Chapter 9: Center of Mass and Linear Momentum

- ✓ **Center of Mass**
- ✓ **Newton's Second Law for a System of Particles**
- ✓ **Linear Momentum**
- ✓ **Impulse**
- ✓ **Collision**

Session 18:

- ✓ **Newton's Second Law for a System of Particles**
- ✓ **Linear Momentum**
- ✓ **Examples**

Newton's Second Law for a System of Particles

$$\vec{\mathbf{F}}_1 = m_1\vec{a}_1 \quad ; \quad \vec{\mathbf{F}}_2 = m_2\vec{a}_2$$

$$\vec{\mathbf{r}}_{com} = \frac{m_1\vec{\mathbf{r}}_1 + m_2\vec{\mathbf{r}}_2}{m_1 + m_2} = \frac{m_1\vec{\mathbf{r}}_1 + m_2\vec{\mathbf{r}}_2}{M}$$

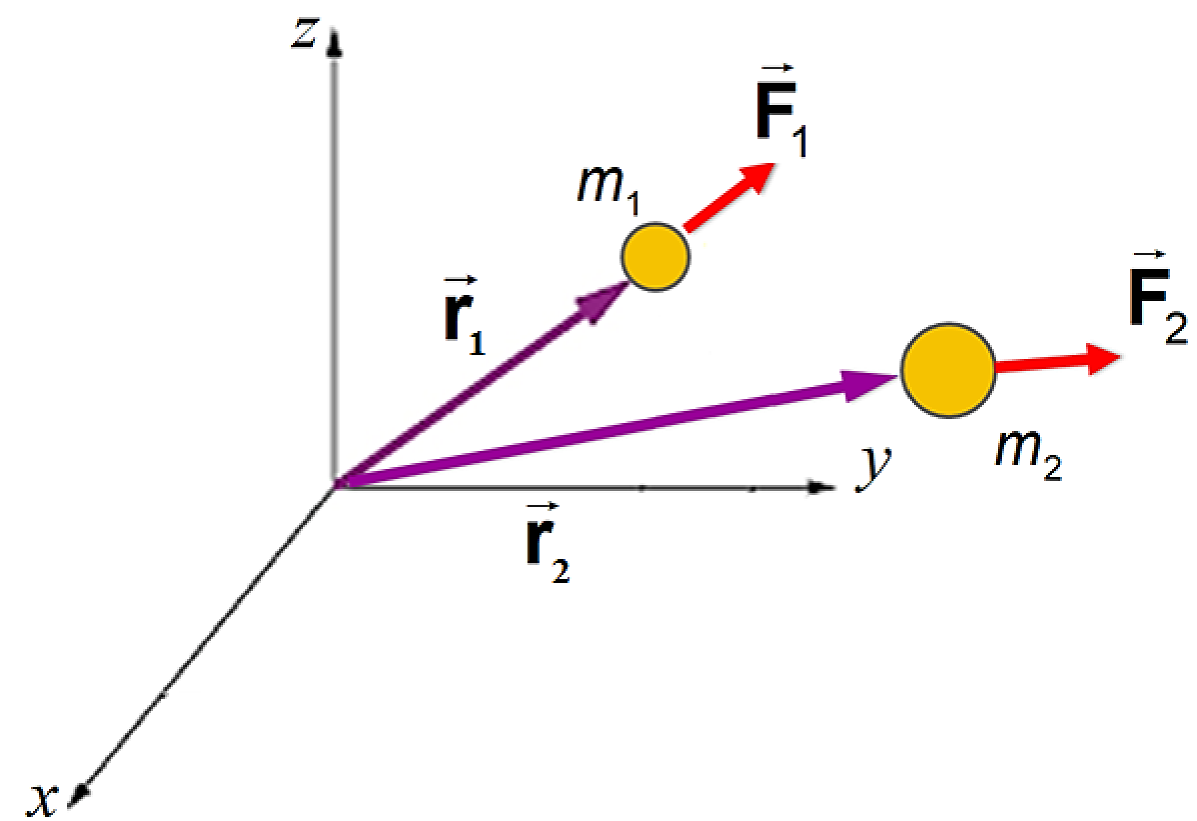
$$M\,\vec{\mathbf{r}}_{com} = m_1\vec{\mathbf{r}}_1 + m_2\vec{\mathbf{r}}_2$$

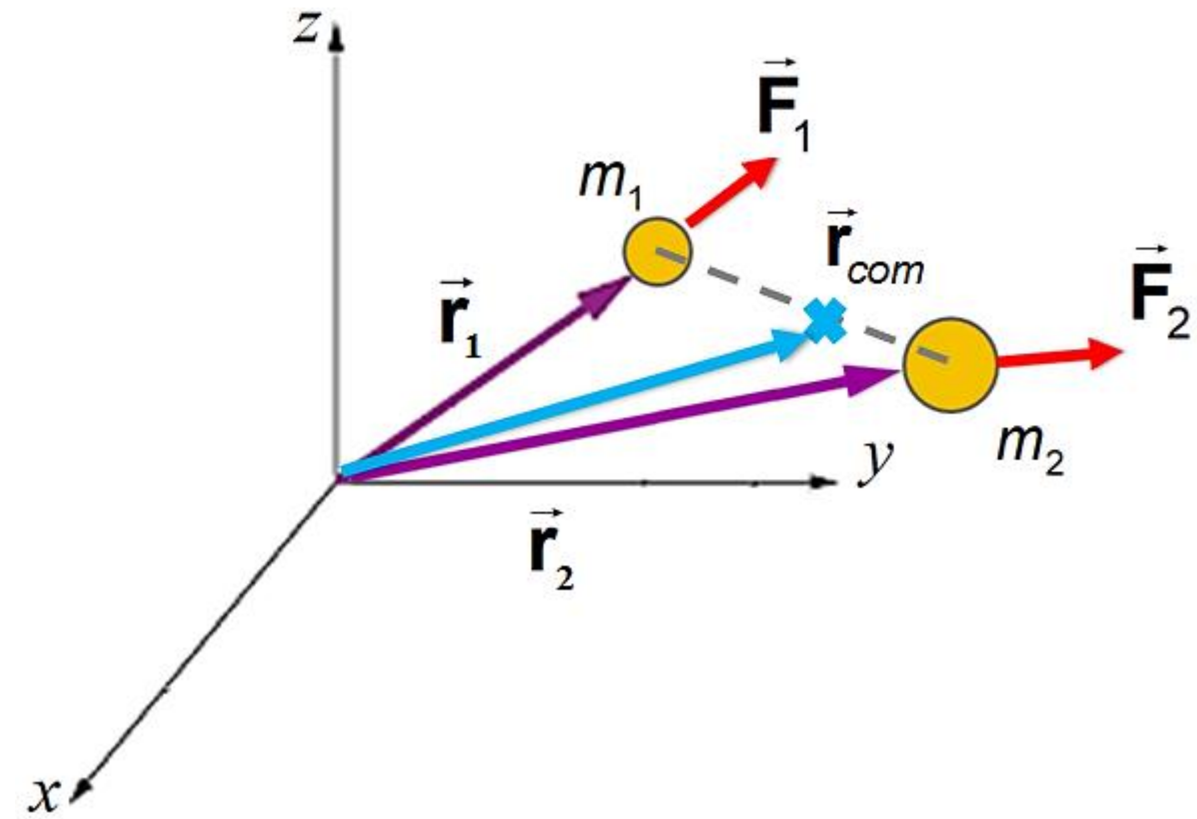
$$M\vec{\mathbf{v}}_{com} = m_1\vec{\mathbf{v}}_1 + m_2\vec{\mathbf{v}}_2$$

$$M\vec{\mathbf{a}}_{com} = m_1\vec{\mathbf{a}}_1 + m_2\vec{\mathbf{a}}_2$$

$$M\vec{\mathbf{a}}_{com} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 = \vec{\mathbf{F}}_{net}$$

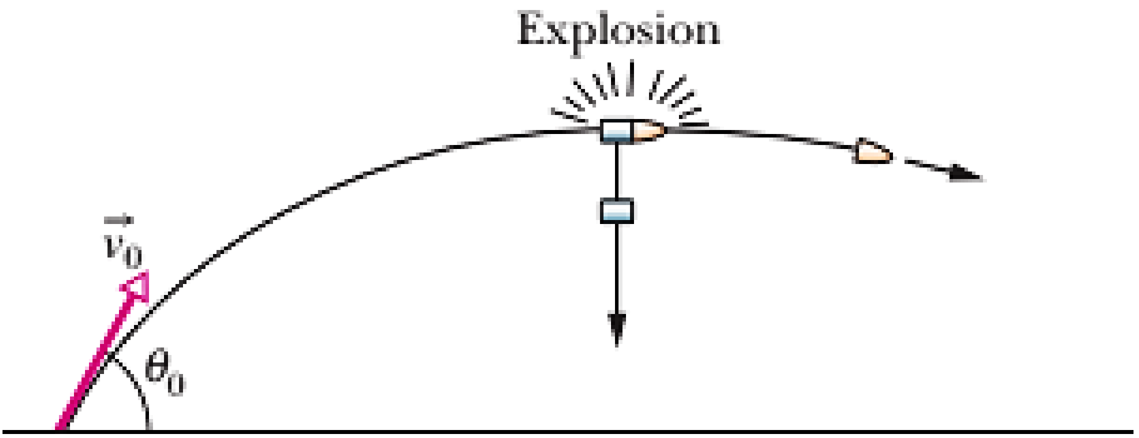
$$\boxed{\vec{\mathbf{F}}_{net} = M\vec{\mathbf{a}}_{com}}$$

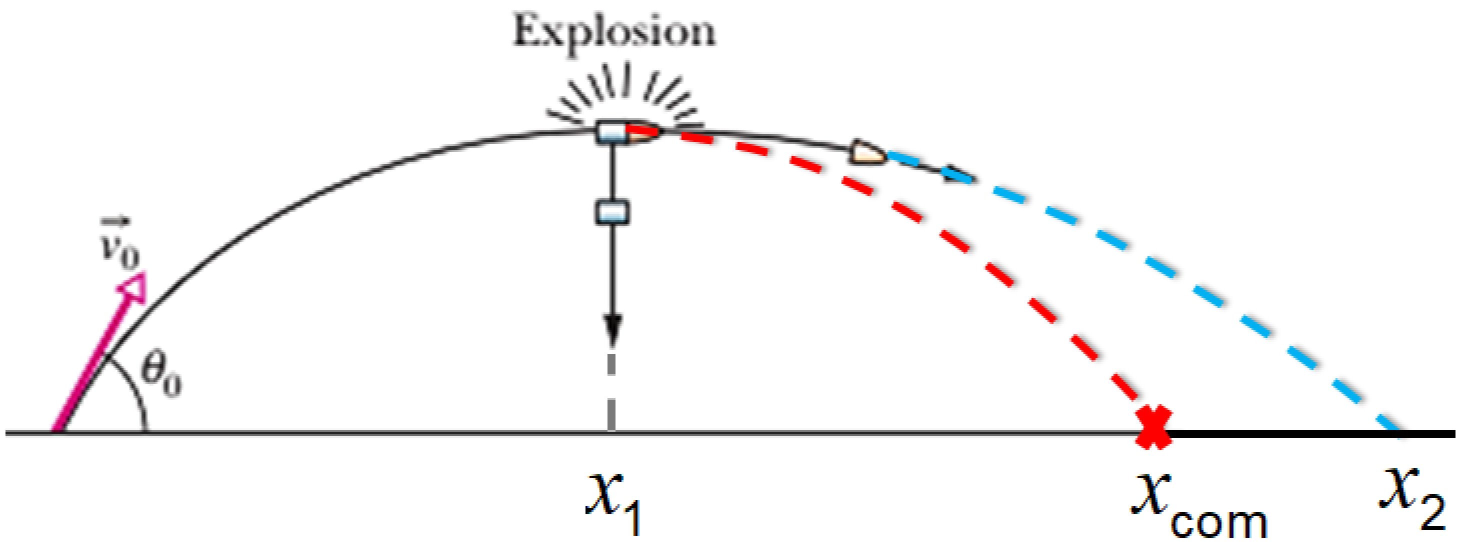




Ex 5: **(Problem 9.13 Halliday)**

A shell is shot with an initial velocity of **20 m/s**, at an angle of **θ= 60 °** with the horizontal. At the top of the trajectory, the **shell explodes into two fragments of equal mass**. One fragment, whose **speed** immediately after the explosion is **zero**, falls vertically. How far from the gun does the other fragment land, assuming that the terrain is level and that air drag is negligible?





Explosion: Internal forces $\vec{\mathbf{F}}_{net} = M\vec{\mathbf{a}}_{com} = 0$ ⇒ $\boxed{a_{x_{com}} = 0 \Rightarrow x_{com} = R}$

$$x_{com} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} \Rightarrow R = \frac{(\frac{M}{2})(\frac{R}{2}) + (\frac{M}{2})x_2}{M} = \frac{R}{4} + \frac{x_2}{2} \Rightarrow \boxed{x_2 = \frac{3}{2}R}$$

$$\boxed{x_2 = \frac{3}{2}(\frac{v_0{}^2}{g}\sin 2\theta_0) = 53.26\ m}$$

Linear Momentum

❖ The **linear momentum** of a particle or an object of mass m moving with a velocity $\vec{\mathbf{v}}$

$$\boxed{\vec{\mathbf{p}} = m\vec{\mathbf{v}}} \Rightarrow \begin{cases} p_x = mv_x \\ p_y = mv_y \\ p_z = mv_z \end{cases}$$

SI unit: **kg · m / s**.

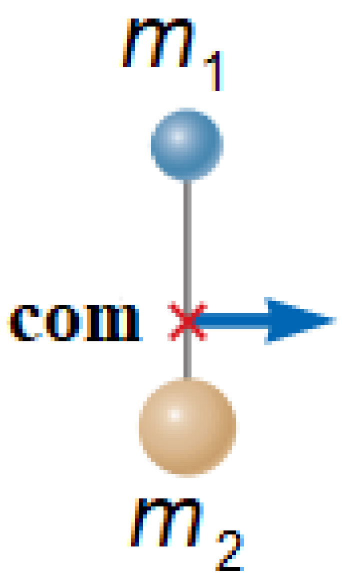
$$\boxed{\vec{\mathbf{F}}_{net} = m\vec{\mathbf{a}} = m\frac{d\vec{\mathbf{v}}}{dt} = \frac{d\left(m\vec{\mathbf{v}}\right)}{dt} = \frac{d\vec{\mathbf{p}}}{dt}}$$

The time rate of **change of the linear momentum** of a particle is equal to the **net force** acting on the particle.

$$\boxed{k = \frac{1}{2}mv^2 = \frac{1}{2m}m^2v^2 = \frac{\left|\vec{\mathbf{p}}\right|^2}{2m}}$$

Linear Momentum

❖ **For a system of particles:**



$$M\vec{\mathbf{v}}_{com} = m_1\vec{\mathbf{v}}_1 + m_2\vec{\mathbf{v}}_2$$

$$\vec{\mathbf{p}}_{com} = \vec{\mathbf{p}}_1 + \vec{\mathbf{p}}_2 = \vec{\mathbf{p}}_{tot}$$

$$\boxed{\vec{\mathbf{F}}_{net} = M\vec{\mathbf{a}}_{com} = \frac{d\vec{\mathbf{p}}_{com}}{dt}}$$

$$if\ \vec{\mathbf{F}}_{net} = 0 \Rightarrow \frac{d\vec{\mathbf{p}}_{com}}{dt} = 0 \Rightarrow \boxed{\vec{\mathbf{p}}_{com} = \text{constant}}$$

$$\vec{\mathbf{p}}_{1i} + \vec{\mathbf{p}}_{2i} = \vec{\mathbf{p}}_{1f} + \vec{\mathbf{p}}_{2f}$$

Conservation of linear momentum

Ex 6: A **60-kg** archer stands at rest on frictionless ice and fires a **0.030-kg** arrow horizontally at **85 m/s**. With what velocity does the archer move across the ice after firing the arrow?

$$\vec{\mathbf{F}}_{net} = 0$$

$$\boxed{\vec{\mathbf{p}}_{com} = \text{constant}}$$

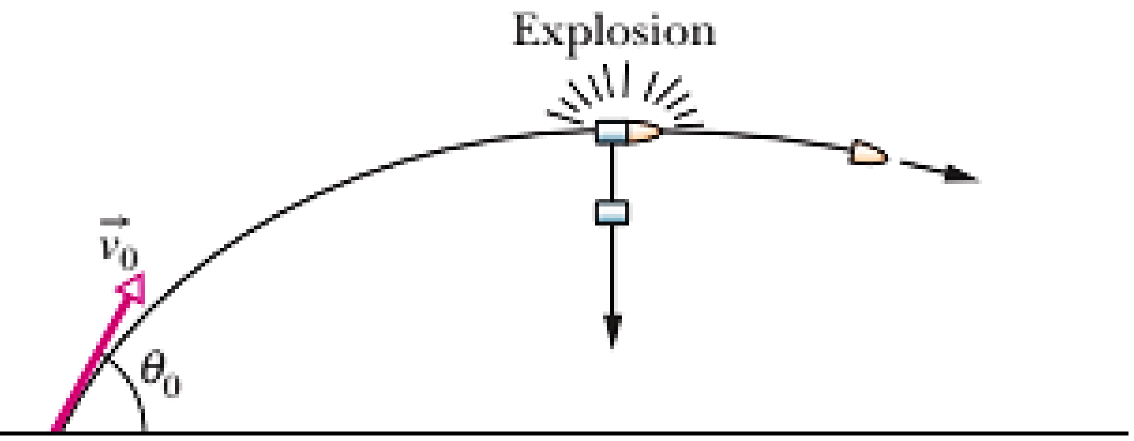
$$\vec{\mathbf{p}}_{1i} + \vec{\mathbf{p}}_{2i} = \vec{\mathbf{p}}_{1f} + \vec{\mathbf{p}}_{2f}$$

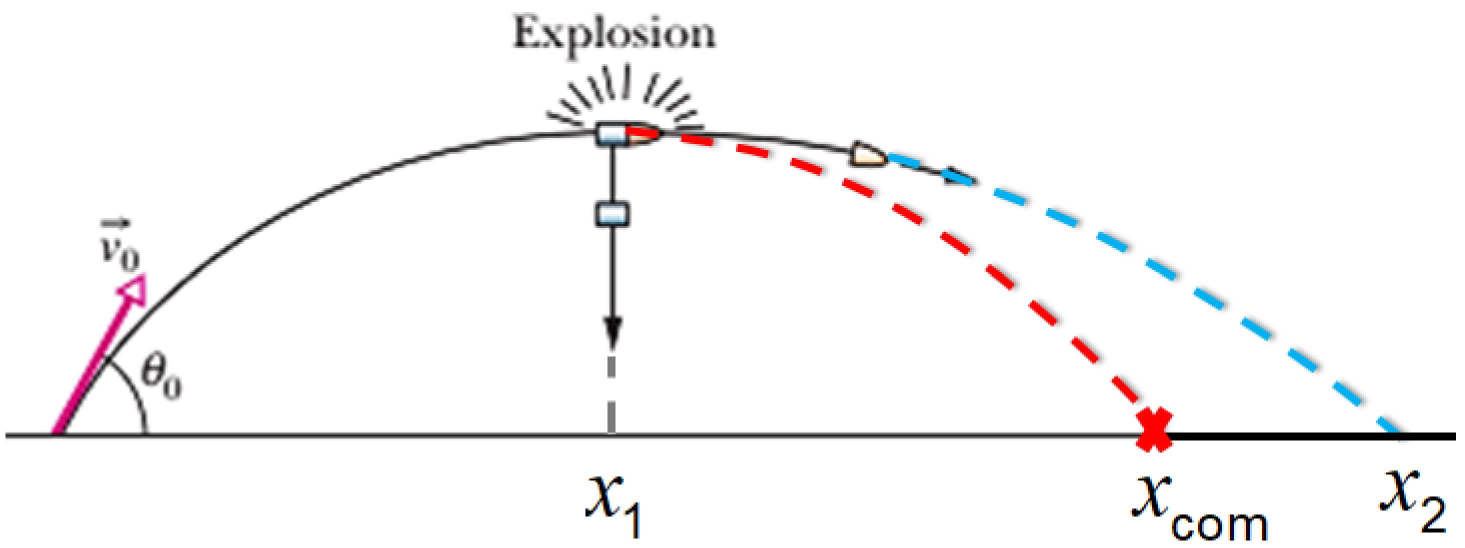
$$0 + 0 = m_1 \vec{\mathbf{V}}_{1f} + m_2 \vec{\mathbf{V}}_{2f}$$

$$\vec{\mathbf{V}}_{2f} = -\frac{m_1}{m_2} \vec{\mathbf{V}}_{1f}$$

$$\boxed{\vec{\mathbf{V}}_{2f} = -\frac{0.03}{60}(85\ \hat{\mathbf{i}}) = -0.042\ \hat{\mathbf{i}} \quad m/s}$$

Ex 7 : Problem 9.13 with conservation of linear momentum.





$$\vec{F}_{net} = 0$$

$$\boxed{\vec{p}_{com} = \text{constant}} \Rightarrow \vec{p}_i = \vec{p}_f \Rightarrow M\vec{V}_{peak} = \frac{M}{2}\underbrace{\vec{V}_{1f}}_{0} + \frac{M}{2}\vec{V}_{2f}$$

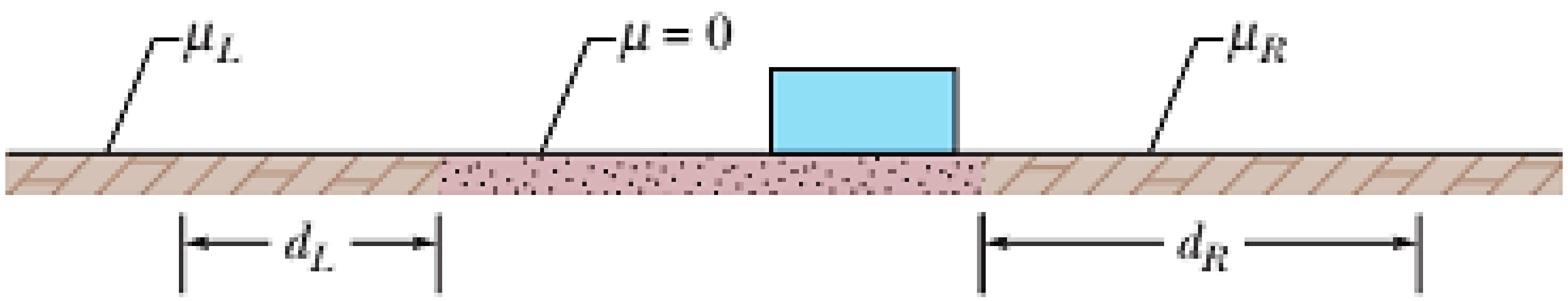
$$\vec{V}_{2f} = 2\vec{V}_{peak} = 2v_0 \cos\theta_0 (\hat{\mathbf{i}}) \Rightarrow x_2 = x_{02} + (2v_0 \cos\theta)t$$

$$x_2 = \frac{R}{2} + (2v_0 \cos\theta_0)\left(\frac{v_0 \sin\theta_0}{g}\right) = \frac{R}{2} + \underbrace{\frac{v_0{}^2 \sin 2\theta_0}{g}}_{R} = \frac{3}{2}R$$

$$\boxed{x_2 = \frac{3}{2}\left(\frac{v_0{}^2}{g}\sin 2\theta_0\right) = 53.26\ m}$$

Ex 8: (Problem 9.44 Halliday)

A stationary block explodes into two pieces **L** and **R** that slide across a frictionless floor and then into regions with friction, where they stop. Piece **L**, with a mass of **2 kg**, encounters a coefficient of kinetic friction $\boldsymbol{\mu_L = 0.40}$ and slides to a stop in distance $\boldsymbol{d_L = 0.15}$ **m**. Piece **R** encounters a coefficient of kinetic friction $\boldsymbol{\mu_R = 0.50}$ and slides to a stop in distance $\boldsymbol{d_R = 0.25}$ **m**. What was the mass of the block?.



$$\vec{\mathbf{F}}_{net} = 0$$

$$\boxed{\vec{\mathbf{p}}_{com} = \text{constant}}$$

$$\vec{\mathbf{p}}_{L_i} + \vec{\mathbf{p}}_{R_i} = \vec{\mathbf{p}}_{L_f} + \vec{\mathbf{p}}_{R_f} \Rightarrow 0 + 0 = m_L \vec{\mathbf{V}}_{L_f} + m_R \vec{\mathbf{V}}_{R_f} \Rightarrow \boxed{\frac{m_R}{m_L} = \left|\frac{\vec{\mathbf{V}}_{L_f}}{\vec{\mathbf{V}}_{R_f}}\right|}$$

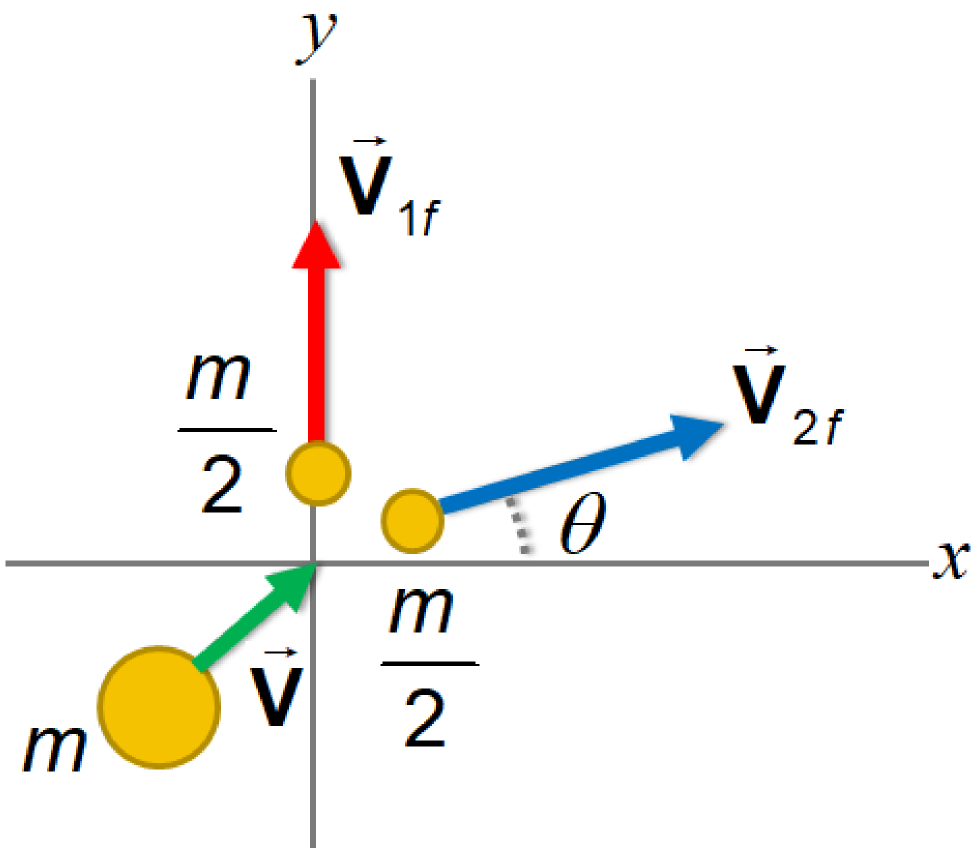
$$v^2 - v_0^2 = 2a_x \Delta x \qquad 0 - v_0^2 = 2(-\mu_k g)d \qquad \boxed{v_0 = \sqrt{2\mu_k g\, d}}$$

$$\frac{m_R}{m_L} = \left|\frac{\vec{\mathbf{V}}_{L_f}}{\vec{\mathbf{V}}_{R_f}}\right| = \frac{\sqrt{2\mu_L g\, d_L}}{\sqrt{2\mu_R g\, d_R}} = \sqrt{\frac{\mu_L d_L}{\mu_R d_R}} \Rightarrow \boxed{m_R = 2\sqrt{\frac{(0.4)(0.15)}{(0.5)(0.25)}} = 1.39\ kg}$$

$$\boxed{m_{block} = m_L + m_R = 3.39\ kg}$$

Ex 9: (Problem 9.46 Halliday)

A **4 kg** mess kit sliding on a frictionless surface **explodes into two 2 kg parts**: **3 m/s, due north**, and **5 m/s, 30° north of east**. What is the original speed of the mess kit?



$$\vec{F}_{net} = 0 \qquad \boxed{\vec{p}_{com} = \text{constant}}$$

$$\vec{p}_i = \vec{p}_f \Rightarrow m\vec{V} = \frac{m}{2}\vec{V}_{1f} + \frac{m}{2}\vec{V}_{2f}$$

$$m(V_x\,\hat{\mathbf{i}} + V_y\,\hat{\mathbf{j}}) = \frac{m}{2}V_{1f}\,(\hat{\mathbf{j}}) + \frac{m}{2}(V_{2f}\cos\theta\,\hat{\mathbf{i}} + V_{2f}\sin\theta\,\hat{\mathbf{j}})$$

$$\begin{cases} mV_x = \dfrac{m}{2}V_{2f}\cos\theta \\ mV_y = \dfrac{m}{2}(V_{1f} + V_{2f}\sin\theta) \end{cases} \Rightarrow \begin{cases} V_x = \dfrac{V_{2f}}{2}\cos\theta = 2.17 \quad m/s \\ V_y = \dfrac{1}{2}(V_{1f} + V_{2f}\sin\theta) = 2.75 \quad m/s \end{cases}$$

$$\boxed{V = \sqrt{V_x^{\,2} + V_y^{\,2}} = 3.50 \quad m/s}$$