Chapter 12: Equilibrium and Elasticity

Session 26:

- ✓ Equilibrium
- ✓ Examples

Equilibrium

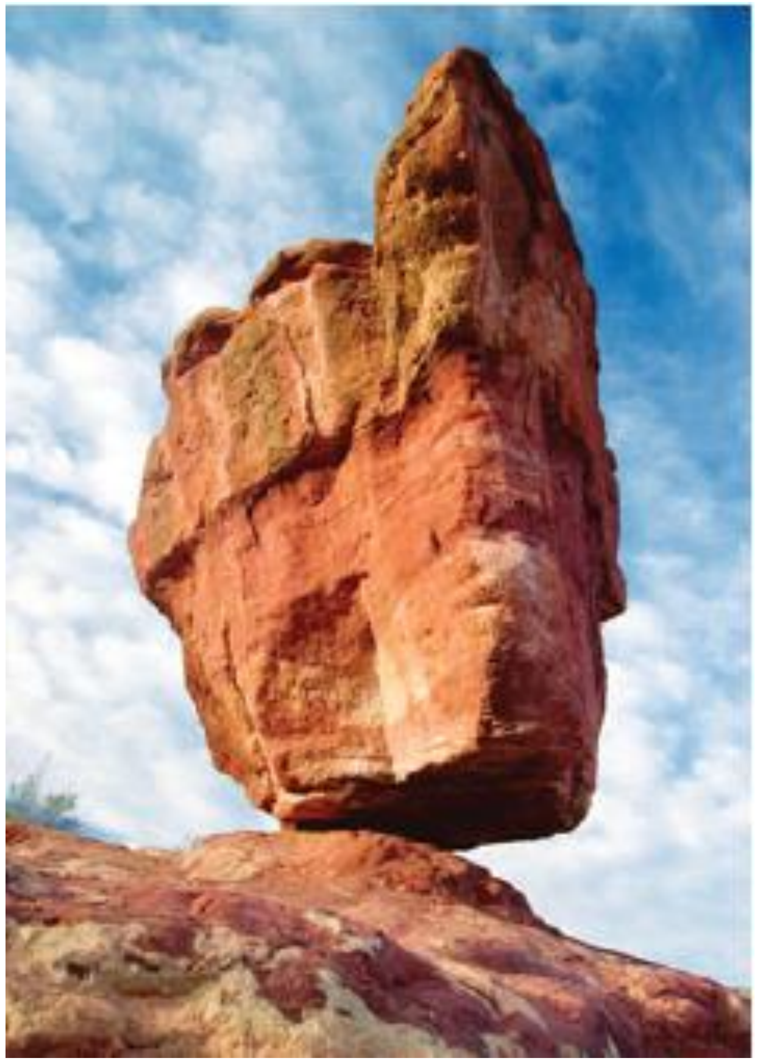
❖ **Equilibrium** implies that the **object moves with both constant velocity and constant angular velocity** relative to an observer in an inertial reference frame.

$$\vec{P} = \text{constant} \quad ; \quad \vec{L} = \text{constant}$$

❖ **Static Equilibrium**:

$$\vec{P} = 0 \quad (\vec{v}_{com} = 0)$$

$$\vec{L} = 0 \quad (\omega = 0)$$



1) Balance of Forces:

$$\vec{\mathbf{F}}_{net} = \frac{d\vec{\mathbf{p}}}{dt} = 0$$

2) Balance of Torques:

$$\vec{\tau}_{net} = \frac{d\vec{L}}{dt} = 0$$

Equilibrium

$$\vec{F}_{net} = 0 \Rightarrow \begin{cases} F_{net,x} = 0 \\ F_{net,y} = 0 \\ F_{net,z} = 0 \end{cases} \qquad \vec{\tau}_{net} = 0 \Rightarrow \begin{cases} \tau_{net,x} = 0 \\ \tau_{net,y} = 0 \\ \tau_{net,z} = 0 \end{cases}$$

Equilibrium in the xy plane: $\boxed{F_{net,x} = 0 \;;\; F_{net,y} = 0 \;;\; \tau_{net,z} = 0}$

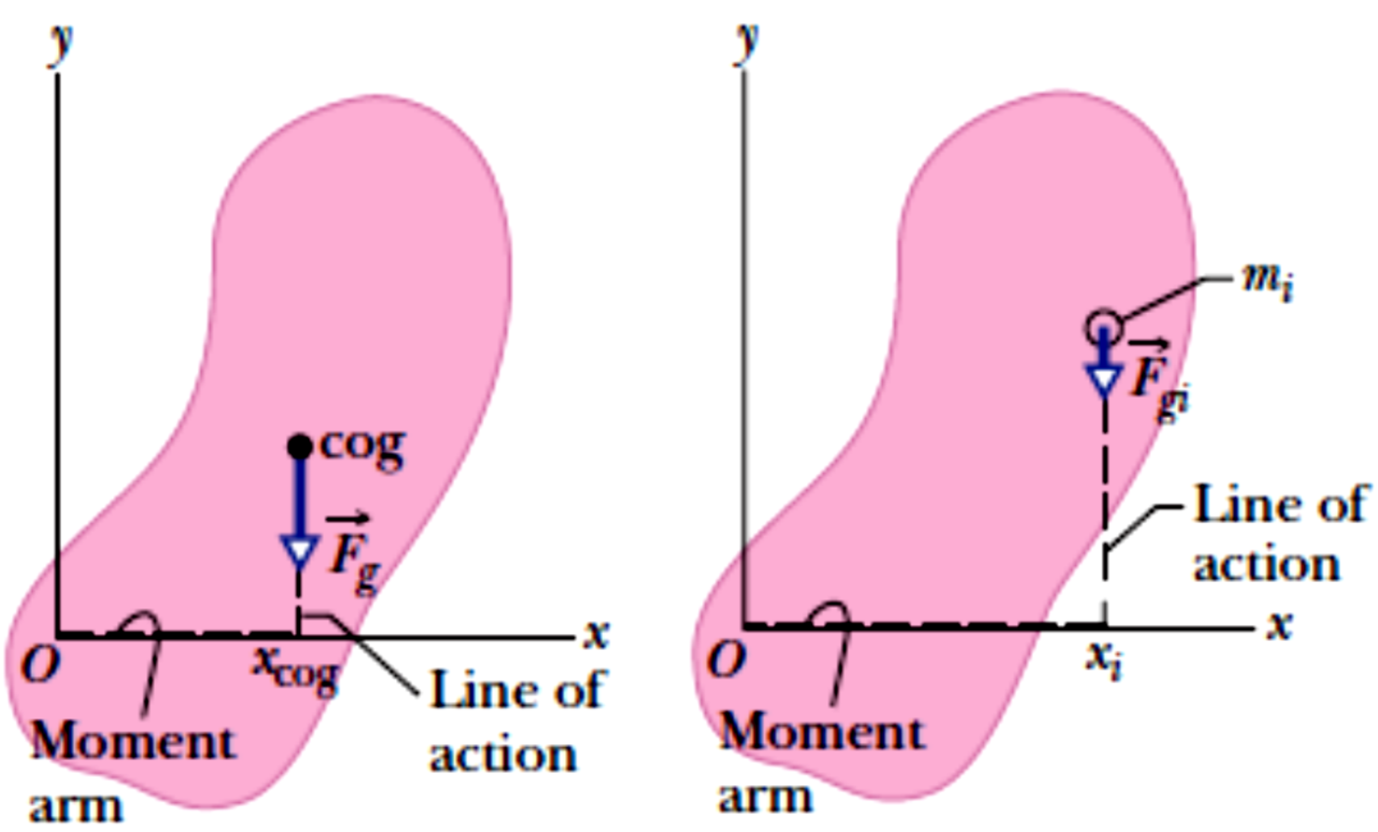
$$\tau_i = x_i F_{gi}$$

$$\tau_{net} = \sum \tau_i = \sum x_i F_{gi} = \sum x_i m_i g_i$$

if $g_i = \text{constant} \Rightarrow \tau_{net} = \left(\sum x_i m_i\right) g$

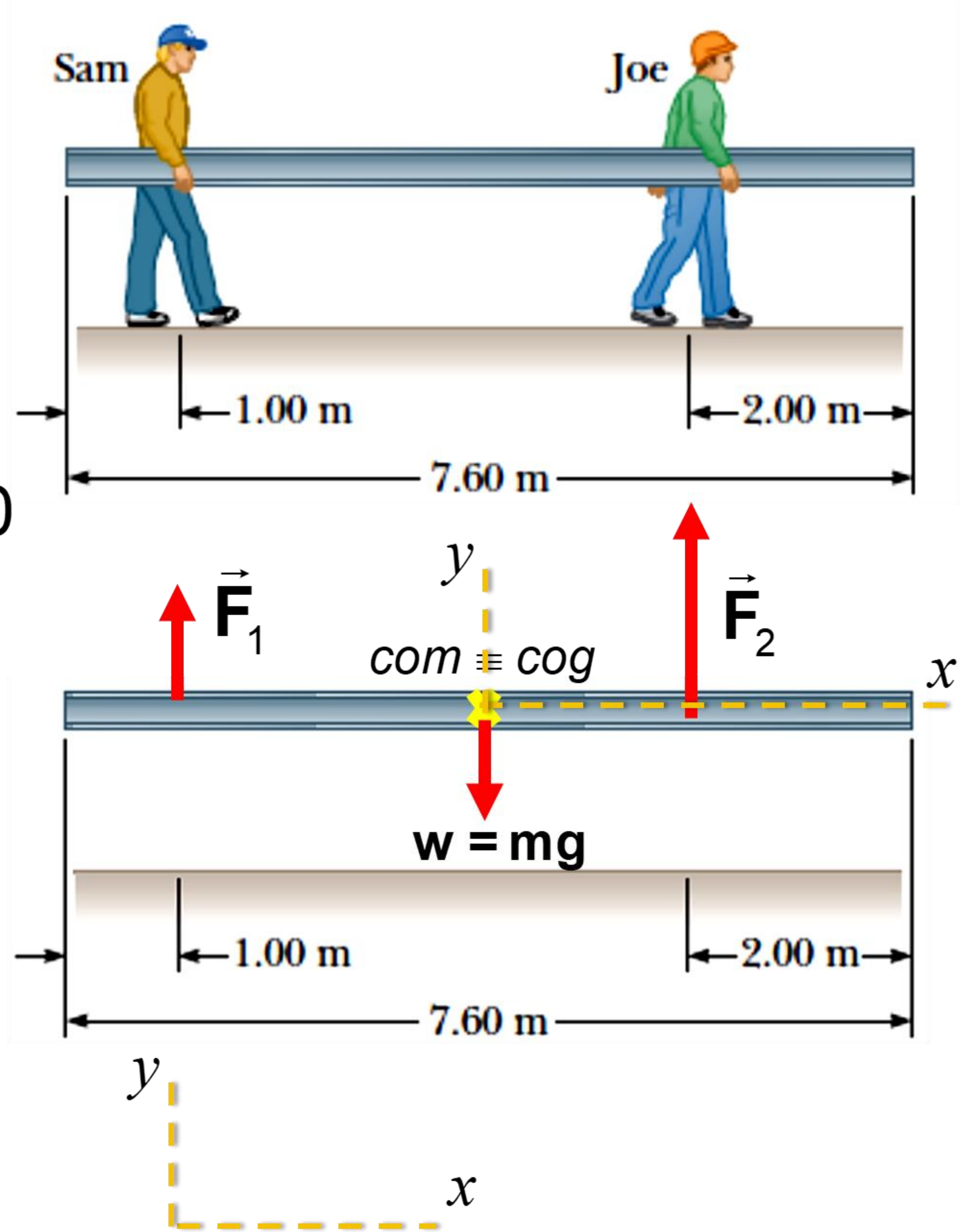
$$\tau_{net} = x_{cog} F_g = x_{cog}(Mg)$$

$$\boxed{x_{cog} = \frac{\sum m_i x_i}{M} = x_{com}}$$



center of gravity ≡ center of mass

Ex 1: A uniform beam of length **L = 7.60 m** and weight **4.50 × 10² N** is carried by two workers, Sam and Joe, as shown in Figure. Determine the force that each person exerts on the beam.



$$F_{net,y} = 0 \quad \Rightarrow \quad \mathbf{F}_1 + \mathbf{F}_2 - w = 0$$

$$\boxed{\mathbf{F}_1 + \mathbf{F}_2 = w = 4.5 \times 10^2}$$

$$\tau_{net,z} = 0 \quad \Rightarrow \quad -\mathbf{F}_1(2.8) + \mathbf{F}_2(1.8) = 0$$

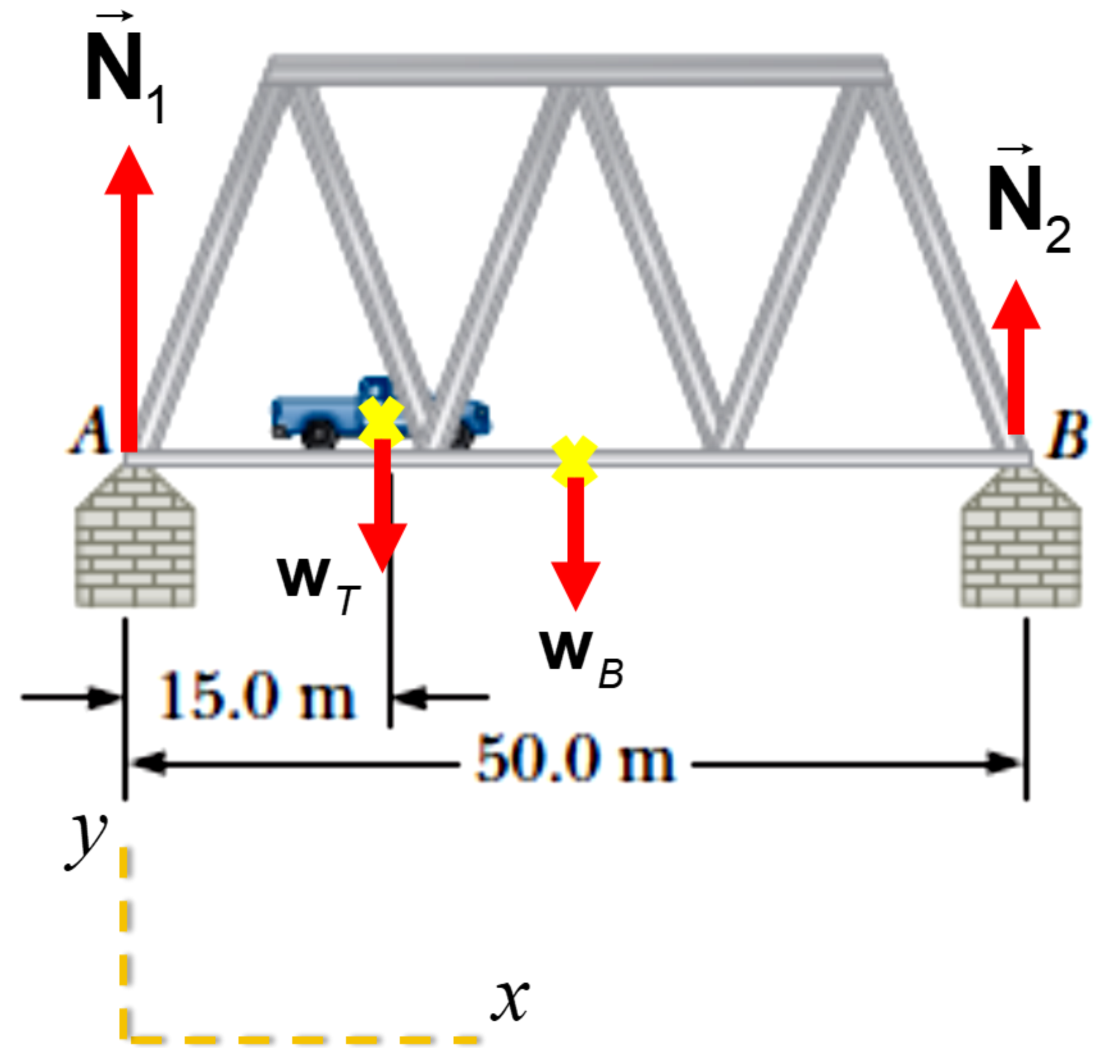
$$\boxed{\mathbf{F}_2 = \frac{2.8}{1.8}\mathbf{F}_1 = 1.56\ \mathbf{F}_1}$$

$$\boxed{\mathbf{F}_1 = 176\ N\ ;\ \mathbf{F}_2 = 274\ N}$$

$$-w(2.8) + \mathbf{F}_2(4.6) = 0$$

$$\boxed{\mathbf{F}_2 = \frac{2.8}{4.6} w = 274\ N\ ;\ \mathbf{F}_1 = 176\ N}$$

Ex 2: A bridge of length **50 m** and mass **8 × 10^4 kg** (with uniform distribution) is supported on a smooth pier at each end as shown in Figure. A truck of mass **3 × 10^4 kg** is located **15 m** from one end. What are the forces on the bridge at the points of support?



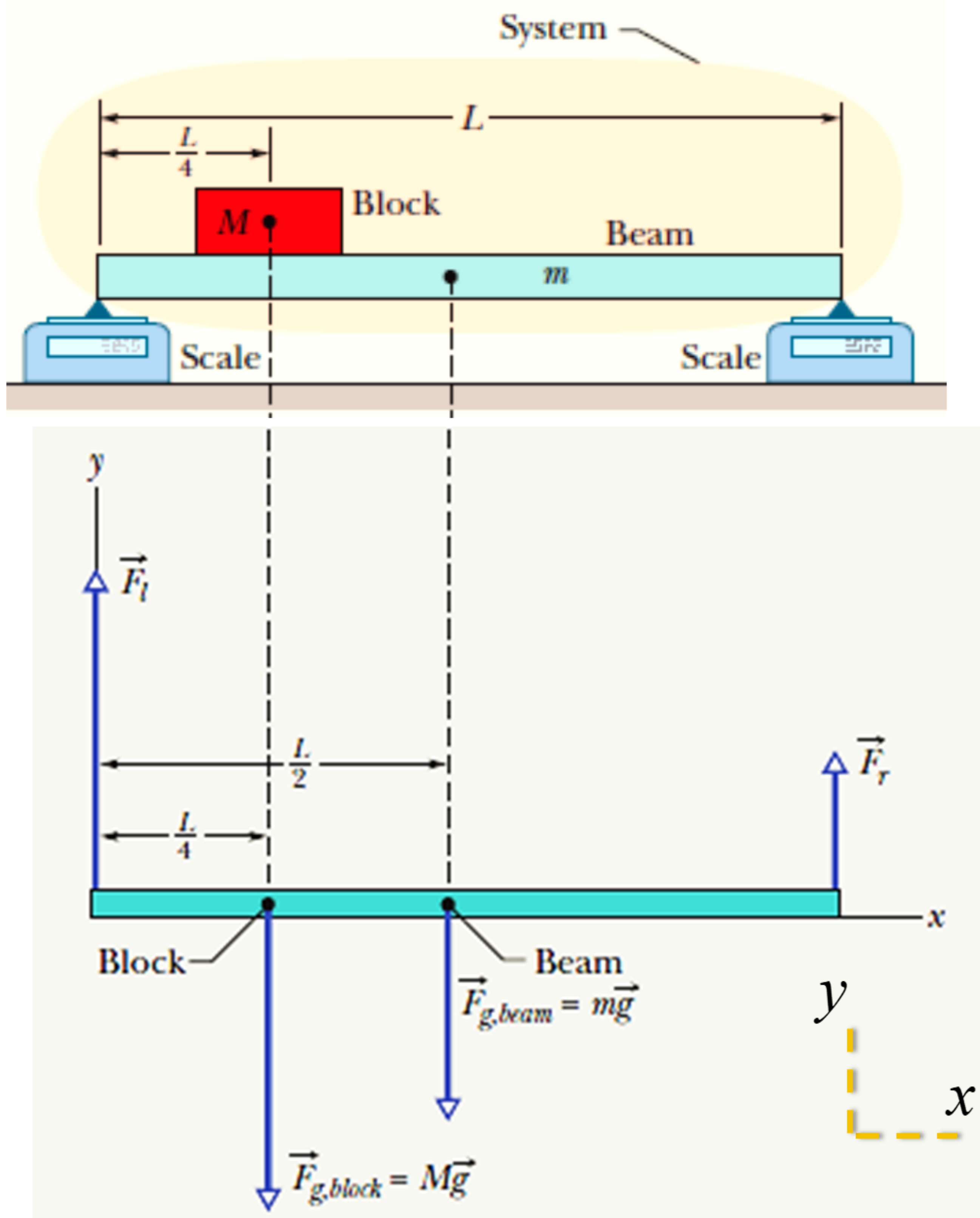
$$F_{net,y} = 0 \Rightarrow N_1 + N_2 - w_B - w_T = 0$$

$$\boxed{N_1 + N_2 = w_B + w_T = 1.078 \times 10^6 \quad N}$$

$$\tau_{net,z} = 0 \Rightarrow -w_T(15) - w_B(25) + N_2(50) = 0$$

$$\boxed{N_2 = 4.80 \times 10^5 \quad N} \qquad \boxed{N_1 = 5.98 \times 10^5 \quad N}$$

Ex 3: A uniform beam, of length **L** and mass **m = 1.8 kg**, is at rest on two scales. A uniform block, with mass **M = 2.7 kg**, is at rest on the beam. with its center a distance **L/4** from the beam's left end. What do the scales read?



$$F_{net,y} = 0 \Rightarrow F_l + F_r - Mg - mg = 0$$

$$\boxed{F_l + F_r = (M+m)g = 44.1\ N}$$

$$\tau_{net,z} = 0$$

$$\boxed{F_l(0) + F_r(L) - Mg\left(\frac{L}{4}\right) - mg\left(\frac{L}{2}\right) = 0}$$

$$\boxed{F_r = \frac{1}{4}Mg + \frac{1}{2}mg = 15.44\ N}$$

$$\boxed{F_l = 28.66\ N}$$

$$\boxed{-F_l(L) + F_r(0) + Mg\left(\frac{3L}{4}\right) + mg\left(\frac{L}{2}\right) = 0} \Rightarrow \boxed{F_l = \frac{3}{4}Mg + \frac{1}{2}mg = 28.66\ N}$$