حل تمرینات فصل۸

تمرینات ۸-۱

$$1)\int (x-1)^{243}dx \Rightarrow x-1=u \Rightarrow dx=du$$

$$\int u^{243}du=\frac{1}{244}(x-1)^{244}+c$$

$$2)\int\frac{1}{\sqrt{1-x}}dx=-2\int\frac{-dx}{2\sqrt{1-x}}=-2\sqrt{1-x}+c$$

$$3)\int x\sqrt{2x^2-1}\,dx \qquad 2x^2-1=u \quad 4x\,dx=du$$

$$\frac{1}{4}\int 4x\sqrt{2x^2-1}dx=\frac{1}{4}\int\sqrt{u}du=$$

$$\frac{1}{4}\times\frac{2}{3}(u)^{3/2}=\frac{1}{6}u^{3/2}=\frac{1}{6}(2x^2-1)^{3/2}$$

راه حل دوم:

$$\int x\sqrt{2x^2-1}dx$$

$$\Rightarrow\frac{1}{4}\int\sqrt{2x^2-1}\;4xdx=\frac{1}{4}\int(2x^2-1)^{\frac{1}{2}}4xdx$$

$$\{2x^2=u\Rightarrow 4xdx=du$$

$$=\frac{1}{4}\int(u-1)^{\frac{1}{2}}du=\frac{1}{4}\times\frac{(u-1)^{\frac{3}{2}}}{\frac{3}{2}}=\frac{1}{6}(u-1)^{\frac{3}{2}}$$

$$4)\int(2-t)^{\frac{2}{3}}dt=-\int-(2-t)^{\frac{2}{3}}dt$$

$$=\frac{-3}{5}(2-t)^{\frac{5}{3}}+c$$

$$5)\int(1+x^3)^2dx=\int(1+x^6+2x^3)dx$$

$$=x+\frac{x^7}{7}+\frac{x^4}{2}+c$$

$$6)\int x(x^2+1)^{10}dx=\frac{1}{2}\int 2x(x^2+1)^{10}dx$$

$$=\frac{1}{2}\times\frac{1}{11}(x^2+1)^{11}+c$$

$$7)\int\frac{x^2}{\sqrt{1+x^3}}dx=\frac{1}{3}\int\frac{3x^2}{\sqrt{1+x^3}}dx$$

$$1+x^3=u \qquad 3x^2dx=du$$

$$\frac{1}{3}\int\frac{du}{\sqrt{u}}=\frac{1}{3}\times 2\times\sqrt{u}=\frac{2}{3}\sqrt{u}+c$$

$$=\frac{2}{3}(1+x^3)^{\frac{1}{2}}+c$$

$$8)\int\frac{dx}{(3x+2)^2}=-\frac{1}{3}\int\frac{-3dx}{(3x+2)^2}=-\frac{1}{3}\times\frac{1}{3x+2}+c$$

$$9)\int\frac{3r}{\sqrt{1-r^2}}dr \qquad 1-r^2=u \quad -2rdr=du$$

$$-\frac{1}{2}\int\frac{-2\times 3r\,dr}{\sqrt{1-r^2}}=\frac{-1}{2}\int\frac{3du}{\sqrt{u}}$$

$$=\frac{-1}{2}\times 2\times 3\sqrt{u}=-3\sqrt{u}=-3\sqrt{1-r^2}$$

$$10)\int x^4(7-x^5)^3dx \quad u=7-x^5 \quad du=-5x^4dx$$

$$=-\frac{1}{5}\int u^3du=\frac{u^4}{20}+c=-\frac{(7-x^5)^4}{20}+c$$

$$11)\int\frac{ds}{(s+1)^3} \qquad s+1=u \qquad ds=du$$

$$\int\frac{du}{u^3}=\frac{1}{-2u^2}=\frac{-1}{2(s+1)^2}+c$$

$$12)\int\frac{1}{x^2+4x+4}dx=-\int\frac{-dx}{(x+2)^2}=\frac{-1}{x+2}+c$$

راه حل دوم

$$\int\frac{1}{x^2+4x+4}dx=\int\frac{1}{u^2}du=\int u^{-2}du=\frac{u^{-1}}{-1}=\frac{1}{u}+c$$

$$x^2+4x+14=(x+2)^2 \quad , \quad x+2=u\rightarrow dx=du$$

$$13)\int \frac{x+1}{2\sqrt{x+1}}dx=\frac{1}{2}\int \sqrt{x+1}dx=$$

$$\frac{1}{2}\times\frac{2}{3}(x+1)^{\frac{3}{2}}=\frac{1}{3}(x+1)^{\frac{3}{2}}+c$$

$$14)\int (y^3+6y^2+12y+8)(y^2+4y+4)dy$$

راه حل اول:

$$=\frac{1}{3}\times\frac{1}{2}(y^3+6y^2+12y+8)^2+c$$

راه حل دوم:

$$=\int (y+2)^5dy=\frac{(y+2)^6}{6}+c$$

$$15)\int \frac{1}{\sqrt{x}(1+\sqrt{x})^2}dx=2\int \frac{dx}{2\sqrt{x}(1+\sqrt{x})^2}=$$

$$1+\sqrt{x}=u \quad \frac{1}{2\sqrt{x}}dx=du$$

$$=2\int \frac{du}{u^2}=\frac{-2}{u}+c=\frac{-2}{1+\sqrt{x}}+c$$

$$16)\int (1-2x)^9dx=\frac{-1}{2}\times\frac{1}{10}(1-2x)^{10}+c$$

$$17)\int \sqrt{4+5x}dx=\frac{1}{5}\int 5\sqrt{4+5x}dx=$$

$$4+5x=u \quad 5dx=du$$

$$=\frac{1}{5}\int \sqrt{u}du=\frac{1}{5}\times\frac{2}{3}u^{\frac{3}{2}}+c$$

$$=\frac{2}{15}(4+5x)^{\frac{3}{2}}+c$$

$$18)\int \frac{x}{\sqrt{3x-1}}dx \qquad \sqrt{3x-1}=t \quad 3x-1=t^2$$

$$3dx=2t\,dt$$

$$x=\frac{t^2+1}{3}$$

$$=\frac{1}{9}\int \frac{t^2+1}{t}\times 2t\,dt$$

$$=\frac{2}{27}(3x-1)\sqrt{3x-1}+\frac{2}{9}\sqrt{3x-1}+c$$

$$19)\int x\sqrt[3]{1-x}dx \Rightarrow 1-x=u \ \Rightarrow\ -dx=du$$

$$=-\int (1-u)\sqrt[3]{u}\quad du=\int (-\sqrt[3]{u}+u^{\frac{4}{3}})du$$

$$=(-\frac{3}{4}u^{\frac{4}{3}}+\frac{3}{7}u^{\frac{7}{3}})+c$$

$$=\frac{-3}{4}(1-x)^{\frac{4}{3}}+\frac{3}{7}(1-x)^{\frac{7}{3}}+c$$

$$20)\int (\frac{2+\sqrt{x}}{x})^{\frac{1}{2}}dx$$

$$2+\sqrt{x}=u \quad \frac{1}{2\sqrt{x}}dx=du$$

$$=\int \frac{\sqrt{2+\sqrt{x}}}{\sqrt{x}}dx=2\int \frac{\sqrt{2+\sqrt{x}}}{2\sqrt{x}}dx=$$

$$=2\int \sqrt{u}du=\frac{4}{3}(2+\sqrt{x})(\sqrt{2+\sqrt{x}})+c=$$

راه حل دوم

$$\int (\frac{2+\sqrt{x}}{x})^{\frac{1}{2}}dx,\ 2+\sqrt{x}=u \ \rightarrow\ \frac{1}{2\sqrt{x}}dx=du$$

$$=2\int \frac{\sqrt{2+\sqrt{x}}}{2\sqrt{x}}dx=2\int u^{\frac{1}{2}}du=2\frac{u^{\frac{3}{2}}}{\frac{3}{2}}+c=\frac{4}{3}u^{\frac{3}{2}}$$

$$+c=\frac{4}{3}(2+\sqrt{x})^{\frac{3}{2}}+c$$

$$21)\int \frac{xdx}{\sqrt{x-1}}=\int \frac{x-1+1}{\sqrt{x-1}}dx=$$

$$\int (\sqrt{x-1}+\frac{1}{\sqrt{x-1}})dx=\int \sqrt{x-1}dx+$$

$$\int \frac{1}{\sqrt{x-1}}dx=\frac{2}{3}(x-1)^{\frac{3}{2}}+2(x-1)^{\frac{1}{2}}+c=$$

$$\frac{2}{3}\sqrt{(x-1)^3}+2\sqrt{x-1}+c$$

روش دوم

$$\int \frac{xdx}{\sqrt{x-1}}$$

$$x-1=u^2 \ \rightarrow\ dx=2udu$$

$$\int (2u^2-2)du=\frac{2}{3}u^3-2u+c=3u(\frac{u^2}{3}-1)+c$$

$$\Rightarrow 2\sqrt{x-1}\left[\frac{(x-1)}{3}-1\right]+c$$

$$22)\int Cos^3xdx=\int Cosx(Cos^2x)dx=$$

$$= \int Cosx(1-Sin^2x)dx = \int (1-u^2)du \qquad (Sinx = u)$$

$$= Sin\,x - \frac{1}{3}Sin^3x + c$$

بیانی دیگر

$$\int Cos^3x\,dx = \int Cos\,x(Cos^2x)dx$$

$$= \int Cosx(1-Sin^2x)dx$$

$$= \int Cosx\,dx - \int Cos\,xSin^2xdx$$

$$\int Cosx\,dx = Sinx + c, \int Cosx\,Sin^2xdx$$

$$\rightarrow Sin^2x = u \rightarrow 2Cosx\,Sinx\,dx = du$$

$$Sinx = \sqrt{u}$$

$$= \frac{1}{2}\int \sqrt{u}du = \frac{1}{2}\frac{u^{\frac{3}{2}}}{\frac{3}{2}} = \frac{1}{3}u^{\frac{3}{2}} = \frac{1}{3}\sqrt{u^3} = \frac{1}{3}Sin^3x$$

جواب کلی $Sinx - \frac{1}{3}Sin^3x + c$

$$23)\int Sin^5x\,dx = \int Sinx(1-Cos^2x)^2dx$$

$$= Sinx(1-2Cos^2x + Cos^4x)dx$$

$$= \int (Sinx - 2SinxCos^2 + Sinx\,Cos^4x)dx$$

$$= -Cosx + \frac{2}{3}Cos^3x - \frac{1}{5}Cos^5x + c$$

$$24)\int \frac{Sinx}{\sqrt{1+2Cosx}}dx = -\frac{1}{2}\int \frac{-2Sinx}{\sqrt{1+2Cosx}}dx$$

$$= -\sqrt{1+2Cosx} + c$$

$$25)\int x\sec x^2.\tan x^2dx$$

$$\tan x^2 = u \qquad 2x\sec x^2dx = du$$

$$= \frac{1}{2}\int 2x\sec x^2\tan x^2dx = \frac{1}{2}\int udu$$

$$= \frac{1}{4}u^2 = \frac{1}{4}\tan^2 x^2 + c$$

CommandB

$$26)\int xe^{x^2}dx = \frac{1}{2}e^{x^2} + c$$

$$27)\int \frac{dx}{x(\ln x)^2} \qquad \ln x = u \qquad \frac{1}{x}dx = du$$

$$= \int \frac{du}{u^2} = \frac{-1}{u} + c = \frac{-1}{\ln x} + c$$

$$28)\int \frac{e^{3x}}{e^{3x}-1}dx = \frac{1}{3}\ln\left|e^{3x}-1\right| + c$$

$$29)\int \frac{e^x}{e^{2x}+1}dx \qquad e^x = u \quad e^xdx = du$$

$$= \int \frac{du}{u^2+1} = Arc\tan e^x + c$$

$$30)\int \frac{2^x}{\sqrt{1-4^x}}dx \qquad 2^x = u$$

$$2^x\ln 2dx = du \qquad dx = \frac{du}{u}(\ln 2)^{-1}$$

$$= \int \frac{2^x}{\sqrt{1-(2^x)^2}}dx = \int \frac{udx}{\sqrt{1-u^2}}$$

$$= \int \frac{u}{\sqrt{1-u^2}} \times \frac{du}{u} \times (\ln 2)^{-1}$$

$$= (\ln 2)^{-1}\int \frac{du}{\sqrt{1-u^2}} = (\ln 2)^{-1} \times Sin^{-1}(2^x) + c$$

$$31)\int \frac{1}{x^2}\tan\frac{1}{x}dx$$

$$\frac{1}{x} = u \qquad \frac{-dx}{x^2} = du \qquad \frac{dx}{x^2} = -du$$

$$= -\int \tan u\,du = \ln\left|Cosu\right| + c$$

$$= \ln\left|Cos(\frac{1}{x})\right| + c$$

تمرینات ۸-۲

$$1)\int xe^{3x}dx = x\frac{e^{3x}}{3} - \int \frac{e^{3x}}{3}dx$$

$$= \frac{xe^{3x}}{3} - \frac{e^{3x}}{9} + c$$

$$2)\int \ln xdx = x\ln x - \int x\times\frac{1}{x} = x\ln x - x + c$$

$$3)\int (\ln x)^2 dx = x(\ln x^2) - 2\int \ln xdx$$

$$= x\ln x^2 - 2(x\ln x - x) + c$$

$$4)\int Sinx\ln(Cosx)dx$$

$$= -\ln(Cosx)Cosx - \int \frac{Sinx}{Cosx}Cosx$$

$$= -\ln(Cosx)Cosx + Cosx + c$$

$$5)\int e^x Cosxdx$$

$$\begin{cases} u = e^x \\ dv = Cosxdx \end{cases} \Rightarrow \begin{cases} du = e^x dx \\ v = Sinx \end{cases}$$

$$\int e^x Cosxdx = e^x Sinx - \int e^x Sinx\,dx$$

$$\begin{cases} e^x = u \\ Sinx\,dx = dv \end{cases} \Rightarrow \begin{cases} du = e^x dx \\ v = -Cosx \end{cases}$$

$$\int e^x Sinx\,dx = -e^x Cosx + \int e^x Cosx\,dx$$

$$\int e^x Cosx\,dx = e^x Sinx + e^x Cosx - \int e^x Cosx\,dx$$

$$\int e^x Cosx\,dx = \frac{1}{2}e^x(Sinx + Cosx) + c$$

$$6)\int \frac{x^3}{\sqrt{1-x^2}}dx \quad \begin{cases} u = 1 - x^2 \Rightarrow du = -2xdx \\ dv = x^3 dx \Rightarrow \frac{x^4}{4} = v \end{cases}$$

$$= \frac{(1-x^2)x^4}{4} + \int \frac{x^5}{2}dx =$$

$$\frac{(1-x^2)x^4}{4} + \frac{x^6}{12} + c$$

$$7)\int \sec^5 xdx = \int \sec^3 x\sec^2 xdx =$$

$$\begin{cases} u_1 = \sec^3 x \Rightarrow du_1 = 3\sec^3 x\tan xdx \\ \sec^2 xdx = dv_1 \Rightarrow v_1 = \tan x \end{cases}$$

$$\Rightarrow \tan x\sec^3 x - 3\int \sec^3 x\tan^2 xdx \Rightarrow$$

$$\tan x\sec^3 x - 3\int \sec^3 x(\sec^2 x - 1)dx \Rightarrow$$

$$\int \sec^5 xdx = \tan x\sec^3 x - 3\int \sec^5 x - 3\int \sec^3 dx \Rightarrow$$

$$\Rightarrow 4\int \sec^5 xdx = \tan x\sec^3 x - 3\int \sec^3 xdx$$

ب) $\int \sec^5 xdx = \frac{\tan x\sec^3 x}{4} - \frac{3}{4}\int \sec^3 xdx$

حال $\int \sec^3 xdx$ را محاسبه می‌کنیم:

ج) $\int \sec^3 xdx = \int \sec x\sec^2 x\,dx$

$$\begin{cases} u_2 = \sec x \Rightarrow du_2 = \sec x\tan xdx \\ \sec^2 xdx = dv_2 \Rightarrow v_2 = \tan x \end{cases}$$

$$\Rightarrow \sec x\tan x - \int \sec x\tan^2 x$$

$$= \sec x\tan x - \int \sec x(\sec^2 x - 1)dx \Rightarrow$$

$$\int \sec^3 xdx = \sec x\tan x - \int \sec^3 xdx - \int \sec xdx \Rightarrow$$

$$2\int \sec^3 xdx = \sec x\tan x - \int \sec xdx \Rightarrow$$

$$\int \sec^3 xdx = \frac{\sec x\tan x}{2} - \frac{1}{2}\int \sec xdx$$

$$\Rightarrow \int \sec^3 xdx = \frac{\sec x\tan x}{2} - \frac{1}{2}(1 + \tan^2 x)$$

مقدار $\int \sec^3 xdx$ را در ب قرار می‌دهیم.

$$\int \sec^5 xdx = \frac{\tan x\sec^3 x}{4} - \frac{3}{4}\left[\frac{\sec x\tan x}{2} - \frac{1}{2}(1 + \tan^2 x)\right] + c$$

$$8)\int \frac{\cot^{-1}\sqrt{z}}{\sqrt{z}}dz$$

$$= 2\int Arc\cot udu = 2(Arc\cot u\times u - \int u\times\frac{-1}{1+u^2}du$$

$$= 2Arc\cot u\times u + 2\int \frac{u}{1+u^2}du$$

$$= 2uArc\cot u + \ln(1+u^2) + c$$

$$= 2\sqrt{x}\cot^{-1}\sqrt{x} + \ln|1+x| + c$$

$9)\int xCosxdx = xSinx - \int Sinx\,dx$

$= xSinx + Cosx + c$

$10)\int x^2 \ln xdx = \frac{x^3}{3}\ln x - \int \frac{1}{x}\times\frac{x^3}{3}$

$= \frac{x^3}{3}\ln x - \int \frac{x^2}{3}dx = \frac{x^3}{3}\ln x - \frac{1}{3}\frac{x^3}{3}$

$11)\int Arc\,Sinx\,dx$

$= xArcSinx - \int \frac{x}{\sqrt{1-x^2}} = xArcSinx + \sqrt{1-x^2}\times c$

$12)\int e^{-x}Sin2xdx = e^{-x}(\frac{-1}{2}Cos2x) +$

$\int e^{-x}\frac{1}{2}Cos2xdx$

$=\frac{1}{2}(e^{-x}\times\frac{1}{2}Sin2x)+\frac{1}{4}\int e^{-x}Sin2xdx$

$=(e^{-x}\times\frac{-1}{2}Cos2x+\frac{1}{4}e^{-x}Sin2x+\frac{1}{4}M)$

$\frac{3}{4}M = e^{-x}\times\frac{-1}{2}Cos2x+\frac{1}{4}e^{-x}Sin2x$

$\Rightarrow M=\frac{2}{3}e^{-x}(\frac{-1}{2}Sin2x - Cos2x)$

$13)\int Sin3xCos5xdx$

$\frac{-1}{2}\int [Sin8x+Sin2x]dx$

$=\frac{1}{16}Cos8x-\frac{1}{4}Cos2x$

$14)\int \sqrt{x}\ln xdx=\frac{2}{3}\ln x\times x^{\frac{3}{2}}-\int\frac{1}{x}\times\frac{2}{3}x\sqrt{x}dx$

$=\frac{2}{3}\ln xx^{\frac{3}{2}}-\frac{2}{3}\int\sqrt{x}dx$

$=\frac{2}{3}\ln xx^{\frac{3}{2}}-\frac{2}{3}\times\frac{2}{3}x^{\frac{3}{2}}$

$=\frac{2}{3}x^{\frac{3}{2}}\ln x-\frac{4}{9}x^{\frac{3}{2}}$

$15)\int Sinx\,Cos2x\,dx$

راه اول:

$\int \underbrace{Sinx}_{u}\cdot\underbrace{Cos2x\,dx}_{dv}$

$= \underbrace{Sinx}_{u}\cdot\underbrace{\frac{1}{2}Sin2x}_{v}-\int(\underbrace{\frac{-1}{2}Sin2x}_{v}\cdot\underbrace{Cosx}_{du})dx$

$=\frac{1}{2}Sinx\,Sin2x+1\int(\frac{1}{2}Sin2xCosx)dx$

$\to\int\underbrace{\frac{1}{2}Sin2x}_{u}\cdot\underbrace{Cosxdx}_{dv}=\frac{1}{2}Sin2xSinx-\int Sinx\cdot Cos2x\cdot x$

$\Rightarrow\int SinxCos2xdx=\frac{1}{2}SinxSin2x+\frac{1}{2}Sn2xSinx$

$-\int SinxCos2x\,dx$

$\Rightarrow 2\int SinxCos2xdx=SinxSin2x$

$\Rightarrow\int SinxCos2x=\frac{1}{2}(SinxSin2x)+c$

راه دوم:

$\int SinxCos2xdx=\int Sinx(2Cos^2x-1)dx$

$Cosx=u\Rightarrow -Sinxdx=du$

$=-2(\frac{Cos^3x}{3})+Cosx+c$

$=\frac{-2}{3}Cos^3x+Cosx+c$

$16)\int \underbrace{x^3}_{u}\underbrace{Cosxdx}_{dv}=\underbrace{x^3}_{u}\underbrace{Sinx}_{v}-\int\underbrace{3x^2}_{du}\underbrace{Sinx}_{v}dx$

$\Rightarrow\int 3x^2Sinxdx=3\int\underbrace{x^2}_{u}\underbrace{Sinxdx}_{dv}$

$=-3x^2Cosx$

$-3\int(-Cosx)2xdx+3\int 2xCosxdx=\int 6xCosxdx$

$=x^3Sinx-3(-x^2Cosx-6xSinx+6Cosx)$

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$\int x\sqrt{2x+3}dx$

$$\begin{cases} x=u\Rightarrow dx=du \\ dv=\sqrt{2x+3}dx\Rightarrow v=\frac{(2x+3)^{\frac{3}{2}}}{3} \end{cases}$$

$$22)\int_0^x x^3 Sinx\,dx$$

$$=-x^3Cosx+3x^2Sinx+6xCosx-6Sinx$$

$$23)\int_1^e (\ln x)^3 dx \Rightarrow \ln x=t \Rightarrow \frac{dx}{x}=dt \Rightarrow x=e^t$$

$$=\int_1^e t^3e^t dt\ ,\begin{cases} t^3=u_1 \Rightarrow du_1=3t^2dt \\ e^t\ dt=dv_1 \Rightarrow v_1=e^t \end{cases}$$

$$=e^tt^3-3\int_1^e e^tt^2dt \qquad (I)$$

$$=e^tt^2-2\int_1^e e^tt\,dt \qquad (II)$$

$$\int_1^e e^t+dt \quad ,\begin{cases} t=u_3 \Rightarrow dt=du_3 \\ e^tdt=dv_3 \Rightarrow v_3=e^t \end{cases}$$

$$e^tt-\int_1^e e^tdt=e^tt-e^t \qquad (III)$$

دو رابطه $(II),(III)$ را در (I) قرار می‌دهیم.

$$(I),(II),(III) \Rightarrow e^tt^3-3\left[e^tt^2-2(e^tt-e^t)\right] \Rightarrow$$

$$e^tt^3-3e^tt^2+6e^tt-6e^t=e^t(t^3-3t^2+6t-6)$$

در حالت کلی‌تری داریم:

$$\int_1^e (\ln x)^3dx=x\left[(\ln x)^3-3(\ln x)^2+6\ln x-6\right]\Big]_0^e+c$$

$$\Rightarrow e\left[(1)^3-3(1)^2=6\times1-6\right]-0 \Rightarrow -2e+c \Rightarrow$$

$$\int_1^e (\ln x)^3dx=-2e+c$$

$$24)\int_{-1}^1 ArcCoszdz \Rightarrow$$

$$ArcCosx=u \qquad du=\frac{-1}{\sqrt{1-x^2}}$$

$$dx=dv \qquad u=v$$

$$\int_{-1}^1 ArcCosxdx=xArcCosx\Big]_{-1}^1+\int_{-1}^1\frac{x\,dx}{\sqrt{1-x^2}}$$

$$=xArcCosx\Big]_{-1}^1+0=\pi$$

$$25)\int x^5e^{x^2}dx=\frac{1}{2}\int x^4\,2xe^{x^2}dx$$

$$x^4=u \qquad 4x^3dx=du$$

$$2xe^{x^2}dx=dv \qquad v=e^{x^2}$$

$$\int x^4\,2xe^{x^2}dx=x^4e^{x^2}\,4\int e^{x^2}x^3dx$$

$$\int x\sqrt{2x+3}dx=\frac{x}{3}(2x+3)^{\frac{3}{2}}-\frac{1}{3}\int (2x+2)^{\frac{3}{2}}dx \Rightarrow$$

$$=\frac{x}{3}(2x+3)^{\frac{3}{2}}-\frac{1}{15}(2x+3)^{\frac{5}{2}}+c$$

$$18)\int \csc^3 xdx=\frac{1}{2}\left[\ln\left|\csc x-\cot gx\right|+\cot gx\csc x\right]$$

$$19)\int Sin(\ln x)dx=$$

$$=xSin(\ln x)-\int x\times\frac{1}{x}Cos(\ln x)dx$$

$$=xSin(\ln x)-\int Cos(\ln x)dx$$

$$\int Cos(\ln x)dx=xCos(\ln x)+$$

$$\int x\times\frac{1}{x}Sin(\ln x)dx=xCos(\ln x)+\int Sin(\ln x)dx$$

$$I=\int Sin(\ln x)dx=xSin(\ln x)-Cos(\ln x)-$$

$$\int Sin(\ln x)dx$$

$$2I=xSin(\ln x)dx$$

$$2I=xSin(\ln x)-xCos(\ln x)$$

$$I=\frac{1}{2}xSin(\ln x)-\frac{1}{2}xCos(\ln x)$$

$$20)\int_0^1 xe^{-x}dx=-xe^{-x}+e^{-x}\Big]_0^1=-1$$

$$\int_0^1 xe^{-x}dx \xrightarrow{-x=t} x=-t\,,dx=-dt$$

$$=\int -te^t(-dt)=\int \overbrace{t}^{u}\overbrace{e^tdt}^{dv}$$

$$=(t-1)e^t=\left[(-x-1)e^{-x}\right]_0^1=-2e^{-1}-(-1)=\frac{-2}{e}+1$$

$$21)\int_0^x x^2Cosxdx=I$$

$$=x^2Sinx-\int 2xSinx\,dx$$

$$2\int xSinx=2(-xCosx+\int Cosxdx)$$

$$I=x^2Sinx+2xCox-\int 2Cosxdx$$

$$=x^2Sinx+2xCosx-2Sinx$$

$$\int e^{x^2}x^3dx=\frac{1}{2}\int 2xe^{x^2}x^2dx$$

$$\int 2xe^{x^2}x^2dx=x^2e^{x^2}-2\int e^{x^2}xdx$$

$$\int e^{x^2}xdx=\frac{1}{2}e^{x^2}$$

$$\int x^5e^{x^2}dx=\frac{1}{2}(x^4e^{x^2}-2x^2e^{x^2}+2e^{x^2})+c$$

$$26)\int (2x+3)e^xdx=(2x+3)e^x-2\int e^xdx$$

$$=(2x+3)e^x-2e^x$$

$$27)\int Sin\theta Cos\theta d\theta$$

$$=\frac{1}{2}\int Sin2\theta\, d\theta=\frac{-1}{4}Cos2\theta+c$$

$$\int \overbrace{Sin\theta}^{u}\overbrace{Cos\theta\, d\theta}^{du},Sin\theta=u\rightarrow Cos\theta\, d\theta=du$$

روش دوم

$$\int udu=\frac{u^2}{2}+c=\frac{Sin^2\theta}{2}+c$$

$$=\frac{\frac{1-Cos2\theta}{2}}{2}=\frac{1}{4}(1-Cos2\theta)+c$$

$$28)\int \theta\sec^2\theta\, d\theta$$

$$\sec^2\theta\, d\theta=dv \qquad v=tg\theta$$

$$\theta=u \qquad du=d\theta$$

$$\int \theta\sec^2\theta\, d\theta=\theta tg\theta-\int tg\theta\, d\theta$$

$$=\theta tg\theta+\ln|Cos\theta|$$

$$29)\int e^{2\theta}Sin3\theta\, d\theta$$

$$=e^{2\theta}\times\frac{-1}{3}Cos3\theta+\frac{2}{3}\int e^{2\theta}Cos3\theta\, d\theta$$

$$\int e^{2\theta}Cos3\theta=e^{2\theta}\times\frac{1}{3}Sin3\theta-\frac{2}{3}\Big]e^{2\theta}Sin3\theta\, d\theta$$

$$\Rightarrow I=-e^{2\theta}\times\frac{1}{3}Cos3\theta+\frac{2}{9}e^{2\theta}Sin3\theta-\frac{4}{9}I$$

$$\frac{13}{9}I=-e^{2\theta}\times\frac{1}{3}Cos3\theta+\frac{2}{9}e^{2\theta}Sin3\theta$$

$$\Rightarrow I=\frac{9}{13}(-e^{2\theta}\times\frac{1}{3}Cos3\theta+\frac{2}{9}e^{2\theta}Sin3\theta)$$

$$30)\int e^{-\theta}Cos3\theta\, d\theta=-e^{-\theta}Cos3\theta-3\int e^{-\theta}Sin3\theta\, d\theta$$

$$=-e^{-\theta}Cos3\theta+3e^{-\theta}Sin3\theta-9\int_{e}^{-\theta}Cos3\theta\, d\theta$$

$$=\frac{1}{10}e^{-\theta}[3Sin3\theta-Cos3\theta]$$

$$31)\int x^5Cos(x^3)dx$$

$$=\frac{1}{3}\int u\,Cosu\,du=\frac{1}{3}(uSin\,u+Cosu)+c$$

$$=\frac{1}{3}(x^3Sinx^3+Cosx^3)$$

تمرینات ۸-۳

1) $\int xe^{2x}dx = \frac{xe^{2x}}{2} - \frac{e^{2x}}{4}$

u و مشتقهایش		dv و انتگرالهایش
x	۱+۱	e^{2x}
۱	۱-۱	$e^{2x}/2$
۰		$e^{2x}/4$

2) $\int xSin4x\,dx = \frac{-xCos4x}{-4} + \frac{Sin4x}{16}$

x^2 $\quad Cos3x$

$2x$ $\quad Sin\frac{3x}{3}$

u و مشتقهایش		dv و انتگرالهایش
۲x	۱+۱	$\frac{-Cos3x}{9}$
۱	۱-۱	$\frac{-Cos4x}{4}$
۰		$\frac{-Sin3x}{27}$
۰		$\frac{-Sin4x}{16}$

3) $\int x^2Cos3xdx$

$= \frac{x^2Sin3x}{3} + \frac{2xCos3x}{9} - \frac{2Sin3x}{27}$

u و مشتقهایش		dv و انتگرالهایش
	+	
	−	
	+	

4) $\int x^2Sin2x\,dx$

$= \frac{-x^2Cos2x}{2} + \frac{2xSin2x}{4} + \frac{2Cos2x}{8}$

6) $\int t^3e^t dt = t^3e^t - 3t^2e^t + 6te^t - 6e^t$

u و مشتقهایش		dv و انتگرالهایش
t^3	+	e^t
	−	e^t
۶t	+	e^t
۶	−	e^t
۰		e^t

7) $\int x^5Cos(x^3)dx \qquad x^3 = u \quad 3x^2dx = du$

$\int x^5Cos(x^3)dx = \frac{1}{3}\int 3x^2x^3Cosx^3dx$

$\Rightarrow \frac{1}{3}\int uCosu\,du = \frac{1}{3}(uSinu - \int Sinu\,du$

$= \frac{1}{3}x^3Sin(x^3) + \frac{1}{3}Cosx^3 + c$

8) $\int x^5e^{x^2}dx = \frac{1}{2}e^u(u^2 - 2u + 2)$

u و مشتقهایش		dv و انتگرالهایش
$u^۲$	+	e^u
x $\;2u$	−	e^{-2x} e^u
$2$۱	+	$\frac{e^{-2x}}{-2}$ e^u
۰		$\frac{e^{-2x}}{4}$ e^u

9) $\int xe^{-2x}dx = \frac{-xe^{-2x}}{2} - \frac{e^{-2x}}{4}$

10) $\int x3^x dx = \frac{x3^x}{\ln 3} - \frac{3^x}{(\ln 3)^2}$

u و مشتقهایش		dv و انتگرالهایش
x	+	3^x
۱	−	$\frac{3x}{\ln 3}$
۰		

11) $\int x^2 Sin5xdx = \frac{-x^2Cos^2x}{2}+\frac{2xSin2x}{4}+\frac{2Cos^2x}{8}$

u و مشتقهایش		dv و انتگرالهایش
x^2		$Sin2x$
$2x$	+	$\frac{-Cos2x}{2}$
2	−	$\frac{-Sin2x}{4}$
۰	+	$\frac{Cos2x}{8}$

$\int x^2 Sin\delta x dx\ ,\ \Delta x = t \to x = \frac{t}{\delta} \to dx = \frac{1}{\delta}dt$

$\int \frac{t^2}{25}Sin\,t\cdot\frac{1}{\delta}dt = \frac{1}{125}\int t^2 Sin\,t\,dt$

$-\frac{x^2Cos\delta x}{\delta}+\frac{2xSin\delta x}{25}+\frac{2Cos\delta x}{125}+c$

t^2	$Sin\,t$
$2t$	$-Cost$
2	$-Sin\,t$
0	$Cost$

12) $\int x^3 Cosxdx$

$= x3Sinx + 3x^2Cosx - 6xSinx - 6Cosx$

u و مشتقهایش		dv و انتگرالهایش
x^3		$Cosx$
$3x^2$	+	$Sinx$
$6x$	−	$-Cosx$
۶	+	$-Sinx$
۰	−	$Cosx$

13) $\int x\sec^2 x\,d\,x$

u و مشتقهایش		dv و انتگرالهایش
x		$\sec^2 x$
۱	+	tgx
۰	−	$\ln\lvert Cosx\rvert$

$\int x\sec^2 x\,d\,x = xtgx - \ln\lvert\cos x\rvert + c$

۱۴) $\int x^{۵}\ln x\,dx$

تمرینات ۸-۴

1) $\frac{1}{(x-1)(x+2)}=\frac{A}{x-1}+\frac{B}{x+2}$

$A=\frac{1}{3} \qquad B=-\frac{1}{3}$

2) $\frac{x+1}{x^2+2x}=\frac{A}{x}+\frac{B}{x+2}$

$A=\frac{1}{2} \qquad B=\frac{1}{2}$

3) $\frac{x^2+3x-4}{(2x-1)^2(2x+3)}=\frac{A}{2x-1}+\frac{B}{(2x-1)^2}+\frac{C}{(2x+3)}$

$=\frac{\frac{41}{64}}{2x-1}+\frac{\frac{-9}{16}}{(2x-1)^2}+\frac{\frac{-25}{64}}{(2x+3)}$

4) $\frac{1}{x^4-x^3}=\frac{A}{x}+\frac{B}{x^2}+\frac{C}{x^3}+\frac{D}{x-1}$

$D=1 \quad A=-1 \quad B=-1 \quad C=-1$

5) $\frac{x^2+1}{x^2-1}=1+\frac{A}{x-1}+\frac{B}{x+1}$

$=1+\frac{1}{x-1}-\frac{1}{x+1}$

6) $\int\frac{x^2}{x+1}dx=\int(x-1+\frac{1}{x+1})dx=$

$\frac{1}{2}x^2-x+\ln|x+1|+c$

$\int(\frac{x^2}{x+1})dx \quad \begin{cases} x+1=u \Rightarrow x=u+1 \\ dx=du \end{cases}$

$\int(u+1)^2\times u^{-1}du \Rightarrow \int(u^2+2u+1)u^{-1}du \Rightarrow$

$\int(u+2+^{-1})du \Rightarrow \int udu+2\int du+\int\frac{du}{u} \Rightarrow$

$\frac{(x+1)^2}{2}+2(x+1)+\ln(x+1)$

7) $\int_2^4\frac{4x-1}{(x-1)(x+2)}dx$

$=\int_2^4(\frac{A}{x-1}+\frac{B}{x+2})dx$

☐ CheckBox1

$=\int_2^4\frac{1}{x-1}+\frac{3}{x+2}dx$

$=(\ln|x-1|+3\ln|x+2|)]_2^4$

$=\ln 3+3\ln 6-6\ln 2=2/3$

8) $\int\frac{6x-5}{2x+3}dx=\int\frac{6x+9-14}{2x+3}dx$

$=\int\frac{3(2x+3)-14}{2x+3}dx=3\int dx-14\int\frac{dx}{2x+3}=$

$3x-7\ln(2x+3)+c$

9) $\int\frac{x^2+1}{x^2-x}dx=\int(1+\frac{x+1}{x^2-x})dx=$

$\int 1-\frac{1}{x}+\frac{2}{x-1}=x-\ln|x|+$

$2\ln|x-1|+c$

10) $\int_0^1\frac{2x+3}{(x+1)^2}dx=\int_0^1(\frac{2}{x+1}+\frac{1}{(x+1)^2})dx$

$=(2\ln|x+1|-\frac{1}{x+1})\Big]_0^1=2\ln 2+\frac{1}{2}=1/89$

11) $\int_1^2\frac{1}{x(x+1)(2x+3)}dx=$

$\int_1^2(\frac{1}{3x}-\frac{1}{x+1}+\frac{4}{3(2x+3)})dx=$

$\frac{1}{3}\ln|x|-\ln|x+1|+\frac{4}{3}\ln|2x+3|\Big]_0^1$

$=-\ln 2+\frac{4}{3}\ln 5+\frac{1}{3}\ln 3$

حل کامل

$\int_1^2\frac{1}{x(x+1)(x+3)}dx \Rightarrow \int_1^2(\frac{a}{x}+\frac{b}{x+1}+\frac{c}{2x+3})dx \Rightarrow$

$\int_1^2(\frac{a(x+1)(2x+3)+b(x)(2x+3)+c(x)(x+1)}{x(x+1)(2x+3)}\Big]dx \Rightarrow$

$\int_1^2\Big[x^2\frac{(2a+2b+c)+x(5a+3b+c)+3a}{x(x+1)(2x+3)}\Big]dx$

$$\begin{cases}2a+2b+c=0\\5a+3b+c=0\\3a=1\Rightarrow a=\frac{1}{3}\ ,\ b=1\ ,\ c=-\frac{8}{3}\end{cases}$$

$$\int_1^2\frac{1}{x(x+1)(2x+3)}dx=\int_1^2\left(\frac{\frac{1}{3}}{x}+\frac{1}{x+1}+\frac{-\frac{8}{3}}{2x+3}\right)dx\Rightarrow$$

$$\int_1^2\frac{1}{3}\left(\frac{1}{x}+\frac{3}{x+1}-\frac{8}{2x+3}\right)dx$$

$$=\frac{1}{3}\left[\int_1^2\frac{dx}{x}+3\int_1^2\frac{dx}{(x+1)}-8\int_1^2\frac{dx}{(2x+3)}\right]$$

$$=\frac{1}{3}\left[\ln x+3\ln(x+1)-8\ln(2x+3)\right]_1^2$$

$$=-\ln 2+\frac{4}{3}\ln 5+\frac{1}{3}\ln 3$$

12) $\displaystyle\int_2^3\frac{6x^2+5x-3}{x^3+2x^2-3x}dx$

$$=\int\frac{A}{x}+\frac{B}{x+3}+\frac{C}{x-1}dx$$

$$=\ln|x|+3\ln|x+3|+2\ln|x-1|+c$$

13) $\displaystyle\int\frac{1}{(x-1)^2(x+4)}dx$

$$=\int\left(\frac{A}{x-1}+\frac{B}{(x-1)^2}+\frac{C}{x+4}\right)dx$$

$$=\int\left(\frac{-1/25}{x-1}+\frac{1/5}{(x-1)^2}+\frac{1/25}{x+4}\right)dx$$

$$=\frac{-1}{25}\ln|x-1|-\frac{1}{5(x-1)}+\frac{1}{25}\ln|x+4|+c$$

14) $\displaystyle\int\frac{5x^2+3x-2}{x^3+2x^2}dx=\int\left(\frac{2}{x}-\frac{1}{x^2}+\frac{3}{x+2}\right)dx$

$$=2\ln|x|+\frac{1}{x}+3\ln|x+2|+c$$

15) $\displaystyle\int\frac{x^2+2x}{x^3+3x^2+4}dx=\frac{1}{3}\int\frac{3x^2+6x}{x^3+3x^2+4}dx$

$$=\frac{1}{3}\ln|x^3+3x^2+4|+c$$

16) $\displaystyle\int\frac{x^2}{(x+1)^3}dx=\int\left(\frac{1}{x+1}-\frac{2}{(x+1)^2}+\frac{1}{(x+1)^3}\right)dx$

$$=\ln|x+1|+\frac{2}{x+1}-\frac{1}{2(x+1)^2}+c$$

راه حل دوم:

$x+1=t$

$$\Rightarrow\int\frac{(t-1)^2}{t^3}dt=\ln t+\frac{2}{t}-\frac{2}{t^2}$$

17) $\displaystyle\int\frac{dx}{x^4-x^2}=\int\left(\frac{A}{x}+\frac{B}{x^2}+\frac{C}{x-1}+\frac{D}{x+1}\right)dx$

$$\Rightarrow\frac{A}{x}+\frac{B}{x^2}+\frac{C}{x-1}+\frac{D}{x+1}$$

$$=\int\left(\frac{-1}{x^2}+\frac{\frac{1}{2}}{x-1}-\frac{\frac{1}{2}}{x+1}\right)dx$$

$$=\frac{1}{x}+\frac{1}{2}\ln|x-1|-\frac{1}{2}\ln(x+1)+c$$

$$=\int\left(\frac{A}{2x-1}+\frac{B}{2x-3}+\frac{c}{2x-5}\right)dx$$

18) $\displaystyle\int\frac{(2Sinx-3)Cosx}{Sin^2x-3Sinx+2}dx$

$Sin^2x-3Sinx+2=u$

$(2CosxSinx-Cosx)dx=du$

$(2Sinx-3)Cosx\,dx=du$

$$\int\frac{2Sinx-3}{Sin^2x-3Sinx+2}dx=\int\frac{du}{u}=\ln|u|$$

$$=\ln|Sin^2x-3Sinx+2|+c$$

۱۹) $\displaystyle\int\frac{8x-3}{4x+1}dx=\int\frac{8x}{4x+1}dx-3\int\frac{dx}{4x+1}$

$$=\left(۲x-\frac{۵}{۴}\ln|۴x+۱|\right)+c$$

روش دوم:

$$\int\frac{۸x-۳}{۴x+۱}dx=\int\frac{۸x+۲-۵}{۴x+۱}dx+\int\frac{-۵}{۴x+۱}dx$$

$$=\int ۲-\frac{۵}{۴}\int\frac{۴}{۴x+۱}dx=۲x-\frac{۵}{۴}\ln|۴x+۱|+c$$

۲۰)

$$\frac{dx}{(x+2)(3x+4)}=\int\left(\frac{\frac{-1}{2}}{x+2}+\frac{\frac{3}{2}}{3x+4}\right)dx$$

$$=-\frac{1}{2}\ln|x+2|+\frac{1}{2}\ln|3x+4|+c$$

۲۱) $\int\frac{x}{x^2-x-6}dx=\int\left(\frac{A}{x-3}+\frac{B}{x+2}\right)dx$

$$\int\left(\frac{\frac{3}{5}}{x-3}+\frac{\frac{2}{5}}{x+2}\right)dx$$

$$=\frac{3}{5}\ln|x-3|+\frac{2}{5}\ln|x+2|$$

۲۲) $\int\left(\frac{x-1}{x+2}\right)^2dx=\int\left(1-\frac{3}{x+2}\right)^2dx$

$$=\int\left(1+\frac{9}{(x+2)^2}-\frac{6}{x+2}\right)dx$$

$u=x+2 \quad\Rightarrow\quad du=dx$

$$=x+9\int u^{-2}-6\ln|x+2|=x-\frac{9}{(x+2)}-6\ln|x+2|$$

۲۳) $\int\frac{32x}{(2x-1)(2x-3)(2x-5)}dx$

$A=2 \qquad B=-12 \qquad c=10$

$$=\int\left(\frac{2}{2x-1}-\frac{12}{2x-3}+\frac{10}{2x-5}\right)dx$$

$$=\ln|2x-1|-6\ln|2x-3|+5\ln|2x-5|+c$$

$\Rightarrow A=1-(-3)=4 \quad\rightarrow A=4$

$C=-4$

$$\Rightarrow\int_1^2\frac{x-3}{x^3+x^2}=\int_1^2\frac{4}{x}+\frac{-3}{x^2}+\frac{-4}{x+1}$$

$$=\left[4\ln x+\frac{3}{x}-4\ln|x+1|\right]_1^2$$

$$=\left(4\ln 2+\frac{3}{2}-4\ln 3\right)-(4\ln 1+3-4\ln 2)$$

$$=4\ln 2+\frac{3}{2}-4\ln 3-3+4\ln 2$$

$$=8\ln 2-4\ln 3-\frac{3}{2}$$

۲۴) $\int_1^2\frac{x-3}{x^3+x^2}dx=\int_1^2\frac{x-3}{x^2(x+1)}dx$

$$\Rightarrow\frac{x-3}{x^2(x+1)}=\frac{A}{x}+\frac{B}{x^2}+\frac{C}{x+1}$$

$A=4 \qquad B=-3 \qquad C=-4$

$x-3=Ax(x+1)+B(x+1)+Cx^2$

$x-3=Ax^2+Ax+Bx+B+Cx^2$

۲۵) $\int_1^3\frac{x^2-4x+3}{x(x+1)^2}dx=\int_1^3\frac{A}{x}+\frac{B}{x+1}+\frac{C}{(x+1)^2}$

$x^2-4x+3=A(x+1)^2+Bx(x+1)+Cx$

$$\begin{cases}A+B=1 & \rightarrow \quad A=-C\\ A+C=0 & \rightarrow \quad A=1-B\\ B=-3\end{cases}$$

$x=0\Rightarrow A=3 \quad x^2-4x+C$

$=Cx^2+C+6x+Bx^2+Bx+8x$

$x=-1\Rightarrow C=8 \;\Rightarrow B+3=1 \quad B=-2$

$$\int_1^3\frac{3}{x}-\frac{2}{x+1}+\frac{8}{(x+1)^2}$$

$$=3\ln|x|-2\ln|x+1|-\frac{8}{(x+1)}\Bigg]_1^3$$

$$=(3\ln 3-2\ln 4-2)-(-2\ln 2-4)$$

$$=3\ln 3-2\ln 2+2$$

روش دیگر

$$\int_1^3\frac{x^2-4x+3}{x(x+1)}dx$$

$$\frac{x^2-4x+3}{x(x+1)^2}=\frac{A}{x}+\frac{B}{x+1}+\frac{C}{(x+1)^2}=$$

$\Rightarrow x^2-4x+3=A(x+1)^2+Bx(x+1)+Cx$

$\Rightarrow x^2-4x+3=Ax^2+2Ax+A+Bx^2+Bx+Cx$

$$\begin{cases}A+B=1 & \Rightarrow A=1-B\overset{A=3}{\Rightarrow}B=-2\\ 2A+B+C=-4 & \Rightarrow 4+C=-4\Rightarrow C=-81\\ A=3\end{cases}$$

$$\Rightarrow\int_1^3\frac{x^2-4x+3}{x(x+1)^2}dx=\int_1^3\left(\frac{3}{x}+\frac{-2}{x+1}+0\right)dx$$

$$=\left[3\ln|x|-2\ln|x+1|\ \right]_1^3$$
$$=(3\ln 3-2\ln 4)-\left(3\frac{\ln 1}{0}-2\ln 2\right)$$
$$=3\ln 3-2\ln 4+2\ln 2=3\ln 3-2\ln 2$$

۲۶) $\int_1^2 \frac{5x^2-3x+18}{9x-x^3}dx$

$$\int_1^2 \frac{5x^2-3x+18}{x(9-x^2)}=\int_1^2 \frac{5x^2-3x+18}{x(3-x)(3+x)}$$
$$=\int_1^2\left(\frac{A}{x}+\frac{B}{3-x}+\frac{C}{3+x}\right)dx$$
$$A=2 \quad B=6 \quad C=-4$$
$$\int_1^2\left(\frac{2}{x}+\frac{6}{3-x}-\frac{4}{3+x}\right)dx$$
$$=\left[2\ln|x|+6\ln|3-x|-4\ln|3+x|\right]_1^2$$
$$=(2\ln 2-4\ln 5)-(6\ln 2-4\ln 4)$$
$$=32\ln 2-24\ln 5$$

۲۷)

$$\int_0^5\left(\frac{(x^2-3)}{x^3+4x^2+5x+2}\right)dx=\int \frac{(x^2-3)}{(x+2)(x+1)^2}dx$$

$$=\int\left(\frac{A}{x+2}+\frac{B}{(x+1)}+\frac{C}{(x+1)^2}\right)dx$$
$$(x^2-3)=A(x+1)^2+B(x^2+3x+2)+C(x+2)$$
$$\begin{cases} A+B=1 & A=1 \\ 2A+3B+C=0 & B=0 \\ A+2B+2C=-3 & C=-2 \end{cases}$$
$$\int_0^5\left(\frac{1}{x+2}\right)dx-2\int_0^5\frac{dx}{(x+1)^2}=$$
$$=\ln(x+2)\Big]_0^5+\frac{2}{(x+1)}\Bigg]_0^5$$
$$=\ln\left(\frac{7}{2}\right)+\frac{7}{3}$$