Subject:
Year. Month. Date. ( )

٨٧٤٢٣٢١٣ لعیا رضایی حل تمرین ۳

$$E(x) = \sum_{1}^{4} 2^{x}\left(\frac{1}{4}\right) = 2\left(\frac{1}{4}\right) + 8\left(\frac{1}{4}\right) + 16\left(\frac{1}{4}\right) = \frac{1}{2} + 1 + 2 + 4 = \frac{15}{2} \quad -1$$

$$\sum_{1}^{4} \frac{1}{2^x}\left(\frac{1}{4}\right) = \frac{1}{2}\left(\frac{1}{4}\right) + \frac{1}{4}\left(\frac{1}{4}\right) + \frac{1}{8}\left(\frac{1}{4}\right) + \frac{1}{16}\left(\frac{1}{4}\right) = \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \frac{1}{64} =$$

$$\frac{8+4+2}{64} = \frac{15}{64}$$

$$E(2^{2x}) = \sum_{1}^{4} 2^{2^x}\left(\frac{1}{4}\right) = 4\left(\frac{1}{4}\right) + 16\left(\frac{1}{4}\right) + 256\left(\frac{1}{4}\right) + 2^{16}\left(\frac{1}{4}\right)$$

$$E(x^2) = \sum_{1}^{4} x^2\left(\frac{1}{4}\right) = 4\left(\frac{1}{4}\right) + 16\left(\frac{1}{4}\right) + 64\left(\frac{1}{4}\right) + 256\left(\frac{1}{4}\right)$$

$$= 1 + 4 + 16 + 64 = 85$$

$$Var(x) = E(x^2) - [E(x)]^2 = 85 - \left(\frac{15}{2}\right)^2 = 28/75$$

$$E(x) = \int_0^2 x\left(\frac{1}{6}(2x)\right)dx = \frac{1}{6}\int_0^2 2x - x^2\, dx = \frac{1}{6}\left(x^2 - \frac{x^3}{3}\right)\Big|_0^2 \quad -2$$

$$= \frac{1}{6}\left(4 - \frac{8}{3}\right) = \frac{2}{3} - \frac{4}{9} = \frac{6-4}{9} = \frac{2}{9}$$

$$E(x^2) = \int_0^2 x^2\left(\frac{1}{6}(2-x)\right)dx = \frac{1}{6}\int_0^2 2x^2 - x^3\, dx = \frac{1}{6}\left(\frac{2x^3}{3} - \frac{x^4}{4}\right)\Big|_0^2$$

$$= \frac{1}{6}\left(\frac{16}{3} - \frac{16}{4}\right) = \frac{1}{6}\left(\frac{64-48}{12}\right) = \frac{1}{6} \times \frac{16}{12} = \frac{16}{72}$$

$$Var(x) = E(x^2) - [E(x)]^2 = \frac{2}{9} - \left(\frac{2}{9}\right)^2 = \frac{2}{9} - \frac{4}{81} = \frac{18-4}{81} = \frac{14}{81}$$

$$\int_{-\infty}^{+\infty} 2xk\, dx = 2k\int_1^4 x\, dx = 2k\left(\frac{x^2}{2}\right)\Big|_1^4 = 2k\left[\frac{16}{2} - \frac{1}{2}\right] = \quad -3$$

$$2k\left[\frac{15}{2}\right] = 15k = 1 \qquad k = \frac{1}{15}$$

$$Var(x) = E(x^2) - (E(x))^2 = \frac{21}{10} - \left(\frac{42}{15}\right)^2$$

$$E(x) = \int_1^4 x f(x) = \int_1^4 x\left(\frac{2}{15}x\right)dx = \frac{2}{15}\int_1^4 x^2\, dx = \frac{2}{15}\left(\frac{64}{3} - \frac{1}{3}\right) =$$

$$\frac{2}{15}\left(\frac{63}{3}\right) = \frac{42}{15}$$

$$E(x^2) = \frac{2}{15}\int_1^4 x^3\, dx = \frac{2}{15}\left[\frac{x^4}{4}\right]_1^4 = \frac{2}{15}\left[64 - \frac{1}{4}\right] = \frac{21}{10}$$

$\int_0^1 (ax+b)\,dx = \left[\frac{ax^2}{2}+bx\right]_0^1 = \frac{a}{2}+b=1 \qquad a+2b=2$ — ۵

$\mu = E(x) = \int_0^1 x(ax+b)\,dx = \int_0^1 (ax^2+bx)\,dx = \left[\frac{ax^3}{3}+\frac{bx^2}{2}\right]_0^1 =$

$= \frac{a}{3}+\frac{b}{2} = \frac{1}{2} \qquad 2a+3b=3$

$\begin{cases} a+2b=2 \\ 2a+3b=3 \end{cases} \rightarrow \begin{cases} -2a-4b=-4 \\ 2a+3b=3 \end{cases} \qquad a+4=2 \qquad \boxed{a=-2} \qquad \boxed{b=2}$

۶ — الف)

$E(x)=\int_0^\infty x\left(\frac{1}{4}xe^{-x/2}\right)dx = \frac{1}{4}\int_0^\infty x^2e^{-x/2}\,dx$

ب)

$\int_{-1}^1 c(1-x^2)\,dx = \int_{-1}^1 c - \int_{-1}^1 x^2 = 2\int_0^1 c - 2\int_0^1 x^2 =$

$2c[x]_0^1 - 2\left[\frac{x^3}{3}\right]_0^1 \rightarrow 2c - 2/3 \rightarrow 2c = 1+\frac{2}{3} \rightarrow 2c = \frac{5}{3} \qquad \boxed{c=\frac{10}{3}}$

$E(x)=\int_{-1}^1 x\,\frac{10}{3}(1-x^2)\,dx = \frac{10}{3}\int_{-1}^1 x-x^3\,dx$

ج)

$\int_5^{+\infty} x\,\frac{5}{x^2}\,dx = 5\int_5^\infty \frac{1}{x}\,dx = 5\times|\mathrm{Ln}\,x|_5^\infty$

۷ —

$E(x) = \sum x f_X(x) = (-3)(\frac{1}{6}) + (6)(\frac{1}{2}) + (9)(\frac{1}{3}) = -\frac{1}{2}+3+2 = \frac{9}{2}$

$E(x^2) = \sum x^2 f_X(x) = (-3)^2(\frac{1}{6}) + (6)^2(\frac{1}{2}) + (9)^2(\frac{1}{3}) = \frac{93}{2}$

$E[(2x+1)^2] = E[4x^2-2x+1] = 4E(x^2)-2E(x)+1 =$

$4(\frac{93}{2}) - 2(\frac{9}{2}) + 1 = 178$

۸ —

$E((x-2)^2)=6 \qquad E(x^2-4x+4)=6 \qquad E(x^2)-4E(x)+4=6$

$E(x^2)-4E(x)=2$

$E((x-1)^2)=10 \qquad E(x^2-2x+1)=10 \qquad E(x^2)-2E(x)+1=10$

$E(x^2)-2E(x)=9 \qquad E(x)=\frac{7}{2} \qquad E(x^2)=16$

$\mu = E(x) = \frac{7}{2} \qquad \sigma^2 = Var(x) = E(x^2)-(E(x))^2 = 16-(\frac{7}{2})^2 = 3{,}75$

۹ —

$E(x) = \sum x f_X(x) = (0)(\frac{64}{216}) + (1)(\frac{42}{216}) + (2)(\frac{16}{216}) + (3)(\frac{8}{216}) = \frac{88}{216}$

$E(x)= \int_0^1 x \frac{1}{2}\theta dx + \int_1^2 x\frac{1}{2} dx + \int_2^3 x \frac{1}{2}(1-\theta)dx$ —۱۰

$= \frac{1}{2}\theta \; \frac{x^2}{2}\Big|_0^1 + \frac{1}{2}\frac{x^2}{2}\Big|_1^2 + \frac{1}{2}(1-\theta)\; \frac{x^2}{2}\Big|_2^3 = \frac{1}{2}(\frac{1}{2}\theta + \frac{3}{2} + \frac{5}{2} - \frac{5}{2}\theta) = \frac{1}{2}(\frac{\theta+3+5-5\theta}{2}) = \frac{1}{2}(\frac{8-4\theta}{2})$

$E(x) = 0\times p + 1 \times 1-2p + 2\times p = 1-2p+2p = 1$ —۱۱

$E(x^2) = 0\times p + 1\times 1-2p + 4\times p = 1-2p+4p = 1+2p$

$Var(x) = E(x^2) - (E(x))^2 = 1+2p-1 = 2p$

به ازای ۱/۲ مقدار p ، $Var(x)$ مقدار max میشود

$E(x) = \int_0^1 x \cdot x^2 dx + \int_1^a x c\, dx = \frac{x^4}{4}\Big|_0^1 + c\frac{x^2}{2}\Big|_1^a = \frac{5}{4}$ —۱۲

$\frac{1}{4} + c(\frac{a^2}{2} - \frac{1}{2}) = \frac{5}{4}$ $\quad \frac{1}{4} + c\frac{a^2}{2} - \frac{c}{2} = \frac{5}{4}$ $\quad 1 + 2ca^2 - 2c = 5$

$2ca^2 - 2c = 4$ $\quad ca^2 - c = 2$

$\int_0^1 x^2 dx + c\int_1^a dx = 1$ $\quad \frac{x^3}{3}\Big|_0^1 + c\, x\Big|_1^a = 1$ $\quad \frac{1}{3} + ca = 1$

$ca = \frac{2}{3}$

$\int_{-1}^1 k(1-|x|)dx = k\left[\int_{-1}^1 1-|x| dx\right]$ —۱۳

$k\int_{-1}^1 dx - \int_{-1}^1 |x| dx = k\left[\int_{-1}^1 dx - \int_{-1}^0 x dx - \int_0^1 x dx\right]$

$k\left[2(x)\Big|_0^1 - 2\frac{x^2}{2}\Big|_0^1\right] = k(2(1) - 2(\frac{1}{2})) = 2-1=1 \quad k=1$

$P(-\frac{1}{2}<x<\frac{1}{2}) = \int_{-\frac{1}{2}}^{\frac{1}{2}} (1-|x|) = \int_{-\frac{1}{2}}^{\frac{1}{2}} dx - \int_{-\frac{1}{2}}^{\frac{1}{2}} |x| dx =$

$2(x)\Big|_0^{\frac{1}{2}} - 2\frac{x^2}{2}\Big|_0^{\frac{1}{2}} = 1 - \frac{1}{8}$

$E(x) = \int_{-1}^1 x(1-|x|)dx = 2\int_{-1}^1 x - x|x| dx = \int_{-1}^1 x - \int_{-1}^1 x(x) = 0$

x	۱۹	۲۰	۲۱
	$\frac{10}{45}$	$\frac{14}{45}$	$\frac{1}{45}$

$Var(x) = E(x^2) - (E(x))^2 = ۳۹/۷۱ - ۴۴/۴۵ =$ —۱۴

$-۱۷/۷۴$

Subject:
Year. Month. Date. / /

$E(x) = 19\left(\frac{10}{25}\right) + (20)\left(\frac{14}{25}\right) + (21)\left(\frac{1}{25}\right) = 19,35 + \frac{240}{25} + \frac{21}{25} = 4,447$

$E(x^2) = (19)^2\left(\frac{10}{25}\right) + (20)^2\left(\frac{14}{25}\right) + (21)^2\left(\frac{1}{25}\right) = 386,71$

۱۷- الف)

y \ x	0	1	2	3	$f_Y(y)$
0	$\frac{1}{8}$	$\frac{1}{8}$	0	0	$\frac{2}{8}$
1	0	$\frac{2}{8}$	$\frac{2}{8}$	0	$\frac{4}{8}$
2	0	0	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{2}{8}$
$f_X(x)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$	

$\rho(x,y) = \frac{Cov(x,y)}{\sqrt{Var(x)\,Var(y)}} = \frac{\frac{1}{2}}{\sqrt{\left(\frac{24}{8} - \frac{12}{8}\right)^2\left(\frac{3}{2}-1\right)}}$

$Cov(x,y) = E(xy) - E(x)E(y) = 2 - \frac{12}{8} = 2 - \frac{3}{2} = \frac{1}{2}$

$E(xy) = \sum\sum xy\, f_{xy}(x,y) = 0\times0\times\frac{1}{8} + 0\times1\times0 + 0\times2\times0 + 1\times0\times\frac{1}{8} +$
$1\times1\times\frac{2}{8} + 1\times2\times0 + 2\times0\times0 + 2\times1\times\frac{2}{8} + 2\times2\times\frac{1}{8} + 3\times0\times0 + 3\times1\times0 +$
$3\times2\times\frac{1}{8} = \frac{1}{4} + \frac{1}{2} + \frac{1}{2} + \frac{6}{8} = \frac{16}{8} = 2$

$E(x) = 0\times\frac{1}{8} + 1\times\frac{3}{8} + 2\times\frac{3}{8} + 3\times\frac{1}{8} = \frac{12}{8}$

$E(y) = 0\times\frac{2}{8} + 1\times\frac{4}{8} + 2\times\frac{2}{8} = 1$

$E(x^2) = \frac{9}{8} + \frac{3}{8} + \frac{12}{8} = \frac{24}{8}$ $\qquad E(y^2) = \frac{3}{2}$

$E(x) = \int_0^1 x \cdot \frac{2}{5}x(x+y)\,dx = \frac{2}{5}\int_0^1 x^2(x+y)\,dx =$ ۱۸-

$\frac{2}{5}\int_0^1 x^3 + x^2 y\,dx = \frac{2}{5}\; \frac{x^4}{4}\Big|_0^1 + y\;\frac{x^3}{3}\Big|_0^1 = \frac{2}{5}\left(\frac{1}{4} + y\,\frac{1}{3}\right)$

$E(y) = \int_0^2 y \cdot \frac{2}{5}x(x+y)\,dy = \frac{2}{5}\int_0^2 x^2 y + x y^2\,dy =$

$\frac{2}{5}\left(x^2\left(\frac{y^2}{2}\right)\Big|_0^2 + \frac{y^3}{3}\Big|_0^2\right) = \frac{2}{5}\left(x^2(2) + \frac{8}{3}\right)$

$E(x+y) = E(x) + E(y) = \frac{2}{5}\left(\frac{1}{4} + y\,\frac{1}{3}\right) + \frac{2}{5}\left(2x^2 + \frac{8}{3}\right)$

$E(xy) = \frac{2}{5}\int_0^1\int_0^2 xy(x+y)\,dy\,dx = \frac{2}{5}\int_0^1\int_0^2 x^2 y + x y^2\,dy\,dx =$

$\frac{2}{5}\int_0^1 \left[\frac{x^2 y^2}{2} + \frac{x y^3}{3}\right]_0^2 = \frac{2}{5}\int_0^1 2x^2 + 8\frac{x}{3}\,dx =$

$\frac{2}{5}\left[2\frac{x^3}{3} + 8\frac{x^2}{6}\right]_0^1 = \frac{2}{5}\left(\frac{2}{3} + \frac{4}{3}\right)$

$\rho = \rho(x,y) = \frac{Cov(x,y)}{\sqrt{Var(x)\,Var(y)}} = 0$ ۱۹-

PAPCO

Subject:

Year. Month. Date. ( )

$Cov(x,y) = ۰ - \frac{۱}{۸} \times ۰ = ۰$

$E(x) = -۱ \times \frac{۴}{۱۶} + ۰ \times \frac{۶}{۱۶} = \frac{۱}{۸}$

$E(y) = -۱ \times \frac{۵}{۱۶} + ۰ + ۱ \times \frac{۵}{۱۶} = ۰$

$E(xy) = (-۱)(-۱)(\frac{۱}{۱۶}) + (-۱)(+۱)(\frac{۱}{۱۶}) + (۱)(-۱)(\frac{۲}{۱۶}) + (۱)(۱)(\frac{۲}{۱۶}) =$

$\frac{۱}{۱۶} - \frac{۱}{۱۶} - \frac{۲}{۱۶} + \frac{۲}{۱۶} = ۰$

$f_{x,y}(x,y) \neq f_x(x) f_y(y)$ مستقل نیستند

$f(۰,۰) - f(۰) f(۰) > ۰ \quad \frac{۲}{۱۶} \neq \frac{۶}{۱۶} \times \frac{۶}{۱۶}$

$E(x) = (۱)(\frac{۴}{۸}) + (۳)(\frac{۸}{۲۴}) + (۹)(\frac{۲}{۱۲}) = \frac{۷۶}{۲۴}$. -۲۱

$E(y) = (۲)(\frac{۷}{۲۴}) + (۴)(\frac{۲}{۴}) + (۶)(\frac{۷}{۲۴}) = \frac{۸۰}{۲۴}$

$Cov(x,y) = \frac{۱۷۴}{۲۴} - \frac{۷۶}{۲۴} \times \frac{۸۰}{۲۴} = \frac{۱۷۴ - ۶۰۸۰}{۲۴}$

$E(xy) = (۱)(۲)(\frac{۱}{۸۱}) + (۱)(۴)(\frac{۱}{۴}) + (۱)(۶)(\frac{۱}{۸}) + (۳)(۲)(\frac{۱}{۲۴}) + (۳)(۴)(\frac{۱}{۲})$

$+ (۳)(۶)(\frac{۱}{۲۴}) + (۹)(۲)(\frac{۱}{۱۲}) + (۹)(۴)(۰) + (۹)(۶)(\frac{۱}{۱۲}) + (\frac{۱}{۲})(۱)(\frac{۶}{۸}) +$

$\frac{۶}{۲۴} + ۳ + \frac{۱۸}{۲۴} + \frac{۱۸}{۱۲} = ۷,۲۵$

$F(x,y) = F(x) \cdot F(y) > f(3,2) = f_x(3) f_y(۲) \quad \frac{۱}{۲۴} \neq \frac{۸}{۲۴} \times \frac{۷}{۲۴}$

$E(x) = \int_۰^۱ x\,dx = \frac{x^۲}{۲} \Big|_۰^۱ = \frac{۱}{۲}$ -۲۲

$E(y) = \int_۰^۱ y\,dy = \frac{y^۲}{۲} \Big|_۰^۱ = \frac{۱}{۲}$

$E(xy) = \int_۰^۱ \int_۰^۱ xy\,dy\,dx = \int_۰^۱ x \frac{y^۲}{۲} \Big|_۰^۱ dx = \int_۰^۱ x \times \frac{۱}{۲}\,dx =$

$\frac{۱}{۲} \times \frac{x^۲}{۲} \Big|_۰^۱ = \frac{۱}{۴}$

$E(x) = ۲ \quad E(y) = ۲ \quad E(x^۲) - (E(x))^۲ = ۴ \quad E(y^۲) - (E(y))^۲ = ۵$ -۲۳

$E((x-y)(x+y)) = E(x^۲ - y^۲) = E(x^۲) - E(y^۲) = ۸ - ۸ = ۰$

$E(x^۲) - (E(x))^۲ = ۴ \quad E(x^۲) - ۴ = ۴ \quad E(x^۲) = ۸$

$E(y^۲) - (E(y))^۲ = ۵ \quad E(y^۲) - ۳ = ۵ \quad E(y^۲) = ۸$

$E(xy) = E(x)E(y) = ۲ \times ۲ = ۴$

$E(y)=\int_0^1 y\,2y\,dy=\int_0^1 2y^2\,dy = 2\,\frac{y^3}{3}\Big|_0^1=\frac{2}{3}$ -۲۴

$E(x)=\int_2^\infty x\,\frac{8}{x^3}\,dx=\int_2^\infty \frac{8}{x^2}\,dx = 8\int_2^\infty x^{-2}\,dx = -8\left[\frac{1}{x}\right]_2^\infty$

$= -8(0-\frac{1}{2}) = 4$

$2 = E(xy) \Rightarrow 2 = E(x)E(y) \Rightarrow 2 = \frac{2}{3}\times 4 \quad 2=\frac{8}{3}$

$E(x)=1 \quad E(y)=2 \quad Var(x)=3 \Rightarrow E(x^2)-(E(x))^2 = 3 \Rightarrow$ -۲۸

$E(x^2)=4$

$Var(y)=4 \quad E(y^2)-E(y)^2 = 4 \Rightarrow E(y^2)=8$

$Cov(x,y)=4 \quad E(xy)-E(x)E(y) = 4 \quad E(xy)=2$

$E(2x-y+1) = 2E(x)-E(y)+1 = 2-2+1=1$

$\rho(x,y) = \frac{Cov(x,y)}{\sqrt{Var(x)Var(y)}}$ [illegible]

$r = mx+y = E(mx+y) = mE(x)+E(y) = m(1)+2 = m+2$

$r = x-my = E(x-my) = E(x) - mE(y) = 1+m(1) = 2m+1$

$Cov(x,y) = E(x,y)-E(x)E(y) = -2m^2-3m+2-2m^2-5m-2 = 4m^2 -$

$m\neq 0 \quad m\neq 2$

$Var(x)=4$

$Cov(xy) = E(xy)-E(y)E(x) = \int_0^1 x\,6(1-x-y)\,dx =$ -۲۶

$6\int_0^1 x-x^2-yx\,dx = 6\left(\frac{x^2}{2}-\frac{x^3}{3}-y\frac{x^2}{2}\right)\Big|_0^1 =$

$6\left(\frac{1}{2}-\frac{1}{3}-\frac{y}{2}\right) = 6\left(\frac{3-2-3y}{6}\right)$

$E(x) = 1-3y$

$E(y) = \int_0^1 6(1-x-y)\,dy = 6\int_0^1 y-xy-y^2\,dy = 6\left[\frac{y^2}{2}-x\frac{y^2}{2}-\frac{y^3}{3}\right]_0^1$

$= 3y^2-3xy^2-2y^3$

$\int_0^1\int_{x^2}^1 k\,dy\,dx = \int_0^1 k[y]_{x^2}^1\,dx = k\int_0^1 (1-x^2)\,dx =$ -۲۸

$k\left[x - \frac{x^2}{2}\right]_0^1 = k\left[1 - \frac{1}{2}\right] = 1 \Rightarrow k\left(\frac{1}{2}\right) = 1 \qquad k = \frac{2}{1}$

$E(x \mid y = y) = \int_{-\infty}^{\infty} x\, f_{x|y}(x \mid y)\, dx = \frac{[illegible]}{[illegible]} = 1$

$E(x) = \int_0^1 \frac{2}{1}\, dx = \frac{2}{1}\, x \Big|_0^1 = \frac{2}{1}$

$E(x) = \int_0^1 x\, \frac{2}{1}\, dx = \frac{2}{1}\left[\frac{x^2}{2}\right]_0^1 = \frac{2}{1} \cdot \frac{1}{2} = \frac{2}{2}$

$E(y) = \int_{x}^{1} y\, \frac{2}{1}\, dy = \frac{2}{1}\left[\frac{y^2}{2}\right]_{x}^{1} = \frac{2}{1}\left[\frac{1}{2} - \frac{x^2}{2}\right] = \frac{2}{2}$