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Neural Adaptive Control of Two-Link Manipulator with Sliding Mode Compensation

Wen Yu†, Alexander S. Poznyak† and Edgar N.Sanchez‡

†CINVESTAV-IPN, Seccion de Control Automatico,

Av.IPN 2508, A.P.14-740,

Mexico D.F., 07000, Mexico,

FAX: +525-747-7089, e-mail: yuw@ctrl.cinvestav.mx.

‡ CINVESTAV, Unidad Guadalajara,

Aparatado Postal 31-438, Gaudalajara, Jalisco, C.P. 45091, Mexico

Abstract— **In this paper we developed a new neuro controller for robot manipulators. A simple dynamic neural network is used to estimate the unknown robot manipulators, then the direct linearization controller is derived via this neuro identifier. Because the approximation capability is limited, another robust sliding mode compensator is addressed. Our main contributions are: first we give a bound for the identification error of the parallel neuro identifier; second we establish a bound for the tracking error of the hybrid controller.**

I. Introduction

Recently many researchers manage to use modern elegant theories for robot control. Adaptive Control is a popular and powerful approach to control systems with unknown parameters [17]. Sliding Mode Control [18] consists a hypersurface switching surface which leads to the asymptotic trajectory convergence to this sliding surface. In spite of that this control is robust with respect to external disturbances, its implementation is never perfect because of "chattering effects" (state oscillation around sliding surface). Robust Feedback Control [3] is usually designed to guarantee the stability and some quality of control in the presence of parametric or unparametric uncertainties. Robust Adaptive Control can be realized by the following two ways: by adding minimax control or saturation-type control to the existing adaptive control [15] or by changing the adaptation law so there is a negative defined term (leakage-like adaptation) [10]. Adaptive-Robust Control (see [1] and [16]) estimates on-line the size of the uncertainties and uses these estimates in the tradi nal robust procedures [1]. Unfortunately, the corresponding theoretical study is still not completed. Optimal Control is applied to design a robust control for manipulators with some uncertainties [8].

Neural Networks (NN) control is a very effective tool to control the robot manipulator when we have no complete model information or, even, consider a controlled plant as "a black box" [7]. Neurocontrol is model-free, it is based on the NN model. To get a neuro model there exists two kinds of structure: *serial-parallel model* and *parallel model* [11]. Serial-parallel model can ensure all the signals bounded if the plant is BIBO stable for both multilayer perceptrons (MLP) [11] and dynamic neural networks (DNN) [14], [5]. Most of published papers used the *series-parallel model* because of its stability result. On the other hand, parallel model is very useful when deals with noisy systems, because it avoids problems of bias caused by noise on the real system output [19]. If the identification model is to be used off-line, obviously the parallel model is more suitable. However *parallel model* lacks theoretical verification, it is difficult to enjoy its advantages. In [5] a high order parallel NN can ensure that the identification error converges to zero, but they need the regressor vector is persistently exciting, for closed-loop control it is not reasonable. Neurocontrol may be classified as indirect (identification-based) and direct control [4]. The direct neurocontrol of MLP [9] suffers from some problems, such as lack of knowledge of the plant Jacobian, local minima and requiring specified training data pairs (off-line training). Many efforts are made to overcome these disadvantages. In [12] a online estimation of plant Jacobian is presented. A modified continuous-time version of backpropagation algorithm is given in [6] which does not need off-line learning and plant Jacobia. These "static" networks are based on the theory of function approximation which are sensitive to the training data. DNN can successfully overcome this disadvantage as well as demonstrate workable behavior in the presence of unmodeled dynamics, because their structure incorporate feedback. The direct and indirect [14] neural adaptive controls use linear two-layer DNN. Because this DNN has a poor approximation capability, if the neurocontrol is derived from this neuro identifier, the control results are not satisfied. In order to improve the identification results, a high-order DNN is proposed in [5], but the neurocontrol is not easy to realize because they use high-order multiply of control input.

In this paper, a simple DNN similar as [14] is used to identify robot manipulator, instead of serial-parallel model we use parallel identification structure. This neuro identifier cannot give a good approximate accuracy. So our controller for the robot manipulator has two parts: a direct linearization neuro controller and a sliding mode compensator. The main contribution of this paper is connected with the extension of our previous results [13] in the following directions: we consider parallel model of dynamic neural network to identify the unknown robot manipulator, the new learning laws assure stability of the identification error; then a bound for the tracking error of robot manipulator is established.

II. Robot Manipulator Dynamics

The dynamics of an $n-$link robot manipulator may be expressed in the Lagrange form [6]

$$M(q)\ddot{q} + V\left(q,\dot{q}\right)\dot{q} + G(q) + F_d\left(\dot{q}\right) = \tau \qquad (1)$$

where q consists of the joint variables, τ is the generalized forces, $M(q)$ is the intertia matrix, $V\left(q,\dot{q}\right)$ is centripetal-Coriolis matrix, $G(q)$ is gravity vector, $F_d\left(\dot{q}\right)$ is the friction vector. $M(q)$ represents inertia matrix. A scheme of the two-link robot manipulator is shown in Fig.1. For

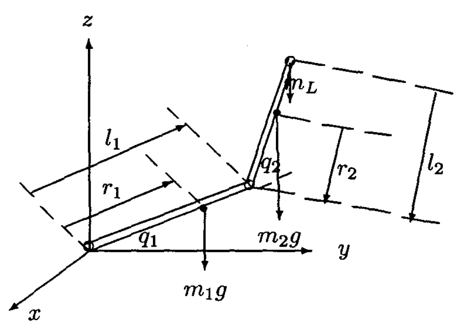

Fig. 1. A scheme of two-links manipulator

the case of two links, the elements can be represented as $M(q) = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}$,

$$M_{11} = m_1\left(r_1^2/4 + l_1^2/3\right) + m_2\left(l_1^2 + r_2^2/4 + l_2^2/3 + l_1 l_2 \cos q_2\right) + m_2\left(l_1^2 + 3/4 l_2^2 + l_1 l_2 \cos q_2\right)$$

$$M_{12} = m_2\left(r_2^2/4 + l_2^2/3 + \tfrac{1}{2} l_1 l_2 \cos q_2\right) + m_L l_2^2 = M_{21}$$

$$M_{22} = m_2\left(r_2^2/4 + l_2^2/3\right) + m_L l_2^2$$

$$V\left(q,\dot{q}\right) = \begin{bmatrix} -V_m \dot{q}_2 \sin q_2 & -V_m \sin q_2\left(\dot{q}_1 + \dot{q}_2\right) \\ V_m \dot{q}_1 \sin q_2 & 0 \end{bmatrix}$$

$$V_m = \left(\tfrac{1}{2} m_2 l_1 l_2 + m_L l_1 l_2\right),$$

$$G(q), = \begin{bmatrix} \left(\tfrac{1}{2} m_1 + m_2\right) g l_1 \cos q_1 + \tfrac{1}{2} m_2 g l_2 \cos(q_1 + q_2) \\ \tfrac{1}{2} m_2 g l_2 \cos(q_1 + q_2) \end{bmatrix}$$

$$F_d\left(\dot{q}\right) = \begin{bmatrix} \nu_1 \dot{q}_1 + \kappa_1 sign(\dot{q}_1) \\ \nu_2 \dot{q}_2 + \kappa_2 sign(\dot{q}_2) \end{bmatrix}$$

The robot dynamics have following standard properties

- $M(q)$ is a positive symmetric matrix bounded by $m_1 I < M(q) < m_2 I$.
- The norm of matrix $V\left(q,\dot{q}\right)$ is bounded an known function $v_b(q)$.

So (1) can be rewritten as

$$\dot{x} = f(x_t,t) + g(x_t,t)u_t \qquad (2)$$

where $x_t = \left[q_1, q_2, \dot{q}_1, \dot{q}_2\right]^T = \left[x_1, x_2, x_3, x_4\right]^T$, $f(x_t,t) = \left[\dot{x}_3, \dot{x}_4, -M^{-1}\left(V\dot{q} + G + F_d\right)\right]^T$, $g(x_t,t)u = \left[0, 0, M^{-1}(q)\tau\right]^T$.

The two-link manipulator system (2) has 4−dimension state and 2−dimension input. In order to design a suitable neuro controller, we define following auxiliary variable

$$\overline{q} = \dot{q} + \Lambda q \qquad (3)$$

where $\overline{q} \in R^2$, Λ is positive defined diagonal matrix. The system can be rewritten as

$$\dot{\overline{q}} = F(\overline{q},t) + G(\overline{q},t)u$$

where F and Gu are 2−dimension vector functions. Or we can even rewrite it as

$$\dot{q} = F'(q,t) + G'(q,u,t) \qquad (4)$$

When we design a neurocontrol we do not need to transfer the original system (1) into the standard form (2) or (4). We give the form of (4) in order to illuminate the neuro identifier. Now we only assume the state outputs q is available.

III. Adaptive Control via Dynamic Neural Network

In this section we derive a neuro controller for the robot dynamics in Section II. This controller has two new contribution: first we use parallel dynamic NN to identify the manipulator, a robust learning algorithm is proposed. Second, a hybrid neuro controller is presented. We construct the following parallel dynamic neural network:

$$\dot{\widehat{x}}_t = A\widehat{x}_t + W_t\sigma(\widehat{x}_t) + u_t \qquad (5)$$

where $\widehat{x}_t \in \Re^n$ is the state of the neural network, $u_t \in \Re^n$, $W_t \in \Re^{n\times n}$ is the weight of neural network. $A \in \Re^{n\times n}$ is a stable matrix. The vector functions $\sigma(\widehat{x}_t)$ is assumed to be $n-$dimensional with the elements increasing monotonically, such as sigmoid functions

$$\sigma_i(x_i) = \frac{a_i}{1 + e^{-b_i x_i}} - c_i. \qquad (6)$$

Remark 1: This neural network does not contain any hidden layers. We use this simplified neural network because we want to make the neuro controller more reliable. This model will cause more unmodeled dynamic, but we may compensation the modelling error.

For the two-link manipulator $n = 2$. The robot manipulator to be control is given in (4), it can be presented as a DNN plus a modelling error, i..e. there exists weight W^* such that the system (4) is complete described by

$$\dot{\overline{q}} = A\overline{q} + W^*\sigma(\overline{q}) + u_t + \Delta f(\overline{q},u_t) \qquad (7)$$

where W^* is bounded as

$$W^*\Lambda_\sigma{}^{-1}W^{*T} \leq \overline{W}. \qquad (8)$$

Here Λ_σ and $\overline{W}$ are priory known matrices.

Let define the identification error as,

$$\Delta_t := \widehat{x}_t - q. \tag{9}$$

Because $\sigma(\cdot)$ is chosen as sigmoid functions, the following general Lipschitz condition is fulfilled.

A1: *The function $\sigma(\cdot)$ satisfies*

$$\widetilde{\sigma}_t^T \Lambda_\sigma \widetilde{\sigma}_t \le \Delta_t^T D_\sigma \Delta_t,$$

where $\widetilde{\sigma}_t := \sigma(\widehat{x}_t) - \sigma(\overline{q})$, Λ_σ and D_σ are known positive constants.

From (4) and (5), if a bounded control input u_t may stabilize the nonlinear system, the unmodeled dynamics $\Delta f(\overline{q}, u_t)$ is bounded, we can assume that

A2: *There exists positive defined matrix Λ_f such that*

$$\Delta f^T \Lambda_f \Delta f := \|\Delta f\|_{\Lambda_f} \le \overline{\eta}$$

where $\overline{\eta}$ is the upper bound of the modeling error.

It is well known that, if the matrix A is stable, the pair $\left(A, R^{\frac{1}{2}}\right)$ is controllable and the pair $\left(Q, R^{\frac{1}{2}}\right)$ is observable and the *local frequency condition* fulfills,

$$A^T R^{-1} A - Q \ge \frac{1}{4}\left[A^T R^{-1} - R^{-1} A\right] R \left[A^T R^{-1} - R^{-1} A\right]^T. \tag{10}$$

then the following matrix Riccati equation:

$$A^T P + PA + PRP + Q = 0 \tag{11}$$

has a positive solution. So we can introduce following assumption:

A3: *For a given matrix A. there exists a strictly positive defined matrix Q_0 such that the matrix Riccati equation (11) with the matrices R and Q given by*

$$R := \overline{W} + \Lambda_f^{-1}, \quad Q := Q_0 + D_\sigma,$$

has a positive solution.

This conditions is easily fulfilled if we select A as diagonal matrix. The next theorem presents stable learning procedure of the parallel DNN.

Theorem 1: Let us consider the unknown robot manipulator (4) and parallel neural network (5) whose weights are adjusted as

$$\dot{W}_t = \dot{\widetilde{W}}_t = -s_t K P \sigma(\widehat{x}_t) \Delta_t^T \tag{12}$$

where $s_t = \begin{cases} 1 & \text{if } \|\Delta_t\| > \sqrt{\overline{\eta}\lambda_{\min}(Q_0)} \\ 0 & \text{if } \|\Delta_t\| \le \sqrt{\overline{\eta}\lambda_{\min}(Q_0)} \end{cases}$, $\widetilde{W}_t := W_t - W^*$, K is a positive define matrix, P is the solution of the matrix Riccati equation given by (11). Assuming also that the assumptions **A1-A3** hold, we conclude that the weight and identification error are bounded, i.e.,

$$\Delta_t, W_t \in L_\infty \tag{13}$$

for any $T \in (0, \infty)$ the identification error Δ_t satisfies the following tracking performance

$$\begin{array}{c} \frac{1}{T}\int_0^T \|\Delta_t\|_{Q_0} dt \le \overline{\eta} + C_0/T \\ C_0 = \Delta_0^T P \Delta_0 + tr\left[\widetilde{W}_0^T K^{-1} \widetilde{W}_0\right] \end{array} \tag{14}$$

Proof: From (9) and (5) we have

$$\dot{\Delta}_t = A\Delta_t + \widetilde{W}_t \sigma(\widehat{x}_t) + W_t \widetilde{\sigma}_t - \Delta f(\overline{q}, u_t). \tag{15}$$

Define Lyapunov function candidate as

$$V_t := \Delta_t^T P \Delta_t + \frac{1}{2} tr\left[\widetilde{W}_t^T K^{-1} \widetilde{W}_t\right]. \tag{16}$$

So, calculating its derivative, we obtain

$$\frac{d}{dt} V_t = 2\Delta_t^T P \dot{\Delta}_t + tr\left[\dot{\widetilde{W}}_t^T K^{-1} \widetilde{W}_t\right]. \tag{17}$$

As

$$\Delta_t^T P \dot{\Delta}_t = \Delta_t^T P A \Delta_t + \Delta_t^T P \left(W_t \widetilde{\sigma}_t - \Delta f(\overline{q}, u_t)\right) + \Delta_t^T P \widetilde{W}_t \sigma(\widehat{x}_t).$$

Using matrix inequality [13]

$$X^T Y + \left(X^T Y\right)^T \le X^T \Lambda^{-1} X + Y^T \Lambda Y \tag{18}$$

which is valid for any $X, Y \in \Re^{n \times k}$ and for any positive defined matrix $0 < \Lambda = \Lambda^T \in \Re^{n \times n}$.

The assumption **A1** leads to

$$\begin{array}{c} 2\Delta_t^T P W^* \widetilde{\sigma}_t \le \Delta_t^T P W^* \Lambda_\sigma^{-1} W_t P \Delta_t \\ + \widetilde{\sigma}_t^T \Lambda_\sigma \widetilde{\sigma}_t \le \Delta_t^T \left(P \overline{W} P + D_\sigma\right) \Delta_t, \end{array} \tag{19}$$

From **A2**

$$-2\Delta_t^T P \Delta f \le \Delta_t^T P \Lambda_f^{-1} P \Delta_t + \Delta f^T \Lambda_f \Delta f \le \Delta_t^T P \Lambda_f^{-1} P \Delta_t + \overline{\eta},$$

we obtain:

$$\begin{array}{c} \dot{V}_t \le \Delta_t^T \left(PA + A^T P + P\left(\overline{W} + \Lambda^{-1}\right) P + D_\sigma + +Q_0\right) \Delta_t \\ \Delta_t^T P \widetilde{W}_t \sigma(\widehat{x}_t) + tr\left[\dot{\widetilde{W}}_t^T K^{-1} \widetilde{W}_t\right] - \Delta_t^T Q_0 \Delta_t + \overline{\eta}. \end{array}$$

Use **A3**

$$\dot{V}_t \le \Delta_t^T P \widetilde{W}_t \sigma(\widehat{x}_t) + tr\left[\dot{\widetilde{W}}_t^T K^{-1} \widetilde{W}_t\right] - \Delta_t^T Q_0 \Delta_t + \overline{\eta}. \tag{20}$$

If

$$\|\Delta_t\| > \sqrt{\frac{\overline{\eta}}{\lambda_{\min}(Q_0)}},$$

according to (12),(20) becomes

$$\dot{V}_t \le -\Delta_t^T Q_0 \Delta_t + \overline{\eta} \le -\lambda_{\min}(Q_0) \|\Delta_t\|^2 + \overline{\eta} \le 0. \tag{21}$$

If

$$\|\Delta_t\| \le \overline{\eta} \sqrt{\frac{1}{\lambda_{\min}(Q_0)}}, \tag{22}$$

W_t is constant matrix and from (22) $\|\Delta_t\|$ is also bounded, so V is bounded. (13) is realized. Integrating (21) from 0 up to T yields

$$V(T)-V(0)\leq-\int_0^T \Delta_t^T Q_0\Delta_t dt+\overline{\eta}T,$$

from which (14) is achieved. ∎

Remark 2: The updating law (12) is similar with the *series-parallel* structure [14], the differences are *series-parallel* structure uses $\sigma(\overline{q})$ and *parallel* structure uses $\sigma(\widehat{x}_t)$; In *series-parallel* structure P is the solution of following Lyapunov equation

$$PA+A^TP=-Q.$$

in p*arallel* structure P is the solution of a matrix Riccati equation (11).

Remark 3: The updating rates $\overline{K}:=KP$ can be achieved by a special selection of the gain matrices K, the solution of matrix Riccati equation (11) does not influence the updating law.

Remark 4: If we have no any unmodeled dynamic ($\Delta f=0$), we obtain $\overline{\eta}=0$ and, hence, from (14) the globally asymptotic stability is guaranteed, i.e.,

$$\limsup_{T\to\infty}\frac{1}{T}\int_0^T\|\Delta\|_{Q_0}dt=0,$$

from which we directly obtain $\|\Delta_t\|\underset{t\to\infty}{\to}0.$

From (7), we know that the nonlinear system (4) may be modeled as

$$\dot{q}=Aq+W_t\sigma(\widehat{x}_t)+u_t+\Delta f(\overline{q},u_t)+W^*\sigma(q)-W_t\sigma(\widehat{x}_t) \quad (23)$$

Using the assumptions **A1** and **A2**, from theorem 1 we have $\Delta f(q,u_t)+W^*\sigma(q)-W_t\sigma(\widehat{x}_t)$ is bounded. (23) can be rewritten as

$$\dot{q}=Aq+W_t\sigma(\widehat{x}_t)+u_t+d_t \quad (24)$$

where

$$d_t=\Delta f(q,u_t)+W^*\sigma(q)-W_t\sigma(\widehat{x}_t) \quad (25)$$

is bounded

Based on the neural network identifier (5), we will force the robot manipulator system (4) to track a optimal trajectory $x^*\in\Re^2$ which is assumed to be smooth enough. This trajectory is regarded as a solution of a nonlinear reference model:

$$\dot{x}^*=\varphi(x^*,t) \quad (26)$$

with a fixed initial condition. If the trajectory has points of discontinuity in some fixed moments, we can use any approximating trajectory which is smooth. In the case of regulation problem $\varphi(x^*,t)=0$, $x^*(0)=c$, c is constant.

Theorem 2: Let the desired trajectory be given by (26), the weights of the parallel DNN (5) is tuned by (12). For the two-link manipulator, if the control input is provided by

$$\begin{aligned} u_t &= u_{1,t}+u_{2,t} && (27)\\ u_{1,t} &= \varphi-Ax^*-W_t\sigma(\widehat{x}_t). && (28)\\ u_{2,t} &= \begin{cases} -kP_c^{-1}\operatorname{sign}(\Delta_t^*) & |\Delta_t^*|\geq\delta \\ -kP_c^{-1}\Delta_t^*/\delta & |\Delta_t^*|<\delta \end{cases},\quad k,\delta>0 && (29)\end{aligned}$$

where P is a solution of the Lyapunov equation $A^TP_c+P_cA=-Q_c$, the tracking error

$$\Delta_t^*=q-x^*$$

is bounded.

Proof: From (23) and (26) we have

$$\dot{\Delta}^*=A\Delta^*+W_t\sigma(\widehat{x}_t)+u_t+d_t-\varphi+Ax^*. \quad (30)$$

As $\varphi(x^*,t)$, x^* and $W_t\sigma(\widehat{x}_t)$ are available, we can select u_t as (27) where $u_{1,t}$ is direct linearization control (28). So

$$\dot{\Delta}^*=A\Delta^*+u_{2,t}+d_t. \quad (31)$$

The Sliding Mode technique can be applied to compensate d_t. Let us define Lyapunov-like function as

$$V_t=\Delta_t^*P_c\Delta_t^*,\quad P_c=P_c^T>0 \quad (32)$$

where P is a solution of the Lyapunov equation $A^TP_c+P_cA=-Q_c$. Using (31) whose time derivative is

$$\dot{V}_t=\Delta_t^*\left(A^TP_c+P_cA\right)\Delta_t^*+2\Delta_t^{*T}P_cU_{2,t}+2\Delta_t^{*T}P_cd_t. \quad (33)$$

According to sliding mode technique, we may select $u_{2,t}$ as

$$U_{2,t}=-kP_c^{-1}\operatorname{sign}(\Delta_t^*),\quad k>0 \quad (34)$$

where k is positive constant,

$$sgn(\Delta_t^*)=\left[sgn(\Delta_{1,t}^*),\cdots sgn(\Delta_{n,t}^*)\right]^T\in\Re^n$$

Substitute (34) into (33)

$$\begin{aligned}\dot{V}_t &= -\Delta_t^{*T}Q_c\Delta_t^*-2k\|\Delta_t^*\|+2\Delta_t^TP_cd_t\\ &\leq -\lambda_{\max}(Q_c)\|\Delta_t^*\|^2-2k\|\Delta_t^*\|+2\lambda_{\max}(P_c)\|\Delta_t^*\|\|d_t\|\\ &= -\lambda_{\max}(Q_c)\|\Delta_t^*\|^2-2\|\Delta_t^*\|(k-\lambda_{\max}(P_c)\|d_t\|)\end{aligned}$$

If we select k is big enough such that

$$k>\lambda_{\max}(P_c)\overline{d}$$

where $\overline{d}$ is upper bound of $\|d_t\|$ ($\overline{d}=\sup_t\|d_t\|$), then $\dot{V}_t<0$. So

$$\lim_{t\to\infty}\Delta_t^*=0.$$

Because the sliding mode control $u_{2,t}$ is inserted in the closed-loop system, chattering occur in the control input which may excite unmodeled high-frequency dynamics. To eliminate chattering, the following boundary layer compensator can be used

$$u_{2,i}=\begin{cases} -kP_i^{-1}\text{sign}\left(\Delta_{i,t}^*\right) & \left|\Delta_{i,t}^*\right|\geq\delta \\ -kP_i^{-1}\Delta_{i,t}^*/\delta & \left|\Delta_{i,t}^*\right|<\delta \end{cases} \quad (35)$$

where $u_{2,t}=[u_{2,1},\dots u_{2,n}]$, δ is small enough positive constant. The above boundary layer controller offers a continuous approximation to the discontinuous sliding mode control law inside the boundary layer and guarantees the output tracking error within any neighborhood of the origin [2]. ■

Remark 5: Theorem 1 assures W_t is bounded. So $u_{1,t}$ in (28) is bounded. From (35) we know $u_{2,t}$ is bounded. So the hybrid control input is bounded. Although we use the sliding mode control [18], we can avoid the "chattering effects" because our hybrid control has two part, the main part $u_{1,t}$ is derived from neuro identifier, and $u_{2,t}$ uses boundary layer compensator to eliminate the chattering.

Remark 6: Compare (5) and (26)

$$\dot{\widehat{x}}-\dot{x}^*=A\left(\widehat{x}-x^*\right)+W_t\sigma(\widehat{x}_t)+u_t-\varphi+Ax^*.$$

If we select $u_t=u_{1,t}$

$$\dot{\widehat{x}}-\dot{x}^*=A\left(\widehat{x}-x^*\right)$$

Because A is stable, $\lim_{t\to\infty}\left(\widehat{x}-x^*\right)=0$. $u_{1,t}$ is used to make neuro identifier (5) follow the reference model (26). Because the neuro identifier (5) cannot follow the robot dynamic (4) exactly, so $u_{2,t}$ is used to compensate the modeling error in order to make the robot (4) follow the reference model (26).

IV. Simulation Results

We take the robot parameters as in [6], we also include friction in (1), i.e. $l_1=l_2=2r_1=2r_2=1$m, $m_1=0.8$kg, $m_2=2.3$kg, $v1=v2=0.4$, $k1=k2=0.8$, $g=9.81$. The initial conditions are $\dot{q}(0)=[\dot{q}_1(0),\dot{q}_2(0)]=[0,0]$, $q(0)=[q_1(0),q_2(0)]=[3.14,0.6]$. We assume the parameters in (4) are unknown, only the position and the velocity of q are available. The neural network used for identification is as (5), where $\widehat{x}_t=[\widehat{q}_1,\widehat{q}_2]^T$, $A=$diag$[-2,-2]$,

$$\sigma(x)=\frac{2}{(1+e^{-2x})}-\frac{1}{2}$$

The initial conditions for the neural network are $\widehat{x}_0=[0,0]^T$, $W(0)=\begin{bmatrix}1&1\\1&1\end{bmatrix}$.

First we check the identification ability of parallel DNN, because the open-loop system is unstable, first we use a PD control as

$$\tau=-10\left(q-q^*\right)-5\left(\dot{q}-\dot{q}^*\right) \quad (36)$$

where the reference inputs are $q_1^*(t)=\sin t$, $q_2^*(t)=\cos t$, $q_1^*(0)=q_2^*(0)=0$. So in (26) $\varphi\left(q_t^*,t\right)=[-\cos t,\sin t]^T$. Let $u_t=\tau$, use (5) to identify the two-link robot. The updating laws are same as (12). we select $KP=$diag$[10,10]$, $\overline{\eta}=\lambda_{\min}(Q_0)=1$. The results are shown in Fig.2 and Fig.3. There exists identification errors because we use second order neural network to model the dynamic of two links robot, there are unmodelled dynamics.

Then we apply the hybrid neurocontrol to the two-link manipulator. For compensation a standard PD control is give as in (36). The result appears in Fig.4 and Fig.5 and is unsatisfactory. If we use parallel DNN, we select $\Lambda=$diag$[1,1]$, so the control input is

$$\tau=u_{1,t}+u_{2,t}$$

$$u_{1,t}=\varphi\left(q^*\right)-\begin{bmatrix}-2&0\\0&-2\end{bmatrix}x^*-W_t\sigma(\widehat{x}_t)$$

$$=\begin{pmatrix}-3\cos t+\sin t\\ \cos t+3\sin t\end{pmatrix}-W_t\sigma(\widehat{x}_t),$$

$$u_{2,t}\begin{cases} -kP_i^{-1}\text{sign}\left(\Delta_{i,t}^*\right) & \left|\Delta_{i,t}^*\right|\geq\delta \\ -kP_i^{-1}\Delta_{i,t}^*/\delta & \left|\Delta_{i,t}^*\right|<\delta \end{cases},\quad i=1,2$$

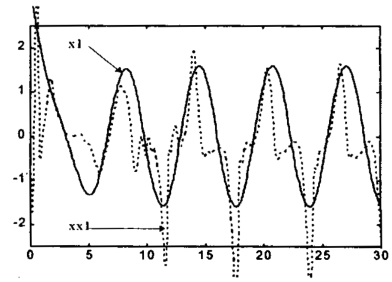

Fig. 2. Identification for q_1

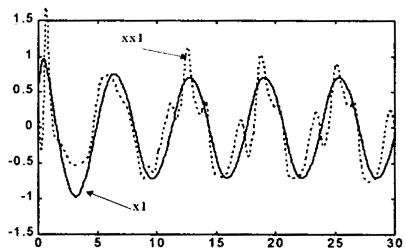

Fig. 3. Identification for q_2

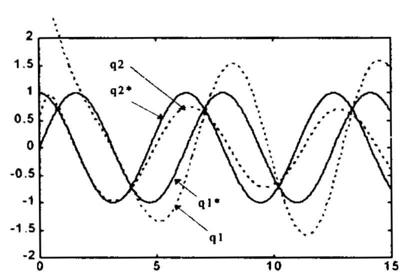

Fig. 4. PD control

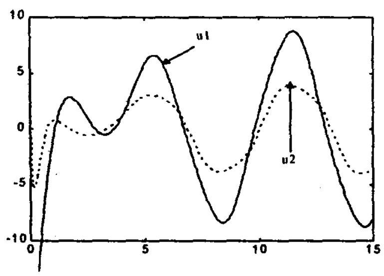

Fig. 5. Control input of PD

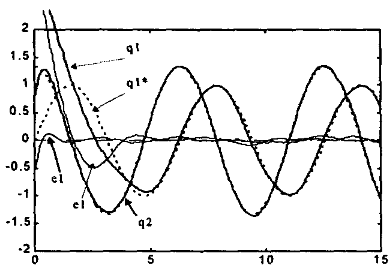

Fig. 6. Response of neurocontrol

where $\delta = 0.1$, $\Delta_t^* = q - q^*$. The neurocontrol presented in this paper are shown in Fig.6 and Fig.7.No initial training or learning phase was need. It is very clear that the addition of DNN makes a significant improvement in the tracking performance.

V. Conclusion

A dynamic neural network was developed for the two-link robot manipulator. First we use the parallel DNN to identify the dynamic of robot, then a direct linearization controller is applied base based on this neuro identifier. Because of the modelling error, a sliding mode compensator is presented. In this paper we proof that both of identification error and tracking error are bounded.

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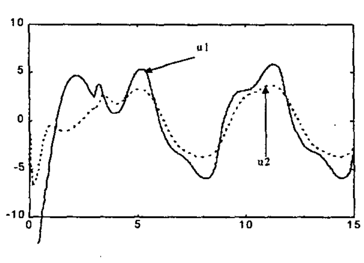

Fig. 7. Control input of neurocontrol

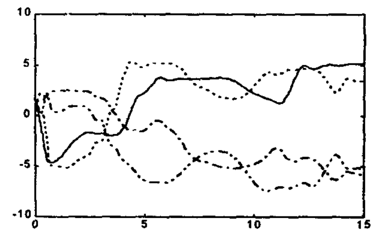

Fig. 8. Representative weightd estimate