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Theory and Methodology

Probabilistic linear programming problems with exponential random variables: A technical note

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Abstract

A method for solving probabilistic linear programming problems with exponential random variables is presented in this paper. Assuming that either some or all of the parameters are exponential random variables a transformation is presented to convert the probabilistic linear programming problem to a deterministic mathematical programming problem. A non-linear programming algorithm can then be used to solve the resulting deterministic problem. © 1998 Elsevier Science B.V. All rights reserved.

Keywords: Probabilistic linear programming; Exponential random variable; Probability density function; Non-linear programming

1. Introduction

Charnes and Cooper [1,2] first introduced chance constrained programming model which is also known as probabilistic programming. They suggested three models that have different objective functions and probabilistic types of constraints, a model that maximizes the expected value of the objective function (the E-model), a model that minimizes the generalized mean square of the objective function (the V-model), and a model that maximizes the probability of exceeding an aspiration level of the objective function (the P-model). In the literature of the stochastic linear programming [3,4], various models have been suggested by several researchers. A bibliography has been presented by Stancu-Minasian and Wets [5]. Most of the probabilistic models assume normal distribution for model coefficients. Goicoechea and Duckstein [6]

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presented some deterministic equivalents for some probabilistic programming with non-normal distributions.

In this paper we investigate a probabilistic linear programming problem with exponential random variables. A mathematical model of a probabilistic linear programming problem can be presented as follows:

$$\max: Z = \sum_{j=1}^n c_j x_j \quad (1)$$

$$\text{s.t. } \text{Prob} \left(\sum_{j=1}^n a_{ij} x_j \leq b_i \right) \geq 1 - \alpha_i, \quad i = 1, 2, \dots, m, \quad (2)$$

$$x_j \geq 0, \quad j = 1, 2, \dots, n, \quad (3)$$

where $0 < \alpha_i < 1$ and is a given constant. It is assumed that a_{ij} and c_j are independent exponential random variables with known distributions for $i = 1, 2, \dots, m$ and $j = 1, 2, \dots, n$. The results in this paper can be readily extended to situations where b_i 's are also exponential random variables [7]. Goicoechea et al. [8] presented a probabilistic model involving only two independent exponential random parameters. The main aim of the paper is to present a solution scheme for general probabilistic linear programming problems involving independent exponential random variables in the probabilistic constraints. At first we find the probability density function (p.d.f.) of the linear combination of n independent exponential random variables. Then using the p.d.f., the probabilistic constraints are transformed to the deterministic constraints. The resulting equivalent non-linear deterministic model can be then solved by some suitable non-linear programming solution scheme [9].

2. Probability distribution of $Y_i = \sum_{j=1}^n a_{ij} x_j$ and the deterministic forms of the probabilistic constraints

In this section we present the p.d.f. of the random variable $Y_i = \sum_{j=1}^n a_{ij} x_j$, where a_{ij} , $j = 1, 2, \dots, n$, are independent exponential random variables with known means and x_j , $j = 1, 2, \dots, n$, are some scalar quantities. Then we can use it to find a deterministic form of the probabilistic constraint in a probabilistic linear programming problem. The density function of Y_i and the proof for general situations will be presented at the end of this section after we study the special cases with $n = 2$ and $n = 3$.

We start by considering a mathematical model where only two random variables are involved. The i th probabilistic constraint can be stated explicitly as

$$\text{Prob}(a_{i1}x_1 + a_{i2}x_2 \leq b_i) \geq 1 - \alpha_i, \quad i = 1, 2, \dots, m, \quad (4)$$

where a_{i1} and a_{i2} are the two independent exponential random variables with known means. The joint probability density function of a_{i1} and a_{i2} can be stated as

$$f_i(a_{i1}, a_{i2}) = \begin{cases} \lambda_{i1} \lambda_{i2} e^{-a_{i1} \lambda_{i1} - a_{i2} \lambda_{i2}}, & a_{i1}, a_{i2} > 0 \\ 0, & \text{elsewhere,} \end{cases} \quad (5)$$

where $E(a_{i1}) = 1/\lambda_{i1}$ and $E(a_{i2}) = 1/\lambda_{i2}$, $\lambda_{i1}, \lambda_{i2} > 0$. The cumulative density function (c.d.f.) of the random variable $Y_i = a_{i1}x_1 + a_{i2}x_2$ can be computed as

$$G_i(y_i) = \int_{a_{i2}=0}^{y_i/x_2} \int_{a_{i1}=0}^{(y_i-a_{i2}x_2)/x_1} \lambda_{i1} \lambda_{i2} e^{-a_{i1}\lambda_{i1}-a_{i2}\lambda_{i2}} da_{i1} da_{i2}. \quad (6)$$

After the integration the above integral can be simplified as

$$G_i(y_i) = 1 + \frac{x_1 \lambda_{i2} e^{-y_i \lambda_{i1}/x_1}}{x_2 \lambda_{i1} - x_1 \lambda_{i2}} - e^{-y_i \lambda_{i2}/x_2} - \frac{x_1 \lambda_{i2} e^{-y_i \lambda_{i2}/x_2}}{x_2 \lambda_{i1} - x_1 \lambda_{i2}}. \quad (7)$$

Differentiating both sides of (7) with respect to y_i the p.d.f. of Y_i can be obtained as

$$g_i(y_i) = \begin{cases} \frac{\lambda_{i1} \lambda_{i2} e^{-y_i \lambda_{i1}/x_1}}{x_1 \lambda_{i2} - x_2 \lambda_{i1}} + \frac{\lambda_{i1} \lambda_{i2} e^{-y_i \lambda_{i2}/x_2}}{x_2 \lambda_{i1} - x_1 \lambda_{i2}}, & \text{if } y_i > 0, \\ 0, & \text{elsewhere.} \end{cases} \quad (8)$$

Hence the i th probabilistic constraint can be made a deterministic constraint by integrating the p.d.f. of Y_i as stated below

$$\int_0^{b_i} g_i(y_i) dy_i \geq 1 - \alpha_i, \quad i = 1, 2, \dots, m. \quad (9)$$

This can be further simplified as follows (see [3]):

$$\lambda_{i1} \lambda_{i2} \left[\frac{x_1 e^{-b_i \lambda_{i1}/x_1}}{\lambda_{i1}(x_1 \lambda_{i2} - x_2 \lambda_{i1})} + \frac{x_2 e^{-b_i \lambda_{i2}/x_2}}{\lambda_{i2}(x_2 \lambda_{i1} - x_1 \lambda_{i2})} \right] \leq \alpha_i, \quad i = 1, 2, \dots, m. \quad (10)$$

We now consider a probabilistic constraint involving three random variables. The i th probabilistic constraint involving three random variables can be presented as

$$\text{Prob} \left(\sum_{j=1}^3 a_{ij} x_j \leq b_i \right) \geq 1 - \alpha_i, \quad i = 1, 2, \dots, m, \quad (11)$$

where a_{i1} , a_{i2} and a_{i3} are three independent exponential random variables with known means. The joint p.d.f. of a_{i1} , a_{i2} and a_{i3} can be stated as

$$g_i(a_{i1}, a_{i2}, a_{i3}) = \begin{cases} \lambda_{i1} \lambda_{i2} \lambda_{i3} e^{-a_{i1} \lambda_{i1} - a_{i2} \lambda_{i2} - a_{i3} \lambda_{i3}}, & a_{i1}, a_{i2}, a_{i3} > 0, \\ 0, & \text{elsewhere,} \end{cases} \quad (12)$$

where $E(a_{i1}) = 1/\lambda_{i1}$, $E(a_{i2}) = 1/\lambda_{i2}$ and $E(a_{i3}) = 1/\lambda_{i3}$, λ_{i1} , λ_{i2} , $\lambda_{i3} > 0$. The c.d.f. of the random variable $Y_i = a_{i1}x_1 + a_{i2}x_2 + a_{i3}x_3$ can be computed as

$$G_i(y_i) = \int_{a_{i3}=0}^{y_i/x_3} \int_{a_{i2}=0}^{(y_i-a_{i3}x_3)/x_2} \int_{a_{i1}=0}^{(y_i-a_{i2}x_2-a_{i3}x_3)/x_1} \lambda_{i1} \lambda_{i2} \lambda_{i3} e^{-a_{i1} \lambda_{i1} - a_{i2} \lambda_{i2} - a_{i3} \lambda_{i3}} da_{i1} da_{i2} da_{i3}. \quad (13)$$

After integrating the above integral, the c.d.f. of Y_i can be obtained as

$$\begin{aligned}
G_i(y_i) = & 1 - e^{-y_i \lambda_{i3}/x_3} + \frac{x_2 \lambda_{i3}}{x_3 \lambda_{i2} - x_2 \lambda_{i3}} e^{-y_i \lambda_{i2}/x_2} - \frac{x_2 \lambda_{i3}}{x_3 \lambda_{i2} - x_2 \lambda_{i3}} e^{-y_i \lambda_{i3}/x_3} \\
& + \frac{x_1^2 \lambda_{i2} \lambda_{i3}}{(x_2 \lambda_{i1} - x_1 \lambda_{i2})(x_3 \lambda_{i1} - x_1 \lambda_{i3})} e^{-y_i \lambda_{i3}/x_3} - \frac{x_1^2 \lambda_{i2} \lambda_{i3}}{(x_2 \lambda_{i1} - x_1 \lambda_{i2})(x_3 \lambda_{i1} - x_1 \lambda_{i3})} e^{-\lambda_{i1} y_i / x_1} \\
& + \frac{x_1 x_2 \lambda_{i2} \lambda_{i3}}{(x_2 \lambda_{i1} - x_1 \lambda_{i2})(x_3 \lambda_{i2} - x_2 \lambda_{i3})} e^{-\lambda_{i2} y_i / x_2} - \frac{x_1 x_2 \lambda_{i2} \lambda_{i3}}{(x_2 \lambda_{i1} - x_1 \lambda_{i2})(x_3 \lambda_{i2} - x_2 \lambda_{i3})} e^{-\lambda_{i3} y_i / x_3}.
\end{aligned} \quad (14)$$

Differentiating the c.d.f. with respect to y_i the p.d.f. of Y_i can be obtained as

$$\begin{aligned}
g_i(y_i) = & \lambda_{i1} \lambda_{i2} \lambda_{i3} \left[\frac{x_1 e^{-\lambda_{i1} y_i / x_1}}{(x_1 \lambda_{i2} - x_2 \lambda_{i1})(x_1 \lambda_{i3} - x_3 \lambda_{i1})} + \frac{x_2 e^{-\lambda_{i2} y_i / x_2}}{(x_2 \lambda_{i1} - x_1 \lambda_{i2})(x_2 \lambda_{i3} - x_3 \lambda_{i2})} \right. \\
& \left. + \frac{x_3 e^{-\lambda_{i3} y_i / x_3}}{(x_3 \lambda_{i1} - x_1 \lambda_{i3})(x_3 \lambda_{i2} - x_2 \lambda_{i3})} \right] \quad \text{if } y_i > 0.
\end{aligned} \quad (15)$$

Hence the i th probabilistic constraint can be converted to a deterministic constraint by integrating the p.d.f. $g_i(y_i)$ as follows:

$$\int_0^{b_i} g_i(y_i) dy_i \geq 1 - \alpha_i, \quad i = 1, 2, \dots, m. \quad (16)$$

This can be simplified as follows:

$$\begin{aligned}
\lambda_{i1} \lambda_{i2} \lambda_{i3} \left[\frac{x_1^2 e^{-\lambda_{i1} b_i / x_1}}{\lambda_{i1} (x_1 \lambda_{i2} - x_2 \lambda_{i1})(x_1 \lambda_{i3} - x_3 \lambda_{i1})} + \frac{x_2^2 e^{-\lambda_{i2} b_i / x_2}}{\lambda_{i2} (x_2 \lambda_{i1} - x_1 \lambda_{i2})(x_2 \lambda_{i3} - x_3 \lambda_{i2})} \right. \\
\left. + \frac{x_3^2 e^{-\lambda_{i3} b_i / x_3}}{\lambda_{i3} (x_3 \lambda_{i1} - x_1 \lambda_{i3})(x_3 \lambda_{i2} - x_2 \lambda_{i3})} \right] \leq \alpha_i, \quad i = 1, 2, \dots, m.
\end{aligned} \quad (17)$$

Finally, we generalize the result for n number of random variables by using mathematical induction.

Theorem 1. If $a_{ij}, j = 1, 2, \dots, n$, are independent exponential random variables with known means, then for some scalars $x_j, j = 1, 2, \dots, n$, the probability density function of the random variable $Y_i = \sum_{j=1}^n a_{ij} x_j$ is given by

$$g_i(y_i) = \prod_{j=1}^n \lambda_{ij} \left[\sum_{k=1}^n \frac{x_k^{n-2} e^{-\lambda_{ik} y_i / x_k}}{\prod_{l \neq k}^n (x_k \lambda_{il} - x_l \lambda_{ik})} \right] \quad \text{if } y_i > 0, \quad (18)$$

where $E(a_{ij}) = 1/\lambda_{ij}, \lambda_{ij} > 0, j = 1, 2, \dots, n$.

Proof. The method of induction is used to prove Theorem 1. For $n = 1$, $Y_i = a_{i1} x_1$. If we set $n = 1$ in Eq. (18), then we find the p.d.f. of Y_i as:

$$g_i(y_i) = \frac{\lambda_1}{x_1} e^{-\lambda_1 y_i / x_1}, \quad y_i > 0, \quad (19)$$

which is true for $n = 1$. For $n = 2$, $Y_i = a_{i1} x_1 + a_{i2} x_2$. If we set $n = 2$ in Eq. (18), then we find the p.d.f. of Y_i as

$$g_i(y_i) = \begin{cases} \frac{\lambda_{i1}\lambda_{i2}e^{-y_i\lambda_{i1}/x_1}}{x_1\lambda_{i2}-x_2\lambda_{i1}} + \frac{\lambda_{i1}\lambda_{i2}e^{-y_i\lambda_{i2}/x_2}}{x_2\lambda_{i1}-x_1\lambda_{i2}}, & \text{if } y_i > 0, \\ 0, & \text{elsewhere} \end{cases} \quad (20)$$

which is also proven in Eq. (8). We further assume that the result is true for $n = s$, i.e., the p.d.f. of $Y_i = \sum_{j=1}^s a_{ij}x_j$ is given by

$$g_i(y_i) = \prod_{j=1}^s \lambda_{ij} \left[\sum_{k=1}^s \frac{x_k^{s-2} e^{-\lambda_{ik}y_i/x_k}}{\prod_{l \neq k}^s (x_k \lambda_{il} - x_l \lambda_{ik})} \right] \quad \text{if } y_i > 0. \quad (21)$$

Finally, it is necessary to prove that the result is true for $n = s + 1$. This can be proved as follows: Let $Y_i = \sum_{j=1}^{s+1} a_{ij}x_j = a + a_{i,s+1}x_{s+1}$, where the p.d.f. of $a = \sum_{j=1}^s a_{ij}x_j$ and $a_{i,s+1}$ are given by $g(a)$ and $f(a_{i,s+1})$, respectively as:

$$g(a) = \prod_{j=1}^s \lambda_{ij} \left[\sum_{k=1}^s \frac{x_k^{s-2} e^{-\lambda_{ik}a/x_k}}{\prod_{l \neq k}^s (x_k \lambda_{il} - x_l \lambda_{ik})} \right] \quad \text{if } a > 0, \quad (22)$$

$$f(a_{i,s+1}) = \begin{cases} \lambda_{i,s+1} e^{-a_{i,s+1}\lambda_{i,s+1}}, & a_{i,s+1} > 0, \\ 0, & \text{elsewhere.} \end{cases} \quad (23)$$

The c.d.f. of the random variable $Y_i = a + a_{i,s+1}x_{s+1}$ is given by:

$$G_i(y_i) = \int_{a_{i,s+1}=0}^{y_i/x_{s+1}} \int_{a=0}^{y_i - a_{i,s+1}x_{s+1}} g(a) f(a_{i,s+1}) da da_{i,s+1} \quad (24)$$

or

$$G_i(y_i) = \int_{a_{i,s+1}=0}^{y_i/x_{s+1}} \int_{a=0}^{y_i - a_{i,s+1}x_{s+1}} \left(\prod_{j=1}^{s+1} \lambda_{ij} \right) e^{-a_{i,s+1}\lambda_{i,s+1}} \times \left[\sum_{k=1}^s \frac{x_k^{s-2} e^{-\lambda_{ik}a/x_k}}{\prod_{l \neq k}^s (x_k \lambda_{il} - x_l \lambda_{ik})} \right] da da_{i,s+1} \quad (25)$$

or

$$G_i(y_i) = \int_{a_{i,s+1}=0}^{y_i/x_{s+1}} \prod_{j=1}^{s+1} \lambda_{ij} \left[\sum_{k=1}^s \frac{x_k^{s-1} \left(e^{-\lambda_{i,s+1}a_{i,s+1}} - \exp \left\{ \frac{-\lambda_{ik}y_i}{x_k} - a_{i,s+1} \left(\frac{x_k \lambda_{i,s+1} - x_{s+1} \lambda_{ik}}{x_k} \right) \right\} \right)}{\lambda_{ik} \prod_{l \neq k}^s (x_k \lambda_{il} - x_l \lambda_{ik})} \right] da_{i,s+1} \quad (26)$$

or

$$G_i(y_i) = \prod_{j=1}^{s+1} \lambda_{ij} \left\{ \sum_{k=1}^s \left[\frac{x_k^{s-1} (1 - e^{-\lambda_{i,s+1}y_i/x_{s+1}})}{\lambda_{ik} \lambda_{i,s+1} \prod_{l \neq k}^s (x_k \lambda_{il} - x_l \lambda_{ik})} - \frac{x_k^s (e^{-\lambda_{ik}y_i/x_k} - e^{-\lambda_{i,s+1}y_i/x_{s+1}})}{\lambda_{ik} \prod_{l \neq k}^s (x_k \lambda_{il} - x_l \lambda_{ik})} \right] \right\}. \quad (27)$$

Differentiating $G_i(y_i)$ with respect to y_i the p.d.f. of the Y_i can be obtained as

$$g_i(y_i) = \prod_{j=1}^{s+1} \lambda_{ij} \left[\sum_{k=1}^{s+1} \frac{x_k^{s-1} e^{-\lambda_{ik} y_i / x_k}}{\prod_{\substack{l=1 \\ l \neq k}}^{s+1} (x_k \lambda_{il} - x_l \lambda_{ik})} \right] \quad \text{if } y_i > 0. \quad (28)$$

This completes the proof. $\square$

As a special case if we assume $Y_i = a_{i1} + a_{i2}$, where a_{i1} and a_{i2} are the two independent exponential random variables with $E(a_{i1}) = E(a_{i2}) = 1/\lambda_i$, then the random variable Y_i can be proven to follow a gamma distribution. If we set $\lambda_{i1} = \lambda_{i2} = \lambda_i$ in Eq. (8) we find

$$g_i(y_i) = \frac{\lambda_i e^{-y_i \lambda_i / x_1}}{x_1 - x_2} + \frac{\lambda_i e^{-y_i \lambda_i / x_2}}{x_2 - x_1} \quad (29)$$

or

$$g_i(y_i) = \frac{\lambda_i}{x_1 - x_2} \left\{ \left(1 - \frac{y_i \lambda_i}{x_1} + \frac{y_i^2 \lambda_i^2}{2! x_1^2} - \dots \right) - \left(1 - \frac{y_i \lambda_i}{x_2} + \frac{y_i^2 \lambda_i^2}{2! x_2^2} - \dots \right) \right\} \quad (30)$$

or

$$g_i(y_i) = \frac{\lambda_i^2 y_i}{(x_1 - x_2)} \left\{ \frac{(x_1 - x_2)}{x_1 x_2} - \frac{y_i \lambda_i (x_1^2 - x_2^2)}{2! x_1^2 x_2^2} + \frac{y_i^2 \lambda_i^2 (x_1^3 - x_2^3)}{3! x_1^3 x_2^3} - \dots \right\}. \quad (31)$$

After cancelling $(x_1 - x_2)$ from the numerator and denominator and setting $x_1 = x_2 = 1$ we find the p.d.f. of $Y_i = a_{i1} + a_{i2}$ as:

$$g_i(y_i) = \lambda_i^2 y_i \left(1 - \frac{\lambda_i y_i}{1!} + \frac{\lambda_i^2 y_i^2}{2!} - \dots \right), \quad y_i > 0, \quad (32)$$

or

$$g_i(y_i) = \lambda_i^2 y_i e^{-\lambda_i y_i}, \quad y_i > 0. \quad (33)$$

Now it is clear that $Y_i = a_{i1} + a_{i2}$ follows gamma distribution.

For general case the mean and the variance of the random variable Y_i can be obtained by using the properties of the independent random variables as

$$E(Y_i) = \sum_{j=1}^n x_j / \lambda_{ij}, \quad (34)$$

$$\text{Var}(Y_i) = \sum_{j=1}^n x_j^2 / \lambda_{ij}^2. \quad (35)$$

Using the p.d.f. of Y_i as stated in Eq. (18) again the mean and the variance of Y_i can be computed as given in Eqs. (34) and (35), respectively.

Finally, for the general case a deterministic constraint can be obtained by integrating the p.d.f. of Y_i as stated below

$$\int_0^{b_i} g_i(y_i) dy_i \geq 1 - \alpha_i, \quad i = 1, 2, \dots, m. \quad (36)$$

The above integral can be simplified as follows:

$$\prod_{j=1}^n \lambda_{ij} \left[\sum_{k=1}^n \frac{x_k^{n-1} e^{-\lambda_{ik} b_i / x_k}}{\lambda_{ik} \prod_{l \neq k}^n (x_k \lambda_{il} - x_l \lambda_{ik})} \right] \leq \alpha_i, \quad i = 1, 2, \dots, m. \quad (37)$$

3. Deterministic model of the probabilistic linear programming problem

If we assume a_{ij} , and c_j are independent exponential random variables with known distributions, then the probabilistic linear programming model as stated in Eqs. (1)–(3) can be transformed to a deterministic non-linear programming problem as

$$\max: \quad E[Z] = \sum_{j=1}^n E[c_j] x_j \quad (38)$$

$$\text{s.t.} \quad \prod_{j=1}^n \lambda_{ij} \left[\sum_{k=1}^n \frac{x_k^{n-1} e^{-\lambda_{ik} b_i / x_k}}{\lambda_{ik} \prod_{l \neq k}^n (x_k \lambda_{il} - x_l \lambda_{ik})} \right] \leq \alpha_i, \quad i = 1, 2, \dots, m, \quad (39)$$

$$x_j \geq 0, \quad j = 1, 2, \dots, n. \quad (40)$$

In this section one numerical example is presented to illustrate the methodology.

Example 1.

$$\max: \quad Z = c_1 x_1 + c_2 x_2 + c_3 x_3 \quad (41)$$

$$\text{s.t.} \quad \text{Prob}(a_{11} x_1 + a_{12} x_2 + a_{13} x_3 \leq 10) \geq 0.95, \quad (42)$$

$$\text{Prob}(a_{21} x_1 + a_{22} x_2 + a_{23} x_3 \leq 20) \geq 0.90, \quad (43)$$

$$x_1, x_2, x_3 \geq 0. \quad (44)$$

In the above numerical example a_{ij} and c_j are independent exponential random variables with known means

$$E(a_{11}) = 5, \quad E(a_{12}) = 4, \quad E(a_{13}) = 8, \quad E(a_{21}) = 10, \quad E(a_{22}) = 2, \quad E(a_{23}) = 20,$$

$$E(c_1) = 5, \quad E(c_2) = 6, \quad E(c_3) = 3, \quad b_1 = 10, \quad b_2 = 20, \quad \alpha_1 = 0.05, \quad \alpha_2 = 0.10.$$

The deterministic model can be obtained using Eqs. (38)–(40) as

$$\max: \quad E[Z] = 5x_1 + 6x_2 + 3x_3 \quad (45)$$

$$\text{s.t. } \frac{0.03125x_1^2e^{-2/x_1}}{(0.25x_1 - 0.2x_2)(0.125x_1 - 0.2x_3)} + \frac{0.025x_2^2e^{-2.5/x_2}}{(0.2x_2 - 0.25x_1)(0.125x_2 - 0.25x_3)} + \frac{0.05x_3^2e^{-1.25/x_3}}{(0.2x_3 - 0.125x_1)(0.25x_3 - 0.125x_2)} \leq 0.05, \quad (46)$$

$$\frac{0.025x_1^2e^{-2/x_1}}{(0.5x_1 - 0.1x_2)(0.05x_1 - 0.1x_3)} + \frac{0.005x_2^2e^{-10/x_2}}{(0.1x_2 - 0.5x_1)(0.05x_2 - 0.5x_3)} + \frac{0.05x_3^2e^{-1/x_3}}{(0.1x_3 - 0.05x_1)(0.5x_3 - 0.05x_2)} \leq 0.10, \quad (47)$$

$$x_1, x_2, x_3 \geq 0. \quad (48)$$

The above deterministic problem is solved by using GINO package [9] and the optimal solution is obtained as

$$x_1 = 0.001177, \quad x_2 = 0.346094, \quad x_3 = 0.000010, \quad \text{and} \quad E[Z] = 2.082476.$$

4. Conclusions

In this paper we have established the p.d.f. of the random variable $Y_i = \sum_{j=1}^n a_{ij}x_j$, where a_{ij} are independent exponential random variables with known means. Then using the derived p.d.f. a probabilistic linear programming problem can be transformed into a deterministic non-linear programming problem. The result in this paper has generalized the existing literature to cases with n independent exponential random variables. If there are multiple objective functions present in the probabilistic model, some extensions can be easily done by using the deterministic model. To solve the multi-objective deterministic problem we may use either non-inferior solution methodology [10] or fuzzy programming approach [11,12]. We may apply the fuzzy programming approach to obtain the optimal compromise solution. If the parameter b_i is the only exponential random variable in a probabilistic constraint, by integrating the probability density function of b_i the deterministic model can be obtained for a probabilistic linear programming problem [7]. For situations where all the a_{ij} and b_i are exponential random variables in a probabilistic constraint with known means, then the p.d.f. of $Y_i = \sum_{j=1}^n a_{ij}x_j - b_i$ need to be obtained and the deterministic model need to be established.

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