Chapter 32

3

(a) Use Gauss' law for magnetism: $\oint \vec{B} \cdot d\vec{A} = 0$. Write $\oint \vec{B} \cdot d\vec{A} = \Phi_1 + \Phi_2 + \Phi_C$, where Φ_1 is the magnetic flux through the first end mentioned, Φ_2 is the magnetic flux through the second end mentioned, and Φ_C is the magnetic flux through the curved surface. Over the first end, the magnetic field is inward, so the flux is $\Phi_1 = -25.0\,\mu\text{Wb}$. Over the second end, the magnetic field is uniform, normal to the surface, and outward, so the flux is $\Phi_2 = AB = \pi r^2 B$, where A is the area of the end and r is the radius of the cylinder. Its value is

$$\Phi_2 = \pi(0.120\,\text{m})^2(1.60 \times 10^{-3}\,\text{T}) = +7.24 \times 10^{-5}\,\text{Wb} = +72.4\,\mu\text{Wb}\,.$$

Since the three fluxes must sum to zero,

$$\Phi_C = -\Phi_1 - \Phi_2 = 25.0\,\mu\text{Wb} - 72.4\,\mu\text{Wb} = -47.4\,\mu\text{Wb}\,.$$

(b) The minus sign indicates that the flux is inward through the curved surface.

5

Consider a circle of radius r (= 6.0 mm), between the plates and with its center on the axis of the capacitor. The current through this circle is zero, so the Ampere-Maxwell law becomes

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}\,,$$

where $\vec{B}$ is the magnetic field at points on the circle and Φ_E is the electric flux through the circle. The magnetic field is tangent to the circle at all points on it, so $\oint \vec{B} \cdot d\vec{s} = 2\pi r B$. The electric flux through the circle is $\Phi_E = \pi R^2 E$, where R (= 3.0 mm) is the radius of a capacitor plate. When these substitutions are made, the Ampere-Maxwell law becomes

$$2\pi r B = \mu_0 \epsilon_0 \pi R^2 \frac{dE}{dt}\,.$$

Thus

$$\frac{dE}{dt} = \frac{2rB}{\mu_0 \epsilon_0 R^2} = \frac{2(6.0 \times 10^{-3}\,\text{m})(2.0 \times 10^{-7}\,\text{T})}{(4\pi \times 10^{-7}\,\text{H/m})(8.85 \times 10^{-12}\,\text{Fm})(3.0 \times 10^{-3}\,\text{m})^2} = 2.4 \times 10^{13}\,\text{V/m}\cdot\text{s}\,.$$

13

The displacement current is given by

$$i_d = \epsilon_0 A \frac{dE}{dt}\,,$$

where A is the area of a plate and E is the magnitude of the electric field between the plates. The field between the plates is uniform, so $E = V/d$, where V is the potential difference across the plates and d is the plate separation. Thus

$$i_d = \frac{\epsilon_0 A}{d} \frac{dV}{dt} .$$

Now $\epsilon_0 A/d$ is the capacitance C of a parallel-plate capacitor without a dielectric, so

$$i_d = C \frac{dV}{dt} .$$

21

(a) For a parallel-plate capacitor, the charge q on the positive plate is given by $q = (\epsilon_0 A/d)V$, where A is the plate area, d is the plate separation, and V is the potential difference between the plates. In terms of the electric field E between the plates, $V = Ed$, so $q = \epsilon_0 AE = \epsilon_0 \Phi_E$, where Φ_E is the total electric flux through the region between the plates. The true current into the positive plate is $i = dq/dt = \epsilon_0\, d\Phi_E/dt = i_{d\text{ total}}$, where $i_{d\text{ total}}$ is the total displacement current between the plates. Thus $i_{d\text{ total}} = 2.0\,\text{A}$.

(b) Since $i_{d\text{ total}} = \epsilon_0\, d\Phi_E/dt = \epsilon_0 A\, dE/dt$,

$$\frac{dE}{dt} = \frac{i_{d\text{ total}}}{\epsilon_0 A} = \frac{2.0\,\text{A}}{(8.85 \times 10^{-12}\,\text{F/m})(1.0\,\text{m})^2} = 2.3 \times 10^{11}\,\text{V/m} \cdot \text{s} .$$

(c) The displacement current is uniformly distributed over the area. If a is the area enclosed by the dashed lines and A is the area of a plate, then the displacement current through the dashed path is

$$i_{d\text{ enc}} = \frac{a}{A} i_{d\text{ total}} = \frac{(0.50\,\text{m})^2}{(1.0\,\text{m})^2}(2.0\,\text{A}) = 0.50\,\text{A} .$$

(d) According to Maxwell's law of induction,

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{d\text{ enc}} = (4\pi \times 10^{-7}\,\text{H/m})(0.50\,\text{A}) = 6.3 \times 10^{-7}\,\text{T} \cdot \text{m} .$$

Notice that the integral is around the dashed path and the displacement current on the right side of the Maxwell's law equation is the displacement current through that path, not the total displacement current.

35

(a) The z component of the orbital angular momentum is given by $L_{\text{orb},\,z} = m_\ell h/2\pi$, where h is the Planck constant. Since $m_\ell = 0$, $L_{\text{orb},\,z} = 0$.

(b) The z component of the orbital contribution to the magnetic dipole moment is given by $\mu_{\text{orb},\,z} = -m_\ell \mu_B$, where μ_B is the Bohr magneton. Since $m_\ell = 0$, $\mu_{\text{orb},\,z} = 0$.

(c) The potential energy associated with the orbital contribution to the magnetic dipole moment is given by $U = -\mu_{\text{orb},\,z} B_{\text{ext}}$, where B_{ext} is the z component of the external magnetic field. Since $\mu_{\text{orb},\,z} = 0$, $U = 0$.

(d) The z component of the spin magnetic dipole moment is either $+\mu_B$ or $-\mu_B$, so the potential energy is either

$$U = -\mu_B B_{\text{ext}} = -(9.27 \times 10^{-24}\,\text{J/T})(35 \times 10^{-3}\,\text{T}) = -3.2 \times 10^{-25}\,\text{J}\,.$$

or $U = +3.2 \times 10^{-25}$ J.

(e) Substitute m_ℓ into the equations given above. The z component of the orbital angular momentum is

$$L_{\text{orb},\,z} = \frac{m_\ell h}{2\pi} = \frac{(-3)(6.626 \times 10^{-34}\,\text{J}\cdot\text{s})}{2\pi} = -3.2 \times 10^{-34}\,\text{J}\cdot\text{s}\,.$$

(f) The z component of the orbital contribution to the magnetic dipole moment is

$$\mu_{\text{orb},\,z} = -m_\ell \mu_B = -(-3)(9.27 \times 10^{-24}\,\text{J/T}) = 2.8 \times 10^{-23}\,\text{J/T}\,.$$

(g) The potential energy associated with the orbital contribution to the magnetic dipole moment is

$$U = -\mu_{\text{orb},\ z} B_{\text{ext}} = -(2.78 \times 10^{-23}\,\text{J/T})(35 \times 10^{-3}\,\text{T}) = -9.7 \times 10^{-25}\,\text{J}\,.$$

(h) The potential energy associated with spin does not depend on m_ℓ. It is $\pm 3.2 \times 10^{-25}$ J.

39

The magnetization is the dipole moment per unit volume, so the dipole moment is given by $\mu = MV$, where M is the magnetization and V is the volume of the cylinder. Use $V = \pi r^2 L$, where r is the radius of the cylinder and L is its length. Thus

$$\mu = M\pi r^2 L = (5.30 \times 10^3\,\text{A/m})\pi(0.500 \times 10^{-2}\,\text{m})^2(5.00 \times 10^{-2}\,\text{m}) = 2.08 \times 10^{-2}\,\text{J/T}\,.$$

45

(a) The number of atoms per unit volume in states with the dipole moment aligned with the magnetic field is $N_+ = Ae^{\mu B/kT}$ and the number per unit volume in states with the dipole moment antialigned is $N_- = Ae^{-\mu B/kT}$, where A is a constant of proportionality. The total number of atoms per unit volume is $N = N_+ + N_- = A\left(e^{\mu B/kT} + e^{-\mu B/kT}\right)$. Thus

$$A = \frac{N}{e^{\mu B/kT} + e^{-\mu B/kT}}\,.$$

The magnetization is the net dipole moment per unit volume. Subtract the magnitude of the total dipole moment per unit volume of the antialigned moments from the total dipole moment per unit volume of the aligned moments. The result is

$$M = \frac{N\mu e^{\mu B/kT} - N\mu e^{-\mu B/kT}}{e^{\mu B/kT} + e^{-\mu B/kT}} = \frac{N\mu\left(e^{\mu B/kT} - e^{-\mu B/kT}\right)}{e^{\mu B/kT} + e^{-\mu B/kT}} = N\mu\tanh(\mu B/kT)\,.$$

(b) If $\mu B \ll kT$, then $e^{\mu B/kT} \approx 1 + \mu B/kT$ and $e^{-\mu B/kT} \approx 1 - \mu B/kT$. (See Appendix E for the power series expansion of the exponential function.) The expression for the magnetization becomes

$$M \approx \frac{N\mu\left[(1 + \mu B/kT) - (1 - \mu B/kT)\right]}{(1 + \mu B/kT) + (1 - \mu B/kT)} = \frac{N\mu^2 B}{kT}\,.$$

(c) If $\mu B \gg kT$, then $e^{-\mu B/kT}$ is negligible compared to $e^{\mu B/kT}$ in both the numerator and denominator of the expression for M. Thus

$$M \approx \frac{N\mu e^{\mu B/kT}}{e^{\mu B/kT}} = N\mu\,.$$

(d) The expression for M predicts that it is linear in B/kT for $\mu B/kT$ small and independent of B/kT for $\mu B/kT$ large. The figure agrees with these predictions.

47

(a) The field of a dipole along its axis is given by Eq. 29–27:

$$\vec{B} = \frac{\mu_0}{2\pi}\frac{\vec{\mu}}{z^3}\,,$$

where μ is the dipole moment and z is the distance from the dipole. Thus the magnitude of the magnetic field is

$$B = \frac{(4\pi \times 10^{-7}\,\mathrm{T\cdot m/A})(1.5 \times 10^{-23}\,\mathrm{J/T})}{2\pi(10 \times 10^{-9}\,\mathrm{m})^3} = 3.0 \times 10^{-6}\,\mathrm{T}\,.$$

(b) The energy of a magnetic dipole with dipole moment $\vec{\mu}$ in a magnetic field $\vec{B}$ is given by $U = -\vec{\mu}\cdot\vec{B} = -\mu B\cos\phi$, where ϕ is the angle between the dipole moment and the field. The energy required to turn it end for end (from $\phi = 0°$ to $\phi = 180°$) is

$$\begin{aligned}\Delta U &= -\mu B(\cos 180° - \cos 0°) = 2\mu B = 2(1.5 \times 10^{-23}\,\mathrm{J/T})(3.0 \times 10^{-6}\,\mathrm{T})\\ &= 9.0 \times 10^{-29}\,\mathrm{J} = 5.6 \times 10^{-10}\,\mathrm{eV}\,.\end{aligned}$$

The mean kinetic energy of translation at room temperature is about 0.04 eV (see Eq. 19–24 or Sample Problem 32–3). Thus if dipole-dipole interactions were responsible for aligning dipoles, collisions would easily randomize the directions of the moments and they would not remain aligned.

53

(a) If the magnetization of the sphere is saturated, the total dipole moment is $\mu_{\text{total}} = N\mu$, where N is the number of iron atoms in the sphere and μ is the dipole moment of an iron atom. We wish to find the radius of an iron sphere with N iron atoms. The mass of such a sphere is Nm, where m is the mass of an iron atom. It is also given by $4\pi\rho R^3/3$, where ρ is the density of iron and R is the radius of the sphere. Thus $Nm = 4\pi\rho R^3/3$ and

$$N = \frac{4\pi\rho R^3}{3m}\,.$$

Substitute this into $\mu_{\text{total}} = N\mu$ to obtain

$$\mu_{\text{total}} = \frac{4\pi\rho R^3 \mu}{3m}\,.$$

Solve for R and obtain

$$R = \left[\frac{3m\mu_{\text{total}}}{4\pi\rho\mu}\right]^{1/3}\,.$$

The mass of an iron atom is

$$m = 56\,\text{u} = (56\,\text{u})(1.66\times10^{-27}\,\text{kg/u}) = 9.30\times10^{-26}\,\text{kg}\,.$$

So

$$R = \left[\frac{3(9.30\times10^{-26}\,\text{kg})(8.0\times10^{22}\,\text{J/T})}{4\pi(14\times10^{3}\,\text{kg/m}^3)(2.1\times10^{-23}\,\text{J/T})}\right]^{1/3} = 1.8\times10^{5}\,\text{m}\,.$$

(b) The volume of the sphere is

$$V_s = \frac{4\pi}{3}R^3 = \frac{4\pi}{3}(1.82\times10^{5}\,\text{m})^3 = 2.53\times10^{16}\,\text{m}^3$$

and the volume of Earth is

$$V_e = \frac{4\pi}{3}(6.37\times10^{6}\,\text{m})^3 = 1.08\times10^{21}\,\text{m}^3\,,$$

so the fraction of Earth's volume that is occupied by the sphere is

$$\frac{2.53\times10^{16}\,\text{m}^3}{1.08\times10^{21}\,\text{m}^3} = 2.3\times10^{-5}\,.$$

The radius of Earth was obtained from Appendix C.

55

(a) The horizontal and vertical directions are perpendicular to each other, so the magnitude of the field is

$$B = \sqrt{B_h^2 + B_v^2} = \frac{\mu_0\mu}{4\pi r^3}\sqrt{\cos^2\lambda_m + 4\sin^2\lambda_m} = \frac{\mu_0\mu}{4\pi r^3}\sqrt{1 - \sin^2\lambda_m + 4\sin^2\lambda_m}$$
$$= \frac{\mu_0\mu}{4\pi r^3}\sqrt{1 + 3\sin^2\lambda_m}\,,$$

where the trigonometric identity $\cos^2\lambda_m = 1 - \sin^2\lambda_m$ was used.

(b) The tangent of the inclination angle is

$$\tan\phi_i = \frac{B_v}{B_h} = \left(\frac{\mu_0\mu}{2\pi r^3\sin\lambda_m}\right)\left(\frac{4\pi r^3}{\mu_0\mu\cos\lambda_m} = \frac{2\sin\lambda_m}{\cos\lambda_m}\right) = 2\tan\lambda_m\,,$$

where $\tan\lambda_m = (\sin\lambda_m)/(\cos\lambda_m)$ was used.

61

(a) The z component of the orbital angular momentum can have the values $L_{\text{orb},z} = m_\ell h/2\pi$, where m_ℓ can take on any integer value from -3 to $+3$, inclusive. There are seven such values (-3, -2, -1, 0, +1, +2, and +3).

(b) The z component of the orbital magnetic moment is given by $\mu_{\text{orb},z} = -m_\ell eh/4\pi m$, where m is the electron mass. Since there is a different value for each possible value of m_ℓ, there are seven different values in all.

(c) The greatest possible value of $L_{\text{orb},z}$ occurs if $m_\ell = +3$ is $3h/2\pi$.

(d) The greatest value of $\mu_{\text{orb},\,z}$ is $3eh/4\pi m$.

(e) Add the orbital and spin angular momenta: $L_{\text{net},z} = L_{\text{orb},z} + L_{s,z} = (m_\ell h/2\pi) + (m_s h/2\pi)$. To obtain the maximum value, set m_ℓ equal to +3 and m_s equal to $+\frac{1}{2}$. The result is $L_{\text{net},z} = 3.5h/2\pi$.

(f) Write $L_{\text{net},z} = Mh/2\pi$, where M is half an odd integer. M can take on all such values from -3.5 to $+3.5$. There are eight of these: -3.5, -2.5, -1.5, -0.5, $+0.5$, $+1.5$, $+2.5$, and $+3.5$.