

Vogel Approximation

Finally, we should point out that there have been a tremendous number of procedures suggested for finding an initial basic feasible solution to the transportation problem. In the previous section, we mentioned four methods: northwest corner, minimum matrix, minimum column, and minimum row.

The first of these ignores the costs altogether, while the remaining methods allocate costs in such a way that the last few assignments of flows often results in very high costs being incurred. The high costs are due to the lack of choice as to how the final flows are to be assigned to routes, once the initial flows have been established. The initial flows are not chosen with sufficient foresight as to how they might impair later flow choices.

The Vogel approximation method was developed to overcome this difficulty and has proved to be so effective that it is sometimes used to obtain an approximation to the optimal solution of the problem. The method, instead of sequentially using the least-cost remaining arc, bases its selection on the difference between the two lowest-cost arcs leaving an origin or entering a destination. This difference indicates where departure from the lowest-cost allocations will bring the highest increase in cost. Therefore, one assigns the maximum possible amount to that arc that has the lowest cost in that row or column having the greatest cost difference. If this assignment exhausts the demand at that destination, the corresponding column is eliminated from further consideration; similarly, if the assignment exhausts the supply at that origin, the corresponding row is eliminated. In either case, the origin and destination cost differences are recomputed, and the procedure continues in the same way.

The Vogel approximation method is applied to our illustrative example in Tableau 12, and the resulting basic feasible solution is given in Fig. 8.13. It is interesting to note that the approximation finds the optimal solution in this particular case, as can be seen by comparing the initial basis from the Vogel approximation in Fig. 8.13 with the optimal basis in Tableau 7. This, of course, does not mean that the Vogel approximation is the best procedure for determining the initial basic feasible solution. Any such comparison among computational procedures must compare total time required, from preprocessing to final optimal

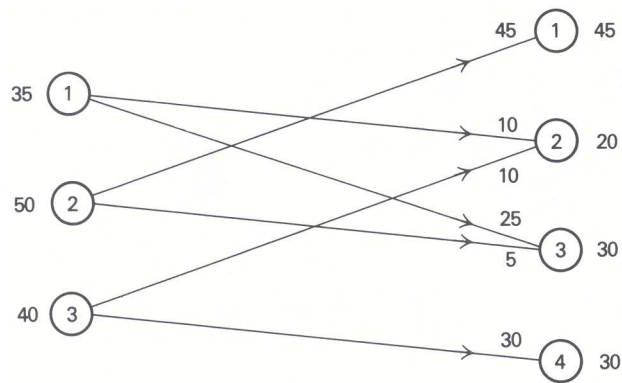


Figure 8.13 Initial basis for Vogel approximation method.

Tableau 12 Applying the Vogel Approximation Method*First iteration:*

	Dallas	Atlanta	San Francisco	Philadelphia	Supply	Row difference
Cleveland	8	6	10	9	35	2
Chicago	9	12	13	7	50	2
Boston	14	9	16	5	40 10	4
Demand	45	20	30	30		
Column difference	1	3	3	2		

Second iteration:

	Dallas	Atlanta	San Francisco	Supply	Row difference
Cleveland	8	6	10	35	2
Chicago	9	12	13	50	3
Boston	14	9 10	16	10	5
Demand	45	20 10	30		
Column difference	1	3	3		

Third iteration:

	Dallas	Atlanta	San Francisco	Supply	Row difference
Cleveland	8	6 10	10	35 25	2
Chicago	9	12	13	50	3
Demand	45	10	30		
Column difference	1	6	3		

Fourth iteration:

	Dallas	San Francisco	Supply	Row difference
Cleveland	8	10 25	25	2
Chicago	9 45	13 5	50 45	4
Demand	45	30		
Column difference	1	3		