Chapter 22

3

Since the magnitude of the electric field produced by a point particle with charge q is given by $E = |q|/4\pi\epsilon_0 r^2$, where r is the distance from the particle to the point where the field has magnitude E, the magnitude of the charge is

$$|q| = 4\pi\epsilon_0 r^2 E = \frac{(0.50\,\text{m})^2(2.0\,\text{N/C})}{8.99 \times 10^9\,\text{N} \cdot \text{m}^2/\text{C}^2} = 5.6 \times 10^{-11}\,\text{C}.$$

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Since the charge is uniformly distributed throughout a sphere, the electric field at the surface is exactly the same as it would be if the charge were all at the center. That is, the magnitude of the field is

$$E = \frac{q}{4\pi\epsilon_0 R^2},$$

where q is the magnitude of the total charge and R is the sphere radius. The magnitude of the total charge is Ze, so

$$E = \frac{Ze}{4\pi\epsilon_0 R^2} = \frac{(8.99 \times 10^9\,\text{N} \cdot \text{m}^2/\text{C}^2)(94)(1.60 \times 10^{-19}\,\text{C})}{(6.64 \times 10^{-15}\,\text{m})^2} = 3.07 \times 10^{21}\,\text{N/C}.$$

The field is normal to the surface and since the charge is positive it points outward from the surface.

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Choose the coordinate axes as shown on the diagram to the right. At the center of the square, the electric fields produced by the particles at the lower left and upper right corners are both along the x axis and each points away from the center and toward the particle that produces it. Since each particle is a distance $d = \sqrt{2}a/2 = a/\sqrt{2}$ away from the center, the net field due to these two particles is

$$E_x = \frac{1}{4\pi\epsilon_0}\left[\frac{2q}{a^2/2} - \frac{q}{a^2/2}\right]$$

$$= \frac{1}{4\pi\epsilon_0}\frac{q}{a^2/2} = \frac{(8.99 \times 10^9\,\text{N} \cdot \text{m}^2/\text{C}^2)(1.0 \times 10^{-8}\,\text{C})}{(0.050\,\text{m})^2/2} = 7.19 \times 10^4\,\text{N/C}.$$
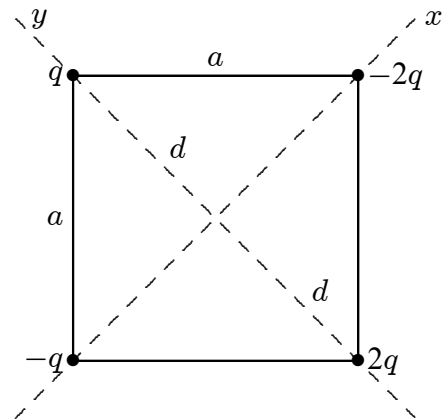
At the center of the square, the field produced by the particles at the upper left and lower right corners are both along the y axis and each points away from the particle that produces it. The net field produced at the center by these particles is

$$E_y = \frac{1}{4\pi\epsilon_0}\left[\frac{2q}{a^2/2} - \frac{q}{a^2/2}\right] = \frac{1}{4\pi\epsilon_0}\frac{q}{a^2/2} = 7.19 \times 10^4\,\text{N/C}.$$

The magnitude of the net field is

$$E = \sqrt{E_x^2 + E_y^2} = \sqrt{2(7.19 \times 10^4\,\text{N/C})^2} = 1.02 \times 10^5\,\text{N/C}$$

and the angle it makes with the x axis is

$$\theta = \tan^{-1}\frac{E_y}{E_x} = \tan^{-1}(1) = 45°\,.$$

It is upward in the diagram, from the center of the square toward the center of the upper side.

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Think of the quadrupole as composed of two dipoles, each with dipole moment of magnitude $p = qd$. The moments point in opposite directions and produce fields in opposite directions at points on the quadrupole axis. Consider the point P on the axis, a distance z to the right of the quadrupole center and take a rightward pointing field to be positive. Then the field produced by the right dipole of the pair is $qd/2\pi\epsilon_0(z - d/2)^3$ and the field produced by the left dipole is $-qd/2\pi\epsilon_0(z + d/2)^3$. Use the binomial expansions $(z - d/2)^{-3} \approx z^{-3} - 3z^{-4}(-d/2)$ and $(z + d/2)^{-3} \approx z^{-3} - 3z^{-4}(d/2)$ to obtain

$$E = \frac{qd}{2\pi\epsilon_0}\left[\frac{1}{z^3} + \frac{3d}{2z^4} - \frac{1}{z^3} + \frac{3d}{2z^4}\right] = \frac{6qd^2}{4\pi\epsilon_0 z^4}\,.$$

Let $Q = 2qd^2$. Then

$$E = \frac{3Q}{4\pi\epsilon_0 z^4}\,.$$

27

(a) The linear charge density λ is the charge per unit length of rod. Since the charge is uniformly distributed on the rod, $\lambda = -q/L = -(4.23 \times 10^{-15}\,\text{C})/(0.0815\,\text{m}) = -5.19 \times 10^{-14}\,\text{C/m}$.

(b) and (c) Position the origin at the left end of the rod, as shown in the diagram. Let dx be an infinitesimal length of rod at x. The charge in this segment is $dq = \lambda\,dx$. Since the segment may be taken to be a point particle, the electric field it produces at point P has only an x component and this component is given by
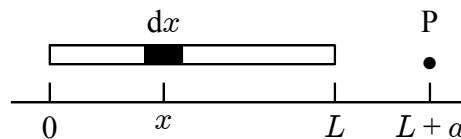
$$dE_x = \frac{1}{4\pi\epsilon_0}\frac{\lambda\,dx}{(L + a - x)^2}\,.$$

The total electric field produced at P by the whole rod is the integral

$$E_x = \frac{\lambda}{4\pi\epsilon_0} \int_0^L \frac{dx}{(L+a-x)^2} = \frac{\lambda}{4\pi\epsilon_0} \frac{1}{L+a-x}\Big|_0^L$$

$$= \frac{\lambda}{4\pi\epsilon_0}\left[\frac{1}{a} - \frac{1}{L+a}\right] = \frac{\lambda}{4\pi\epsilon_0}\frac{L}{a(L+a)}.$$

When $-q/L$ is substituted for λ the result is

$$E_x = -\frac{1}{4\pi\epsilon_0}\frac{q}{a(L+a)} = -\frac{(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)(4.23\times10^{-15}\,\text{C})}{(0.120\,\text{m})(0.0815\,\text{m}+0.120\,\text{m})} = -1.57\times10^{-3}\,\text{N/C}.$$

The negative sign indicates that the field is toward the rod and makes an angle of $180°$ with the positive x direction.

(d) Now

$$E_x = -\frac{1}{4\pi\epsilon_0}\frac{q}{a(L+a)} = -\frac{(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)(4.23\times10^{-15}\,\text{C})}{(50\,\text{m})(0.0815\,\text{m}+50\,\text{m})} = -1.52\times10^{-8}\,\text{N/C}.$$

(e) The field of a point particle at the origin is

$$E_x = -\frac{q}{4\pi\epsilon_0 a^2} = -\frac{(8.99\times10^9\,\text{N}\cdot\text{m}^2/\text{C}^2)(4.23\times10^{-15}\,\text{C})}{(50\,\text{m})^2} = -1.52\times10^{-8}\,\text{N/C}.$$

35

At a point on the axis of a uniformly charged disk a distance z above the center of the disk, the magnitude of the electric field is

$$E = \frac{\sigma}{2\epsilon_0}\left[1 - \frac{z}{\sqrt{z^2+R^2}}\right],$$

where R is the radius of the disk and σ is the surface charge density on the disk. See Eq. 22–26. The magnitude of the field at the center of the disk ($z = 0$) is $E_c = \sigma/2\epsilon_0$. You want to solve for the value of z such that $E/E_c = 1/2$. This means

$$\frac{E}{E_c} = 1 - \frac{z}{\sqrt{z^2+R^2}} = \frac{1}{2}$$

or

$$\frac{z}{\sqrt{z^2+R^2}} = \frac{1}{2}.$$

Square both sides, then multiply them by z^2+R^2 to obtain $z^2 = (z^2/4)+(R^2/4)$. Thus $z^2 = R^2/3$ and $z = R/\sqrt{3} = (0.600\,\text{m})/\sqrt{3} = 0.346\,\text{m}$.

39

The magnitude of the force acting on the electron is $F = eE$, where E is the magnitude of the electric field at its location. The acceleration of the electron is given by Newton's second law:

$$a = \frac{F}{m} = \frac{eE}{m} = \frac{(1.60\times10^{-19}\,\text{C})(2.00\times10^4\,\text{N/C})}{9.11\times10^{-31}\,\text{kg}} = 3.51\times10^{15}\,\text{m/s}^2.$$

43

(a) The magnitude of the force on the particle is given by $F = qE$, where q is the magnitude of the charge carried by the particle and E is the magnitude of the electric field at the location of the particle. Thus

$$E = \frac{F}{q} = \frac{3.0 \times 10^{-6} \, \text{N}}{2.0 \times 10^{-9} \, \text{C}} = 1.5 \times 10^3 \, \text{N/C}.$$

The force points downward and the charge is negative, so the field points upward.

(b) The magnitude of the electrostatic force on a proton is

$$F_e = eE = (1.60 \times 10^{-19} \, \text{C})(1.5 \times 10^3 \, \text{N/C}) = 2.4 \times 10^{-16} \, \text{N}.$$

(c) A proton is positively charged, so the force is in the same direction as the field, upward.

(d) The magnitude of the gravitational force on the proton is

$$F_g = mg = (1.67 \times 10^{-27} \, \text{kg})(9.8 \, \text{m/s}^2) = 1.64 \times 10^{-26} \, \text{N}.$$

The force is downward.

(e) The ratio of the force magnitudes is

$$\frac{F_e}{F_g} = \frac{2.4 \times 10^{-16} \, \text{N}}{1.64 \times 10^{-26} \, \text{N}} = 1.5 \times 10^{10}.$$

45

(a) The magnitude of the force acting on the proton is $F = eE$, where E is the magnitude of the electric field. According to Newton's second law, the acceleration of the proton is $a = F/m = eE/m$, where m is the mass of the proton. Thus

$$a = \frac{(1.60 \times 10^{-19} \, \text{C})(2.00 \times 10^4 \, \text{N/C})}{1.67 \times 10^{-27} \, \text{kg}} = 1.92 \times 10^{12} \, \text{m/s}^2.$$

(b) Assume the proton starts from rest and use the kinematic equation $v^2 = v_0^2 + 2ax$ (or else $x = \frac{1}{2}at^2$ and $v = at$) to show that

$$v = \sqrt{2ax} = \sqrt{2(1.92 \times 10^{12} \, \text{m/s}^2)(0.0100 \, \text{m})} = 1.96 \times 10^5 \, \text{m/s}.$$

57

(a) If q is the positive charge in the dipole and d is the separation of the charged particles, the magnitude of the dipole moment is $p = qd = (1.50 \times 10^{-9} \, \text{C})(6.20 \times 10^{-6} \, \text{m}) = 9.30 \times 10^{-15} \, \text{C·m}$.

(b) If the initial angle between the dipole moment and the electric field is θ_0 and the final angle is θ, then the change in the potential energy as the dipole swings from $\theta = 0$ to $\theta = 180°$ is

$$\Delta U = -pE(\cos \theta - \cos \theta_0) = -(9.30 \times 10^{-15} \, \text{C} \cdot \text{m})(1100 \, \text{N/C})(\cos 180° - \cos 0)$$
$$= 2.05 \times 10^{-11} \, \text{J}.$$

79

(a) and (b) Since the field at the point on the x axis with coordinate $x = 2.0\,\text{cm}$ is in the positive x direction you know that the charged particle is on the x axis. The line through the point with coordinates $x = 3.0\,\text{cm}$ and $y = 3.0\,\text{cm}$ and parallel to the field at that point must pass through the position of the particle. Such a line has slope $(3.0)/(4.0) = 0.75$ and its equation is $y = 0.57 + (0.75)x$. The solution for $y = 0$ is $x = -1.0\,\text{cm}$, so the particle is located at the point with coordinates $x = -1.0\,\text{cm}$ and $y = 0$.

(c) The magnitude of the field at the point on the x axis with coordinate $x = 2.0\,\text{cm}$ is given by $E = (1/4\pi\epsilon_0)q/(2.0\,\text{cm} - x)^2$, so

$$q = 4\pi\epsilon_0 x^2 E = \frac{(0.020\,\text{m} + 0.010\,\text{m})^2(100\,\text{N/C})}{8.99 \times 10^9\,\text{N} \cdot \text{m}^2/\text{C}^2} = 1.0 \times 10^{-11}\,\text{C}.$$

81

(a) The potential energy of an electric dipole with dipole moment $\vec{p}$ in an electric field $\vec{E}$ is

$$U = -\vec{p} \cdot \vec{E} = (1.24 \times 10^{-30}\,\text{C} \cdot \text{m})(3.00\,\hat{\imath} + 4.00\,\hat{\jmath}) \cdot (4000\,\text{N/C})\,\hat{\imath}$$
$$= -(1.24 \times 10^{-30}\,\text{C} \cdot \text{m})(3.00)(4000\,\text{N/C}) = -1.49 \times 10^{-26}\,\text{J}.$$

Here we used $\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z$ to evaluate the scalar product.

(b) The torque is

$$\vec{\tau} = \vec{p} \times \vec{E} = (p_x\,\hat{\imath} + p_y\,\hat{\jmath}) \times (E_x\,\hat{\imath}) = -p_y E_x\,\hat{k}$$
$$= -(4.00)(1.24 \times 10^{-30}\,\text{C} \cdot \text{m})(4000\,\text{N/C}) = -(1.98 \times 10^{-26}\,\text{N} \cdot \text{m})\,\hat{k}.$$

(c) The work done by the agent is equal to the change in the potential energy of the dipole. The initial potential energy is $U_i = -1.49 \times 10^{-26}\,\text{J}$, as computed in part (a). The final potential energy is
$$U_f = (1.24 \times 10^{-30}\,\text{C} \cdot \text{m})(-4.00\,\hat{\imath} + 3.00\,\hat{\jmath}) \cdot (4000\,\text{N/C})\,\hat{\imath}$$
$$= -(1.24 \times 10^{-30}\,\text{C} \cdot \text{m})(-4.00)(4000\,\text{N/C}) = +1.98 \times 10^{-26}\,\text{J}.$$

The work done by the agent is $W = (1.98 \times 10^{-26}\,\text{J}) - (-1.49 \times 10^{-26}\,\text{J}) = 3.47 \times 10^{-26}\,\text{J}.$