Chapter 5: Capacitance

- ✓ Capacitor
- ✓ Parallel Plate Capacitor
- ✓ Cylindrical Capacitor
- ✓ Spherical Capacitor
- ✓ Combinations of Capacitors
- ✓ Energy in a Capacitor
- ✓ Capacitors with Dielectrics

Session 11:

- ✓ Capacitor
- ✓ Parallel Plate Capacitor
- ✓ Cylindrical Capacitor
- ✓ Spherical Capacitor
- ✓ Combinations of Capacitors
- ✓ Examples

Introduction

- Capacitors are devices that store electric charge.

- Examples of where capacitors are used include:
  - radio receivers
  - filters in power supplies
  - to eliminate sparking in automobile ignition systems
  - energy-storing devices in electronic flashes

Capacitor

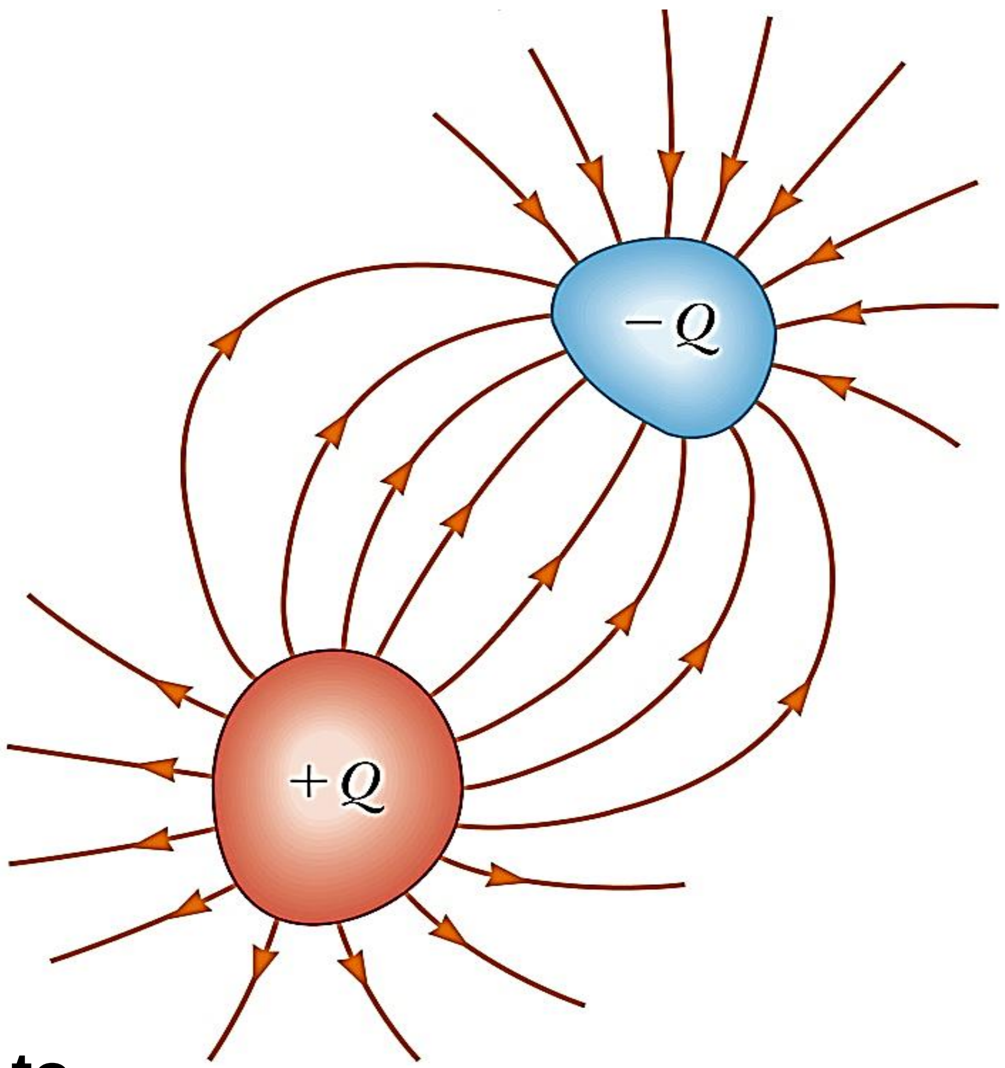
❖ **A capacitor consists of two conductors.**

❖ **When the conductor is charged, the plates carry charges of equal magnitude and opposite directions.**

❖ A **potential difference** exists between the plates due to the charge.

The **capacitance**, *C*, of a capacitor is defined as:

$$\boxed{C = \frac{Q}{\Delta V}}$$

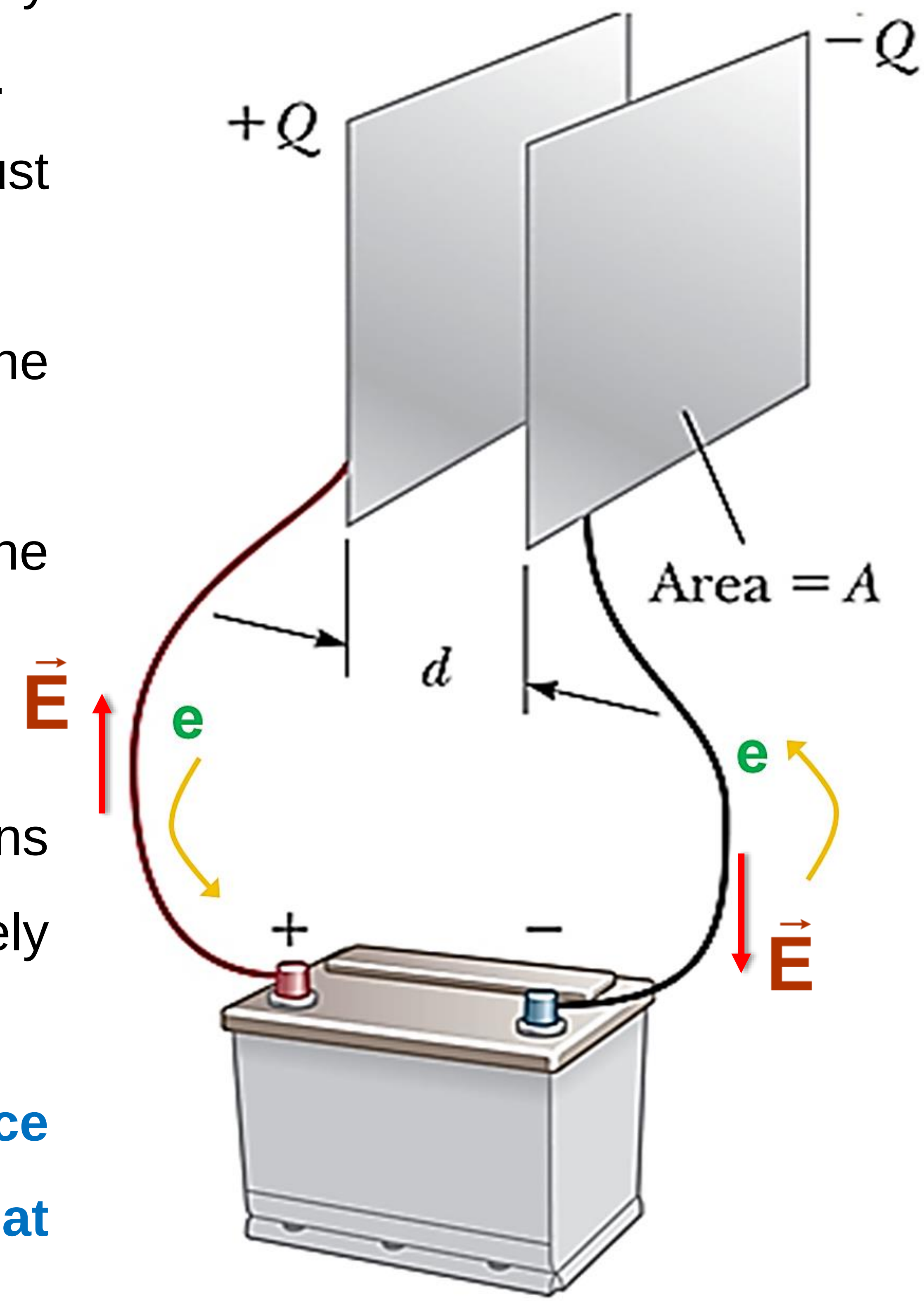


- The SI unit of capacitance is the **farad** (F).
- Capacitance will always be a **positive quantity**
- The capacitance of a given capacitor is **constant**.
- The capacitance is a measure of the capacitor's **ability to store charge .**

Parallel Plate Capacitor

- ✓ If the capacitor is initially uncharged, the battery establishes an **electric field in the connecting wires**.
- ✓ This field applies **a force on electrons in the wire** just outside of the plates.
- ✓ The force causes the electrons to move onto the negative plate.
- ✓ This continues **until equilibrium** is achieved and the plate is now negatively charged.

- ✓ A similar process occurs at the other plate, electrons moving away from the plate and leaving it positively charged.
- ✓ In its final configuration, the **potential difference across the capacitor plates is the same as that between the terminals of the battery.**



Capacitance of a Parallel Plate Capacitor

$$\varepsilon_0 \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = q$$

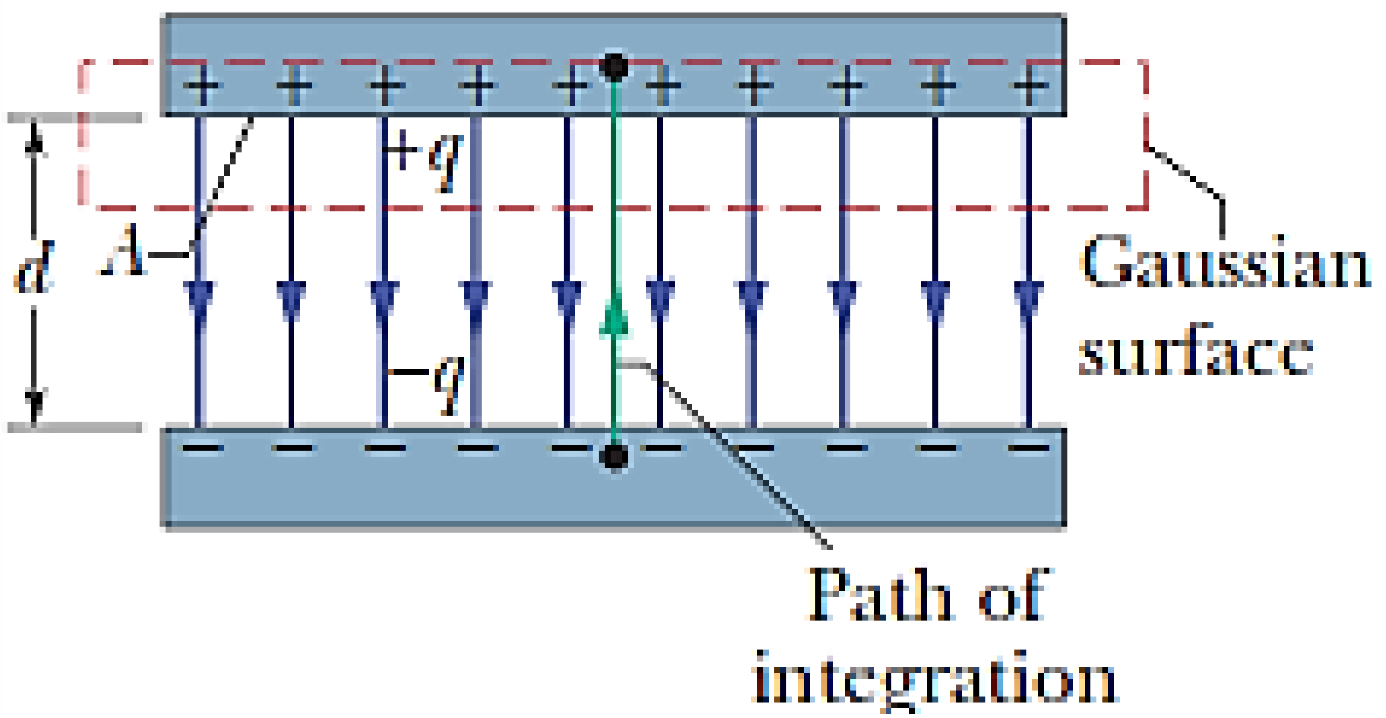
$$q = \varepsilon_0 E\ A$$

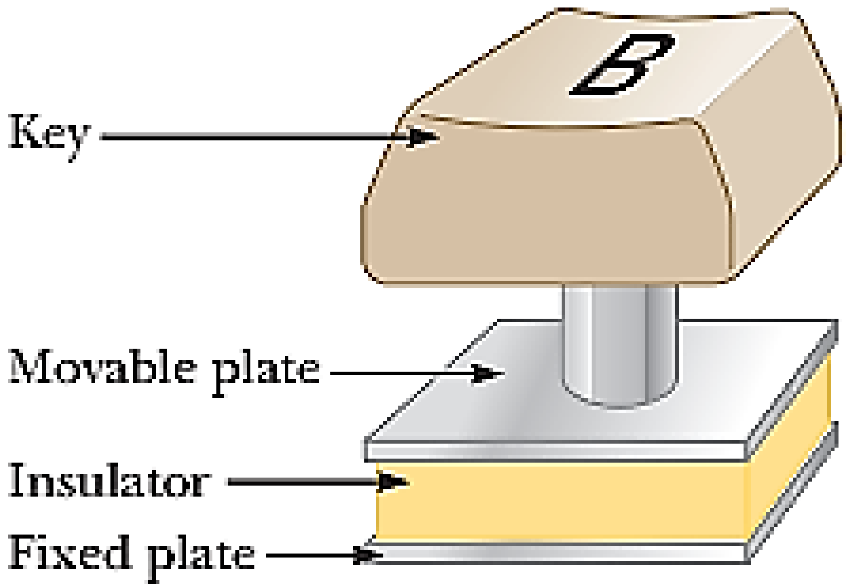
$$V_f - V_i = -\int_i^f \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$$

$$V = -\int_i^f E\, ds\,(\cos 180) = \int_-^+ E\, ds$$

$$V = E \int_-^+ ds = E\, d$$

$$C = \frac{q}{V} = \frac{q}{E\,d} = \frac{q}{\dfrac{q}{\varepsilon_0 A} d} \Rightarrow \boxed{C = \varepsilon_0 \frac{A}{d}}$$

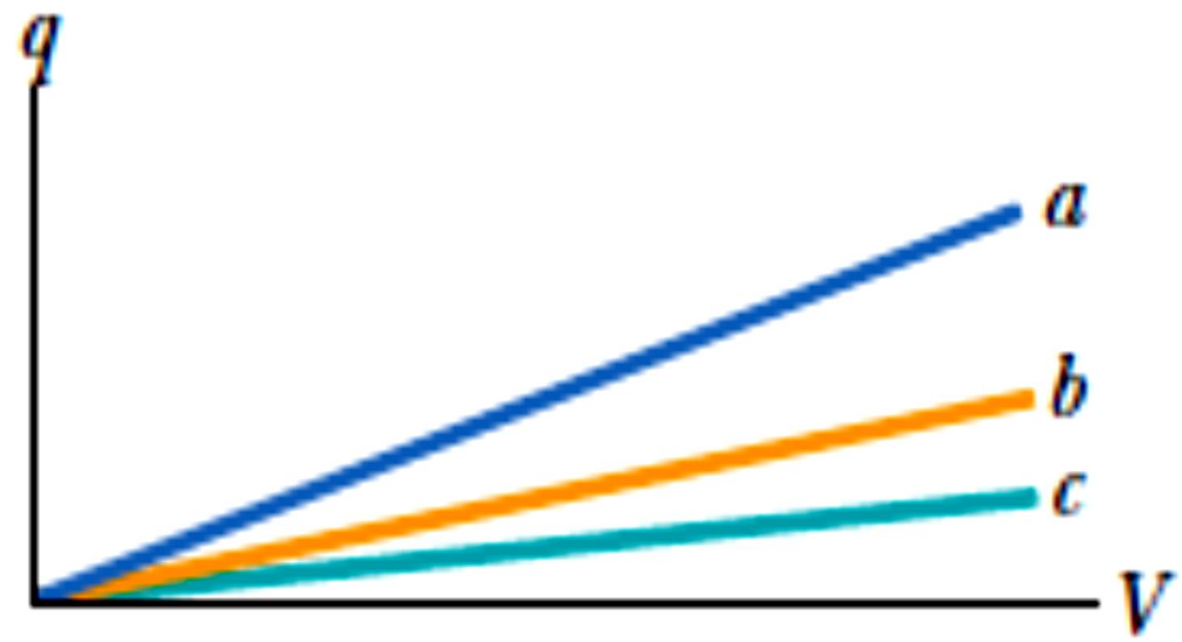




$$\varepsilon_0 = 8.85 \times 10^{-12}\ \mathrm{F/m}$$

Ex 1. Figure below shows plots of charge versus potential difference for **three parallel-plate capacitors** that have the plate areas and separations given in the table. Which plot goes with which capacitor?

Capacitor	Area	Separation
1	A	d
2	$2A$	d
3	A	$2d$



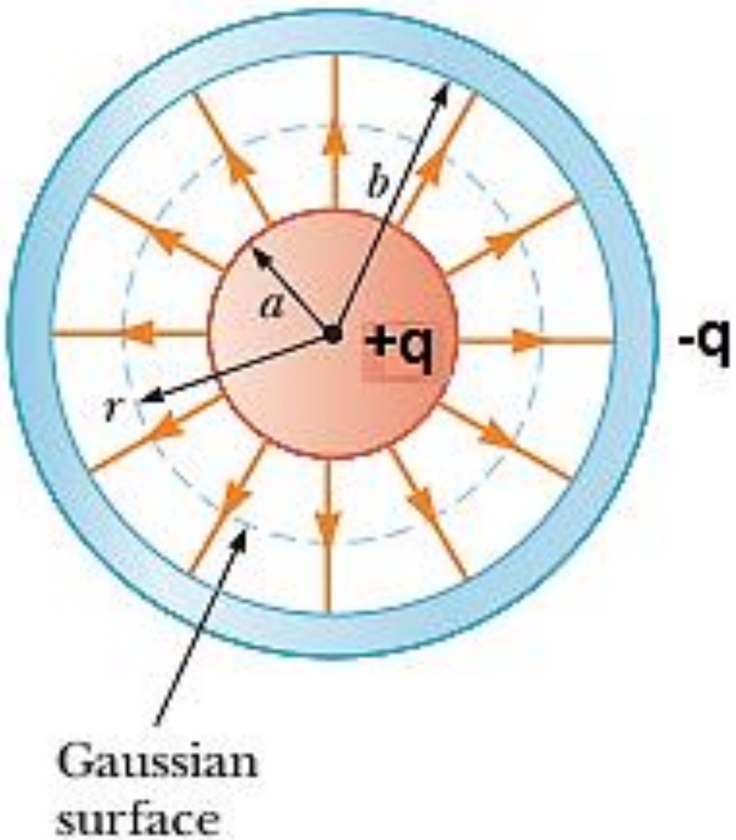
$$C_1 = \varepsilon_0 \frac{A}{d} \quad (b)$$

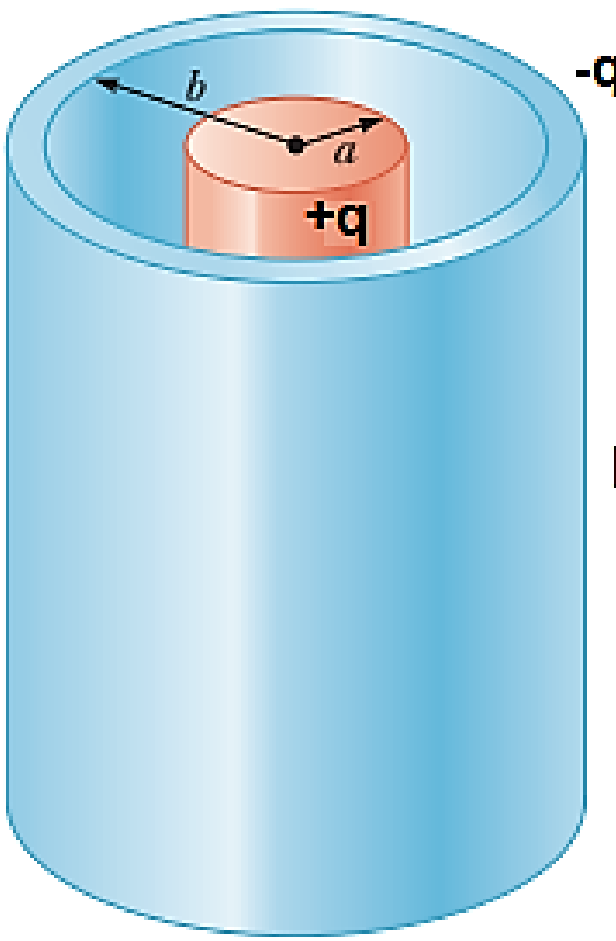
$$C_2 = \varepsilon_0 \frac{2A}{d} \quad (a)$$

$$C_3 = \varepsilon_0 \frac{A}{2d} \quad (c)$$

Capacitance of a Cylindrical Capacitor

Ex 2. A **solid cylindrical conductor** of radius **a** and charge **+q** is coaxial with a **cylindrical shell** of negligible thickness, radius **b > a** and charge **-q**. Find the **capacitance of this cylindrical capacitor** if its length is ***L***, ***L* >> a,b**.



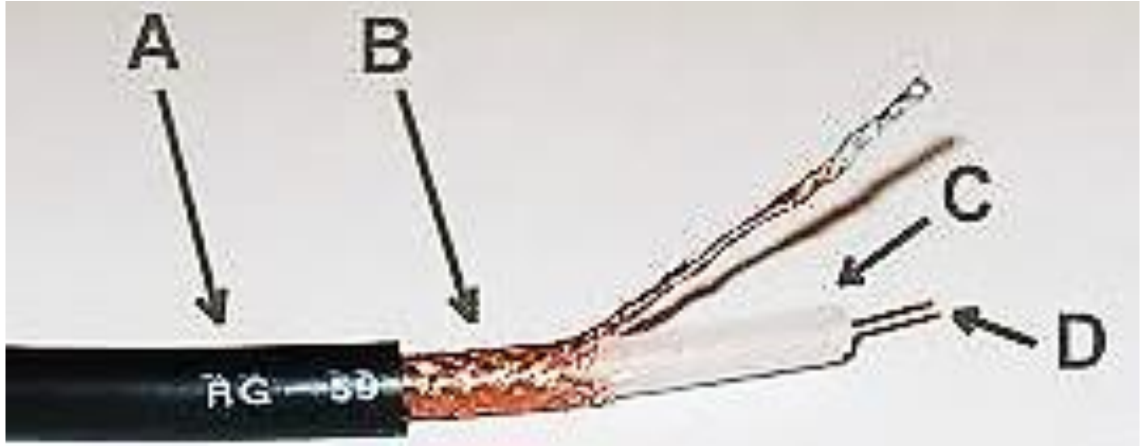


$$q = \varepsilon_0 \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \varepsilon_0 E\, A$$

$$E = \frac{q}{2\pi\varepsilon_0 L r}$$

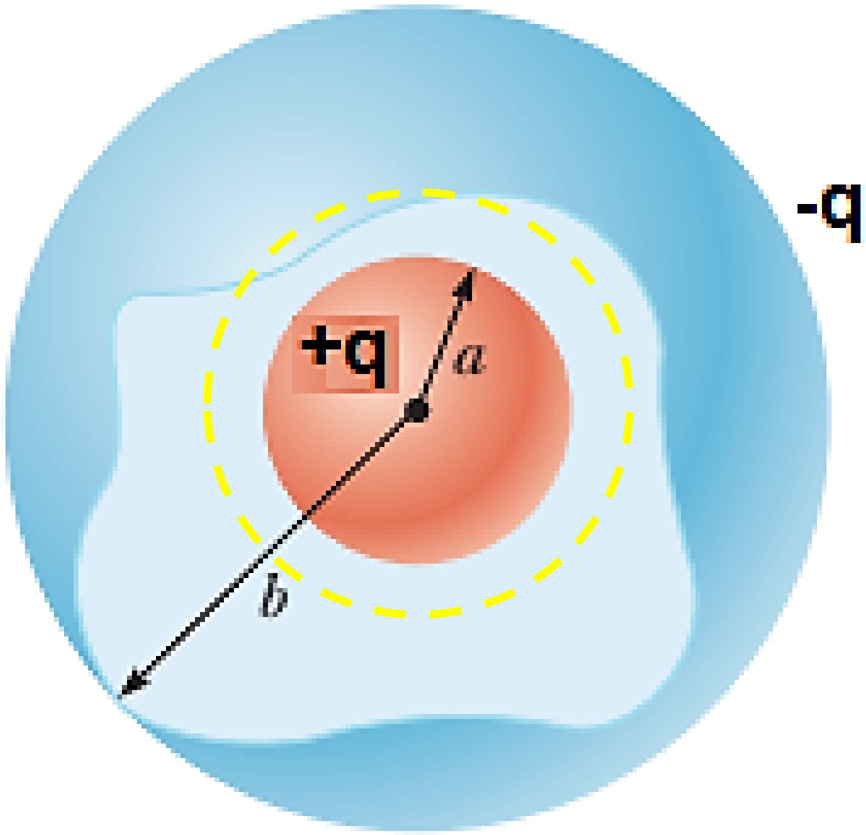
$$V = \int_{-}^{+} E\, ds = -\frac{q}{2\pi\varepsilon_0 L}\int_b^a \frac{dr}{r} = \frac{q}{2\pi\varepsilon_0 L}\ln\left(\frac{b}{a}\right)$$

$$\boxed{C = \frac{q}{V} = \frac{2\pi\varepsilon_0 L}{\ln\left(\frac{b}{a}\right)}}$$



Capacitance of a Spherical Capacitor

Ex 3. A spherical capacitor consists of a spherical conducting shell of radius **b** and charge **-q** concentric with a smaller conducting sphere of radius **a** and charge **+q**. Find the **capacitance** of this device.



$$q = \varepsilon_0 \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \varepsilon_0 E\ A$$

$$E = \frac{q}{4\pi\varepsilon_0 r^2}$$

$$V = \int_{-}^{+} E\, ds = -\frac{q}{4\pi\varepsilon_0}\int_b^a \frac{dr}{r^2} = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)$$

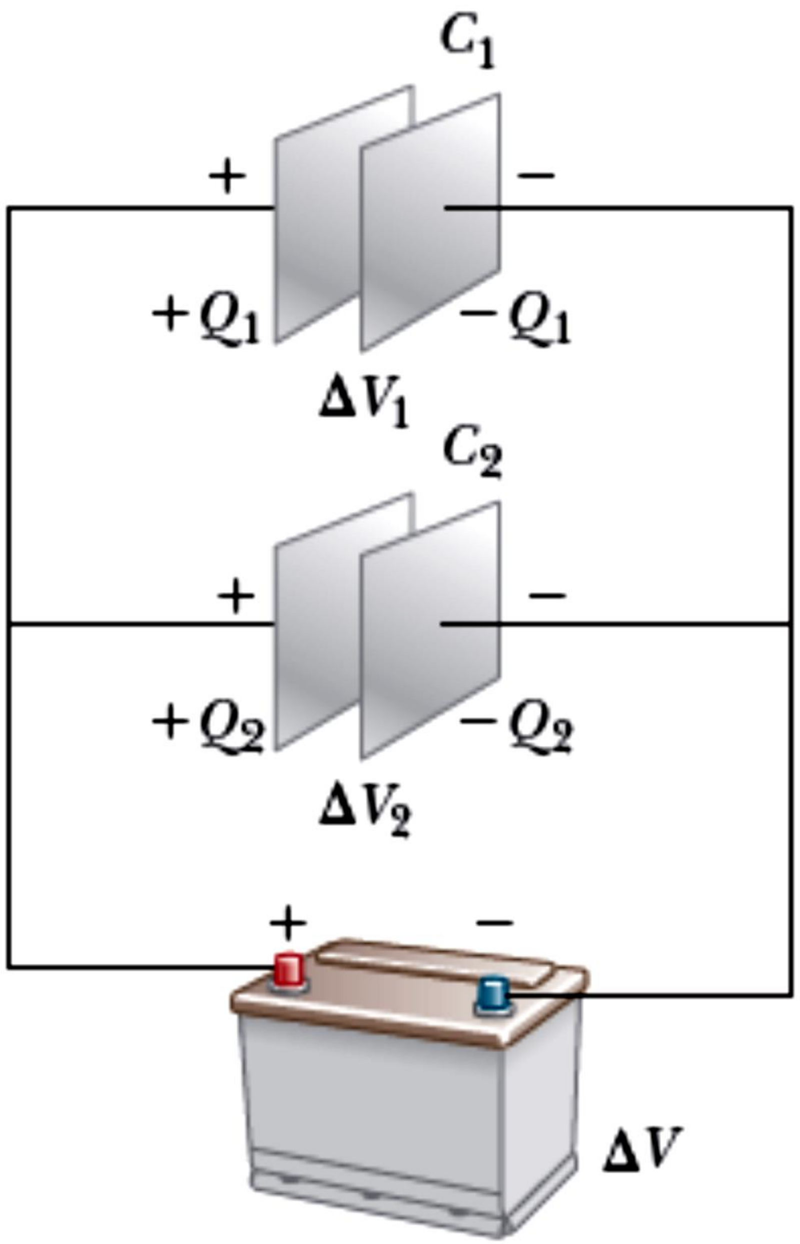
$$\boxed{C = \frac{q}{V} = \frac{4\pi\varepsilon_0}{\left(\frac{1}{a} - \frac{1}{b}\right)}}$$

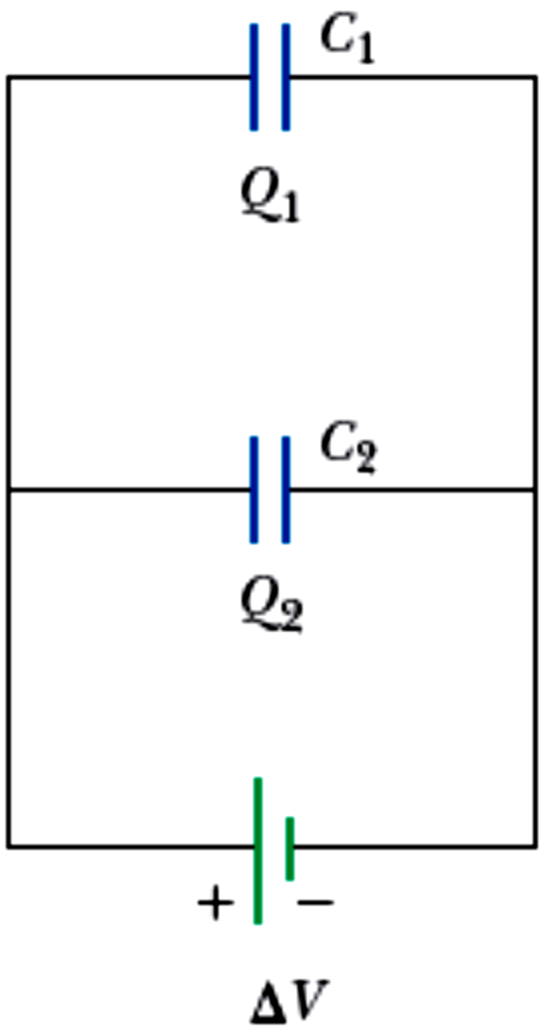
For Isolated Sphere:

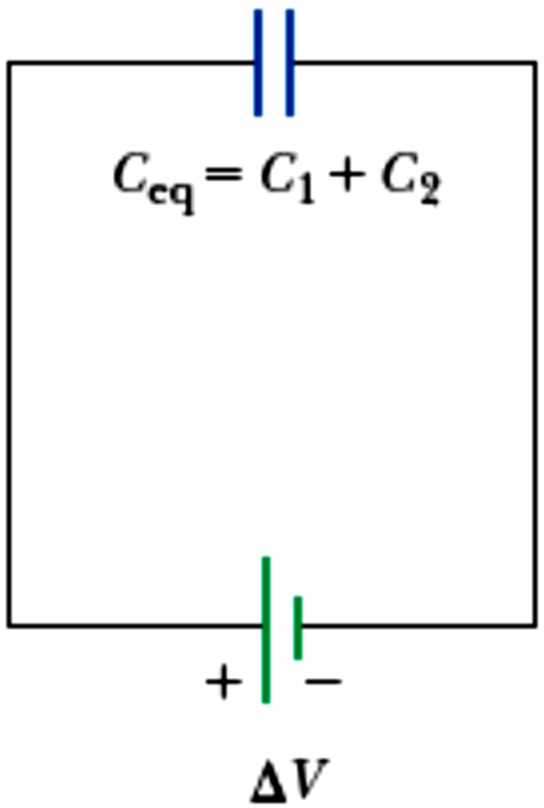
$$C = \lim_{b\to\infty} \frac{4\pi\varepsilon_0}{\left(\frac{1}{a} - \frac{1}{b}\right)} \Rightarrow \boxed{C = 4\pi\varepsilon_0 a}$$

Combinations of Capacitors

1) Capacitors in Parallel:





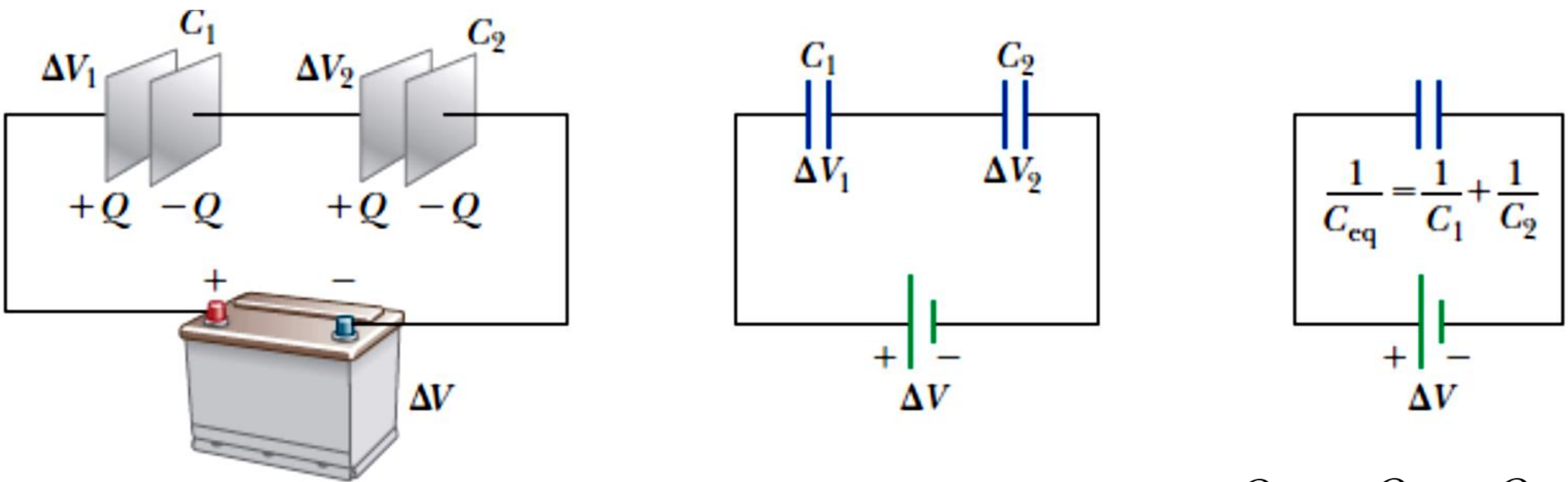


$$\Delta V_1 = \Delta V_2 = \Delta V \qquad Q_{tot} = Q_1 + Q_2 = C_1 \Delta V_1 + C_2 \Delta V_2 \qquad Q_{tot} = C_{eq} \Delta V$$

$$\boxed{C_{eq} = C_1 + C_2} \qquad \boxed{C_{eq} = C_1 + C_2 + C_3 + \ldots}$$

COMBINATIONS OF CAPACITORS

2) Capacitors in Series:



$$Q = Q_1 = Q_2 \qquad \Delta V_{tot} = \Delta V_1 + \Delta V_2 = \frac{Q_1}{C_1} + \frac{Q_2}{C_2} \qquad \frac{Q}{C_{eq}} = \frac{Q_1}{C_1} + \frac{Q_2}{C_2}$$

$$\boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}}$$

$$\boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots}$$

Ex 4. Find **the equivalent capacitance** between **a** and **b** for the combination of capacitors shown in. All capacitances are in microfarads.