Chapter 29

7

(a) If the currents are parallel, the two magnetic fields are in opposite directions in the region between the wires. Since the currents are the same, the net field is zero along the line that runs halfway between the wires. There is no possible current for which the field does not vanish. If there is to be a field on the bisecting line the currents must be in opposite directions. Then the fields are in the same direction in the region between the wires.

(b) At a point halfway between the wires, the fields have the same magnitude, $\mu_0 i/2\pi r$. Thus the net field at the midpoint has magnitude $B = \mu_0 i/\pi r$ and

$$i = \frac{\pi r B}{\mu_0} = \frac{\pi(0.040\,\text{m})(300 \times 10^{-6}\,\text{T})}{4\pi \times 10^{-7}\,\text{T} \cdot \text{m/A}} = 30\,\text{A}\,.$$

15

Sum the fields of the two straight wires and the circular arc. Look at the derivation of the expression for the field of a long straight wire, leading to Eq. 29–6. Since the wires we are considering are infinite in only one direction, the field of either of them is half the field of an infinite wire. That is, the magnitude is $\mu_0 i/4\pi R$, where R is the distance from the end of the wire to the center of the arc. It is the radius of the arc. The fields of both wires are out of the page at the center of the arc.

Now find an expression for the field of the arc at its center. Divide the arc into infinitesimal segments. Each segment produces a field in the same direction. If ds is the length of a segment, the magnitude of the field it produces at the arc center is $(\mu_0 i/4\pi R^2)\,ds$. If θ is the angle subtended by the arc in radians, then $R\theta$ is the length of the arc and the net field of the arc is $\mu_0 i\theta/4\pi R$. For the arc of the diagram, the field is into the page. The net field at the center, due to the wires and arc together, is

$$B = \frac{\mu_0 i}{4\pi R} + \frac{\mu_0 i}{4\pi R} - \frac{\mu_0 i\theta}{4\pi R} = \frac{\mu_0 i}{4\pi R}(2 - \theta)\,.$$

For this to vanish, θ must be exactly 2 radians.

19

Each wire produces a field with magnitude given by $B = \mu_0 i/2\pi r$, where r is the distance from the corner of the square to the center. According to the Pythagorean theorem, the diagonal of the square has length $\sqrt{2}a$, so $r = a/\sqrt{2}$ and $B = \mu_0 i/\sqrt{2}\pi a$. The fields due to the wires at the upper left and lower right corners both point toward the upper right corner of the square. The

fields due to the wires at the upper right and lower left corners both point toward the upper left corner. The horizontal components cancel and the vertical components sum to

$$B_{net} = 4\frac{\mu_0 i}{\sqrt{2}\,2\pi a}\cos 45° = \frac{2\mu_0 i}{\pi a}$$

$$= \frac{2(4\pi \times 10^{-7}\,\text{T} \cdot \text{m/A})(20\,\text{A})}{\pi(0.20\,\text{m})} = 8.0 \times 10^{-5}\,\text{T}.$$

In the calculation $\cos 45°$ was replaced with $1/\sqrt{2}$. In unit vector notation $\vec{B} = (8.0 \times 10^{-5}\,\text{T})\hat{\jmath}$.

21

Follow the same steps as in the solution of Problem 17 above but change the lower limit of integration to $-L$, and the upper limit to 0. The magnitude of the net field is

$$B = \frac{\mu_0 i R}{4\pi}\int_{-L}^{0}\frac{dx}{(x^2+R^2)^{3/2}} = \frac{\mu_0 i R}{4\pi}\frac{1}{R^2}\frac{x}{(x^2+R^2)^{1/2}}\bigg|_{-L}^{0} = \frac{\mu_0 i}{4\pi R}\frac{L}{\sqrt{L^2+R^2}}$$

$$= \frac{4\pi \times 10^{-7}\,\text{T} \cdot \text{m/A}(0.693\,\text{A})}{4\pi(0.251\,\text{m})}\frac{0.136\,\text{m}}{\sqrt{(0.136\,\text{m})^2+(0.251\,\text{m})^2}} = 1.32 \times 10^{-7}\,\text{T}.$$

31

The current per unit width of the strip is i/w and the current through a width dx is $(i/w)\,dx$. Treat this as a long straight wire. The magnitude of the field it produces at a point that is a distance d from the edge of the strip is $dB = (\mu_0/2\pi)(i/w)\,dx/x$ and the net field is

$$B = \frac{\mu_0 i}{2\pi w}\int_{d}^{d+w}\frac{dx}{x} = \frac{\mu_0}{2\pi w}\ln\frac{d+w}{d}$$

$$= \frac{(4\pi \times 10^{-7}\,\text{T} \cdot \text{m/A})(4.61 \times 10^{-6}\,\text{A})}{2\pi(0.0491\,\text{m})}\ln\frac{0.0216\,\text{m}+0.0491\,\text{m}}{0.0216\,\text{m}}$$

$$= 2.23 \times 10^{-11}\,\text{T}.$$

35

The magnitude of the force of wire 1 on wire 2 is given by $\mu_0 i_1 i_2/2\pi r$, where i_1 is the current in wire 1, i_2 is the current in wire 2, and r is the separation of the wires. The distance between the wires is $r = \sqrt{d_1^2 + d_2^2}$. Since the currents are in opposite directions the wires repel each other so the force on wire 2 is along the line that joins the wires and is away from wire 1.

To find the x component of the force, multiply the magnitude of the force by the cosine of the angle θ that the force makes with the x axis. This is $\cos\theta = d_2/\sqrt{d_1^2 + d_2^2}$. Thus the x component
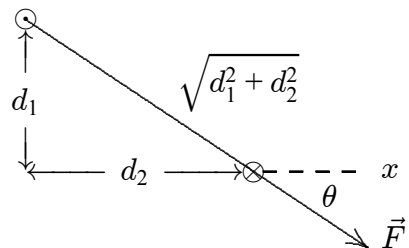
of the force is

$$F_x = \frac{\mu_0 i_1 i_2}{2\pi} \frac{d_2}{d_1^2 + d_2^2}$$

$$= \frac{(2\pi \times 10^{-7}\,\text{T} \cdot \text{m/A})(4.00 \times 10^{-3}\,\text{A})(6.80 \times 10^{-3}\,\text{A})}{2\pi} \frac{0.0500\,\text{m}}{(0.024\,\text{m})^2 + (5.00\,\text{m})^2}$$

$$= 8.84 \times 10^{-11}\,\text{T}.$$

43

(a) Two of the currents are out of the page and one is into the page, so the net current enclosed by the path is 2.0 A, out of the page. Since the path is traversed in the clockwise sense, a current into the page is positive and a current out of the page is negative, as indicated by the right-hand rule associated with Ampere's law. Thus $i_{\text{enc}} = -i$ and

$$\oint \vec{B} \cdot d\vec{s} = -\mu_0 i = -(4\pi \times 10^{-7}\,\text{T} \cdot \text{m/A})(2.0\,\text{A}) = -2.5 \times 10^{-6}\,\text{T} \cdot \text{m}.$$

(b) The net current enclosed by the path is zero (two currents are out of the page and two are into the page), so $\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{\text{enc}} = 0$.

53

(a) Assume that the point is inside the solenoid. The field of the solenoid at the point is parallel to the solenoid axis and the field of the wire is perpendicular to the solenoid axis. The net field makes an angle of 45° with the axis if these two fields have equal magnitudes.

The magnitude of the magnetic field produced by a solenoid at a point inside is given by $B_{\text{sol}} = \mu_0 i_{\text{sol}} n$, where n is the number of turns per unit length and i_{sol} is the current in the solenoid. The magnitude of the magnetic field produced by a long straight wire at a point a distance r away is given by $B_{\text{wire}} = \mu_0 i_{\text{wire}}/2\pi r$, where i_{wire} is the current in the wire. We want $\mu_0 n i_{\text{sol}} = \mu_0 i_{\text{wire}}/2\pi r$. The solution for r is

$$r = \frac{i_{\text{wire}}}{2\pi n i_{\text{sol}}} = \frac{6.00\,\text{A}}{2\pi(10.0 \times 10^2\,\text{m}^{-1})(20.0 \times 10^{-3}\,\text{A})} = 4.77 \times 10^{-2}\,\text{m} = 4.77\,\text{cm}.$$

This distance is less than the radius of the solenoid, so the point is indeed inside as we assumed.

(b) The magnitude of the either field at the point is

$$B_{\text{sol}} = B_{\text{wire}} = \mu_0 n i_{\text{sol}} = (4\pi \times 10^{-7}\,\text{T} \cdot \text{m/A})(10.0 \times 10^2\,\text{m}^{-1})(20.0 \times 10^{-3}\,\text{A}) = 2.51 \times 10^{-5}\,\text{T}.$$

Each of the two fields is a vector component of the net field, so the magnitude of the net field is the square root of the sum of the squares of the individual fields: $B = \sqrt{2(2.51 \times 10^{-5}\,\text{T})^2} = 3.55 \times 10^{-5}\,\text{T}$.

57

The magnitude of the dipole moment is given by $\mu = NiA$, where N is the number of turns, i is the current, and A is the area. Use $A = \pi R^2$, where R is the radius. Thus

$$\mu = Ni\pi R^2 = (200)(0.30\,\text{A})\pi(0.050\,\text{m})^2 = 0.47\,\text{A} \cdot \text{m}^2.$$

65

(a) Take the magnetic field at a point within the hole to be the sum of the fields due to two current distributions. The first is the solid cylinder obtained by filling the hole and has a current density that is the same as that in the original cylinder with the hole. The second is the solid cylinder that fills the hole. It has a current density with the same magnitude as that of the original cylinder but it is in the opposite direction. Notice that if these two situations are superposed, the total current in the region of the hole is zero.

Recall that a solid cylinder carrying current i, uniformly distributed over a cross section, produces a magnetic field with magnitude $B = \mu_0 ir/2\pi R^2$ a distance r from its axis, inside the cylinder. Here R is the radius of the cylinder.

For the cylinder of this problem, the current density is

$$J = \frac{i}{A} = \frac{i}{\pi(a^2 - b^2)},$$

where A ($= \pi(a^2 - b^2)$) is the cross-sectional area of the cylinder with the hole. The current in the cylinder without the hole is

$$i_1 = JA_1 = \pi J a^2 = \frac{ia^2}{a^2 - b^2}$$

and the magnetic field it produces at a point inside, a distance r_1 from its axis, has magnitude

$$B_1 = \frac{\mu_0 i_1 r_1}{2\pi a^2} = \frac{\mu_0 i r_1 a^2}{2\pi a^2 (a^2 - b^2)} = \frac{\mu_0 i r_1}{2\pi (a^2 - b^2)}.$$

The current in the cylinder that fills the hole is

$$i_2 = \pi J b^2 = \frac{ib^2}{a^2 - b^2}$$

and the field it produces at a point inside, a distance r_2 from the its axis, has magnitude

$$B_2 = \frac{\mu_0 i_2 r_2}{2\pi b^2} = \frac{\mu_0 i r_2 b^2}{2\pi b^2 (a^2 - b^2)} = \frac{\mu_0 i r_2}{2\pi (a^2 - b^2)}.$$

At the center of the hole, this field is zero and the field there is exactly the same as it would be if the hole were filled. Place $r_1 = d$ in the expression for B_1 and obtain

$$B = \frac{\mu_0 i d}{2\pi(a^2 - b^2)} = \frac{(4\pi \times 10^{-7}\,\text{T} \cdot \text{m/A})(5.25\,\text{A})(0.0200\,\text{m})}{2\pi[(0.0400\,\text{m})^2 - (0.0150\,\text{m})^2]} = 1.53 \times 10^{-5}\,\text{T}$$

for the field at the center of the hole. The field points upward in the diagram if the current is out of the page.

(b) If $b = 0$, the formula for the field becomes

$$B = \frac{\mu_0 i d}{2\pi a^2}.$$

This correctly gives the field of a solid cylinder carrying a uniform current i, at a point inside the cylinder a distance d from the axis. If $d = 0$, the formula gives $B = 0$. This is correct for the field on the axis of a cylindrical shell carrying a uniform current.

(c) The diagram shows the situation in a cross-sectional plane of the cylinder. P is a point within the hole, A is on the axis of the cylinder, and C is on the axis of the hole. The magnetic field due to the cylinder without the hole, carrying a uniform current out of the page, is labeled $\vec{B}_1$ and the magnetic field of the cylinder that fills the hole, carrying a uniform current into the page, is labeled $\vec{B}_2$. The line from A to P makes the angle θ_1 with the line that joins the centers of the cylinders and the line from C to P makes the angle θ_2 with that line, as shown. $\vec{B}_1$ is perpendicular to the line from A to P and so makes the angle θ_1 with the vertical. Similarly, $\vec{B}_2$ is perpendicular to the line from C to P and so makes the angle θ_2 with the vertical.
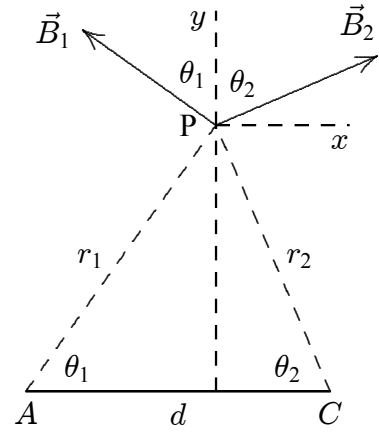
The x component of the total field is

$$B_x = B_2 \sin \theta_2 - B_1 \sin \theta_1 = \frac{\mu_0 i r_2}{2\pi(a^2 - b^2)} \sin \theta_2 - \frac{\mu_0 i r_1}{2\pi(a^2 - b^2)} \sin \theta_1$$

$$= \frac{\mu_0 i}{2\pi(a^2 - b^2)} [r_2 \sin \theta_2 - r_1 \sin \theta_1] .$$

As the diagram shows, $r_2 \sin \theta_2 = r_1 \sin \theta_1$, so $B_x = 0$. The y component is given by

$$B_y = B_2 \cos \theta_2 + B_1 \cos \theta_1 = \frac{\mu_0 i r_2}{2\pi(a^2 - b^2)} \cos \theta_2 + \frac{\mu_0 i r_1}{2\pi(a^2 - b^2)} \cos \theta_1$$

$$= \frac{\mu_0 i}{2\pi(a^2 - b^2)} [r_2 \cos \theta_2 + r_1 \cos \theta_1] .$$

The diagram shows that $r_2 \cos \theta_2 + r_1 \cos \theta_1 = d$, so

$$B_y = \frac{\mu_0 i d}{2\pi(a^2 - b^2)} .$$

This is identical to the result found in part (a) for the field on the axis of the hole. It is independent of r_1, r_2, θ_1, and θ_2, showing that the field is uniform in the hole.

71

Use the Biot-Savart law in the form

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{i \Delta\vec{s} \times \vec{r}}{r^3} .$$

Take $\Delta\vec{s}$ to be $\Delta s\,\hat{\jmath}$, and $\vec{r}$ to be $x\,\hat{\imath} + y\,\hat{\jmath} + z\,\hat{k}$. Then $\Delta\vec{s} \times \vec{r} = \Delta s\,\hat{\jmath} \times (x\,\hat{\imath} + y\,\hat{\jmath} + z\,\hat{k}) = \Delta s(z\,\hat{\imath} - x\,\hat{k})$, where $\hat{\jmath} \times \hat{\imath} = -\hat{k}$, $\hat{\jmath} \times \hat{\jmath} = 0$, and $\hat{\jmath} \times \hat{k} = \hat{\imath}$ were used. In addition, $r = \sqrt{x^2 + y^2 + z^2}$. The Biot-Savart equation becomes

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{i\,\Delta s(z\,\hat{\imath} - z\,\hat{k})}{(x^2 + y^2 + z^2)^{3/2}} .$$

(a) For $x = 0$, $y = 0$, and $z = 5.0\,\mathrm{m}$,

$$\vec{B} = \frac{4\pi \times 10^{-7}\,\mathrm{T \cdot m/A}}{4\pi} \frac{(2.0\,\mathrm{A})(0.030\,\mathrm{m})(5.0\,\mathrm{m})\,\hat{\imath}}{(5.0\,\mathrm{m})^3} = (2.4 \times 10^{-10}\,\mathrm{T})\,\hat{\imath}.$$

(b) For $x = 0$, $y = 6.0\,\mathrm{m}$, and $z = 0$, $\vec{B} = 0$.

(c) For $x = 7.0\,\mathrm{m}$, $y = 7.0\,\mathrm{m}$, and $z = 0$,

$$\vec{B} = \frac{4\pi \times 10^{-7}\,\mathrm{T \cdot m/A}}{4\pi} \frac{(2.0\,\mathrm{A})(0.030\,\mathrm{m})(-7.0\,\mathrm{m})\,\hat{k}}{[(7.0\,\mathrm{m})^2 + (7.0\,\mathrm{m})^2]^{3/2}} = (4.3 \times 10^{-11}\,\mathrm{T})\,\hat{k}.$$

(d) For $x = -3.0\,\mathrm{m}$, $y = -4.0\,\mathrm{m}$, and $z = 0$,

$$\vec{B} = \frac{4\pi \times 10^{-7}\,\mathrm{T \cdot m/A}}{4\pi} \frac{(2.0\,\mathrm{A})(0.030\,\mathrm{m})(3.0\,\mathrm{m})\,\hat{k}}{[(-3.0\,\mathrm{m})^2 + (-4.0\,\mathrm{m})^2]^{3/2}} = (1.4 \times 10^{-10}\,\mathrm{T})\,\hat{k}.$$

77

First consider the finite wire segment shown on the right. It extends from $y = -d$ to $y = a - d$, where a is the length of the segment, and it carries current i in the positive y direction. Let dy be an infinitesimal length of wire at coordinate y. According to the Biot-Savart law the magnitude of the magnetic field at P due to this infinitesimal length is $dB = (\mu_0/4\pi)(i \sin\theta/r^2)\,dy$. Now $r^2 = y^2 + R^2$ and $\sin\theta = R/r = R/\sqrt{y^2 + R^2}$, so

$$dB = \frac{\mu_0}{4\pi} \frac{iR}{(y^2 + R^2)^{3/2}}\,dy$$
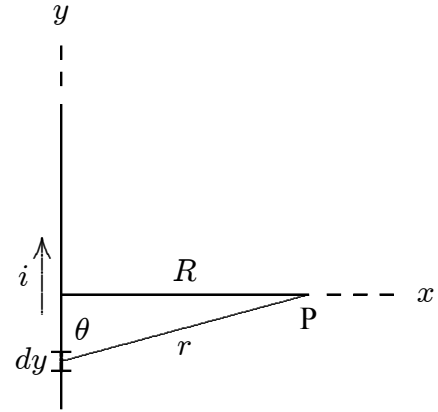
and the field of the entire segment is

$$B = \frac{\mu_0}{4\pi} iR \int_{-d}^{a-d} \frac{y}{(y^2 + R^2)^{3/2}}\,dy = \mu_0/4\pi \frac{i}{R}\left[\frac{a - d}{\sqrt{R^2 + (a-d)^2}} + \frac{d}{\sqrt{R^2 + d^2}}\right],$$

where integral 19 of Appendix E was used.

All four sides of the square produce magnetic fields that are into the page at P, so we sum their magnitudes. To calculate the field of the left side of the square put $d = 3a/4$ and $R = a/4$. The result is

$$B_{\text{left}} = \frac{\mu_0}{4\pi} \frac{4i}{a}\left[\frac{1}{\sqrt{2}} + \frac{3}{\sqrt{10}}\right] = \frac{\mu_O}{3\pi} \frac{4i}{a}(1.66).$$

The field of the upper side of the square is the same. To calculate the field of the right side of the square put $d = a/4$ and $R = 3a/4$. The result is

$$B_{\text{right}} = \frac{\mu_0}{4\pi} \frac{4i}{3a}\left[\frac{3}{\sqrt{18}} + \frac{1}{\sqrt{10}}\right] = \frac{\mu_O}{3\pi} \frac{4i}{a}(0.341).$$

The field of the bottom side is the same. The total field at P is

$$B = B_{\text{left}} + B_{\text{upper}} + B_{\text{right}} + B_{\text{lower}} = \frac{\mu_0}{4\pi} \frac{4i}{a} (1.66 + 1.66 + 0.341 + 0.341)$$

$$= \frac{4\pi \times 10^{-7}\,\text{T} \cdot \text{m/A}}{4\pi} \frac{4(10\,\text{A})}{0.080\,\text{m}}(4.00) = 2.0 \times 10^{-4}\,\text{T}.$$

79

(a) Suppose the field is not parallel to the sheet, as shown in the upper diagram. Reverse the direction of the current. According to the Biot-Savart law, the field reverses, so it will be as in the second diagram. Now rotate the sheet by 180° about a line that is perpendicular to the sheet. The field, of course, will rotate with it and end up in the direction shown in the third diagram. The current distribution is now exactly as it was originally, so the field must also be as it was originally. But it is not. Only if the field is parallel to the sheet will be final direction of the field be the same as the original direction. If the current is out of the page, any infinitesimal portion of the sheet in the form of a long straight wire produces a field that is to the left above the sheet and to the right below the sheet. The field must be as drawn in Fig. 29–85.
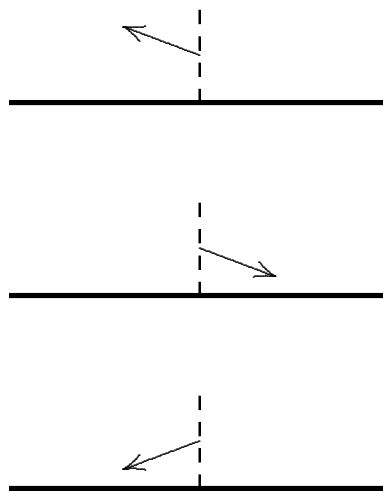
(b) Integrate the tangential component of the magnetic field around the rectangular loop shown with dotted lines. The upper and lower edges are the same distance from the current sheet and each has length L. This means the field has the same magnitude along these edges. It points to the left along the upper edge and to the right along the lower.
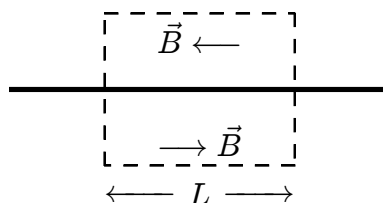
If the integration is carried out in the counterclockwise sense, the contribution of the upper edge is BL, the contribution of the lower edge is also BL, and the contribution of each of the sides is zero because the field is perpendicular to the sides. Thus $\oint \vec{B} \cdot d\vec{s} = 2BL$. The total current through the loop is λL. Ampere's law yields $2BL = \mu_0 \lambda L$, so $B = \mu_0 \lambda / 2$.

81

(a) Use a circular Amperian path that has radius r and is concentric with the cylindrical shell as shown by the dotted circle on Fig. 29–86. The magnetic field is tangent to the path and has uniform magnitude on it, so the integral on the left side of the Ampere's law equation is $\oint \vec{B} \cdot d\vec{s} = 2\pi r B$. The current through the Amperian path is the current through the region outside the circle of radius b and inside the circle of radius r. Since the current is uniformly distributed through a cross section of the shell, the enclosed current is $i(r^2 - b^2)/(a^2 - b^2)$. Thus
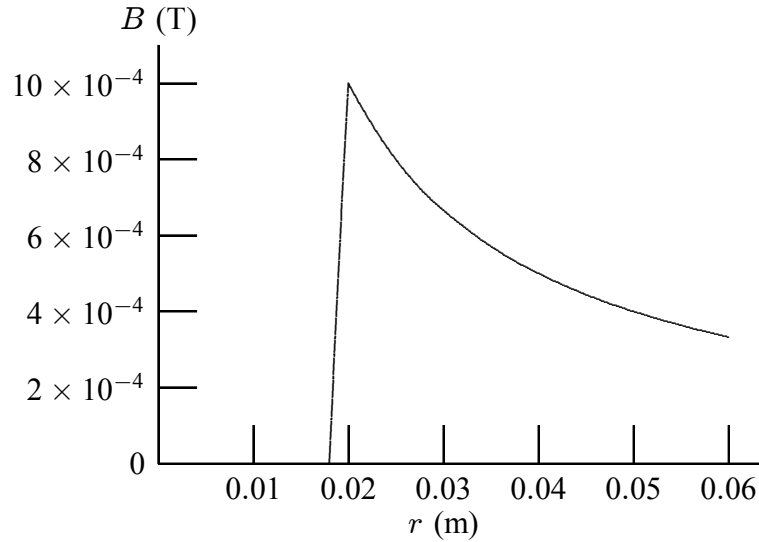
$$2\pi r B = \frac{r^2 - b^2}{a^2 - b^2} i$$

and

$$B = \frac{\mu_0 i}{2\pi(a^2 - b^2)} \frac{r^2 - b^2}{r}.$$

(b) When $r = a$ this expression reduce to $B = \mu_0 i/2\pi r$, which is the correct expression for the field of a long straight wire. When $r = b$ it reduces to $B = 0$, which is correct since there is no field inside the shell. When $b = 0$ it reduces to $B = \mu_0 i r/2\pi a^2$, which is correct for the field inside a cylindrical conductor.

(c) The graph is shown below.

89

The result of Problem 11 is used four times, once for each of the sides of the square loop. A point on the axis of the loop is also on a perpendicular bisector of each of the loop sides. The diagram shows the field due to one of the loop sides, the one on the left. In the expression found in Problem 11, replace L with a and R with $\sqrt{x^2 + a^2/4} = \frac{1}{2}\sqrt{4x^2 + a^2}$. The field due to the side is therefore

$$B = \frac{\mu_0 i a}{\pi\sqrt{4x^2 + a^2}\sqrt{4x^2 + 2a^2}}.$$

The field is in the plane of the dotted triangle shown and is perpendicular to the line from the midpoint of the loop side to the point P. Therefore it makes the angle θ with the vertical.

When the fields of the four sides are summed vectorially the horizontal components add to zero. The vertical components are all the same, so the total field is given by

$$B_{\text{total}} = 4B \cos\theta = \frac{4Ba}{2R} = \frac{4Ba}{\sqrt{4x^2 + a^2}}.$$
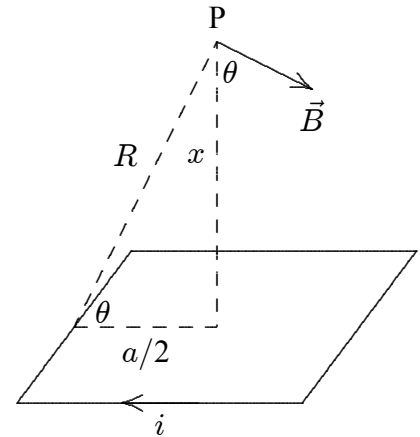
Thus

$$B_{\text{total}} = \frac{4\mu_0 i a^2}{\pi(4x^2 + a^2)\sqrt{4x^2 + 2a^2}} \, .$$

For $x = 0$, the expression reduces to

$$B_{\text{total}} = \frac{4\mu_0 i a^2}{\pi a^2 \sqrt{2} a} = \frac{2\sqrt{2}\mu_0 i}{\pi a} \, ,$$

in agreement with the result of Problem 12.

91

Use Ampere's law: $\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{\text{enc}}$, where the integral is around a closed loop and i_{enc} is the net current through the loop. For the dashed loop shown on the diagram $i = 0$. Assume the integral $\int \vec{B} \cdot d\vec{s}$ is zero along the bottom, right, and top sides of the loop as it would be if the field lines are as shown on the diagram. Along the right side the field is zero and along the top and bottom sides the field is perpendicular to $d\vec{s}$. If ℓ is the length of the left edge, then direct integration yields $\oint \vec{B} \cdot d\vec{s} = B\ell$, where B is the magnitude of the field at the left side of the loop. Since neither B nor ℓ is zero, Ampere's law is contradicted. We conclude that the geometry shown for the magnetic field lines is in error. The lines actually bulge outward and their density decreases gradually, not precipitously as shown.