

Chapter 37

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(a) The rest length L_0 ($= 130$ m) of the spaceship and its length L as measured by the timing station are related by $L = L_0/\gamma = L_0\sqrt{1-\beta^2}$, where $\gamma = 1/\sqrt{1-\beta^2}$ and $\beta = v/c$. Thus $L = (130 \text{ m})\sqrt{1-(0.740)^2} = 87.4$ m.

(b) The time interval for the passage of the spaceship is

$$\Delta t = \frac{L}{v} = \frac{87.4 \text{ m}}{(0.740)(2.9979 \times 10^8 \text{ m/s})} = 3.94 \times 10^{-7} \text{ s}.$$

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The proper time is not measured by clocks in either frame S or frame S' since a single clock at rest in either frame cannot be present at the origin and at the event. The full Lorentz transformation must be used:

$$x' = \gamma[x - vt]$$

$$t' = \gamma[t - \beta x/c],$$

where $\beta = v/c = 0.950$ and $\gamma = 1/\sqrt{1-\beta^2} = 1/\sqrt{1-(0.950)^2} = 3.2026$. Thus

$$\begin{aligned} x' &= (3.2026) [100 \times 10^3 \text{ m} - (0.950)(2.9979 \times 10^8 \text{ m/s})(200 \times 10^{-6} \text{ s})] \\ &= 1.38 \times 10^5 \text{ m} = 138 \text{ km} \end{aligned}$$

and

$$t' = (3.2026) \left[200 \times 10^{-6} \text{ s} - \frac{(0.950)(100 \times 10^3 \text{ m})}{2.9979 \times 10^8 \text{ m/s}} \right] = -3.74 \times 10^{-4} \text{ s} = -374 \mu\text{s}.$$

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(a) The Lorentz factor is

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-(0.600)^2}} = 1.25.$$

(b) In the unprimed frame, the time for the clock to travel from the origin to $x = 180$ m is

$$t = \frac{x}{v} = \frac{180 \text{ m}}{(0.600)(2.9979 \times 10^8 \text{ m/s})} = 1.00 \times 10^{-6} \text{ s}.$$

The proper time interval between the two events (at the origin and at $x = 180$ m) is measured by the clock itself. The reading on the clock at the beginning of the interval is zero, so the reading at the end is

$$t' = \frac{t}{\gamma} = \frac{1.00 \times 10^{-6} \text{ s}}{1.25} = 8.00 \times 10^{-7} \text{ s}.$$

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Use Eq. 37–29 with $u' = 0.40c$ and $v = 0.60c$. Then

$$u = \frac{0.40c + 0.60c}{1 + (0.40c)(0.60c)/c^2} = 0.81c.$$

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Calculate the speed of the micrometeorite relative to the spaceship. Let S' be the reference frame for which the data is given and attach frame S to the spaceship. Suppose the micrometeorite is going in the positive x direction and the spaceship is going in the negative x direction, both as viewed from S' . Then, in Eq. 37–29, $u' = 0.82c$ and $v = 0.82c$. Notice that v in the equation is the velocity of S' relative to S . Thus the velocity of the micrometeorite in the frame of the spaceship is

$$u = \frac{u' + v}{1 + u'v/c^2} = \frac{0.82c + 0.82c}{1 + (0.82c)(0.82c)/c^2} = 0.9806c.$$

The time for the micrometeorite to pass the spaceship is

$$\Delta t = \frac{L}{u} = \frac{350 \text{ m}}{(0.9806)(2.9979 \times 10^8 \text{ m/s})} = 1.19 \times 10^{-6} \text{ s}.$$

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The spaceship is moving away from Earth, so the frequency received is given by

$$f = f_0 \sqrt{\frac{1 - \beta}{1 + \beta}},$$

where f_0 is the frequency in the frame of the spaceship, $\beta = v/c$, and v is the speed of the spaceship relative to Earth. See Eq. 37–31. Thus

$$f = (100 \text{ MHz}) \sqrt{\frac{1 - 0.9000}{1 + 0.9000}} = 22.9 \text{ MHz}.$$

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The spaceship is moving away from Earth, so the frequency received is given by

$$f = f_0 \sqrt{\frac{1 - \beta}{1 + \beta}},$$

where f_0 is the frequency in the frame of the spaceship, $\beta = v/c$, and v is the speed of the spaceship relative to Earth. See Eq. 37–31. The frequency f and wavelength λ are related by $f\lambda = c$, so if λ_0 is the wavelength of the light as seen on the spaceship and λ is the wavelength detected on Earth, then

$$\lambda = \lambda_0 \sqrt{\frac{1 + \beta}{1 - \beta}} = (450 \text{ nm}) \sqrt{\frac{1 + 0.20}{1 - 0.20}} = 550 \text{ nm}.$$

This is in the yellow-green portion of the visible spectrum.

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Use the two expressions for the total energy: $E = mc^2 + K$ and $E = \gamma mc^2$, where m is the mass of an electron, K is the kinetic energy, and $\gamma = 1/\sqrt{1 - \beta^2}$. Thus $mc^2 + K = \gamma mc^2$ and

$$\gamma = 1 + \frac{K}{mc^2} = 1 + \frac{(100.000 \times 10^6 \text{ eV})(1.602 176 462 \text{ J/eV})}{(9.109 381 88 \times 10^{-31} \text{ kg})(2.997 924 58 \times 10^8 \text{ m/s})^2} = 196.695.$$

Now $\gamma^2 = 1/(1 - \beta^2)$, so

$$\beta = \sqrt{1 - \frac{1}{\gamma^2}} = \sqrt{1 - \frac{1}{(196.695)^2}} = 0.999 987.$$

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The energy equivalent of one tablet is $mc^2 = (320 \times 10^{-6} \text{ kg})(2.9979 \times 10^8 \text{ m/s})^2 = 2.88 \times 10^{13} \text{ J}$. This provides the same energy as $(2.88 \times 10^{13} \text{ J})/(3.65 \times 10^7 \text{ J/L}) = 7.89 \times 10^5 \text{ L}$ of gasoline. The distance the car can go is $d = (7.89 \times 10^5 \text{ L})(12.75 \text{ km/L}) = 1.01 \times 10^7 \text{ km}$.

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The energy of the electron is given by $E = mc^2/\sqrt{1 - (v/c)^2}$, which yields

$$v = \sqrt{1 - \left[\frac{mc^2}{E}\right]^2} c = \sqrt{1 - \left[\frac{(9.11 \times 10^{-31} \text{ kg})(2.9979 \times 10^8 \text{ m/s})^2}{(1533 \text{ MeV})(1.602 \times 10^{-13} \text{ J/MeV})}\right]^2} = 0.999999994c \approx c$$

for the speed v of the electron. In the rest frame of Earth the trip took time $t = 26 \text{ y}$. A clock traveling with the electron records the proper time of the trip, so the trip in the rest frame of the electron took time $t' = t/\gamma$. Now

$$\gamma = \frac{E}{mc^2} = \frac{1533 \text{ MeV}(1.602 \times 10^{-13} \text{ J/MeV})}{(9.11 \times 10^{-31} \text{ kg})(2.9979 \times 10^8 \text{ m/s})^2} = 3.0 \times 10^3$$

and $t' = (26 \text{ y})/(3.0 \times 10^3) = 8.7 \times 10^{-3} \text{ y}$. The distance traveled is $8.7 \times 10^{-3} \text{ ly}$.

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Start with $(pc)^2 = K^2 + 2Kmc^2$, where p is the momentum of the particle, K is its kinetic energy, and m is its mass. For an electron $mc^2 = 0.511 \text{ MeV}$, so

$$pc = \sqrt{K^2 + 2Kmc^2} = \sqrt{(2.00 \text{ MeV})^2 + 2(2.00 \text{ MeV})(0.511 \text{ MeV})} = 2.46 \text{ MeV}.$$

Thus $p = 2.46 \text{ MeV}/c$.

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The work required is the increased in the energy of the proton. The energy is given by $E = mc^2/[1 - (v/c)^2]$. Let v_1 be the initial speed and v_2 be the final speed. Then the work is

$$W = \frac{mc^2}{\sqrt{1 - (v_2/c)^2}} - \frac{mc^2}{\sqrt{1 - (v_1/c)^2}} = \frac{938 \text{ MeV}}{\sqrt{1 - (0.9860)^2}} - \frac{938 \text{ MeV}}{\sqrt{1 - (0.9850)^2}} = 189 \text{ MeV},$$

where $mc^2 = 938 \text{ MeV}$ was used.

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(a) Let v be the speed of either satellite, relative to Earth. According to the Galilean velocity transformation equation the relative speed is $v_{\text{rel}} = 2v = 2(2.7 \times 10^4 \text{ km/h}) = 5.4 \times 10^4 \text{ km/h}$.

(b) The correct relativistic transformation equation is

$$v_{\text{rel}} = \frac{2v}{1 + \frac{v^2}{c^2}}.$$

The fractional error is

$$\text{fract err} = \frac{2v - v_{\text{rel}}}{2v} = 1 - \frac{1}{1 + \frac{v^2}{c^2}}.$$

The speed of light is $1.08 \times 10^9 \text{ km/h}$, so

$$\text{fract err} = \frac{1}{1 + \frac{(2.7 \times 10^4 \text{ km/h})^2}{(1.08 \times 10^9 \text{ km/h})^2}} = 6.3 \times 10^{-10}.$$