1.

For each of the following systems construct the phase portrait:

a) $$\begin{cases} \dot{x}_1 = x_1(x_1^2 + x_2^2 - 1) - x_2(x_1^2 + x_2^2 + 1) \\ \dot{x}_2 = x_2(x_1^2 + x_2^2 - 1) + x_1(x_1^2 + x_2^2 + 1) \end{cases}$$

b) $$\begin{cases} \dot{x}_1 = -x_1(x_1^2 + x_2^2 - 1) - x_2(x_1^2 + x_2^2 + 1) \\ \dot{x}_2 = -x_2(x_1^2 + x_2^2 - 1) + x_1(x_1^2 + x_2^2 + 1) \end{cases}$$

2.

Consider the nonlinear system:

$$\dot{x} = y + x(x^2 + y^2 - 1)\sin\frac{1}{x^2 + y^2 - 1}$$

$$\dot{y} = -x + y(x^2 + y^2 - 1)\sin\frac{1}{x^2 + y^2 - 1}$$

Use polar coordinates to show that this system has infinite number of limit cycles and also study the stability of them.

3.

Find the equilibrium point of the following system and its stability completely.

$$\dot{x}_1 = x_2 + x_1(\beta^2 - x_1^2 - x_2^2)$$

$$\dot{x}_2 = -x_1 + x_2(\beta^2 - x_1^2 - x_2^2)$$

4.

According to the shape of the nonlinear block in the following system, this system could be investigated using piecewise linear systems:

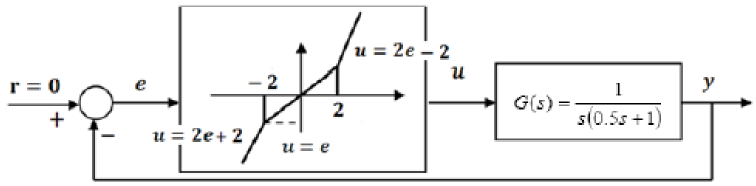

Figure 2: close loop system in Exercise 1.4.

a) For different regions of the nonlinear block extract the state space model of the closed loop system and plot the phase portrait for them.

b) Construct phase portrait of the main system using the plots of part (a)

5&6.

Draw the phase portraits of the following systems, using isoclines

(*a*) $\ddot{\theta} + \dot{\theta} + 0.5\,\theta = 0$

(*b*) $\ddot{\theta} + \dot{\theta} + 0.5\,\theta = 1$

(*c*) $\ddot{\theta} + \dot{\theta}^2 + 0.5\,\theta = 0$

Consider the nonlinear system

$$\dot{x} = y + x\,(x^2 + y^2 - 1)\,\sin\frac{1}{x^2 + y^2 - 1}$$

$$\dot{y} = -x + y\,(x^2 + y^2 - 1)\,\sin\frac{1}{x^2 + y^2 - 1}$$

Without solving the above equations explicitly, show that the system has infinite number of limit cycles. Determine the stability of these limit cycles. (*Hint*: Use polar coordinates.)

7.

Which phase portrait in Figure 1 belongs to what system? No motivat required.

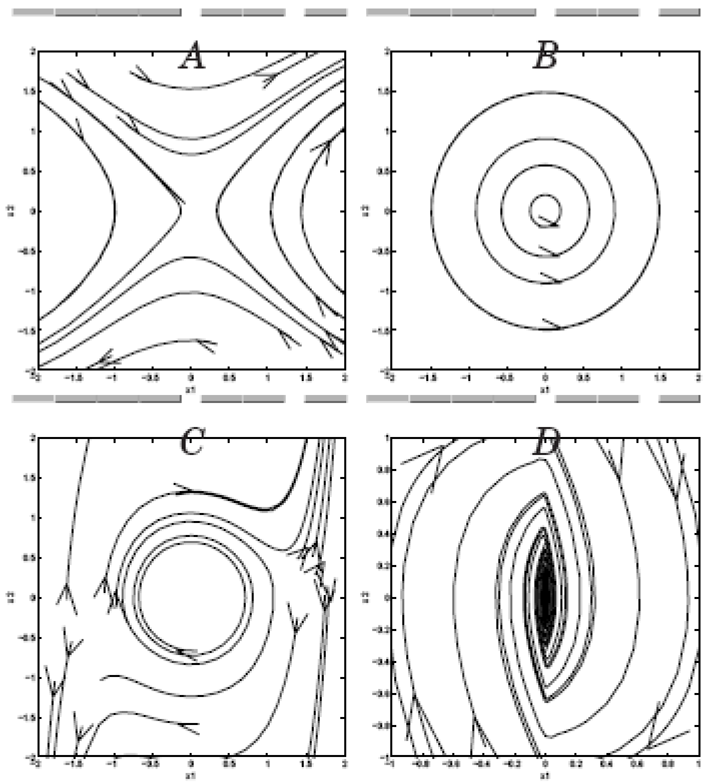

Figure 1 The phase portraits in Problem 1

(i) $\dot{x}_1 = x_2$

$\dot{x}_2 = -x_2 - x_1 - \text{sign}(x_1)$

(ii) $\dot{x}_1 = -x_2$

$\dot{x}_2 = x_1$

(iii) $\dot{x}_1 = x_2$

$\dot{x}_2 = -x_1 + x_2 x_1^4$

(iv) $\dot{x}_1 = x_2$

$\dot{x}_2 = \sin(x_1)$

8.

The system shown in Figure 2.10 represents a satellite control system with rate feedback provided by a gyroscope. Draw the phase portrait of the system, and determine the system's stability.

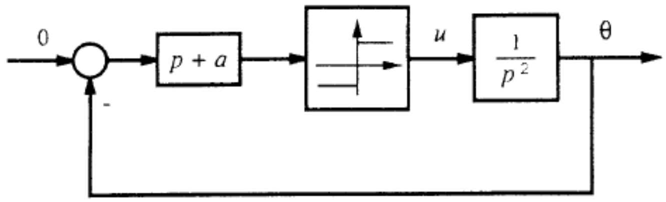

Figure 2.10 : Satellite control system with rate feedback