Chapter 21

1

The magnitude of the force that either charge exerts on the other is given by

$$F = \frac{1}{4\pi\epsilon_0} \frac{|q_1||q_2|}{r^2},$$

where r is the distance between them. Thus

$$r = \sqrt{\frac{|q_1||q_2|}{4\pi\epsilon_0 F}}$$

$$= \sqrt{\frac{(8.99 \times 10^9 \, \text{N} \cdot \text{m}^2/\text{C}^2)(26.0 \times 10^{-6} \, \text{C})(47.0 \times 10^{-6} \, \text{C})}{5.70 \, \text{N}}} = 1.38 \, \text{m}.$$

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The magnitude of the force of either of the charges on the other is given by

$$F = \frac{1}{4\pi\epsilon_0} \frac{q(Q-q)}{r^2},$$

where r is the distance between the charges. You want the value of q that maximizes the function $f(q) = q(Q-q)$. Set the derivative df/dq equal to zero. This yields $Q - 2q = 0$, or $q = Q/2$.

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Assume the spheres are far apart. Then the charge distribution on each of them is spherically symmetric and Coulomb's law can be used. Let q_1 and q_2 be the original charges and choose the coordinate system so the force on q_2 is positive if it is repelled by q_1. Take the distance between the charges to be r. Then the force on q_2 is

$$F_a = -\frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}.$$

The negative sign indicates that the spheres attract each other.

After the wire is connected, the spheres, being identical, have the same charge. Since charge is conserved, the total charge is the same as it was originally. This means the charge on each sphere is $(q_1 + q_2)/2$. The force is now one of repulsion and is given by

$$F_b = \frac{1}{4\pi\epsilon_0} \frac{(q_1 + q_2)^2}{4r^2}.$$

Solve the two force equations simultaneously for q_1 and q_2. The first gives

$$q_1 q_2 = -4\pi\epsilon_0 r^2 F_a = -\frac{(0.500\,\text{m})^2(0.108\,\text{N})}{8.99 \times 10^9\,\text{N}\cdot\text{m}^2/\text{C}^2} = -3.00 \times 10^{-12}\,\text{C}^2$$

and the second gives

$$q_1 + q_2 = 2r\sqrt{4\pi\epsilon_0 F_b} = 2(0.500\,\text{m})\sqrt{\frac{0.0360\,\text{N}}{8.99 \times 10^9\,\text{N}\cdot\text{m}^2/\text{C}^2}} = 2.00 \times 10^{-6}\,\text{C}.$$

Thus

$$q_2 = \frac{-(3.00 \times 10^{-12}\,\text{C}^2)}{q_1}$$

and substitution into the second equation gives

$$q_1 + \frac{-3.00 \times 10^{-12}\,\text{C}^2}{q_1} = 2.00 \times 10^{-6}\,\text{C}.$$

Multiply by q_1 to obtain the quadratic equation

$$q_1^2 - (2.00 \times 10^{-6}\,\text{C})q_1 - 3.00 \times 10^{-12}\,\text{C}^2 = 0.$$

The solutions are

$$q_1 = \frac{2.00 \times 10^{-6}\,\text{C} \pm \sqrt{(-2.00 \times 10^{-6}\,\text{C})^2 + 4(3.00 \times 10^{-12}\,\text{C}^2)}}{2}.$$

If the positive sign is used, $q_1 = 3.00 \times 10^{-6}\,\text{C}$ and if the negative sign is used, $q_1 = -1.00 \times 10^{-6}\,\text{C}$. Use $q_2 = (-3.00 \times 10^{-12})/q_1$ to calculate q_2. If $q_1 = 3.00 \times 10^{-6}\,\text{C}$, then $q_2 = -1.00 \times 10^{-6}\,\text{C}$ and if $q_1 = -1.00 \times 10^{-6}\,\text{C}$, then $q_2 = 3.00 \times 10^{-6}\,\text{C}$. Since the spheres are identical, the solutions are essentially the same: one sphere originally had charge $-1.00 \times 10^{-6}\,\text{C}$ and the other had charge $+3.00 \times 10^{-6}\,\text{C}$.

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If the system of three particles is to be in equilibrium, the force on each particle must be zero. Let the charge on the third particle be q_0. The third particle must lie on the x axis since otherwise the two forces on it would not be along the same line and could not sum to zero. Thus the y coordinate of the particle must be zero. The third particle must lie between the other two since otherwise the forces acting on it would be in the same direction and would not sum to zero. Suppose the third particle is a distance x from the particle with charge q, as shown on the diagram to the right. The force acting on it is then given by
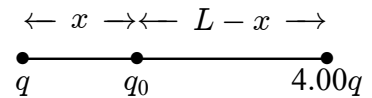
$$F_0 = \frac{1}{4\pi\epsilon_0}\left[\frac{qq_0}{x^2} - \frac{4.00qq_0}{(L - x)^2}\right] = 0,$$

where the positive direction was taken to be toward the right. Solve this equation for x. Canceling common factors yields $1/x^2 = 4.00/(L-x)^2$ and taking the square root yields $1/x = 2.00/(L-x)$. The solution is $x = 0.333L$.

The force on q is

$$F_q = \frac{1}{4\pi\epsilon_0}\left[\frac{qq_0}{x^2} + \frac{4.00q^2}{L^2}\right] = 0.$$

Solve for q_0: $q_0 = -4.00qx^2/L^2 = -0.444q$, where $x = 0.333L$ was used.

The force on the particle with charge $4.00q$ is

$$F_{4q} = \frac{1}{4\pi\epsilon_0}\left[\frac{4.00q^2}{L^2} + \frac{4.00qq_0}{(L-x)^2}\right] = \frac{1}{4\pi\epsilon_0}\left[\frac{4.00q^2}{L^2} + \frac{4.00(0.444)q^2}{(0.444)L^2}\right]$$

$$= \frac{1}{4\pi\epsilon_0}\left[\frac{4.00q^2}{L^2} - \frac{4.00q^2}{L^2}\right] = 0.$$

With $q_0 = -0.444q$ and $x = 0.333L$, all three charges are in equilibrium.

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(a) The magnitude of the force between the ions is given by

$$F = \frac{q^2}{4\pi\epsilon_0 r^2},$$

where q is the charge on either of them and r is the distance between them. Solve for the charge:

$$q = r\sqrt{4\pi\epsilon_0 F} = (5.0 \times 10^{-10}\,\text{m})\sqrt{\frac{3.7 \times 10^{-9}\,\text{N}}{8.99 \times 10^9\,\text{N} \cdot \text{m}^2/\text{C}^2}} = 3.2 \times 10^{-19}\,\text{C}.$$

(b) Let N be the number of electrons missing from each ion. Then $Ne = q$ and

$$N = \frac{q}{e} = \frac{3.2 \times 10^{-19}\,\text{C}}{1.60 \times 10^{-19}\,\text{C}} = 2.$$

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(a) Every cesium ion at a corner of the cube exerts a force of the same magnitude on the chlorine ion at the cube center. Each force is a force of attraction and is directed toward the cesium ion that exerts it, along the body diagonal of the cube. We can pair every cesium ion with another, diametrically positioned at the opposite corner of the cube. Since the two ions in such a pair exert forces that have the same magnitude but are oppositely directed, the two forces sum to zero and, since every cesium ion can be paired in this way, the total force on the chlorine ion is zero.

(b) Rather than remove a cesium ion, superpose charge $-e$ at the position of one cesium ion. This neutralizes the ion and, as far as the electrical force on the chlorine ion is concerned, it is equivalent to removing the ion. The forces of the eight cesium ions at the cube corners sum to zero, so the only force on the chlorine ion is the force of the added charge.

The length of a body diagonal of a cube is $\sqrt{3}a$, where a is the length of a cube edge. Thus the distance from the center of the cube to a corner is $d = (\sqrt{3}/2)a$. The force has magnitude

$$F = \frac{1}{4\pi\epsilon_0}\frac{e^2}{d^2} = \frac{1}{4\pi\epsilon_0}\frac{e^2}{(3/4)a^2}$$

$$= \frac{(8.99 \times 10^9\,\text{N} \cdot \text{m}^2/\text{C}^2)(1.60 \times 10^{-19}\,\text{C})^2}{(3/4)(0.40 \times 10^{-9}\,\text{m})^2} = 1.9 \times 10^{-9}\,\text{N}.$$

Since both the added charge and the chlorine ion are negative, the force is one of repulsion. The chlorine ion is pulled away from the site of the missing cesium ion.

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None of the reactions given include a beta decay, so the number of protons, the number of neutrons, and the number of electrons are each conserved. Atomic numbers (numbers of protons and numbers of electrons) and molar masses (combined numbers of protons and neutrons) can be found in Appendix F of the text.

(a) ^{1}H has 1 proton, 1 electron, and 0 neutrons and ^{9}Be has 4 protons, 4 electrons, and $9 - 4 = 5$ neutrons, so X has $1 + 4 = 5$ protons, $1 + 4 = 5$ electrons, and $0 + 5 - 1 = 4$ neutrons. One of the neutrons is freed in the reaction. X must be boron with a molar mass of $5\,\text{g/mol} + 4\,\text{g/mol} = 9\,\text{g/mol}$: ^{9}B.

(b) ^{12}C has 6 protons, 6 electrons, and $12 - 6 = 6$ neutrons and ^{1}H has 1 proton, 1 electron, and 0 neutrons, so X has $6 + 1 = 7$ protons, $6 + 1 = 7$ electrons, and $6 + 0 = 6$ neutrons. It must be nitrogen with a molar mass of $7\,\text{g/mol} + 6\,\text{g/mol} = 13\,\text{g/mol}$: ^{13}N.

(c) ^{15}N has 7 protons, 7 electrons, and $15 - 7 = 8$ neutrons; ^{1}H has 1 proton, 1 electron, and 0 neutrons; and ^{4}He has 2 protons, 2 electrons, and $4 - 2 = 2$ neutrons; so X has $7 + 1 - 2 = 6$ protons, 6 electrons, and $8 + 0 - 2 = 6$ neutrons. It must be carbon with a molar mass of $6\,\text{g/mol} + 6\,\text{g/mol} = 12\,\text{g/mol}$: ^{12}C.

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The magnitude of the force of particle 1 on particle 2 is

$$F = \frac{1}{4\pi\epsilon_0} \frac{|q_1||q_2|}{d_1^2 + d_2^2}.$$

The signs of the charges are the same, so the particles repel each other along the line that runs through them. This line makes an angle θ with the x axis such that $\cos\theta = d_2/\sqrt{d_1^2 + d_2^2}$, so the x component of the force is

$$F_x = \frac{1}{4\pi\epsilon_0} \frac{|q_1||q_2|}{d_1^2 + d_2^2}\cos\theta = \frac{1}{4\pi\epsilon_0} \frac{|q_1||q_2|d_2}{(d_1^2 + d_2^2)^{3/2}}$$

$$= (8.99 \times 10^9\,\text{C}^2/\text{N}\cdot\text{m}^2)\frac{24(1.60 \times 10^{-19}\,\text{C})^2(6.00 \times 10^{-3}\,\text{m})}{[(2.00 \times 10^{-3}\,\text{m})^2 + (6.00 \times 10^{-3}\,\text{m})^2]^{3/2}} = 1.31 \times 10^{-22}\,\text{N}.$$

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The magnitude of the gravitational force on a proton near Earth's surface is mg, where m is the mass of the proton (1.67×10^{-27} kg from Appendix B). The electrostatic force between two protons is $F = (1/4\pi\epsilon_0)(e^2/d^2)$, where d is their separation. Equate these forces to each other and solve for d. The result is

$$d = \sqrt{\frac{1}{4\pi\epsilon_0}\frac{e^2}{mg}} = \sqrt{(8.99 \times 10^{-9}\,\text{N}\cdot\text{m}^2/\text{C}^2)\frac{(1.60 \times 10^{-19}\,\text{C})^2}{(1.67 \times 10{-27}\,\text{kg})(9.8\,\text{m/s}^2)}} = 0.119\,\text{m}.$$

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The magnitude of the force of particle 1 on particle 4 is

$$F_1 = \frac{1}{4\pi\epsilon_0}\frac{|q_1||q_4|}{d_1^2} = (8.99 \times 10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)\frac{(3.20 \times 10^{-19}\,\text{C})(3.20 \times 10^{-19}\,\text{C})}{(0.0300\,\text{m})^2} = 1.02 \times 10^{-24}\,\text{N}.$$

The charges have opposite signs, so the particles attract each other and the vector force is

$$\vec{F_1} = -(1.02^{-24}\,\text{N})(\cos 35.0°)\hat{i} - (1.02 \times 10^{-24}\,\text{N})(\sin 35.0°)\hat{j}$$
$$= -(8.36 \times 10^{-25}\,\text{N})\hat{i} - (5.85 \times 10^{-25}\,\text{N})\hat{j}.$$

Particles 2 and 3 repel each other. The force of particle 2 on particle 4 is

$$\vec{F_2} = -\frac{1}{4\pi\epsilon_0}\frac{|q_2||q_4|}{d_2^2}\hat{j}$$
$$= -(8.99 \times 10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)\frac{(3.20 \times 10^{-19}\,\text{C})(3.20 \times 10^{-19}\,\text{C})}{(0.0200\,\text{m})^2}\hat{j} = -(2.30 \times 10^{-24}\,\text{N})\hat{j}.$$

Particles 3 and 4 repel each other and the force of particle 3 on particle 4 is

$$\vec{F_3} = -\frac{1}{4\pi\epsilon_0}\frac{|2q_3||q_4|}{d_3^2}\hat{i}$$
$$= -(8.99 \times 10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)\frac{(6.40 \times 10^{-19}\,\text{C})(3.20 \times 10^{-19}\,\text{C})}{(0.0200\,\text{m})^2} = -(4.60 \times 10^{-24}\,\text{C})\hat{i}.$$

The net force is the vector sum of the three forces. The x component is $F_x = -18.36 \times 10^{-25}\,\text{N} - 4.60\,\text{N} = -5.44 \times 10^{-24}\,\text{N}$ and the y component is $F_y = -5.85 \times 10^{-25}\,\text{N} - 2.30 \times 10^{-24}\,\text{N} = -2.89 \times 10^{-24}\,\text{N}$. The magnitude of the force is

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{(-5.44 \times 10^{-24}\,\text{N})^2 + (-2.89 \times 10^{-24}\,\text{N})^2} = 6.16 \times 10^{24}\,\text{N}.$$

The tangent of the angle θ between the net force and the positive x axis is $\tan\theta = F_y/F_x = (-2.89 \times 10^{-24}\,\text{N})/(-5.44 \times 10^{-24}\,\text{N}) = 0.531$ and the angle is either 28° or 208°. The later angle is associated with a vector that has negative x and y components and so is the correct angle.

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The net force on particle 3 is the vector sum of the forces of particles 1 and 2 and for this to be zero the two forces must be along the same line. Since electrostatic forces are along the lines that join the particles, particle 3 must be on the x axis. Its y coordinate is zero.

Particle 3 is repelled by one of the other charges and attracted by the other. As a result, particle 3 cannot be between the other two particles and must be either to the left of particle 1 or to the right of particle 2. Since the magnitude of q_1 is greater than the magnitude of q_2, particle 3 must

be closer to particle 2 than to particle 1 and so must be to the right of particle 2. Let x be the coordinate of particle 3. The the x component of the force on it is

$$F_x = \frac{1}{4\pi\epsilon_0} \left[\frac{q_1 q_3}{x^2} + \frac{q_2 q_3}{(x-L)^2} \right].$$

If $F_x = 0$ the solution for x is

$$x = \frac{\sqrt{-q_1/q_2}}{\sqrt{-q_1/q_2} - 1} L = \frac{\sqrt{-(-5.00q)/(2.00q)}}{\sqrt{-(-5.00q)/(2.00q)} - 1} L = 2.72L .$$