جدول کامل فرمول های انتگرال

Rules for integration of general functions

$$\int af(x)\,dx = a\int f(x)\,dx \qquad (a \neq 0,\ \text{constant})$$

$$\int [f(x)+g(x)]\,dx = \int f(x)\,dx + \int g(x)\,dx$$

$$\int f'(x)g(x)\,dx = f(x)g(x) - \int f(x)g'(x)\,dx$$

$$\int f'(x)f(x)\,dx = \frac{1}{2}[f(x)]^2 + C$$

$$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + C$$

$$\int [f(x)]^n f'(x)\,dx = \frac{[f(x)]^{n+1}}{n+1} + C \qquad (\text{for } n \neq -1)$$

Rational functions

$$\int \mathrm{d}x = x + C$$

$$\int x^n\,\mathrm{d}x = \frac{x^{n+1}}{n+1} + C \qquad \text{if } n \neq -1$$

$$\int \frac{dx}{x} = \ln|x| + C$$

$$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\arctan\frac{x}{a} + C$$

Irrational functions

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \frac{x}{a} + C$$

$$\int \frac{-dx}{\sqrt{a^2 - x^2}} = \cos^{-1} \frac{x}{a} + C$$

$$\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a} \sec^{-1} \frac{|x|}{a} + C$$

Logarithms

$$\int \ln x \, dx = x \ln x - x + C$$

$$\int \log_b x \, dx = x \log_b x - x \log_b e + C$$

Exponential functions

$$\int e^x \, dx = e^x + C$$

$$\int a^x \, dx = \frac{a^x}{\ln a} + C$$

Trigonometric functions

$$\int \sin x \, dx = -\cos x + C$$

$$\int \cos x \, dx = \sin x + C$$

$$\int \tan x \, dx = -\ln |\cos x| + C$$

$$\int \cot x \, dx = \ln |\sin x| + C$$

$$\int \sec x \, dx = \ln|\sec x + \tan x| + C$$

$$\int \csc x \, \cot x \, dx = -\csc x + C$$

$$\int \sec^2 x \, dx = \tan x + C$$

$$\int \csc^2 x \, dx = -\cot x + C$$

$$\int \sec x \, \tan x \, dx = \sec x + C$$

$$\int \csc x \, dx = \ln|\csc x - \cot x| + C$$

$$\int \sin^2 x \, dx = \frac{1}{2}(x - \sin x \cos x) + C$$

$$\int \cos^2 x \, dx = \frac{1}{2}(x + \sin x \cos x) + C$$

$$\int \sec^3 x \, dx = \frac{1}{2}\sec x \tan x + \frac{1}{2}\ln|\sec x + \tan x| + C$$

$$\int \sin^n x \, dx = -\frac{\sin^{n-1} x \cos x}{n} + \frac{n-1}{n}\int \sin^{n-2} x \, dx$$

$$\int \cos^n x \, dx = \frac{\cos^{n-1} x \sin x}{n} + \frac{n-1}{n}\int \cos^{n-2} x \, dx$$

$$\int \arctan x \, dx = x \, \arctan x - \frac{1}{2}\ln\left|1 + x^2\right| + C$$

Hyperbolic functions

$$\int \sinh x \, dx = \cosh x + C$$

$$\int \cosh x \, dx = \sinh x + C$$

$$\int \tanh x \, dx = \ln|\cosh x| + C$$

$$\int \coth x \, dx = \ln|\sinh x| + C$$

$$\int \operatorname{sech} x \, dx = \arctan(\sinh x) + C$$

$$\int \operatorname{csch} x \, dx = \ln\left|\tanh \frac{x}{2}\right| + C$$

$$\int \operatorname{sech}^2 x \, dx = \tanh x + C$$

Inverse hyperbolic functions

$$\int \operatorname{arcsinh} x \, dx = x \operatorname{arcsinh} x - \sqrt{x^2 + 1} + C$$

$$\int \operatorname{arccosh} x \, dx = x \operatorname{arccosh} x - \sqrt{x^2 - 1} + C$$

$$\int \operatorname{arctanh} x \, dx = x \operatorname{arctanh} x + \frac{1}{2} \log(1 - x^2) + C$$

$$\int \operatorname{arccoth} x \, dx = x \operatorname{arccoth} x + \frac{1}{2} \log(x^2 - 1) + C$$

$$\int \operatorname{arccsch} x \, dx = x \operatorname{arccsch} x + \log\left[x\left(\sqrt{1 + \frac{1}{x^2}} + 1\right)\right] + C$$

$$\int \operatorname{arcsech} x \, dx = x \operatorname{arcsech} x - \arctan\left(\frac{x}{x-1}\sqrt{\frac{1-x}{1+x}}\right) + C$$

Definite integrals lacking closed-form ant derivatives

$$\int_0^\infty \sqrt{x}\, e^{-x} \, dx = \frac{1}{2}\sqrt{\pi}$$

$$\int_0^\infty e^{-x^2} \, dx = \frac{1}{2}\sqrt{\pi}$$

$$\int_0^\infty \frac{\sin(x)}{x}\,dx = \frac{\pi}{2}$$

$$\int_0^\infty \frac{x}{e^x-1}\,dx = \frac{\pi^2}{6}$$

$$\int_0^\infty \frac{x^3}{e^x-1}\,dx = \frac{\pi^4}{15}$$

$$\int_0^{2\pi} e^{x\cos\theta} d\theta = 2\pi I_0(x)$$

$$\int_0^\infty \frac{\sin^2 x}{x^2}\,dx = \frac{\pi}{2}$$

$$\int_{-\infty}^\infty (1+x^2/\nu)^{-(\nu+1)/2} dx = \frac{\sqrt{\nu\pi}\ \Gamma(\nu/2)}{\Gamma((\nu+1)/2))}$$

$$\int_0^{2\pi} e^{x\cos\theta+y\sin\theta} d\theta = 2\pi I_0\left(\sqrt{x^2+y^2}\right)$$

$$\int_0^{\frac{\pi}{2}} \sin^n x\,dx = \int_0^{\frac{\pi}{2}} \cos^n x\,dx = \frac{1\cdot 3\cdot 5\cdot\cdots\cdot(n-1)}{2\cdot 4\cdot 6\cdot\cdots\cdot n}\frac{\pi}{2}$$

$$\int_0^\infty x^{z-1}\,e^{-x}\,dx = \Gamma(z)$$

$$\int_0^{\frac{\pi}{2}} \sin^n x\,dx = \int_0^{\frac{\pi}{2}} \cos^n x\,dx = \frac{2\cdot 4\cdot 6\cdot\cdots\cdot(n-1)}{3\cdot 5\cdot 7\cdot\cdots\cdot n}$$

$$\int_{-\infty}^\infty e^{-(ax^2+bx+c)}\,dx = \sqrt{\frac{\pi}{a}}\exp\left[\frac{b^2-4ac}{4a}\right]$$

$$\int_a^b f(x)\,dx = (b-a)\sum_{n=1}^\infty \sum_{m=1}^{2^n-1} (-1)^{m+1} 2^{-n} f(a+m\,(b-a)\,2^{-n})$$

$$\int_0^1 x^{-x}\,dx = \sum_{n=1}^\infty n^{-n} \qquad (= 1.291285997\dots)$$

$$\int_0^1 x^x\,dx = \sum_{n=1}^\infty -(-1)^n n^{-n} \quad (= 0.783430510712\dots)$$

INTEGRATION TABLE (INTEGRALS)

Notation: $f(x)$ and $g(x)$ are any continuous functions; $u = u(x)$ is differentiable function of x; $du = \frac{du}{dx}\,dx = u'\,dx$; c, n, and $a > 0$ are constants

(1) $\int (f(x) + g(x))\,dx = \int f(x)\,dx + \int g(x)\,dx$

(2) $\int cf(x)\,dx = c\int f(x)\,dx$

(3) $\int u^n\,du = \frac{u^{n+1}}{n+1} + C,\ n \neq -1$

(a) $\int \frac{1}{u}\,du = \int \frac{du}{u} = \ln|u| + C$

(b) $\int \frac{1}{\sqrt{u}}\,du = \int \frac{du}{\sqrt{u}} = 2\sqrt{u} + C$

(c) $\int du = u + C$

(4) $\int e^u\,du = e^u + C$

(5) $\int \sin u\,du = -\cos u + C$

(6) $\int \cos u\,du = \sin u + C$

(7) $\int \sec^2 u\,du = \int \frac{1}{\cos^2 u}\,du = \tan u + C$

(8) $\int \csc^2 u\,du = \int \frac{1}{\sin^2 u}\,du = -\cot u + C$

(9) $\int \frac{1}{u^2 + a^2}\,du = \int \frac{du}{u^2 + a^2} = \frac{1}{a}\ \arctan\frac{u}{a} + C$

(10) $\int \frac{1}{\sqrt{a^2 - u^2}}\,du = \int \frac{du}{\sqrt{a^2 - u^2}} = \frac{1}{a}\ \arcsin\frac{u}{a} + C$

• انتگرال

انتگرال های زیر جزء انتگرال های مهمی هستن که سر کلاس اثباتشون کردیم و دیگه آخرین مهلت برای حفظ کردنشونه :

1. $$\int \frac{dx}{a^2+x^2} = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + C$$

2. $$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln\left|\frac{1+\left(\frac{x}{a}\right)}{1-\left(\frac{x}{a}\right)}\right| + C$$

3. $$\int \frac{dx}{\sqrt{x^2-a^2}} = \ln\left|\left(\frac{x}{a}\right)+\sqrt{\left(\frac{x}{a}\right)^2-1}\right| + C$$

4. $$\int \frac{dx}{\sqrt{x^2+a^2}} = \ln\left(\frac{x}{a}+\sqrt{\left(\frac{x}{a}\right)^2+1}\right) + C$$

5. $$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C$$

6. $\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a} \cos^{-1}\left|\frac{a}{x}\right| + C$

7. $\int \frac{dx}{x\sqrt{x^2+a^2}} = \frac{-1}{a} \sinh^{-1}\left|\frac{a}{x}\right| + C$

8. $\int \frac{dx}{x\sqrt{a^2-x^2}} = \frac{-1}{a} \cosh^{-1}\left|\frac{a}{x}\right| + C$

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9. $\int \frac{dx}{\sin x} = \ln\left|\tan \frac{x}{2}\right| + C$

10. $\int \frac{dx}{\cos x} = \ln\left|\tan\left(\frac{x}{2}+\frac{\pi}{4}\right)\right| + C$

11. $\int \tan x \, dx = \ln|\sec x| + C$

12. $\int \cot x \, dx = \ln|\sin x| + C$

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13. $\int \frac{dx}{(x^2+a^2)^{\frac{3}{2}}} = \frac{x}{a^2\sqrt{x^2+a^2}} + C$

Table of Integrals*

Basic Forms

$$\int x^n dx = \frac{1}{n+1}x^{n+1} + c \tag{1}$$

$$\int \frac{1}{x} dx = \ln x + c \tag{2}$$

$$\int u dv = uv - \int v du \tag{3}$$

$$\int \frac{1}{ax+b} dx = \frac{1}{a}\ln|ax+b| + c \tag{4}$$

Integrals of Rational Functions

$$\int \frac{1}{(x+a)^2} dx = -\frac{1}{x+a} + c \tag{5}$$

$$\int (x+a)^n dx = \frac{(x+a)^{n+1}}{n+1} + c, n \neq -1 \tag{6}$$

$$\int x(x+a)^n dx = \frac{(x+a)^{n+1}((n+1)x-a)}{(n+1)(n+2)} + c \tag{7}$$

$$\int \frac{1}{1+x^2} dx = \tan^{-1} x + c \tag{8}$$

$$\int \frac{1}{a^2+x^2} dx = \frac{1}{a}\tan^{-1}\frac{x}{a} + c \tag{9}$$

$$\int \frac{x}{a^2+x^2} dx = \frac{1}{2}\ln|a^2+x^2| + c \tag{10}$$

$$\int \frac{x^2}{a^2+x^2} dx = x - a\tan^{-1}\frac{x}{a} + c \tag{11}$$

$$\int \frac{x^3}{a^2+x^2} dx = \frac{1}{2}x^2 - \frac{1}{2}a^2\ln|a^2+x^2| + c \tag{12}$$

$$\int \frac{1}{ax^2+bx+c} dx = \frac{2}{\sqrt{4ac-b^2}}\tan^{-1}\frac{2ax+b}{\sqrt{4ac-b^2}} + C \tag{13}$$

$$\int \frac{1}{(x+a)(x+b)} dx = \frac{1}{b-a}\ln\frac{a+x}{b+x}, \ a \neq b \tag{14}$$

$$\int \frac{x}{(x+a)^2} dx = \frac{a}{a+x} + \ln|a+x| + C \tag{15}$$

$$\int \frac{x}{ax^2+bx+c} dx = \frac{1}{2a}\ln|ax^2+bx+c| - \frac{b}{a\sqrt{4ac-b^2}}\tan^{-1}\frac{2ax+b}{\sqrt{4ac-b^2}} + C \tag{16}$$

Integrals with Roots

$$\int \sqrt{x-a} dx = \frac{2}{3}(x-a)^{3/2} + C \tag{17}$$

$$\int \frac{1}{\sqrt{x\pm a}} dx = 2\sqrt{x\pm a} + C \tag{18}$$

$$\int \frac{1}{\sqrt{a-x}} dx = -2\sqrt{a-x} + C \tag{19}$$

$$\int x\sqrt{x-a} dx = \frac{2}{3}a(x-a)^{3/2} + \frac{2}{5}(x-a)^{5/2} + C \tag{20}$$

$$\int \sqrt{ax+b} dx = \left(\frac{2b}{3a} + \frac{2x}{3}\right)\sqrt{ax+b} + C \tag{21}$$

$$\int (ax+b)^{3/2} dx = \frac{2}{5a}(ax+b)^{5/2} + C \tag{22}$$

$$\int \frac{x}{\sqrt{x\pm a}} dx = \frac{2}{3}(x\mp 2a)\sqrt{x\pm a} + C \tag{23}$$

$$\int \sqrt{\frac{x}{a-x}} dx = -\sqrt{x(a-x)} - a\tan^{-1}\frac{\sqrt{x(a-x)}}{x-a} + C \tag{24}$$

$$\int \sqrt{\frac{x}{a+x}} dx = \sqrt{x(a+x)} - a\ln\left[\sqrt{x} + \sqrt{x+a}\right] + C \tag{25}$$

$$\int x\sqrt{ax+b} dx = \frac{2}{15a^2}(-2b^2 + abx + 3a^2x^2)\sqrt{ax+b} + C \tag{26}$$

$$\int \sqrt{x(ax+b)} dx = \frac{1}{4a^{3/2}}\left[(2ax+b)\sqrt{ax(ax+b)} - b^2\ln\left|a\sqrt{x} + \sqrt{a(ax+b)}\right|\right] + C \tag{27}$$

$$\int \sqrt{x^3(ax+b)} dx = \left[\frac{b}{12a} - \frac{b^2}{8a^2x} + \frac{x}{3}\right]\sqrt{x^3(ax+b)} + \frac{b^3}{8a^{5/2}}\ln\left|a\sqrt{x} + \sqrt{a(ax+b)}\right| + C \tag{28}$$

$$\int \sqrt{x^2 \pm a^2}dx = \frac{1}{2}x\sqrt{x^2 \pm a^2} \pm \frac{1}{2}a^2 \ln\left|x + \sqrt{x^2 \pm a^2}\right| + C \qquad (29)$$

$$\int \sqrt{a^2 - x^2}dx = \frac{1}{2}x\sqrt{a^2 - x^2} + \frac{1}{2}a^2 \tan^{-1}\frac{x}{\sqrt{a^2 - x^2}} + C \qquad (30)$$

$$\int x\sqrt{x^2 \pm a^2}dx = \frac{1}{3}\left(x^2 \pm a^2\right)^{3/2} + C \qquad (31)$$

$$\int \frac{1}{\sqrt{x^2 \pm a^2}}dx = \ln\left|x + \sqrt{x^2 \pm a^2}\right| + C \qquad (32)$$

$$\int \frac{1}{\sqrt{a^2 - x^2}}dx = \sin^{-1}\frac{x}{a} + C \qquad (33)$$

$$\int \frac{x}{\sqrt{x^2 \pm a^2}}dx = \sqrt{x^2 \pm a^2} + C \qquad (34)$$

$$\int \frac{x}{\sqrt{a^2 - x^2}}dx = -\sqrt{a^2 - x^2} + C \qquad (35)$$

$$\int \frac{x^2}{\sqrt{x^2 \pm a^2}}dx = \frac{1}{2}x\sqrt{x^2 \pm a^2} \mp \frac{1}{2}a^2 \ln\left|x + \sqrt{x^2 \pm a^2}\right| + C \qquad (36)$$

$$\int \sqrt{ax^2 + bx + c}dx = \frac{b + 2ax}{4a}\sqrt{ax^2 + bx + c} + \frac{4ac - b^2}{8a^{3/2}} \ln\left|2ax + b + 2\sqrt{a(ax^2 + bx^+c)}\right| + C \qquad (37)$$

$$\int x\sqrt{ax^2 + bx + c} = \frac{1}{48a^{5/2}}\left(2\sqrt{a}\sqrt{ax^2 + bx + c} - \left(3b^2 + 2abx + 8a(c + ax^2)\right) + 3(b^3 - 4abc)\ln\left|b + 2ax + 2\sqrt{a}\sqrt{ax^2 + bx + x}\right|\right) \qquad (38)$$

$$\int \frac{1}{\sqrt{ax^2 + bx + c}}dx = \frac{1}{\sqrt{a}}\ln\left|2ax + b + 2\sqrt{a(ax^2 + bx + c)}\right| + C \qquad (39)$$

$$\int \frac{x}{\sqrt{ax^2 + bx + c}}dx = \frac{1}{a}\sqrt{ax^2 + bx + c} + \frac{b}{2a^{3/2}}\ln\left|2ax + b + 2\sqrt{a(ax^2 + bx + c)}\right| + C \qquad (40)$$

Integrals with Logarithms

$$\int \ln ax dx = x\ln ax - x + C \qquad (41)$$

$$\int \frac{\ln ax}{x}dx = \frac{1}{2}(\ln ax)^2 + C \qquad (42)$$

$$\int \ln(ax + b)dx = \left(x + \frac{b}{a}\right)\ln(ax + b) - x + C, a \neq 0 \qquad (43)$$

$$\int \ln\left(a^2x^2 \pm b^2\right)dx = x\ln\left(a^2x^2 \pm b^2\right) + \frac{2b}{a}\tan^{-1}\frac{ax}{b} - 2x + C \qquad (44)$$

$$\int \ln\left(a^2 - b^2x^2\right)dx = x\ln\left(ar - b^2x^2\right) + \frac{2a}{b}\tan^{-1}\frac{bx}{a} - 2x + C \qquad (45)$$

$$\int \ln\left(ax^2 + bx + c\right)dx = \frac{1}{a}\sqrt{4ac - b^2}\tan^{-1}\frac{2ax + b}{\sqrt{4ac - b^2}} - 2x + \left(\frac{b}{2a} + x\right)\ln\left(ax^2 + bx + c\right) + C \qquad (46)$$

$$\int x\ln(ax + b)dx = \frac{bx}{2a} - \frac{1}{4}x^2 + \frac{1}{2}\left(x^2 - \frac{b^2}{a^2}\right)\ln(ax + b) + C \qquad (47)$$

$$\int x\ln\left(a^2 - b^2x^2\right)dx = -\frac{1}{2}x^2 + \frac{1}{2}\left(x^2 - \frac{a^2}{b^2}\right)\ln\left(a^2 - b^2x^2\right) + C \qquad (48)$$

Integrals with Exponentials

$$\int e^{ax}dx = \frac{1}{a}e^{ax} + C \qquad (49)$$

$$\int \sqrt{x}e^{ax}dx = \frac{1}{a}\sqrt{x}e^{ax} + \frac{i\sqrt{\pi}}{2a^{3/2}}\text{erf}\left(i\sqrt{ax}\right) + C, \quad \text{where } \text{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dtet \qquad (50)$$

$$\int xe^x dx = (x - 1)e^x + C \qquad (51)$$

$$\int xe^{ax}dx = \left(\frac{x}{a} - \frac{1}{a^2}\right)e^{ax} + C \qquad (52)$$

$$\int x^2e^x dx = \left(x^2 - 2x + 2\right)e^x + C \qquad (53)$$

$$\int x^2 e^{ax} dx = \left(\frac{x^2}{a} - \frac{2x}{a^2} + \frac{2}{a^3}\right) e^{ax} + C \tag{54}$$

$$\int x^3 e^x dx = \left(x^3 - 3x^2 + 6x - 6\right) e^x + C \tag{55}$$

$$\int x^n e^{ax} \, \mathrm{d}x = \frac{x^n e^{ax}}{a} - \frac{n}{a} \int x^{n-1} e^{ax} \, \mathrm{d}x \tag{56}$$

$$\int x^n e^{ax} \, \mathrm{d}x = \frac{(-1)^n}{a^{n+1}} \Gamma[1+n, -ax] + C, \quad \text{where } \Gamma(a,x) = \int_x^\infty t^{a-1} e^{-t} \, \mathrm{d}t \tag{57}$$

$$\int e^{ax^2} \, \mathrm{d}x = -\frac{i\sqrt{\pi}}{2\sqrt{a}} \mathrm{erf}\left(ix\sqrt{a}\right) + C \tag{58}$$

$$\int e^{-ax^2} \, \mathrm{d}x = \frac{\sqrt{\pi}}{2\sqrt{a}} \mathrm{erf}\left(x\sqrt{a}\right) + C \tag{59}$$

Integrals with Trigonometric Functions

$$\int \sin ax dx = -\frac{1}{a} \cos ax + C \tag{60}$$

$$\int \sin^2 ax dx = \frac{x}{2} - \frac{\sin 2ax}{4a} + C \tag{61}$$

$$\int \sin^n ax dx = -\frac{1}{a} \cos ax \;\; {}_2F_1\left[\frac{1}{2}, \frac{1-n}{2}, \frac{3}{2}, \cos^2 ax\right] + C \tag{62}$$

$$\int \sin^3 ax dx = -\frac{3\cos ax}{4a} + \frac{\cos 3ax}{12a} + C \tag{63}$$

$$\int \cos ax dx = \frac{1}{a} \sin ax + C \tag{64}$$

$$\int \cos^2 ax dx = \frac{x}{2} + \frac{\sin 2ax}{4a} + C \tag{65}$$

$$\int \cos^p ax dx = -\frac{1}{a(1+p)} \cos^{1+p} ax \times {}_2F_1\left[\frac{1+p}{2}, \frac{1}{2}, \frac{3+p}{2}, \cos^2 ax\right] + C \tag{66}$$

$$\int \cos^3 ax dx = \frac{3\sin ax}{4a} + \frac{\sin 3ax}{12a} + C \tag{67}$$

$$\int \cos ax \sin bx dx = \frac{\cos[(a-b)x]}{2(a-b)} - \frac{\cos[(a+b)x]}{2(a+b)} + C, a \neq b \tag{68}$$

$$\int \sin^2 ax \cos bx dx = -\frac{\sin[(2a-b)x]}{4(2a-b)} + \frac{\sin bx}{2b} - \frac{\sin[(2a+b)x]}{4(2a+b)} + C \tag{69}$$

$$\int \sin^2 x \cos x dx = \frac{1}{3} \sin^3 x + C \tag{70}$$

$$\int \cos^2 ax \sin bx dx = \frac{\cos[(2a-b)x]}{4(2a-b)} - \frac{\cos bx}{2b} - \frac{\cos[(2a+b)x]}{4(2a+b)} + C \tag{71}$$

$$\int \cos^2 ax \sin ax dx = -\frac{1}{3a} \cos^3 ax + C \tag{72}$$

$$\int \sin^2 ax \cos^2 bx dx = \frac{x}{4} - \frac{\sin 2ax}{8a} - \frac{\sin[2(a-b)x]}{16(a-b)} + \frac{\sin 2bx}{8b} - \frac{\sin[2(a+b)x]}{16(a+b)} + C \tag{73}$$

$$\int \sin^2 ax \cos^2 ax dx = \frac{x}{8} - \frac{\sin 4ax}{32a} + C \tag{74}$$

$$\int \tan ax dx = -\frac{1}{a} \ln \cos ax + C \tag{75}$$

$$\int \tan^2 ax dx = -x + \frac{1}{a} \tan ax + C \tag{76}$$

$$\int \tan^n ax dx = \frac{\tan^{n+1} ax}{a(1+n)} \times {}_2F_1\left(\frac{n+1}{2}, 1, \frac{n+3}{2}, -\tan^2 ax\right) + C \tag{77}$$

$$\int \tan^3 ax dx = \frac{1}{a} \ln \cos ax + \frac{1}{2a} \sec^2 ax + C \tag{78}$$

$$\int \sec x dx = \ln|\sec x + \tan x| + C = 2\tanh^{-1}\left(\tan \frac{x}{2}\right) + C \tag{79}$$

$$\int \sec^2 ax dx = \frac{1}{a} \tan ax + C \tag{80}$$

$$\int \sec^3 x \, \mathrm{d}x = \frac{1}{2} \sec x \tan x + \frac{1}{2} \ln|\sec x + \tan x| + C \tag{81}$$

$$\int \sec x \tan x dx = \sec x + C \tag{82}$$

$$\int \sec^2 x \tan x dx = \frac{1}{2} \sec^2 x + C \tag{83}$$

$$\int \sec^n x \tan x dx = \frac{1}{n} \sec^n x + C, n \neq 0 \tag{84}$$

$$\int \csc x dx = \ln\left|\tan\frac{x}{2}\right| + C = \ln|\csc x - \cot x| + C \tag{85}$$

$$\int \csc^2 ax dx = -\frac{1}{a}\cot ax + C \tag{86}$$

$$\int \csc^3 x dx = -\frac{1}{2}\cot x \csc x + \frac{1}{2}\ln|\csc x - \cot x| + C \tag{87}$$

$$\int \csc^n x \cot x dx = -\frac{1}{n}\csc^n x + C, n \neq 0 \tag{88}$$

$$\int \sec x \csc x dx = \ln|\tan x| + C \tag{89}$$

Products of Trigonometric Functions and Monomials

$$\int x\cos x dx = \cos x + x\sin x + C \tag{90}$$

$$\int x\cos ax dx = \frac{1}{a^2}\cos ax + \frac{x}{a}\sin ax + C \tag{91}$$

$$\int x^2\cos x dx = 2x\cos x + \left(x^2 - 2\right)\sin x + C \tag{92}$$

$$\int x^2\cos ax dx = \frac{2x\cos ax}{a^2} + \frac{a^2x^2 - 2}{a^3}\sin ax + C \tag{93}$$

$$\int x^n cosx dx = -\frac{1}{2}(i)^{n+1}\left[\Gamma(n+1, -ix) + (-1)^n\Gamma(n+1, ix)\right] + C \tag{94}$$

$$\int x^n cosax dx = \frac{1}{2}(ia)^{1-n}\left[(-1)^n\Gamma(n+1, -iax) - \Gamma(n+1, ixa)\right] + C \tag{95}$$

$$\int x\sin x dx = -x\cos x + \sin x + C \tag{96}$$

$$\int x\sin ax dx = -\frac{x\cos ax}{a} + \frac{\sin ax}{a^2} + C \tag{97}$$

$$\int x^2\sin x dx = \left(2 - x^2\right)\cos x + 2x\sin x + C \tag{98}$$

$$\int x^2\sin ax dx = \frac{2 - a^2x^2}{a^3}\cos ax + \frac{2x\sin ax}{a^2} + C \tag{99}$$

$$\int x^n\sin x dx = -\frac{1}{2}(i)^n\left[\Gamma(n+1, -ix) - (-1)^n\Gamma(n+1, -ix)\right] + C \tag{100}$$

Products of Trigonometric Functions and Exponentials

$$\int e^x\sin x dx = \frac{1}{2}e^x(\sin x - \cos x) + C \tag{101}$$

$$\int e^{bx}\sin ax dx = \frac{1}{a^2 + b^2}e^{bx}(b\sin ax - a\cos ax) + C \tag{102}$$

$$\int e^x\cos x dx = \frac{1}{2}e^x(\sin x + \cos x) + C \tag{103}$$

$$\int e^{bx}\cos ax dx = \frac{1}{a^2 + b^2}e^{bx}(a\sin ax + b\cos ax) + C \tag{104}$$

$$\int xe^x\sin x dx = \frac{1}{2}e^x(\cos x - x\cos x + x\sin x) + C \tag{105}$$

$$\int xe^x\cos x dx = \frac{1}{2}e^x(x\cos x - \sin x + x\sin x) + C \tag{106}$$

Integrals of Hyperbolic Functions

$$\int \cosh ax dx = \frac{1}{a}\sinh ax + C \tag{107}$$

$$\int e^{ax}\cosh bx dx = \begin{cases} \dfrac{e^{ax}}{a^2 - b^2}[a\cosh bx - b\sinh bx] + C & a \neq b \\ \dfrac{e^{2ax}}{4a} + \dfrac{x}{2} + C & a = b \end{cases} \tag{108}$$

$$\int \sinh ax dx = \frac{1}{a}\cosh ax + C \tag{109}$$

$$\int e^{ax}\sinh bx dx = \begin{cases} \dfrac{e^{ax}}{a^2 - b^2}[-b\cosh bx + a\sinh bx] + C & a \neq b \\ \dfrac{e^{2ax}}{4a} - \dfrac{x}{2} + C & a = b \end{cases} \tag{110}$$

$$\int e^{ax}\tanh bx dx = \begin{cases} \dfrac{e^{(a+2b)x}}{(a+2b)}\,{}_2F_1\left[1 + \dfrac{a}{2b}, 1, 2 + \dfrac{a}{2b}, -e^{2bx}\right] \\ \qquad -\dfrac{1}{a}e^{ax}\,{}_2F_1\left[\dfrac{a}{2b}, 1, 1E, -e^{2bx}\right] + C & a \neq b \\ \dfrac{e^{ax} - 2\tan^{-1}[e^{ax}]}{a} + C & a = b \end{cases} \tag{111}$$

$$\int \tanh bx dx = \frac{1}{a}\ln\cosh ax + C \tag{112}$$

$$\int \cos ax \cosh bx dx = \frac{1}{a^2+b^2} [a \sin ax \cosh bx + b \cos ax \sinh bx] + C \quad (113)$$

$$\int \cos ax \sinh bx dx = \frac{1}{a^2+b^2} [b \cos ax \cosh bx + a \sin ax \sinh bx] + C \quad (114)$$

$$\int \sin ax \cosh bx dx = \frac{1}{a^2+b^2} [-a \cos ax \cosh bx + b \sin ax \sinh bx] + C \quad (115)$$

$$\int \sin ax \sinh bx dx = \frac{1}{a^2+b^2} [b \cosh bx \sin ax - a \cos ax \sinh bx] + C \quad (116)$$

$$\int \sinh ax \cosh ax dx = \frac{1}{4a} [-2ax + \sinh 2ax] + C \quad (117)$$

$$\int \sinh ax \cosh bx dx = \frac{1}{b^2-a^2} [b \cosh bx \sinh ax - a \cosh ax \sinh bx] + C \quad (118)$$