Chapter 25

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(a) The capacitance of a parallel-plate capacitor is given by $C = \epsilon_0 A/d$, where A is the area of each plate and d is the plate separation. Since the plates are circular, the plate area is $A = \pi R^2$, where R is the radius of a plate. Thus

$$C = \frac{\epsilon_0 \pi R^2}{d} = \frac{(8.85 \times 10^{-12}\,\mathrm{F/m})\pi(8.20 \times 10^{-2}\,\mathrm{m})^2}{1.30 \times 10^{-3}\,\mathrm{m}} = 1.44 \times 10^{-10}\,\mathrm{F} = 144\,\mathrm{pF}\,.$$

(b) The charge on the positive plate is given by $q = CV$, where V is the potential difference across the plates. Thus $q = (1.44 \times 10^{-10}\,\mathrm{F})(120\,\mathrm{V}) = 1.73 \times 10^{-8}\,\mathrm{C} = 17.3\,\mathrm{nC}$.

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The charge initially on the charged capacitor is given by $q = C_1 V_0$, where C_1 (= 100 pF) is the capacitance and V_0 (= 50 V) is the initial potential difference. After the battery is disconnected and the second capacitor wired in parallel to the first, the charge on the first capacitor is $q_1 = C_1 V$, where v (= 35 V) is the new potential difference. Since charge is conserved in the process, the charge on the second capacitor is $q_2 = q - q_1$, where C_2 is the capacitance of the second capacitor. Substitute $C_1 V_0$ for q and $C_1 V$ for q_1 to obtain $q_2 = C_1(V_0 - V)$. The potential difference across the second capacitor is also V, so the capacitance is

$$C_2 = \frac{q_2}{V} = \frac{V_0 - V}{V} C_1 = \frac{50\,\mathrm{V} - 35\,\mathrm{V}}{35\,\mathrm{V}}(100\,\mathrm{pF}) = 43\,\mathrm{pF}\,.$$

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(a) After the switches are closed, the potential differences across the capacitors are the same and the two capacitors are in parallel. The potential difference from a to b is given by $V_{ab} = Q/C_{\mathrm{eq}}$, where Q is the net charge on the combination and C_{eq} is the equivalent capacitance.

The equivalent capacitance is $C_{\mathrm{eq}} = C_1 + C_2 = 4.0 \times 10^{-6}\,\mathrm{F}$. The total charge on the combination is the net charge on either pair of connected plates. The charge on capacitor 1 is

$$q_1 = C_1 V = (1.0 \times 10^{-6}\,\mathrm{F})(100\,\mathrm{V}) = 1.0 \times 10^{-4}\,\mathrm{C}$$

and the charge on capacitor 2 is

$$q_2 = C_2 V = (3.0 \times 10^{-6}\,\mathrm{F})(100\,\mathrm{V}) = 3.0 \times 10^{-4}\,\mathrm{C}\,,$$

so the net charge on the combination is $3.0 \times 10^{-4}\,\mathrm{C} - 1.0 \times 10^{-4}\,\mathrm{C} = 2.0 \times 10^{-4}\,\mathrm{C}$. The potential difference is

$$V_{ab} = \frac{2.0 \times 10^{-4}\,\mathrm{C}}{4.0 \times 10^{-6}\,\mathrm{F}} = 50\,\mathrm{V}\,.$$

(b) The charge on capacitor 1 is now $q_1 = C_1 V_{ab} = (1.0 \times 10^{-6}\,\mathrm{F})(50\,\mathrm{V}) = 5.0 \times 10^{-5}\,\mathrm{C}$.

(c) The charge on capacitor 2 is now $q_2 = C_2V_{ab} = (3.0 \times 10^{-6}\,\text{F})(50\,\text{V}) = 1.5 \times 10^{-4}\,\text{C}$.

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The total energy is the sum of the energies stored in the individual capacitors. Since they are connected in parallel, the potential difference V across the capacitors is the same and the total energy is $U = \frac{1}{2}(C_1 + C_2)V^2 = \frac{1}{2}(2.0 \times 10^{-6}\,\text{F} + 4.0 \times 10^{-6}\,\text{F})(300\,\text{V})^2 = 0.27\,\text{J}$.

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(a) Let q be the charge on the positive plate. Since the capacitance of a parallel-plate capacitor is given by $\epsilon_0 A/d$, the charge is $q = CV = \epsilon_0 AV/d$. After the plates are pulled apart, their separation is d' and the potential difference is V'. Then $q = \epsilon_0 AV'/d'$ and

$$V' = \frac{d'}{\epsilon_0 A}\,q = \frac{d'}{\epsilon_0 A}\,\frac{\epsilon_0 A}{d}\,V = \frac{d'}{d}V = \frac{8.00\,\text{mm}}{3.00\,\text{mm}}(6.00\,\text{V}) = 16.0\,\text{V}\,.$$

(b) The initial energy stored in the capacitor is

$$U_i = \frac{1}{2}CV^2 = \frac{\epsilon_0 AV^2}{2d} = \frac{(8.85 \times 10^{-12}\,\text{F/m})(8.50 \times 10^{-4}\,\text{m}^2)(6.00\,\text{V})}{2(3.00 \times 10^{-3}\,\text{mm})} = 4.51 \times 10^{-11}\,\text{J}$$

and the final energy stored is

$$U_f = \frac{1}{2}C'(V')^2 = \frac{1}{2}\,\frac{\epsilon_0 A}{d'}\,(V')^2 = \frac{(8.85 \times 10^{-12}\,\text{F/m})(8.50 \times 10^{-4}\,\text{m}^2)(16.0\,\text{V})}{2(8.00 \times 10^{-3}\,\text{mm})} = 1.20 \times 10^{-10}\,\text{J}\,.$$

(c) The work done to pull the plates apart is the difference in the energy: $W = U_f - U_i = 1.20 \times 10^{-10}\,\text{J} - 4.51 \times 10^{-11}\,\text{J} = 7.49 \times 10^{-11}\,\text{J}$.

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The capacitance of a cylindrical capacitor is given by

$$C = \kappa C_0 = \frac{2\pi\kappa\epsilon_0 L}{\ln(b/a)}\,,$$

where C_0 is the capacitance without the dielectric, κ is the dielectric constant, L is the length, a is the inner radius, and b is the outer radius. See Eq. 25–14. The capacitance per unit length of the cable is

$$\frac{C}{L} = \frac{2\pi\kappa\epsilon_0}{\ln(b/a)} = \frac{2\pi(2.6)(8.85 \times 10^{-12}\,\text{F/m})}{\ln\left[(0.60\,\text{mm})/(0.10\,\text{mm})\right]} = 8.1 \times 10^{-11}\,\text{F/m} = 81\,\text{pF/m}\,.$$

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The capacitance is given by $C = \kappa C_0 = \kappa\epsilon_0 A/d$, where C_0 is the capacitance without the dielectric, κ is the dielectric constant, A is the plate area, and d is the plate separation. The electric field between the plates is given by $E = V/d$, where V is the potential difference between the plates. Thus $d = V/E$ and $C = \kappa\epsilon_0 AE/V$. Solve for A:

$$A = \frac{CV}{\kappa\epsilon_0 E}\,.$$

For the area to be a minimum, the electric field must be the greatest it can be without breakdown occurring. That is,

$$A = \frac{(7.0 \times 10^{-8}\,\mathrm{F})(4.0 \times 10^{3}\,\mathrm{V})}{2.8(8.85 \times 10^{-12}\,\mathrm{F/m})(18 \times 10^{6}\,\mathrm{V/m})} = 0.63\,\mathrm{m}^2\,.$$

51

(a) The electric field in the region between the plates is given by $E = V/d$, where V is the potential difference between the plates and d is the plate separation. The capacitance is given by $C = \kappa\epsilon_0 A/d$, where A is the plate area and κ is the dielectric constant, so $d = \kappa\epsilon_0 A/C$ and

$$E = \frac{VC}{\kappa\epsilon_0 A} = \frac{(50\,\mathrm{V})(100 \times 10^{-12}\,\mathrm{F})}{5.4(8.85 \times 10^{-12}\,\mathrm{F/m})(100 \times 10^{-4}\,\mathrm{m}^2)} = 1.0 \times 10^{4}\,\mathrm{V/m}\,.$$

(b) The free charge on the plates is $q_f = CV = (100 \times 10^{-12}\,\mathrm{F})(50\,\mathrm{V}) = 5.0 \times 10^{-9}\,\mathrm{C}$.

(c) The electric field is produced by both the free and induced charge. Since the field of a large uniform layer of charge is $q/2\epsilon_0 A$, the field between the plates is

$$E = \frac{q_f}{2\epsilon_0 A} + \frac{q_f}{2\epsilon_0 A} - \frac{q_i}{2\epsilon_0 A} - \frac{q_i}{2\epsilon_0 A}\,,$$

where the first term is due to the positive free charge on one plate, the second is due to the negative free charge on the other plate, the third is due to the positive induced charge on one dielectric surface, and the fourth is due to the negative induced charge on the other dielectric surface. Note that the field due to the induced charge is opposite the field due to the free charge, so the fields tend to cancel. The induced charge is therefore

$$\begin{aligned} q_i &= q_f - \epsilon_0 AE \\ &= 5.0 \times 10^{-9}\,\mathrm{C} - (8.85 \times 10^{-12}\,\mathrm{F/m})(100 \times 10^{-4}\,\mathrm{m}^2)(1.0 \times 10^{4}\,\mathrm{V/m}) \\ &= 4.1 \times 10^{-9}\,\mathrm{C} = 4.1\,\mathrm{nC}\,. \end{aligned}$$

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Capacitors 3 and 4 are in parallel and may be replaced by a capacitor with capacitance $C_{34} = C_3 + C_4 = 30\,\mu\mathrm{F}$. Capacitors 1, 2, and the equivalent capacitor that replaced 3 and 4 are all in series, so the sum of their potential differences must equal the potential difference across the battery. Since all of these capacitors have the same capacitance the potential difference across each of them is one-third the battery potential difference or 3.0 V. The potential difference across capacitor 4 is the same as the potential difference across the equivalent capacitor that replaced 3 and 4, so the charge on capacitor 4 is $q_4 = C_4 V_4 = (15 \times 10^{-6}\,\mathrm{F})(3.0\,\mathrm{V}) = 45 \times 10^{-6}\,\mathrm{C}$.

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(a) and (b) The capacitors have the same plate separation d and the same potential difference V across their plates, so the electric field are the same within them. The magnitude of the field in either one is $E = V/d = (600\,\text{V})/(3.00 \times 10^{-3}\,\text{m}) = 2.00 \times 10^5\,\text{V/m}$.

(c) Let A be the area of a plate. Then the surface charge density on the positive plate is $\sigma_A = q_A/A = C_A V/A = (\epsilon_0 A/d)V/A = \epsilon_0 V/d = \epsilon_0 E = (8.85 \times 10^{-12}\,\text{N}\cdot\text{m}^2/\text{C}^2)(2.00 \times 10^5\,\text{V/m}) = 1.77 \times 10^{-6}\,\text{C/m}^2$, where CV was substituted for q and the expression $\epsilon_0 A/d$ for the capacitance of a parallel-plate capacitor was substituted for C.

(d) Now the capacitance is $\kappa\epsilon_0 A/d$, where κ is the dielectric constant. The surface charge density on the positive plate is $\sigma_B = \kappa\epsilon_0 E = \kappa\sigma_A = (2.60)(1.77 \times 10^{-6}\,\text{C/m}^2) = 4.60 \times 10^{-6}\,\text{C/m}^2$.

(e) The electric field in B is produced by the charge on the plates and the induced charge together while the field in A is produced by the charge on the plates alone. since the fields are the same $\sigma_B + \sigma_{\text{induced}} = \sigma_A$, so $\sigma_{\text{induced}} = \sigma_A - \sigma_B = 1.77 \times 10^{-6}\,\text{C/m}^2 - 4.60 \times 10^{-6}\,\text{C/m}^2 = -2.83 \times 10^{-6}\,\text{C/m}^2$.

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The electric field in the lower region is due to the charge on both plates and the charge induced on the upper and lower surfaces of the dielectric in the region. The charge induced on the dielectric surfaces of the upper region has the same magnitude but opposite sign on the two surfaces and so produces a net field of zero in the lower region. Similarly, the electric field in the upper region is due to the charge on the plates and the charge induced on the upper and lower surfaces of dielectric in that region. Thus the electric field in the upper region has magnitude $E_{\text{upper}} = q\kappa_{\text{upper}}\epsilon_0 A$ and the potential difference across that region is $V_{\text{upper}} = E_{\text{upper}}d$, where d is the thickness of the region. The electric field in the lower region is $E_{\text{lower}} = q\kappa_{\text{lower}}\epsilon_0 A$ and the potential difference across that region is $V_{\text{lower}} = E_{\text{lower}}d$. The sum of the potential differences must equal the potential difference V across the entire capacitor, so

$$V = E_{\text{upper}}d + E_{\text{lower}}d = \frac{qd}{\epsilon_0 A}\left[\frac{1}{\kappa_{\text{upper}}} + \frac{1}{\kappa_{\text{lower}}}\right] .$$

The solution for q is

$$q = \frac{\kappa_{\text{upper}}\kappa_{\text{lower}}}{\kappa_{\text{upper}} + \kappa_{\text{lower}}}\frac{\epsilon_0 A}{d}V = \frac{(3.00)(4.00)}{3.00 + 4.00}\,\frac{(8.85 \times 10^{-12}\,\text{N}\cdot m^2/\text{C}^2)(2.00 \times 10^{-2}\,\text{m}^2)}{2.00 \times 10^{-3}\,\text{m}}(7.00\,\text{V})$$
$$= 1.06 \times 10^{-9}\,\text{C} .$$