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Sliding Mode Observer-Based FTC for Markovian Jump Systems with Actuator and Sensor Faults

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Abstract—**This work addresses the stabilization problem for nonlinear Markovian jump systems (MJS) with output disturbances, actuator and sensor faults simultaneously. This kind of plants are common in practical systems, such as mobile manipulators with switching joints. In this work, a sliding mode observer design scheme is proposed for a new descriptor augmented plant. By employing the developed observer, the effects of actuator and sensor faults can be eliminated. It is shown that the stabilization of the overall closed-loop plant can be guaranteed by the proposed fault tolerant control (FTC) scheme. Finally, an example concerning mobile manipulators with Markovian switching joints is presented to show the effectiveness and applicability of the theoretical results.**

Index Terms— **Sliding mode observer, Fault tolerant control, Markovian jump systems, Actuator faults, Sensor faults.**

I. Introduction

In recent years, mobile manipulators have been extensively employed in practice, especially in some hazardous environments, such as space operations and manufacturing sectors. Moreover, mobile manipulators with hybrid joints exhibit better performance than ordinary ones in practical applications. Recognizing that the robots with hybrid joints can be better in energy saving than the ones without hybrid joints, a lot of researchers focused on the issue of joint switching. A second-order nonholonomic constraint is obtained by considering the zero torque at the hybrid joints in [1]. Furthermore, in [2], the Markovian jump linear system (MJLS) method is applied to analyze the joint switching. During the past few years, Markovian jump systems (MJS) have played an important role as they can be employed to model amounts of physical systems, such as networked systems and power systems [3], [4]. There has been a tremendous progress made in the field of MJS. In [5], an asynchronous switching controller is proposed for stochastic hybrid retarded systems. Then, the authors of [6] pay attention to the output feedback controller design problem for discrete-time MJS. Furthermore, the stabilization can be guaranteed by the method proposed in [7] for MJS with partly unknown transition probabilities. Recently, the authors in [8] propose a novel adaptive FTC method for MJS.

In practice, faults such as actuator faults and sensor faults usually occur inevitably. Markovian jump systems are no exception. These faults may cause discontented system behaviors and even lead to instability or catastrophic accidents. Due to this reason, considerable research efforts have been devoted to the subject of fault tolerant control [9]–[11] (passive FTC and active FTC) and fault detection and isolation in both theory and practice [8], [12]–[15]. It is worth noting that the fault diagnosis techniques in [13] and [14] are mainly data-driven.

On the other hand, control strategies concerning the problem of observer design have been studied for years. Several superior observer schemes have been proposed for various kinds of plants, such as stochastic systems [16], nonlinear plants [17], Markovian jump systems [18], [19] and so on. Among the aforementioned works, [18] provided the scheme of sliding mode observer (SMO). In fact, the SMO [20] has been widely employed due to its strong robustness in regard to uncertainties and the possibility of uncertainty estimation. Furthermore, in [17], the design of SMO was employed to nonlinear systems with actuator faults. Besides, there has been a tremendous progress made in sliding mode control (SMC). The problem of SMC with soft computing has been addressed in the survey [21]. In a follow-up work [22], the terminal SMC for multi-input plants is discussed. However, in practical plants such as mobile manipulators, disturbances, actuator and sensor faults often occur in control systems simultaneously. The negative effect they may cause requires us to take all of them into consideration in designing the control systems. Although some researchers have attempted on the control problem for mobile manipulators, the existing results have been either focused on only disturbances [2] or only actuator faults [8]. Until now, little related works have fully investigated the problem of the design of observers for mobile manipulator systems with output disturbances and both actuator and sensor faults. In other words, this problem is still open and challenging.

Based on the observation above, in this paper, we study the FTC problem for Markovian Jump Systems against actuator and sensor faults simultaneously. Firstly, by introducing an augmented vector consisting of the state, output disturbance, actuator and sensor fault vectors, a SMO is proposed to obtain the estimation of disturbances, actuator and sensor faults simultaneously. Secondly, an observer-based FTC strategy is proposed to stabilize the resulted fault plant. Then, an example concerned with mobile manipulators with Markovian switching joints is provided to demonstrate the effectiveness of the developed FTC approach. The main contributions of this paper lie in: (i) designing a sliding mode observer(SMO)-based FTC strategy for Markovian Jump Systems with Lipschitz nonlinearities, output disturbances, actuator and sensor faults simultaneously; (ii) considering actuator and sensor faults simultaneously for mobile manipulators with hybrid joints in the simulation part.

This paper is organized as follows. In Section 2, the problem formulation is given. The main results are presented

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in Section 3. In Section 4, an example concerned with mobile manipulators with Markovian switching joints is provided and a conclusion in Section 5 ends this paper.

II. Problem formulation

Consider the following MJS with Lipschitz nonlinearities, output disturbances, actuator and sensor faults defined in fixed probability space $(\Omega, \mathcal{F}, \mathcal{P})$

$$\begin{cases} \dot{x}(t) = & A(r_t)x(t) + B(r_t)u(t) + F_a(r_t)f_a(t) \\ & +g(t,x,r_t) \\ y(t) = & C(r_t)x(t) + F_s(r_t)f_s(t) + D(r_t)u(t) \\ & +G(r_t)f_a(t) + D_d(r_t)d(t), \end{cases} \quad (1)$$

where $x(t) \in \mathbb{R}^n$ denotes the state variable, $y(t) \in \mathbb{R}^p$ denotes the measurement output and $u(t) \in \mathbb{R}^m$ is the control input. $f_a(t) \in \mathbb{R}^a$, $f_s(t) \in \mathbb{R}^s$ and $d(t) \in \mathbb{R}^d$ represent the actuator and sensor faults and the bounded disturbance, respectively. $g(t, x, r_t)$ denotes the nonlinear Lipschitz vector function; $\{r_t, t \geq 0\}$ denotes a Markov chain on $(\Omega, \mathcal{F}, \mathcal{P})$ with right continuous trajectories, which takes values in $\mathbb{S} \triangleq \{1, 2, \cdots, N\}$. The generator matrix $\Pi \triangleq [\pi_{ij}], i, j \in \mathbb{S}$ with the transition probability is given by

$$P_{ij} = \Pr\{r_{t+\Delta} = j | r_t = i\} = \begin{cases} \pi_{ij}\Delta + o(\Delta) & if\ i \neq j \\ 1 + \pi_{ii}\Delta + o(\Delta) & if\ i = j. \end{cases}$$

where $\Delta > 0$ and $\lim_{\Delta \to 0} o(\Delta)/\Delta = 0$; $\pi_{ij} > 0$, $i \neq j$ and $\pi_{ii} = -\sum_{j \neq i} \pi_{ij} < 0$ for $i \in \mathbb{S}$. $A(r_t) \in \mathbb{R}^{n\times n}$, $B(r_t) \in \mathbb{R}^{n\times m}$, $F_a(r_t) \in \mathbb{R}^{n\times a}$, $C(r_t) \in \mathbb{R}^{p\times n}$, $F_s(r_t) \in \mathbb{R}^{p\times s}$, $D(r_t) \in \mathbb{R}^{p\times m}$, $D_d(r_t) \in \mathbb{R}^{p\times d}$, $G(r_t) \in \mathbb{R}^{p\times a}$ refer to the system matrices. For notational simplicity, the matrices $A(r_t)$, $B(r_t)$, $C(r_t)$, $F_a(r_t)$, $F_s(r_t)$, $D(r_t)$,$G(r_t)$,$D_d(r_t)$ will be represented by A_i, B_i, C_i, F_{ai}, F_{si}, D_i, G_i,D_{di} when $r_t = i, i \in \mathbb{S}$.

The assumptions below are required for each $r_t = i \in \mathbb{S}$ in this paper.

(A1) (A_i, C_i) is observable. Besides, there exists a scalar $\theta_i > 0$ such that the following holds

$$rank \begin{bmatrix} \theta_i I_n + A_i & F_{ai} \\ C_i & G_i \end{bmatrix} = n + a. \quad (2)$$

(A2) The matrices F_{ai}, F_{si} and D_{di} are of full row rank.

(A3) The output disturbances, sensor faults and actuator faults considered in this paper refer to small bounded ones and satisfy: $||d(t)|| \leq r_d$, $||f_s(t)|| \leq r_{s1}$, $||\dot{f}_s(t)|| \leq r_{s2}$, $||f_a(t)|| \leq r_{a1}$, $||\dot{f}_a(t)|| \leq r_{a2}$, where $r_d > 0$, $r_{s1} > 0$, $r_{s2} > 0$, $r_{a1} > 0$ and $r_{a2} > 0$ represent known constants.

(A4) $g(t, x, r_t)$ satisfies the constraint of Lipschitz:

$$\begin{aligned} ||g(t,\hat{x},r_t) - g(t,x,r_t)|| &\leq ||T_i(\hat{x}(t) - x(t))|| \\ &\leq \eta||\hat{x}(t) - x(t)||, \quad (3) \end{aligned}$$

for any $x(t)$ and $\hat{x}(t) \in \mathbb{R}^n$, where $\eta = \max_{i\in\mathbb{S}} ||T_i||$ and $T_i \in \mathbb{R}^{n\times n}$ denotes a known constant matrix.

(A5) The system matrix dimensions satisfy:

$$a + s + d \leq p$$

.

Remark 1. In practice, most of faults often occur abruptly and the real functions of them are often unknown. In order to proceed the FTC procedure, we provide assumption (A3), in which the condition is general in the existing FTC results.

In the following discussion, the following preliminaries will be adopted.

Definition 1. [7] The MJS (1) is said to be stochastically stable, if for any initial condition $x_0 \in \mathbb{R}^n$, $r_0 \in \mathbb{S}$ and $u(t) \equiv 0$, the following holds:

$$\mathbb{E}\{\int_0^\infty ||x(t)||^2 | x_0, r_0\} < \infty. \quad (4)$$

Definition 2. [23] Let $C_2(\mathbb{R}^n \times \mathbb{S}; \mathbb{R}_+)$ represent the family of nonnegative functions $V(x(t), i)$ on $\mathbb{R}^n \times \mathbb{S}$ which are twice differentiable continuously in $x(t)$. For $V \in C_2(\mathbb{R}^n \times \mathbb{S}; \mathbb{R}_+)$, define an infinitesimal operator in the following form:

$$\begin{aligned} \mathcal{L}V(x(t), i) &= \lim_{\Delta\to 0^+} \tfrac{1}{\Delta}[\mathbb{E}\{V(x(t+\Delta), r_{t+\Delta})|x(t), \\ &\qquad r_t = i\} - V(x(t), i)]. \quad (5) \end{aligned}$$

III. Main results

In this paper, the main goals are to obtain the estimations of $x(t)$, $f_a(t)$, $f_s(t)$ and $d(t)$ and then to synthesize a FTC strategy for the MJS (1). To achieve the goals, firstly, we define some augmented matrices and variables:

$$\bar{x}(t) \triangleq \begin{bmatrix} x(t) \\ f_a(t) \\ F_{si}f_s(t) \\ D_{di}d(t) \end{bmatrix}, \quad \bar{f}(t) \triangleq \begin{bmatrix} \theta_i f_a(t) + \dot{f}_a(t) \\ \theta_i f_s(t) + \dot{f}_s(t) \\ d(t) \end{bmatrix},$$

$$\bar{E}_i \triangleq \begin{bmatrix} I_n & \theta_i^{-1}F_{ai} & 0_{n\times p} & 0_{n\times p} \\ 0_{a\times n} & I_a & 0_{a\times p} & 0_{a\times p} \\ 0_{p\times n} & 0_{p\times a} & I_p & 0_{p\times p} \\ 0_{p\times n} & 0_{p\times a} & 0_{p\times p} & 0_{p\times p} \end{bmatrix},$$

$$\bar{A}_i \triangleq \begin{bmatrix} A_i & 0_{n\times a} & 0_{n\times p} & 0_{n\times p} \\ 0_{a\times n} & -\theta_i I_a & 0_{a\times p} & 0_{a\times p} \\ 0_{p\times n} & 0_{p\times a} & -\theta_i I_p & 0_{p\times p} \\ 0_{p\times n} & 0_{p\times a} & 0_{p\times p} & -I_p \end{bmatrix},$$

$$\bar{B}_{fi} \triangleq \begin{bmatrix} \theta_i^{-1}F_{ai} & 0_{n\times s} & 0_{n\times d} \\ I_a & 0_{a\times s} & 0_{a\times d} \\ 0_{p\times a} & F_{si} & 0_{p\times d} \\ 0_{p\times a} & 0_{p\times s} & D_{di} \end{bmatrix}, \quad \bar{B}_i \triangleq \begin{bmatrix} B_i \\ 0_{a\times m} \\ 0_{p\times m} \\ 0_{p\times m} \end{bmatrix},$$

$$\bar{C}_i \triangleq \begin{bmatrix} C_i & G_i & I_p & I_p \end{bmatrix}, \quad \bar{F} \triangleq \begin{bmatrix} I_n \\ 0_{a\times n} \\ 0_{p\times n} \\ 0_{p\times n} \end{bmatrix},$$

where $\theta_i > 0$ is a parameter which stratifies A1.

Furthermore, the augmented plant can be constructed as follows:

$$\begin{cases} \bar{E}_i\dot{\bar{x}}(t) = \bar{A}_i\bar{x}(t) + \bar{B}_i u(t) + \bar{B}_{fi}\bar{f}(t) + \bar{F}g(t,x,i) \\ y(t) = \bar{C}_i\bar{x}(t) + D_i u(t). \end{cases} \quad (6)$$

It can be seen that the plant (6) is a singular system. Then, we know that the matrices $\bar{C}_i$ and $\bar{E}_i$ have the property below

$$rank \begin{bmatrix} \bar{E}_i \\ \bar{C}_i \end{bmatrix} = n + a + 2p = \bar{n}. \quad (7)$$

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Define two matrices in the following form:

$$\bar{L}_{Di} = \begin{bmatrix} 0_{p\times n} & 0_{p\times a} & 0_{p\times p} & W_i^T \end{bmatrix}^T,$$

$$\bar{S}_i \triangleq (\bar{E}_i + \bar{L}_{D_i}\bar{C}_i), \tag{8}$$

where $W = diag\{l_1, l_2, \ldots, l_p\}$ and $l_i > 0$ for $i = 1, 2, \ldots, p$.

Following the proof of Lemma 1 in [18], it can be calculated easily that

$$\bar{S}_i = \begin{bmatrix} I_n & \theta_i^{-1}F_{ai} & 0_{n\times p} & 0_{n\times p} \\ 0_{a\times n} & I_a & 0_{a\times p} & 0_{a\times p} \\ 0_{p\times n} & 0_{p\times a} & I_p & 0_{p\times p} \\ W_iC_i & W_iG_i & W_i & W_i \end{bmatrix},$$
$$\bar{S}_i^{-1} = \begin{bmatrix} I_n & -\theta_i^{-1}F_{ai} & 0_{n\times p} & 0_{n\times p} \\ 0_{a\times n} & I_a & 0_{a\times p} & 0_{a\times p} \\ 0_{p\times n} & 0_{p\times a} & I_p & 0_{p\times p} \\ -C_i & \theta_i^{-1}C_iF_{ai} - G_i & -I_p & W_i^{-1} \end{bmatrix}.$$

Furthermore, one has

$$\bar{C}_i\bar{S}_i^{-1}\bar{L}_{Di} = I_p, \tag{9}$$

$$\bar{A}_i\bar{S}_i^{-1}\bar{L}_{Di} = -\bar{N}. \tag{10}$$

Remark 2. Note that the estimation of the vector $\bar{x}_i$ in singular system (6) will lead to an estimation of the unmeasured state vector x_i, the actuator fault vector $f_a(t)$, the sensor fault vector $F_{si}f_s(t)$ and the disturbance vector $F_{di}d(t)$. If an effective state observer can be constructed for the singular system, the asymptotic estimations of state, the actuator fault, the sensor fault and disturbance will be achieved simultaneously. Thus, based on such an idea, the original problem of state and fault estimation is transferred into an observer design issue for the descriptor system (6).

Based upon the above discussion, the descriptor SMO is proposed for (6)

$$\begin{cases} \bar{S}_i\dot{\bar{z}}(t) &= (\bar{A}_i - \bar{L}_{pi}\bar{C}_i)\bar{z}(t) + \bar{B}_iu(t) \\ &\quad +\bar{N}(y(t) - D_iu(t)) + \bar{L}_{si}u_s(t) + \bar{F}g(t,\hat{x},i) \\ \hat{\bar{x}}(t) &= \bar{z}(t) + \bar{S}_i^{-1}\bar{L}_{Di}(y(t) - D_iu(t)), \end{cases} \tag{11}$$

where $\bar{z}(t) \triangleq [z_x^T(t), z_a^T(t), z_s^T(t), z_d^T(t)]^T$ denotes the intermediate variable and $\hat{\bar{x}} \triangleq [\hat{x}^T(t), \hat{f}_a^T(t), \hat{f}_s^T(t), \hat{d}^T(t)]^T$ denotes the estimation of $\bar{x}(t)$; $g(t,\hat{x},i)$ is a known function, $\bar{L}_{Di} \in \mathbb{R}^{\bar{n}\times p}$, $\bar{L}_{pi} \in \mathbb{R}^{\bar{n}\times p}$ and $\bar{L}_{si} \in \mathbb{R}^{\bar{n}\times(a+s+d)}$ are the observer gains to be designed. Based upon the discussion above, $\bar{S}_i \triangleq (\bar{E}_i + \bar{L}_{D_i}\bar{C}_i)$ is non-singular by selecting appropriate $\bar{L}_{D_i}$.

It is noticed that D_{di} and F_{si} denote full column rank matrices. Thus $(D_{di}^TD_{di})^{-1}$ and $(F_{si}^TF_{si})^{-1}$ exist. Then, we know that the real estimation of $d(t)$ and $f_s(t)$ are $(D_{di}^TD_{di})^{-1}D_{di}^T\hat{d}(t)$ and $(F_{si}^TF_{si})^{-1}F_{si}^T\hat{f}_s(t)$.

For further analysis in the remaining work, two crucial lemmas are introduced as follows:

Lemma 1. [24] Given a pair $(\tilde{A}, \tilde{C})$ with $\tilde{A} \in \mathbb{R}^{n\times n}$, $\tilde{C} \in \mathbb{R}^{p\times n}$, the following are equivalent:(1) $\tilde{A}$ is stable. (2) If the pair $(\tilde{A}, \tilde{C})$ is observable, then the Lyapunov equation $\tilde{A}^T\tilde{P} + \tilde{P}\tilde{A} = -\tilde{C}^T\tilde{C}$ has a unique positive definite symmetric solution.

Lemma 2. [18] For each $i \in \mathbb{S}$, under (A1), there exists a gain matrix $\bar{L}_{pi}$ with appropriate dimension such that the matrix $\bar{S}_i^{-1}(\bar{A}_i - \bar{L}_{pi}\bar{C}_i)$ is Hurwitz.

3.1 Derivation of the error dynamics

In this part, we construct the error plant. From observer (11), we have

$$\begin{aligned} \bar{S}_i\dot{\hat{\bar{x}}}(t) &= \bar{S}_i\dot{\bar{z}}(t) + \bar{L}_{Di}\bar{C}_i\dot{\bar{x}}(t) \\ &= (\bar{A}_i - \bar{L}_{pi}\bar{C}_i)\hat{\bar{x}}(t) + \bar{L}_{pi}\bar{C}_i\bar{x}(t) + \bar{B}_iu(t) \\ &\quad +\bar{L}_{si}u_s(t) + \bar{L}_{Di}\bar{C}_i\dot{\bar{x}}(t) + \bar{F}g(t,\hat{x},i). \end{aligned} \tag{12}$$

Furthermore, to obtain the error plant, we should get $\bar{S}_i\dot{\bar{x}}(t)$ firstly. Thus, by adding $\bar{L}_{Di}\bar{C}_i\dot{\bar{x}}(t)$ to both sides of (6), one can obtain

$$\begin{aligned} \bar{S}_i\dot{\bar{x}}(t) &= \bar{A}_i\bar{x}(t) + \bar{F}g(t,x,i) + \bar{B}_iu(t) \\ &\quad +\bar{B}_{fi}\bar{f}(t) + \bar{L}_{Di}\bar{C}_i\dot{\bar{x}}(t). \end{aligned} \tag{13}$$

Define

$$\bar{e}(t) \triangleq \hat{\bar{x}}(t) - \bar{x}(t) = [e_x^T(t), e_a^T(t), e_s^T(t), e_d^T(t)]^T, \tag{14}$$

and subtract (13) from (12), the following error plant can be obtained:

$$\begin{aligned} \bar{S}_i\dot{\bar{e}}(t) &= (\bar{A}_i - \bar{L}_{pi}\bar{C}_i)\bar{e}(t) + \bar{F}g_e(t,e,i) + \bar{L}_{si}u_s(t) \\ &\quad -\bar{B}_{fi}\bar{f}(t), \end{aligned} \tag{15}$$

where $e_x^T(t) \triangleq \hat{x}(t) - x(t)$, $g_e(t,e,i) \triangleq g(t,\hat{x},i) - g(t,x,i)$, $e_a(t) \triangleq \hat{f}_a(t) - f_a(t)$.

In order to design $u_s(t)$, for each $i \in \mathbb{S}$, we firstly define the sliding mode surface as $s(t,i) = \bar{B}_{fi}^T\bar{S}_i^{-T}\bar{P}_i\bar{e}(t)$ with the positive-definite Lyapunov matrix $\bar{P}_i$ [25], [26] satisfying:

$$\bar{B}_{fi}^T\bar{S}_i^{-T}\bar{P}_i = H_i\bar{C}_i, \bar{P}_i > 0, \tag{16}$$

where $H_i \in \mathbb{R}^p$ will be determined later. Then, $u_s(t)$ is designed in the following form:

$$\begin{aligned} u_s(t) &= -(\theta_i(r_{a1} + r_{s1}) + r_{a2} + r_{s2} + r_d + \epsilon)sgn(s(t,i)) \\ &\quad -0.5\sum_{j=1}^N \pi_{ij}(\bar{B}_{fi}\bar{S}_i^{-T}\bar{P}_j\bar{S}_i\bar{B}_{fi})^{-1}s(t,i)sgn(\lambda_i), \end{aligned} \tag{17}$$

where $r_{a1}, r_{a2}, r_{s1}, r_{s2}$ and r_d are defined as in Assumption A3. $\lambda_i = s(t,i)^T\sum_{j=1}^N \pi_{ij}(\bar{B}_{fi}\bar{S}_i^{-T}\bar{P}_j\bar{S}_i\bar{B}_{fi})^{-1}s(t,i)$ and $\epsilon > 0$ denotes a parameter to be designed. Finally, the overall closed-loop plant can be formulated as follows:

$$\begin{cases} \dot{x}(t) &= A_ix(t) + B_iu(t) + F_{ai}f_a(t) + g(t,x,i) \\ \dot{\bar{e}}(t) &= \bar{S}_i^{-1}[(\bar{A}_i - \bar{L}_{pi}\bar{C}_i)\bar{e}(t) + \bar{F}g_e(t,e,i) \\ &\quad +\bar{L}_{si}u_s(t) - \bar{B}_{fi}\bar{f}(t)]. \end{cases} \tag{18}$$

3.2 Stability analysis of the overall closed-loop system

We are in a position to establish the observer-based controller for the plant (18). Firstly, we construct

$$u(t) = K_i\hat{x}(t) = \bar{K}_i\hat{\bar{x}}(t), \tag{19}$$

with $\bar{K}_i = [K_i, -B_i^\dagger F_{ai}, 0_{m\times p}, 0_{m\times d}]$. For each $i \in \mathbb{S}$, $B_i^\dagger$ is the generalized inverse of B_i. The gain K_i will be designed in

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the discussion such that $A_i + B_i K_i$ is Hurwitz. By applying (19) to the plant (1), we obtain

$$\begin{aligned} \dot{x}(t) &= A_i x(t) + B_i u(t) + F_{ai} f_a(t) + g(t,x,i) \\ &= A_i x(t) + B_i K_i \hat{x}(t) - B_i B_i^\dagger F_{ai} f_a(t) \\ &\quad + F_{ai} f_a(t) + g(t,x,i). \end{aligned} \tag{20}$$

Suppose that $rank B_i = rank([B_i, F_{ai}])$, we have the following property

$$F_{ai} = B_i B_i^\dagger F_{ai}, \tag{21}$$

then the following closed-loop plant is obtained

$$\begin{cases} \dot{x}(t) &= (A_i + B_i K_i)x(t) + F_i \bar{e}(t) + g(t,x,i) \\ \dot{\bar{e}}(t) &= \bar{S}_i^{-1}[(\bar{A}_i - \bar{L}_{pi}\bar{C}_i)\bar{e}(t) + \bar{F} g_e(t,e,i) \\ &\quad + \bar{L}_{si} u_s(t) - \bar{B}_{fi} \bar{f}(t)], \end{cases} \tag{22}$$

where $F_i = [B_i K_i, -F_{ai}, 0_{n\times s}, 0_{n\times d}]$.

We now present our first main result for establishing the stability condition of the plant (22).

Theorem 1 Applying $u_s(t)$ (17) to the error dynamics (22), if for each $i \in \mathbb{S}$, there exist positive and definite matrices $\bar{P}_i \in \mathbb{R}^{\bar{n}\times\bar{n}}, \bar{R}_i \in \mathbb{R}^{n\times n}$, $\bar{Y}_i \in \mathbb{R}^{\bar{n}\times\bar{n}}$ and $H_i \in \mathbb{R}^{(a+s+d)\times p}$ with appropriate dimensions, such that the following equality and the LMI hold

$$\Gamma_i = \begin{bmatrix} \Gamma_{11i} & \Gamma_{12i} \\ * & \Gamma_{22i} \end{bmatrix} < 0,$$

$$\bar{B}_{fi}^T \bar{S}_i^{-T} \bar{P}_i = H_i \bar{C}_i, \tag{23}$$

with $\Gamma_{11i} = \bar{R}_i(A_i + B_i K_i) + (A_i + B_i K_i)^T \bar{R}_i + \sum_{j=1}^N \pi_{ij} \bar{R}_j + I_n + \eta^2 \bar{R}_i^T \bar{R}_i$, $\Gamma_{22i} = \bar{P}_i \bar{S}_i^{-1} \bar{A}_i - \bar{Y}_i \bar{C}_i + \bar{A}_i^T \bar{S}_i^{-T} \bar{P}_i - \bar{C}_i^T \bar{Y}_i^T + \sum_{j=1}^N \pi_{ij} \bar{P}_j + I_{\bar{n}} + \eta^2 \bar{F}^T \bar{S}_i^{-T} \bar{P}_i \bar{P}_i \bar{S}_i^{-1} \bar{F}$, $\Gamma_{12i} = \begin{bmatrix} \bar{R}_i & -\bar{R}_i F_{ai} & 0_{n\times 2p} \end{bmatrix}$, $\tilde{F} = \begin{bmatrix} \bar{F}, 0_{\bar{n}\times(a+2p)} \end{bmatrix}$. Then the plant (22) is stochastically stable. Furthermore, for each $i \in \mathbb{S}$, the observer gains $\bar{L}_{pi}$ and $\bar{L}_{si}$ are given by

$$\bar{L}_{pi} = \bar{S}_i \bar{P}_i^{-1} \bar{Y}_i, \bar{L}_{si} = \bar{S}_i \bar{P}_i^{-1} \bar{C}_i^{-T} H_i^T = \bar{B}_{fi}. \tag{24}$$

Proof: Consider the error plant (22). We construct the Lyapunov function as follows:

$$V(t) = V_x(t) + V_e(t), \tag{25}$$

with $V_x(t) = x^T(t)\bar{R}_i x(t)$, and $V_e(t) = \bar{e}^T(t)\bar{P}_i \bar{e}(t)$. For each $i \in \mathbb{S}$, by Definition 2, we get the infinitesimal operator along the trajectories of (22)

$$\begin{aligned} \mathcal{L}V_x(t) &= 2x^T(t)\bar{R}_i[(A_i + B_i K_i)x(t) + F_i \bar{e}(t) + g(t,x,i)] \\ &\quad + x^T(t)(\sum_{j=1}^N \pi_{ij} \bar{R}_j)x(t), \end{aligned} \tag{26}$$

$$\begin{aligned} \mathcal{L}V_e(t) &= \bar{e}^T(t)[\bar{P}_i \bar{S}_i^{-1}(\bar{A}_i - \bar{L}_{pi}\bar{C}_i) + (\bar{A}_i \\ &\quad - \bar{L}_{pi}\bar{C}_i)^T \bar{S}_i^{-T} \bar{P}_i + \sum_{j=1}^N \pi_{ij} \bar{P}_j]\bar{e}(t) \\ &\quad + 2\bar{e}^T(t)\bar{P}_i \bar{S}_i^{-1}[\bar{L}_{si} u_s(t) - \bar{B}_{fi} \bar{f}(t) \\ &\quad + \bar{F} g_e(t,e,i)]. \end{aligned} \tag{27}$$

Recall that $\bar{L}_{si} = (\bar{P}_i \bar{S}_i^{-1})^{-1} \bar{C}_i^T H_i^T$ and $\bar{B}_{fi}^T \bar{S}_i^{-T} \bar{P}_i = H_i \bar{C}_i$, one gets

$$\begin{aligned} 2\bar{e}^T(t)\bar{P}_i \bar{S}_i^{-1}[\bar{L}_{si} u_s(t) - \bar{B}_{fi} \bar{f}(t)] &= 2\bar{e}^T(t)\bar{C}_i^T H_i^T \\ &\leq -2\epsilon||s(t,i)||. \end{aligned} \tag{28}$$

On the other hand, following A3, it is derived that

$$\begin{aligned} 2x^T(t)\bar{R}_i g(t,x,i) &\leq x^T(t)x(t) + g^T(t,x,i)\bar{R}^T \bar{R}_i g(t,x,i) \\ &= x^T(t)x(t) + \eta^2 x^T(t)\bar{R}_i^T \bar{R}_i x(t), \end{aligned} \tag{29}$$

$$\begin{aligned} 2\bar{e}^T(t)\bar{P}_i \bar{S}_i^{-1} \bar{F} g_e(t,e,i) &\leq \bar{e}^T(t)\bar{e}(t) + g_e^T(t,e,i) \\ &\quad \bar{F}^T \bar{S}_i^{-T} \bar{P}_i \bar{P}_i \bar{S}_i^{-1} \bar{F} g_e(t,e,i) \\ &\leq \bar{e}^T(t)\bar{e}(t) + \eta^2 \bar{e}^T(t) \\ &\quad \tilde{F}^T \bar{S}_i^{-T} \bar{P}_i \bar{P}_i \bar{S}_i^{-1} \tilde{F} \bar{e}(t), \end{aligned} \tag{30}$$

where $\tilde{F}$ has been defined in Theorem 1.

Substituting (28), (29) and (30) into (25) yields

$$\begin{aligned} \mathcal{L}V(t) &= \mathcal{L}V_x(t) + \mathcal{L}V_e(t) \\ &\leq \begin{bmatrix} x^T(t) & \bar{e}^T(t) \end{bmatrix} \Gamma_i \begin{bmatrix} x^T(t) & \bar{e}^T(t) \end{bmatrix}^T. \end{aligned} \tag{31}$$

It is noticed that $\Gamma_i < 0$ from (23). Furthermore, we can derive that $\mathcal{L}V(t) < 0$. Therefore, we conclude that the closed-plant (22) is stochastically stable.

In addition, the linear equality condition in (23) can be converted into a minimization problem, i.e. finding the minimum β_i under the following constrain:

$$\begin{bmatrix} -\beta_i I_{\bar{n}} & (\bar{B}_{fi}^T \bar{S}_i^{-T} \bar{P}_i - H_i \bar{C}_i)^T \\ * & -I_{a+s+d} \end{bmatrix} < 0. \tag{32}$$

The details of dealing with the linear equality condition can be found in [18]. The proof is completed.

3.3 Reachability condition of the sliding mode surface

The second result of this paper will be derived in this part.

Theorem 2 For each $i \in \mathbb{S}$, if there exist positive and define matrices $\bar{P}_i$, $\bar{R}_i$ and parameter matrix H_i such that the conditions in Theorem 1 hold, then $u_s(t)$ guarantees that the sliding motion is driven on the sliding surfaces $s(t,i) = 0$ in finite time.

Proof. Construct the Lyapunov function as follows:

$$V_s(t) = 0.5 s^T(t,i)(\bar{B}_{fi}^T \bar{S}_i^{-T} \bar{P}_i \bar{S}_i^{-1} \bar{B}_{fi})^{-1} s(t,i). \tag{33}$$

For notational simplicity, we define $G_i = \bar{B}_{fi}^T \bar{S}_i^{-T}$, and the following holds

$$s(t,i) = \bar{B}_{fi}^T \bar{S}_i^{-T} \bar{P}_i \bar{e}(t) = G_i \bar{P}_i \bar{e}(t) = H_i \bar{C}_i \bar{e}(t). \tag{34}$$

Following (22), we have

$$\begin{aligned} \mathcal{L}V_s(t,i) &= s^T(t,i)(G_i \bar{P}_i G_i^T)^{-1} G_i \bar{P}_i \bar{S}_i^{-1} \\ &\quad \times[(\bar{A}_i - \bar{L}_{pi}\bar{C}_i)\bar{e}(t) + \bar{F} g_e(t,e,i) \\ &\quad + \bar{L}_{si} u_s(t) - \bar{B}_{fi} \bar{f}(t) + 0.5 s^T(t,i) \\ &\quad \times \sum_{j=1}^N \pi_{ij} (\bar{B}_{fi}^T \bar{S}_i^{-T} \bar{P}_i \bar{S}_i^{-1} \bar{B}_{fi})^{-1} s(t,i)]. \end{aligned} \tag{35}$$

Based on A3, it is derived that

$$\bar{F} g_e(t,e,i) \leq \eta||\bar{e}(t)||. \tag{36}$$

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On the other hand, recall that $\bar{L}_{si} = \bar{B}_{fi}$, $\bar{B}_{fi}\bar{S}_i^{-T}\bar{P}_i = H_i\bar{C}_i$, and it can be derived that

$$\begin{aligned}
& s^T(t,i)(G_i\bar{P}_iG_i^T)^{-1}G_i\bar{P}_i\bar{S}_i^{-1} \times [\bar{L}_{si}u_s(t) - \bar{B}_{fi}\bar{f}(t)] \\
= \; & s^T(t,i)(G_i\bar{P}_iG_i^T)^{-1}G_i\bar{P}_i\bar{S}_i^{-1}\bar{B}_{fi}[u_s(t) - \bar{f}(t)] \\
= \; & s^T(t,i)(G_i\bar{P}_iG_i^T)^{-1}G_i\bar{P}_iG_i^Tu_s(t) \\
& -s^T(t,i)(G_i\bar{P}_iG_i^T)^{-1}G_i\bar{P}_iG_i^T\bar{f}(t) \\
= \; & s^T(t,i)(u_s(t) - \bar{f}(t)) - 0.5s^T(t,i) \\
& \sum_{j=1}^{N}\pi_{ij}(\bar{B}_{fi}^T\bar{S}_i^{-T}\bar{P}_i\bar{S}_i^{-1}\bar{B}_{fi})^{-1}s(t,i) \\
< \; & -\epsilon||s(t,i)|| - 0.5s^T(t,i) \\
& \times\sum_{j=1}^{N}\pi_{ij}(\bar{B}_{fi}^T\bar{S}_i^{-T}\bar{P}_i\bar{S}_i^{-1}\bar{B}_{fi})^{-1}s(t,i). \qquad (37)
\end{aligned}$$

Substituting (36) and (37) into (35), one has

$$\begin{aligned}
\mathcal{L}V_s(t,i) \; < \; & -\epsilon||s(t,i)|| + ||s(t,i)||||(G_i\bar{P}_iG_i^T)^{-1}G_i\bar{P}_i\bar{S}_i^{-1} \\
& (\bar{A}_i - \bar{L}_{pi}\bar{C}_i)||||\bar{e}(t)|| + \eta||s(t,i)||||(G_i\bar{P}_iG_i^T)^{-1} \\
& \times G_i\bar{P}_i\bar{S}_i^{-1}||||\bar{e}(t)||. \qquad (38)
\end{aligned}$$

We define

$$\begin{aligned}
\delta_i \; = \; & \eta||(G_i\bar{P}_iG_i^T)^{-1}G_i\bar{P}_i\bar{S}_i^{-1}||||(G_i\bar{P}_iG_i^T)^{-1} \\
& \times G_i\bar{P}_i\bar{S}_i^{-1}(\bar{A}_i - \bar{L}_{pi}\bar{C}_i)||. \qquad (39)
\end{aligned}$$

From (38), we can obtain

$$\mathcal{L}V_s(t,i) \; < \; -||s(t,i)||(\epsilon - \delta_i||\bar{e}(t)||). \qquad (40)$$

Then, for each $i \in \mathbb{S}$, define following domain

$$\Omega \triangleq \bigcap_{i=1}^{s}\Omega_i(\delta_i), \qquad (41)$$

$$\Omega_i(\delta_i) \triangleq \{\epsilon - \delta_i||\bar{e}(t)|| > 0\}. \qquad (42)$$

It is obvious that $\mathcal{L}V_s(t,i) < 0$ in the domain Ω. Furthermore, in Theorem 1, the stochastic stabilization of the error plant (22) has been proved. This implies that the trajectories of $\bar{e}(t)$ will enter Ω in finite time and remains there. Until now, we complete the proof.

IV. Simulation Result

In this section, the theoretical results proposed in this paper will be applied to mobile manipulators with hybrid joints against actuator and sensor faults, simultaneously. Consider a wheeled mobile manipulator model [2] shown in Fig.1.

$$M(q)\ddot{q} + V(q,\dot{q})\dot{q} + G(q) + d(t) = B(q)\tau, \qquad (43)$$

where $M(q)$ represents the symmetric positive definite inertia matrix, $G(q)$ is the gravitational torque vector, $V(q,\dot{q})$ represents the Centripetal and Coriolis torques, $B(q)$ is the input transformation matrix, $d(t)$ represents the bounded external disturbance. τ denotes the control inputs, $q = [q_v^T, q_a^T]^T \in \mathbb{R}^n$ with $q_v = [x, y, v]^T \in \mathbb{R}^{n_v}$ represents the generalized coordinates for the mobile platform, $n = n_v + n_a$, $q_a \in \mathbb{R}^{n_a}$ denotes the coordinates of the manipulator joints. The details of this model can be found in [2].

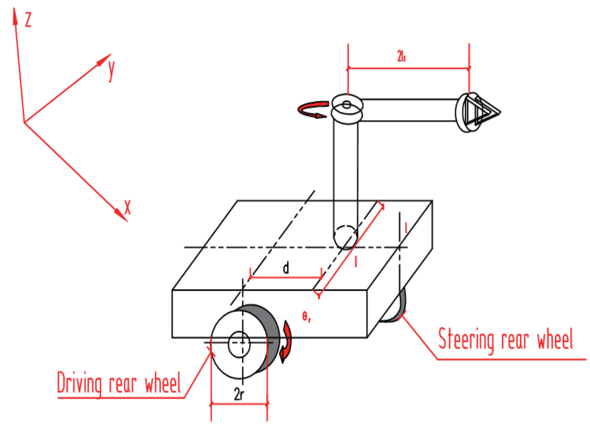

Fig. 1. The wheeled mobile manipulator.

Then, the dynamic of (43) can be described as follows:

$$A_1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1.0000 \\ 0.0040 & 0.0012 & 0.0653 & -0.0728 \\ -0.0047 & -0.0010 & -0.0717 & 0.0647 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1.0000 \\ 0.0040 & 0.0012 & 0.0653 & -0.0728 \\ -0.0047 & -0.0010 & -0.0717 & 0.0647 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1.0000 \\ 0.0057 & 0.0014 & 0.0725 & -0.0764 \\ -0.0064 & -0.0011 & -0.0790 & 0.0676 \end{bmatrix},$$

$$A_4 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1.0000 \\ 0.0057 & 0.0014 & 0.0725 & -0.0764 \\ -0.0064 & -0.0011 & -0.0790 & 0.0676 \end{bmatrix},$$

$$A_5 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1.0000 \\ 0.0042 & 0.0016 & 0.0628 & -0.0686 \\ -0.0048 & -0.0013 & -0.0691 & 0.0606 \end{bmatrix},$$

$$A_6 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1.0000 \\ 0.0042 & 0.0016 & 0.0628 & -0.0686 \\ -0.0048 & -0.0013 & -0.0691 & 0.0606 \end{bmatrix},$$

$$A_7 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1.0000 \\ 0.0055 & 0.0010 & 0.0753 & -0.0809 \\ -0.0062 & -0.0008 & -0.0819 & 0.0719 \end{bmatrix},$$

$$A_8 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1.0000 \\ 0.0055 & 0.0010 & 0.0753 & -0.0809 \\ -0.0062 & -0.0008 & -0.0819 & 0.0719 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0.0003 & 0.3354 \\ -0.0003 & -0.0020 \end{bmatrix}, B_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0.0003 & 0.3354 \\ -0.0003 & -0.3333 \end{bmatrix},$$

$$B_3 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0.0035 & 0.3825 \\ -0.0035 & -0.0249 \end{bmatrix}, B_4 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0.0035 & 0.3825 \\ -0.0035 & -0.3333 \end{bmatrix},$$

$$B_5 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0.0022 & 0.3175 \\ -0.0022 & -0.0158 \end{bmatrix}, B_6 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0.0022 & 0.3175 \\ -0.0022 & -0.3333 \end{bmatrix},$$

$$B_7 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0.0068 & 0.3808 \\ -0.0068 & -0.0475 \end{bmatrix}, B_8 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0.0068 & 0.3808 \\ -0.0068 & -0.3333 \end{bmatrix},$$

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$$
\begin{aligned}
C_1 &= \begin{bmatrix} 0.6541 & -0.3565 & 1 & 0 \\ 0.5137 & 0.5698 & 0 & 1 \end{bmatrix}, \\
C_2 &= \begin{bmatrix} 0.6075 & -0.2904 & 0.001 & 0 \\ 0.2939 & 0.5533 & 0 & 0.001 \end{bmatrix}, \\
C_3 &= \begin{bmatrix} 0.8858 & -0.7980 & 1 & 0 \\ 0.7751 & 0.8795 & 0 & 1 \end{bmatrix}, \\
C_4 &= \begin{bmatrix} 1.4812 & 1.0992 & 1 & 0 \\ -0.9700 & 1.2748 & 0 & 1 \end{bmatrix}, \\
C_5 &= \begin{bmatrix} -0.6003 & 0.8012 & 1 & 0 \\ -0.8193 & -0.5905 & 0 & 1 \end{bmatrix}, \\
C_6 &= \begin{bmatrix} -0.6361 & 0.9155 & 1 & 0 \\ -0.8544 & -0.5695 & 0 & 1 \end{bmatrix}, \\
C_7 &= \begin{bmatrix} -0.5967 & 1.2628 & 1 & 0 \\ -1.1860 & -0.6170 & 0 & 1 \end{bmatrix}, \\
C_8 &= \begin{bmatrix} -0.6096 & 0.5122 & 1 & 0 \\ -0.4224 & -0.6055 & 0 & 1 \end{bmatrix}.
\end{aligned}
$$

The matrices F_{ai}, F_{si}, G_{ai} and D_{di} $(i = 1, 2, \ldots, 8)$ are chosen in the following forms:

$$
F_{ai} = \begin{bmatrix} 0.1 \\ 0 \\ 0.4 \\ 0 \end{bmatrix}, F_{si} = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix}, G_{ai} = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix}, D_{di} = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix}.
$$

The transition rate matrix Π is given in the following form:

$$
\begin{bmatrix}
-0.72 & 0.15 & 0.22 & 0.21 & 0.14 & 0 & 0 & 0 \\
0.2 & -0.7 & 0.2 & 0.2 & 0 & 0.1 & 0 & 0 \\
0.16 & 0.22 & -0.68 & 0.2 & 0 & 0 & 0.1 & 0 \\
0.22 & 0.3 & 0.2 & -0.82 & 0 & 0 & 0 & 0.1 \\
0 & 0 & 0 & 0 & -0.78 & 0.26 & 0.26 & 0.26 \\
0 & 0 & 0 & 0 & 0.26 & -0.78 & 0.26 & 0.26 \\
0 & 0 & 0 & 0 & 0.26 & 0.26 & -0.78 & 0.26 \\
0 & 0 & 0 & 0 & 0.26 & 0.26 & 0.26 & -0.78
\end{bmatrix}.
$$

Assume that the sensor faults $f_s(t)$, actuator faults $f_a(t)$ and disturbance $d(t)$ are set as $f_s(t) = 0.2 + 0.4\sin(10t)$, $f_a(t) = 0.1 + 0.5\cos(10t)$ and $d(t) = \sin(t) + \cos(t)$.

The norm bounds of $f_s(t)$, $f_a(t)$ and $d(t)$ are $r_s = 0.6$, $r_a = 0.6$ and $r_d = 1.5$. It should be noticed that the faults are small and bounded. The parameter is chosen as $\theta_i = 0.1$, respectively. According to the design procedure in Section 3, the derivative gain $\bar{L}_{Di}, (i = 1, \ldots, 8)$ is selected as $\bar{L}_{Di} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.5 \end{bmatrix}^T$. The observer matrices $\bar{L}_{pi}$ and the design matrices K_i, $(i = 1, \ldots, 8)$ are respectively designed in this section.

Furthermore, by solving (32), we have $\beta_i \approx 6.2502 \times 10^{-6}, i = 1, \ldots, 8$. Then, based on the parameter in SMO chosen as $\varepsilon = 0.1$, the discontinuous input is designed as $u_s(t) = -2.92 \times sgn(s(t))$.

The initial condition is set as $x(0) = [-2,\ 2,\ 1,\ -2]^T$. By employing the controller (19), the simulation results are reported in Figs. 2-7 below. It is obvious that a continuous approximation of the sliding mode control has been utilized and the stability performance of the closed-loop plant is ideal.

V. Conclusion

In this paper, the FTC problems have been studied for Markovian jump systems with simultaneous Lipschitz nonlinearity and out disturbances subject to actuator and sensor faults. To solve these problems, a novel SMO which can eliminate the effects of simultaneous disturbances, actuator and sensor faults has been developed. Based upon the state estimation, a FTC strategy is designed to stabilize the overall closed-loop plant. An example related to mobile manipulators with switching joints has been given to demonstrate the effectiveness and applicability of the theoretical results.

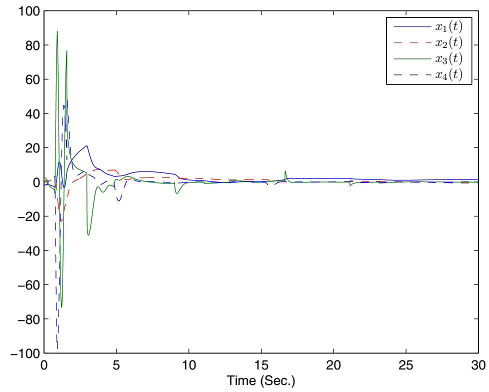

Fig. 2. Trajectories of $x(t)$.

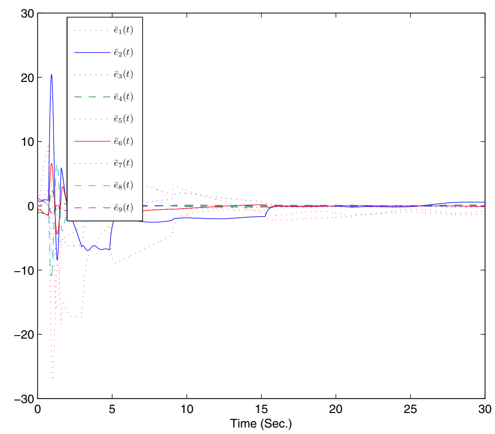

Fig. 3. Trajectories of $\bar{e}(t)$.

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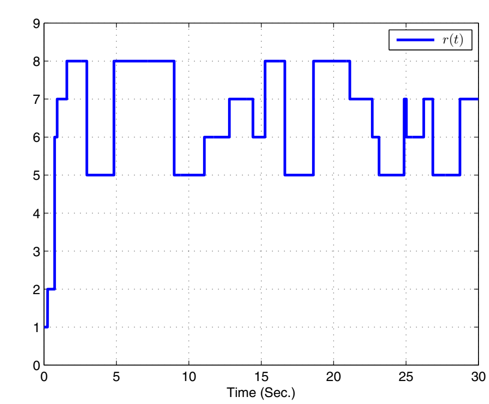

Fig. 4. Trajectories of $r(t)$.

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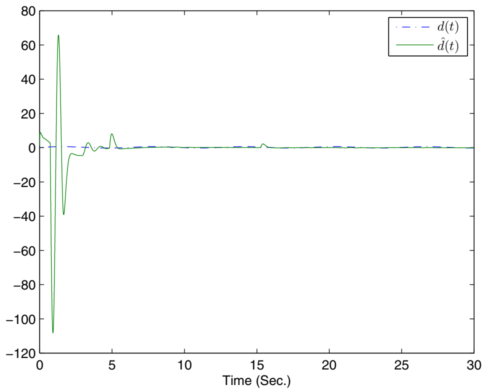

Fig. 5. Bounded disturbance $d(t)$ and its estimation $\hat{d}(t)$.

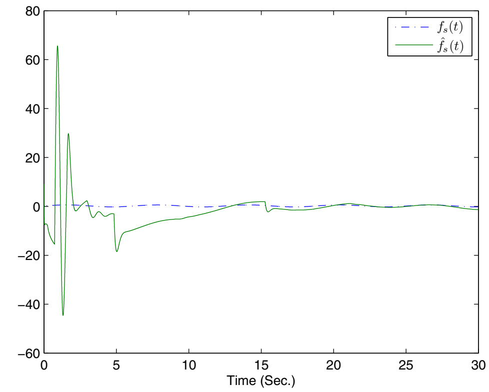

Fig. 7. Sensor fault $f_s(t)$ and its estimation $\hat{f}_s(t)$.

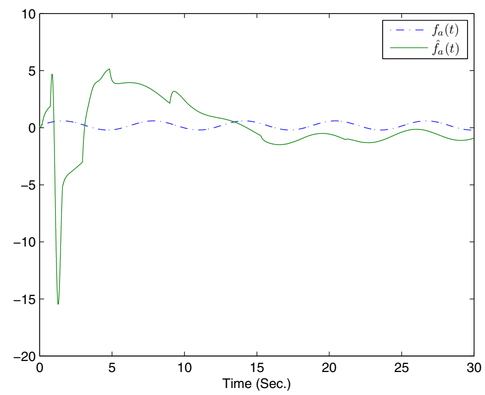

Fig. 6. Actuator fault $f_a(t)$ and its estimation $\hat{f}_a(t)$.

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