Chapter 38

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(a) Let R be the rate of photon emission (number of photons emitted per unit time) and let E be the energy of a single photon. Then the power output of a lamp is given by $P = RE$ if all the power goes into photon production. Now $E = hf = hc/\lambda$, where h is the Planck constant, f is the frequency of the light emitted, and λ is the wavelength. Thus $P = Rhc/\lambda$ and $R = \lambda P/hc$. The lamp emitting light with the longer wavelength (the 700-nm lamp) emits more photons per unit time. The energy of each photon is less so it must emit photons at a greater rate.

(b) Let R be the rate of photon production for the 700 nm lamp Then

$$R = \frac{\lambda P}{hc} = \frac{(700 \times 10^{-9}\,\text{m})(400\,\text{J/s})}{(6.626 \times 10^{-34}\,\text{J}\cdot\text{s})(2.9979 \times 10^{8}\,\text{m/s})} = 1.41 \times 10^{21}\,\text{photon/s}\,.$$

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The energy of an incident photon is $E = hf = hc/\lambda$, where h is the Planck constant, f is the frequency of the electromagnetic radiation, and λ is its wavelength. The kinetic energy of the most energetic electron emitted is $K_m = E - \Phi = (hc/\lambda) - \Phi$, where Φ is the work function for sodium. The stopping potential V_0 is related to the maximum kinetic energy by $eV_0 = K_m$, so $eV_0 = (hc/\lambda) - \Phi$ and

$$\lambda = \frac{hc}{eV_0 + \Phi} = \frac{(6.626 \times 19^{-34}\,\text{J}\cdot\text{s})(2.9979 \times 10^{8}\,\text{m/s})}{(5.0\,\text{eV} + 2.2\,\text{eV})(1.602 \times 10^{-19}\,\text{J/eV})} = 1.7 \times 10^{-7}\,\text{m}\,.$$

Here $eV_0 = 5.0\,\text{eV}$ was used.

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(a) The kinetic energy K_m of the fastest electron emitted is given by $K_m = hf - \Phi = (hc/\lambda) - \Phi$, where Φ is the work function of aluminum, f is the frequency of the incident radiation, and λ is its wavelength. The relationship $f = c/\lambda$ was used to obtain the second form. Thus

$$K_m = \frac{(6.626 \times 10^{-34}\,\text{J}\cdot\text{s})(2.9979 \times 10^{8}\,\text{m/s})}{(200 \times 10^{-9}\,\text{m})(1.602 \times 10^{-19}\,\text{J/eV})} - 4.20\,\text{eV} = 2.00\,\text{eV}\,.$$

(b) The slowest electron just breaks free of the surface and so has zero kinetic energy.

(c) The stopping potential V_0 is given by $K_m = eV_0$, so $V_0 = K_m/e = (2.00\,\text{eV})/e = 2.00\,\text{V}$.

(d) The value of the cutoff wavelength is such that $K_m = 0$. Thus $hc/\lambda = \Phi$ or

$$\lambda = \frac{hc}{\Phi} = \frac{(6.626 \times 10^{-34}\,\text{J}\cdot\text{s})(2.9979 \times 10^{8}\,\text{m/s})}{(4.2\,\text{eV})(1.602 \times 10^{-19}\,\text{J/eV})} = 2.95 \times 10^{-7}\,\text{m}\,.$$

If the wavelength is longer, the photon energy is less and a photon does not have sufficient energy to knock even the most energetic electron out of the aluminum sample.

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(a) When a photon scatters from an electron initially at rest, the change in wavelength is given by $\Delta\lambda = (h/mc)(1 - \cos\phi)$, where m is the mass of an electron and ϕ is the scattering angle. Now $h/mc = 2.43 \times 10^{-12}$ m $= 2.43$ pm, so $\Delta\lambda = (2.43\,\text{pm})(1 - \cos 30°) = 0.326$ pm. The final wavelength is $\lambda' = \lambda + \Delta\lambda = 2.4\,\text{pm} + 0.326\,\text{pm} = 2.73$ pm.

(b) Now $\Delta\lambda = (2.43\,\text{pm})(1 - \cos 120°) = 3.645$ pm and $\lambda' = 2.4\,\text{pm} + 3.645\,\text{pm} = 6.05$ pm.

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Since the kinetic energy K and momentum p are related by $K = p^2/2m$, the momentum of the electron is $p = \sqrt{2mK}$ and the wavelength of its matter wave is $\lambda = h/p = h/\sqrt{2mK}$. Replace K with eV, where V is the accelerating potential and e is the fundamental charge, to obtain

$$\lambda = \frac{h}{\sqrt{2meV}} = \frac{6.626 \times 10^{-34}\,\text{J}\cdot\text{s}}{\sqrt{2(9.109 \times 10^{-31}\,\text{kg})(1.602 \times 10^{-19}\,\text{C})(25.0 \times 10^3\,\text{V})}}$$
$$= 7.75 \times 10^{-12}\,\text{m} = 7.75\,\text{pm}.$$

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(a) The kinetic energy acquired is $K = qV$, where q is the charge on an ion and V is the accelerating potential. Thus $K = (1.602 \times 10^{-19}\,\text{C})(300\,\text{V}) = 4.80 \times 10^{-17}\,\text{J}$. The mass of a single sodium atom is, from Appendix F, $m = (22.9898\,\text{g/mol})/(6.02 \times 10^{23}\,\text{atom/mol}) = 3.819 \times 10^{-23}\,\text{g} = 3.819 \times 10^{-26}\,\text{kg}$. Thus the momentum of an ion is

$$p = \sqrt{2mK} = \sqrt{2(3.819 \times 10^{-26}\,\text{kg})(4.80 \times 10^{-17}\,\text{J})} = 1.91 \times 10^{-21}\,\text{kg}\cdot\text{m/s}.$$

(b) The de Broglie wavelength is

$$\lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34}\,\text{J}\cdot\text{s}}{1.91 \times 10^{-21}\,\text{kg}\cdot\text{m/s}} = 3.47 \times 10^{-13}\,\text{m}.$$

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Since the kinetic energy K and momentum p are related by $K = p^2/2m$, the momentum of the electron is $p = \sqrt{2mK}$ and the wavelength of its matter wave is $\lambda = h/p = h/\sqrt{2mK}$. Thus

$$K = \frac{1}{2m}\left(\frac{h}{\lambda}\right)^2 = \frac{1}{2(9.11 \times 10^{-31}\,\text{kg})}\left(\frac{6.626 \times 10^{-34}\,\text{J}\cdot\text{s}}{590 \times 10^{-9}\,\text{m}}\right)^2$$
$$= 6.92 \times 10^{-25}\,\text{J} = 4.33 \times 10^{-6}\,\text{eV}.$$

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The angular wave number k is related to the wavelength λ by $k = 2\pi/\lambda$ and the wavelength is related to the particle momentum p by $\lambda = h/p$, so $k = 2\pi p/h$. Now the kinetic energy K and

the momentum are related by $K = p^2/2m$, where m is the mass of the particle. Thus $p = \sqrt{2mK}$ and

$$k = \frac{2\pi\sqrt{2mK}}{h} .$$

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For $U = U_0$, Schrödinger's equation becomes

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}[E - U_0]\psi = 0 .$$

Substitute $\psi = \psi_0 e^{ikx}$. The second derivative is $d^2\psi/dx^2 = -k^2\psi_0 e^{ikx} = -k^2\psi$. The result is

$$-k^2\psi + \frac{8\pi^2 m}{h^2}[E - U_0]\psi = 0 .$$

Solve for k and obtain

$$k = \sqrt{\frac{8\pi^2 m}{h^2}[E - U_0]} = \frac{2\pi}{h}\sqrt{2m[E - U_0]} .$$

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(a) If m is the mass of the particle and E is its energy, then the transmission coefficient for a barrier of height U and width L is given by

$$T = e^{-2kL} ,$$

where

$$k = \sqrt{\frac{8\pi^2 m(U - E)}{h^2}} .$$

If the change ΔU in U is small (as it is), the change in the transmission coefficient is given by

$$\Delta T = \frac{dT}{dU}\Delta U = -2LT\frac{dk}{dU}\Delta U .$$

Now

$$\frac{dk}{dU} = \frac{1}{2\sqrt{U - E}}\sqrt{\frac{8\pi^2 m}{h^2}} = \frac{1}{2(U - E)}\sqrt{\frac{8\pi^2 m(U - E)}{h^2}} = \frac{k}{2(U - E)} .$$

Thus

$$\Delta T = -LTk\frac{\Delta U}{U - E} .$$

For the data of Sample Problem 38–7, $2kL = 10.0$, so $kL = 5.0$ and

$$\frac{\Delta T}{T} = -kL\frac{\Delta U}{U - E} = -(5.0)\frac{(0.010)(6.8\,\text{eV})}{6.8\,\text{eV} - 5.1\,\text{eV}} = -0.20 .$$

There is a 20% decrease in the transmission coefficient.

(b) The change in the transmission coefficient is given by

$$\Delta T = \frac{dT}{dL}\,\Delta L = -2ke^{-2kL}\,\Delta L = -2kT\,\Delta L$$

and

$$\frac{\Delta T}{T} = -2k\,\Delta L = -2(6.67 \times 10^9\,\mathrm{m}^{-1})(0.010)(750 \times 10^{-12}\,\mathrm{m}) = -0.10\,.$$

There is a 10% decrease in the transmission coefficient.

(c) The change in the transmission coefficient is given by

$$\Delta T = \frac{dT}{dE}\,\Delta E = -2Le^{-2kL}\frac{dk}{dE}\,\Delta E = -2LT\frac{dk}{dE}\,\Delta E\,.$$

Now $dk/dE = -dk/dU = -k/2(U - E)$, so

$$\frac{\Delta T}{T} = kL\frac{\Delta E}{U - E} = (5.0)\frac{(0.010)(5.1\,\mathrm{eV})}{6.8\,\mathrm{eV} - 5.1\,\mathrm{eV}} = 0.15\,.$$

There is a 15% increase in the transmission coefficient.

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The uncertainty in the momentum is $\Delta p = m\,\Delta v = (0.50\,\mathrm{kg})(1.0\,\mathrm{m/s}) = 0.50\,\mathrm{kg \cdot m/s}$, where Δv is the uncertainty in the velocity. Solve the uncertainty relationship $\Delta x\,\Delta p \geq \hbar$ for the minimum uncertainty in the coordinate x: $\Delta x = \hbar/\Delta p = (0.60\,\mathrm{J \cdot s})/2\pi(0.50\,\mathrm{kg \cdot m/s}) = 0.19\,\mathrm{m}$.