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# Integrated fault estimation and fault tolerant attitude control for rigid spacecraft with multiple actuator faults and saturation

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**Abstract:** This study proposes an integrated fault estimation (FE) and fault-tolerant control design for a rigid spacecraft attitude system with inertia uncertainties, external disturbances, input saturation, and different type multiple actuator faults. A barrier function is first introduced to eliminate the effects of inertia uncertainties and disturbances in the design of the sliding mode FE observer. By using the non-singular fast terminal sliding mode control technology, a finite-time fault-tolerant attitude stabilisation controller and a finite-time fault-tolerant attitude tracking controller are designed to guarantee that the closed-loop attitude system has a good fault-tolerant performance under actuator faults. Furthermore, when considering actuator saturation, an auxiliary system is utilised to compensate for the saturation. The stability of the closed-loop system is analysed by Lyapunov theory. Finally, the effectiveness of the proposed control approach is demonstrated via simulation results.

## 1 Introduction

In recent years, rigid spacecraft has attracted significant attention in the fields of theoretical research and various military/civilian applications. It is mainly because that rigid spacecraft could be utilised to conduct deep space exploration and positioning navigation associated with weather monitoring etc. However, in the above proceeding, the rigid spacecraft's unexpected actuator/sensor faults may occur inevitably, which can cause severe deterioration of the system. For example, when an actuator is stuck and failed to deflect the certain control state, it may result in a catastrophic accident. It is widely believed that fault diagnosis (FD) and fault-tolerant control (FTC) are effective techniques to improve the safe and reliable operation of spacecraft. Therefore FD and FTC have become a relatively active researching field in spacecraft [1–4].

Generally, the existing approaches of FTC can be classified into two main categories, namely, passive FTC (PFTC) and active FTC (AFTC). The PFTC of spacecraft has been investigated to be robust against actuator faults with fixed structured controllers [5–8]. Li and Yang [5] designed an adaptive state feedback fault-tolerant controller, in which the effect of actuator fault, exogenous disturbance and parameter uncertainty can be eliminated completely. Xiao *et al*. [6] proposed an integral-type sliding mode scheme to enhance the fault-tolerant attitude control performance of the rigid spacecraft with uncertain inertia parameters and external disturbances. Xiao *et al*. [7] investigated an adaptive backstepping control strategy to improve the attitude tracking performance of the flexible spacecraft, where the unknown fault is approximated by an adaptive sliding mode fault estimation FE observer. Huo and Xia [8] proposed an adaptive fuzzy fault-tolerant tracking control algorithm for rigid spacecraft attitude systems with actuator faults, where the unknown control signals could be approximated directly by fuzzy logic systems. However, these control algorithms do not accurately acquire fault information, which greatly reduces the FTC performance. In contrast to the PFTC, the AFTC could obtain FE information more accurately [9–11]. The result information of fault detection and FE is added to the controller as a feedback signal, such that the influence of the unknown faults can be compensated effectively. This method not only guarantees the stability of the system but also optimises the performance of the control system. In [9], the problem of robust fault detection and isolation (FDI) was studied for a class of uncertain single output non-linear systems with faults. Gao *et al*. [10] proposed an active fault-tolerant attitude control approach under the framework of both backstepping control and adaptive control theory. An active FTC scheme for satellite attitude control was designed in [11], which used an FDI mechanism to improve FTC performance of the attitude system.

However, all of the above methods have some serious drawbacks. The AFTC means that the controller changes in an active way according to the effects that faults have on the control reconfiguration. The PFTC is just an extension of robust control in which the faults are considered as an additional form of uncertainty affecting the closed-loop system. Therefore, many integrated FE/FTC designs for some control systems have also been published [12–16]. The integrated FE/FTC strategy means that the direct use of FE without the need for a reconfigurable mechanism brings significant convenience and application potential to the subject of FTC system design. An integrated FE and non-fragile FTC design approach was proposed for uncertain Takagi-Sugeno fuzzy system with actuator fault and sensor fault [12]. Lan and Patton [13] proposed an integrated FE/FTC design for Lipschitz non-linear systems subject to uncertainties, disturbances, and actuator/sensor faults. Lan and Patton [14] proposed a decoupling approach to the integrated design of FE/FTC for linear systems in the presence of unknown bounded actuator faults and perturbations. The FTC approach using FE and fault compensation was proposed for a class of linear systems with system state uncertainty [15]. Lan *et al*. [16] proposed FE-based FTC output tracking strategy for a linearised three degree of freedom helicopter with perturbations and oscillatory and drift actuator faults. To the best of the authors' knowledge, until now few applications of FE/FTC strategy in spacecraft attitude control systems have been reported in the published literature works. The innovation of this paper is the application of the integrated FE/FTC theory in a rigid spacecraft attitude system.

From the above described AFTC and FE/FTC, it can be found that the FE observer is a very important key link for the implementation of fault reconfiguration. In the field of FE, many important results have been obtained [17–19], such as the popular adaptive observer, the unknown input observer (UIO), and the sliding mode observer. The adaptive observer could achieve asymptotic estimation for constant faults and bounded estimation for time-varying faults in [17]. The UIO has been presented to guarantee that the residuals are completely decoupled from disturbances in [18]. Based on the sliding mode control method, an adaptive sliding mode observer was provided for a class of uncertain mechanical systems with unknown parameters and faults in [19]. It is important to note that although in [19] the stabilities of the observer was analysed, the knowledge of the upper bound of disturbances was required to design observer. In practice, this bound is not constant and it is unknown. It often follows that this bound is overestimated, which products a chattering phenomenon to damage actuators and systems.

In an actual system, the actuator saturation is also a critical factor which affects the attitude control performance. If it is ignored during the controller design process, many negative influences such as system performance degradation and even instability will be caused on the spacecraft when the actuator saturation occurs. It should be pointed out that the actuator saturation is considered in [20–23]. Han *et al*. [20] designed a finite time fault-tolerant attitude controller, in which the actuator-magnitude constraints were rigorously enforced and the attitude of the rigid spacecraft converged to the equilibrium in finite time even in the presence of external disturbances and actuator faults. A similar problem was considered in [21], in which an integral sliding mode fault-tolerant attitude controller was presented for the spacecraft, and the control allocation was used to solve effectively the actuator saturation problem. In addition, a finite time attitude stabilisation controller for a rigid spacecraft subject to external disturbance, actuator faults, and input saturation has been presented by Jiang *et al*. [22]. Then, [22] have been further studied by Esmailzadeh and Golestani [23]. So, the consideration of actuator saturation in control problems is meaningful. The main contributions of this study relative to other existing works are as follows:

- This paper is different from previous research papers [16, 24, 25] that only studied a kind of actuator FE or multiple actuator FE without considering actuator saturation. In this study, the proposed multiple actuator FE strategy for rigid spacecraft attitude systems with inertial uncertainties, external disturbances and actuator saturation is a major contribution.
- In comparison to the FE observer design approaches in literature [12, 14], a new adaptive strategy based on barrier function (BF) is introduced into an adaptive sliding mode FE observer which does not need any upper bound information of uncertainties and external disturbances. The proposed barrier strategy can ensure the convergence of the output variable and maintain it in a neighbourhood of zero, without overestimating the sliding mode gain. Therefore, it improves the rapidity and accuracy of FE.
- Based on the integrated FE/FTC strategy, by using non-singular fast terminal sliding mode (NFTSM) control techniques and introducing a dynamic auxiliary system, a finite time fault tolerant attitude stabilisation controller and a finite time fault-tolerant attitude tracking controller are designed to accurately compensate for the multiple actuator faults and avoid actuator saturation. Furthermore, a neural network (NN) algorithm [26, 27] is introduced into two fault-tolerant controllers, such that no longer require the prior knowledge of uncertainties and external disturbances.
- In most modern spacecraft attitude control schemes [10, 20], their control inputs contain the discontinuous term $k\,\mathrm{sign}(s)$. In order to have a faster reaching time, good robustness and tracking performance, $k$ must be increased. However, this will directly increase the chattering level on the control input. In order to solve this dilemma, a novel reaching law containing an exponential term function of the sliding surface $s$ is proposed in this study, such that the interdependence between the reaching time and the chattering level could be removed. The exponential term smoothly adapts to the variations of $s$ and improves sliding mode dynamics reaching performance.

The remainder of the paper is organised as follows. Section 2 presents the mathematical models for the rigid spacecraft and actuator fault. An adaptive sliding mode FE observer is designed and analysed in Section 3. The NFTSM-based fault-tolerant stabilisation controller and tracking controller are designed in Section 4. Section 5 provides simulation results. Finally, the main conclusions are summarised in Section 6.

## 2 Preliminaries and mathematical models

In this section, the modified Rodrigues parameters (MRPs) are used to describe the spacecraft attitude. A vector of MRPs representing the spacecraft attitude can be defined as [4, 8]:

$$\sigma = \frac{\sin(\theta/2)\mathbf{n}}{1+\cos(\theta/2)} = \tan(\theta/4)\mathbf{n} \tag{1}$$

where $\sigma = [\sigma_1, \sigma_2, \sigma_3]^{\mathrm{T}} \in R^3$ represents the spacecraft attitude in body frame with respect to the inertial frame, $\theta \in R$ and $\mathbf{n} \in R^3$ (a unit vector) denote the Euler eigenangle and eigenaxis.

Note that the MRPs allow non-singular attitude representation for $-2\pi < \theta < 2\pi$ and singularities arise at $\theta = \pm 2\pi$. As is shown in [8], it is possible to map the MRP vector $\sigma$ to its shadow counterpart $\sigma^s$ through $\sigma^s = (1/\sigma^{\mathrm{T}}\sigma)\sigma$. By switching the MRPs to $\sigma^s$ when $\sigma^{\mathrm{T}}\sigma > 1$, the MRP vector remains bounded within a unit sphere, global rotation representation without singularity can thus be ensured. The kinematic equation represented by MRPs is [8]

$$\dot{\sigma} = \frac{1}{4}\left[(1-\sigma^{\mathrm{T}}\sigma)\mathbf{I}_{3\times 3} + 2\sigma^{\times} + 2\sigma\sigma^{\mathrm{T}}\right]\omega = G(\sigma)\omega \tag{2}$$

where $\mathbf{I}_{3\times 3} \in R^{3\times 3}$ is the identity matrix, $\omega = [\omega_1, \omega_2, \omega_3]^{\mathrm{T}} \in R^3$ is the angular velocity of body frame with respect to inertial frame, $\sigma^{\times}$ is the vector cross-product operator of a skew-symmetric matrix $\sigma^{\times} = [0, -\sigma_3, \sigma_2; \sigma_3, 0, -\sigma_1; -\sigma_2, \sigma_1, 0]$. Simple algebraic manipulation shows that $G(\sigma)$ has a following property.

*Property 1 (P1):* The matrix $G(\sigma)$ is such that

$$G^{-1}(\sigma) = \frac{16}{(1+\sigma^{\mathrm{T}}\sigma)^2}G^T(\sigma)$$

The dynamic equation of the rigid spacecraft with respect to the uncertainty of the inertia matrix is described as [4]

$$(J+\Delta J)\dot{\omega} = -\omega^{\times}(J+\Delta J)\omega + D\tau + T_d \tag{3}$$

where $J \in R^{3\times 3}$ is the symmetric inertia matrix of rigid spacecraft; $\Delta J$ is the parameter uncertainty; $\tau = [\tau_1, \tau_2, \ldots \tau_n]^{\mathrm{T}}$ is the control torque vector generated by $n$ reaction wheels (RWs); $D \in R^{3\times n}$ is the RW distribution matrix; $T_d = [d_1, d_2, d_3]^{\mathrm{T}}$ represents the external disturbance torques; $\omega^{\times}$ represents the cross-product operator and has the similar form of $\sigma^{\times}$.

After some manipulations, (3) can be transformed into the following form

$$\dot{\omega} = -J^{-1}\omega^{\times}J\omega + J^{-1}D\tau + J^{-1}d \tag{4}$$

where $d = T_d - \omega^{\times}\Delta J\omega - \Delta J\dot{\omega}$ and it can be viewed as a generalised perturbation for the dynamics equation.

The output torque of the $i$th RW in the presence of possible faults and input saturation can be written as [28]

$$\tau_i = e_i \mathrm{sat}(u_{ci}) + u_{fi} \tag{5}$$

where $u_{ci}$ is the torque command from attitude controller, $0 < e_i \le 1$ is the actuator loss of effectiveness fault (i.e. multiplicative fault), and $u_{fi}$ represents the bias torque (i.e. additive fault). The $i$th RW partially loses its effectiveness if $0 < e_i < 1$ and completely fails if $e_i = 0$. With no faults, it follows that $e_i = 1$ and $u_{fi} = 0$. Saturation function $\mathrm{sat}(u_{ci})$ is described by [16, 29]

$$\mathrm{sat}(u_{ci}) = \begin{cases} u_{ci}, & |u_{ci}| \le u_M \\ u_M \cdot \mathrm{sign}(u_{ci}), & |u_{ci}| > u_M \end{cases} \tag{6}$$

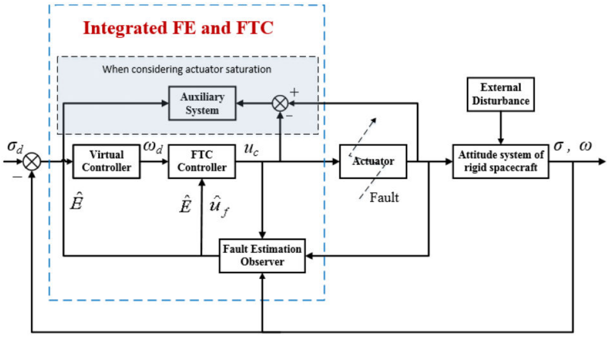


**Fig. 1** *The structure of FE/FTC developed in this paper*

where $u_M$ is the maximum control torque, and $i = 1, \ldots, n$.

Note that $\mathrm{sat}(u_{ci})$ can also be written as $\mathrm{sat}(u_{ci}) = u_{ci} + \Theta_i(u_{ci})$, where $\Theta_i(u_{ci})$ represents the excess portion of $u_{ci}$ over its limit and is given by

$$\Theta_i(u_{ci}) = \begin{cases} u_M - u_{ci}, & \text{if} \quad u_{ci} > u_M \\ 0, & \text{if} \quad |u_{ci}| \le u_M \\ -u_M - u_{ci}, & \text{if} \quad u_{ci} < -u_M \end{cases} \tag{7}$$

Let $E = \mathrm{diag}\{e_1, \ldots, e_n\}$, the control torque in actuator fault and input saturation case could be expressed as the following form [28]

$$\tau = Eu_c + E\Theta(u_c) + u_f \tag{8}$$

where $u_c = [u_{c1}, \ldots, u_{cn}]^{\mathrm{T}}$, $u_f = [u_{f1}, \ldots, u_{fn}]^{\mathrm{T}}$, and $\Theta(u_c) = [\Theta_1(u_{c1}), \ldots, \Theta_n(u_{cn})]^{\mathrm{T}}$.

Based on the above description, the faulty dynamics equation of rigid spacecraft could be rewritten as the following form

$$\dot{\omega} = -J^{-1}\omega^{\times}J\omega + J^{-1}DEu_c + J^{-1}Du_f + J^{-1}d_{\Theta} \tag{9}$$

where $d_{\Theta} = DE\Theta(u_c) + d$ is regarded as the total time-varying uncertainties and satisfying $\| d_{\Theta} \| < d_M$.

The objectives of this study are listed as follows:

• Design an adaptive sliding mode FE observer for the attitude system of the faulty rigid spacecraft to obtain the accurate estimated value of the actuator LOE fault and bias fault under actuator saturation, external disturbances, and inertia uncertainties.
• Based on the integrated FE/FTC strategy, design a finite time fault-tolerant attitude stabilisation controller and a finite time fault-tolerant attitude tracking controller to guarantee that the closed-loop attitude system has a good fault-tolerant performance under actuator faults and saturation.

The whole integrated FE/FTC strategy developed in this study is shown in Fig. 1. To achieve the objectives described above, we recall the following assumptions and lemmas, which will be needed to prove the main results.

*Assumption 1:* The attitude angle $\sigma$ and the angular velocity $\omega$ are measurable. The desired attitude angle $\sigma_d$, the desired angular velocity $\omega_d$, and its first time derivative (i.e. $\dot{\sigma}_d$, and $\dot{\omega}_d$) are limited.

*Assumption 2:* The non-linear function $\omega^{\times}J\omega$ is locally Lipschitz bounded with a Lipschitz constant $\varepsilon_0$, which can be formulated in the following

$$\| \omega^{\times}J\omega - \hat{\omega}^{\times}J\hat{\omega} \| \le \varepsilon_0 \| \omega - \hat{\omega} \| = \varepsilon_0 \| \tilde{\omega} \| \tag{10}$$

*Assumption 3:* In this paper, FTC of faults is implemented by installing redundant actuators, i.e. the number of actuators required is $n > 3$.

*Lemma 1:* The extended Lyapunov description of finite-time stability with faster finite time convergence is given as

$$\dot{V}(x) + \lambda_1 V(x) + \lambda_2 V^{\lambda_0}(x) \le 0 \tag{11}$$

and the convergence time is given by

$$T_k \le 1/(\lambda_1(1 - \lambda_0))\ln(\lambda_1 V^{1-\lambda_0}(x_0) + \lambda_2)/\lambda_2 \tag{12}$$

where $\lambda_1 > 0$, $\lambda_2 > 0$ and $0 < \lambda_0 < 1$ [29, 31].

*Remark 1:* As discussed in [20, 28], Assumption 1 is reasonable because spacecraft attitude angle and angular velocity can be measured directly according to the position sensitive detector and gyro, and the expected angular velocity and its first derivative are usually continuous and bounded when the spacecraft carries out tracking tasks. Assumption 2 is more general and have been given in [7, 32], where the state $\omega$ is first-order differentiable and its derivative is bounded. According to [4], Assumption 3 means that, although the $n$ actuators ($n > 3$) may suffer from partial loss of actuator effectiveness or even complete failure, the number of totally failed actuators is no more than $n - 3$, such that $DED^{\mathrm{T}}$ remains positive definite. If more than $n - 3$ actuators have totally failed, the matrix $DED^{\mathrm{T}}$ becomes singular and the system is underactuated.

## 3 Fault estimation observer design

In this position, by introducing a BF, a new adaptive sliding mode FE observer is designed to achieve accurately multiple actuator FE.

### *3.1 BFs preliminaries*

*Definition 1:* Suppose that $\varepsilon > 0$ is given and fixed, the BF can be defined as an even continuous function $K_b$: $x \in [-\varepsilon, \varepsilon] \to K_b(x) \in [b, \infty]$ strictly increasing on $[0, \varepsilon]$ [33].

- $\lim_{|x| \to \varepsilon} K_b(x) = +\infty$.
- $K_b(x)$ has a unique minimum at zero and $K_b(0) = b \ge 0$.

In this paper, the following BF is considered

- Positive Semi-definite BF: $K_{\mathrm{psb}}(x) = |x|/(\varepsilon - |x|)$, i.e. $k_{\mathrm{psb}}(0) = 0$.

### *3.2 Adaptive sliding mode fault estimation observer design*

The actuator loss of effectiveness fault $E$ is a diagonal matrix, and $Eu_c$ of the formula (9) can be written

$$Eu_c = Ue \tag{13}$$

where $U = \mathrm{diag}\{u_{c1}, \ldots, u_{cn}\}$ and $e = [e_1, \ldots, e_n]^{\mathrm{T}}$.

The faulty dynamics equation of rigid spacecraft (9) can be transformed into

$$\dot{\omega} = -J^{-1}\omega^{\times}J\omega + J^{-1}DUe + J^{-1}Du_f + J^{-1}d_{\Theta} \tag{14}$$

A novel adaptive sliding mode FE observer with parameter updated algorithms are designed as

$$\begin{aligned} \dot{\hat{\omega}} = {} & \Lambda(\omega - \hat{\omega}) - J^{-1}\hat{\omega}^{\times}J\hat{\omega} + J^{-1}DU\hat{e} \\ & + J^{-1}D\hat{u}_f + J^{-1}K(t)\mathrm{sign}(\omega - \hat{\omega}) \end{aligned} \tag{15}$$

$$\dot{\hat{e}} = \gamma_1 U^{\mathrm{T}}D^{\mathrm{T}}J^{-\mathrm{T}}\tilde{\omega}, \quad \dot{\hat{u}}_f = \gamma_2 D^{\mathrm{T}}J^{-T}\tilde{\omega} \tag{16}$$

where $\tilde{\omega} = \omega - \hat{\omega}$, $\Lambda > 0$ is a diagonal matrix, which is determined in advance. $\gamma_1$ and $\gamma_2$ are two positive constants.

Time-varying switching gain $K(t)$ is defined by as follow:

- if $0 < t \le \bar{t}$, $K(t)$ is the solution of

$$K(t) = K_a(t) \ \text{with} \ \dot{K}_a = \gamma_3 \parallel J^{-1} \parallel \tilde{\omega}^{\mathrm{T}}\mathrm{sign}(\tilde{\omega}) \quad (17)$$

- if $t > \bar{t}$, then

$$K(t) = \frac{\tilde{\omega}^{\mathrm{T}}\tilde{\omega}}{\varepsilon - \tilde{\omega}^{\mathrm{T}}\tilde{\omega}} \quad (18)$$

Where there exists $\bar{t}$, for all $t \ge \bar{t}$, the inequality $\tilde{\omega}^{\mathrm{T}}\mathrm{sign}(\tilde{\omega}) < \varepsilon$ holds.

Let $\tilde{e} = e - \hat{e}$, the following observer error equation is obtained by subtracting (13) from (12)

$$\begin{aligned}\dot{\tilde{\omega}} &= -\Lambda\tilde{\omega} - J^{-1}(\omega^{\times}J\omega - \hat{\omega}^{\times}J\hat{\omega}) + J^{-1}DU\tilde{e} \\ &+ J^{-1}D\tilde{u}_f + J^{-1}d_{\Theta} - J^{-1}K(t)\mathrm{sign}(\tilde{\omega})\end{aligned} \quad (19)$$

where for any $\tilde{\omega}(0)$ and $\varepsilon > 0$.

*Lemma 2:* For the adaptive sliding mode FE observer (15), the adaptive gain $K_a$ in (17) has an upper bound $K^*$ for all $t > 0$ with $K^* > d_M$ [33].

*Theorem 1:* For the faulty spacecraft attitude system (2) and (9), suppose that Assumptions 1 and 2 hold, if there exists an appropriate $\Lambda$ such that the following condition is satisfied $\lambda_{\min}(\Lambda) - \varepsilon_0 \parallel J^{-1} \parallel \ \ge 0$, then the proposed adaptive sliding mode FE observer (15) and FE laws (16) can provide an accurate FE under actuator saturation. Meanwhile, the error dynamical system (19) for the FE observer is practically stable (i.e. final and uniform bound). It means that all estimation error trajectories converge to a neighbourhood of the origin.

*Proof:* In this position, the Lyapunov candidate is selected as follows

$$V_0 = \frac{1}{2}\tilde{\omega}^{\mathrm{T}}\tilde{\omega} + \frac{1}{2\gamma_1}\tilde{e}^{\mathrm{T}}\tilde{e} + \frac{1}{2\gamma_2}\tilde{u}_f^{\mathrm{T}}\tilde{u}_f + \frac{1}{2\gamma_3}(K - K^*)^2 \quad (20)$$

1. Supposing that $0 < t \le \bar{t}$, i.e. $\tilde{\omega}^{\mathrm{T}}\mathrm{sign}(\tilde{\omega}(\bar{t})) > \varepsilon$ which means that $K(t)$ is adjusted by the adaptive update law (17). Then the derivative of $V_0$ with respect to time is

$$\begin{aligned}\dot{V}_0 &= \tilde{\omega}^{\mathrm{T}}\dot{\tilde{\omega}} + \frac{1}{\gamma_1}\tilde{e}^{\mathrm{T}}\dot{\tilde{e}} + \frac{1}{\gamma_2}\tilde{u}_f^{\mathrm{T}}\dot{\tilde{u}}_f + \frac{1}{\gamma_3}(K - K^*)\dot{K} \\ &= \tilde{\omega}^{\mathrm{T}}[-\Lambda\tilde{\omega} - J^{-1}(\omega^{\times}J\omega - \hat{\omega}^{\times}J\hat{\omega}) + J^{-1}DU\tilde{e} \\ &+ J^{-1}D\tilde{u}_f + J^{-1}d_{\Theta} - J^{-1}K(t)\mathrm{sign}(\tilde{\omega})] \\ &- \frac{1}{\gamma_1}\tilde{e}^{\mathrm{T}}\dot{\hat{e}} - \frac{1}{\gamma_2}\tilde{u}_f^{\mathrm{T}}\dot{\hat{u}}_f + \frac{1}{\gamma_3}(K - K^*)\dot{K}\end{aligned} \quad (21)$$

Substituting the first equation of (16) into (21), one has

$$\begin{aligned}\dot{V}_0 &= \tilde{\omega}^{\mathrm{T}}[-\Lambda\tilde{\omega} - J^{-1}(\omega^{\times}J\omega - \hat{\omega}^{\times}J\hat{\omega}) + J^{-1}d_{\Theta} \\ &- J^{-1}K(t)\mathrm{sign}(\tilde{\omega})] + \frac{1}{\gamma_3}(K - K^*)\dot{K}\end{aligned} \quad (22)$$

Substituting the first equation of (17) into (22), it obtains

$$\begin{aligned}\dot{V}_0 &\le -(\lambda_{\min}(\Lambda) - \varepsilon_0 \parallel J^{-1} \parallel) \parallel \tilde{\omega} \parallel^2 + \parallel \tilde{\omega}^{\mathrm{T}} \parallel \ \parallel J^{-} \\ &\times (d_M - K_a) + (K_a - K^*) \parallel J^{-1} \parallel \tilde{\omega}^{\mathrm{T}}\mathrm{sign}(\tilde{\omega}) \\ &\le -(\lambda_{\min}(\Lambda) - \varepsilon_0 \parallel J^{-1} \parallel) \parallel \tilde{\omega} \parallel^2 - (K^* - d_M) \\ &\times \parallel \tilde{\omega}^{\mathrm{T}}J^{-1} \parallel \ < 0\end{aligned} \quad (23)$$

It can be seen that there always exist $K^*$ such that $K^* > d_M$. The main result of Theorem 1 can be achieved.

2. Supposing that $t > \bar{t}$, i.e. $\tilde{\omega}^{\mathrm{T}}\mathrm{sign}(\tilde{\omega}(\bar{t})) < \varepsilon$. According to [33], the proposed adaptive sliding mode observer with the equation (18) can maintain that $\tilde{\omega}^{\mathrm{T}}\mathrm{sign}(\tilde{\omega}(\bar{t})) < \varepsilon$ for all $t > \bar{t}$. Therefore, the proof of Theorem 1 can be easily obtained.

□

*Remark 2:* A FE approach is proposed in [15] to only achieve the actuator bias FE in the presence of external disturbances, but actuator LOE fault and actuator saturation is ignored. The other references [16, 25] treat external disturbances, the actuator LOE fault and the bias fault as a lumped fault in the design of the FE observer, but it does not take into account the FE in the case of actuator saturation. Compared with the results obtained [15, 16, 25], an adaptive sliding mode FE observer is developed in this paper for the rigid spacecraft attitude system, such that the actuator LOE fault and bias fault could be estimated simultaneously under actuator saturation, external disturbances and inertia uncertainties.

*Remark 3:* A new BF-based adaptive strategy is proposed firstly for a sliding mode FE observer (15). The sliding mode term $K\mathrm{sign}(\tilde{\omega})$ is used to offset the effects of the total time-varying uncertainties for a sliding mode FE observer. Unlike the general sliding mode FE observer [13], this strategy allows the adaptive gain to increase and decrease based on the current value of the output variable. When the output variable is going to zero, the adaptive gain decreases till the value which allows to compensate for the total time-varying uncertainties.

- On the one hand, the proposed adaptive strategy is to first increase the adaptive gain until the output variable reaches a small neighbourhood of zero $\varepsilon$ at time $\bar{t}$ by using a derivative gain.
- On the other hand, a new BF-based adaptive strategy can achieve the convergence of the output variable to a neighbourhood of zero, with an adaptive gain that is not overestimated, and without using any information about the upper bound of the total time-varying uncertainties, nor the use of the low pass filter.

## 4 Fault-tolerant controller design

In this section, according to fault information from the FE observer (15), a finite time fault-tolerant attitude stabilisation controller and a finite time fault-tolerant attitude tracking controller are designed respectively to compensate for the multiple actuator faults and eliminate of the actuator saturation. In order to relax the requirement on disturbances bound and attenuate chattering problem in control forces, a NN technique is used to estimate the lumped disturbances online.

### *4.1 NNs theory*

The structure of a radial basis function (RBF) NN is a three-lawyer feedforward network. The input layer passes input signals without any operation; the hidden layer performs activation function in each node of the layer and the output layer gives the output.

The output of the NN can be described as [26]

$$y = W^{\mathrm{T}}\phi \quad (24)$$

where $W = [W_1, W_2, \ldots, W_N]^{\mathrm{T}}$ is the weight vector and $W_N$ is the neural weight connecting the $N$th neuron in the hidden layer and the output neuron. $\phi$ represents the activation function that is performed in every node in the hidden layer; $\phi = [\phi_1, \phi_2 \ldots, \phi_N]^{\mathrm{T}}$ is the output vector of the hidden layer. The Gaussian function is usually chosen as the activation function and the output of the $N$th node in the hidden layer is given by

$$\phi_N(x) = \exp\left(-\frac{\parallel x - C_N \parallel^2}{\eta_N^2}\right) \quad (25)$$

where $C_N$ and $\eta_N$ is the centre and the width of the Gaussian function.

In this study, the following RBF NNs are used to reconstruct the generalised perturbation $d(t)$ in the first equation of (9), which can be expressed as

$$d(t) = W^{\mathrm{T}}\phi \tag{26}$$

where $W$ is the vector of adjustable weights.

The optimal weight value $W$ of RBF NN is given by

$$W = \arg\ \min_{\hat{W} \in R^{N \times 3}} \{\sup_{\omega \in \Gamma} \left| d(t) - \hat{W}^T \phi(t) \right| \} \tag{27}$$

where $\hat{W}$ is the estimated matrix of $W$.

Since the optimal weight value $W$ is unknown, it follows that

$$\hat{d}(t) = \hat{W}^T \phi(t) + \epsilon_\theta \tag{28}$$

where $\hat{d}(t)$ is the estimated value of $d(t)$, $\epsilon_\theta$ is the very small known estimation error.

### *4.2 Finite time fault tolerant attitude stabilisation control*

In this section, a finite time fault tolerant attitude stabilisation controller is proposed for the faulty attitude systems (2) and (9) with actuator saturation. First, a fast terminal sliding model surface is designed as

$$s = \omega + a\sigma + b\mathrm{sig}^{\frac{p}{q}}(\sigma) \tag{29}$$

where $s = [s_1, s_2, s_3]^{\mathrm{T}} \in R^3$ is the sliding mode variable, $a > 0$, $b > 0$ are two positive scalars, $0 < \frac{p}{q} < 1$, and $p$, $q$ are positive odd integers. the function $\mathrm{sig}^{\frac{p}{q}}(\sigma)$ is defined as

$$\mathrm{sig}^{\frac{p}{q}}(\sigma) = [\left|\sigma_1\right|^{\frac{p}{q}} \mathrm{sign}(\sigma_1), \left|\sigma_2\right|^{\frac{p}{q}} \mathrm{sign}(\sigma_2), \left|\sigma_3\right|^{\frac{p}{q}} \mathrm{sign}(\sigma_3)]^{\mathrm{T}}. \tag{30}$$

Taking the derivative of sliding variable $s$ with respect to time yields

$$\dot{s} = \dot{\omega} + a\dot{\sigma} + b\frac{p}{q}\mathrm{diag}^{p/q}(\left|\sigma\right|^{(p/q)-1})\dot{\sigma} \tag{31}$$

It is noted that (31) contains a negative fractional term $(p/q) - 1$, the singularity will occurs while $\sigma_j = 0$ and $\dot{\sigma}_j \neq 0$. To avoid the singularity problem, the derivative of $s$ is modified as

$$\dot{s} = \dot{\omega} + a\dot{\sigma} + b\bar{s}(\sigma) \tag{32}$$

where $\bar{s}(\sigma) = [\bar{s}_1(\sigma), \bar{s}_2(\sigma), \bar{s}_3(\sigma)]^{\mathrm{T}} \in R^3$ with

$$\bar{s}_i(\sigma) = \begin{cases} \dfrac{p}{q}|\dot{\sigma}_i|^{(p/q)-1}\dot{\sigma}_i, & \text{if } \dot{\sigma}_i \neq 0, |\dot{\sigma}_i| \geq \epsilon \\ \dfrac{p}{q}|\epsilon_i|^{(p/q)-1}\dot{\sigma}_i, & \text{if } \dot{\sigma}_i \neq 0, |\dot{\sigma}_i| < \epsilon \\ 0, & \text{if } \dot{\sigma}_i = 0 \end{cases} \tag{33}$$

where $\bar{s}_i(\sigma)$ is the $i$th component of $\bar{s}(\sigma)$ and $\epsilon$ is a small positive constant.

Considering a faulty rigid spacecraft attitude system with actuator saturation, it is easily known that

$$\begin{aligned} \dot{s} = &-J^{-1}\omega^{\times}J\omega + J^{-1}DEu_c + J^{-1}DE\Theta(u_c) \\ &+J^{-1}Du_f + J^{-1}d + a\dot{\sigma} + b\bar{s}(\sigma) \end{aligned} \tag{34}$$

The reaching law proposed in this paper is based on the choice of an exponential term that adapts to the variations of the switching function. A new exponential reaching law (ERL) is designed as follows [34]:

$$\dot{s} = -\varepsilon_1 s - \frac{\varepsilon_2}{N(s)}\mathrm{sign}(s) \tag{35}$$

where $N(s) = \delta_0 + (1 - \delta_0)e^{-\alpha \| s \|^{P_0}}$, $\varepsilon_1 = \mathrm{diag}\{\varepsilon_{11}, \varepsilon_{12}, \varepsilon_{13}\} > 0$ and

$$\frac{\varepsilon_2}{N(s)} = \mathrm{diag}\left\{\frac{\varepsilon_{2i}}{\delta_0 + (1 - \delta_0)e^{-\alpha|s_i|^{P_0}}}\right\} > 0, \ i = 1, 2, 3.$$

$\delta_0$ is a strictly positive offset that is less than one, $P_0$ is a strictly positive integer, and $\alpha$ is also strictly positive. Note that the ERL is given by (35) does not affect the stability of the control because $N(s)$ is always strictly positive.

*Assumption 4:* Similar to Assumption 4 in [20], there exist unknown positive constants $\chi$ and $\rho$ such that

$$\| a\dot{\sigma} + b\bar{s}(\sigma) \| \leq \chi \| \dot{\sigma} \|, \quad \| \tilde{E}\Theta(u_c) + \tilde{u}_f \| \leq \rho$$

*Remark 4:* According to the Assumption 1 in and (33) in this paper, $\| a\dot{\sigma} + b\bar{s}(\sigma) \| \leq \chi \| \dot{\sigma} \|$ can be obtained. In addition, Theorem 1 means the estimates of the actuator LOE fault and the bias fault are accurate, such that the errors of faults are almost zero and bounded. It easily known that $\| \tilde{E}\Theta(u_c) + \tilde{u}_f \| \leq \rho$ is satisfied. Xiao *et al*. [7] and Han *et al*. [20] also pointed out that Assumption 4 holds for the rigid spacecraft.

To achieve the desired fault-tolerant attitude control performance, the control inputs $u_c$ are designed as follows:

$$u_c = \hat{E}^2 D^{\mathrm{T}}(J^{-1}D\hat{E}^3 D^{\mathrm{T}})^{-1}(u_n + u_m) \tag{36}$$

where

$$u_n = -\varepsilon_1 s - \frac{\varepsilon_2}{N(s)}\mathrm{sign}(s) \tag{37}$$

$$\begin{aligned} u_m = &J^{-1}\omega^{\times}J\omega - l_1\varphi - J^{-1}D\hat{u}_f - (\hat{\chi} \| \dot{\sigma} \| \\ &+\hat{\rho} \| J^{-1} \| \ \| D \|)\mathrm{sign}(s) - J^{-1}(\hat{W}^{\mathrm{T}}\phi + \epsilon_\theta) \end{aligned} \tag{38}$$

An auxiliary system [1] is introduced to deal with the actuator saturation as

$$\dot{\varphi} = \begin{cases} -l_2\varphi - \dfrac{\| H \|^2 \ \| \Theta(u_c) \|^2}{\| \varphi \|^2}\varphi - H\Theta(u_c), & \| \varphi \| \geq \delta_k \\ 0, & \| \varphi \| < \delta_k \end{cases} \tag{39}$$

where $\varphi$ is an auxiliary variable, $\delta_k$ is a small positive scalar, $l_2$ is a gain matrix, and $H = J^{-1}D\hat{E}$.

$\hat{\chi}$, $\hat{W}$ and $\hat{\rho}$ are the estimates of $\chi$, $W$ and $\rho$, respectively, updated by the following adaptive laws:

$$\dot{\hat{\chi}} = \mu_1 \| s \| \ \| \dot{\sigma} \| \tag{40}$$

$$\dot{\hat{\rho}} = \mu_2 \| s \| \ \| J^{-1} \| \ \| D \| \tag{41}$$

$$\dot{\hat{W}} = \mu_3 \phi s^{\mathrm{T}} J^{-1} \tag{42}$$

where $l_1$, $l_2$, $\mu_1$, $\mu_2$, and $\mu_3$ are positive constants.

In the following, the second result of this study is given in the form of Theorem 2.

*Theorem 2:* Consider a faulty rigid spacecraft attitude system described by (2) and (9) in the presence of the three types of the actuator faults and actuator saturation. Suppose that Assumptions 1–4 are satisfied, and the control parameters are chosen such that $l_2 - (l_1^2/2) - (1/2) > 0$. By applying the designed control laws (36)–(38) and the adaptive laws (39)–(42), the following results are achieved:

1. The closed-loop attitude system is asymptotically stable.
2. The attitude angle $\sigma$ and the attitude angular velocity $\omega$ will reach zero in a finite time, that is, $\lim_{t \to T_1}\sigma \to 0$ and $\lim_{t \to T_1}\omega \to 0$.

*Proof:* Consider the case that $\| \varphi \| \geq \delta_k$. Define the following Lyapunov function:

$$V_1 = V_0 + \frac{s^{\mathrm{T}}s}{2} + \frac{\varphi^{\mathrm{T}}\varphi}{2} + \frac{\tilde{\chi}^2}{2\mu_1} + \frac{\tilde{\rho}^2}{2\mu_2} + \frac{\mathrm{tr}(\tilde{W}^{\mathrm{T}}\tilde{W})}{2\mu_3} \tag{43}$$

where $\tilde{\chi} = \chi - \hat{\chi}$, $\tilde{\rho} = \rho - \hat{\rho}$ and $\tilde{W} = W - \hat{W}$.

The derivative of $V_1$ with respect to time is

$$\begin{aligned} \dot{V}_1 &= \dot{V}_0 + s^{\mathrm{T}}\dot{s} + \varphi^{\mathrm{T}}\dot{\varphi} - \frac{\tilde{\chi}\dot{\hat{\chi}}}{\mu_1} - \frac{\tilde{\rho}\dot{\hat{\rho}}}{\mu_2} - \frac{\mathrm{tr}(\tilde{W}^{\mathrm{T}}\dot{\hat{W}})}{\mu_3} \\ &= \dot{V}_0 + s^{\mathrm{T}}[-J^{-1}\omega^{\times}J\omega + J^{-1}D\hat{E}u_c + J^{-1}D\hat{E} \\ &\quad \times \Theta(u_c) + J^{-1}D\tilde{E}\mathrm{sat}(u_c) + J^{-1}Du_f + J^{-1}d \\ &\quad + a\dot{\sigma} + b\bar{s}(\sigma)] + \varphi^{\mathrm{T}}\dot{\varphi} - \frac{\tilde{\chi}\dot{\hat{\chi}}}{\mu_1} - \frac{\tilde{\rho}\dot{\hat{\rho}}}{\mu_2} - \frac{\mathrm{tr}(\tilde{W}^{\mathrm{T}}\dot{\hat{W}})}{\mu_3} \end{aligned} \tag{44}$$

Substituting the control laws (36)–(38) into (44) results in

$$\begin{aligned} \dot{V}_1 &\leq \dot{V}_0 + s^{\mathrm{T}}[J^{-1}D\hat{E}\Theta(u_c) + J^{-1}D\tilde{E}\mathrm{sat}(u_c) - l_1 \\ &\quad \times \varphi + J^{-1}D\tilde{u}_f + J^{-1}d + a\dot{\sigma} + b\bar{s}(\sigma) - (\hat{\chi} \| \dot{\sigma} \| \\ &\quad + \hat{\rho} \| J^{-1} \| \| D \|)\mathrm{sign}(s) - J^{-1}(\hat{W}^{\mathrm{T}}\phi + \epsilon_\theta)] \\ &\quad + s^{\mathrm{T}}[-\varepsilon_1 s - \frac{\varepsilon_2}{N(s)}\mathrm{sign}(s)] + \varphi^{\mathrm{T}}\dot{\varphi} - \frac{\tilde{\chi}\dot{\hat{\chi}}}{\mu_1} \\ &\quad - \frac{\tilde{\rho}\dot{\hat{\rho}}}{\mu_2} - \frac{\mathrm{tr}(\tilde{W}^{\mathrm{T}}\dot{\hat{W}})}{\mu_3} \end{aligned} \tag{45}$$

From (39), it can be seen that

$$\begin{aligned} \dot{V}_1 &\leq \dot{V}_0 + s^{\mathrm{T}}[J^{-1}D\hat{E}\Theta(u_c) + J^{-1}D\tilde{E}\mathrm{sat}(u_c) - l_1\varphi \\ &\quad + J^{-1}D\tilde{u}_f + J^{-1}d + a\dot{\sigma} + b\bar{s}(\sigma) - (\hat{\chi} \| \dot{\sigma} \| \\ &\quad + \| J^{-1} \| \hat{\rho})\mathrm{sign}(s) - J^{-1}(\hat{W}^{\mathrm{T}}\phi + \epsilon_\theta)] + s^{\mathrm{T}} \\ &\quad \times [-\varepsilon_1 s - \frac{\varepsilon_2}{N(s)}\mathrm{sign}(s)] - l_2 \| \varphi \|^2 + \| J^{-1}D \|^2 \\ &\quad \times \| \hat{E}(t) \|^2 \| \Theta(u_c) \|^2 - \varphi^{\mathrm{T}}J^{-1}D\hat{E}\Theta(u_c) \\ &\quad - \frac{\tilde{\chi}\dot{\hat{\chi}}}{\mu_1} - \frac{\tilde{\rho}\dot{\hat{\rho}}}{\mu_2} - \frac{\mathrm{tr}(\tilde{W}^{\mathrm{T}}\dot{\hat{W}})}{\mu_3} \end{aligned} \tag{46}$$

Further, using the specific case of Young's inequality $x^{\mathrm{T}}y \leq \frac{1}{2}x^{\mathrm{T}}x + \frac{1}{2}y^{\mathrm{T}}y$ for all $x, y \in R^3$, the following inequalities may be established:

$$-l_1 s^{\mathrm{T}}\varphi \leq \frac{l_1^2}{2} \| \varphi \|^2 + \frac{1}{2} \| s \|^2 \tag{47}$$

$$s^{\mathrm{T}}J^{-1}D\hat{E}\Theta(u_c) \leq \frac{\| J^{-1}D \|^2 \| \hat{E} \|^2 \| \Theta(u_c) \|^2}{2} + \frac{\| s \|^2}{2} \tag{48}$$

$$-\varphi^{\mathrm{T}}J^{-1}D\hat{E}\Theta(u_c) \leq \frac{\| J^{-1}D \|^2 \| \hat{E} \|^2 \| \Theta(u_c) \|^2}{2} + \frac{\| \varphi \|^2}{2} \tag{49}$$

Therefore, inserting inequalities (47)–(49) into (46) yields

$$\begin{aligned} \dot{V}_1 &\leq s^{\mathrm{T}}[J^{-1}D\tilde{E}\mathrm{sat}(u_c) + J^{-1}D\tilde{u}_f + J^{-1}d + a\dot{\sigma} \\ &\quad + b\bar{s}(\sigma) - (\hat{\chi} \| \dot{\sigma} \| + \hat{\rho} \| J^{-1} \| \| D \|)\mathrm{sign}(s) - J^{-1} \\ &\quad \times (\hat{W}^{\mathrm{T}}\phi + \epsilon_\theta)] + s^{\mathrm{T}}\left[(I_3 - \varepsilon_1)s - \frac{\varepsilon_2}{N(s)}\mathrm{sign}(s)\right] \\ &\quad - \left(l_2 - \frac{l_1^2}{2} - \frac{1}{2}\right) \| \varphi \|^2 - \frac{\tilde{\chi}\dot{\hat{\chi}}}{\mu_1} - \frac{\tilde{\rho}\dot{\hat{\rho}}}{\mu_2} - \frac{\mathrm{tr}(\tilde{W}^{\mathrm{T}}\dot{\hat{W}})}{\mu_3} \end{aligned} \tag{50}$$

According to $l_2 - (l_1^2/2) - (1/2) > 0$ and (40)–(42), it follows that

$$\begin{aligned} \dot{V}_3 &\leq s^{\mathrm{T}}J^{-1}(\tilde{W}^{\mathrm{T}}\phi - \epsilon_\theta) + s^{\mathrm{T}}[(I_3 - \varepsilon_1)s - \frac{\varepsilon_2}{N(s)} \\ &\quad \times \mathrm{sign}(s)] - \mathrm{tr}(\tilde{W}^{\mathrm{T}}\phi s^{\mathrm{T}}J^{-1}) \end{aligned} \tag{51}$$

The following property for tracing the matrix is used:

$$\mathrm{tr}(s^{\mathrm{T}}J^{-1}\tilde{W}^{\mathrm{T}}\phi) = \mathrm{tr}(\tilde{W}^{\mathrm{T}}\phi s^{\mathrm{T}}J^{-1}) \tag{52}$$

Similar to the proof of [18, 30], it follows that

$$\begin{aligned} \dot{V}_3 &\leq -s^{\mathrm{T}}(\varepsilon_1 - I_3)s - 2^{1/2}\left(\frac{s^{\mathrm{T}}\varepsilon_2 s}{2N(s)}\right)^{1/2} \\ &\leq -2\lambda_{\min}(\varepsilon_1 - I_3)\left(1 - \frac{\Psi}{V_1}\right)V_1 \\ &\quad - 2^{1/2}\lambda_{\min}\left(\frac{\varepsilon_2}{N(s)}\right)\left(1 - \frac{\Psi}{V_1}\right)^{1/2}V_1^{1/2} \end{aligned} \tag{53}$$

where

$\Psi = V_0 + \frac{\varphi^{\mathrm{T}}\varphi}{2} + \frac{\tilde{\chi}^2}{2\mu_1} + \frac{\tilde{\rho}^2}{2\mu_2} + \frac{\mathrm{tr}(\tilde{W}^{\mathrm{T}}\tilde{W})}{2\mu_3}$, $\frac{\Psi}{V_1} < 1$, and $\left(\frac{\Psi}{V_1}\right)^{1/2} < 1$.

Define

$\rho_0 = 1/2$, $\rho_1 = 2\lambda_{\min}(\varepsilon_1 - I_3)\left(1 - \frac{\Psi}{V_1}\right)$, and $\rho_2 = 2^{1/2}\lambda_{\min}\left(\frac{\varepsilon_2}{N(s)}\right)\left(1 - \frac{\Psi}{V_1}\right)^{1/2}$,

then

$$\dot{V}_1 + \rho_1 V_1 + \rho_2 V_1^{\rho_0} \leq 0 \tag{54}$$

and the convergence time is given as:

$$T_1 \leq \frac{1}{\rho_1(1 - \rho_0)}\ln\frac{\rho_1 V_1^{1-\rho_0}(s_0) + \rho_2}{\rho_2} \tag{55}$$

The proof of the case when $\| \varphi \| < \delta_k$ is similar to the above case and the same conclusion can be drawn. Therefore, the state trajectory of the faulty closed-loop attitude system can converge into origin in finite time. This completes the proof. □

*Remark 5:* In [10], an active FTC scheme is proposed for a rigid spacecraft using both sliding FE observer and terminal sliding mode control techniques, but it could only handle the composite fault effects of actuator partial LOE fault and bias fault, without considering actuator saturation. In [16], the integrated FTC strategy for attitude stabilisation control estimates and compensates the total effect of the actuator faults and saturation. In [35], a finite time attitude stabilisation controller is presented for a rigid spacecraft by using the sliding mode control techniques, it has both the advantage of high convergence rate and high steady accuracy, but the actuator fault problems are not considered in [35]. Inspired by the above results, a novel finite-time attitude stabilisation fault-tolerant controller is proposed in this paper, it not only could compensate for different type actuator faults, including partial LOE fault, bias fault and complete failure fault, but also could avoid actuator saturation.

*Remark 6:* The parameter $\delta_k$ in (39) is designed to avoid the singularity. In the practical application, $\delta_k$ is usually selected as a very small positive constant, while the initial value of the auxiliary variable $\varphi(0)$ is given to satisfy $\| \varphi(0) \| \geq \delta_k$ which ensures the auxiliary system works at the beginning.

*Remark 7:* When considering actuator saturation, an auxiliary system (39) is utilised to compensate for the saturation. The main contributions of (39) are as follows

- When $\| \varphi \| \geq \delta_k$, it means that the auxiliary system works (even when $\triangle u = 0$).
- When $\| \varphi \| < \delta_k$ and $\triangle u = 0$, it means the auxiliary system does not work, and meanwhile the actuator saturation does not exist.
- Especially, the case $0 < \| \varphi \| < \delta_k$ and $\triangle u \neq 0$ represents that the auxiliary system is not working at this moment but the actuator saturation occurs. The case $0 < \| \varphi \| < \delta_k$ and $\triangle u \neq 0$ usually happens when the spacecraft tends to be stable but suddenly requires a large control torque to perform the attitude adjustment. On that occasion, $\varphi$ will quickly increase and satisfy the condition $\| \varphi \| \geq \delta_k$ owing to (39) and $\triangle u$, such that the auxiliary system starts to work to compensate the actuator saturation. Therefore, this case is just a transient stage.

*Remark 8:* From the reaching law stated in (35), one can see that if $\| s \|$ increases, $N(s)$ approaches $\delta_0$, and therefore, $\varepsilon_2/N(s)$ converges to $\varepsilon_2/\delta_0$, which is greater than $\varepsilon_2$. This means that $\varepsilon_2/N(s)$ increases in reaching phase, and consequently, the attraction of the sliding surface will be faster. On the other hand, if $\| s \|$ decreases, then $N(s)$ approaches one, and $\varepsilon_2/N(s)$ converges to $\varepsilon_2$. This means that when the system approaches the sliding surface, $\varepsilon_2/N(s)$ gradually decreases in order to limit the chattering. Therefore, the ERL allows the controller to dynamically adapt to the variations of the switching function by letting $\varepsilon_2/N(s)$ to vary between $\varepsilon_2$ and $\varepsilon_2/\delta_0$.

### *4.3 Finite-time fault tolerant attitude tracking control*

A finite time fault-tolerant attitude tracking controller design is similar to a finite time fault-tolerant attitude stabilisation controller. However, the design of finite time fault-tolerant attitude tracking controller is more complex since it is designed by using back-stepping techniques and sliding mode control methods. The design steps of NFTSM fault-tolerant tracking controller are as follows.

*Step 1:* Two new error variables are defined as follows:

$$z_1 = \sigma - \sigma_d, \ z_2 = \omega - \omega_d \tag{56}$$

where $\sigma_d$ is the desired attitude angle command, and $\omega_d$ is the desired angular velocity command.

For the outer loop which is also known as an angular loop, the virtual control law is selected as follows:

$$\dot{z}_1 = G(\sigma)(\omega_d + z_2) - \dot{\sigma}_d \tag{57}$$

Consider a Lyapunov function

$$V_2 = \frac{1}{2} z_1^{\mathrm{T}} z_1 \tag{58}$$

The derivative of $V_1$ is

$$\dot{V}_2 = z_1^{\mathrm{T}} \dot{z}_1 = z_1^{\mathrm{T}} [G(\sigma)(\omega_d + z_2) - \dot{\sigma}_d] \tag{59}$$

It can be further obtained from *P1* that $G(\sigma)$ has an inverse matrix. Therefore, the virtual control $\omega_d$ is designed as

$$\omega_d = -G^{-1}(\sigma)(cz_1 + \dot{\sigma}_d) \tag{60}$$

where $c$ is a design parameter.

According to P1, it can be found that

$$\dot{V}_2 \leq -c \| z_1 \|^2 + z_1^{\mathrm{T}} G(\sigma) z_2 \tag{61}$$

*Step 2:* Note that

$$\begin{aligned} \dot{z}_2 &= \dot{\omega} - \dot{\omega}_d \\ &= -J^{-1}\omega^{\times} J\omega + J^{-1} DE \mathrm{sat}(u_c) + J^{-1} D u_f + J^{-1} d - \dot{\omega}_d \end{aligned} \tag{62}$$

According to the attitude error and the angular velocity error, a fast non-singular terminal sliding mode surface is selected as follows:

$$s = z_2 + k_1 z_1 + k_2 S_{au} \tag{63}$$

where $S_{au} = [S_{au1}, S_{au2}, S_{au3}]^{\mathrm{T}}$, $k_{ji} > 0$ $(j = 1, 2, i = 1, 2, 3)$.

$$S_{aui} = \begin{cases} c_1 z_{1i} + c_2 \mathrm{sign}(z_{1i}) z_{1i}^2, & \text{if } \bar{S}_i \neq 0, |z_{1i}| < \mu_\epsilon \\ z_{1i}^r, & \text{otherwise} \end{cases} \tag{64}$$

where $\bar{S}_i = z_{2i} + k_{1i} z_{1i} + k_{2i} z_{1i}^r$, $r = r_1/r_2$, $r_1$, $r_2$ are positive odd number, and $0 < r < 1$. $c_1 > 0$, $c_2 > 0$.

It is easily known that

$$\begin{aligned} \dot{s} = &-J^{-1}\omega^{\times} J\omega + J^{-1} DE u_c + J^{-1} DE \Theta(u_c) \\ &+ J^{-1} D u_f + J^{-1} d - \dot{\omega}_d + (k_1 + k_2 \delta) \dot{z}_1 \end{aligned} \tag{65}$$

where

$$\delta = \begin{cases} c_1 I_3 + 2c_2 \mathrm{diag}\{\mathrm{sign}(z_{1i}) z_{1i}\}, & \text{if } \bar{S}_i \neq 0, |z_{1i}| < \mu_\epsilon \\ r \mathrm{diag}\{z_{1i}^{r-1}\}, & \text{otherwise} \end{cases}.$$

A new exponential approximation law is designed as follows:

$$\dot{s} = -\varepsilon_3 s - \frac{\varepsilon_4}{N(s)} \mathrm{sign}(s) \tag{66}$$

where the parameters are similar to (35).

*Assumption 5:* There exists an unknown constant $\Omega > 0$, such that the following inequality is satisfied

$$\| J^{-1} D[\tilde{E}\Theta(u_c) + \tilde{u}_f] + k_2 \delta(-cz_1 + G(\sigma) z_2) \| \leq \Omega \tag{67}$$

*Remark 9:* According to Assumption 4,

$$\begin{aligned} &\| J^{-1} D[\tilde{E}\Theta(u_c) + \tilde{u}_f] \| \leq \| J^{-1} D \| \ \| (\tilde{E}\Theta(u_c) + \tilde{u}_f) \| \leq \\ &. \ \| J^{-1} D \| \rho \end{aligned} \tag{68}$$

Moreover, due to $\sigma_d$, $\dot{\sigma}_d$, $\omega_d$ and $\dot{\omega}_d$ are bounded in Assumption 1, it is obvious that $z_1$ and $z_2$ are bounded, such that $\| k_2 \delta(-cz_1 + G(\sigma) z_2) \| \leq \rho_k$ is satisfied. Therefore,

$$\begin{aligned} &\| J^{-1} D[\tilde{E}\Theta(u_c) + \tilde{u}_f] + k_2 \delta(-cz_1 + G(\sigma) z_2) \| \\ &\leq \| J^{-1} D[\tilde{E}\Theta(u_c) + \tilde{u}_f] \| + \| k_2 \delta(-cz_1 + G(\sigma) z_2) \| \\ &\leq \| J^{-1} D \| \rho + \rho_k = \Omega \end{aligned} \tag{69}$$

is satisfied, which is similar to Assumption 4 in [20].

To achieve the desired tracking performance, the control inputs $u_c$ are designed as follows:

$$u_c = \hat{E}^2 D^{\mathrm{T}}(J^{-1}D\hat{E}^3 D^{\mathrm{T}})^{-1}(u_1 + u_2) \tag{70}$$

where

$$u_1 = -\varepsilon_3 s - \frac{\varepsilon_4}{N(s)}\mathrm{sign}(s) \tag{71}$$

$$\begin{aligned} u_2 = {} & J^{-1}\omega^{\times}J\omega - L_1\xi - J^{-1}D\hat{u}_f + \dot{\omega}_d - k_1(-cz_1 + G(\sigma)z_2) \\ & - \frac{s(1+\eta)\,\| z_1^{\mathrm{T}}G(\sigma)z_2 \|}{\| s \|^2 + \delta_n} - \hat{\Omega}\mathrm{sign}(s) - J^{-1}(\hat{W}^{\mathrm{T}}\phi + \epsilon_\theta) \end{aligned} \tag{72}$$

The following auxiliary system is designed to compensate for the saturation,

$$\dot{\xi} = \begin{cases} -L_2\xi - \dfrac{\| H \|^2 \, \| \Theta(u_c) \|^2}{\| \xi \|^2}\xi - H\Theta(u_c), & \| \xi \| \ge \delta_k \\ 0, & \| \xi \| < \delta_k \end{cases} \tag{73}$$

where $L_2$, $\delta_k$, $\Theta(u_c)$, and $H$ are similar to parameters of (39).

$\hat{\Omega}$, $\hat{W}$ and $\eta$ are updated by the following adaptive laws:

$$\dot{\hat{\Omega}} = \theta_1 \| s \| \tag{74}$$

$$\dot{\hat{W}} = \theta_2 \phi s^{\mathrm{T}} J^{-1} \tag{75}$$

$$\dot{\eta} = \begin{cases} \dfrac{\theta_3}{\eta} \cdot \dfrac{\eta \| s \|^2 - \delta_m}{\| s \|^2 + \delta_n} \| z_1^{\mathrm{T}}G(\sigma)z_2 \|, & \eta \ne 0 \\ \delta_n, & \eta = 0 \end{cases} \tag{76}$$

where $\delta_m > \delta_n > 0$. $L_1$, $L_2$, $\theta_1$, $\theta_2$, and $\theta_3$ are positive constants.

It is ready to present the third main result in the following theorem.

*Theorem 3:* Consider a faulty rigid spacecraft attitude system (2) and (9) involving actuator saturation, actuator faults, inertia matrix uncertainties, external disturbances under Assumptions 1–3, Assumption 5, and $L_2 - (L_1^2/2) - (1/2) > 0$. By applying the designed control laws in (70)–(72) and adaptive laws in (73)–(76), the following results are achieved:

1. The closed-loop system is asymptotically stable.
2. The states such as the error attitude angle $z_1$ and error attitude angular velocity $z_2$ converge to an arbitrary small set containing the origin in a finite time, that is, $\lim_{t\to T_2} z_1 \to 0$ and $\lim_{t\to T_2} z_2 \to 0$.

*Proof :* When $\| \xi \| \ge \delta_k$, define the following Lyapunov function:

$$V_3 = V_0 + V_2 + \frac{s^{\mathrm{T}}s}{2} + \frac{\xi^{\mathrm{T}}\xi}{2} + \frac{\tilde{\Omega}^2}{2\theta_1} + \frac{\mathrm{tr}(\tilde{W}^{\mathrm{T}}\tilde{W})}{2\theta_2} + \frac{\eta^2}{2\theta_3} \tag{77}$$

where $\tilde{\Omega} = \Omega - \hat{\Omega}$.

The derivative of $V_1$ with respect to time is

$$\begin{aligned} \dot{V}_3 = {} & \dot{V}_0 + \dot{V}_2 + s^{\mathrm{T}}\dot{s} + \xi^{\mathrm{T}}\dot{\xi} - \frac{\tilde{\Omega}\dot{\hat{\Omega}}}{\theta_1} - \frac{\mathrm{tr}(\tilde{W}^{\mathrm{T}}\dot{\hat{W}})}{\theta_2} + \frac{\eta\dot{\eta}}{\theta_3} \\ = {} & \dot{V}_0 + \dot{V}_2 + s^{\mathrm{T}}[-J^{-1}\omega^{\times}J\omega + J^{-1}D\hat{E}u_c \\ & + J^{-1}D\hat{E}\Theta(u_c) + J^{-1}D\tilde{E}\mathrm{sat}(u_c) \\ & + J^{-1}Du_f + J^{-1}d - \dot{\omega}_d + (k_1 + k_2\delta)\dot{z}_1] \\ & + \xi^{\mathrm{T}}\dot{\xi} - \frac{\tilde{\Omega}\dot{\hat{\Omega}}}{\theta_1} - \frac{\mathrm{tr}(\tilde{W}^{\mathrm{T}}\dot{\hat{W}})}{\theta_2} + \frac{\eta\dot{\eta}}{\theta_3} \end{aligned} \tag{78}$$

Substituting the control laws (71) and (72) into (78) results in

$$\begin{aligned} \dot{V}_3 \le {} & -c \| z_1 \|^2 + z_1^{\mathrm{T}}G(\sigma)z_2 + s^{\mathrm{T}}[J^{-1}D\hat{E}\Theta(u_c) \\ & + J^{-1}D\tilde{E}\mathrm{sat}(u_c) - L_1\xi + J^{-1}D\tilde{u}_f + J^{-1}d \\ & + k_2\delta(-cz_1 + G(\sigma)z_2) - \frac{s(1+\eta)\,\| z_1^{\mathrm{T}}G(\sigma)z_2 \|}{\| s \|^2 + \delta_n} \\ & - (\hat{\Omega} + \hat{W}^{\mathrm{T}}\phi)\mathrm{sign}(s)] + s^{\mathrm{T}}[-\varepsilon_3 s - \frac{\varepsilon_4}{N(s)}\mathrm{sign}(s)] \\ & + \xi^{\mathrm{T}}\dot{\xi} - \frac{\tilde{\Omega}\dot{\hat{\Omega}}}{\theta_1} - \frac{\mathrm{tr}(\tilde{W}^{\mathrm{T}}\dot{\hat{W}})}{\theta_2} + \frac{\eta\dot{\eta}}{\theta_3} \end{aligned} \tag{79}$$

Substituting (73) into the above equation (79), it follows that

$$\begin{aligned} \dot{V}_3 \le {} & z_1^{\mathrm{T}}G(\sigma)z_2 + s^{\mathrm{T}}[J^{-1}D\hat{E}\Theta(u_c) + J^{-1}D\tilde{E}\mathrm{sat}(u_c) \\ & - L_1\xi + J^{-1}D\tilde{u}_f + J^{-1}d + k_2\delta(-cz_1 + G(\sigma)z_2) \\ & - \frac{s(1+\eta)}{\| s \|^2 + \delta_n} \| z_1^{\mathrm{T}}G(\sigma)z_2 \| - J^{-1}(\hat{W}^{\mathrm{T}}\phi + \epsilon_\theta)] \\ & - s^{\mathrm{T}}[\varepsilon_3 s + \frac{\varepsilon_4}{N(s)}\mathrm{sign}(s)] - L_2 \| \xi \|^2 + \| J^{-1}D \|^2 \\ & \times \| \hat{E} \|^2 \, \| \Theta(u_c) \|^2 - \hat{\Omega}\mathrm{sign}(s) - \xi^{\mathrm{T}}J^{-1}D\hat{E}\Theta(u_c) \\ & - \frac{\tilde{\Omega}\dot{\hat{\Omega}}}{\theta_1} - \frac{\mathrm{tr}(\tilde{W}^{\mathrm{T}}\dot{\hat{W}})}{\theta_2} + \frac{\eta\dot{\eta}}{\theta_3} \end{aligned} \tag{80}$$

The essence of auxiliary system (73) is same as the auxiliary system (39) in Section 4.2. Therefore, the proof is similar with Section 4.2, it can be attained

$$\begin{aligned} \dot{V}_3 \le {} & z_1^{\mathrm{T}}G(\sigma)z_2 + s^{\mathrm{T}}[J^{-1}D\tilde{E}\mathrm{sat}(u_c) + J^{-1}D\tilde{u}_f \\ & + J^{-1}d + k_2\delta(-cz_1 + G(\sigma)z_2) - \hat{\Omega}\mathrm{sign}(s) \\ & - \frac{s(1+\eta)}{\| s \|^2 + \delta_n} \| z_1^{\mathrm{T}}G(\sigma)z_2 \| - J^{-1}(\hat{W}^{\mathrm{T}}\phi + \epsilon_\theta)] \\ & + s^{\mathrm{T}}\Big[(I_3 - \varepsilon_3)s - \frac{\varepsilon_4}{N(s)}\mathrm{sign}(s)\Big] - \Big(L_2 - \frac{L_1^2}{2} - \frac{1}{2}\Big) \\ & \times \| \xi \|^2 - \frac{\tilde{\Omega}\dot{\hat{\Omega}}}{\theta_1} - \frac{\mathrm{tr}(\tilde{W}^{\mathrm{T}}\dot{\hat{W}})}{\theta_2} + \frac{\eta\dot{\eta}}{\theta_3} \end{aligned} \tag{81}$$

According to $L_2 - (L_1^2/2) - (1/2) > 0$ and (74)–(76), one has

$$\begin{aligned} \dot{V}_3 \le {} & \| s \| \; \| J^{-1}D(\tilde{E}\Theta(u_c) + \tilde{u}_f) + k_2\delta(-cz_1 + G(\sigma)z_2) \| \\ & - \| s \| \hat{\Omega} + s^{\mathrm{T}}J^{-1}[d(t) - \hat{W}^{\mathrm{T}}\phi - \epsilon_\theta] + s^{\mathrm{T}}(I_3 - \varepsilon_3) \\ & \times s - s^{\mathrm{T}}\frac{\varepsilon_4}{N(s)}\mathrm{sign}(s) - \tilde{\Omega} \| s \| - \mathrm{tr}(\tilde{W}^{\mathrm{T}}\phi s^{\mathrm{T}}J^{-1}) \end{aligned} \tag{82}$$

Then by using Lemma 1 and the similar proof of Theorem 2, it follows from (82) that

$$\begin{aligned} \dot{V}_3 \le {} & -2\lambda_{\min}(\varepsilon_3 - I_3)\Big(1 - \frac{\Upsilon}{V_3}\Big)V_3 \\ & -2^{1/2}\lambda_{\min}\Big(\frac{\varepsilon_4}{N(s)}\Big) \times \Big(1 - \frac{\Upsilon}{V_3}\Big)^{1/2} V_3^{1/2} \end{aligned} \tag{83}$$

where

$$\begin{aligned} \Upsilon = {} & V_0 + V_2 + (1/2)\xi^{\mathrm{T}}\xi + (1/2\theta_1)\tilde{\Omega}^2 \\ & + (1/2\theta_2)\mathrm{tr}(\tilde{W}^{\mathrm{T}}\tilde{W}) + (1/2\theta_3)\eta^2, \\ & (\Upsilon/V_3) < 1, \text{ and } (\Upsilon/V_3)^{1/2} < 1. \end{aligned} \tag{84}$$

Define

$\chi_0 = 1/2$, $\quad \chi_1 = 2\lambda_{\min}(\varepsilon_3 - I_3)\Big(1 - \dfrac{\Upsilon}{V_3}\Big)$ and $\quad \chi_2 = 2^{1/2}\lambda_{\min}\Big(\dfrac{\varepsilon_4}{N(s)}\Big)\Big(1 - \dfrac{\Upsilon}{V_3}\Big)^{1/2}$

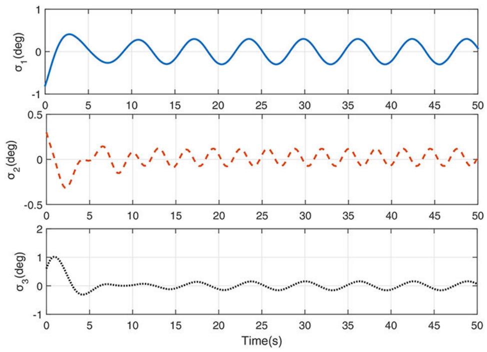


**Fig. 2** *The attitude angle in a healthy case under the tracking scheme in [4]*

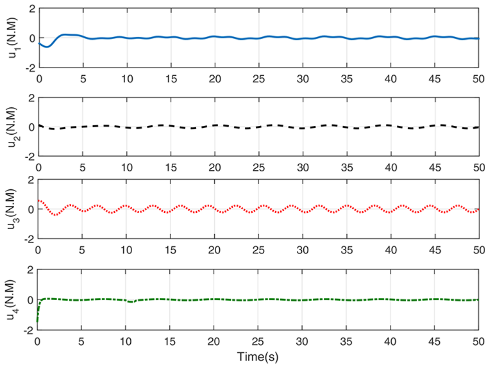


**Fig. 3** *The control input torque in a healthy case under the tracking scheme [4]*

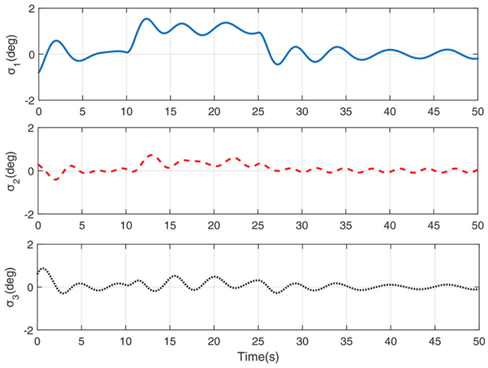


**Fig. 4** *The attitude angle in a faulty case under the tracking scheme in [4]*

then,

$$\dot{V}_3 + \chi_1 V_3 + \chi_2 V_3^{\chi_0} \le 0 \tag{85}$$

and the convergence time is given as:

$$T_2 \le \frac{1}{\chi_1(1-\chi_0)} \ln \frac{\chi_1 V_3^{1-\chi_0}(s_0) + \chi_2}{\chi_2} \tag{86}$$

The proof of the case when $\| \xi \| < \delta_k$ is similar to the above case. Therefore, the faulty closed-loop attitude system can track the desired attitude in finite time. This completes the proof.

*Remark 10:* Compared with the existing fault-tolerant attitude tracking controller design [1, 4, 24, 25], the fault-tolerant attitude tracking controller of this paper compensates the actuator by using accurate information from the FE observer, which greatly improves FTC performance. In addition, a NN algorithm for fault-tolerant attitude tracking controller is presented to compensate for uncertainties and external disturbances to better achieve desired tracking performance. In this case, this paper relaxed the hypothesis that the perturbation has a known upper bound in [1, 4, 24, 25].

*Remark 11:* Although the fast terminal sliding mode surface (FTSM) $S = \dot{X} + K_1 X + K_2 X^{R_1/R_2}(K_1 > 0, K_2 > 0)$ in [27] provides fast convergence when the spacecraft attitude is equilibrium, it cannot avoid singularity. In [21], an integral sliding mode fault-tolerant controller is proposed to deal with faults with matched uncertainties, unmatched uncertainties, and input saturation, but its convergence rate is slow and does not achieve attitude tracking in av finite time. Compared with the above references, this paper designed a fast NTSM surface as shown in (63), which ensures that the convergence rate is faster than the traditional TSM and avoid the singularity. Therefore, it can improve the transient performance of the rigid spacecraft attitude system.

## 5 Numerical simulation

To verify the effectiveness and performance of the proposed FTC strategy, we select a finite time fault-tolerant attitude tracking controller (71) as numerical simulation. The physical parameters of the rigid spacecraft attitude system are listed in this section.

$$J = \begin{bmatrix} 30 & 5.3 & 6.4 \\ 5.3 & 27 & 10 \\ 6.4 & 10 & 19 \end{bmatrix} \mathrm{kg \cdot m^2} \quad D = \begin{bmatrix} 1 & 0 & 0 & \frac{1}{\sqrt{3}} \\ 0 & 1 & 0 & \frac{1}{\sqrt{3}} \\ 0 & 0 & 1 & \frac{1}{\sqrt{3}} \end{bmatrix}$$

The inertial uncertainty $\triangle J$ is proposed as

$$\triangle J = \mathrm{diag}\{\sin(0.1t), 2\sin(0.2t), \sin(0.3t)\}$$

and the external disturbances vector is chosen as $T_d = 0.1[\sin(t) + 1, 3\sin(t) + 1, 5\sin(t) + 1]^{\mathrm{T}} \mathrm{N \cdot m}$. The initial attitude of the rigid spacecraft is $\sigma(0) = [-0.8, 0.3, 0.5]^{\mathrm{T}}$, with an initial angular velocity of $\omega(0) = [0.5, -0.6, 0.4]^{\mathrm{T}}$. The desired attitude angle is selected as $\sigma_d = [0.3\sin(t); 0.1\sin(2t); 0.2\sin(t)]^{\mathrm{T}}$.

In this article, it is assumed that there are four RWs used for the spacecraft attitude control. The actuator saturation is addressed in the position, namely, the maximum torque of each actuator is constrained to be the value of 5.0N $\cdot$ m. The actuator fault scenario used in the simulation is described as follows. The first RW decreases 50% control effectiveness after $t = 10$ s, the second RW occurs a bias fault $u_{f2} = 0.06\,\mathrm{N \cdot m}$ after $t = 10$ s, the third RW and the fourth RW work normal.

The observer gains were ultimately chosen as $\Lambda = 5\mathrm{diag}\{1, 1, 1\}$, $\gamma_1 = 0.5$, $\gamma_2 = 1$ and $\gamma_3 = 0.1$. As for the finite time attitude tracking controller part, the parameters are selected as $k_1 = k_2 = \mathrm{diag}[0.6, 0.6, 0.6]$, $\varepsilon_3 = \varepsilon_4 = 1.5\mathrm{diag}\{1, 1, 1\}$, $\alpha = 20$, $P_0 = 1$, $\hat{W}(0) = \mathrm{diag}[1, 1, 1]$, $L_1 = 0.5$, $L_2 = 1$, $\delta_m = 0.75$, $\delta_n = 0.001$, $\theta_1 = 0.1$, $\theta_2 = 0.3$, and $\theta_3 = 0.01$.

We first present the simulation results. Fig. 2 gives the curves of attitude angle in [4] for the healthy closed-loop attitude system of rigid spacecraft, and the input torque curve is depicted in Fig. 3. It can be seen that the closed-loop attitude system in actuator fault free case has satisfactory tracking performance. Fig. 4 gives the curves of attitude angle under the actuator faults in [4]. Fig. 5 gives the actual control input torque curve under actuator faults. Fig. 6 portrays given the estimation of the actuator loss of effectiveness fault $e_1$ and the estimation of additive bias fault effect of $u_{f2}$, it illustrates that the proposed adaptive sliding mode observer can successfully estimate the actuator faults. Fig. 7 depicts curves of attitude angle by using the NFTSM control law and illustrates that the control input is maintained within the defined constraint $|u_{ci}| < 2\,\mathrm{N \cdot m}$. NFTSM control law successfully eliminates the fault effect and ensures the tracking performance in finite time. Fig. 8 shows the attitude control performance with the application of the FTC in the given an actuator loss of effectiveness fault, a bias

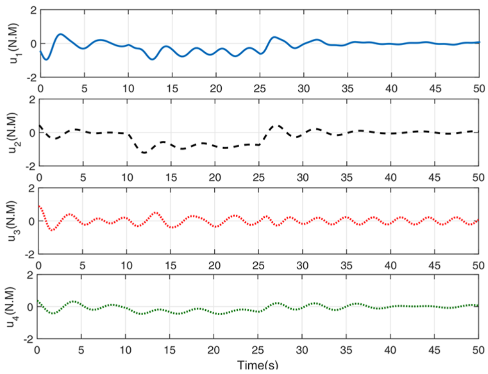


**Fig. 5** *The control input torque in a faulty case under the tracking scheme in [4]*

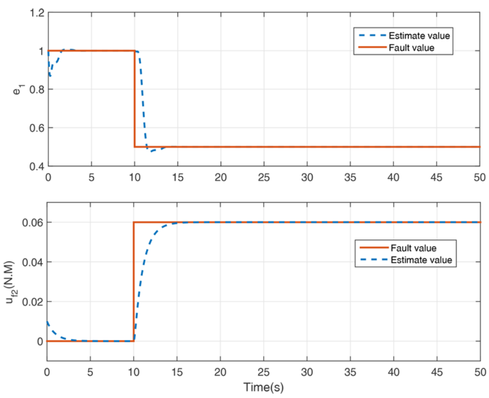


**Fig. 6** *The actual actuator faults and its estimated value in this paper*

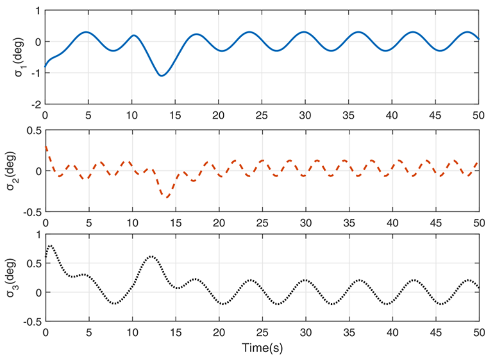


**Fig. 7** *The attitude angle using integrated FE/FTC strategy in this paper*

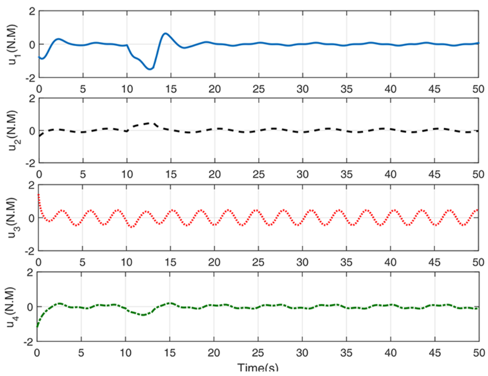


**Fig. 8** *The actual control input torque using integrated FE/FTC strategy in this paper*

fault and actuator saturation. It can see clearly that the proposed finite time fault-tolerant attitude tracking controller (67) has accomplished the attitude tracking manoeuver.

Table 1 presents the results on the comparison of the FTC law in [4] with NFTSM control law in this paper. As mentioned in Table 1 and illustrated in Figs. 2–5, the FTC law guarantees the steady precision in 0.008, with the convergence time being 36 s. The analysis of the comparison results shows that the NFTSM control laws in this paper need less time to track the desired attitude than FTC laws [4] and has higher steady precision.

**Table 1** The control performance comparisons under different control schemes

| FTC scheme | FTC in [4] | FE/FTC in this paper |
|---|---|---|
| $\sigma_1$ tracking accuracy, deg | $\pm 0.008$ | $\pm 1.3 \times 10^{-4}$ |
| $\sigma_2$ tracking accuracy, deg | $\pm 0.006$ | $\pm 3.2 \times 10^{-4}$ |
| $\sigma_3$ tracking accuracy, deg | $\pm 0.005$ | $\pm 8.5 \times 10^{-4}$ |
| stabilisation time, s | 36 s | 20 s |

## 6 Conclusion

In this study, an integrated FE/FTC strategy is developed for a rigid spacecraft attitude system with inertia uncertainties, external disturbances, input saturation and different type multiple actuator faults. The BF is used to eliminate the effects of inertia uncertainties and disturbances in sliding mode FE observer. Based on the FE information, by using NFTSM techniques and introducing a dynamic auxiliary system, a finite time fault-tolerant attitude stabilisation controller and a finite time fault-tolerant attitude tracking controller are designed to accurately compensate for the multiple actuator faults and avoid actuator saturation. Furthermore, to achieve the desirable FTC performance, a NN algorithm is introduced into two fault-tolerant controllers, such that no longer require the prior knowledge of uncertainties and external disturbances. In addition, the integrated FE/FTC strategy in this paper has a simple structure and be easily implemented on other uncertain non-linear systems such as robotic manipulator, aircraft etc.


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