Chapter 1: Coulomb's Law

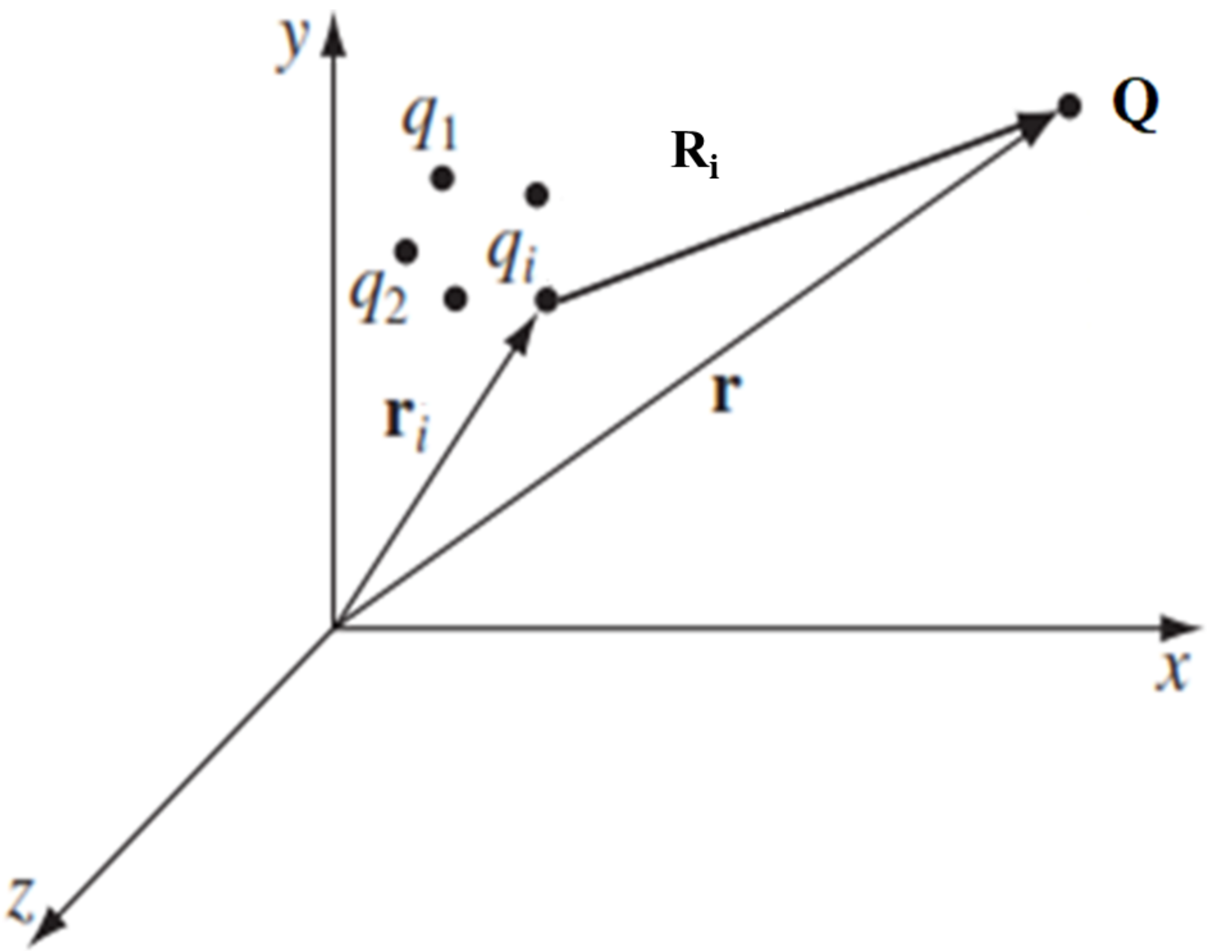
Session 2:

- ✓ Coulomb's Law - Multiple Charges
- ✓ Examples

MULTIPLE CHARGES

The resultant force on any one charge equals the vector sum of the forces exerted by the other individual charges that are present.

For example, if four charges are present:



$$\vec{\mathbf{F}}_Q = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 + \vec{\mathbf{F}}_3 + \ldots = k_e \sum_i \frac{q_i Q}{R_i^{\,2}} (\hat{\mathbf{r}}_i)$$

Ex 3. **Three point charges lie along the *x axis as shown in Figure. The positive charge q_1=15 μC is at x = 2.00 m, the positive charge q_2=6 μC is at the origin, and the net force acting on q_3 is zero. What is the x coordinate of q_3 ?***

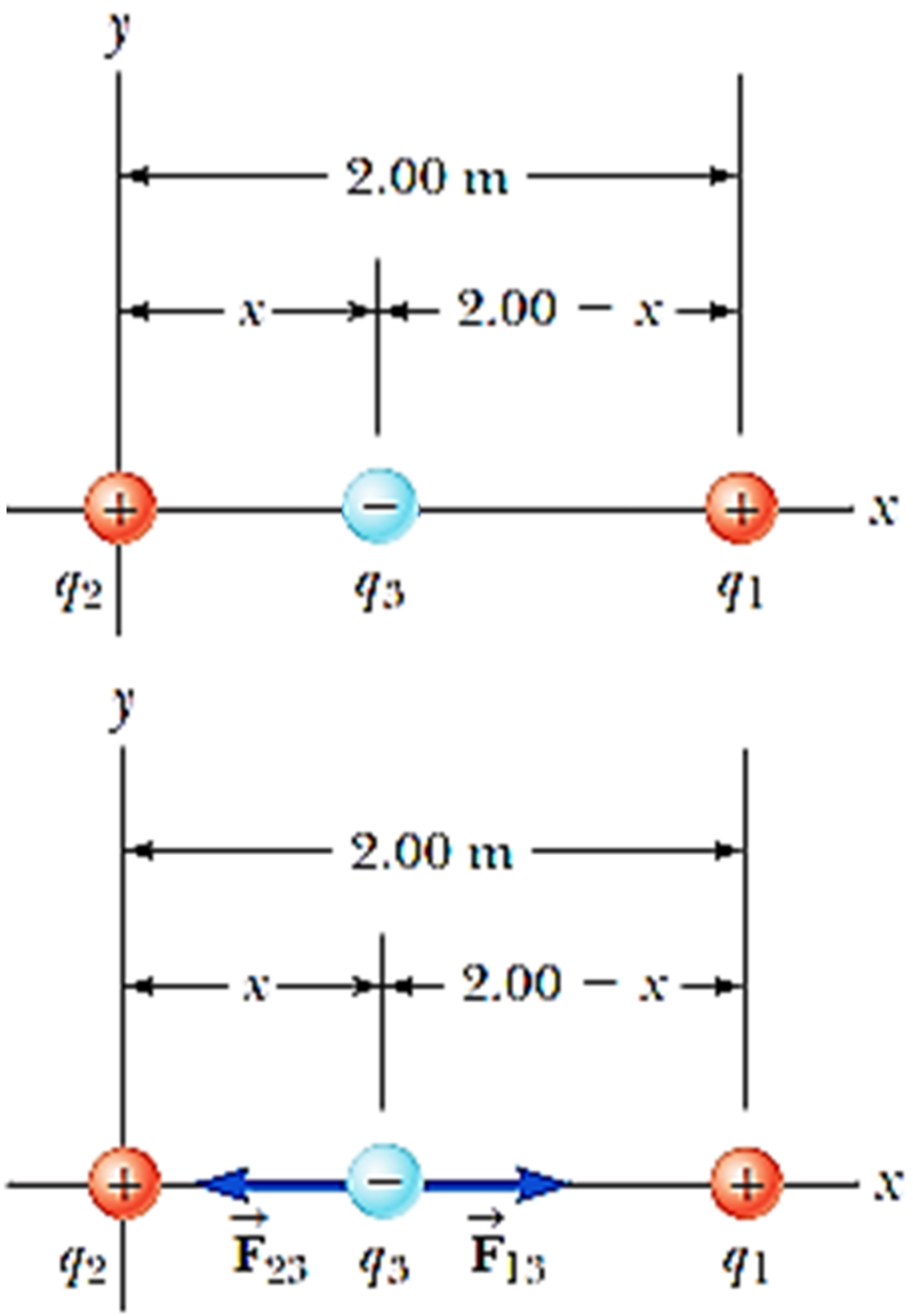
$$\vec{F}_3 = \vec{F}_{13} + \vec{F}_{23} = 0$$

$$k_e \frac{|q_1||q_3|}{(2-x)^2}\hat{i} - k_e \frac{|q_2||q_3|}{x^2}\hat{i} = 0$$

$$k_e \frac{|q_1||q_3|}{(2-x)^2} = k_e \frac{|q_2||q_3|}{x^2}$$

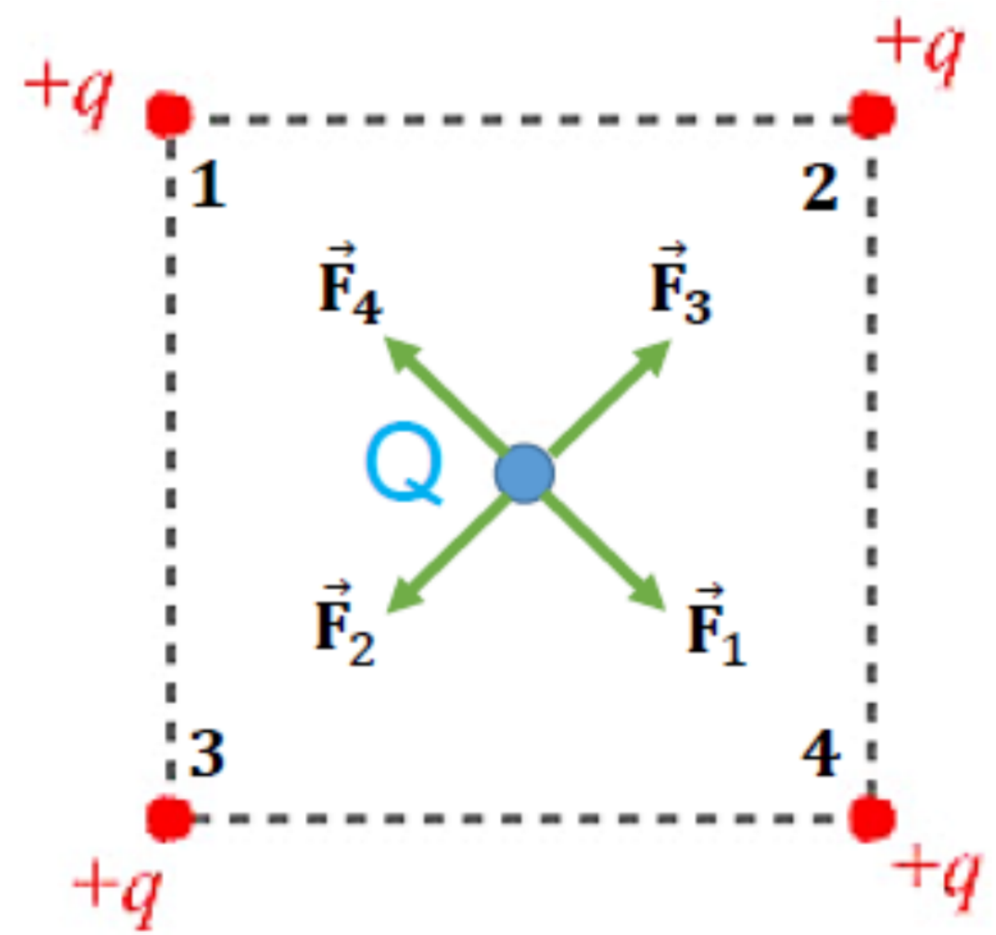
$$\frac{|q_1|}{(2-x)^2} = \frac{|q_2|}{x^2}$$

$$x = \frac{2\sqrt{|q_2|}}{\sqrt{|q_2|} + \sqrt{|q_1|}} = 0.775 \ m$$



Ex 4. Four 10-n*C* positive charges are located in the z = 0 plane at the corners of a square 8 cm on a side. Calculate the magnitude of the total force on a fifth 10-n*C* positive charge when : a) this charge is located at the center of square. b) this charge is located at a point 8 cm distant from each of the other charges..

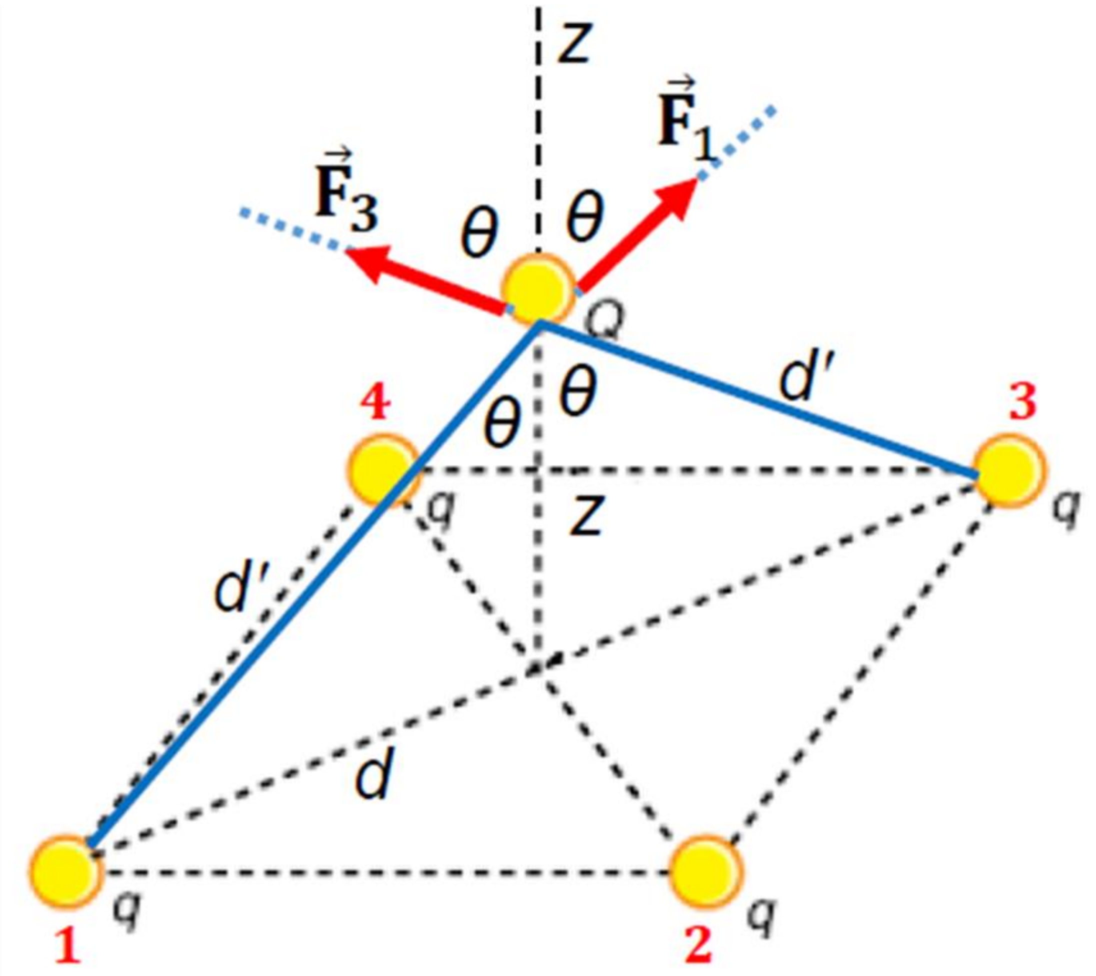
$$\vec{F}_Q = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4$$



$$\left|\vec{F}_1\right| = \left|\vec{F}_2\right| = \left|\vec{F}_3\right| = \left|\vec{F}_4\right| = k_e \frac{qQ}{d^2}$$

$$d = \frac{8\sqrt{2}}{2} = 4\sqrt{2}\ cm$$

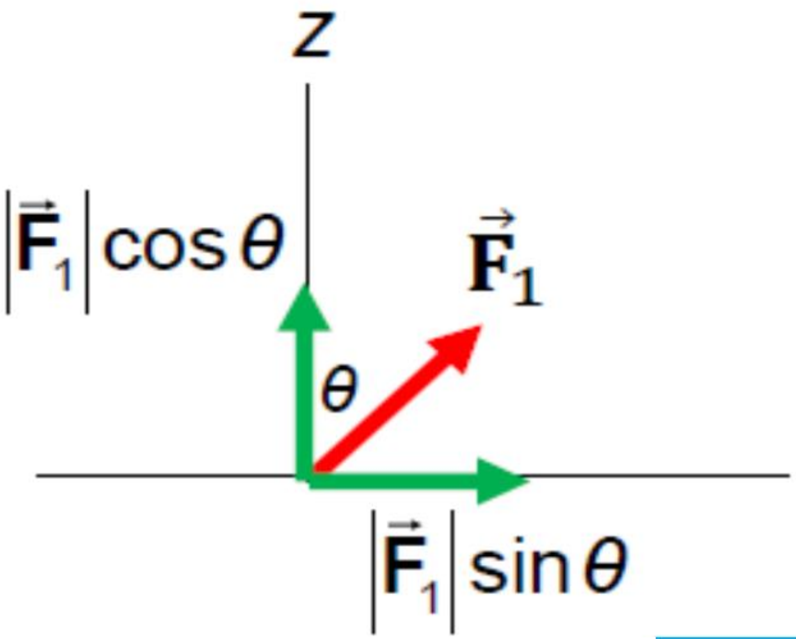
$$\vec{F}_Q = 0$$



$$d'^2 = d^2 + z^2 \Rightarrow 64 = 32 + z^2 \Rightarrow z = \pm 4\sqrt{2}\ cm$$

$$\cos\theta = \frac{z}{d'} = \frac{\sqrt{2}}{2}$$

$$\left|\vec{\mathbf{F}}_1\right| = \left|\vec{\mathbf{F}}_2\right| = \left|\vec{\mathbf{F}}_3\right| = \left|\vec{\mathbf{F}}_4\right| = \left|\vec{\mathbf{F}}\right| = k_e \frac{qQ}{d'^2}$$

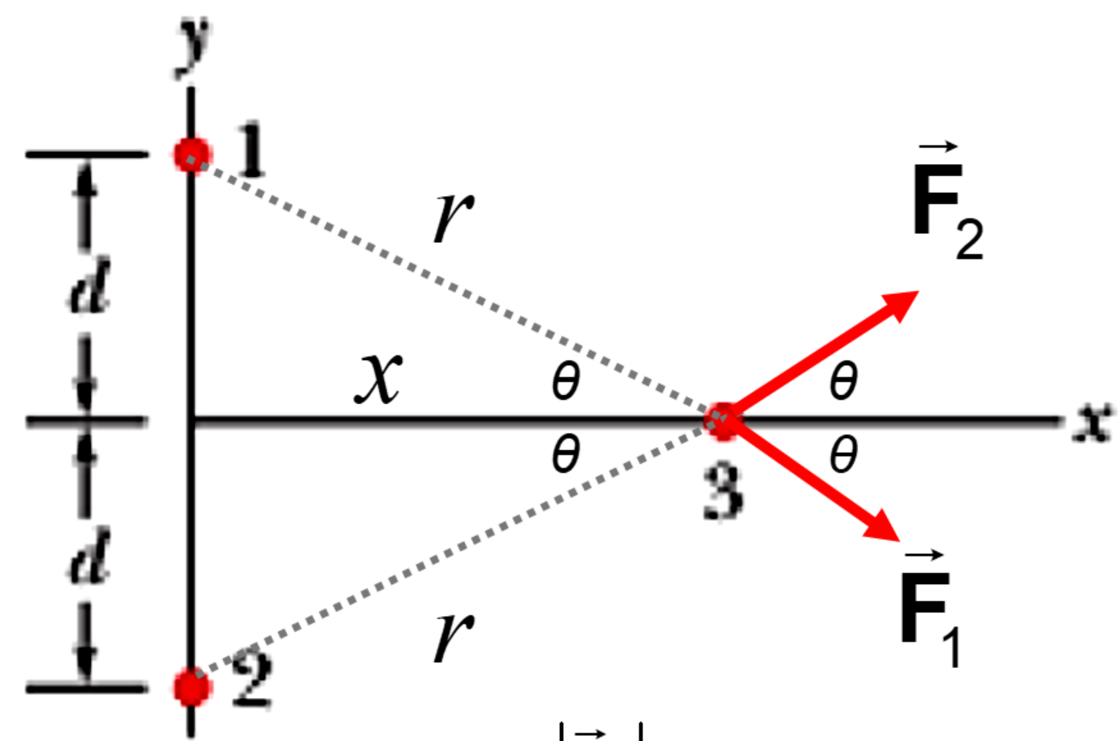


$$\vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_3 = 2\left|\vec{\mathbf{F}}\right|\cos\theta\ \hat{z} \qquad \vec{\mathbf{F}}_2 + \vec{\mathbf{F}}_4 = 2\left|\vec{\mathbf{F}}\right|\cos\theta\ \hat{z}$$

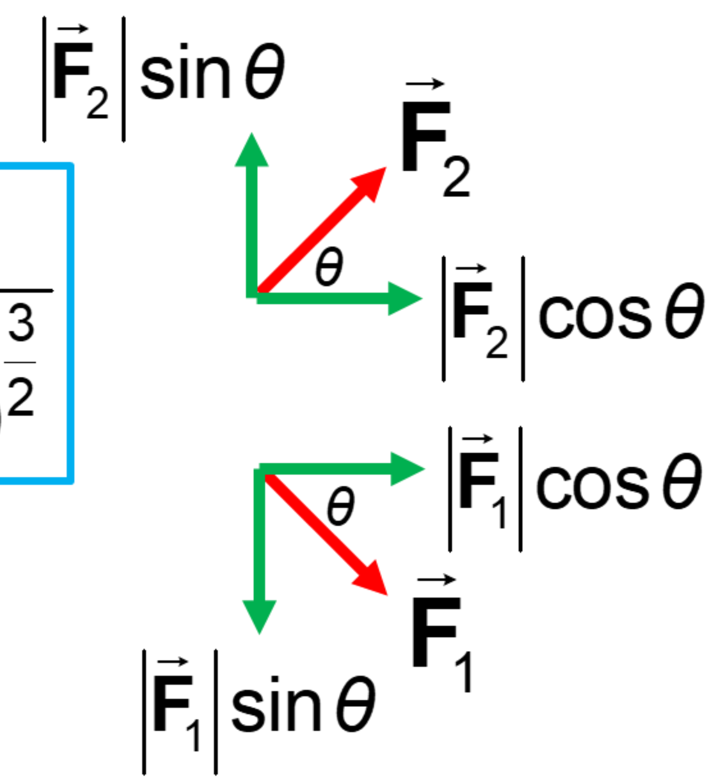
$$\vec{\mathbf{F}}_Q = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 + \vec{\mathbf{F}}_3 + \vec{\mathbf{F}}_4 = 4\left|\vec{\mathbf{F}}\right|\cos\theta\ \hat{z} \approx 4\times 10^{-4}\ \hat{z}\ (N)$$

Ex 5. (Prob 21.23) In Figure, particles 1 and 2 of charge $q_1 = q_2 = +3.2\times 10^{-19}$ C are on a y axis at distance $d = 17$ cm from the origin. Particle 3 of charge $q_3 = +6.4\times 10^{-19}$ C is moved gradually along the x axis from $x = 0$ to $x = 5$ m. At what values of x will the magnitude of the electrostatic force on the third particle from the other two particles be (a) minimum and (b) maximum? What are the (c) minimum and (d) maximum magnitudes?

$$r^2 = x^2 + d^2 \qquad \cos\theta = \frac{x}{r}$$



$$\left|\vec{F}_1\right| = \left|\vec{F}_2\right| = k_e \frac{q_1 q_3}{r^2}$$



$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 = 2k_e \frac{(2e)(4e)x}{r^3}(\hat{i}) = (\hat{i})16k_e e^2 \frac{x}{(x^2+d^2)^{\frac{3}{2}}}$$

$$x = 0 \Rightarrow \vec{F}_{net} = 0 \quad \text{(Minimum)}$$

$$\frac{d\vec{\mathbf{F}}_{net}}{dx} = 0 \;\Rightarrow\; 16k_e e^2 \frac{d}{dx}\left(\frac{x}{(x^2+d^2)^{\frac{3}{2}}}\right) = 0$$

$$\frac{(x^2+d^2)^{\frac{3}{2}} - 3x^2(x^2+d^2)^{\frac{1}{2}}}{(x^2+d^2)^3} = 0$$

$$(x^2+d^2)^{\frac{1}{2}}(\underbrace{x^2+d^2-3x^2}_{d^2-2x^2}) = 0$$

$$x_{max} = \frac{d}{\sqrt{2}} \approx 12\ cm \quad , \quad \theta_{max} = 54.7^\circ$$

$$x_{max} \;\Rightarrow\; \vec{\mathbf{F}}_{net} = 4.9\times 10^{-26}\ N \quad \text{(Maximum)}$$

What is the magnitude of the total electrostatic force on a the charge q?