Chapter 6: Current and Resistance

- ✓ **Electric Current**
- ✓ **Current Density and Drift Speed**
- ✓ **Resistance and Resistor**
- ✓ **Electrical Power**

Session 13:

- ✓ **Electric Current**
- ✓ **Current Density and Drift Speed**
- ✓ **Resistance and Resistor**
- ✓ **Electrical Power**
- ✓ **Examples**

Introduction

- **Most practical applications of electricity deal with electric currents.**
  - The **electric charges move** through some region of space.
- **The resistor is a new element added to circuits.**
- **Energy can be transferred to a device in an electric circuit.**

Electric Current

- **Electric current** is the rate of flow of charge through some region of space.
- The SI unit of current is the **ampere** (A).
- 1 A = 1 C / s
- The symbol for electric current is **I**.

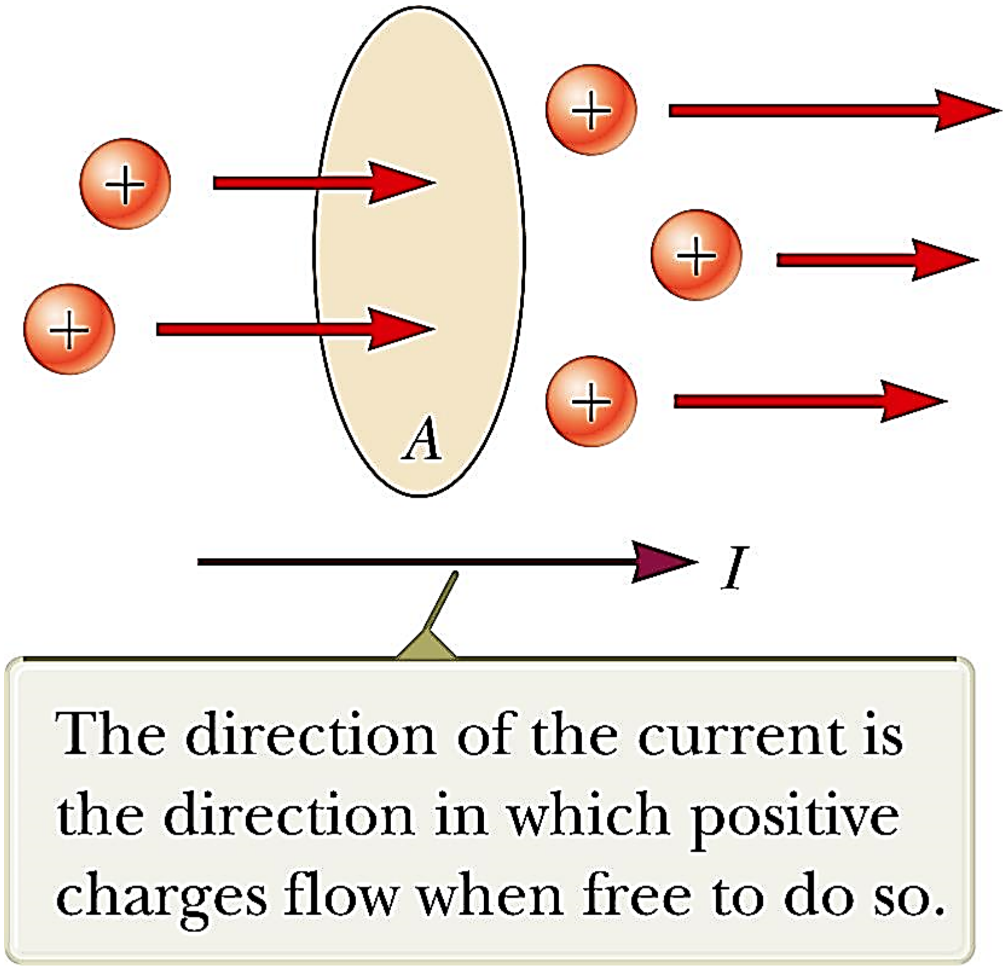
Andre-Marie Ampere

Average Electric Current:

$$\boxed{I_{avg} = \frac{\Delta Q}{\Delta t}}$$

Instantaneous Electric Current:

$$\boxed{I = \frac{dQ}{dt}}$$



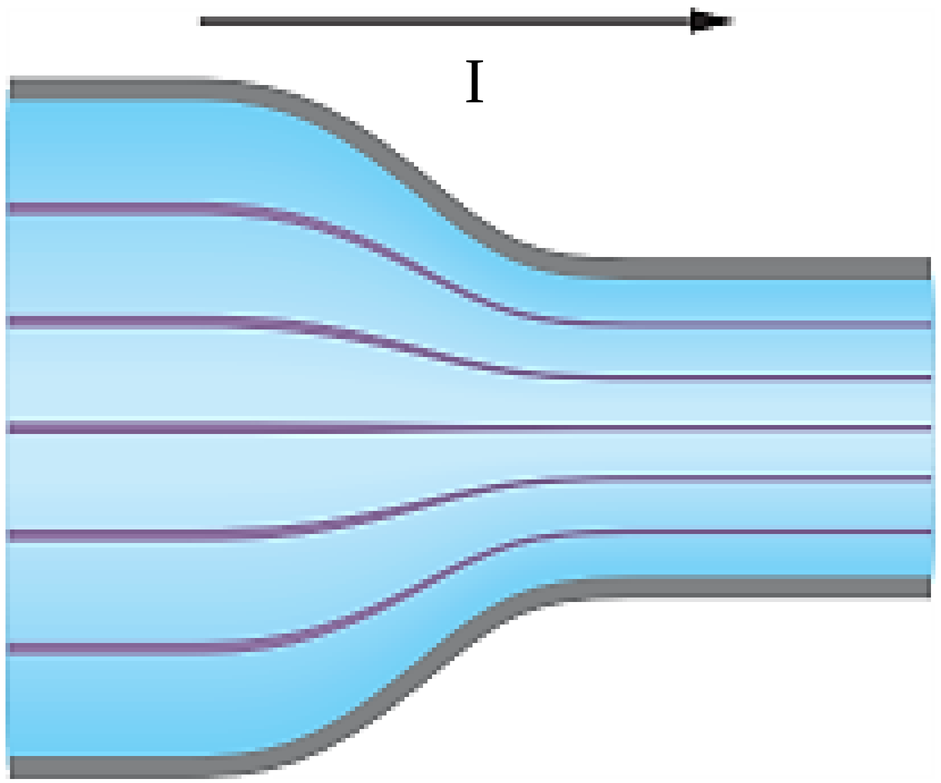
Electric Current Density

$$I = \int \vec{J} \cdot d\vec{A}$$

$$if \;\; \vec{J} = constant \;, \;\; \vec{J} \parallel d\vec{A}$$

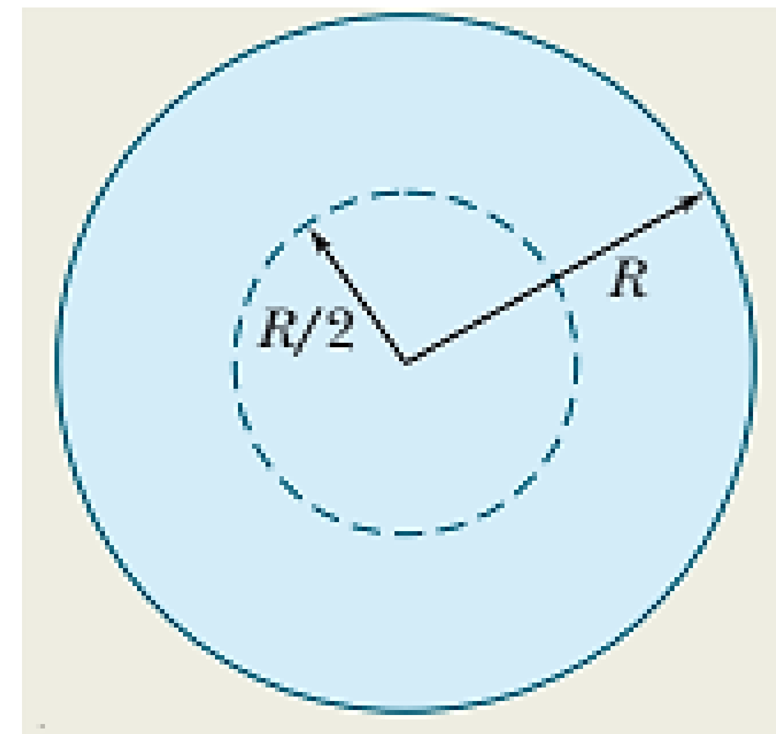
$$\Rightarrow \qquad I = \int J \; dA = J \int dA = J \; A$$

$$\Rightarrow \qquad J = \frac{I}{A}$$



- *J* is the **current density** of a conductor.
- It is defined as the **current per unit area**.
- *J* has SI units of **A/m²**
- The current density is **in the direction of the positive charge** carriers.

Ex 1. The **current density in a cylindrical wire** of radius **R = 2 mm** is **uniform across a cross section** of the wire and is **J = 2 × 10 5 A/m²**. What is the current through the outer portion of the wire between radial distances **R/2** and **R**? (b) Suppose, instead, that the current density through a cross section **varies with radial distance r** as **J = ar²**, in which **a = 3 × 10¹¹ A/m⁴** and **r** is in meters. What now is the current through the same outer portion of the wire?

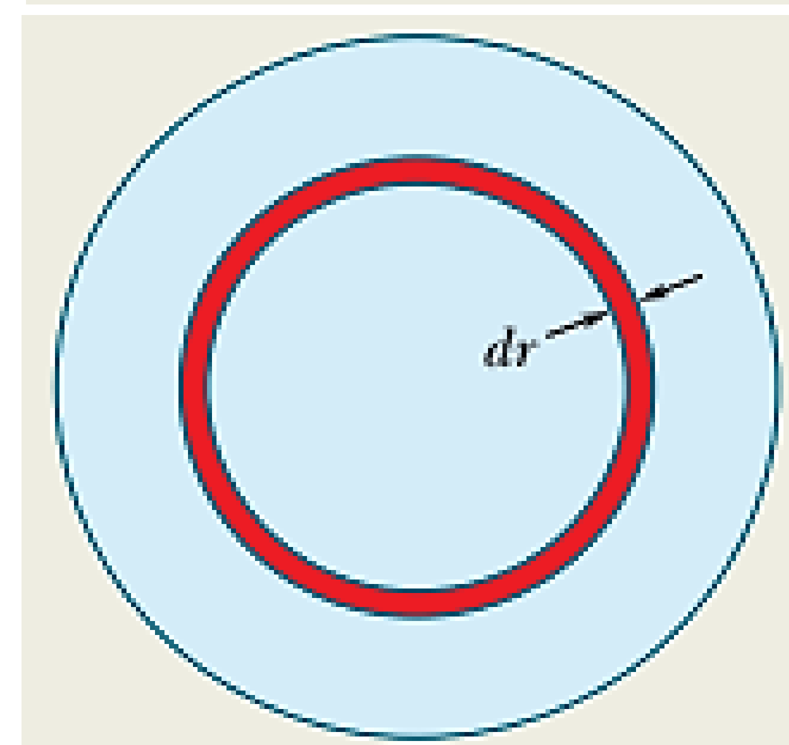


$$I = \int \vec{J}.d\vec{A} \quad \Rightarrow \quad I = JA'$$

$$A' = \pi R^2 - \pi\left(\frac{R}{2}\right)^2 = \frac{3}{4}\pi R^2$$

$$A' = \frac{3}{4}\pi(2\times 10^{-3})^2 = 9.424\times 10^{-6} \quad m^2$$

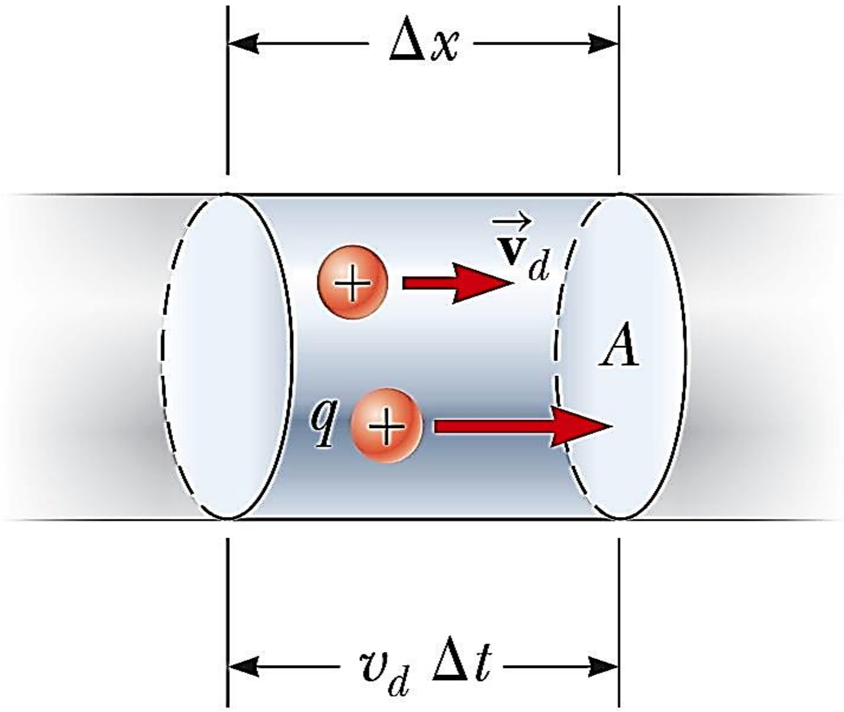
$$\boxed{I = (2\times 10^5)(9.424\times 10^{-6}) = 1.9 \ A}$$

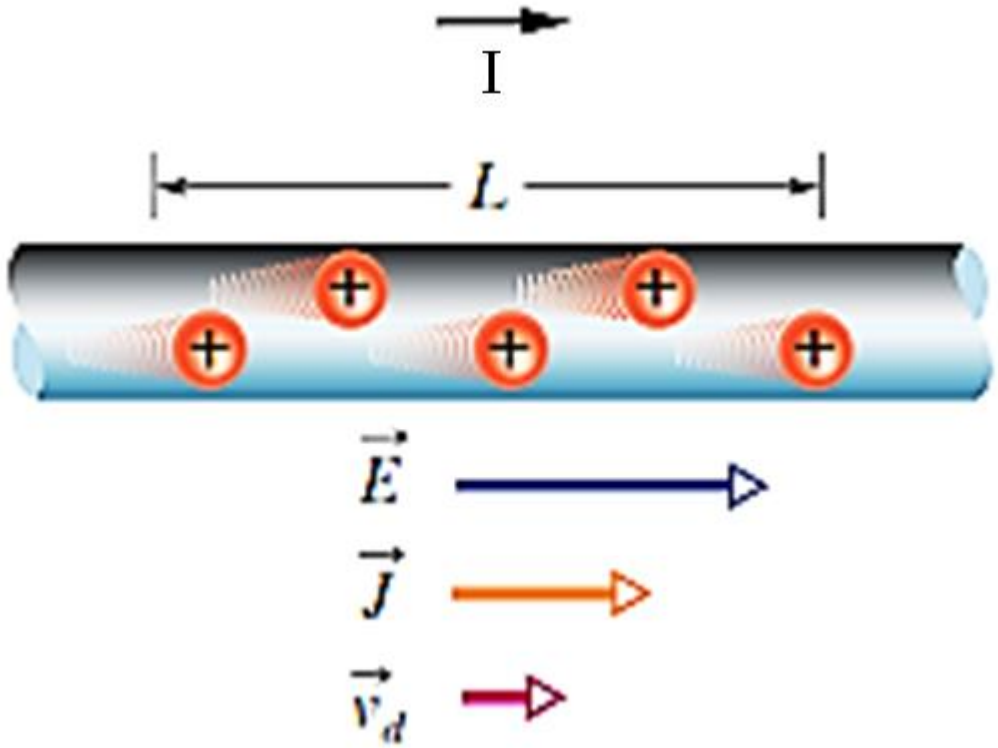


$$I = \int \vec{J}.d\vec{A} = \int_{\frac{R}{2}}^{R} ar^2(2\pi r \ dr) = 2\pi a\int_{\frac{R}{2}}^{R} r^3 \, dr = 2\pi a \left.\frac{r^4}{4}\right|_{\frac{R}{2}}^{R}$$

$$\boxed{I = \frac{15}{32}\pi a R^4 = 7.1 \ A}$$

Drift Speed





- Charged particles move through **a cylindrical conductor of cross-sectional area *A***.
- ***n*** is the number of **mobile charge carriers per unit volume**.
- ***nAΔx*** is the **total number of charge** carriers in a segment.

$$\Delta Q = (n\,A\,\Delta x)\,q \qquad V_d = \frac{\Delta x}{\Delta t} \Rightarrow \Delta x = V_d\,\Delta t \qquad \Delta Q = (n\,A\,V_d\,\Delta t)\,q$$

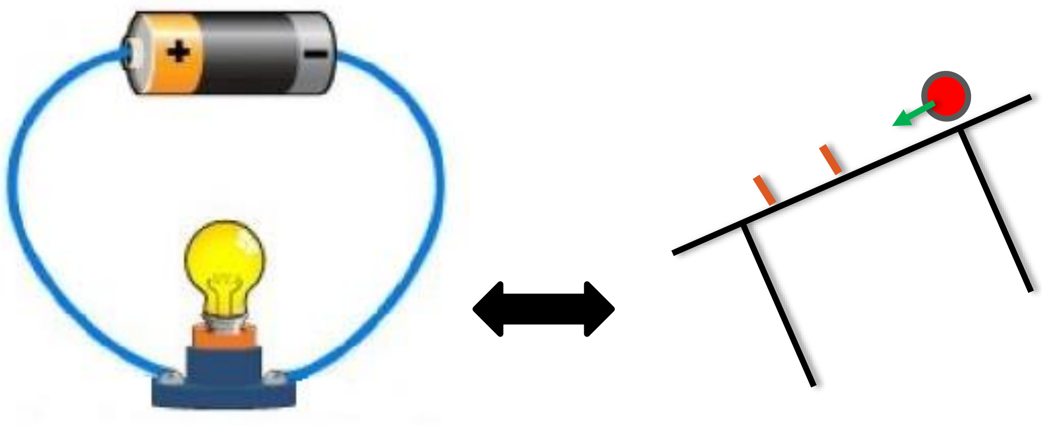
$$I_{avg} = \frac{\Delta Q}{\Delta t} = n\,A\,V_d\,q \qquad \boxed{\vec{J} = n\,q\,\vec{V}_d}$$

v_d is an average speed called the **drift speed**.

- The drift velocity for a 12-gauge **copper wire carrying a current of 10 A** is:

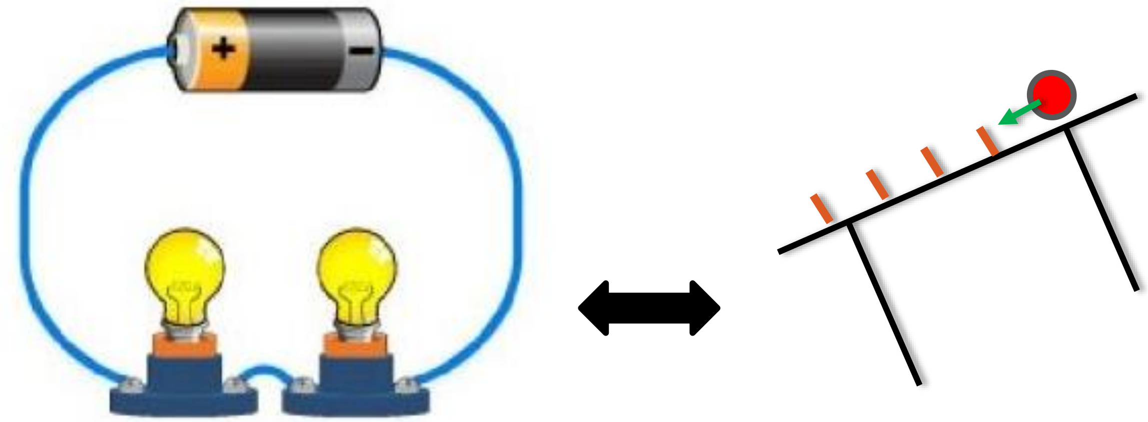
$$v_d = 2.23 \times 10^{-4}\ \text{m/s}$$

Resistance and Resistor



(1)

$$I_1 > I_2$$

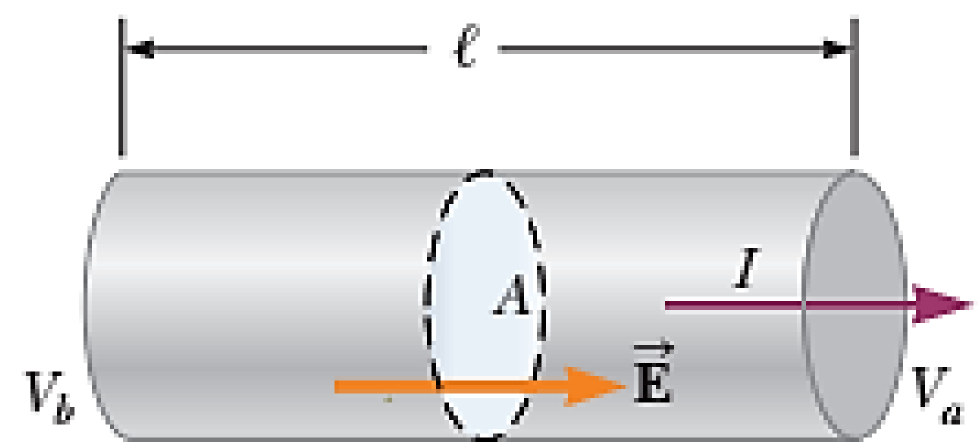


(2)

Georg Simon Ohm

$$J = \frac{I}{A}$$

$$J = \sigma E$$ **(σ** is **conductivity)**



$$(V_b - V_a) = V = E\,l \;\Rightarrow\; J = \sigma \frac{V}{l}$$

$$V = \frac{l}{\sigma} J = \frac{l}{\sigma}\frac{I}{A} = \left(\frac{l}{\sigma A}\right) I \;\Rightarrow\; \boxed{V = RI} \qquad \boxed{R = \frac{l}{\sigma A}}$$

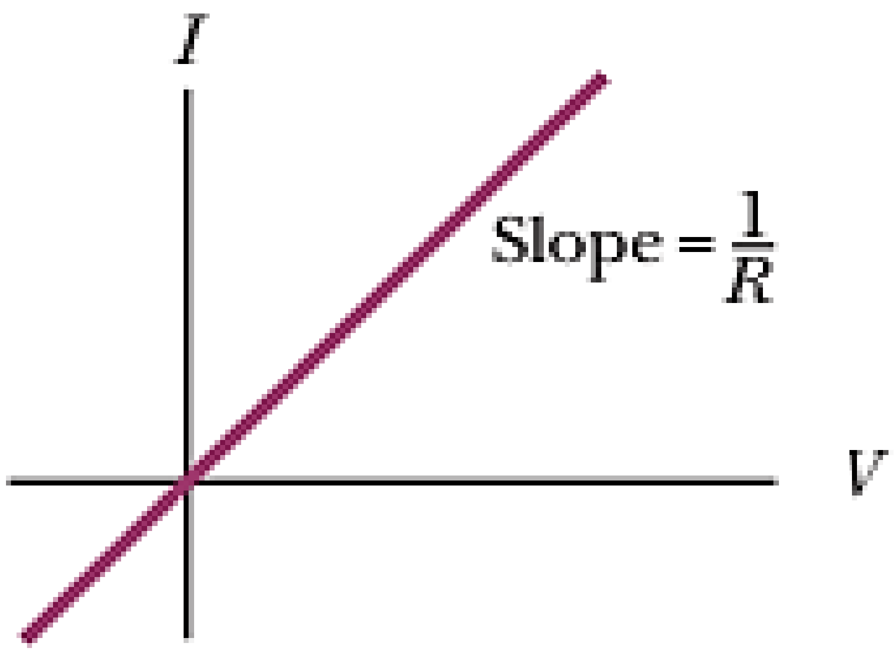
Ohm's Law: For many materials (including most metals), the **ratio of the current density to the electric field is a constant σ** that is independent of the electric field producing the current.

Resistance and Resistor

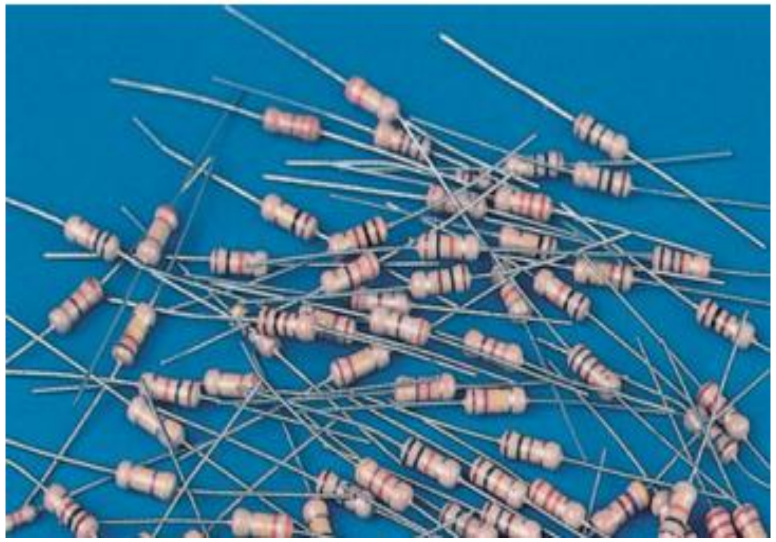
definition of Resistance:

$$R = \frac{V}{I}$$

1 ohm = 1 Ω = 1 volt per ampere = 1 V/A



❖ Most electric circuits use circuit elements called **resistors** to **control the current** in the various parts of the circuit.



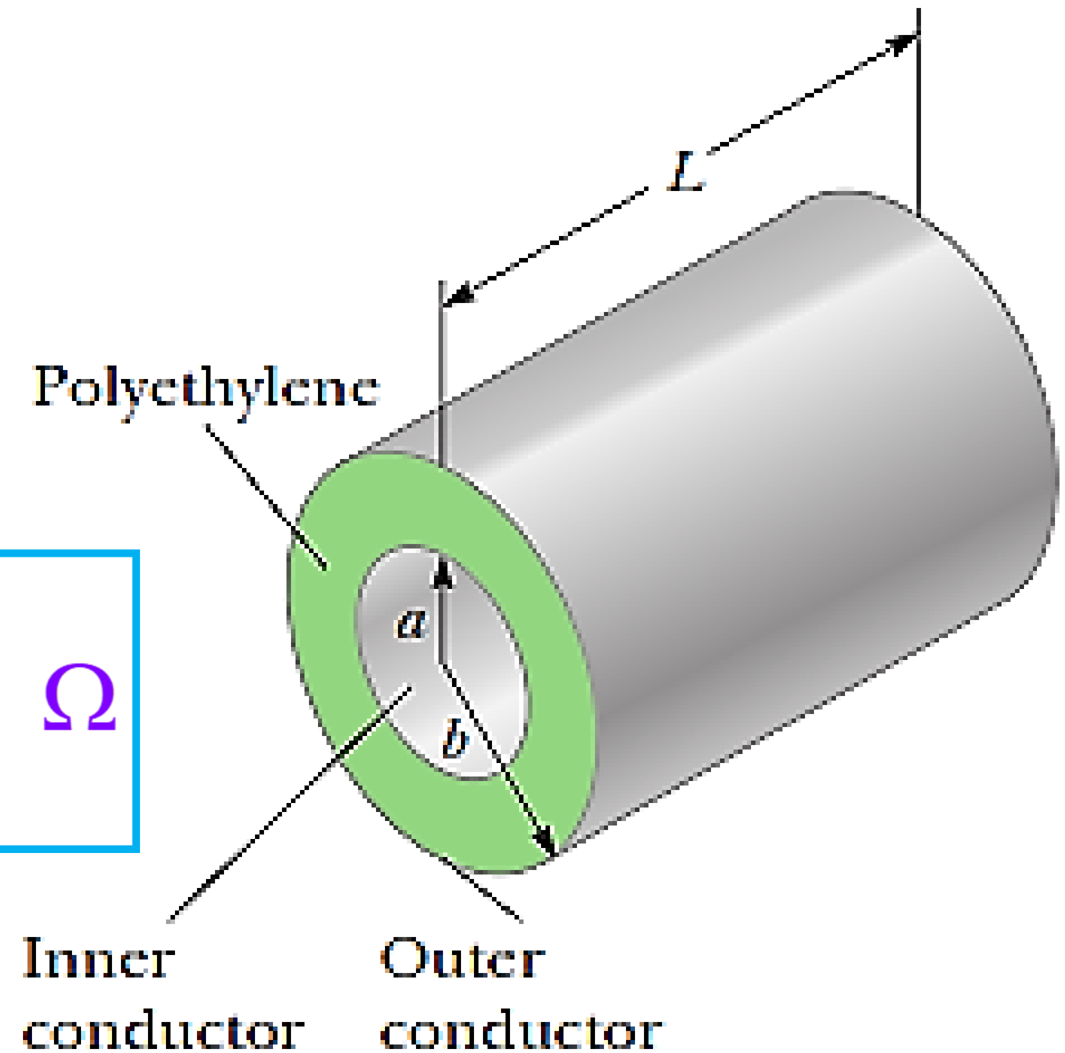
Resistivity: $\rho = \frac{1}{\sigma} \quad (\Omega.m)$ ➡ $\boxed{R = \frac{l}{\sigma A} = \rho \frac{l}{A}}$

Resistance is a property of an object. Resistivity is a property of a material.

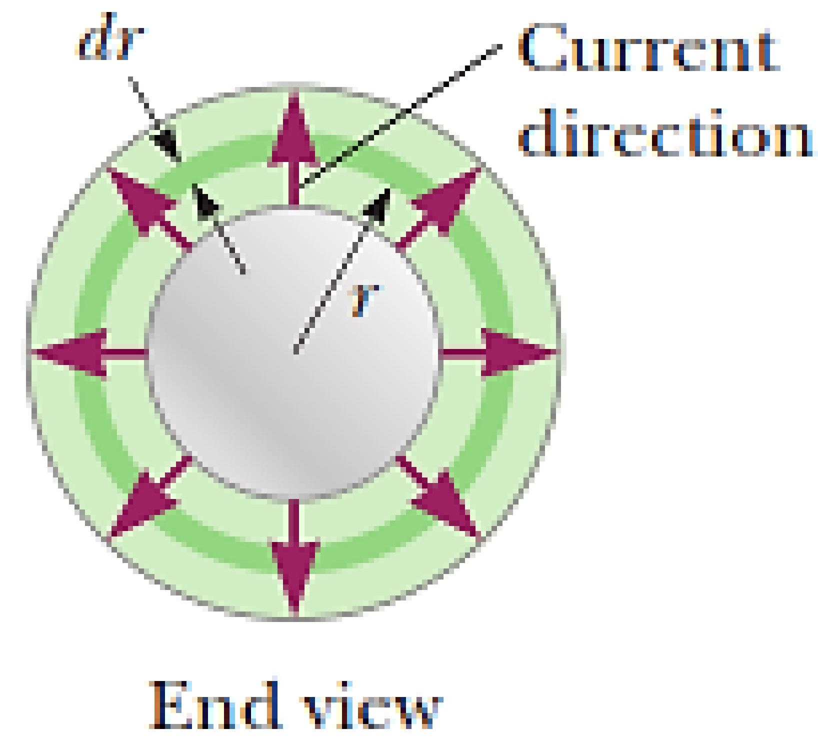
Ex 2. **Coaxial cables** are used extensively for cable television and other electronic applications. A coaxial cable consists of **two concentric cylindrical conductors (copper)**. The region between the conductors is completely **filled with polyethylene plastic** as shown in Figure. Current leakage through the plastic, in the radial direction, is unwanted. The cable is designed to **conduct current along its length**, with **$\rho_{cu} = 1.7 \times 10^{-8}\ \Omega$ m.** The radius of the inner conductor is **a = 0.5 cm**, the radius of the outer conductor is **b = 1.75 cm**, and the length is **L = 15.0 cm**. The resistivity of the plastic is **$\rho_p = 1.0 \times 10^{13}\ \Omega$ m**. Calculate the resistance of the **(a) inner copper conductor** and **(b) plastic between the two conductors**.

$$R_{cu} = \frac{L}{\sigma_{cu} A} = \rho_{cu} \frac{L}{A}$$

$$\boxed{R_{cu} = (1.7\times10^{-8})\frac{(15\times10^{-2})}{\pi(5\times10^{-3})^2} = 3.2\times10^{-5}\ \Omega}$$



Current leakage through the plastic:



$$dR_p = \rho_p \frac{dr}{A} = \rho_p \frac{dr}{2\pi r L}$$

$$R_p = \int dR_p = \int_a^b \rho_p \frac{dr}{2\pi r L} = \frac{\rho_p}{2\pi L} \int_a^b \frac{dr}{r} = \frac{\rho_p}{2\pi L} \ln(\frac{b}{a})$$

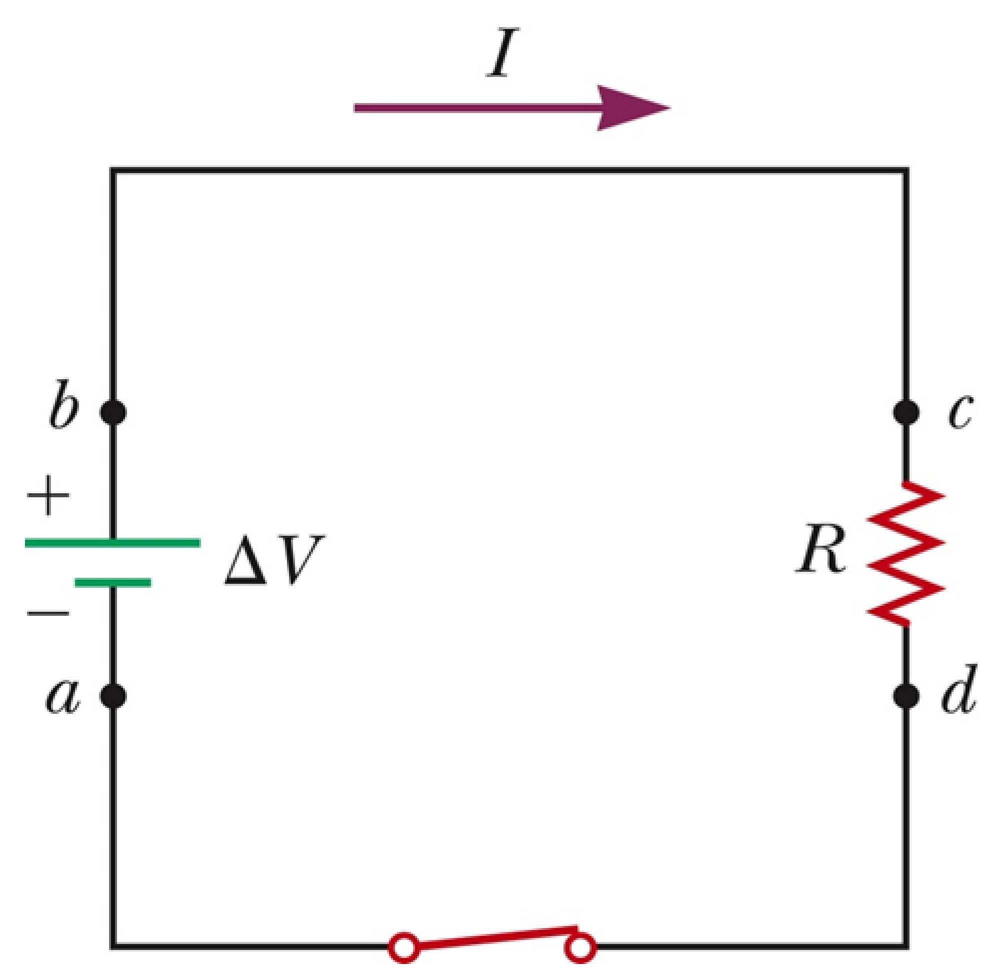
$$\boxed{R_p = \frac{1 \times 10^{13}}{2\pi(15 \times 10^{-2})} \ln(\frac{1.75}{0.5}) = 1.33 \times 10^{13}\ \Omega}$$

Electrical Power

- The entire **circuit is the system**.
- As a charge moves from *a* to *b*, the **electric potential energy** of the system increases by:

$$U = Q\,\Delta V$$

- This **electric potential energy is transformed into internal energy in the resistor** (increased vibrational motion of the atoms in the resistor).
- The **power** is the rate at which the energy is delivered to the resistor.



$$P = \frac{dU}{dt} = \frac{d}{dt}(Q\,\Delta V) = \frac{dQ}{dt}\Delta V = I\,\Delta V$$

- Units: I is in A, R is in Ω, ΔV is in V, and P is in W

$$\boxed{P = I\,\Delta V = I^2 R = \frac{(\Delta V)^2}{R}}$$