

Chapter 24

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(a) An ampere is a coulomb per second, so

$$84 \text{ A} \cdot \text{h} = \left(84 \frac{\text{C} \cdot \text{h}}{\text{s}}\right) \left(3600 \frac{\text{s}}{\text{h}}\right) = 3.0 \times 10^5 \text{ C}.$$

(b) The change in potential energy is $\Delta U = q \Delta V = (3.0 \times 10^5 \text{ C})(12 \text{ V}) = 3.6 \times 10^6 \text{ J}$.

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The electric field produced by an infinite sheet of charge has magnitude $E = \sigma/2\epsilon_0$, where σ is the surface charge density. The field is normal to the sheet and is uniform. Place the origin of a coordinate system at the sheet and take the x axis to be parallel to the field and positive in the direction of the field. Then the electric potential is

$$V = V_s - \int_0^x E dx = V_s - Ex,$$

where V_s is the potential at the sheet. The equipotential surfaces are surfaces of constant x ; that is, they are planes that are parallel to the plane of charge. If two surfaces are separated by Δx then their potentials differ in magnitude by $\Delta V = E\Delta x = (\sigma/2\epsilon_0)\Delta x$. Thus

$$\Delta x = \frac{2\epsilon_0 \Delta V}{\sigma} = \frac{2(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(50 \text{ V})}{0.10 \times 10^{-6} \text{ C/m}^2} = 8.8 \times 10^{-3} \text{ m}.$$

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(a) The electric potential V at the surface of the drop, the charge q on the drop, and the radius R of the drop are related by $V = q/4\pi\epsilon_0 R$. Thus

$$R = \frac{q}{4\pi\epsilon_0 V} = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(30 \times 10^{-12} \text{ C})}{500 \text{ V}} = 5.4 \times 10^{-4} \text{ m}.$$

(b) After the drops combine the total volume is twice the volume of an original drop, so the radius R' of the combined drop is given by $(R')^3 = 2R^3$ and $R' = 2^{1/3}R$. The charge is twice the charge of original drop: $q' = 2q$. Thus

$$V' = \frac{1}{4\pi\epsilon_0} \frac{q'}{R'} = \frac{1}{4\pi\epsilon_0} \frac{2q}{2^{1/3}R} = 2^{2/3}V = 2^{2/3}(500 \text{ V}) = 790 \text{ V}.$$

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The disk is uniformly charged. This means that when the full disk is present each quadrant contributes equally to the electric potential at P , so the potential at P due to a single quadrant is one-fourth the potential due to the entire disk. First find an expression for the potential at P due to the entire disk.

Consider a ring of charge with radius r and width dr . Its area is $2\pi r dr$ and it contains charge $dq = 2\pi\sigma r dr$. All the charge in it is a distance $\sqrt{r^2 + D^2}$ from P , so the potential it produces at P is

$$dV = \frac{1}{4\pi\epsilon_0} \frac{2\pi\sigma r dr}{\sqrt{r^2 + D^2}} = \frac{\sigma r dr}{2\epsilon_0\sqrt{r^2 + D^2}}.$$

The total potential at P is

$$V = \frac{\sigma}{2\epsilon_0} \int_0^R \frac{r dr}{\sqrt{r^2 + D^2}} = \frac{\sigma}{2\epsilon_0} \sqrt{r^2 + D^2} \Big|_0^R = \frac{\sigma}{2\epsilon_0} [\sqrt{R^2 + D^2} - D].$$

The potential V_{sq} at P due to a single quadrant is

$$\begin{aligned} V_{sq} &= \frac{V}{4} = \frac{\sigma}{8\epsilon_0} [\sqrt{R^2 + D^2} - D] \\ &= \frac{7.73 \times 10^{-15} \text{ C/m}^2}{8(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)} [\sqrt{(0.640 \text{ m})^2 + (0.259 \text{ m})^2} - 0.259 \text{ m}] \\ &= 4.71 \times 10^{-5} \text{ V}. \end{aligned}$$

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Take the negatives of the partial derivatives of the electric potential with respect to the coordinates and evaluate the results for $x = 3.00 \text{ m}$, $y = -2.00 \text{ m}$, and $z = 4.00 \text{ m}$. This yields

$$\begin{aligned} E_x &= -\frac{\partial V}{\partial x} = -(2.00 \text{ V/m}^4)yz^2 = -(2.00 \text{ V/m}^4)((-2.00 \text{ m})(4.00 \text{ m})^2) = 64.0 \text{ V/m}, \\ E_y &= -\frac{\partial V}{\partial y} = -(2.00 \text{ V/m}^4)xz^2 = -(2.00 \text{ V/m}^4)(3.00 \text{ m})(4.00 \text{ m})^2 = -96.0 \text{ V/m}, \\ E_z &= -\frac{\partial V}{\partial z} = -2(2.00 \text{ V/m}^4)xyz = -2(2.00 \text{ V/m}^4)(3.00 \text{ m})(-2.00 \text{ m})(4.00 \text{ m}) = 96.0 \text{ V/m}. \end{aligned}$$

The magnitude of the electric field is

$$E = \sqrt{E_x^2 + E_y^2 + E_z^2} = \sqrt{(64.0 \text{ V/m})^2 + (-96.0 \text{ V/m})^2 + (96.0 \text{ V/m})^2} = 1.50 \times 10^2 \text{ V/m}.$$

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The work required is equal to the potential energy of the system, relative to a potential energy of zero for infinite separation. Number the particles 1, 2, 3, and 4, in clockwise order starting with the particle in the upper left corner of the arrangement. The potential energy of the interaction of particles 1 and 2 is

$$\begin{aligned} U_{12} &= \frac{q_1 q_2}{4\pi\epsilon_0 a} = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(2.30 \times 10^{-12} \text{ C})(-2.30 \times 10^{-12} \text{ C})}{0.640 \text{ m}} \\ &= -7.43 \times 10^{-14} \text{ J}. \end{aligned}$$

The distance between particles 1 and 3 is $\sqrt{2}a$ and both these particles are positively charged, so the potential energy of the interaction between particles 1 and 3 is $U_{13} = -U_{12}/\sqrt{2} = +5.25 \times$

10^{-14} J. The potential energy of the interaction between particles 1 and 4 is $U_{14} = U_{12} = -7.43 \times 10^{-14}$ J. The potential energy of the interaction between particles 2 and 3 is $U_{23} = U_{12} = -7.43 \times 10^{-14}$ J. The potential energy of the interaction between particles 2 and 4 is $U_{24} = U_{13} = 5.25 \times 10^{-14}$ J. The potential energy of the interaction between particles 3 and 4 is $U_{34} = U_{12} = -7.43 \times 10^{-14}$ J.

The total potential energy of the system is

$$\begin{aligned} U &= U_{12} + U_{13} + U_{14} + U_{23} + U_{24} + U_{34} \\ &= -7.43 \times 10^{-14} \text{ J} + 5.25 \times 10^{-14} \text{ J} - 7.43 \times 10^{-14} \text{ J} - 7.43 \times 10^{-14} \text{ J} \\ &\quad - 7.43 \times 10^{-14} \text{ J} + 5.25 \times 10^{-14} \text{ J} = -1.92 \times 10^{-13} \text{ J}. \end{aligned}$$

This is equal to the work that must be done to assemble the system from infinite separation.

59

(a) Use conservation of mechanical energy. The potential energy when the moving particle is at any coordinate y is qV , where V is the electric potential produced at that place by the two fixed particles. That is,

$$U = q \frac{2Q}{4\pi\epsilon_0 \sqrt{x^2 + y^2}},$$

where x is the coordinate and Q is the charge of either one of the fixed particles. The factor 2 appears since the two fixed particles produce the same potential at points on the y axis. Conservation of mechanical energy yields

$$K_f = K_i + q \frac{2Q}{4\pi\epsilon_0 \sqrt{x^2 + y_i^2}} - q \frac{2Q}{4\pi\epsilon_0 \sqrt{x^2 + y_f^2}} = K_i + \frac{2qQ}{4\pi\epsilon_0} \left(\frac{1}{\sqrt{x^2 + y_i^2}} - \frac{1}{\sqrt{x^2 + y_f^2}} \right),$$

where K is the kinetic energy of the moving particle, the subscript i refers to the initial position of the moving particle, and the subscript f refers to the final position. Numerically

$$K_f = 1.2 \text{ J} + \frac{2(-15 \times 10^{-6} \text{ C})(50 \times 10^{-6} \text{ C})}{4\pi(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)} \left[\frac{1}{\sqrt{(3.0 \text{ m})^2 + (4.0 \text{ m})^2}} - \frac{1}{\sqrt{(3.0 \text{ m})^2}} \right] = 3.0 \text{ J}.$$

(b) Now $K_f = 0$ and we solve the energy conservation equation for y_f . Conservation of energy first yields $U_f = K_i + U_i$. The initial potential energy is

$$U_i = \frac{2qQ}{4\pi\epsilon_0 \sqrt{x^2 + y_i^2}} = \frac{2(-15 \times 10^{-6} \text{ C})(50 \times 10^{-6} \text{ C})}{4\pi(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2) \sqrt{(3.0 \text{ m})^2 + (4.0 \text{ m})^2}} = -2.7 \text{ J}.$$

Thus $K_f = 1.2 \text{ J} - 2.7 \text{ J} = -1.5 \text{ J}$.

Now

$$U_f = \frac{2qQ}{4\pi\epsilon_0 \sqrt{x^2 + y_f^2}}$$

so

$$y = -\sqrt{\left(\frac{2qQ}{4\pi\epsilon_0 U_f}\right)^2 - x^2} = \sqrt{\left(\frac{2(-15 \times 10^{-6} \text{ C})(50 \times 10^{-6} \text{ C})}{4\pi(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(-1.5 \text{ J})}\right)^2 - (3.0 \text{ m})^2} = -8.5 \text{ m}.$$

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If the electric potential is zero at infinity, then the electric potential at the surface of the sphere is given by $V = q/4\pi\epsilon_0 r$, where q is the charge on the sphere and r is its radius. Thus

$$q = 4\pi\epsilon_0 r V = \frac{(0.15 \text{ m})(1500 \text{ V})}{8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2} = 2.5 \times 10^{-8} \text{ C}.$$

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(a) The electric potential is the sum of the contributions of the individual spheres. Let q_1 be the charge on one, q_2 be the charge on the other, and d be their separation. The point halfway between them is the same distance $d/2$ ($= 1.0 \text{ m}$) from the center of each sphere, so the potential at the halfway point is

$$V = \frac{q_1 + q_2}{4\pi\epsilon_0 d/2} = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(1.0 \times 10^{-8} \text{ C} - 3.0 \times 10^{-8} \text{ C})}{1.0 \text{ m}} = -1.80 \times 10^2 \text{ V}.$$

(b) The distance from the center of one sphere to the surface of the other is $d - R$, where R is the radius of either sphere. The potential of either one of the spheres is due to the charge on that sphere and the charge on the other sphere. The potential at the surface of sphere 1 is

$$\begin{aligned} V_1 &= \frac{1}{4\pi\epsilon_0} \left[\frac{q_1}{R} + \frac{q_2}{d - R} \right] \\ &= (8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \left[\frac{1.0 \times 10^{-8} \text{ C}}{0.030 \text{ m}} - \frac{3.0 \times 10^{-8} \text{ C}}{2.0 \text{ m} - 0.030 \text{ m}} \right] \\ &= 2.9 \times 10^3 \text{ V}. \end{aligned}$$

(c) The potential at the surface of sphere 2 is

$$\begin{aligned} V_2 &= \frac{1}{4\pi\epsilon_0} \left[\frac{q_1}{d - R} + \frac{q_2}{R} \right] \\ &= (8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \left[\frac{1.0 \times 10^{-8} \text{ C}}{2.0 \text{ m} - 0.030 \text{ m}} - \frac{3.0 \times 10^{-8} \text{ C}}{0.030 \text{ m}} \right] \\ &= -8.9 \times 10^3 \text{ V}. \end{aligned}$$

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The initial potential energy of the three-particle system is $U_i = 2(q^2/4\pi\epsilon_0 L) + U_{\text{fixed}}$, where q is the charge on each particle, L is the length of a triangle side, and U_{fixed} is the potential energy associated with the interaction of the two fixed particles. The factor 2 appears since the potential

energy is the same for the interaction of the movable particle and each of the fixed particles. The final potential energy is $U_f = 2[q^2/4\pi\epsilon_0(L/2)] + U_{\text{fixed}}$ and the change in the potential energy is

$$\Delta U = U_f - U_i = \frac{2q^2}{4\pi\epsilon_0} \left(\frac{2}{L} - \frac{1}{L} \right) = \frac{2q^2}{4\pi\epsilon_0 L}.$$

This is the work that is done by the external agent. If P is the rate with energy is supplied by the agent and t is the time for the move, then $Pt = \Delta U$, and

$$t = \frac{\Delta U}{P} = \frac{2q^2}{4\pi\epsilon_0 LP} = \frac{2(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(0.12 \text{ C})^2}{(1.7 \text{ m})(0.83 \times 10^3 \text{ W})} = 1.83 \times 10^5 \text{ s}.$$

This is 2.1 d.

77

(a) Use Gauss' law to find an expression for the electric field. The Gaussian surface is a cylindrical surface that is concentric with the cylinder and has a radius r that is greater than the radius of the cylinder. The electric field is normal to the Gaussian surface and has uniform magnitude on it, so the integral in Gauss' law is $\oint \vec{E} \cdot d\vec{A} = 2\pi rEL$, where L is the length of the Gaussian surface. The charge enclosed is λL , where λ is the charge per unit length on the cylinder. Thus $2\pi rLE = \lambda L/\epsilon_0$ and $E = \lambda/2\pi\epsilon_0 r$.

Let E_B be the magnitude of the field at B and r_B be the distance from the central axis to B. Let E_C be the magnitude of the field at C and r_C be the distance from the central axis to C. Since E is inversely proportional to the distance from the central axis,

$$E_C = \frac{r_B}{r_C} E_B = \frac{2.0 \text{ cm}}{5.0 \text{ cm}} (160 \text{ N/C}) = 64 \text{ N/C}.$$

(b) The magnitude of the field a distance r from the central axis is $E = (r_B/r)E_B$, so the potential difference of points B and C is

$$\begin{aligned} V_B - V_C &= - \int_{r_C}^{r_B} \frac{r_B}{r} E_B dr = -r_B E_B \ln \left(\frac{r_B}{r_C} \right) \\ &= -(0.020 \text{ m})(160 \text{ N/C}) \ln \left(\frac{0.020 \text{ m}}{0.050 \text{ m}} \right) = 2.9 \text{ V}. \end{aligned}$$

(c) The cylinder is conducting, so all points inside have the same potential, namely V_B , so $V_A - V_B = 0$.

85

Consider a point on the z axis that has coordinate z . All points on the ring are the same distance from the point. The distance is $r = \sqrt{R^2 + z^2}$, where R is the radius of the ring. If the electric potential is taken to be zero at points that are infinitely far from the ring, then the potential at the point is

$$V = \frac{Q}{4\pi\epsilon_0 \sqrt{R^2 + z^2}},$$

where Q is the charge on the ring. Thus

$$\begin{aligned} V_B - V_A &= \frac{Q}{4\pi\epsilon_0} \left[\frac{1}{\sqrt{R^2 + z^2}} - \frac{1}{R} \right] \\ &= (8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(16.0 \times 10^{-6} \text{ C}) \left[\frac{1}{\sqrt{(0.0300 \text{ m})^2 + (0.0400 \text{ m})^2}} - 0.300 \text{ m} \right] \\ &= -1.92 \times 10^6 \text{ V}. \end{aligned}$$

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(a) For $r > r_2$ the field is like that of a point charge and

$$V = \frac{1}{4\pi\epsilon_0} \frac{Q}{r},$$

where the zero of potential was taken to be at infinity.

(b) To find the potential in the region $r_1 < r < r_2$, first use Gauss's law to find an expression for the electric field, then integrate along a radial path from r_2 to r . The Gaussian surface is a sphere of radius r , concentric with the shell. The field is radial and therefore normal to the surface. Its magnitude is uniform over the surface, so the flux through the surface is $\Phi = 4\pi r^2 E$. The volume of the shell is $(4\pi/3)(r_2^3 - r_1^3)$, so the charge density is

$$\rho = \frac{3Q}{4\pi(r_2^3 - r_1^3)}$$

and the charge enclosed by the Gaussian surface is

$$q = \left(\frac{4\pi}{3} \right) (r^3 - r_1^3) \rho = Q \left(\frac{r^3 - r_1^3}{r_2^3 - r_1^3} \right).$$

Gauss' law yields

$$4\pi\epsilon_0 r^2 E = Q \left(\frac{r^3 - r_1^3}{r_2^3 - r_1^3} \right)$$

and the magnitude of the electric field is

$$E = \frac{Q}{4\pi\epsilon_0} \frac{r^3 - r_1^3}{r^2(r_2^3 - r_1^3)}.$$

If V_s is the electric potential at the outer surface of the shell ($r = r_2$) then the potential a distance r from the center is given by

$$\begin{aligned} V &= V_s - \int_{r_2}^r E \, dr = V_s - \frac{Q}{4\pi\epsilon_0} \frac{1}{r_2^3 - r_1^3} \int_{r_2}^r \left(r - \frac{r_1^3}{r^2} \right) \, dr \\ &= V_s - \frac{Q}{4\pi\epsilon_0} \frac{1}{r_2^3 - r_1^3} \left(\frac{r^2}{2} - \frac{r_2^2}{2} + \frac{r_1^3}{r} - \frac{r_1^3}{r_2} \right). \end{aligned}$$

The potential at the outer surface is found by placing $r = r_2$ in the expression found in part (a). It is $V_s = Q/4\pi\epsilon_0 r_2$. Make this substitution and collect like terms to find

$$V = \frac{Q}{4\pi\epsilon_0} \frac{1}{r_2^3 - r_1^3} \left(\frac{3r_2^2}{2} - \frac{r^2}{2} - \frac{r_1^3}{r} \right).$$

Since $\rho = 3Q/4\pi(r_2^3 - r_1^3)$ this can also be written

$$V = \frac{\rho}{3\epsilon_0} \left(\frac{3r_2^2}{2} - \frac{r^2}{2} - \frac{r_1^3}{r} \right).$$

(c) The electric field vanishes in the cavity, so the potential is everywhere the same inside and has the same value as at a point on the inside surface of the shell. Put $r = r_1$ in the result of part (b). After collecting terms the result is

$$V = \frac{Q}{4\pi\epsilon_0} \frac{3(r_2^2 - r_1^2)}{2(r_2^3 - r_1^3)},$$

or in terms of the charge density

$$V = \frac{\rho}{2\epsilon_0} (r_2^2 - r_1^2).$$

(d) The solutions agree at $r = r_1$ and at $r = r_2$.

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The electric potential of a dipole at a point a distance r away is given by Eq. 24–30:

$$V = \frac{1}{4\pi\epsilon_0} \frac{p \cos \theta}{r^2},$$

where p is the magnitude of the dipole moment and θ is the angle between the dipole moment and the position vector of the point. The potential at infinity was taken to be zero. Take the z axis to be the dipole axis and consider a point with z positive (on the positive side of the dipole). For this point $r = z$ and $\theta = 0$. The z component of the electric field is

$$E_z = -\frac{\partial V}{\partial z} = -\frac{\partial}{\partial z} \left(\frac{p}{4\pi\epsilon_0 z^2} \right) = \frac{p}{2\pi\epsilon_0 z^3}.$$

This is the only nonvanishing component at a point on the dipole axis.

For a point with a negative value of z , $r = -z$ and $\cos \theta = -1$, so

$$E_z = -\frac{\partial}{\partial z} \left(\frac{-p}{4\pi\epsilon_0 z^2} \right) = -\frac{p}{2\pi\epsilon_0 z^3}.$$

103

(a) The electric potential at the surface of the sphere is given by $V = q/4\pi\epsilon_0 R$, where q is the charge on the sphere and R is the sphere radius. The charge on the sphere when the potential reaches 1000 V is

$$q = 4\pi\epsilon_0 R V = \frac{(0.010 \text{ m})(1000 \text{ V})}{8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2} = 1.11 \times 10^{-9} \text{ C}.$$

The number of electrons that enter the sphere is $N = q/e = (1.11 \times 10^{-9} \text{ C})/(1.60 \times 10^{-19} \text{ C}) = 6.95 \times 10^9$. Let R be the decay rate and t be the time for the potential to reach its final value. Since half the resulting electrons enter the sphere $N = (P/2)t$ and $t = 2N/P = 2(6.95 \times 10^9)/(3.70 \times 10^8 \text{ s}^{-1}) = 38 \text{ s}$.

(b) The increase in temperature is $\Delta T = N\Delta E/C$, where E is the energy deposited by a single electron and C is the heat capacity of the sphere. Since $N = (P/2)t$, this is $\Delta T = (P/2)t\Delta E/C$ and

$$t = \frac{2C \Delta T}{P \Delta E} = \frac{2(14 \text{ J/K})(5.0 \text{ K})}{(3.70 \times 10^8 \text{ s}^{-1})(100 \times 10^3 \text{ eV})(1.60 \times 10^{-19} \text{ J/eV})} = 2.4 \times 10^7 \text{ s}.$$

This is about 280 d.