

Vogel Approximation Method (VAM). VAM is an improved version of the least-cost method that generally, but not always, produces better starting solutions.

Step 1. For each row (column), determine a penalty measure by subtracting the *smallest* unit cost element in the row (column) from the *next smallest* unit cost element in the same row (column).

TABLE 5.18 Least-Cost Starting Solution

	1	2	3	4	Supply
1	10	(start) 2	20	11	15
2	12	7	9	(end) 20	25
3	4	14	16	18	10
Demand	5	15	15	15	

- Step 2.** Identify the row or column with the largest penalty. Break ties arbitrarily. Allocate as much as possible to the variable with the least unit cost in the selected row or column. Adjust the supply and demand, and cross out the satisfied row or column. If a row and a column are satisfied simultaneously, only one of the two is crossed out, and the remaining row (column) is assigned zero supply (demand).
- Step 3.** (a) If exactly one row or column with zero supply or demand remains uncrossed out, stop.
- (b) If one row (column) with *positive* supply (demand) remains uncrossed out, determine the basic variables in the row (column) by the least-cost method. Stop.
- (c) If all the uncrossed out rows and columns have (remaining) zero supply and demand, determine the *zero* basic variables by the least-cost method. Stop.
- (d) Otherwise, go to step 1.

Example 5.3-4

VAM is applied to Example 5.3-1. Table 5.19 computes the first set of penalties. Because row 3 has the largest penalty (= 10) and cell (3, 1) has the smallest unit cost in that row, the amount 5 is assigned to x_{31} . Column 1 is now satisfied and must be crossed out. Next, new penalties are recomputed as in Table 5.20.

Table 5.20 shows that row 1 has the highest penalty (= 9). Hence, we assign the maximum amount possible to cell (1, 2), which yields $x_{12} = 15$ and simultaneously satisfies both row 1 and column 2. We arbitrarily cross out column 2 and adjust the supply in row 1 to zero.

Continuing in the same manner, row 2 will produce the highest penalty (= 11), and we assign $x_{23} = 15$, which crosses out column 3 and leaves 10 units in row 2. Only column 4 is left, and it has a positive supply of 15 units. Applying the least-cost method to that column, we successively assign $x_{14} = 0$, $x_{34} = 5$, and $x_{24} = 10$ (verify!). The associated objective value for this solution is

$$z = 15 \times 2 + 0 \times 11 + 15 \times 9 + 10 \times 20 + 5 \times 4 + 5 \times 18 = \$475$$

This solution happens to have the same objective value as in the least-cost method.

TABLE 5.19 Row and Column Penalties in VAM

	1	2	3	4	Row penalty
1	10	2	20	11	$10 - 2 = 8$
2	12	7	9	20	$9 - 7 = 2$
3	4	14	16	18	$14 - 4 = 10$
	5	15	15	15	
Column penalty	$10 - 4 = 6$	$7 - 2 = 5$	$16 - 9 = 7$	$18 - 11 = 7$	

TABLE 5.20 First Assignment in VAM ($x_{31} = 5$)

	1	2	3	4	Row penalty
1	10	2	20	11	9
2	12	7	9	20	2
3	4	14	16	18	2
	5	15	15	15	
Column penalty	—	5	7	7	