قلم را آن زبان نبود که سرّ عشق گوید باز
ورای حد تقریر است شرح آرزومندی
درین بازار اگر سودی ست با درویش خرسند است
خدایا منعمم گردان به درویشی و خرسندی

فصل چهارم: خمش

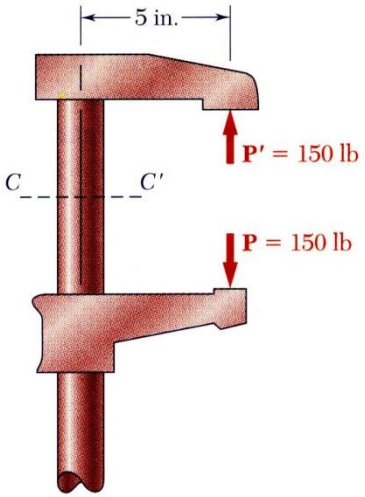
Pure Bending

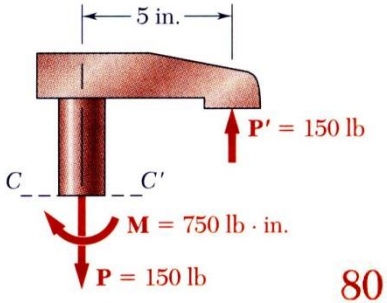
اکبر اقبالی

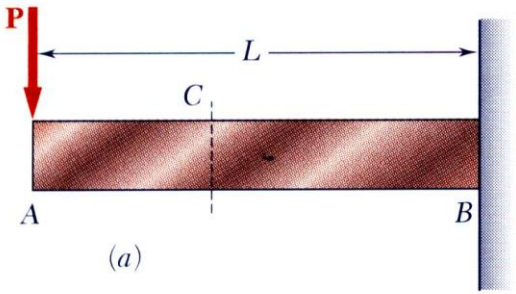
WWW.AEGHBALI.BLOG.IR
EGHBALI.D@GMAIL.COM

1393

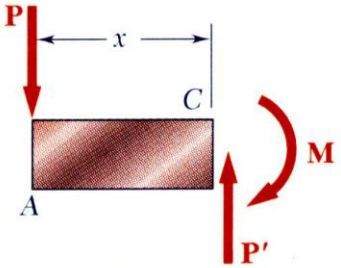
خمش



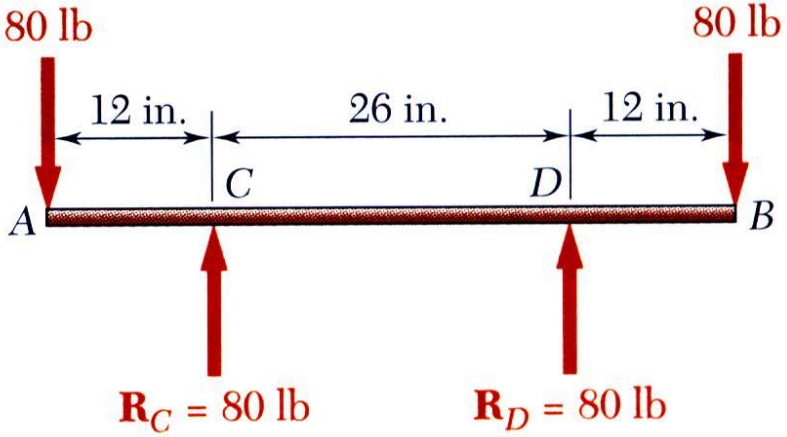




(a)



(b)



(a)

(b)

خمش

تغییر شکل

کرنش و خمش

تیرها

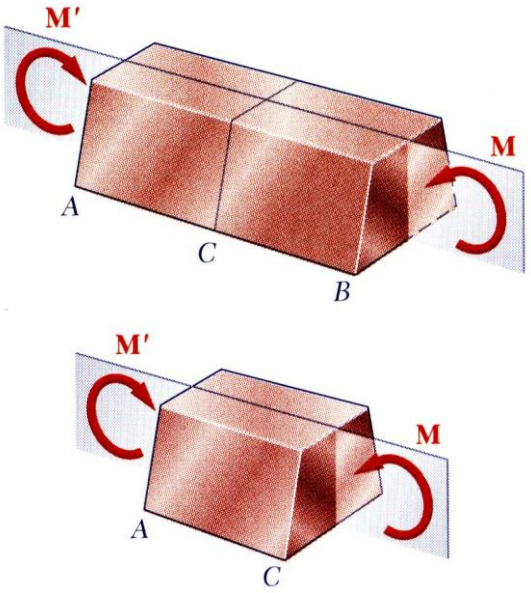
بتون مسلح

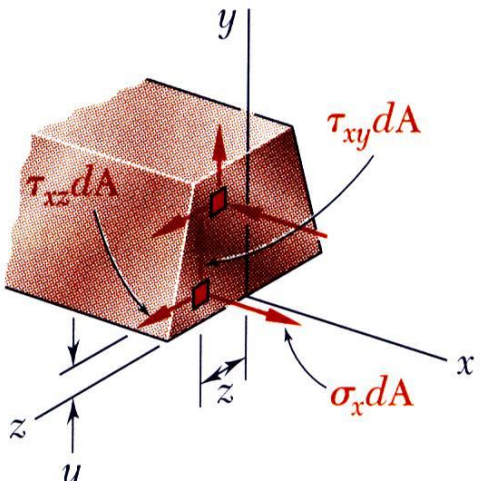
حوزه پلاستیک

مباحث تکمیلی

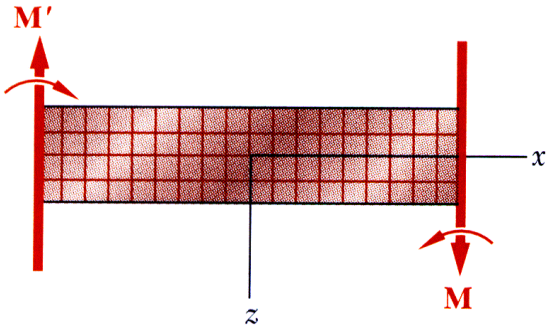
خمش نامتقارن

خمش و تغییر شکل ناشی از آن

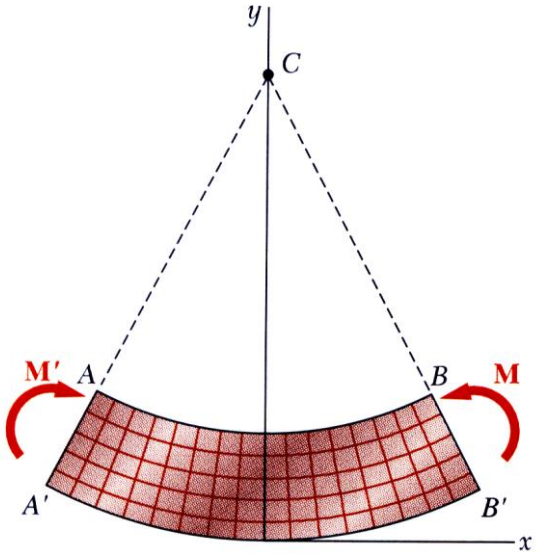




$$F_x = \int \sigma_x \, dA = 0$$
$$M_y = \int z\sigma_x \, dA = 0$$
$$M_z = \int -y\sigma_x \, dA = M$$



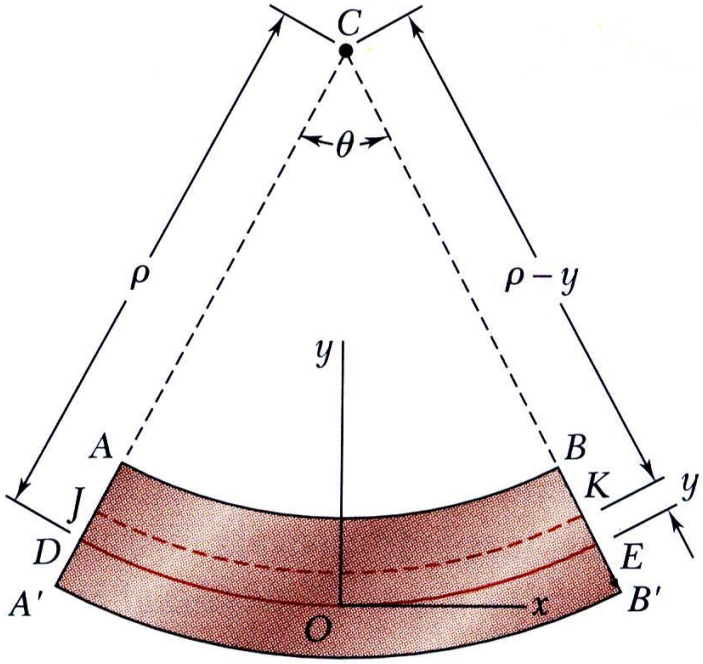
(b) Longitudinal, horizontal section

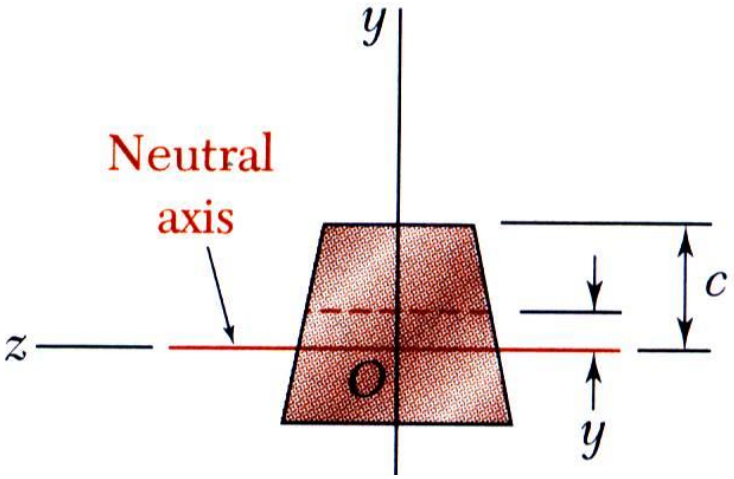


(a) Longitudinal, vertical section (plane of symmetry)

خمش
تغییر شکل
کرنش و خمش
تیرها
بتون مسلح
حوزه پلاستیک
مباحث تکمیلی
خمش نامتقارن

کرنش ناشی از خمش





$$L' = (\rho - y)\theta$$

$$\delta = L - L' = (\rho - y)\theta - \rho\theta = -y\theta$$

$$\varepsilon_x = \frac{\delta}{L} = -\frac{y\theta}{\rho\theta} = -\frac{y}{\rho}$$ (strain varies linearly)

$$\varepsilon_m = \frac{c}{\rho} \quad \text{or} \quad \rho = \frac{c}{\varepsilon_m}$$

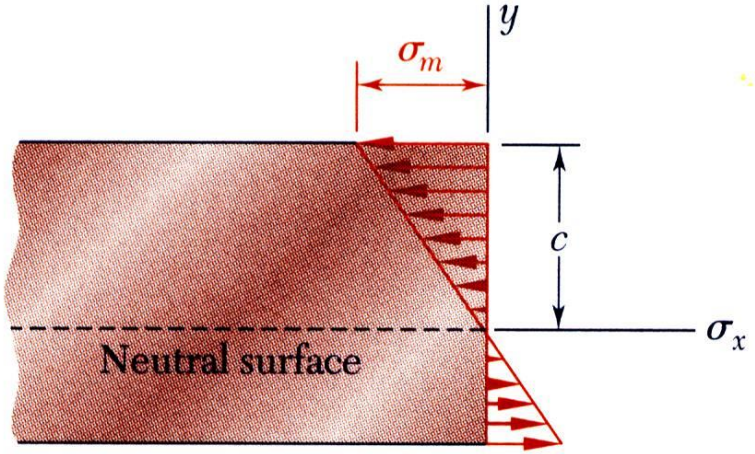
$$\varepsilon_x = -\frac{y}{c}\varepsilon_m$$

خمش
تغییر شکل
کرنش و خمش
تیرها
بتون مسلح
موزه پلاستیک
مباحث تکمیلی
خمش نامتقارن

تنش ناشی از خمش

For a linearly elastic material,

$$\sigma_x = E\varepsilon_x = -\frac{y}{c}E\varepsilon_m = -\frac{y}{c}\sigma_m \quad \text{(stress varies linearly)}$$



For static equilibrium,

$$F_x = 0 = \int \sigma_x \, dA = \int -\frac{y}{c}\sigma_m \, dA$$

$$0 = -\frac{\sigma_m}{c}\int y \, dA$$

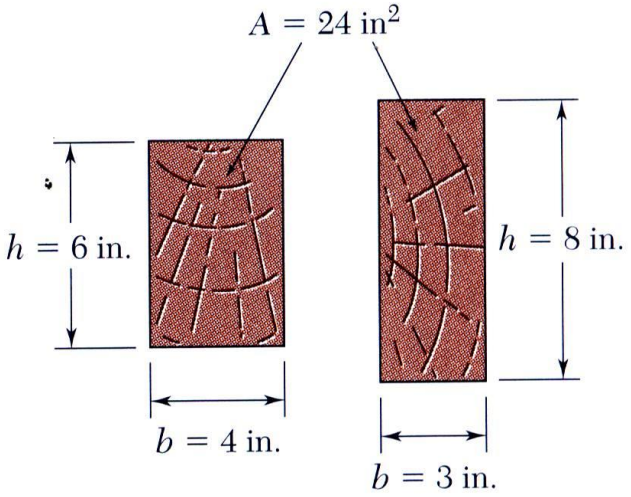
$$M = \int -y\sigma_x \, dA = \int -y\left(-\frac{y}{c}\sigma_m\right) dA = \frac{\sigma_m}{c}\int y^2 \, dA = \frac{\sigma_m I}{c}$$

$$\sigma_m = \frac{Mc}{I} = \frac{M}{S}$$

$$\sigma_x = -\frac{y}{c}\sigma_m \qquad \sigma_x = -\frac{My}{I}$$

خمش
تغییر شکل
کرنش و خمش
تیرها
بتون مسلح
موزه پلاستیک
مباحث تکمیلی
خمش نامتقارن

خصوصیات مقطع تیر

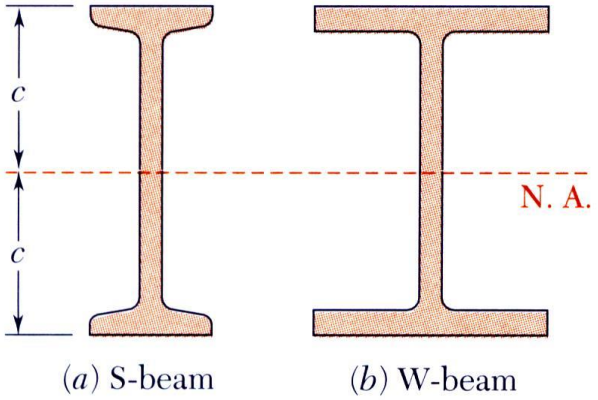


The maximum normal stress due to bending,

$$\sigma_m = \frac{Mc}{I} = \frac{M}{S}$$

I = section moment of inertia

$$S = \frac{I}{c} = \text{section modulus}$$



(a) S-beam (b) W-beam

$$S = \frac{I}{c} = \frac{\frac{1}{12}bh^3}{h/2} = \frac{1}{6}bh^3 = \frac{1}{6}Ah$$

خمش

تغییر شکل

کرنش و خمش

تیرها

بتون مسلح

حوزه پلاستیک

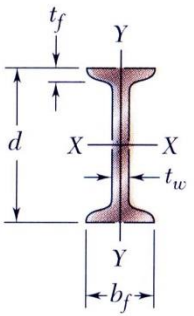
مباحث تکمیلی

خمش نامتقارن

خصوصیات مقطع تیر

Appendix C. Properties of Rolled-Steel Shapes
(SI Units)

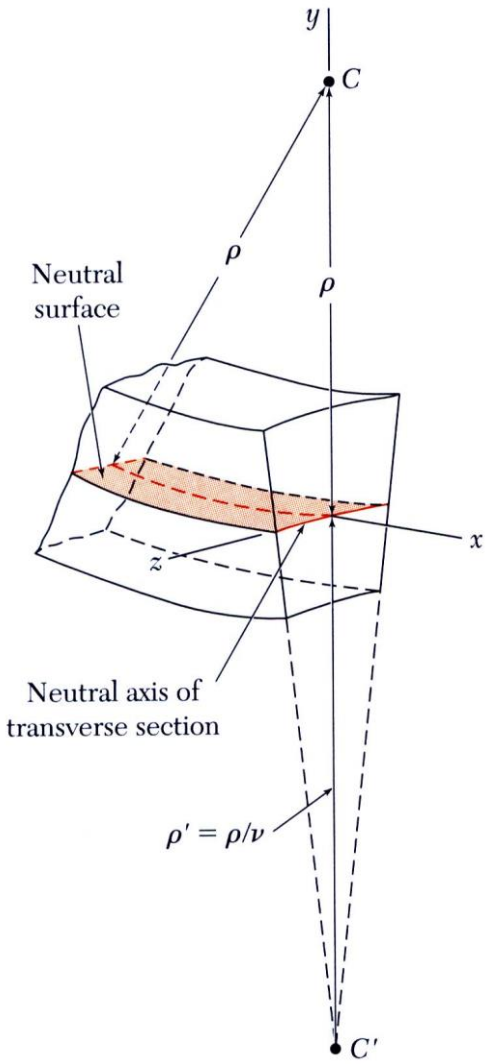
S Shapes
(American Standard Shapes)



Designation†	Area A, mm²	Depth d, mm	Flange Width b_f, mm	Flange Thickness t_f, mm	Web Thickness t_w, mm	Axis X-X I_x 10^6 mm⁴	Axis X-X S_x 10^3 mm³	Axis X-X r_x mm	Axis Y-Y I_y 10^6 mm⁴	Axis Y-Y S_y 10^3 mm³	Axis Y-Y r_y mm
S610 × 180	22900	622	204	27.7	20.3	1320	4240	240	34.9	341	39.0
158	20100	622	200	27.7	15.7	1230	3950	247	32.5	321	39.9
149	19000	610	184	22.1	18.9	995	3260	229	20.2	215	32.3
134	17100	610	181	22.1	15.9	938	3080	234	19.0	206	33.0
119	15200	610	178	22.1	12.7	878	2880	240	17.9	198	34.0
S510 × 143	18200	516	183	23.4	20.3	700	2710	196	21.3	228	33.9
128	16400	516	179	23.4	16.8	658	2550	200	19.7	216	34.4
112	14200	508	162	20.2	16.1	530	2090	193	12.6	152	29.5
98.3	12500	508	159	20.2	12.8	495	1950	199	11.8	145	30.4
S460 × 104	13300	457	159	17.6	18.1	385	1685	170	10.4	127	27.5
81.4	10400	457	152	17.6	11.7	333	1460	179	8.83	113	28.8
S380 × 74	9500	381	143	15.6	14.0	201	1060	145	6.65	90.8	26.1
64	8150	381	140	15.8	10.4	185	971	151	6.15	85.7	27.1

خمش
تغییر شکل
کرنش و خمش
تیرها
بتون مسلح
حوزه پلاستیک
مباحث تکمیلی
خمش نامتقارن

تغییر شکل ناشی از خمش در مقطع عرضی



$$\frac{1}{\rho}=\frac{\varepsilon_m}{c}=\frac{\sigma_m}{Ec}=\frac{1}{Ec}\frac{Mc}{I}=\frac{M}{EI}$$

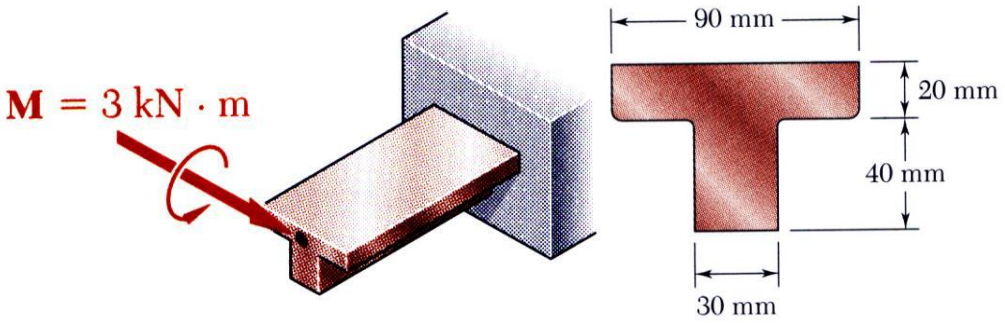
$$\varepsilon_y=-\nu\varepsilon_x=\frac{\nu y}{\rho}$$

$$\varepsilon_z=-\nu\varepsilon_x=\frac{\nu y}{\rho}$$

$$\frac{1}{\rho'}=\frac{\nu}{\rho}:\text{anticlastic curvature}$$

خمش
تغییر شکل
کرنش و خمش
تیرها
بتون مسلح
موزه پلاستیک
مباحث تکمیلی
خمش نامتقارن

مثال

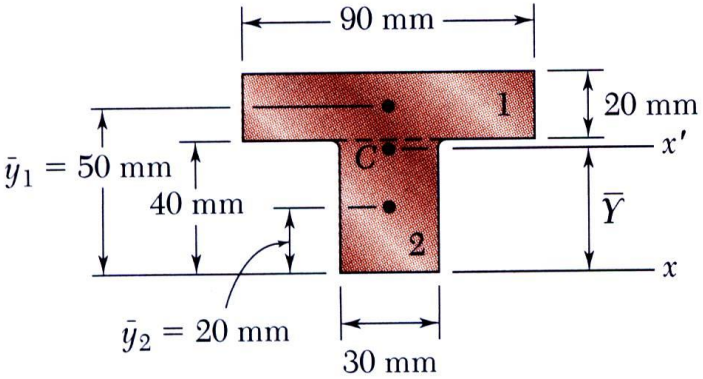


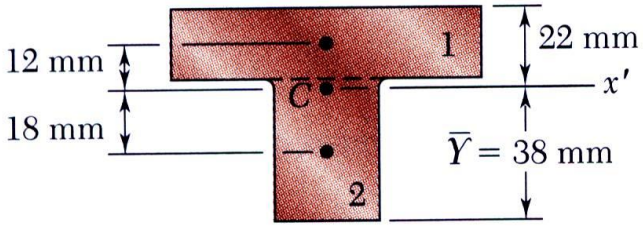
E = 165 GPa

(a) The maximum tensile

(b) The compressive stresses

(c) The radius of curvature





	Area, mm^2	$\bar{y}$, mm	$\bar{y}A$, mm^3
1	$20\times90=1800$	50	90×10^3
2	$40\times30=1200$	20	24×10^3
	$\sum A=3000$		$\sum \bar{y}A=114\times10^3$

$$\bar{Y}=\frac{\sum \bar{y}A}{\sum A}=\frac{114\times10^3}{3000}=38\text{ mm}$$

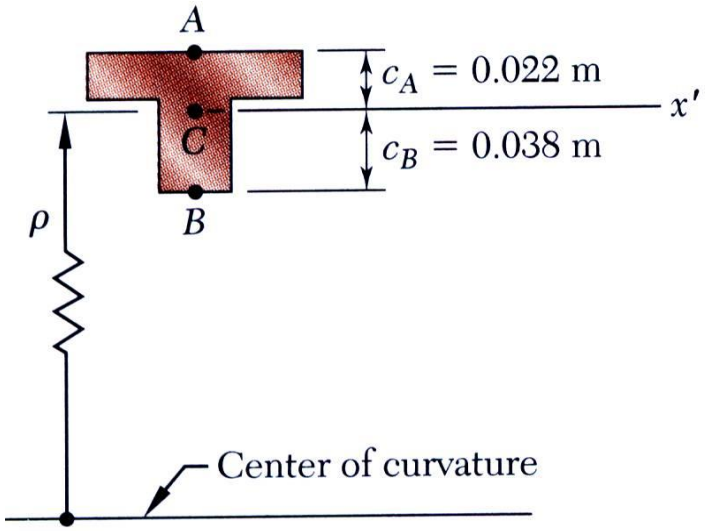
$$I_{x'}=\sum\left(\bar{I}+Ad^2\right)=\sum\left(\tfrac{1}{12}bh^3+Ad^2\right)$$
$$=\left(\tfrac{1}{12}90\times20^3+1800\times12^2\right)$$
$$+\left(\tfrac{1}{12}30\times40^3+1200\times18^2\right)$$
$$I=868\times10^3\text{ mm}=868\times10^{-9}\text{ m}^4$$

مثال

$$\sigma_m = \frac{Mc}{I}$$

$$\sigma_A = \frac{M\,c_A}{I} = \frac{3\,\text{kN}\cdot\text{m}\times 0.022\,\text{m}}{868\times10^{-9}\,\text{mm}^4} = +76.0MPa$$

$$\sigma_B = -\frac{M\,c_B}{I} = -\frac{3\,\text{kN}\cdot\text{m}\times 0.038\,\text{m}}{868\times10^{-9}\,\text{mm}^4} = -131.3MPa$$



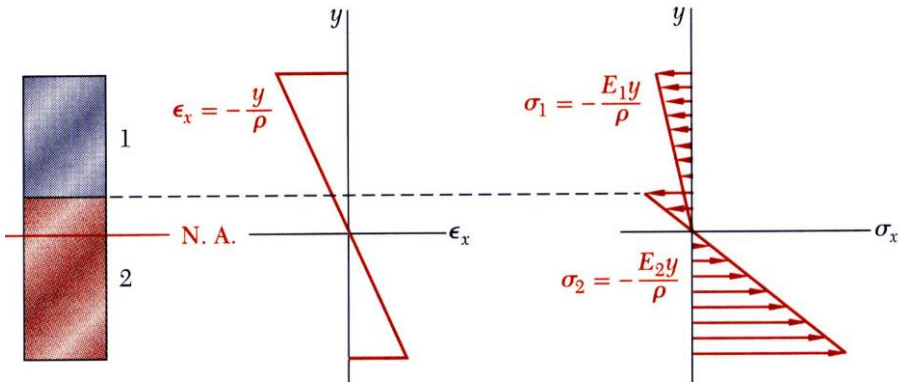
$$\frac{1}{\rho} = \frac{M}{EI} = \frac{3\,\text{kN}\cdot\text{m}}{(165\,\text{GPa})(868\times10^{-9}\,\text{m}^4)}$$

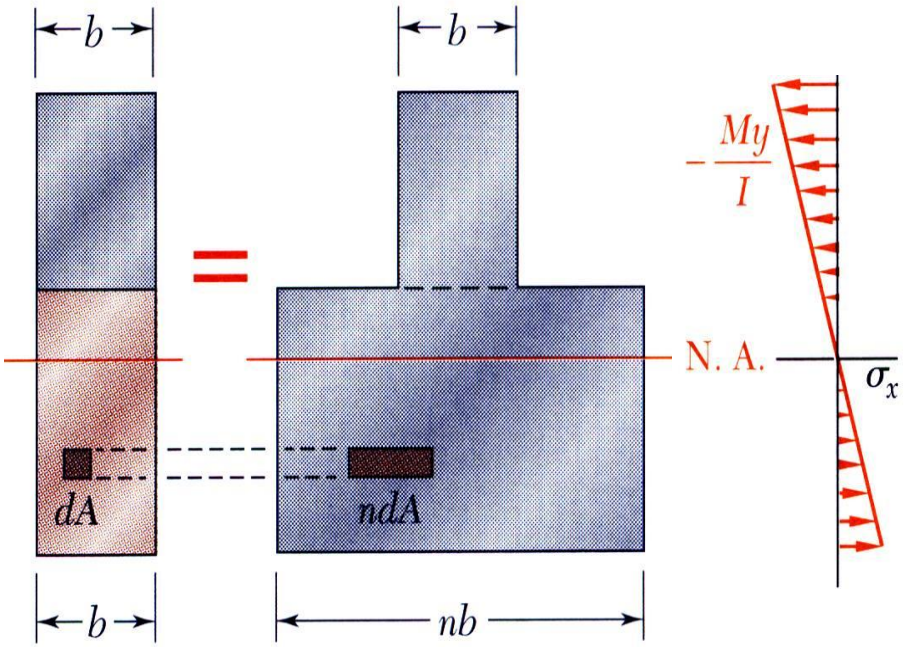
$$\frac{1}{\rho} = 20.95\times10^{-3}\,\text{m}^{-1}$$

$$\rho = 47.7\,\text{m}$$

خمش
تغییر شکل
کرنش و خمش
تیرها
بتون مسلح
حوزه پلاستیک
مباحث تکمیلی
خمش نامتقارن

خمش در اعضایی با مواد مختلف





$$\sigma_x = -\frac{My}{I}$$

$$\sigma_1 = \sigma_x \qquad \sigma_2 = n\sigma_x$$

$$\sigma_1 = E_1\varepsilon_x = -\frac{E_1 y}{\rho}$$

$$\sigma_2 = E_2\varepsilon_x = -\frac{E_2 y}{\rho}$$

$$dF_1 = \sigma_1 dA = -\frac{E_1 y}{\rho} dA$$

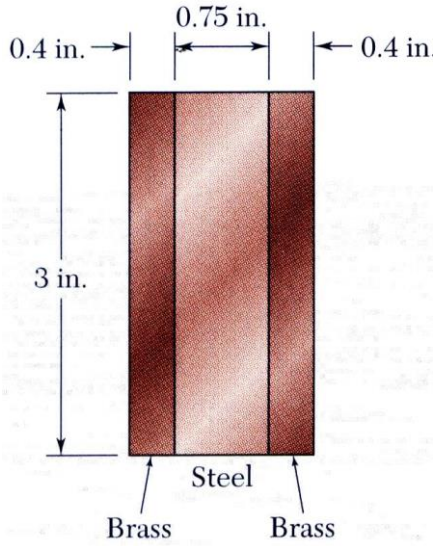
$$dF_2 = \sigma_2 dA = -\frac{E_2 y}{\rho} dA$$

$$dF_2 = -\frac{(nE_1)y}{\rho} dA = -\frac{E_1 y}{\rho}(n\,dA)$$

$$n = \frac{E_2}{E_1}$$

خمش
تغییر شکل
کرنش و خمش
تیرها
بتون مسلح
موزه پلاستیک
مباحث تکمیلی
خمش نامتقارن

مثال



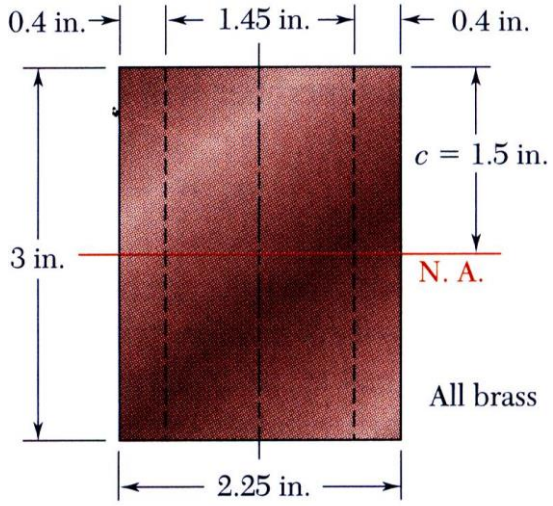
(Es = 29x106 psi) and (Eb = 15x106 psi)

*Determine the maximum stress in the steel and brass when a moment of 40 kip*in is applied.*

$$n = \frac{E_s}{E_b} = \frac{29\times10^6\,\text{psi}}{15\times10^6\,\text{psi}} = 1.933$$

$$b_T = 0.4\,\text{in} + 1.933\times0.75\,\text{in} + 0.4\,\text{in} = 2.25\,\text{in}$$

$$I = \tfrac{1}{12}b_T h^3 = \tfrac{1}{12}(2.25\,\text{in.})(3\,\text{in})^3 = 5.063\,\text{in}^4$$



$$\sigma_m = \frac{Mc}{I} = \frac{(40\,\text{kip}\cdot\text{in})(1.5\,\text{in})}{5.063\,\text{in}^4} = 11.85\,\text{ksi}$$

$$(\sigma_b)_{\max} = \sigma_m$$

$$(\sigma_s)_{\max} = n\sigma_m = 1.933\times11.85\,\text{ksi}$$

$$(\sigma_b)_{\max} = 11.85\,\text{ksi}$$

$$(\sigma_s)_{\max} = 22.9\,\text{ksi}$$

خمش

تغییر شکل

کرنش و خمش

تیرها

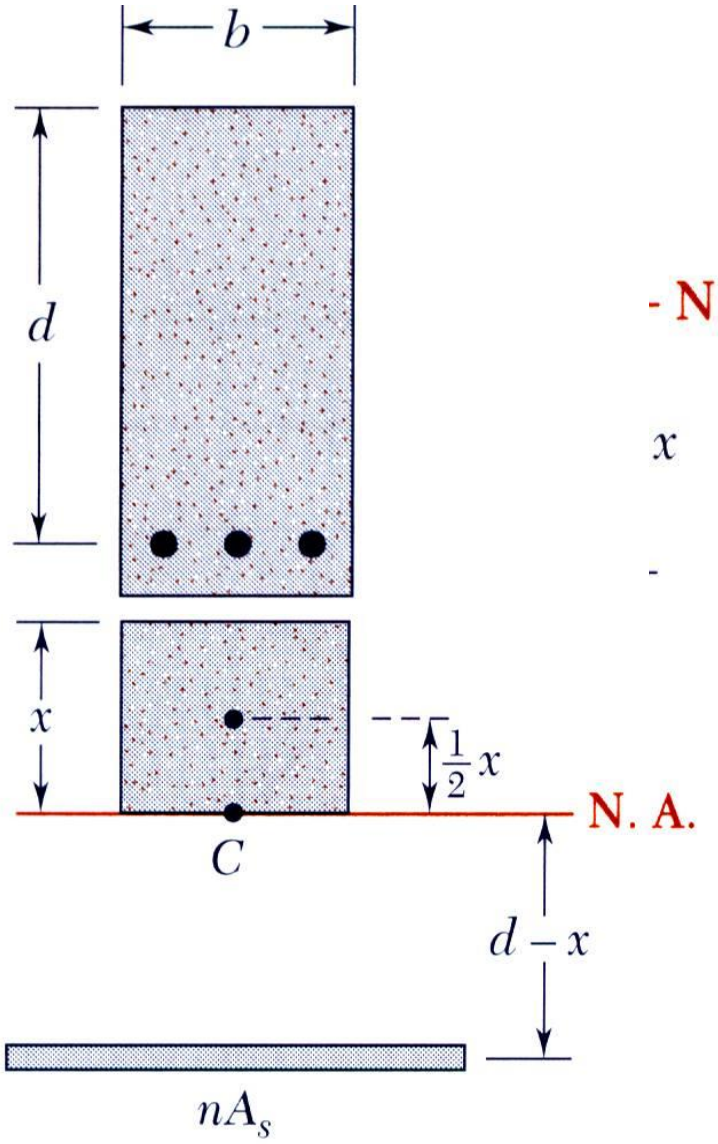
بتون مسلح

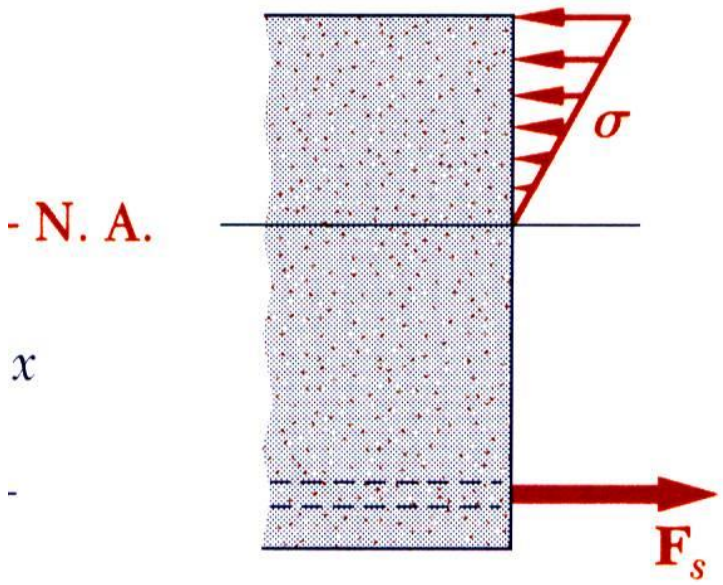
حوزه پلاستیک

مباحث تکمیلی

خمش نامتقارن

تیرهای ساخته شده از بتون مسلح



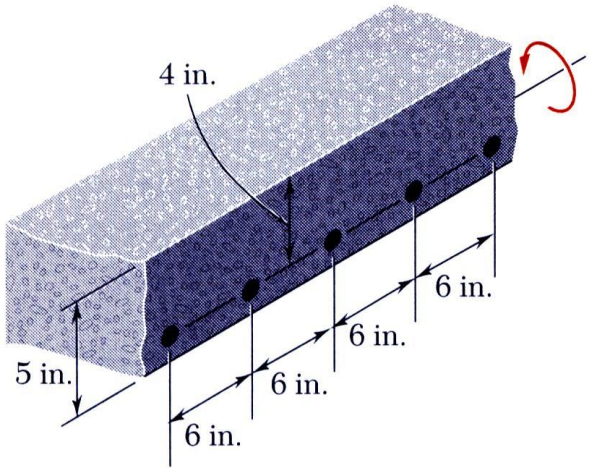


$$\sigma_x = -\frac{My}{I}$$

$$\sigma_c = \sigma_x \qquad \sigma_s = n\sigma_x$$

خمش
تغییر شکل
کرنش و خمش
تیرها
بتون مسلح
حوزه پلاستیک
مباحث تکمیلی
خمش نامتقارن

مثال



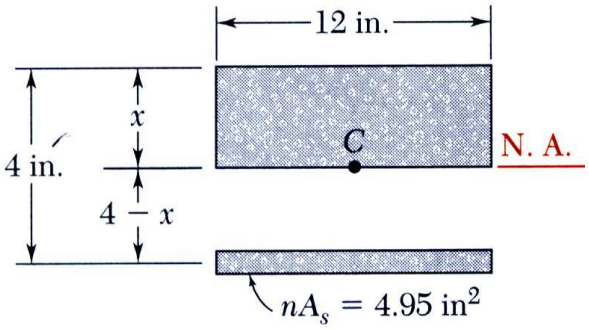
5/8-in-diameter steel rods

E = 29x106psi for steel

E = 3.6x106psi for concrete

*Bending moment of 40 kip*in for 1-ft width*

Determine the max. stress in Both

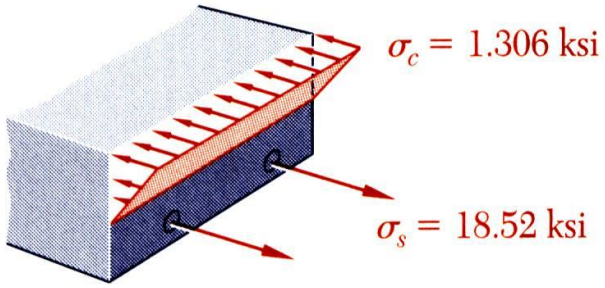


$$n = \frac{E_s}{E_c} = \frac{29\times10^6\ \text{psi}}{3.6\times10^6\ \text{psi}} = 8.06$$

$$nA_s = 8.06\times 2\left[\frac{\pi}{4}\left(\frac{5}{8}\text{ in}\right)^2\right] = 4.95\,\text{in}^2$$

$$12x\left(\frac{x}{2}\right) - 4.95(4-x) = 0 \qquad x = 1.450\,\text{in}$$

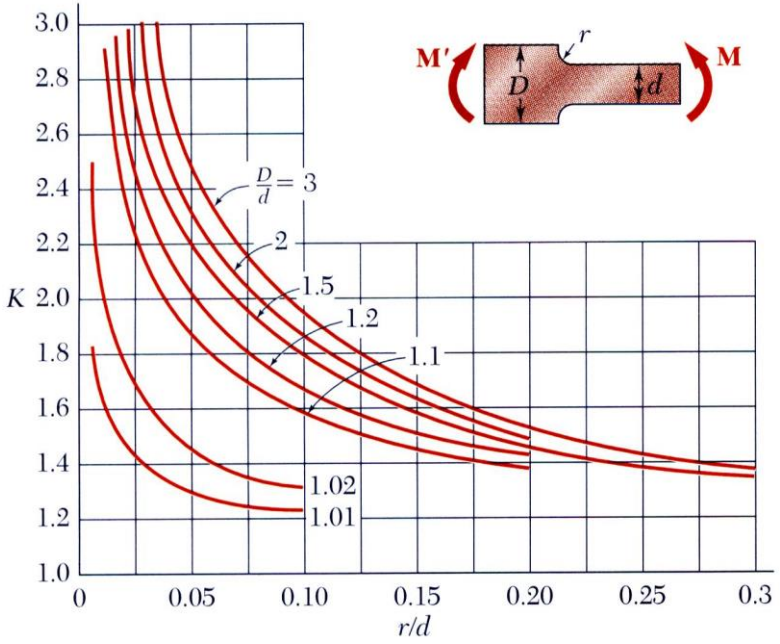
$$I = \frac{1}{3}(12\,\text{in})(1.45\,\text{in})^3 + (4.95\,\text{in}^2)(2.55\,\text{in})^2 = 44.4\,\text{in}^4$$



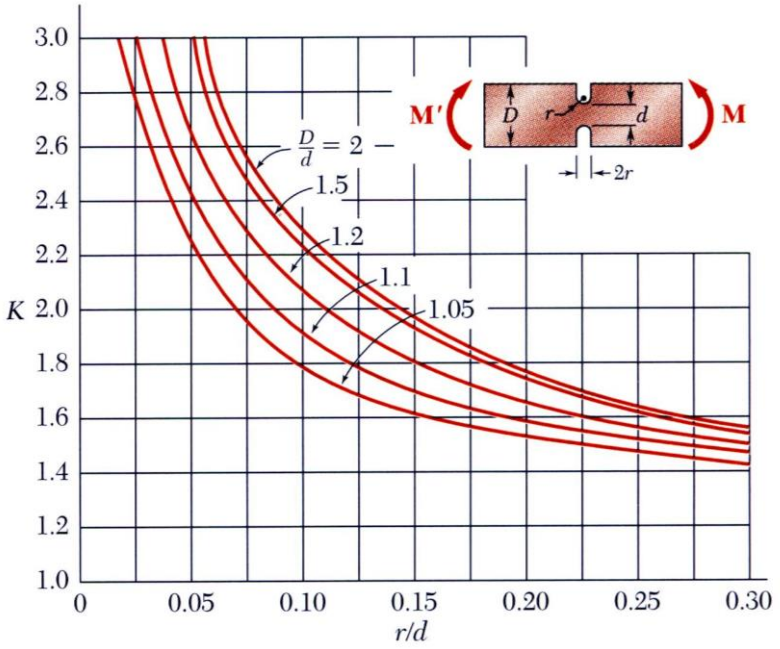
$$\sigma_c = \frac{Mc_1}{I} = \frac{40\,\text{kip}\cdot\text{in}\times 1.45\,\text{in}}{44.4\,\text{in}^4} = 1.306 ksi$$

$$\sigma_s = n\frac{Mc_2}{I} = 8.06\,\frac{40\,\text{kip}\cdot\text{in}\times 2.55\,\text{in}}{44.4\,\text{in}^4} = 18.52 ksi$$

تمرکز تنش



$$\sigma_m = K\frac{Mc}{I}$$



خمش

تغییر شکل

کرنش و خمش

تیرها

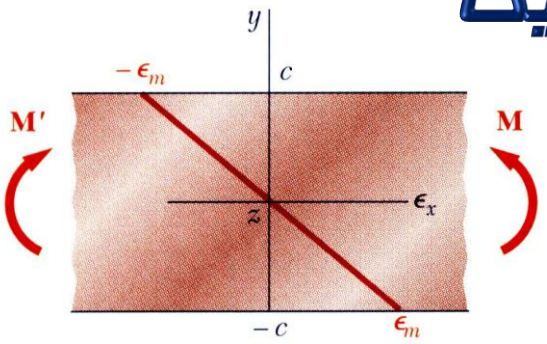
بتون مسلح

حوزه پلاستیک

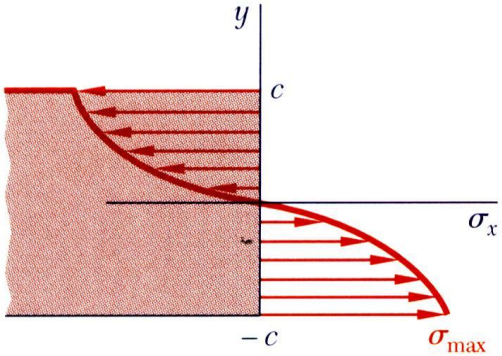
مباحث تکمیلی

خمش نامتقارن

تغییر شکل پلاستیک

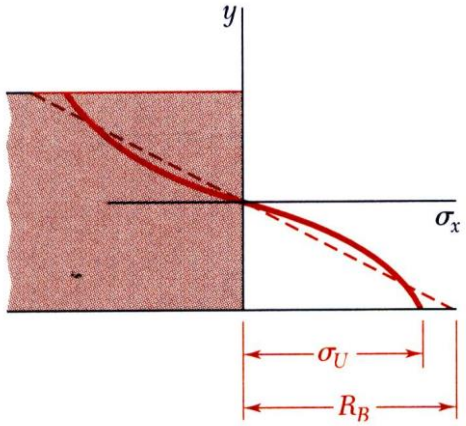


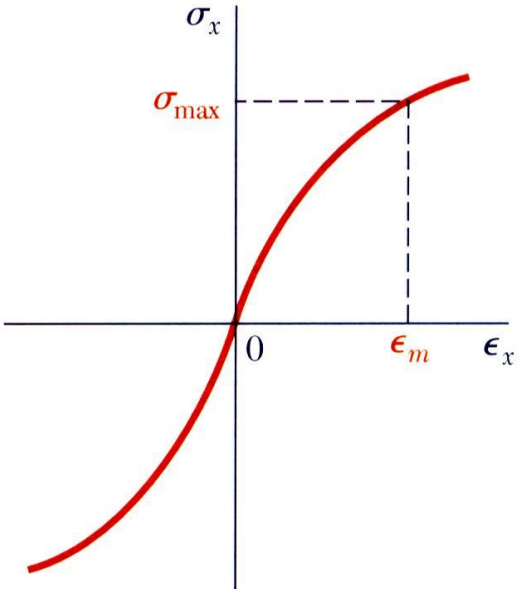
$$\varepsilon_x = -\frac{y}{c}\varepsilon_m \qquad \sigma_x = -\frac{My}{I}$$



$$F_x = \int \sigma_x \, dA = 0$$

$$M = \int -y\sigma_x \, dA$$





$$R_B = \frac{M_U c}{I}$$

خمش

تغییر شکل

کرنش و خمش

تیرها

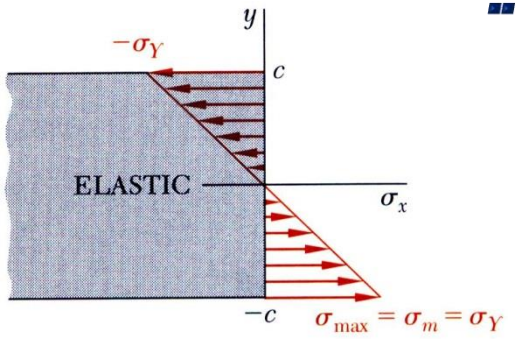
بتون مسلح

حوزه پلاستیک

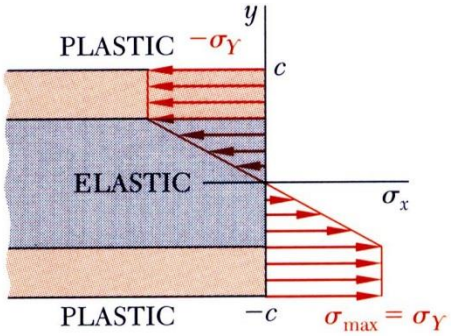
مباحث تکمیلی

خمش نامتقارن

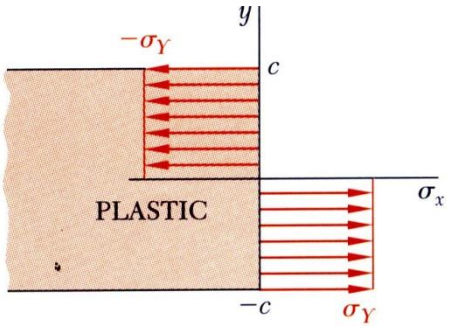
مواد الاستوپلاستیک



(b) $M = M_Y$



(c) $M > M_Y$



(d) $M = M_p$

$$\sigma_x \le \sigma_Y \qquad \sigma_m = \frac{Mc}{I}$$

$$\sigma_m = \sigma_Y \qquad M_Y = \frac{I}{c}\sigma_Y = \text{maximum elastic moment}$$

$$M = \tfrac{3}{2} M_Y \left(1 - \tfrac{1}{3}\frac{y_Y^2}{c^2}\right)$$

$$y_Y = \text{elastic core half - thickness}$$

$$M_p = \tfrac{3}{2} M_Y = \text{plastic moment}$$

$$k = \frac{M_p}{M_Y} = \text{shape factor}$$

(depends only on cross section shape)

خمش نامتقارن

خمش
تغییر شکل
کرنش و خمش
تیرها
بتون مسلح
موزه پلاستیک
مباحث تکمیلی
خمش نامتقارن

ای مردم!

امیر مؤمنان، امام علی علیه السلام