

Chapter 35

5

(a) Take the phases of both waves to be zero at the front surfaces of the layers. The phase of the first wave at the back surface of the glass is given by $\phi_1 = k_1 L - \omega t$, where $k_1 (= 2\pi/\lambda_1)$ is the angular wave number and λ_1 is the wavelength in glass. Similarly, the phase of the second wave at the back surface of the plastic is given by $\phi_2 = k_2 L - \omega t$, where $k_2 (= 2\pi/\lambda_2)$ is the angular wave number and λ_2 is the wavelength in plastic. The angular frequencies are the same since the waves have the same wavelength in air and the frequency of a wave does not change when the wave enters another medium. The phase difference is

$$\phi_1 - \phi_2 = (k_1 - k_2)L = 2\pi \left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2} \right) L.$$

Now $\lambda_1 = \lambda_{\text{air}}/n_1$, where λ_{air} is the wavelength in air and n_1 is the index of refraction of the glass. Similarly, $\lambda_2 = \lambda_{\text{air}}/n_2$, where n_2 is the index of refraction of the plastic. This means that the phase difference is $\phi_1 - \phi_2 = (2\pi/\lambda_{\text{air}})(n_1 - n_2)L$. The value of L that makes this 5.65 rad is

$$L = \frac{(\phi_1 - \phi_2)\lambda_{\text{air}}}{2\pi(n_1 - n_2)} = \frac{5.65(400 \times 10^{-9} \text{ m})}{2\pi(1.60 - 1.50)} = 3.60 \times 10^{-6} \text{ m}.$$

(b) 5.65 rad is less than 2π rad ($= 6.28$ rad), the phase difference for completely constructive interference, and greater than π rad ($= 3.14$ rad), the phase difference for completely destructive interference. The interference is therefore intermediate, neither completely constructive nor completely destructive. It is, however, closer to completely constructive than to completely destructive.

15

Interference maxima occur at angles θ such that $d \sin \theta = m\lambda$, where d is the separation of the sources, λ is the wavelength, and m is an integer. Since $d = 2.0$ m and $\lambda = 0.50$ m, this means that $\sin \theta = 0.25m$. You want all values of m (positive and negative) for which $|0.25m| \leq 1$. These are $-4, -3, -2, -1, 0, +1, +2, +3$, and $+4$. For each of these except -4 and $+4$, there are two different values for θ . A single value of θ (-90°) is associated with $m = -4$ and a single value ($+90^\circ$) is associated with $m = +4$. There are sixteen different angles in all and therefore sixteen maxima.

17

The angular positions of the maxima of a two-slit interference pattern are given by $d \sin \theta = m\lambda$, where d is the slit separation, λ is the wavelength, and m is an integer. If θ is small, $\sin \theta$ may be approximated by θ in radians. Then $d\theta = m\lambda$. The angular separation of two adjacent maxima

is $\Delta\theta = \lambda/d$. Let λ' be the wavelength for which the angular separation is 10.0% greater. Then $1.10\lambda/d = \lambda'/d$ or $\lambda' = 1.10\lambda = 1.10(589 \text{ nm}) = 648 \text{ nm}$.

19

The condition for a maximum in the two-slit interference pattern is $d \sin \theta = m\lambda$, where d is the slit separation, λ is the wavelength, m is an integer, and θ is the angle made by the interfering rays with the forward direction. If θ is small, $\sin \theta$ may be approximated by θ in radians. Then $d\theta = m\lambda$ and the angular separation of adjacent maxima, one associated with the integer m and the other associated with the integer $m + 1$, is given by $\Delta\theta = \lambda/d$. The separation on a screen a distance D away is given by $\Delta y = D \Delta\theta = \lambda D/d$. Thus

$$\Delta y = \frac{(500 \times 10^{-9} \text{ m})(5.40 \text{ m})}{1.20 \times 10^{-3} \text{ m}} = 2.25 \times 10^{-3} \text{ m} = 2.25 \text{ mm}.$$

21

The maxima of a two-slit interference pattern are at angles θ that are given by $d \sin \theta = m\lambda$, where d is the slit separation, λ is the wavelength, and m is an integer. If θ is small, $\sin \theta$ may be replaced by θ in radians. Then $d\theta = m\lambda$. The angular separation of two maxima associated with different wavelengths but the same value of m is $\Delta\theta = (m/d)(\lambda_2 - \lambda_1)$ and the separation on a screen a distance D away is

$$\begin{aligned} \Delta y &= D \tan \Delta\theta \approx D \Delta\theta = \left[\frac{mD}{d} \right] (\lambda_2 - \lambda_1) \\ &= \left[\frac{3(1.0 \text{ m})}{5.0 \times 10^{-3} \text{ m}} \right] (600 \times 10^{-9} \text{ m} - 480 \times 10^{-9} \text{ m}) = 7.2 \times 10^{-5} \text{ m}. \end{aligned}$$

The small angle approximation $\tan \Delta\theta \approx \Delta\theta$ was made. $\Delta\theta$ must be in radians.

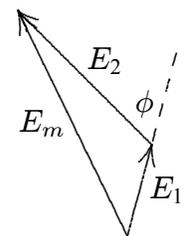
29

The phasor diagram is shown to the right. Here $E_1 = 1.00$, $E_2 = 2.00$, and $\phi = 60^\circ$. The resultant amplitude E_m is given by the trigonometric law of cosines:

$$E_m^2 = E_1^2 + E_2^2 - 2E_1E_2 \cos(180^\circ - \phi),$$

so

$$E_m = \sqrt{(1.00)^2 + (2.00)^2 - 2(1.00)(2.00) \cos 120^\circ} = 2.65.$$



39

For complete destructive interference, you want the waves reflected from the front and back of the coating to differ in phase by an odd multiple of π rad. Each wave is incident on a medium of higher index of refraction from a medium of lower index, so both suffer phase changes of π rad

on reflection. If L is the thickness of the coating, the wave reflected from the back surface travels a distance $2L$ farther than the wave reflected from the front. The phase difference is $2L(2\pi/\lambda_c)$, where λ_c is the wavelength in the coating. If n is the index of refraction of the coating, $\lambda_c = \lambda/n$, where λ is the wavelength in vacuum, and the phase difference is $2nL(2\pi/\lambda)$. Solve

$$2nL \left(\frac{2\pi}{\lambda} \right) = (2m + 1)\pi$$

for L . Here m is an integer. The result is

$$L = \frac{(2m + 1)\lambda}{4n}.$$

To find the least thickness for which destructive interference occurs, take $m = 0$. Then

$$L = \frac{\lambda}{4n} = \frac{600 \times 10^{-9} \text{ m}}{4(1.25)} = 1.2 \times 10^{-7} \text{ m}.$$

41

Since n_1 is greater than n_2 there is no change in phase on reflection from the first surface. Since n_2 is less than n_3 there is a change in phase of π rad on reflection from the second surface. One wave travels a distance $2L$ further than the other, so the difference in the phases of the two waves is $4\pi L/\lambda_2 + \pi$, where λ_2 is the wavelength in medium 2. Since interference produces a minimum the phase difference must be an odd multiple of π . Thus $4\pi L/\lambda_2 + \pi = (2m + 1)\pi$, where m is an integer or zero. Replace λ_2 with λ/n_2 , where λ is the wavelength in air, and solve for λ . The result is

$$\lambda = \frac{4Ln_2}{2m} = \frac{2(380 \text{ nm})(1.1.34)}{m} = \frac{1018 \text{ nm}}{m}.$$

For $m = 1$, $\lambda = 1018 \text{ nm}$ and for $m = 2$, $\lambda = (1018 \text{ nm})/2 = 509 \text{ nm}$. Other wavelengths are shorter. Only $\lambda = 509 \text{ nm}$ is in the visible range.

47

There is a phase shift on reflection of π for both waves and one wave travels a distance $2L$ further than the other, so the phase difference of the reflected waves is $4\pi L/\lambda_2$, where λ_2 is the wavelength in medium 2. Since the result of the interference is a minimum of intensity the phase difference must be an odd multiple of π . Thus $4\pi L/\lambda_2 = (2m + 1)\pi$, where m is an integer or zero. Replace λ_2 with λ/n_2 , where λ is the wavelength in air, and solve for λ . The result is

$$\lambda = \frac{4Ln_2}{2m + 1} = \frac{4(210 \text{ nm})(1.46)}{2m + 1} = \frac{1226 \text{ nm}}{2m + 1}.$$

For $m = 1$, $\lambda = (1226 \text{ nm})/3 = 409 \text{ nm}$. This is in the visible range. Other values of m are associated with wavelengths that are not in the visible range.

53

(a) Oil has a greater index of refraction than air and water has a still greater index of refraction. There is a change of phase of π rad at each reflection. One wave travels a distance $2L$ further

than the other, where L is the thickness of the oil. The phase difference of the two reflected waves is $4\pi L/\lambda_o$, where λ is the wavelength in oil, and this must be equal to a multiple of 2π for a bright reflection. Thus $4\pi L/\lambda_o = 2m\pi$, where m is an integer. Use $\lambda = n_o\lambda_o$, where n_o is the index of refraction for oil, to find the wavelength in air. The result is

$$\lambda = \frac{2n_oL}{m} = \frac{2(1.20)(460 \text{ nm})}{m} = \frac{1104 \text{ nm}}{m}.$$

For $m = 1$, $\lambda = 1104 \text{ nm}$; for $m = 2$, $\lambda = (1104 \text{ nm})/2 = 552 \text{ nm}$; and for $m = 3$, $\lambda = (1104 \text{ nm})/3 = 368 \text{ nm}$. Other wavelengths are shorter. Only $\lambda = 552 \text{ nm}$ is in the visible range.

(b) A maximum in transmission occurs for wavelengths for which the reflection is a minimum. The phases of the two reflected waves then differ by an odd multiple of π rad. This means $4\pi L/\lambda_o = (2m + 1)\pi$ and

$$\lambda = \frac{4n_oL}{2m + 1} = \frac{4(1.20)(460 \text{ nm})}{2m + 1} = \frac{2208 \text{ nm}}{2m + 1}.$$

For $m = 0$, $\lambda = 2208 \text{ nm}$; for $m = 1$, $\lambda = (2208 \text{ nm})/3 = 736 \text{ nm}$; and for $m = 3$, $\lambda = (2208 \text{ nm})/5 = 442 \text{ nm}$. Other wavelengths are shorter. Only $\lambda = 442 \text{ nm}$ falls in the visible range.

63

One wave travels a distance $2L$ further than the other. This wave is reflected twice, once from the back surface and once from the front surface. Since n_2 is greater than n_3 there is no change in phase at the back-surface reflection. Since n_1 is greater than n_2 there is a phase change of π at the front-surface reflection. Thus the phase difference of the two waves as they exit material 2 is $4\pi L/\lambda_2 + \pi$, where λ_2 is the wavelength in material 2. For a maximum in intensity the phase difference is a multiple of 2π . Thus $4\pi L/\lambda_2 + \pi = 2m\pi$, where m is an integer. The solution for λ_2 is

$$\lambda_2 = \frac{4L}{2m - 1} = \frac{4(415 \text{ nm})}{2m - 1} = \frac{1660 \text{ nm}}{2m - 1}.$$

The wavelength in air is

$$\lambda = n_2\lambda_2 = \frac{(1.59)(1660 \text{ nm})}{2m - 1} = \frac{2639 \text{ nm}}{2m - 1}.$$

For $m = 1$, $\lambda = 2639 \text{ nm}$; for $m = 2$, $\lambda = 880 \text{ nm}$; for $m = 3$, $\lambda = 528 \text{ nm}$; and for $m = 4$, $\lambda = 377 \text{ nm}$. Other wavelengths are shorter. Only $\lambda = 528 \text{ nm}$ is in the visible range.

71

Consider the interference of waves reflected from the top and bottom surfaces of the air film. The wave reflected from the upper surface does not change phase on reflection but the wave reflected from the bottom surface changes phase by π rad. At a place where the thickness of the air film is L , the condition for fully constructive interference is $2L = (m + \frac{1}{2})\lambda$, where λ ($= 683 \text{ nm}$) is the wavelength and m is an integer. This is satisfied for $m = 140$:

$$L = \frac{(m + \frac{1}{2})\lambda}{2} = \frac{(140.5)(683 \times 10^{-9} \text{ m})}{2} = 4.80 \times 10^{-5} \text{ m} = 0.048 \text{ mm}.$$

At the thin end of the air film, there is a bright fringe. It is associated with $m = 0$. There are, therefore, 140 bright fringes in all.

75

Consider the interference pattern formed by waves reflected from the upper and lower surfaces of the air wedge. The wave reflected from the lower surface undergoes a π -rad phase change while the wave reflected from the upper surface does not. At a place where the thickness of the wedge is d , the condition for a maximum in intensity is $2d = (m + \frac{1}{2})\lambda$, where λ is the wavelength in air and m is an integer. Thus $d = (2m + 1)\lambda/4$. As the geometry of Fig. 35–47 shows, $d = R - \sqrt{R^2 - r^2}$, where R is the radius of curvature of the lens and r is the radius of a Newton's ring. Thus $(2m + 1)\lambda/4 = R - \sqrt{R^2 - r^2}$. Solve for r . First rearrange the terms so the equation becomes

$$\sqrt{R^2 - r^2} = R - \frac{(2m + 1)\lambda}{4}.$$

Now square both sides and solve for r^2 . When you take the square root, you should get

$$r = \sqrt{\frac{(2m + 1)R\lambda}{2} - \frac{(2m + 1)^2\lambda^2}{16}}.$$

If R is much larger than a wavelength, the first term dominates the second and

$$r = \sqrt{\frac{(2m + 1)R\lambda}{2}}.$$

81

Let ϕ_1 be the phase difference of the waves in the two arms when the tube has air in it and let ϕ_2 be the phase difference when the tube is evacuated. These are different because the wavelength in air is different from the wavelength in vacuum. If λ is the wavelength in vacuum, then the wavelength in air is λ/n , where n is the index of refraction of air. This means

$$\phi_1 - \phi_2 = 2L \left[\frac{2\pi n}{\lambda} - \frac{2\pi}{\lambda} \right] = \frac{4\pi(n - 1)L}{\lambda},$$

where L is the length of the tube. The factor 2 arises because the light traverses the tube twice, once on the way to a mirror and once after reflection from the mirror.

Each shift by one fringe corresponds to a change in phase of 2π rad, so if the interference pattern shifts by N fringes as the tube is evacuated,

$$\frac{4\pi(n - 1)L}{\lambda} = 2N\pi$$

and

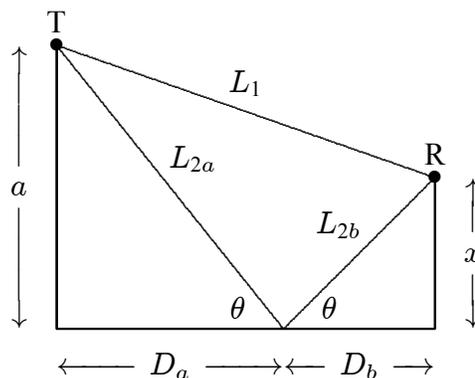
$$n = 1 + \frac{N\lambda}{2L} = 1 + \frac{60(500 \times 10^{-9} \text{ m})}{2(5.0 \times 10^{-2} \text{ m})} = 1.00030.$$

87

Suppose the wave that goes directly to the receiver travels a distance L_1 and the reflected wave travels a distance L_2 . Since the index of refraction of water is greater than that of air this last wave suffers a phase change on reflection of half a wavelength. To obtain constructive interference at the receiver the difference $L_2 - L_1$ in the distances traveled must be an odd multiple of a half wavelength.

Look at the diagram on the right. The right triangle on the left, formed by the vertical line from the water to the transmitter T, the ray incident on the water, and the water line, gives $D_a = a / \tan \theta$ and the right triangle on the right, formed by the vertical line from the water to the receiver R, the reflected ray, and the water line gives $D_b = x / \tan \theta$. Since $D_a + D_b = D$,

$$\tan \theta = \frac{a + x}{D}.$$



Use the identity $\sin^2 \theta = \tan^2 \theta / (1 + \tan^2 \theta)$ to show that $\sin \theta = (a + x) / \sqrt{D^2 + (a + x)^2}$. This means

$$L_{2a} = \frac{a}{\sin \theta} = \frac{a \sqrt{D^2 + (a + x)^2}}{a + x}$$

and

$$L_{2b} = \frac{x}{\sin \theta} = \frac{x \sqrt{D^2 + (a + x)^2}}{a + x},$$

so

$$L_2 = L_{2a} + L_{2b} = \frac{(a + x) \sqrt{D^2 + (a + x)^2}}{a + x} = \sqrt{D^2 + (a + x)^2}.$$

Use the binomial theorem, with D^2 large and $a^2 + x^2$ small, to approximate this expression: $L_2 \approx D + (a + x)^2 / 2D$.

The distance traveled by the direct wave is $L_1 = \sqrt{D^2 + (a - x)^2}$. Use the binomial theorem to approximate this expression: $L_1 \approx D + (a - x)^2 / 2D$. Thus

$$L_2 - L_1 \approx D + \frac{a^2 + 2ax + x^2}{2D} - D - \frac{a^2 - 2ax + x^2}{2D} = \frac{2ax}{D}.$$

Set this equal to $(m + \frac{1}{2})\lambda$, where m is zero or a positive integer. Solve for x . The result is $x = (m + \frac{1}{2})(D/2a)\lambda$.

89

Bright fringes occur at an angle θ such that $d \sin \theta = m\lambda$, where d is the slit separation, λ is the wavelength in the medium of propagation, and m is an integer. Near the center of the pattern the angles are small and $\sin \theta$ can be approximated by θ in radians. Thus $\theta = m\lambda/d$ and the angular separation of two adjacent bright fringes is $\Delta\theta = \lambda/d$. When the arrangement is immersed in water the angular separation of the fringes becomes $\Delta\theta' = \lambda_w/d$, where λ_w is the wavelength in

water. Since $\lambda_w = \lambda/n_w$, where n_w is the index of refraction of water, $\Delta\theta' = \lambda/n_w d = (\Delta\theta)/n_w$. Since the units of the angles cancel from this equation we may substitute the angles in degrees and obtain $\Delta\theta' = 0.30^\circ/1.33 = 0.23^\circ$.

93

(a) For wavelength λ dark bands occur where the path difference is an odd multiple of $\lambda/2$. That is, where the path difference is $(2m + 1)\lambda/2$, where m is an integer. The fourth dark band from the central bright fringe is associated with $m = 3$ and is $7\lambda/2 = 7(500 \text{ nm})/2 = 1750 \text{ nm}$.

(b) The angular position θ of the first bright band on either side of the central band is given by $\sin \theta = \lambda/d$, where d is the slit separation. The distance on the screen is given by $\Delta y = D \tan \theta$, where D is the distance from the slits to the screen. Because θ is small its sine and tangent are very nearly equal and $\Delta y = D \sin \theta = D\lambda/d$.

Dark bands have angular positions that are given by $\sin \theta = (m + \frac{1}{2})\lambda/d$ and, for the fourth dark band, $m = 3$ and $\sin \theta_4 = (7/2)\lambda/d$. Its distance on the screen from the central fringe is $\Delta y_4 = D \tan \theta_4 = D \sin \theta_4 = 7D\lambda/2d$. This means that $D\lambda/d = 2\Delta y_4/7 = 2(1.68 \text{ cm})/7 = 0.48 \text{ cm}$. Note that this is Δy .

99

Minima occur at angles θ for which $\sin \theta = (m + \frac{1}{2})\lambda/d$, where λ is the wavelength, d is the slit separation, and m is an integer. For the first minimum, $m = 0$ and $\sin \theta_1 = \lambda/2d$. For the tenth minimum, $m = 9$ and $\sin \theta_{10} = 19\lambda/2d$.

The distance on the screen from the central fringe to a minimum is $y = D \tan \theta$, where D is the distance from the slits to the screen. Since the angle is small we may approximate its tangent with its sine and write $y = D \sin \theta = D(m + \frac{1}{2})\lambda/d$. Thus the separation of the first and tenth minima is

$$\Delta y = \frac{D}{d} \left(\frac{19\lambda}{2} - \frac{\lambda}{2} \right) = \frac{9D\lambda}{d}$$

and

$$\lambda = \frac{d \Delta y}{9D} = \frac{(0.150 \times 10^{-3} \text{ m})(18.0 \times 10^{-3} \text{ m})}{9(50.0 \times 10^{-2} \text{ m})} = 6.00 \times 10^{-7} \text{ m}.$$

103

The difference in the path lengths of the two beams is $2x$, so their difference in phase when they reach the detector is $\phi = 4\pi x/\lambda$, where λ is the wavelength. Assume their amplitudes are the same. According to Eq. 35-22 the intensity associated with the addition of two waves is proportional to the square of the cosine of half their phase difference. Thus the intensity of the light observed in the interferometer is proportional to $\cos^2(2\pi x/\lambda)$. Since the intensity is maximum when $x = 0$ (and the arms have equal lengths), the constant of proportionality is the maximum intensity I_m and $I = I_m \cos^2(2\pi x/\lambda)$.