بسم الله الرحمن الرحیم

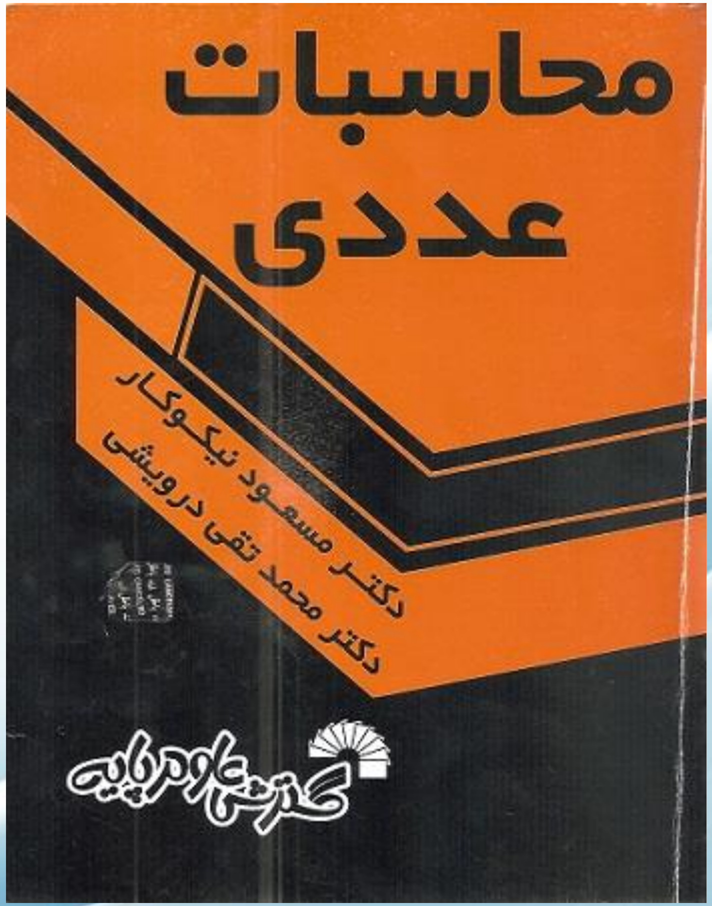
دانشگاه صنعتی خواجه نصیرالدین طوسی

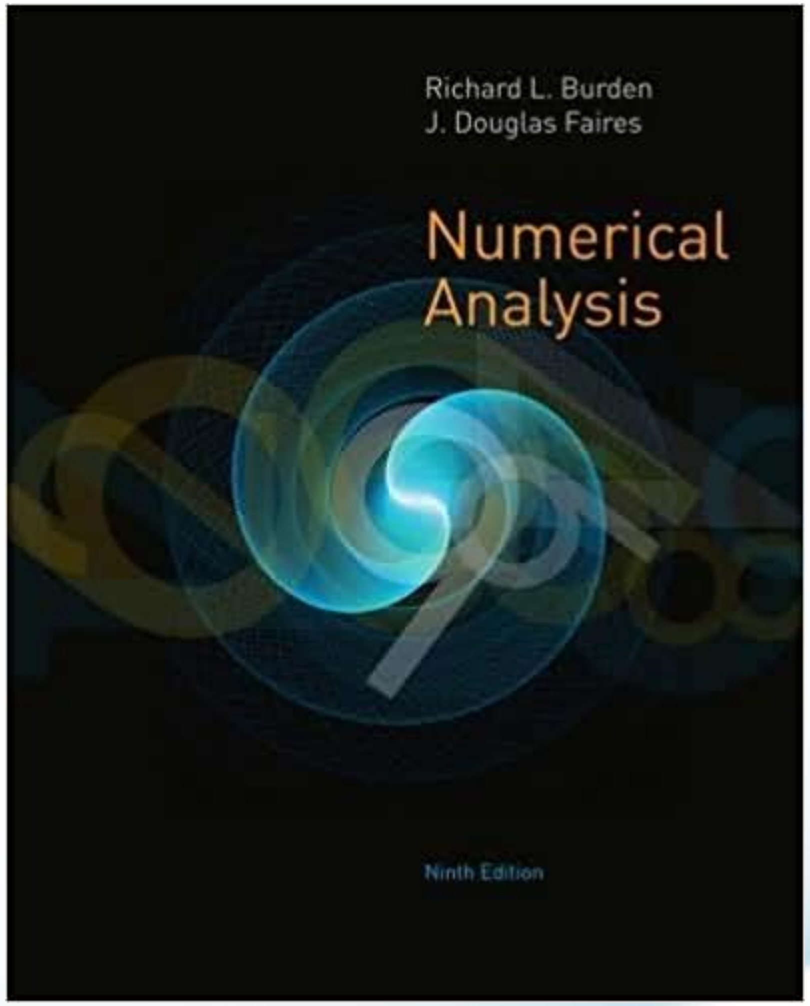
محاسبات عددی

ابراهیم شاه ابراهیمی

نوروز ۹۹

منابع :





Numerical Analysis
Book by J. Douglas Faires and Richard L. Burden

1 Solutions of Equations in One Variable

In this chapter we consider

one of the most basic problems

of numerical approximation.

the **root-finding problem**

This process involves

finding a **root**,

or solution,

of an equation

of the form $f(x) = 0$,

1 The Bisection Method

2 Fixed-Point Iteration

3 Newton's Method

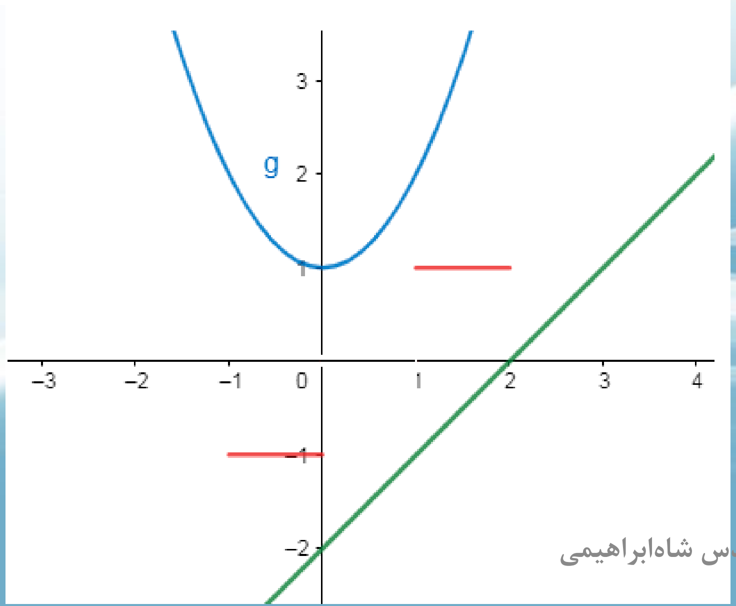
1 Solutions of Equations in One Variable

Bisection Technique

Suppose $\begin{cases} f \text{ is a continuous function} & \text{defined on the interval } [a, b] \\ \text{with } f(a) \text{ and } f(b) \text{ of opposite sign} & f(a) \cdot f(b) < 0 \end{cases}$

The first technique
based on
the Intermediate Value Theorem,
is called
the **Bisection method**

The Intermediate Value Theorem
implies that
a number p exists in (a, b)
with $f(p) = 0$



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Bisection Technique

To begin set $a_1 = a$ and $b_1 = b$ and let p_1 be the midpoint of $[a, b]$ $p_1 = \dfrac{a_1 + b_1}{2}$

- If $f(p_1) = 0$ then $p = p_1$ and we are done.
- If $f(p_1) \neq 0$ then $f(p_1)$ has the same sign as either $f(a_1)$ or $f(b_1)$
- If $f(p_1)$ and $f(a_1)$ have the same sign $p \in (p_1, b_1)$ Set $a_2 = p_1$ and $b_2 = b_1$
- If $f(p_1)$ and $f(a_1)$ have opposite signs $p \in (a_1, p_1)$ Set $a_2 = a_1$ and $b_2 = p_1$

Then reapply the process to the interval $[a_2, b_2]$.

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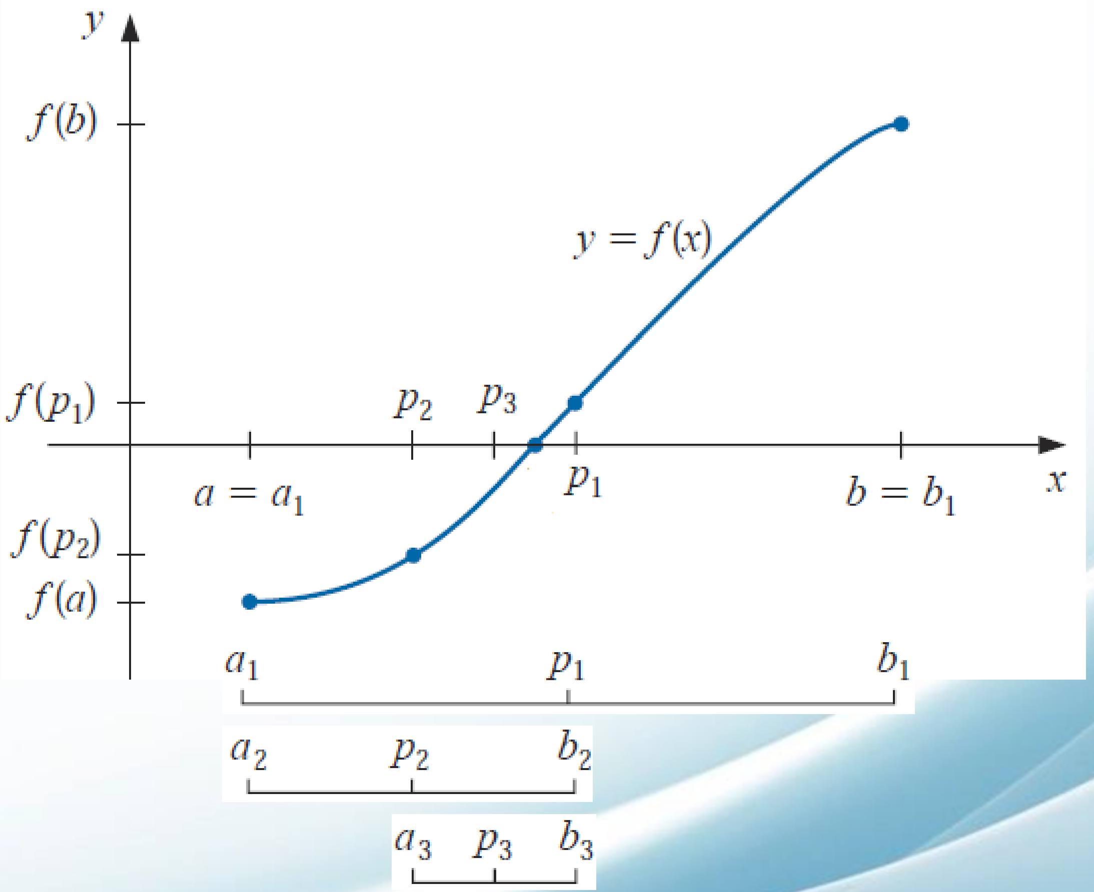
Bisection Technique



we can select a tolerance $\varepsilon > 0$ and generate $p_1, \ldots, p_N$ until one of the following conditions is met:

$$|p_N - p_{N-1}| < \varepsilon \qquad |f(p_N)| < \varepsilon$$

Example 1

Show that $f(x) = x^3 + 4x^2 - 10 = 0$ has a root in $[1, 2]$, and use the Bisection method to determine an approximation to the root that is accurate to at least within 10^{-4}.

Solution

Because $f(1) = -5$ and $f(2) = 14$ the Intermediate Value Theorem ensures that this continuous function has a root in $[1, 2]$.

For the first iteration of the Bisection method we use the fact that at the midpoint of $[1, 2]$ we have $f(1.5) = 2.375 > 0$.

This indicates that we should select the interval $[1, 1.5]$ for our second iteration

Then we find that $f(1.25) = -1.796875$ so our new interval becomes $[1.25, 1.5]$ whose midpoint is 1.375.

Bisection Technique

n	a_n	b_n	p_n	$f(p_n)$
1	1.0	2.0	1.5	2.375
2	1.0	1.5	1.25	−1.79687
3	1.25	1.5	1.375	0.16211
4	1.25	1.375	1.3125	−0.84839
5	1.3125	1.375	1.34375	−0.35098
6	1.34375	1.375	1.359375	−0.09641
7	1.359375	1.375	1.3671875	0.03236
8	1.359375	1.3671875	1.36328125	−0.03215
9	1.36328125	1.3671875	$\boxed{1.365234375}$	0.000072

$$|p_n - p| \le \frac{b-a}{2^n}, \quad \text{when} \quad n \ge 1.$$

Example 2 Determine the number of iterations necessary to solve $f(x) = x^3 + 4x^2 - 10 = 0$ with accuracy 10^{-3} using $a_1 = 1$ and $b_1 = 2$.

$$|p_N - p| \le 2^{-N}(b-a) = 2^{-N} < 10^{-3}$$

$$\log_{10} 2^{-N} < \log_{10} 10^{-3} \qquad -N \log_{10} 2 < -3 \qquad N > \frac{3}{\log_{10} 2} \approx 9.96$$

EXERCISE SET 1

1. Use the Bisection method to find p_3 for $f(x) = \sqrt{x} - \cos x$ on $[0, 1]$.

2. Use the Bisection method to find solutions accurate to within 10^{-5} for the following problems.
   a. $x - 2^{-x} = 0$ for $0 \le x \le 1$
   b. $2x\cos(2x) - (x+1)^2 = 0$ for $-3 \le x \le -2$

3. a. Sketch the graphs of $y = x$ and $y = 2\sin x$.
   b. Use the Bisection method to find an approximation to within 10^{-5} to the first positive value of x with $x = 2\sin x$.

پایان جلسه اول

۱۰ فروردین ۹۹

با تشکر از توجه شما