Chapter 2: Electric Field

- ✓ **Electric Field Due to a Point Charge**
- ✓ **Electric Fields Due to Multiple Charges**
- ✓ **Electric Field Lines**
- ✓ **Electric Field of a Continuous Charge Distribution**

Session 5:

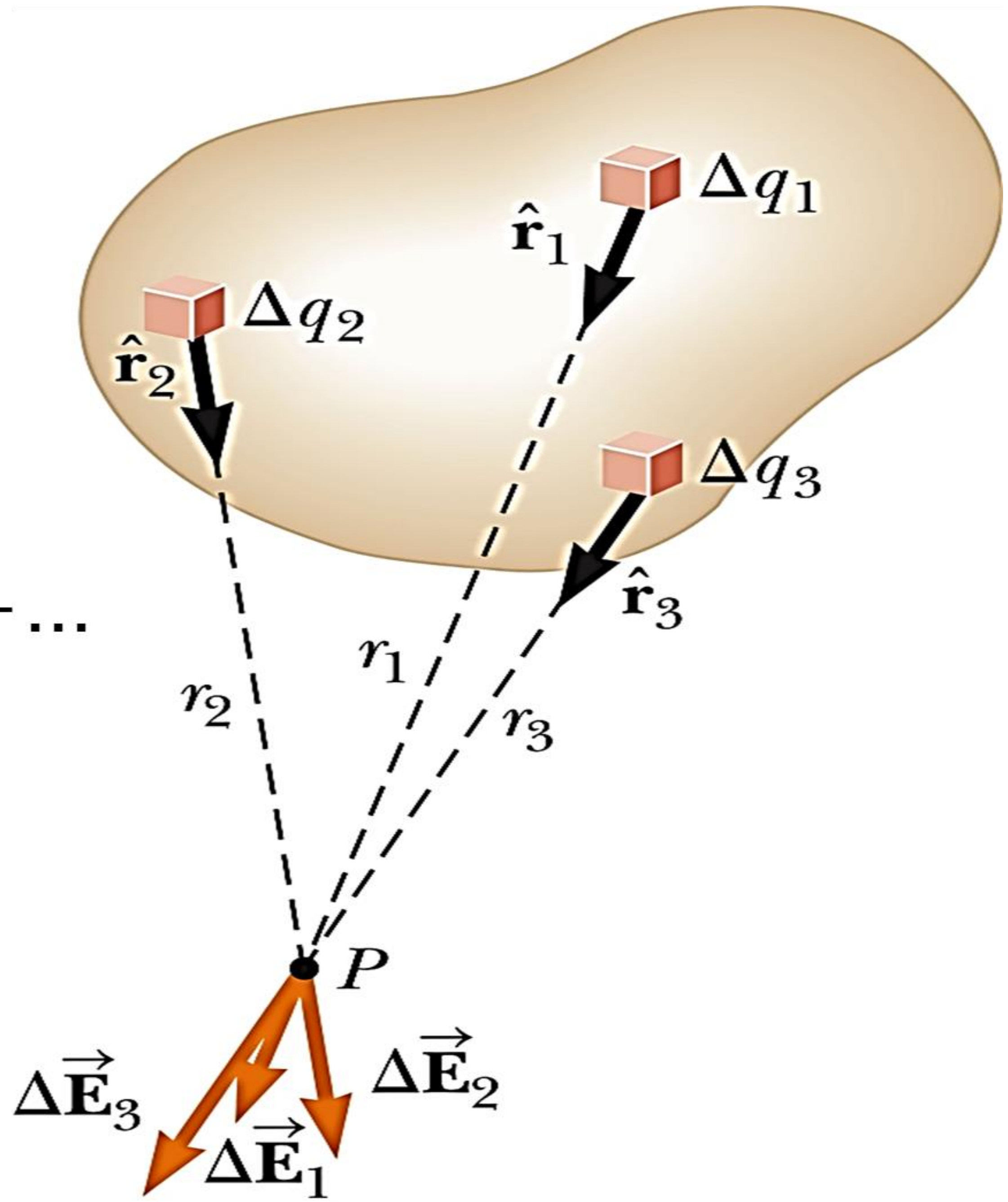
- ✓ **Electric Field of a Continuous Charge Distribution**
- ✓ **Examples**

Electric Field of a Continuous Charge Distribution

$$\vec{\mathbf{E}} = \Delta\vec{\mathbf{E}}_1 + \Delta\vec{\mathbf{E}}_2 + \ldots = k_e \frac{\Delta q_1}{r_1^{\,2}}\hat{\mathbf{r}}_1 + k_e \frac{\Delta q_2}{r_2^{\,2}}\hat{\mathbf{r}}_2 + \ldots$$

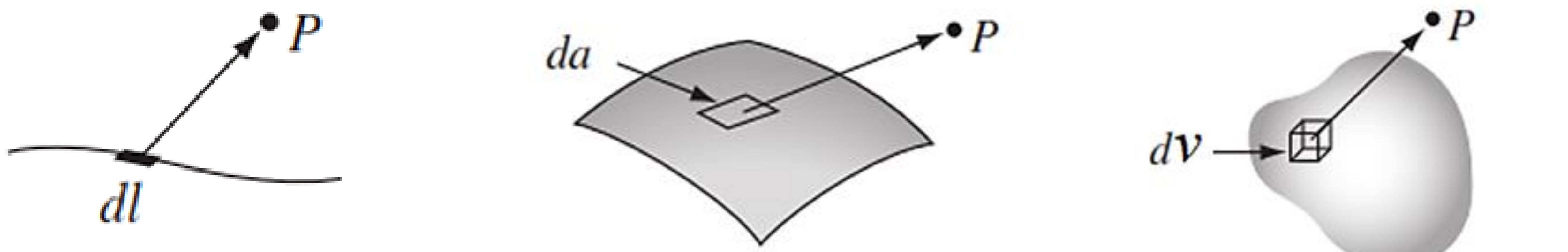
$$\vec{\mathbf{E}} = k_e \lim_{\Delta q_i \to 0} \sum_i \frac{\Delta q_i}{r_i^{\,2}}\hat{\mathbf{r}}_i = k_e \int \frac{dq}{r^2}\hat{\mathbf{r}}$$

$$\boxed{\vec{\mathbf{E}} = k_e \int \frac{dq}{r^2}\hat{\mathbf{r}}}$$



Charge Densities

1. **Linear charge density: when a charge is distributed along a line**
   - $\lambda \equiv Q / \ell$ with units C/m
2. **Surface charge density: when a charge is distributed evenly over a surface area**
   - $\sigma \equiv Q / A$ with units C/m^2
3. **Volume charge density: when a charge is distributed evenly throughout a volume**
   - $\rho \equiv Q / V$ with units C/m^3

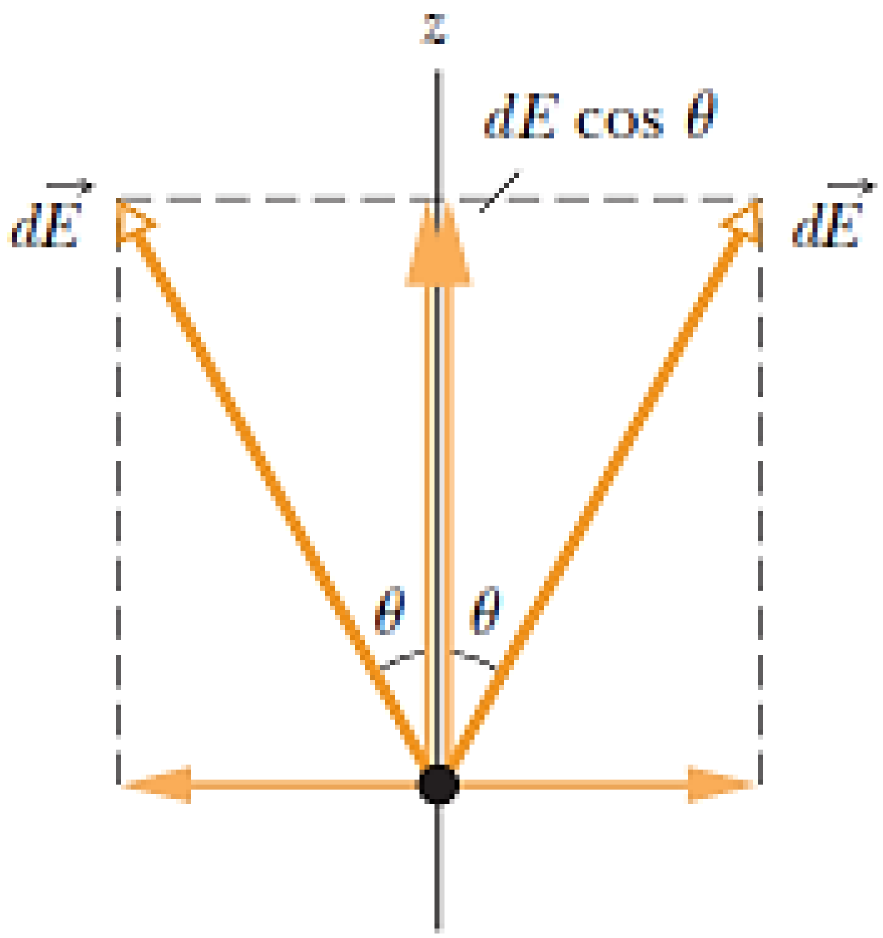


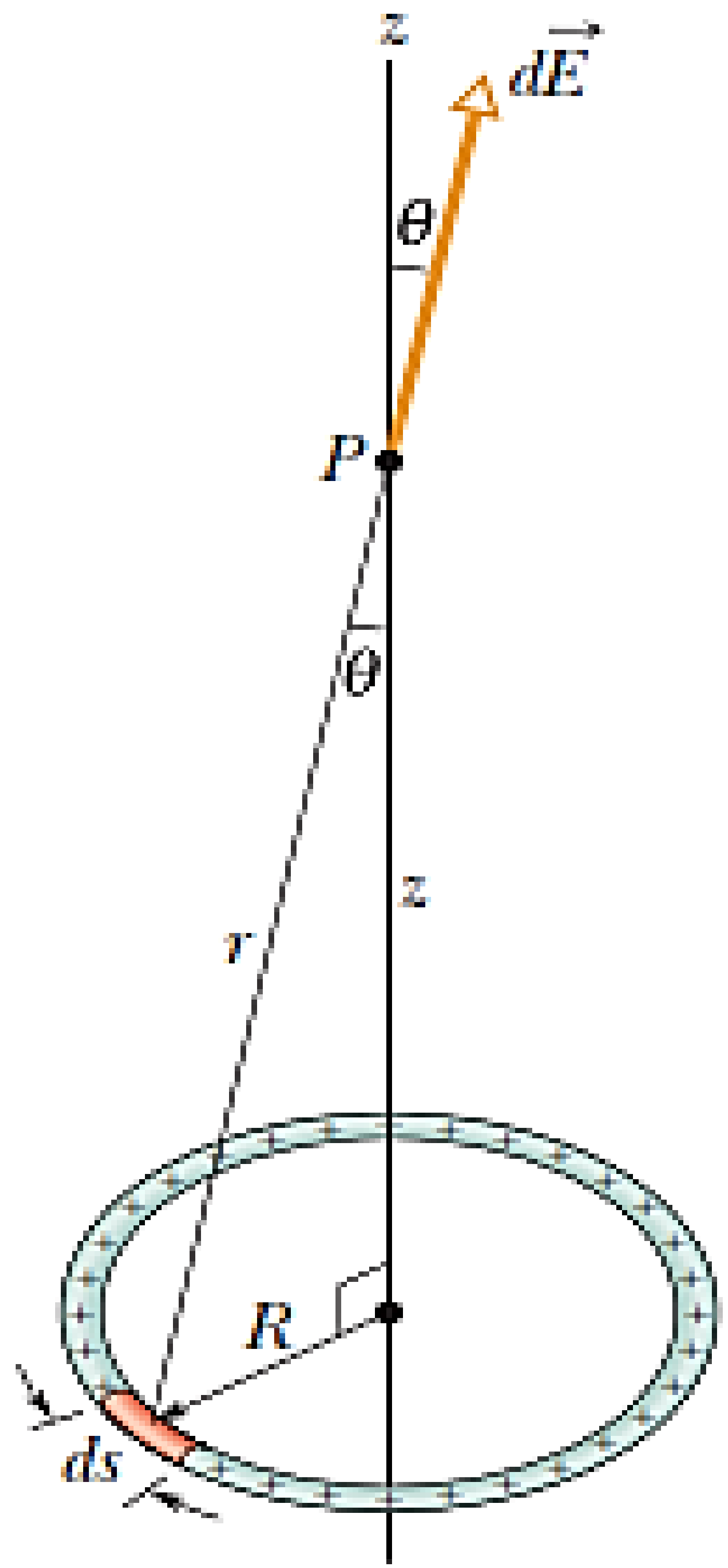
Line charge, λ　　Surface charge, σ　　Volume charge, ρ

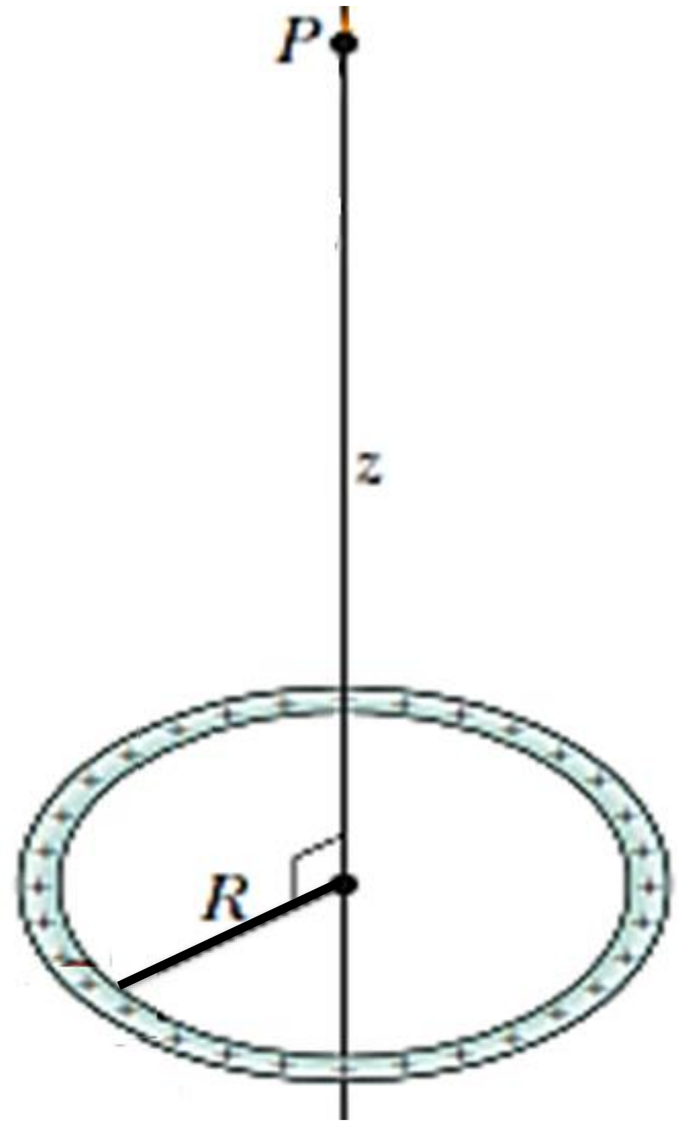
$$\boxed{\vec{\mathbf{E}} = k_e \int \frac{dq}{r^2}\hat{\mathbf{r}}}$$

- **For the length element:** $dq = \lambda\, d\ell$
- **For the surface:** $dq = \sigma\, dA$
- **For the volume:** $dq = \rho\, dV$

Ex 7. A ring of radius **R** carries a uniformly distributed positive total charge **q**. Calculate the electric field due to the ring at a point P lying a distance **z** from its center along the central axis perpendicular to the plane of the ring.







$$\left|d\vec{\mathbf{E}}\right| = k_e \frac{dq}{r^2} = k_e \frac{\lambda\, ds}{r^2}$$

$$\cos\theta = \frac{z}{r}$$

$$\lambda = \frac{q}{l} = \frac{q}{2\pi R}$$

$$\mathbf{dE}_z = \left|\mathbf{d}\vec{\mathbf{E}}\right|\cos\theta = k_e \frac{\lambda \cos\theta\, ds}{r^2} = k_e \frac{\lambda z\; ds}{r^3} = k_e \frac{\lambda z\; ds}{(R^2 + z^2)^{\frac{3}{2}}}$$

$$\mathbf{E}_z = \int \mathbf{dE}_z = \int k_e \frac{\lambda z\; ds}{(R^2 + z^2)^{\frac{3}{2}}} = k_e \frac{\lambda z}{(R^2 + z^2)^{\frac{3}{2}}} \int_0^{2\pi R} ds = k_e \frac{\lambda z (2\pi R)}{(R^2 + z^2)^{\frac{3}{2}}}$$

$$\boxed{\mathbf{E}_z = k_e \frac{q\, z}{(R^2 + z^2)^{\frac{3}{2}}}}$$

$$\boxed{z \gg R \;\Rightarrow\; \mathbf{E}_z \approx k_e \frac{q}{z^2} \quad \text{(point charge)}}$$

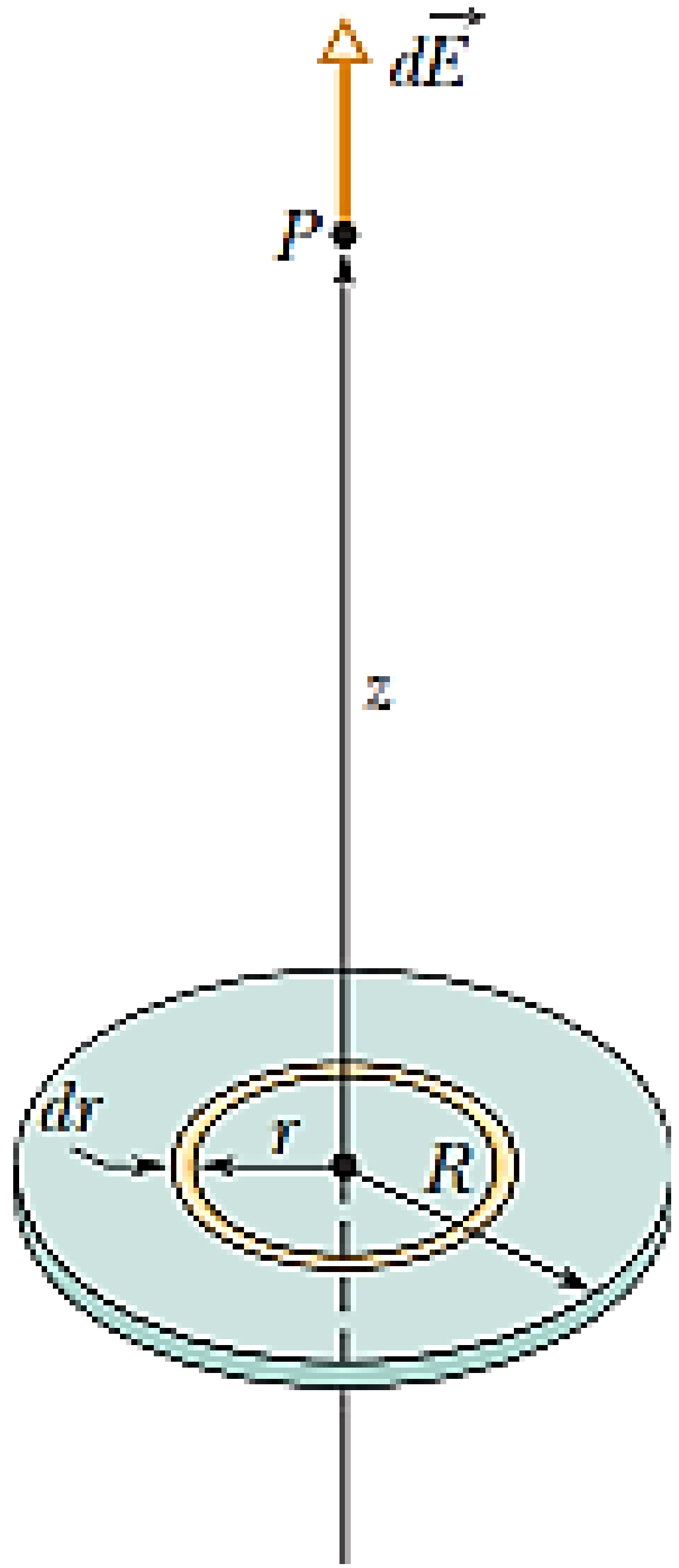
$$\boxed{\text{Prob (22.24)} \;\Rightarrow\; \mathbf{E}_{max} = ? \;\Rightarrow\; z = \frac{R}{\sqrt{2}}}$$

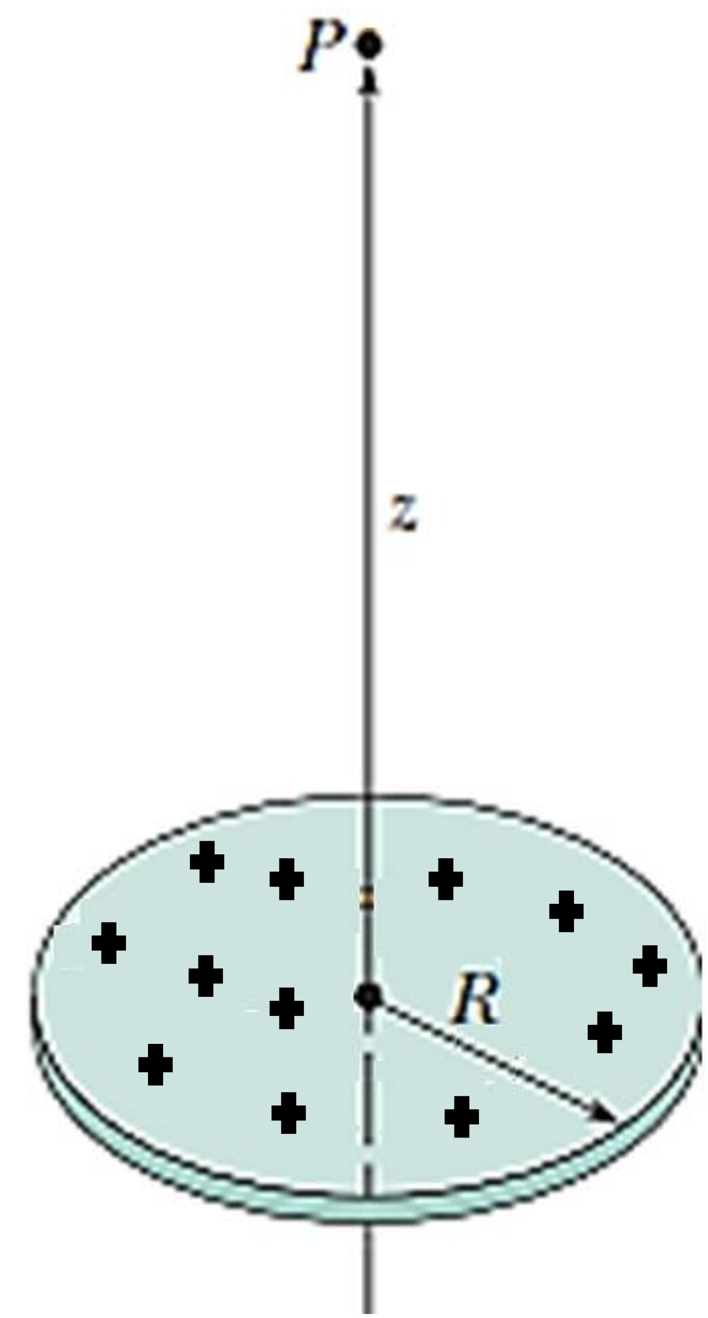
Ex 8. A disk of radius **R** has a uniform surface charge density **σ**. Calculate the electric field at a point P that lies along the central perpendicular axis of the disk and a distance **z** from the center of the disk.

$$d\mathbf{E}_z = k_e \frac{(dq)z}{(r^2+z^2)^{\frac{3}{2}}}$$

$$dq = \sigma\, dA = \sigma\,(2\pi r\, dr)$$

$$\mathbf{E}_z = \int d\mathbf{E}_z$$





$$\boxed{\sigma = \frac{q}{\pi R^2}}$$

$$\mathbf{E}_z = \int d\mathbf{E}_z = \int_0^R k_e \frac{\sigma\ (2\pi r\, dr)\ z}{(r^2+z^2)^{\frac{3}{2}}} = k_e \sigma \pi z \int_0^R \frac{2r\, dr}{(r^2+z^2)^{\frac{3}{2}}}$$

$$u = (r^2+z^2) \Rightarrow \quad du = 2r\, dr$$

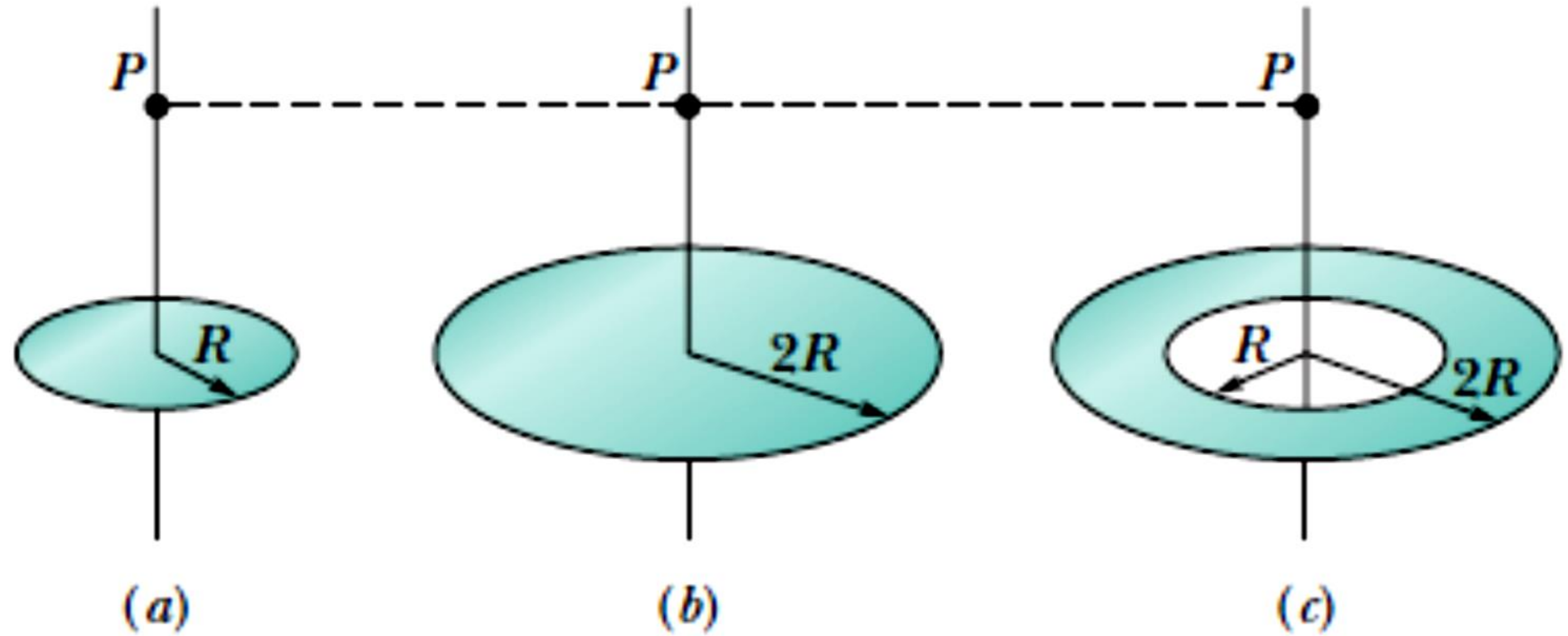
$$\mathbf{E}_z = k_e \sigma \pi z \int \frac{du}{u^{\frac{3}{2}}} = k_e \sigma \pi z (-\frac{1}{2} u^{-\frac{1}{2}}) = k_e \sigma \pi z (-\frac{1}{2} (r^2+z^2)^{-\frac{1}{2}} \Big|_0^R)$$

$$\boxed{\mathbf{E}_z = \frac{\sigma}{2\varepsilon_0}(1 - \frac{z}{\sqrt{R^2+z^2}})}$$

$$\boxed{R \to \infty \ \Rightarrow \ \mathbf{E}_z = \frac{\sigma}{2\varepsilon_0} \quad \text{(Infinite sheet)}}$$

Ex 9. **Figure below shows two disks and a flat ring, each with the same uniform charge density σ. Rank the objects according to the magnitude of the electric field they create at points P (which are at the same vertical heights z = R), greatest first.**

$$\boxed{E_b > E_a > E_c}$$



$$E_a = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{R^2 + z^2}}\right) = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{1}{\sqrt{2}}\right) = (0.29)\frac{\sigma}{2\varepsilon_0}$$

$$E_b = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{4R^2 + z^2}}\right) = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{1}{\sqrt{5}}\right) = (0.55)\frac{\sigma}{2\varepsilon_0}$$

$$E_c = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{4R^2 + z^2}}\right) - \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{R^2 + z^2}}\right) = \frac{\sigma}{2\varepsilon_0}\left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{5}}\right) = (0.25)\frac{\sigma}{2\varepsilon_0}$$