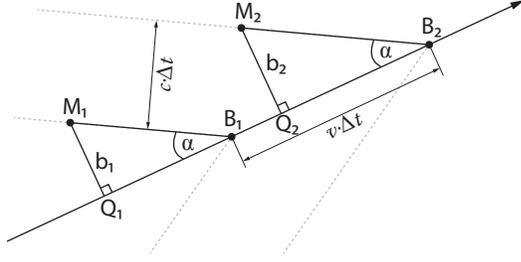


Figure 6. Shockwave angle calculation using TDOA.

microphone positions M_i and the corresponding shockwave length measurements T_i , find the trajectory L that minimizes

$$\sum w_i(b_i - kT_i^4)^2, \quad i = 1..n \quad (7)$$

where n is the number of microphones that reported shockwave detections, b_i is the distance of M_i from the trajectory, and $w_i \in \{0, 1\}$ are binary weights assigned such that only the N smallest error values are considered.

While the above technique is sufficient to find the trajectory, the second component of the error function relies on the shockwave TOAs that provide additional information to refine the trajectory estimate, as well as to compute the speed of the bullet.

Assuming a given trajectory, the shockwave TDOA of a pair of microphones can be used to compute the angle of the conical shockwave front (precisely the angle of the axis and the surface of the cone). This angle α is related to the bullet velocity as described in Equation 4.

Consider Figure 6. Microphones M_1 and M_2 with miss distances b_1 and b_2 detect the shockwave at times t_{sw1} and t_{sw2} , respectively. For the sake of simplicity, let us assume that the TDOA, denoted by $\Delta t = t_{sw2} - t_{sw1}$, is positive (if not, we can change the labeling of the two microphones to make Δt positive). It is easy to see that at time t_{sw1} (i.e. when the shockwave front hits microphone M_1) the bullet is at position B_1 , and similarly, at t_{sw2} , the bullet is at point B_2 . Therefore, the bullet, traveling at speed v , covers the $|B_1B_2|$ distance in Δt time. Q_1 and Q_2 denoting the orthogonal projections of the respective microphone positions to the trajectory, we can write the following equality:

$$|Q_1B_1| + v\Delta t = |Q_1Q_2| + |Q_2B_2| \quad (8)$$

Here, Δt and $|Q_1Q_2|$ are known, while $|Q_1B_1|$ and $|Q_2B_2|$ can be expressed with the shockwave angle and miss distances b_1 and b_2 :

$$|Q_1B_1| = \frac{b_1}{\tan\alpha} \quad |Q_2B_2| = \frac{b_2}{\tan\alpha} \quad (9)$$

From Equation 4, we can express v in terms of $\tan\alpha$, as well. Substituting back to Equation 10, we get the following equation that is quadratic in $\tan\alpha$ and has a unique solution:

$$\frac{b_1}{\tan\alpha} + c\sqrt{1 + \frac{1}{\tan^2\alpha}}\Delta t = |Q_1Q_2| + \frac{b_2}{\tan\alpha} \quad (10)$$

The shockwave angle computed this way may contain errors if the speed of sound, or the assumed trajectory are incorrect, or if the TDOA is noisy. However, we know that the bullet speed must have been higher than the speed of sound (since there is no acoustic shockwave for subsonic projectiles), and lower than v_{max} , the maximum bullet speed of the weapons being considered. Therefore, the corresponding shockwave angle must be less than $\pi/2$ but higher than $\sin^{-1}(c/v_{max})$. These bounds can be used to validate the computed results and allow for discarding a subset of the outliers.

The second component of the error function is then the variance of the shockwave angles computed for all pairs of microphones. The final error function is the product of these two components. Formally, it is defined for a trajectory L and a scaling factor k as

$$\sum w_i(b_i - kT_i^4)^2 \text{var}(\{\alpha_{i,j}\}), \quad i, j = 1..n \quad (11)$$

where n is the number of microphones that reported shockwave detections, b_i is the distance of M_i from the trajectory, $\alpha_{i,j}$ is the shockwave angle computed from the shockwave TDOA of microphones M_i and M_j , and $w_i \in \{0, 1\}$ are binary weights assigned such that only the N smallest error values are considered.

Notice that once the best fit trajectory is found, the computed shockwave angles give the bullet velocity (see Equation 4).

The search procedure to minimize the error function is carried out as follows. If the approximate shooter position is known (this is the output of the initial shooter localization step), the space of possible trajectories is defined by a single parameter, the slope (angle) of the line that passes through the known shooter location. If an initial shooter position estimate is not available, it is not sufficient to search for just the trajectory angle, but also for a point on the trajectory, since a point and a slope are required to define a line (in two dimensions). This way, the coordinates of the point would increase the dimension of the search space by two.

Using the following observation, however, it is possible to express all geometrically feasible trajectories, i.e. those that either cross the sensor field, or pass close to the edge of the sensor field with just two parameters. Consider Figure 7. The maximum shockwave sensing distance b_{max} as radius is used to construct circles centered at the microphone positions. It is easy to notice that a trajectory that intersects at least N of these circles must also intersect a large circle that encompasses all small ones. (The center of the large circle can be chosen to be the center of mass of the microphone positions reporting a detection.) The geometrically feasible trajectories can be expressed by an angle variable ϕ , running from 0 to 2π that is used to define a pivot point on the circumference of the large circle, and the slope angle. Notice that for each pivot point P_i , it is sufficient to sweep the angles right of the diameter of the large circle, since the left side will be swept from the pivot point on the other end of the diagonal.

The search for the trajectory is carried out as follows. If no shooter position estimate is available, we generate a set of

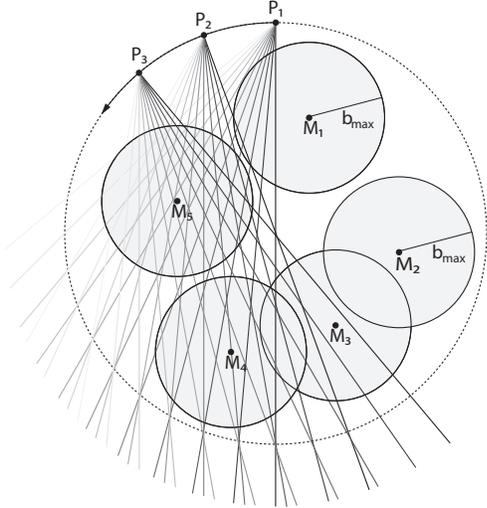


Figure 7. Trajectory search procedure.

pivot points on the circumference of a large circle surrounding the sensor field, constructed as described above. For all of these pivot points, we sweep half of the circle with the slope angle in discrete steps, minimizing the error function given in Formula 11 for all trajectory candidates defined by the pivot points and slope angles. The trajectory candidate for which the formula gives the minimum value is returned. As a side effect, k in Equation 6 and the bullet speed are also determined.

If a shooter location estimate is available, the search becomes simpler. Since we know that the trajectory passes through the shooter position, we only need to search for the slope angle, but not for the pivot points. The search space for the slope angle is constrained: we only have to consider those angles that define trajectory estimates which either cross the sensor field or pass close by (not further than the shockwave sensing range of the microphones). Similarly to the previous case, the search returns the trajectory candidate for which Formula 11 yielded the minimum value.

4.4 Final shooter localization

At this point, we may or may not have an approximate shooter position, depending on whether or not the first step of the sensor fusion succeeded, but we do have a good trajectory estimate. The purpose of this final position refinement step is to find an accurate shooter position estimate. Here we use range estimates to carry out constrained trilateration along the trajectory, which does not suffer from the radial inaccuracy of the multilateration applied in the first step.

Once the trajectory estimate and the bullet speed are available, we refine the shooter position estimate based on this additional information. As described in Section 3, the distance of the shooter from the microphone can be estimated if the miss distance, the bullet speed and the time difference of the shockwave and the muzzle blast are known. That is, we can compute individual microphone-shooter distance estimates using the microphones' distances from the trajectory estimate, the bullet speed estimate and the microphones'

shockwave and muzzle blast detection times. At this point, these are all known without assuming a known weapon type.

Given a set of microphone-shooter distances, the shooter position can be trivially found by trilateration. But since there is a trajectory estimate available already, it is sufficient to search along the trajectory for the position that is most consistent with the range estimates. The consistency function is the mean square error of the range estimates for the position that is being evaluated. Since the range estimates may contain large outliers, we discard all but the N range estimates that have the smallest absolute errors.

Formally, the problem formulation to find the shooter position on the trajectory estimate is the following. Given a set of microphone positions M_i , the corresponding range estimates r_i computed using Equation 5, find the point S on the trajectory that minimizes

$$\sum w_i (r_i - d_{S, M_i})^2, \quad i = 1..n \quad (12)$$

where n is the number of microphones for which range estimates are available, d_{S, M_i} is the Euclidean distance of microphone M_i from the position being evaluated, and $w_i \in \{0, 1\}$ are binary weights assigned such that only the N smallest absolute error values are considered.

It is interesting to consider the nature of errors of multilateration vs. trilateration, if the set of sensors are relatively close to each other, while the shooter is far away. Multilateration is based on the intersection of hyperbolas defined by pairwise TDOAs of the muzzle blast observations. Small errors can cause large displacement in the intersection of the arms at shallow angles as shown in Figure 8a causing a radial error pattern. Trilateration is based on the intersection of circles defined by the individual range estimates. In the same setup, the angles are shallow again, but the error pattern they create are orthogonal to the direction of the shooter as shown in Figure 8b. Using multilateration to help find the initial shooter position estimate to cut the search space for trajectory localization and then constraining the shooter estimate to the trajectory while applying trilateration consequently minimizes the effects of the geometric dilution of precision. This is illustrated in Figure 8c.

4.5 Caliber and weapon classification

We have two types of information available to carry out weapon classification. First, as a side effect of the trajectory estimation algorithm, the scaling factor k has been computed that tells us how the miss distances are related to the fourth power of the shockwave lengths. The value of k carries information about the projectile, and, together with the bullet speed, allows for projectile classification (caliber and length). Second, the bullet speed at the muzzle (also referred to as the *muzzle velocity*), is characteristic of the particular weapon-ammunition pair. Since an estimate of the bullet speed over the sensor field has been computed in the trajectory estimation step, the shooter position has been estimated, and the bullet-specific deceleration is known after the projectile is identified, the muzzle speed can be computed. From here, weapon classification becomes possible.

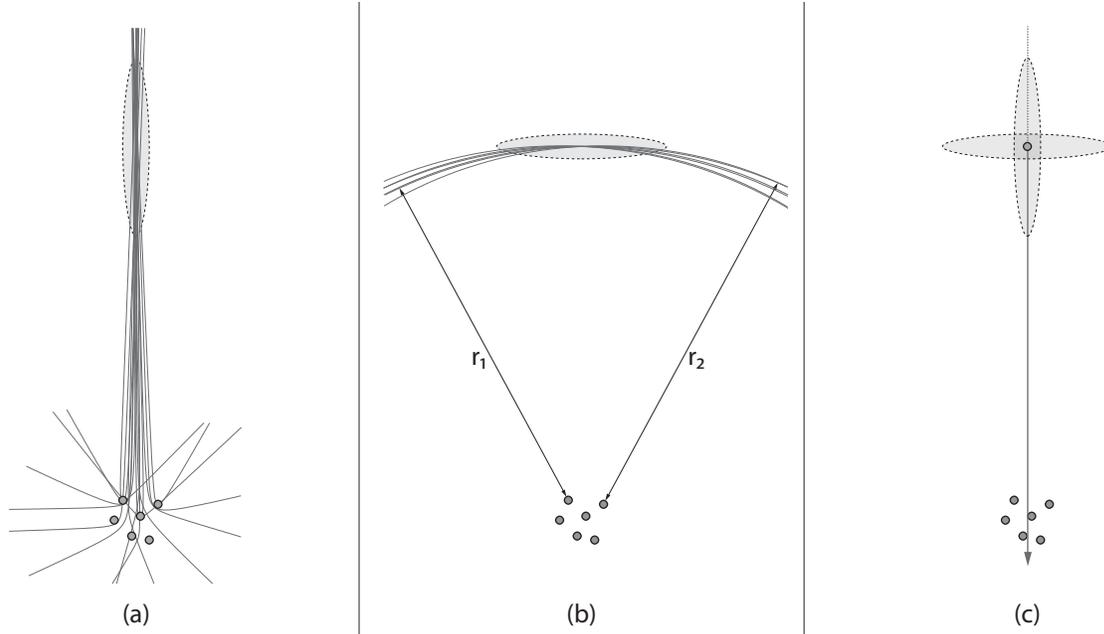


Figure 8. The nature of multilateration (a) and trilateration (b) errors and the result of their combined application (c).

4.5.1 Projectile classification

Beside the trajectory itself, the trajectory estimation step optimizes the scaling factor k that is used in the formula that relates the shockwave length T and the miss distance b . Rearranging Equation 6 which itself is derived from Whitham's formula (Equation 1):

$$b \approx kT^4 \quad (13)$$

In particular, k can be expanded as

$$k = \frac{lc^5}{1.82^4 d^4 v} \quad (14)$$

where l is the length and d is the caliber of the bullet, and c and v are the sound and bullet speeds, respectively. Here, the projectile specific quantity is d^4/l , which we call the bullet coefficient C_{bullet} , computed as follows:

$$C_{bullet} = \frac{c^5}{1.82^4 vk} \quad (15)$$

Since the bullet speed v is also computed in the trajectory estimation step, we have all the information available to compute the bullet coefficient, which makes it feasible to differentiate between possible projectile types.

4.5.2 Weapon classification

Weapon classification involves identifying the type of ammunition used and computing the muzzle speed of the bullet, as the combination of these values is often uniquely characteristic of the particular weapon used.

Calculation of the muzzle speed is carried out as follows. First, having identified the projectile as described above, its deceleration is looked up in a database (it can be computed from the ballistic coefficient (BC) typically specified by the manufacturer). Given the deceleration a (a negative number), the average bullet speed over the sensor field v_{bullet} and the

estimated range to the shooter position, the muzzle velocity can be approximated by:

$$v_{muzzle} = \sqrt{v_{bullet}^2 - 2ar} \quad (16)$$

Note that the above caliber estimation and weapon classification methodology is very similar to the technique presented in [14]. The novelty lies in the fact that our system uses single-channel sensors as opposed to multi-channel ones that provide the AOA of the acoustic events.

5 Evaluation

The evaluation of our sensor fusion approach has been carried out using the shot library created during the evaluation of our multi-channel system in 2006 [14]. It contains 196 shots taken from ranges between 50 and 300 meters using six different rifles of three different calibers. They are the AK-47 (7.62 mm projectile), M240 (7.62 mm), M16 (5.56 mm), M4 (5.56 mm) and M249 (5.56 mm) and M107 (12.7 mm or .50 caliber). The miss distances vary between 0 and 28 meters. The measurements were taken by 10 multi-channel sensors distributed in a 30x30 m area. The sensor locations were known with a few-centimeter error. For our purposes, we picked one of the microphones from each node. Furthermore, in the data set, muzzle blast detections become relatively rare at a range of 200 m and beyond. Therefore, we only utilize the 108 shots that are below this limit. The ranges of these shots from the different sensors vary between 50 and 130 m. The latter measure is the sum of the range to the 100 m firing line and the 30 m depth of the sensor field.

The sensor fusion was implemented in Matlab and it ran on a regular desktop PC. When the first step of the sensor fusion finds an approximate shooter position, the fusion completes in 2 sec. However, when no such estimate exists, the extensive trajectory search causes the fusion to slow down.

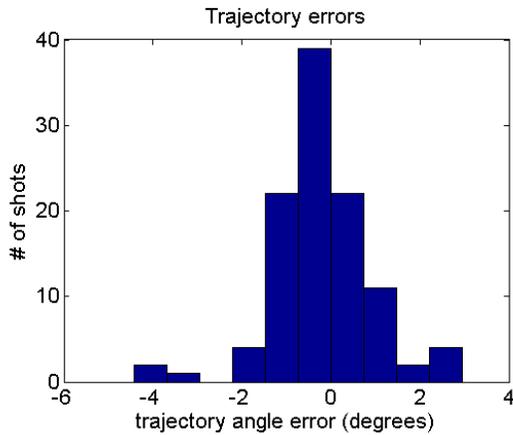


Figure 9. Histogram of trajectory angle errors.

It completes in about 20 sec. This will need to be optimized significantly to achieve a more reasonable latency.

All 108 shots were evaluated. The cluster size for the trajectory error function was set at 5. That is, we were looking for five consistent sensor observations. We set a threshold for the error function: if the minimum exceeded 4, we decided that the observations are not consistent enough to locate the trajectory. Trajectory localization were successful for 107 of 108 shots. The mean of the trajectory angle error, that is, the angle between the real trajectory and the estimated one, was -0.1° . The standard deviation was 1.3° . The mean absolute error was 0.8° . This is about the same as was reported in [14] using the same dataset. Figure 9 shows the histogram of trajectory angle errors.

Shooter position estimation needs a trajectory estimate and at least one muzzle blast detection. However, if that single muzzle blast detection is erroneous, the shooter location can exhibit huge errors. Therefore, we set the minimum number of muzzle blast detections at three since two detections would not be enough to identify one of them as an outlier. Also, we set the threshold in the constrained trilateration procedure for the least squares error at 150. If it was exceeded, we concluded that the muzzle blast detections are

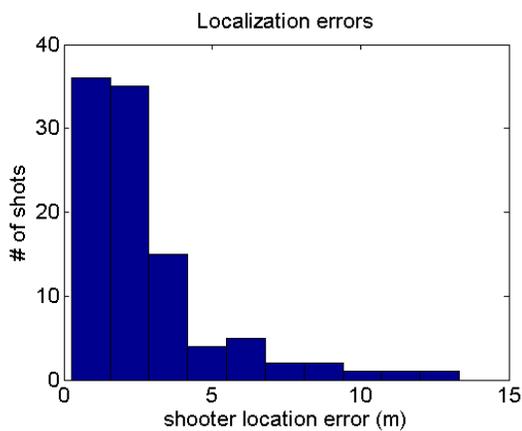


Figure 10. Histogram of shooter position errors.

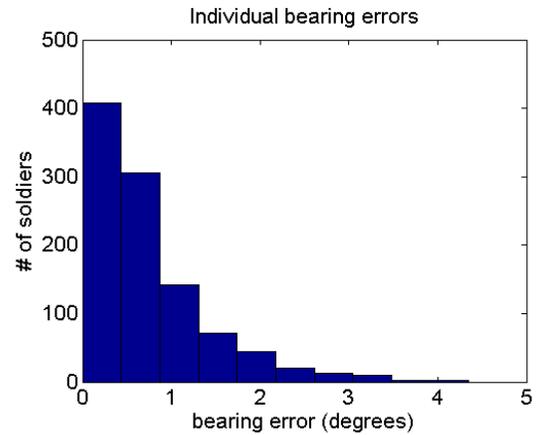


Figure 11. Errors in bearing relative to each sensor.

not consistent enough for shooter localization. In this case, only the trajectory is reported. Out of the 107 shots with localized trajectories, the sensor fusion located 104 shots.

The shooter position error is measured as the distance between the ground truth and the estimated position. As such, it has only positive values. The mean position error was 2.96 m. This is better than the approximately 5.45 m mean error achieved with the multi-channel system with the same shots, that is, the shots from the 50 and 100 m firing lines [14]. Figure 10 shows the histogram of shooter position errors.

Traditional non-networked countersniper systems report their result as a bearing and range pair. For a networked system, such as ours, there is no such thing as bearing and range because there is no single reference point. However, the results will be reported to the individual soldiers on their smartphones. As such, it makes sense to translate the shooter position estimate to a bearing/range pair relative to each individual sensor's location. Figures 11 and 12 show the histograms of the corresponding absolute bearing error and the range error, respectively.

The mean of the range errors is 0.2 m, while the standard deviation is 3.3 m. The mean absolute range error is

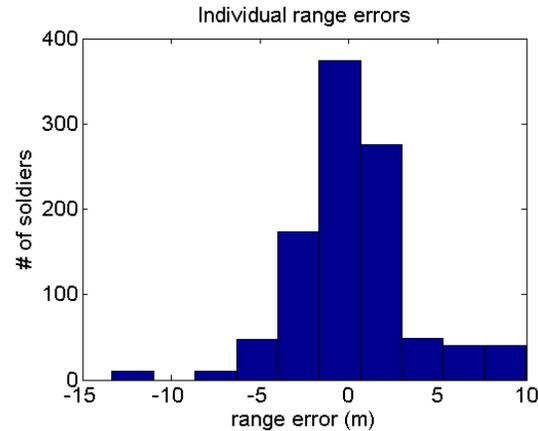


Figure 12. Errors in range relative to each sensor.

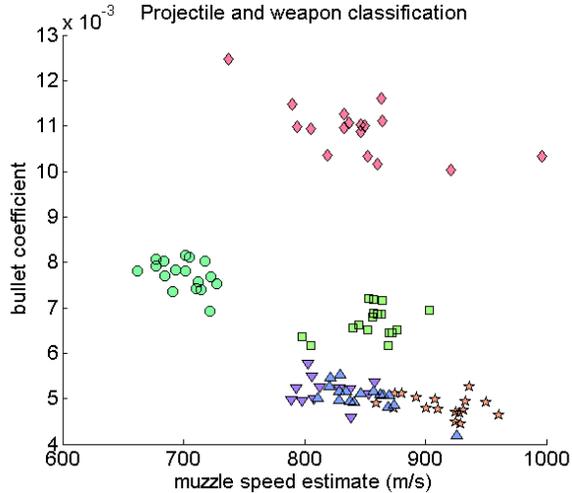


Figure 13. Projectile and weapon classification.

2.3 m, while the bearing error is 0.75° . These are an order of magnitude better results than those of the latest commercial countersniper systems based on publicly available knowledge [1, 9]. Note, however, that our results correspond to a stationary test and not the much more challenging mobile case.

Figure 13 shows how the 104 shots with trajectory and shooter position estimates were classified. The bullet coefficient (C_{bullet} defined in Equation 15) clearly separates the different caliber projectiles. The 0.50 caliber M107 are shown with red diamonds near the top, the 762 mm AK-47 (green circle) and M240 (green square) are clustered in the middle, while the 5.56 mm M16 (orange star), M4 (blue triangle) and M249 (purple triangle) are at the bottom. Hence, caliber estimation is 100% accurate.

Within each caliber, weapon classification works based on the estimated muzzle velocity. The AK-47 and M240 are clearly separated with no overlap, hence, their classification is perfect using this data set. The M16 and M4 overlap somewhat, however, all but one M16 shot are classified correctly with a 875 m/s threshold. However, the M4 and M249 shots are completely intermingled making their separation impossible with this method. These results are in line with those in [14].

5.1 Error sensitivity to sensor position error

The biggest open problem before our system can be built and deployed is a reliable and accurate sensor location service. This will be based on a high-precision GPS, such as the u-Blox LEA6 [13] family which costs around \$200. Under ideal operating conditions, 1 to 2 meter errors can be expected. Furthermore, the errors of nearby units tend to be correlated. The sensor fusion relies on the relative location of the nodes, therefore, the absolute error of the receivers is not that important. Once the shooter location is estimated, the result needs to be provided relative to the location of the given user, otherwise, the absolute location error would have a detrimental effect on the accuracy. In urban areas, multipath causes additional errors that tend not to be correlated

across receivers if they are not located very close to each other. To test how sensitive the accuracy of our approach is to uncorrelated sensor location errors, we conducted the following experiment.

Using the same 108 shots, we ran the sensor fusion multiple times with altered sensor locations. We shifted each sensor in a random direction uniformly distributed between 0 and 2π with a uniformly distributed random displacement of a maximum amount of up to 1.5 m in 0.25 m increments. Interestingly, the trajectory estimation and shooter localization rates remained almost unchanged (approximately 99% and 96%, respectively). The trajectory angle and shooter position errors are shown in Figures 14 and 15.

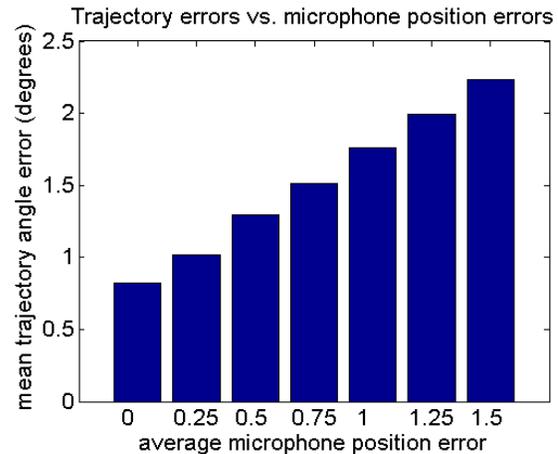


Figure 14. Trajectory angle vs. sensor location errors.

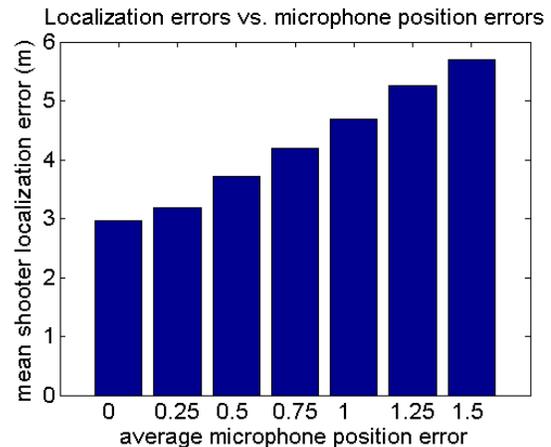


Figure 15. Shooter position vs. sensor location errors.

The results are encouraging. Both the trajectory angle and the shooter position errors display an increasing trend with increasing sensor location errors, as expected. However, the worst mean angle error of 2.5° and the largest average shooter position error of 5.7 m at 1.5 m sensor position errors can still be considered excellent performance.

6 Conclusion

Even though the field of acoustic shooter localization is quite mature, we have presented significant and novel contributions in this paper. These were made possible by the distributed sensing provided by the WSN-based approach and the utilization of the shockwave length for trajectory estimation. To the best of our knowledge, the latter has not been done before. The major design drivers of our approach were 1) the need to use wearable sensors that limits the number of nodes available and 2) the requirement of single-channel nodes, so that the orientation of the sensors do not need to be tracked.

Our results indicate that it is possible to locate shooters relying on as few as five single-channel wireless sensor nodes. However, missed detections and blocked LOS can happen frequently in practice, so the current approach would need more sensors to work reliably. We utilized ten sensors in our evaluation and the sensor fusion selected the five most consistent ones out of the subset that detected it. An additional experiment that we plan to conduct in the near future is to select fixed subsets of the ten sensors before running the sensor fusion and see how the localization rate and the accuracy of the system is affected.

Nevertheless, the demonstrated performance is excellent and it is due to the consistency function-based sensor fusion. The results are comparable to that of the previous generation system [14], even though that had four microphones per node. Even caliber estimation and weapon classification works well. However, our evaluation is based on a static deployment. While we showed that the effects of sensor location errors are manageable, nevertheless, a field test with mobile sensors is absolutely necessary to verify our results in real-world deployments.

By relying on smartphones soldiers are carrying today or are expected to carry in the near future and the fact that only a single acoustic channel is needed in our custom sensor node, the cost of our system is forecasted to be an order of magnitude less than current commercial wearable systems. The tradeoff is that our sensor node alone can only tell the miss distance of the bullet and the range to the shooter and not a bearing and it needs to assume that the weapon type is known, e.g., that it is an AK-47. Once observations from multiple nodes are available, however, our system surpasses the performance of centralized approaches and it does need to know the weapon fired. On the contrary, it classifies the projectile and rifle. It is also noteworthy, that to the best of our knowledge, our system is the first to provide accurate range estimate based on a single shot using a single microphone (assuming a known weapon type).

We have plenty of work left before we can field our system. We are in the process of designing a custom sensor node that will provide the detection and signal processing tasks as well as handle the GPS data. Only event detections with location- and time-stamps will be sent to the smartphone which is expected to provide the wireless networking capability. Finally, the sensor fusion will be optimized and ported

to native code on the phone from the current Matlab-based desktop implementation.

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