Chapter 36

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The condition for a minimum of intensity in a single-slit diffraction pattern is $a \sin\theta = m\lambda$, where a is the slit width, λ is the wavelength, and m is an integer. To find the angular position of the first minimum to one side of the central maximum, set $m = 1$:

$$\theta_1 = \sin^{-1}\left(\frac{\lambda}{a}\right) = \sin^{-1}\left(\frac{589 \times 10^{-9}\,\mathrm{m}}{1.00 \times 10^{-3}\,\mathrm{m}}\right) = 5.89 \times 10^{-4}\,\mathrm{rad}\,.$$

If D is the distance from the slit to the screen, the distance on the screen from the center of the pattern to the minimum is $y_1 = D \tan\theta_1 = (3.00\,\mathrm{m}) \tan(5.89 \times 10^{-4}\,\mathrm{rad}) = 1.767 \times 10^{-3}\,\mathrm{m}$.

To find the second minimum, set $m = 2$:

$$\theta_2 = \sin^{-1}\left[\frac{2(589 \times 10^{-9}\,\mathrm{m})}{1.00 \times 10^{-3}\,\mathrm{m}}\right] = 1.178 \times 10^{-3}\,\mathrm{rad}\,.$$

The distance from the pattern center to the minimum is $y_2 = D \tan\theta_2 = (3.00\,\mathrm{m}) \tan(1.178 \times 10^{-3}\,\mathrm{rad}) = 3.534 \times 10^{-3}\,\mathrm{m}$. The separation of the two minima is $\Delta y = y_2 - y_1 = 3.534\,\mathrm{mm} - 1.767\,\mathrm{mm} = 1.77\,\mathrm{mm}$.

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(a) The intensity for a single-slit diffraction pattern is given by

$$I = I_m \frac{\sin^2\alpha}{\alpha^2}\,,$$

where $\alpha = (\pi a/\lambda)\sin\theta$, a is the slit width and λ is the wavelength. The angle θ is measured from the forward direction. You want $I = I_m/2$, so

$$\sin^2\alpha = \frac{1}{2}\alpha^2\,.$$

(b) Evaluate $\sin^2\alpha$ and $\alpha^2/2$ for $\alpha = 1.39\,\mathrm{rad}$ and compare the results. To be sure that 1.39 rad is closer to the correct value for α than any other value with three significant digits, you should also try 1.385 rad and 1.395 rad.

(c) Since $\alpha = (\pi a/\lambda)\sin\theta$,

$$\theta = \sin^{-1}\left(\frac{\alpha\lambda}{\pi a}\right)\,.$$

Now $\alpha/\pi = 1.39/\pi = 0.442$, so

$$\theta = \sin^{-1}\left(\frac{0.442\lambda}{a}\right)\,.$$

The angular separation of the two points of half intensity, one on either side of the center of the diffraction pattern, is

$$\Delta\theta = 2\theta = 2\sin^{-1}\left(\frac{0.442\lambda}{a}\right).$$

(d) For $a/\lambda = 1.0$,

$$\Delta\theta = 2\sin^{-1}(0.442/1.0) = 0.916\,\text{rad} = 52.5^\circ.$$

(e) For $a/\lambda = 5.0$,

$$\Delta\theta = 2\sin^{-1}(0.442/5.0) = 0.177\,\text{rad} = 10.1^\circ.$$

(f) For $a/\lambda = 10$,

$$\Delta\theta = 2\sin^{-1}(0.442/10) = 0.0884\,\text{rad} = 5.06^\circ.$$

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(a) Use the Rayleigh criteria. To resolve two point sources, the central maximum of the diffraction pattern of one must lie at or beyond the first minimum of the diffraction pattern of the other. This means the angular separation of the sources must be at least $\theta_R = 1.22\lambda/d$, where λ is the wavelength and d is the diameter of the aperture. For the headlights of this problem,

$$\theta_R = \frac{1.22(550 \times 10^{-9}\,\text{m})}{5.0 \times 10^{-3}\,\text{m}} = 1.3 \times 10^{-4}\,\text{rad}.$$

(b) If D is the distance from the headlights to the eye when the headlights are just resolvable and ℓ is the separation of the headlights, then $\ell = D\tan\theta_R \approx D\theta_R$, where the small angle approximation $\tan\theta_R \approx \theta_R$ was made. This is valid if θ_R is measured in radians. Thus $D = \ell/\theta_R = (1.4\,\text{m})/(1.34 \times 10^{-4}\,\text{rad}) = 1.0 \times 10^4\,\text{m} = 10\,\text{km}$.

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(a) Use the Rayleigh criteria: two objects can be resolved if their angular separation at the observer is greater than $\theta_R = 1.22\lambda/d$, where λ is the wavelength of the light and d is the diameter of the aperture (the eye or mirror). If D is the distance from the observer to the objects, then the smallest separation ℓ they can have and still be resolvable is $\ell = D\tan\theta_R \approx D\theta_R$, where θ_R is measured in radians. The small angle approximation $\tan\theta_R \approx \theta_R$ was made. Thus

$$\ell = \frac{1.22D\lambda}{d} = \frac{1.22(8.0 \times 10^{10}\,\text{m})(550 \times 10^{-9}\,\text{m})}{5.0 \times 10^{-3}\,\text{m}} = 1.1 \times 10^7\,\text{m} = 1.1 \times 10^4\,\text{km}.$$

This distance is greater than the diameter of Mars. One part of the planet's surface cannot be resolved from another part.

(b) Now $d = 5.1$ m and

$$\ell = \frac{1.22(8.0 \times 10^{10}\,\text{m})(550 \times 10^{-9}\,\text{m})}{5.1\,\text{m}} = 1.1 \times 10^4\,\text{m} = 11\,\text{km}.$$

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(a) The first minimum in the diffraction pattern is at an angular position θ, measured from the center of the pattern, such that $\sin\theta = 1.22\lambda/d$, where λ is the wavelength and d is the diameter of the antenna. If f is the frequency, then the wavelength is

$$\lambda = \frac{c}{f} = \frac{3.00 \times 10^8\,\mathrm{m/s}}{220 \times 10^9\,\mathrm{Hz}} = 1.36 \times 10^{-3}\,\mathrm{m}\,.$$

Thus

$$\theta = \sin^{-1}\left(\frac{1.22\lambda}{d}\right) = \sin^{-1}\left(\frac{1.22(1.36 \times 10^{-3}\,\mathrm{m})}{55.0 \times 10^{-2}\,\mathrm{m}}\right) = 3.02 \times 10^{-3}\,\mathrm{rad}\,.$$

The angular width of the central maximum is twice this, or 6.04×10^{-3} rad (0.346°).

(b) Now $\lambda = 1.6$ cm and $d = 2.3$ m, so

$$\theta = \sin^{-1}\left(\frac{1.22(1.6 \times 10^{-2}\,\mathrm{m})}{2.3\,\mathrm{m}}\right) = 8.5 \times 10^{-3}\,\mathrm{rad}\,.$$

The angular width of the central maximum is 1.7×10^{-2} rad (0.97°).

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(a) The angular positions θ of the bright interference fringes are given by $d\sin\theta = m\lambda$, where d is the slit separation, λ is the wavelength, and m is an integer. The first diffraction minimum occurs at the angle θ_1 given by $a\sin\theta_1 = \lambda$, where a is the slit width. The diffraction peak extends from $-\theta_1$ to $+\theta_1$, so you want to count the number of values of m for which $-\theta_1 < \theta < +\theta_1$, or what is the same, the number of values of m for which $-\sin\theta_1 < \sin\theta < +\sin\theta_1$. This means $-1/a < m/d < 1/a$ or $-d/a < m < +d/a$. Now $d/a = (0.150 \times 10^{-3}\,\mathrm{m})/(30.0 \times 10^{-6}\,\mathrm{m}) = 5.00$, so the values of m are $m = -4, -3, -2, -1, 0, +1, +2, +3$, and $+4$. There are nine fringes.

(b) The intensity at the screen is given by

$$I = I_m\,\left(\cos^2\beta\right)\left(\frac{\sin\alpha}{\alpha}\right)^2,$$

where $\alpha = (\pi a/\lambda)\sin\theta$, $\beta = (\pi d/\lambda)\sin\theta$, and I_m is the intensity at the center of the pattern. For the third bright interference fringe, $d\sin\theta = 3\lambda$, so $\beta = 3\pi$ rad and $\cos^2\beta = 1$. Similarly, $\alpha = 3\pi a/d = 3\pi/5.00 = 0.600\pi$ rad and

$$\left(\frac{\sin\alpha}{\alpha}\right)^2 = \left(\frac{\sin 0.600\pi}{0.600\pi}\right)^2 = 0.255\,.$$

The intensity ratio is $I/I_m = 0.255$.

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The ruling separation is $d = 1/(400\,\mathrm{mm}^{-1}) = 2.5 \times 10^{-3}$ mm. Diffraction lines occur at angles θ such that $d\sin\theta = m\lambda$, where λ is the wavelength and m is an integer. Notice that for a given order, the line associated with a long wavelength is produced at a greater angle than the line associated with a shorter wavelength. Take λ to be the longest wavelength in the visible spectrum (700 nm) and find the greatest integer value of m such that θ is less than 90°. That is, find the greatest integer value of m for which $m\lambda < d$. Since $d/\lambda = (2.5\times10^{-6}\,\text{m})/(700\times10^{-9}\,\text{m}) = 3.57$, that value is $m = 3$. There are three complete orders on each side of the $m = 0$ order. The second and third orders overlap.

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(a) Maxima of a diffraction grating pattern occur at angles θ given by $d\sin\theta = m\lambda$, where d is the slit separation, λ is the wavelength, and m is an integer. The two lines are adjacent, so their order numbers differ by unity. Let m be the order number for the line with $\sin\theta = 0.2$ and $m+1$ be the order number for the line with $\sin\theta = 0.3$. Then $0.2d = m\lambda$ and $0.3d = (m+1)\lambda$. Subtract the first equation from the second to obtain $0.1d = \lambda$, or $d = \lambda/0.1 = (600\times10^{-9}\,\text{m})/0.1 = 6.0\times10^{-6}\,\text{m}$.

(b) Minima of the single-slit diffraction pattern occur at angles θ given by $a\sin\theta = m\lambda$, where a is the slit width. Since the fourth-order interference maximum is missing, it must fall at one of these angles. If a is the smallest slit width for which this order is missing, the angle must be given by $a\sin\theta = \lambda$. It is also given by $d\sin\theta = 4\lambda$, so $a = d/4 = (6.0\times10^{-6}\,\text{m})/4 = 1.5\times10^{-6}\,\text{m}$.

(c) First, set $\theta = 90°$ and find the largest value of m for which $m\lambda < d\sin\theta$. This is the highest order that is diffracted toward the screen. The condition is the same as $m < d/\lambda$ and since $d/\lambda = (6.0\times10^{-6}\,\text{m})/(600\times10^{-9}\,\text{m}) = 10.0$, the highest order seen is the $m = 9$ order.

(d) and (e) The fourth and eighth orders are missing so the observable orders are $m = 0$, 1, 2, 3, 5, 6, 7, and 9. The second highest order is the $m = 7$ order and the third highest order is the $m = 6$ order.

61

If a grating just resolves two wavelengths whose average is λ_{avg} and whose separation is $\Delta\lambda$, then its resolving power is defined by $R = \lambda_{\text{avg}}/\Delta\lambda$. The text shows this is Nm, where N is the number of rulings in the grating and m is the order of the lines. Thus $\lambda_{\text{avg}}/\Delta\lambda = Nm$ and

$$N = \frac{\lambda_{\text{avg}}}{m\,\Delta\lambda} = \frac{656.3\,\text{nm}}{(1)(0.18\,\text{nm})} = 3.65\times10^{3}\ \text{rulings}\,.$$

73

We want the reflections to obey the Bragg condition $2d\sin\theta = m\lambda$, where θ is the angle between the incoming rays and the reflecting planes, λ is the wavelength, and m is an integer. Solve for θ:

$$\theta = \sin^{-1}\left[\frac{m\lambda}{2d}\right] = \sin^{-1}\left[\frac{(0.125\times10^{-9}\,\text{m})m}{2(0.252\times10^{-9}\,\text{m})}\right] = \sin^{-1}(0.2480m)\,.$$

For $m = 1$ this gives $\theta = 14.4°$. The crystal should be turned $45° - 14.4° = 30.6°$ clockwise.

For $m = 2$ it gives $\theta = 29.7°$. The crystal should be turned $45° - 29.7° = 15.3°$ clockwise.

For $m = 3$ it gives $\theta = 48.1°$. The crystal should be turned $48.1° - 45° = 3.1°$ counterclockwise.

For $m = 4$ it gives $\theta = 82.8°$. The crystal should be turned $82.8° - 45° = 37.8°$ counterclockwise. There are no intensity maxima for $m > 4$ as you can verify by noting that $m\lambda/2d$ is greater than 1 for m greater than 4. For clockwise turns the smaller value is 15.3° and the larger value is 30.6°. For counterclockwise turns the smaller value is 3.1° and the larger value is 37.8°.

<u>**77**</u>

Intensity maxima occur at angles θ such that $d \sin\theta = m\lambda$, where d is the separation of adjacent rulings and λ is the wavelength. Here the ruling separation is $1/(200\,\text{mm}^{-1}) = 5.00 \times 10^{-3}\,\text{mm} = 5.00 \times 10^{-6}\,\text{m}$. Thus

$$\lambda = \frac{d \sin\theta}{m} = \frac{(5.00 \times 10^{-6}\,\text{m}) \sin 30.0°}{m} = \frac{2.50 \times 10^{-6}\,\text{m}}{m}.$$

For $m = 1$, $\lambda = 2500\,\text{nm}$; for $m = 2$, $\lambda = 1250\,\text{nm}$; for $m - 3$, $\lambda = 833\,\text{nm}$; for $m = 4$, $\lambda = 625\,\text{nm}$; for $m = 5$, $\lambda = 500\,\text{nm}$, and for $m = 6$, $\lambda = 417\,\text{nm}$. Only the last three are in the visible range, so the longest wavelength in the visible range is 625 nm, the next longest is 500 nm, and the third longest is 417 nm.

<u>**79**</u>

Suppose m_o is the order of the minimum for orange light, with wavelength λ_o, and m_{bg} is the order of the minimum for blue-green light, with wavelength λ_{bg}. Then $a \sin\theta = m_o\lambda_o$ and $a \sin\theta = m_{bg}\lambda_{bg}$. Thus $m_o\lambda_o = m_{bg}\lambda_{bg}$ and $m_{bg}/m_o = \lambda_o/\lambda_{bg} = (600\,\text{nm})/(500\,\text{nm}) = 6/5$. The smallest two integers with this ratio are $m_{bg} = 6$ and $m_o = 5$. The slit width is

$$a = \frac{m_o\lambda_o}{\sin\theta} = \frac{5(600 \times 10^{-9}\,\text{m}}{\sin(1.00 \times 10^{-3}\,\text{rad})} = 3.0 \times 10^{-3}\,\text{m}.$$

Other values for m_o and m_{bg} are possible but these are associated with a wider slit.

<u>**81**</u>

(a) Since the first minimum of the diffraction pattern occurs at the angle θ such that $\sin\theta = \lambda/a$, where λ is the wavelength and a is the slit width, the central maximum extends from $\theta_1 = -\sin^{-1}(\lambda/a)$ to $\theta_2 = +\sin^{-1}(\lambda/a)$. Maxima of the two-slit interference pattern are at angles θ such that $\sin\theta = m\lambda/d$, where d is the slit separation and m is an integer. We wish to know the number of values of m such that $\sin^{-1}(m\lambda/d)$ lies between $-\sin^{-1}(\lambda/a)$ and $+\sin(\lambda/a)$ or, what is the same, the number of values of m such that m/d lies between $-1/a$ and $+1/a$. The greatest m can be is the greatest integer that is smaller than $d/a = (14\,\mu\text{m})/(2.0\,\mu\text{m}) = 7$. (The $m = 7$ maximum does not appear since it coincides with a minimum of the diffraction pattern.) There are 13 such values: 0, ±1, ±2, ±3; ±4; ±5, and ±6. Thus 13 interference maxima appear in the central diffraction envelope.

(b) The first diffraction envelope extends from $\theta_1 = \sin^{-1}(\lambda/a)$ to $\theta_2 = \sin^{-1}(2\lambda/a)$. Thus we wish to know the number of values of m such that m/d is greater than $1/a$ and less than $2/a$. Since $d = 7.0a$, m can be 8, 9, 10, 11, 12, or 13. That is, there are 6 interference maxima in the first diffraction envelope.

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If you divide the original slit into N strips and represent the light from each strip, when it reaches the screen, by a phasor, then at the central maximum in the diffraction pattern you add N phasors, all in the same direction and each with the same amplitude. The intensity there is proportional to N^2. If you double the slit width, you need $2N$ phasors if they are each to have the amplitude of the phasors you used for the narrow slit. The intensity at the central maximum is proportional to $(2N)^2$ and is, therefore, four times the intensity for the narrow slit. The energy reaching the screen per unit time, however, is only twice the energy reaching it per unit time when the narrow slit is in place. The energy is simply redistributed. For example, the central peak is now half as wide and the integral of the intensity over the peak is only twice the analogous integral for the narrow slit.

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(a) Since the resolving power of a grating is given by $R = \lambda/\Delta\lambda$ and by Nm, the range of wavelengths that can just be resolved in order m is $\Delta\lambda = \lambda/Nm$. Here N is the number of rulings in the grating and λ is the average wavelength. The frequency f is related to the wavelength by $f\lambda = c$, where c is the speed of light. This means $f\,\Delta\lambda + \lambda\,\Delta f = 0$, so

$$\Delta\lambda = -\frac{\lambda}{f}\,\Delta f = -\frac{\lambda^2}{c}\,\Delta f\,,$$

where $f = c/\lambda$ was used. The negative sign means that an increase in frequency corresponds to a decrease in wavelength. We may interpret Δf as the range of frequencies that can be resolved and take it to be positive. Then

$$\frac{\lambda^2}{c}\,\Delta f = \frac{\lambda}{Nm}$$

and

$$\Delta f = \frac{c}{Nm\lambda}\,.$$

(b) The difference in travel time for waves traveling along the two extreme rays is $\Delta t = \Delta L/c$, where ΔL is the difference in path length. The waves originate at slits that are separated by $(N-1)d$, where d is the slit separation and N is the number of slits, so the path difference is $\Delta L = (N-1)d\sin\theta$ and the time difference is

$$\Delta t = \frac{(N-1)d\sin\theta}{c}\,.$$

If N is large, this may be approximated by $\Delta t = (Nd/c)\sin\theta$. The lens does not affect the travel time.

(c) Substitute the expressions you derived for Δt and Δf to obtain

$$\Delta f\,\Delta t = \left(\frac{c}{Nm\lambda}\right)\left(\frac{Nd\sin\theta}{c}\right) = \frac{d\sin\theta}{m\lambda} = 1\,.$$

The condition $d\sin\theta = m\lambda$ for a diffraction line was used to obtain the last result.

101

The dispersion of a grating is given by $D = d\theta/d\lambda$, where θ is the angular position of a line associated with wavelength λ. The angular position and wavelength are related by $\ell \sin\theta = m\lambda$, where ℓ is the slit separation and m is an integer. Differentiate this with respect to θ to obtain $(d\theta/d\lambda)\,\ell\cos\theta = m$ or

$$D = \frac{\ell\theta}{\ell\lambda} = \frac{m}{\ell\cos\theta}\,.$$

Now $m = (\ell/\lambda)\sin\theta$, so

$$D = \frac{\ell\sin\theta}{\ell\lambda\cos\theta} = \frac{\tan\theta}{\lambda}\,.$$

The trigonometric identity $\tan\theta = \sin\theta/\cos\theta$ was used.