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Study of Nonsingular Fast Terminal Sliding-Mode Fault-Tolerant Control

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***Abstract*—This paper studies fault-tolerant control (FTC) designs based on nonsingular terminal sliding-mode control (NTSMC) and nonsingular fast terminal sliding-mode control (NFTSMC). The proposed active FTC laws are shown to be able to achieve fault-tolerant objectives and maintain stabilization performance even when some of the actuators fail to operate. In comparison to existing sliding mode control (SMC) fault-tolerant designs, the proposed schemes not only can retain the advantages of traditional SMC, including fast response, easy implementation and robustness to disturbances/uncertainties, but also make the system states reach the control objective point in a finite amount of time. Moreover, they also resolve the potential singularity phenomena in traditional terminal and faster terminal sliding-mode control designs; meanwhile, the proposed NFTSMC fault-tolerant scheme also possesses the benefit of faster state convergence speed of NFTSMC. Finally, the proposed analytical results are also applied to the attitude control of a spacecraft. Simulation results demonstrate the benefits of the proposed schemes.**

***Index Terms*—Fault detection, fault-tolerant control (FTC), nonsingular fast terminal sliding-mode control (NFTSMC), nonsingular terminal sliding-mode control (NTSMC), nonlinear systems, sliding mode control (SMC).**

I. Introduction

With the rapid development of advanced technologies and complex industrial systems, it is necessary to maintain ever increasing requirements for reliability and safety of control systems. The issues concerning fault-tolerant control (FTC) are thus widely discussed and applied in various areas (see, e.g., [1]-[44], and the references therein). It is worth noting that the Fault Detection and Diagnosis (FDD) (e.g., see [5], [8]-[9], [12]-[18], [23]-[26], [31], [35]-[38], [42]-[44]) module is a cornerstone of an FTC control scheme [9], [12]. The module should be able to provide: 1) Fault Detection and Isolation (FDI); or 2) Fault Detection, Isolation and Estimation, which is referred to as an FDD module. The performance of the overall FTC scheme mainly depends on the performance of this FDD (or FDI) module. The effect of the detection delay and gyroscopic effect on the performance of the overall FTC scheme are also discussed and explored [23]-[26]. Some researchers studied the FDD module and the overall FTC in the presence of different kinds of faults, e.g., sinusoidal fault [5], [8], [12], [23]-[26]. Ossmann and Varga [37]-[38] discussed the Detection and Identification of loss of efficiency (LOE) faults in flight actuators. Moreover, the dynamic disturbances can mask small faults. Therefore, the size of the fault [12] will be discussed in FDD (or FDI). Persis and Isidori [1] presented the theoretical development of FDI with disturbance decoupling by means of the nonlinear geometric approach. Baldi et al. [43] discussed the application and development of such theory in the case of FDI with aerodynamic disturbance decoupling for satellite reaction wheels.

The main objective of FTC is to design an appropriate control law such that the closed-loop system can tolerate some faults in specific control components and maintain the whole system stability with acceptable performance. In some critical system, e.g., spacecraft, the actuator faults may make the overall performance deteriorate severely and become unstable and, thus, may terminate the control mission. This is also why researchers and scholars are continuously interested in developing the reliable control with fault-tolerant methodologies for spacecraft missions. With the aim of tackling these challenges, the main goal of this study is to develop proper FTC laws that possess the advantages of high control precision, robustness and rapid response, even in the presence of actuator failures, such that the control mission of the system can be maintained.

Traditional sliding mode control (SMC) has been widely applied in nonlinear systems [23]-[26], [45]-[51] because of its benefits, including fast response, easy implementation, and robustness to disturbances/uncertainties. Recently, Liang et al. [2], [4], [10] applied the SMC techniques to the FTC design for spacecraft attitude stabilization control. In their application, the T-S fuzzy systems are also incorporated to the nonlinear systems so that the online computational burden, which has been widely discussed in [49], can be reduced. In general, the system under traditional SMC design has only asymptotic stable performance. To enhance the asymptotic convergence to a finite-time state convergence, Zak [52]-[53] first proposed the so-called terminal sliding-mode control (TSMC) [53]. In comparison to traditional SMC, one nonlinear term is added to the sliding surface TSMC to improve the convergence performance. By suitably designing the parameters of TSMC, the system states can reach the control objective point on the sliding surface in a finite amount of time. Therefore, TSMC not only has the advantages of rapid response and robustness, but also offers the benefit of finite-time convergence. However, it was reported that common TSMC design

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suffers a singular problem, and the control magnitude may become unbounded whenever the state reaches zero. This potential drawback has been resolved by so-called nonsingular terminal sliding-mode control (NTSMC) [55]. More recently, by utilizing new-type sliding surface design, some researchers have extended the results of NTSMC to the new-type NTSMC with faster convergence time [58]-[65], i.e., nonsingular fast terminal sliding-mode control (NFTSMC). The NFTSMC not only retains the advantages of NTSMC, but also provides a faster state convergence than that of NTSMC. In light of the remarkable importance and advantages mentioned above, this paper will investigate FTC design by using NTSMC and NFTSMC, respectively.

The organization of this paper is as follows. Section II states the problem formulation and main goal of this paper. Section III describes the FTC design by using NTSMC and NFTSMC, respectively. The analytical results are then applied to the attitude control of a spacecraft in Section IV. Finally, Section V gives the conclusions.

II. Problem Statement

Consider a class of second-order nonlinear control systems

$$\dot{\mathbf{x}}_1 = \mathbf{x}_2 \tag{1}$$
$$\dot{\mathbf{x}}_2 = \mathbf{f}(\mathbf{x}) + G(\mathbf{x})\mathbf{u} + \mathbf{d} \tag{2}$$

where $\mathbf{x}_1 = (x_1, \ldots, x_n)^T \in \mathbb{R}^n$, $\mathbf{x}_2 = (x_{n+1}, \ldots, x_{2n})^T \in \mathbb{R}^n$ and $\mathbf{x} = (\mathbf{x}_1, \mathbf{x}_2)^T$ represent the system states, $\mathbf{u} = (u_1, \ldots, u_m)^T \in \mathbb{R}^m$ with $m \geq n$ are the control inputs, $\mathbf{d} = (d_1, \ldots, d_n)^T \in \mathbb{R}^n$ represent the possible model uncertainties and/or disturbances, $\mathbf{f}(\mathbf{x}) \in \mathbb{R}^n$ and $G(\mathbf{x}) \in \mathbb{R}^{n\times m}$ are smooth functions with $\mathbf{f}(\mathbf{0}) = \mathbf{0}$, and $(\cdot)^T$ denotes the transpose of a vector or a matrix.

In this study, we assume that the actuators' faults have been successfully detected and diagnosed by a suitable FDD scheme. From the FDD information, the overall actuators can be divided into two groups, $\mathcal{H}$ and $\mathcal{F}$, in which we assume that all of the actuators in $\mathcal{H}$ are healthy, while those in $\mathcal{F}$ experience faults. Thus, System (1)-(2) can be rewritten as

$$\dot{\mathbf{x}}_1 = \mathbf{x}_2 \tag{3}$$
$$\dot{\mathbf{x}}_2 = \mathbf{f}(\mathbf{x}) + G_{\mathcal{H}}(\mathbf{x})\mathbf{u}_{\mathcal{H}} + G_{\mathcal{F}}(\mathbf{x})\mathbf{u}_{\mathcal{F}} + \mathbf{d} \tag{4}$$

where $G(\mathbf{x}) = (G_{\mathcal{H}}(\mathbf{x}) \vdots G_{\mathcal{F}}(\mathbf{x}))$ and $\mathbf{u} = (\mathbf{u}_{\mathcal{H}}^T \vdots \mathbf{u}_{\mathcal{F}}^T)^T$. In the rest of this paper, we assume that $\mathbf{u}_{\mathcal{H}} \in \mathbb{R}^k$, $\mathbf{u}_{\mathcal{F}} \in \mathbb{R}^{m-k}$, and $m \geq k \geq n$. For the succeeding FTC law design, we impose the following assumptions:

Assumption 1: For any system state $\mathbf{x} \in \mathbb{R}^{2n}$, the matrix $G_{\mathcal{H}}(\mathbf{x}) \in \mathbb{R}^{n\times k}$ has full row rank, i.e., $\text{rank}(G_{\mathcal{H}}(\mathbf{x})) = n$.

Note that Assumption 1 implies that there exist sufficient healthy actuators to perform the fault-tolerant control task. In addition, we assume that the control inputs in the set of $\mathcal{F}$ are diagnosed as

$$\mathbf{u}_{\mathcal{F}} = \hat{\mathbf{u}}_{\mathcal{F}} + \Delta\mathbf{u}_{\mathcal{F}} \tag{5}$$

where $\hat{\mathbf{u}}_{\mathcal{F}}$ and $\Delta\mathbf{u}_{\mathcal{F}}$ represent the estimated value and estimated error of the faulty actuator $\mathbf{u}_{\mathcal{F}}$, respectively. Therefore, System (3)-(4) can be described as

$$\dot{\mathbf{x}}_1 = \mathbf{x}_2 \tag{6}$$
$$\dot{\mathbf{x}}_2 = \mathbf{f}(\mathbf{x}) + G_{\mathcal{H}}(\mathbf{x})\mathbf{u}_{\mathcal{H}} + G_{\mathcal{F}}(\mathbf{x})\left(\hat{\mathbf{u}}_{\mathcal{F}} + \Delta\mathbf{u}_{\mathcal{F}}\right) + \mathbf{d}. \tag{7}$$

Based on System (6)-(7), an appropriate $\mathbf{u}_{\mathcal{H}}$ will be organized so that the states of the closed-loop system converge to the origin in a finite amount of time, even when the actuators in $\mathcal{F}$ are detected and diagnosed as experiencing faults by an FDD mechanism.

III. Design of Fault-Tolerant Controllers

A. Nonsingular Terminal Sliding-Mode Fault-Tolerant Controller (NTSM-FTC) Design

Considering System (6)-(7), in this section we will employ the NTSMC technique to design the FTC law so that the closed-loop system trajectory can reach the origin in a finite amount of time. According to the concepts of NTSMC, we design the sliding surface of NTSMC as

$$\mathbf{s} = \mathbf{x}_1 + \beta^{-1}\mathbf{sig}^{p/q}(\mathbf{x}_2) = \mathbf{0} \tag{8}$$

where $\mathbf{s} = (s_1, \ldots, s_n)^T \in \mathbb{R}^n$ is the sliding variable, $\beta^{-1} = \text{diag}\{\beta_1^{-1}, \ldots, \beta_n^{-1}\} \in \mathbb{R}^{n\times n}$ is a positive definite matrix, p and q are positive odd numbers satisfying the relation $1 < p/q < 2$, and $\mathbf{sig}^{p/q}(\mathbf{x}_2) := (|\dot{x}_1|^{p/q}\,\text{sign}(\dot{x}_1), \ldots, |\dot{x}_n|^{p/q}\,\text{sign}(\dot{x}_n))^T \in \mathbb{R}^n$. Then, the time derivative of sliding variable $\mathbf{s}$ along the trajectories of System (6)-(7) is

$$\dot{\mathbf{s}} = \mathbf{x}_2 + \beta^{-1}\frac{p}{q}\text{sig}^{(p/q)-1}(\mathbf{x}_2)\cdot\Big[\mathbf{f}(\mathbf{x}) + G_{\mathcal{H}}(\mathbf{x})\mathbf{u}_{\mathcal{H}} + G_{\mathcal{F}}(\mathbf{x})\cdot(\hat{\mathbf{u}}_{\mathcal{F}} + \Delta\mathbf{u}_{\mathcal{F}}) + \mathbf{d}\Big] \tag{9}$$

where

$$\text{sig}^{(p/q)-1}(\mathbf{x}_2) := \text{diag}\left\{|\dot{x}_1|^{(p/q)-1}, \ldots, |\dot{x}_n|^{(p/q)-1}\right\} \in \mathbb{R}^{n\times n}.$$

It is worth noting from Eq. (9) that the estimated value $\hat{\mathbf{u}}_{\mathcal{F}}$ is available from FDD information. Hence, the total uncertainties take the form of $G_{\mathcal{F}}(\mathbf{x})\Delta\mathbf{u}_{\mathcal{F}} + \mathbf{d}$. Moreover, due to the success of fault diagnosis, the estimation error $\Delta\mathbf{u}_{\mathcal{F}}$ can be assumed to be bounded. This can be described by the assumption below:

Assumption 2: There exists a nonnegative function $\rho(\mathbf{x}, t)$ such that

$$\|G_{\mathcal{F}}(\mathbf{x})\Delta\mathbf{u}_{\mathcal{F}} + \mathbf{d}\| \leq \rho(\mathbf{x}, t) \text{ for all } \mathbf{x} \in \mathbb{R}^{2n} \text{ and } t \in \mathbb{R}^+.$$

Under Assumptions 1 and 2, we design the NTSMC-FTC law as follows:

$$\mathbf{u}_{\mathcal{H}} = \mathbf{u}_{\mathcal{H}eq} + \mathbf{u}_{\mathcal{H}re} \tag{10}$$
$$\mathbf{u}_{\mathcal{H}eq} = -G_{\mathcal{H}}^{+}(\mathbf{x})\Big[\mathbf{f}(\mathbf{x}) + \beta\frac{q}{p}\mathbf{sig}^{2-(p/q)}(\mathbf{x}_2) + G_{\mathcal{F}}(\mathbf{x})\hat{\mathbf{u}}_{\mathcal{F}}\Big] \tag{11}$$
$$\mathbf{u}_{\mathcal{H}re} = -\Big[\eta + \rho(\mathbf{x}, t)\Big]\cdot G_{\mathcal{H}}^{+}(\mathbf{x})\mathbf{sign}(\mathbf{s}) \tag{12}$$

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where $G_{\mathcal{H}}^{+}(\mathbf{x}) = G_{\mathcal{H}}^{T}(\mathbf{x})\left[G_{\mathcal{H}}(\mathbf{x})G_{\mathcal{H}}^{T}(\mathbf{x})\right]^{-1} \in \mathbb{R}^{k\times n}$, $\eta > 0$, $\beta := (\beta^{-1})^{-1} \in \mathbb{R}^{n\times n}$, $\rho(\mathbf{x}, t)$ is given by Assumption 2, $\mathbf{sign}(\mathbf{s}) = (\mathrm{sign}(s_1), \ldots, \mathrm{sign}(s_n))^T \in \mathbb{R}^n$, and

$$\mathbf{sig}^{2-(p/q)}(\mathbf{x}_2) := \Big(|\dot{x}_1|^{2-(p/q)}\,\mathrm{sign}(\dot{x}_1), \ldots, |\dot{x}_n|^{2-(p/q)}\,\mathrm{sign}(\dot{x}_n)\Big)^T \in \mathbb{R}^n.$$

Here, we summarize our main result in the following theorem.

Theorem 1: Suppose that System (1)-(2) experiences actuator faults at the control channel denoted by $\mathcal{F}$ with estimated value $\hat{\mathbf{u}}_{\mathcal{F}}$ and estimation error $\Delta\mathbf{u}_{\mathcal{F}}$ given by Eq. (5). Then, under Assumptions 1 and 2, and controller defined by Eqs. (10)-(12), the system states of System (1)-(2) can converge to the origin, i.e., the equilibrium point, in a finite amount of time.

Proof: Denote $(\cdot)_i$ as the (i, i) element of a matrix. From Assumptions 1 and 2, Eqs. (9)-(12), and the fact that $\mathbf{x}_2 = (|\dot{x}_1|\,\mathrm{sign}(\dot{x}_1), \ldots, |\dot{x}_n|\,\mathrm{sign}(\dot{x}_n))^T$, we have

$$\begin{aligned}\mathbf{s}^T\dot{\mathbf{s}} &= \mathbf{s}^T\cdot\beta^{-1}\frac{p}{q}\mathrm{sig}^{(p/q)-1}(\mathbf{x}_2)\Big\{G_{\mathcal{F}}(\mathbf{x})\Delta\mathbf{u}_{\mathcal{F}} + \mathbf{d} \\ &\quad - \Big[\eta + \rho(\mathbf{x}, t)\Big]\cdot\mathbf{sign}(\mathbf{s})\Big\} \\ &\le \sum_{i=1}^{n}\Big\{\Big(\beta^{-1}\frac{p}{q}\mathrm{sig}^{(p/q)-1}(\mathbf{x}_2)\Big)_i\cdot|s_i|\cdot\Big\|G_{\mathcal{F}}(\mathbf{x})\Delta\mathbf{u}_{\mathcal{F}} + \mathbf{d}\Big\|_\infty \\ &\quad - \Big[\eta + \rho(\mathbf{x}, t)\Big]\cdot\Big(\beta^{-1}\frac{p}{q}\mathrm{sig}^{(p/q)-1}(\mathbf{x}_2)\Big)_i\cdot|s_i|\Big\} \\ &\le -\eta\cdot\Pi(\mathbf{x}_2)\cdot\|\mathbf{s}\| \end{aligned} \tag{13}$$

where

$$\Pi(\mathbf{x}_2) := \inf_{1\le i\le n}\left\{\Big(\beta^{-1}\frac{p}{q}\mathrm{sig}^{(p/q)-1}(\mathbf{x}_2)\Big)_i\right\}. \tag{14}$$

Then, from Ineq. (13) and the fact that $\mathbf{x}_2 = \mathbf{0}$ is not an attractor [55], we can conclude that the sliding mode can be reached in finite amount of time, i.e., $\mathbf{s}(t) = \mathbf{0}$ for all $t \ge t_0 + t_r$ where t_0 is the initial time and t_r is the time horizon of reaching phase. Next, when the sliding mode is achieved, it can be found from Eq. (8) that the reduced model takes the form $\dot{\mathbf{x}}_1 = -\beta'\mathbf{sig}^{\frac{q}{p}}(\mathbf{x}_1)$ for all $t \ge t_0 + t_r$, where $\beta' = \mathrm{diag}\{\beta_1^{\frac{q}{p}}, \ldots, \beta_n^{\frac{q}{p}}\}$ and $\mathbf{sig}^{\frac{q}{p}}(\mathbf{x}_1) = (|x_1|^{q/p}\,\mathrm{sign}(x_1), \ldots, |x_n|^{q/p}\,\mathrm{sign}(x_n))^T \in \mathbb{R}^n$; moreover, the largest finite time t_s that is taken to force $\mathbf{x}_1(t_0 + t_r)$ to $\mathbf{x}_1(t_0 + t_r + t_s) = \mathbf{0}$ is easily determined as

$$t_s = \max_{1\le i\le n}\left\{\frac{p}{(\beta_i)^{\frac{q}{p}}(p-q)}|x_i(t_0 + t_r)|^{(p-q)/p}\right\}. \tag{15}$$

Once $\mathbf{x}_1 = \mathbf{0}$ is achieved, from Eq. (8) one can find that $\mathbf{x}_2 = \mathbf{0}$ is also achieved. From the analysis given above, we can conclude that the system state can converge to zero in a finite amount of time, i.e., $t_0 + t_r + t_s$. This completes the proof. ■

B. Nonsingular Fast Terminal Sliding-Mode Fault-Tolerant Controller (NFTSM-FTC) Design

In this section, the NFTSMC [61] will be employed to design the FTC law. Following the idea of NFTSMC, we introduce the sliding surface of NFTSMC as below:

$$\mathbf{s} = \mathbf{x}_1 + \alpha^{-1}\mathbf{sig}^{\gamma}(\mathbf{x}_1) + \beta^{-1}\mathbf{sig}^{p/q}(\mathbf{x}_2) = \mathbf{0}. \tag{16}$$

where $\mathbf{s} \in \mathbb{R}^n$ is the sliding variable, $\alpha^{-1} = \mathrm{diag}\{\alpha_1^{-1}, \ldots, \alpha_n^{-1}\} \in \mathbb{R}^{n\times n}$ and $\beta^{-1} = \mathrm{diag}\{\beta_1^{-1}, \ldots, \beta_n^{-1}\} \in \mathbb{R}^{n\times n}$ are positive definite matrices, respectively, p and q are positive odd numbers satisfying the relation $1 < p/q < 2$ and $\gamma > p/q$, $\mathbf{sig}^{p/q}(\mathbf{x}_2) := (|\dot{x}_1|^{p/q}\,\mathrm{sign}(\dot{x}_1), \ldots, |\dot{x}_n|^{p/q}\,\mathrm{sign}(\dot{x}_n))^T \in \mathbb{R}^n$, and $\mathbf{sig}^{\gamma}(\mathbf{x}_1) := (|x_1|^{\gamma}\,\mathrm{sign}(x_1), \ldots, |x_n|^{\gamma}\,\mathrm{sign}(x_n))^T \in \mathbb{R}^n$. In this case, the time derivative of sliding variable along the trajectories of System (6)-(7) is

$$\begin{aligned}\dot{\mathbf{s}} &= \mathbf{x}_2 + \alpha^{-1}\gamma\mathrm{sig}^{\gamma-1}(\mathbf{x}_1)\cdot\mathbf{x}_2 \\ &\quad + \beta^{-1}\frac{p}{q}\mathrm{sig}^{(p/q)-1}(\mathbf{x}_2)\cdot\Big[\mathbf{f}(\mathbf{x}) + G_{\mathcal{H}}(\mathbf{x})\mathbf{u}_{\mathcal{H}} \\ &\quad + G_{\mathcal{F}}(\mathbf{x})(\hat{\mathbf{u}}_{\mathcal{F}} + \Delta\mathbf{u}_{\mathcal{F}}) + \mathbf{d}\Big]\end{aligned} \tag{17}$$

where

$$\begin{aligned}\mathrm{sig}^{\gamma-1}(\mathbf{x}_1) &:= \mathrm{diag}\left\{|x_1|^{\gamma-1}, \ldots, |x_n|^{\gamma-1}\right\} \in \mathbb{R}^{n\times n} \\ \mathrm{sig}^{(p/q)-1}(\mathbf{x}_2) &:= \mathrm{diag}\Big\{|\dot{x}_1|^{(p/q)-1}, \ldots, |\dot{x}_n|^{(p/q)-1}\Big\} \in \mathbb{R}^{n\times n}.\end{aligned}$$

Again, the total uncertainties have the form of $G_{\mathcal{F}}(\mathbf{x})\Delta\mathbf{u}_{\mathcal{F}} + \mathbf{d}$; thus, Assumption 2 is also imposed in this case because of the success of fault diagnosed. Under Assumptions 1 and 2, we design the NTSMC-FTC law as below:

$$\mathbf{u}_{\mathcal{H}} = \mathbf{u}_{\mathcal{H}eq} + \mathbf{u}_{\mathcal{H}re} \tag{18}$$

$$\begin{aligned}\mathbf{u}_{\mathcal{H}eq} &= -G_{\mathcal{H}}^{+}(\mathbf{x})\Big[\mathbf{f}(\mathbf{x}) + G_{\mathcal{F}}(\mathbf{x})\hat{\mathbf{u}}_{\mathcal{F}} + \beta\frac{q}{p}\mathbf{sig}^{2-(p/q)}(\mathbf{x}_2) \\ &\quad + \alpha^{-1}\gamma\mathrm{sig}^{\gamma-1}(\mathbf{x}_1)\cdot\beta\frac{q}{p}\mathbf{sig}^{2-(p/q)}(\mathbf{x}_2)\Big]\end{aligned} \tag{19}$$

$$\mathbf{u}_{\mathcal{H}re} = -\Big[\eta + \rho(\mathbf{x}, t)\Big]\cdot G_{\mathcal{H}}^{+}(\mathbf{x})\mathbf{sign}(\mathbf{s}) \tag{20}$$

where $G_{\mathcal{H}}^{+}(\mathbf{x}) = G_{\mathcal{H}}^{T}(\mathbf{x})\left[G_{\mathcal{H}}(\mathbf{x})G_{\mathcal{H}}^{T}(\mathbf{x})\right]^{-1} \in \mathbb{R}^{k\times n}$, $\eta > 0$, $\beta := (\beta^{-1})^{-1} \in \mathbb{R}^{n\times n}$, $\rho(\mathbf{x}, t)$ is given by Assumption 2, $\mathbf{sign}(\mathbf{s}) = (\mathrm{sign}(s_1), \ldots, \mathrm{sign}(s_n))^T \in \mathbb{R}^n$ and

$$\mathbf{sig}^{2-(p/q)}(\mathbf{x}_2) := \Big(|\dot{x}_1|^{2-(p/q)}\,\mathrm{sign}(\dot{x}_1), \ldots, |\dot{x}_n|^{2-(p/q)}\,\mathrm{sign}(\dot{x}_n)\Big)^T \in \mathbb{R}^n.$$

From the above design, we have the following result.

Theorem 2: Suppose that System (1)-(2) experiences actuator faults at the control channel denoted by $\mathcal{F}$ with estimated value $\hat{\mathbf{u}}_{\mathcal{F}}$ and estimation error $\Delta\mathbf{u}_{\mathcal{F}}$ given by Eq. (5). Then, under Assumptions 1 and 2, and controller defined by Eqs. (18)-(20), the system states of System (1)-(2) can converge to the origin, i.e., the equilibrium point, in a finite amount of time.

Proof: The proof is similar to that of Theorem 1, except that the largest finite time t_s is taken to force $\mathbf{x}_1(t_0 + t_r)$ to

$\mathbf{x}_1(t_0 + t_r + t_s)$ having the value of (for details, please refer to [61]):

$$t_s = \max_{1 \le i \le n} \left\{ \frac{1}{\beta_i^{q/p}} \int_0^{|x_i(t_0+t_r)|} \frac{1}{\left(\tau - \alpha_i^{-1}\tau^{\gamma}\right)^{q/p}}\, d\tau \right\}. \quad (21)$$

Thus, as the result of Theorem 1, one can conclude that the system state will converge to zero in a finite amount of time $t_0 + t_r + t_s$. The proof is completed. ■

Remark 1: In the NTSMC and NFTSMC design, the selection of p, q, and γ should satisfy the rules $1 < p/q < 2$ and $\gamma > p/q$. In general, the relative parameters in controller design can be tuned by some adaptive methods [58], [61].

IV. Simulation and Discussion

Consider the following dynamics for satellite attitude stabilization control from [2], [10], and [34] as described in the same form as (1)-(2) with $n = 3$, in which $\mathbf{f}(\mathbf{x}) = [f_1(\mathbf{x}), f_2(\mathbf{x}), f_3(\mathbf{x})]^T$, $\mathbf{x} = (\mathbf{x}_1, \mathbf{x}_2)^T$, $\mathbf{x}_1 = (x_1, x_2, x_3)^T = (\phi, \theta, \psi)^T$, $\mathbf{x}_2 = (x_4, x_5, x_6)^T = (\dot\phi, \dot\theta, \dot\psi)^T$, $\mathbf{u} = (u_1, u_2, u_3, u_4)^T$, and $\mathbf{d} = [d_1, d_2, d_3]^T$. Here, ϕ, θ, and ψ are Euler's angles for x, y, z axes, respectively, $\mathbf{u}$ denote actuators, providing the force torques by reaction wheels or thrusters in four directions, and $\mathbf{f}(\mathbf{x})$ and $G(\mathbf{x})$ are described as below:

$$\begin{aligned} f_1(\mathbf{x}) =\ & \omega_0 x_6 c x_3 c x_2 - \omega_0 x_5 s x_3 s x_2 + \frac{I_y - I_z}{I_x}\Big[x_5 x_6 \\ & + \omega_0 x_5 c x_1 s x_3 s x_2 + \omega_0 x_5 c x_3 s x_1 + \omega_0 x_6 c x_3 c x_1 \\ & + \frac{1}{2}\omega_0^2 s(2x_3) c^2 x_1 s x_2 + \frac{1}{2} w_0^2 c^2 x_3 s(2x_1) \\ & - \omega_0 x_6 s x_3 s x_2 s x_1 - \frac{1}{2}\omega_0^2 s^2 x_2 s^2 x_3 s(2x_1) \\ & - \frac{1}{2}\omega_0^2 s(2x_3) s x_2 s^2 x_1 - \frac{3}{2}\omega_0^2 c^2 x_2 s(2x_1) \Big] \end{aligned} \quad (22)$$

$$\begin{aligned} f_2(\mathbf{x}) =\ & \omega_0 x_6 s x_3 c x_1 + \omega_0 x_4 c x_3 s x_1 + \omega_0 x_6 c x_3 s x_2 s x_1 \\ & + \omega_0 x_5 s x_3 c x_2 s x_1 + \omega_0 x_4 s x_3 s x_2 c x_1 \\ & + \frac{I_z - I_x}{I_y}\Big[x_4 x_6 + \omega_0 x_4 c x_1 s x_3 s x_2 \\ & + \omega_0 x_4 c x_3 s x_1 - \omega_0 x_6 s x_3 c x_2 \\ & - \frac{1}{2}\omega_0^2 s(2x_2) s^2 x_3 c x_1 - \frac{1}{2} w_0^2 c x_2 s x_1 s(2x_3) \\ & + \frac{3}{2}\omega_0^2 s(2x_2) c x_1 \Big] \end{aligned} \quad (23)$$

$$\begin{aligned} f_3(\mathbf{x}) =\ & \omega_0 x_4 s x_1 s x_3 s x_2 - \omega_0 x_6 c x_1 c x_3 s x_2 \\ & - \omega_0 x_5 c x_1 s x_3 c x_2 + \omega_0 x_6 s x_3 s x_1 - \omega_0 x_4 c x_3 c x_1 \\ & + \frac{I_x - I_y}{I_z}\Big[x_4 x_5 + \omega_0 x_4 c x_3 c x_1 \\ & - \omega_0 x_4 s x_3 s x_2 s x_1 - \omega_0 x_5 s x_3 c x_2 \\ & - \frac{1}{2}\omega_0^2 s(2x_3) c x_2 c x_1 + \frac{1}{2} w_0^2 s^2 x_3 s x_1 s(2x_2) \\ & - \frac{3}{2}\omega_0^2 s(2x_2) s x_1 \Big] \end{aligned} \quad (24)$$

$$G(\mathbf{x}) = \begin{pmatrix} 0.67 & 0.67 & 0.67 & 0.67 \\ 0.69 & -0.69 & -0.69 & 0.69 \\ 0.28 & 0.28 & -0.28 & -0.28 \end{pmatrix}. \quad (25)$$

Here I_x, I_y, and I_z are the inertia with respect to the three body coordinate axes, ω_0 denotes the constant orbital rate, and c and s denote the cos and sin functions, respectively. Details of these parameters can be referred to [2]. Moreover, we adopt an FDD mechanism from [2, Eq. (10) and Eq. (11), p. 335] for active FTC mission. For completeness, we briefly recall the mechanism as follows. The main idea of the FDD mechanism is to decouple the control input through a coordinate transformation so that any fault associated with a channel can be diagnosed. Following this idea, the coordinate transformation is chosen to have the form $\mathbf{z}_1 = \mathbf{x}_1$ and $\mathbf{z}_2 = P\mathbf{x}_2$, where $P := (g_1, g_2, g_3)^{-1}$ and g_i denotes the ith column of $G(\mathbf{x})$. With the new state variables, Eq. (1) will be transformed as

$$\dot z_1 = P^{-1}\mathbf{z}_2, \quad \dot{\mathbf{z}}_2 = \mathbf{f}^{new}(\mathbf{z}) + G^{new}(\mathbf{z})\mathbf{u} + P\mathbf{d} \quad (26)$$

where

$$\mathbf{f}^{new}(\mathbf{z}) = P\mathbf{f}(\mathbf{z}_1, P^{-1}\mathbf{z}_2) \quad (27)$$

$$G^{new}(\mathbf{z}) = PG\left(\mathbf{z}_1, P^{-1}\mathbf{z}_2\right) = \begin{pmatrix} 1 & 0 & 0 & l_1 \\ 0 & 1 & 0 & l_2 \\ 0 & 0 & 1 & l_3 \end{pmatrix}. \quad (28)$$

Since every three columns taken out from $G(\mathbf{x})$ are linearly independent, all constants l_1, l_2, and l_3 are nonzero. Based on the transformed system (27), observer and the associated residual signals γ_i are chosen as below:

$$\dot\xi_i = f_i^{new}(\mathbf{z}) + u_i + l_i u_4 + k_i \cdot (z_{i+3} - \xi_i) \quad (29)$$

$$\gamma_i = z_{i+3} - \xi_i \quad (30)$$

for $1 \le i \le 3$, where $k_i > 0$ for all $1 \le i \le 3$ and $f_i^{new}(\mathbf{z})$ represents the ith entry of $\mathbf{f}^{new}(\mathbf{z})$. It was shown in [2] that the observer presented in Eqs. (29)-(30) can achieve fault detection and diagnosis for any single actuator fault, and estimate the output value of the faulty actuator, e.g., the ith actuator, to be $u_i + k_i \cdot \gamma_i$, where u_i is the designed output value of the ith actuator. With these settings, an alarm will be triggered if the magnitude of the ith residual signal r_i from the observer is greater than a threshold set by the designer. In this example, we assume that the actuator u_2 fails to work at time $t = 4$ sec. In order to alleviate the chattering phenomena in the presented FTC, we use the saturation function to replace the original ideal switching function with boundary layer to be $\varepsilon = 0.002$. Moreover, for demonstration, the initial condition and disturbance are set as $\mathbf{x}(0) = [0.8, 0.15, -1.2, 0.15, -1.2, -0.3]^T$ and $\mathbf{d} = 0.05 \cdot [\sin(t), \cos(2t), \sin(3t)]$, respectively. The alarm threshold for each actuator is chosen as 0.015. Other parameters are set as follows: $p = 9$, $q = 7$, $\gamma = 1.4$, $\alpha = \mathrm{diag}\{1, 1, 1\}$, $\beta = \mathrm{diag}\{1.2, 0.9, 0.8\}$, $\rho(\mathbf{x}, t) = \|\mathbf{d}\|_\infty := \sup_t \|\mathbf{d}\|$, and $\eta = 1$.

Simulation results are shown in Figs. 1-7. In this example, we use three types of active FTC laws, including sliding-mode FTC law (labeled SMC) (which can be referred to [2]) and presented NTSMC-FTC (labeled NTSMC), and NFTMSC-FTC laws (labeled NFTSMC). In this example, Figs. 1 and 2 show the time history of six system states, while Figs. 4 and 6, respectively, show the time history of the three

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sliding variables and the control inputs. From Figs. 1 (a)-(c), we can see that the proposed FTC laws can still make the closed-loop system achieve stabilization task under the allowable disturbances and actuator fault; moreover, Figs. 2 (a)-(c) indicate that the angular speeds can also converge to zero. Figure 3, a zoom in on Figs. 1 (a)-(c) and 2 (a)-(c), clearly show that NFTSMC will result in faster state convergence time in comparison with NTSMC and SMC. This also demonstrates that NFTSMC possesses the characteristic of fast state convergence speed after the closed-loop system is in sliding mode. From Fig. 4, three FTC laws can make the system states reach the sliding surface in a finite amount of time. From Fig. 6, we can see that u_2 fails at $t = 4$ sec and its output value becomes 0; moreover, the value of the residual signal γ_2 reaches the threshold value 0.015, which results in FDD triggering the alarm$_2$ near $t = 4$ sec (as shown in Fig. 7). After the success of fault detection and diagnosis, the FTC law is reconfigured so that the control mission is continuously performed. This action corresponds to the jumping of control inputs curves near $t = 4$ sec, as shown in Figs. 6 (a), (c)-(d). From the simulation results, we can verify the effectiveness of the proposed methods. It is worth noting that the time delay of the FDD mechanism is included in this example. A more detailed discussion concerning possible effects in FTC with FDD, such as time delay effect, gyroscopic effect, and the effect by different kinds of input forms, can be found in [5], [8], [12], and [23]-[26].

V. Conclusion

In this study, new-type nonsingular terminal sliding-mode fault-tolerant control (NTSM-FTC) and nonsingular fast terminal sliding-mode fault-tolerant control (NFTSM-FTC) laws have been presented for a set of second order nonlinear control systems. It was shown that the closed-loop system can achieve the stabilization mission even when some of the actuators fail to operate while retaining the advantages of the traditional sliding mode control (SMC) approach. Moreover, the system states of the closed-loop system were shown to be able to converge to the equivalent point in a finite amount of time; meanwhile, the potential singularity phenomena that exist in the both traditional terminal and faster terminal sliding-mode designs are resolved. Through application to spacecraft attitude control, the simulation results demonstrate the effectiveness of the proposed schemes.

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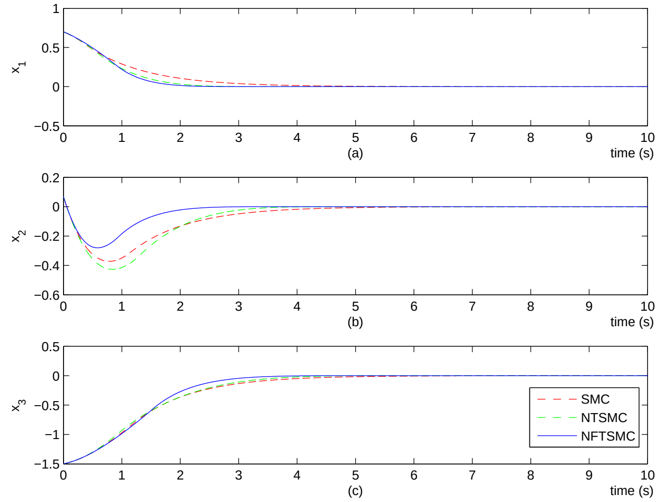

Fig. 1. Time history of system states x_1, x_2, and x_3.

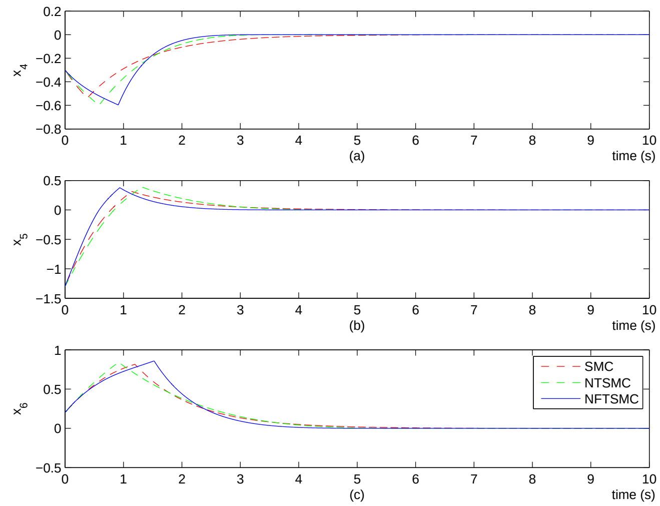

Fig. 2. Time history of system states x_4, x_5, and x_6.

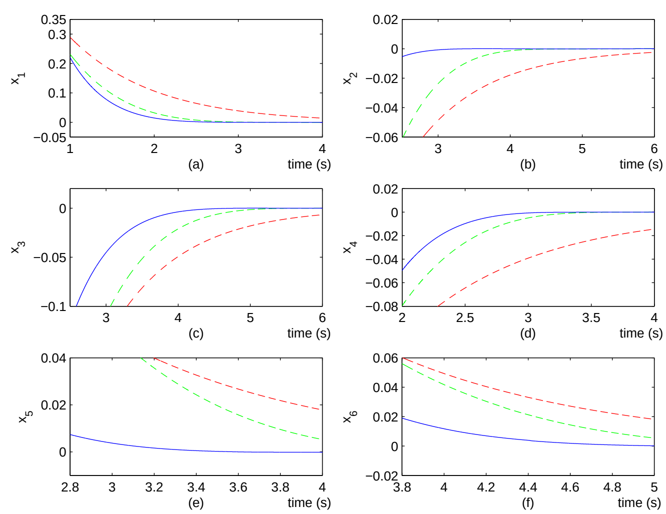

Fig. 3. Zoom in on time history of system states x_1 to x_6.

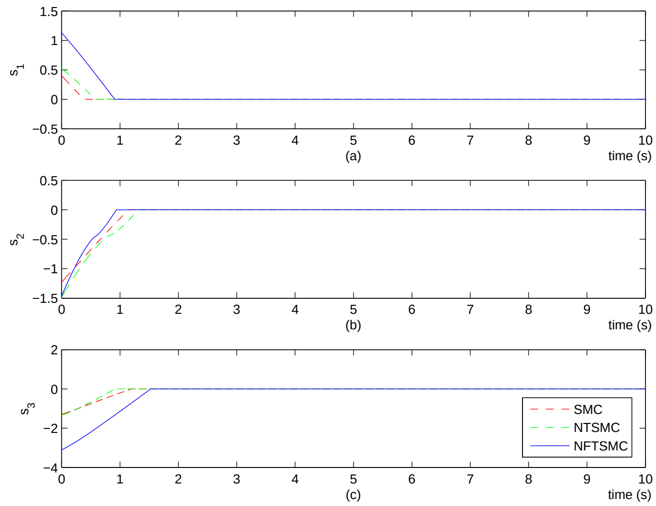

Fig. 4. Time history of the three sliding variables.

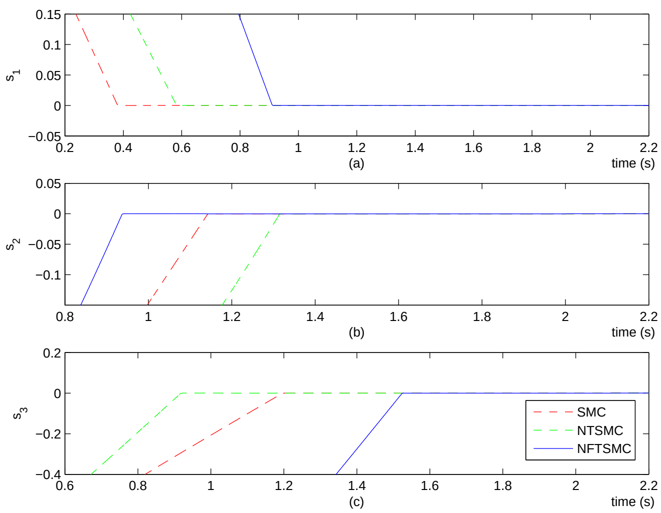

Fig. 5. Zoom in on time history of the three sliding variables.

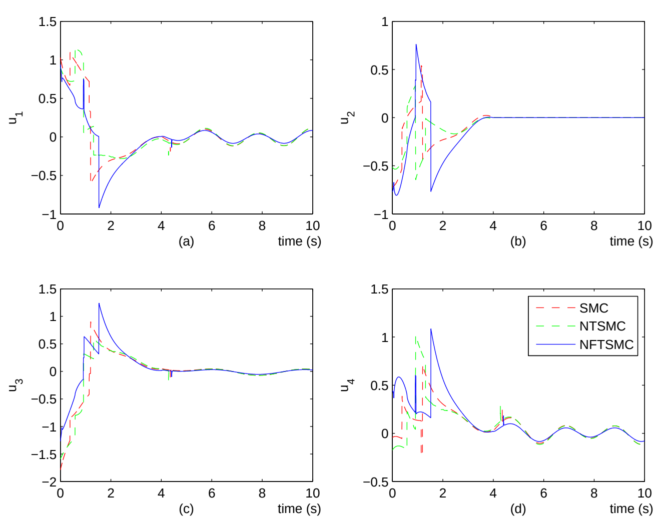

Fig. 6. Time history of the four control inputs.

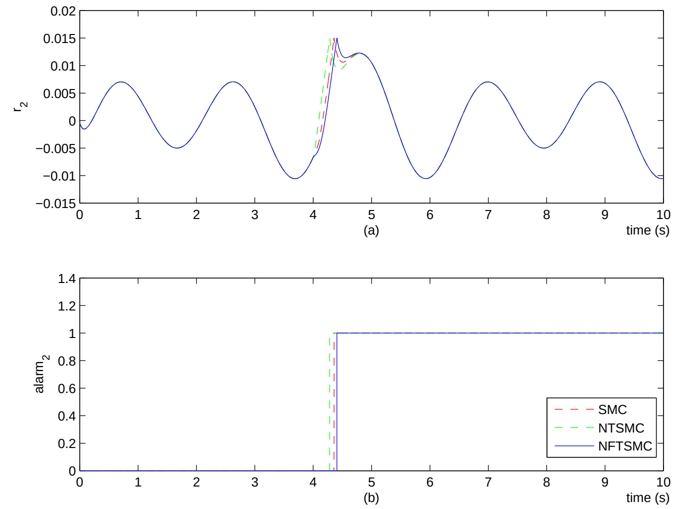

Fig. 7. (a) Residual signal r_2; (b) Alarm signal $alarm_2$.

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