Recap: Tensor Algebra - Intro

* Quantity Types:

1. Scalar: v

2. vector: $\vec{v} = v_x \vec{i} + v_y \vec{j}$

   or

   $\vec{V} = V_X \bar{I} + V_Y \bar{J}$

3. Tensor

$$\vec{\vec{V}} = \vec{V}_X \bar{I} + \vec{V}_Y \bar{J}$$

$$= (V_{Xx} \vec{i} + V_{Xy} \vec{j}) \bar{I} + (V_{Yx} \vec{i} + V_{Yy} \vec{j}) \bar{J}$$

or

$$\vec{\vec{V}} = \vec{\vec{V}}_x \vec{i} + \vec{\vec{V}}_y \vec{j} =$$

$$= (V_{Xx} \bar{I} + V_{Yx} \bar{J}) \vec{i} + (V_{Xy} \bar{I} + V_{Yy} \bar{J}) \vec{j}$$

** differential Directional Operators:

∇ : del operator or Gradient

$$= \left(\frac{\partial}{\partial x} \vec{i} + \frac{\partial}{\partial y} \vec{j}\right) \text{ of a scalar}$$

ای باد حدیث من نهانش میگو   سر دل من به صد زبانش میگو

میگو نه بدان سان که ملالش گیرد   میگو سخنی و در میانش میگو

Or

$$\nabla = \frac{\partial}{\partial X}\bar{I} + \frac{\partial}{\partial Y}\bar{J}$$

*** Operator Interpretations:

∇ : increase the quantity for one dimension or and interpretes differentiation as vector component. It is used ~~when~~ to interprete differentiation as direction or vector component and vice versa. Therefor, we can use the differentiation rules for directional calculations or vector algebra.

$\nabla\cdot$: divergence operator, unlike gradient operator reduces a ~~vector~~ quantitie's dimensions by one order.

به مژگان سیه کردی هزاران رخنه در دینم   بیا کز چشم بیمارت هزاران درد برچینم

الا ای همنشین دل که یارانت برفت از یاد   مرا روزی مباد آن دم که بی یاد تو بنشینم

therefore, Gradient and Divergence operators could be looked as inverse of each other.

it could ~~also~~ be illustrated ~~using~~ by Double Derham theory.

$$\text{scalar} \underset{\nabla\cdot}{\overset{\cancel{\nabla}}{\rightleftarrows}} \text{vector} \underset{\nabla\times}{\overset{\nabla\times}{\rightleftarrows}} \cdot \;\text{Scalar} \underset{\nabla}{\overset{\nabla\cdot}{\rightleftarrows}}$$

~~more~~ physicall interpretations could be found in fluid mechanics.

**** some vector algebra equalities & identities

$$(\vec{V}\cdot\nabla)\vec{V} - \vec{V}\times(\nabla\times\vec{V}) = \frac{1}{2}\nabla\left(\frac{|\vec{V}|^2}{1}\right)$$

$$\nabla(\nabla\cdot\vec{V}) - \nabla\times(\nabla\times\vec{V}) = \nabla\cdot\nabla(\vec{V})$$

دیشب بسیل اشک ره خواب میزدم   نقشی بیاد خط تو بر آب میزدم
ابروی یار در نظر و خرقه سوخته   جامی بیاد گوشه محراب میزدم

How a vector is produced?

$$\vec{v} = \nabla \cdot v = v_x \vec{i} + v_y \vec{j} = \frac{\partial v}{\partial x} \bar{i} + \frac{\partial v}{\partial y} \bar{j}$$

therefore, using the previous definition, the index x, for instance, could be interpreted

both as { direction/ or xth element of a vector
or as { ~~$\overline{\partial}$~~ Partial derivative of a scalar.

in other words:

~~How a tensor is produced?~~

index

(I) partial derivative

(II) ith direction or component of a vector

?

How a tensor is produced? gradient of a vector

$$\vec{\vec{v}} = \nabla \vec{\vec{v}} \quad \text{while} \quad \vec{v} = \blacktriangledown v$$

therefore a tensor =

$$\vec{\vec{V}} = \nabla \vec{\vec{V}} = \nabla (\blacktriangledown v)$$

where: $\nabla = \frac{\partial}{\partial x} \vec{i} + \frac{\partial}{\partial y} \vec{j}$

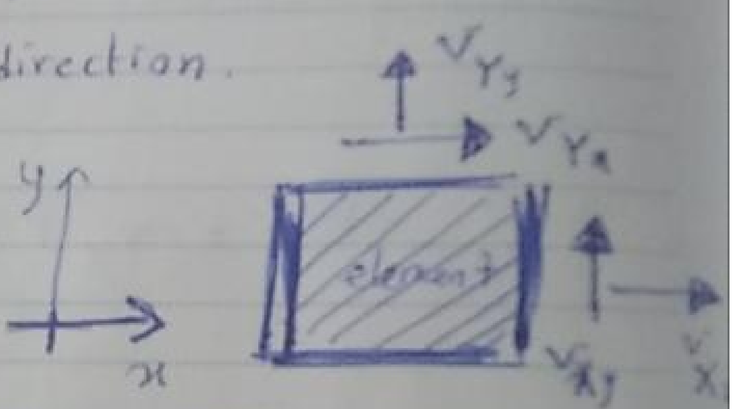
and $\blacktriangledown = \frac{\partial}{\partial X} \vec{I} + \frac{\partial}{\partial Y} \vec{J}$

ای سایه‌ی سنبلت سمن پرورده — یاقوت لبت در عدن پرورده
همچون لب خود مدام جان میپرور — زان راح که روح بیت تن پرورده

- Here upper case letters account for directions or differentiations of plane / surface of action (where a force is applied)
- and lower case letter show directions of vectors.

for example V_{Xy} is a component of a force in y direction which is applied to the surface ~~X~~ with normal in X direction.

Therefor, upper case and lower case indices are of different nature



and might not ~~be operate on ea~~ impact each other.

در خرابات مغان نور خدا می‌بینم   این عجب بین که چه نوری ز کجا می‌بینم

جلوه بر من مفروش ای ملک الحاج که تو   خانه می‌بینی و من خانه خدا می‌بینم

vector - tensor - scalar productions :

(I) consider a scalar v :

$$\vec{v} = \nabla v = v_x \vec{i} + v_y \vec{j} \qquad (1)$$

v_x and v_y ~~are~~ represent both x and y direction components as well as partial derivatives of v in x and y directions in order.

$$\vec{\vec{v}} = \nabla v = v_X \vec{I} + v_Y \vec{J} \qquad (2)$$

(II) consider vector $\vec{v}$:

we know that $\vec{v} = \nabla v = v_x \vec{i} + v_y \vec{j}$

$$\nabla \vec{v} = \vec{v}_X \vec{I} + \vec{v}_Y \vec{J} = \frac{\partial \vec{v}}{\partial X} I + \frac{\partial \vec{v}}{\partial Y} J$$

if we ~~[illegible]~~ treat $\vec{v}$ as a scalar for ∇

ز دست کوته خود زیر بارم	که از بالابلندان شرمسارم
مگر زنجیر مویی گیردم دست	وگرنه سر به شیدایی برآرم

۲۹ شوال ۱۴۴۳

(1) $\nabla v = v_x i + v_y j$ (2) $\nabla v = v_X I + v_Y J$

(3) $\nabla . v = \nabla . (v_x i + v_y j) = v_{xx} + v_{yy}$

$\nabla . \vec{v} = (\frac{\partial}{\partial X} I + \frac{\partial}{\partial Y} J) . (v_x i + v_y j)$

→ scalar: X X

→ vector: ✓

→ tensor ← is not

→ vector product.

$\nabla . \vec{v} = \nabla . \nabla \tilde{v} = (\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}) v = v_{xx} + v_{yy}$

$\nabla . \vec{v} = \nabla . \nabla v = (\frac{\partial^2}{\partial X \partial x} i . I + \frac{\partial^2}{\partial X \partial y} I_j + \frac{\partial^2}{\partial Y \partial x} J_i$

$+ \frac{\partial^2}{\partial Y \partial y} J . j \Rightarrow$

در دشتی گشته‌ام که می‌پرسی   زیر چرخی چشیده‌ام که مپرس
گشته‌ام در جهان و آخر کار   دلبری برگزیده‌ام که مپرس

Result: net volume specific force exerting on a fluid volume equals:

$$\nabla\cdot\underbrace{\vec{\vec{\tau}}}_{\text{tensor}} = \nabla\cdot\nabla\underbrace{\vec{\tau}}_{\text{vector}} = \nabla\cdot\nabla(\nabla\underbrace{\tau}_{\text{scalar}})$$

Operator $\nabla ⊛$

- [illegible]
- مشتق
- [illegible]

Operand V

- scalar
- vector
- tensor
- vector product