Analytical Calculation Method of Reliability Sensitivity Indexes for Distribution Systems Based on Fault Incidence Matrix

Tianyu Zhang, Chengshan Wang, Fengzhang Luo, Peng Li, and Liangzhong Yao

***Abstract*—An analytical calculation method for the reliability sensitivity indexes of distribution systems is proposed to explicitly quantify the impact of various influence factors on system reliability. Firstly, the analytical calculation formulas for the reliability indexes of distribution systems are derived based on the fault incidence matrix (FIM). Secondly, the factors that affect system reliability are divided into two categories: quantifiable parameter factors and non-quantifiable network structure factors. The sensitivity indexes for the quantifiable parameter factors are derived using the direct partial derivation of the reliability calculation formulas. The sensitivity indexes for the non-quantifiable network structure factors are derived using the transformation of FIMs. Finally, the accuracy and efficiency of the proposed sensitivity calculation method are verified by applying them to an IEEE 6-bus RBTS system. This paper sums up the factors that influence system reliability in detail and gives the explicit analytical calculation method for the sensitivity of each factor. Repetitive calculation of the reliability index can be avoided during the sensitivity analysis. The bottleneck that affects the reliability level of distribution systems can be identified efficiently, and valuable information and guidance can be provided to enhance the reliability of distribution systems.**

***Index Terms*—Distribution system reliability, fault incidence matrix, reliability sensitivity, analytical calculation.**

I. INTRODUCTION

THE reliability level of a distribution system has a significant impact on customers [1], [2]. With the development of electricity market, the continuity of service has become one of the most important competitive factors for power grid operators. Therefore, it is important to accurately identify the influcing factors of system reliability and enhance the reliability efficiency [3].

Sensitivity analysis is a necessary step to quantify the impact of each component's failure event on system reliability and identify the weak link of the system, which can provide valuable information for reliability enhancement. Currently, sensitivity calculation methods can be classified into two categories: the finite difference method or perturbation method [4] and the partial derivation method [5]. In the finite difference method, one small perturbation is added to one influence factor during each calculation process of the indexes for system reliability. After this analyzed factor is changed several times, the corresponding sensitivity can be obtained by comparing the obtained system reliability indexes. For example, in [6] and [7], iterative micro-adjustments are made to the failure rate of a component, after which the system reliability index is calculated. After several reliability calculations and index comparisons, the failure rate sensitivity of this component is obtained. The finite difference method has no requirement for an explicit expression or analytical relationship between the component parameter and whole-system reliability indexes. Only several iterations of parameter changes and reliability index calculations are required. However, as the number of parameters increases, the repetitive calculation will become time-consuming. Another disadvantage of this method is the lack of adaptability to changes in the parameters. The sensitivity result of one parameter is only valid if the other parameters remain unchanged. If the other parameters change, the sensitivity result is invalid and should be re-calculated [8].

The partial derivation method is based on an explicit calculation formula for a system reliability index. The sensitivity of one parameter can be directly deduced using partial derivation despite changes in other parameters. Reference [9] presents the explicit calculation formula for reliability indexes of transmission systems. Reliability sensitivity indexes are then calculated using partial derivation with respect to the parameters such as the component failure rate, the fault repair time, the power line capacity, and the generator capacity. Reference [10] establishes a fault tree for high-voltage direct current (HVDC) transmission systems and the probability sensitivity with respect to the fault parameters of DC com-

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Manuscript received: November 26, 2018; accepted: October 11, 2019. Date of CrossCheck: October 11, 2019. Date of online publication: March 9, 2020.

This work was supported in part by the National Key Research and Development Program of China (No. 2016YFB0900100), in part by the National Natural Science Foundation of China (No. 51977140, No. U1866207, No. 51207101), in part by the Natural Science Foundation of Tianjin (No. 19JCYBJC21300), and in part by the Science and Technology Projects of China Southern Power Grid (No. 060100KK52170118).

T. Zhang, C. Wang, F. Luo (corresponding author), and P. Li are with the Key Laboratory of Smart Grid of Ministry of Education, Tianjin University, Tianjin 300072, China, and T. Zhang is also with State Grid Tianjin Electric Power Company Economic and Technological Research Institute, Tianjin 300171, China (e-mail: zhangtianyu@tju.edu.cn; cswang@tju.edu.cn; luofengzhang@tju.edu.cn; lip@tju.edu.cn).

L. Z. Yao is with China Electric Power Research Institute, Beijing 100192, China (e-mail: yaoliangzhong@epri.sgcc.com.cn).

DOI: 10.35833/MPCE.2018.000750

ponents is calculated using partial derivation. Reference [11] takes the loop in the transmission system as the basic structural unit, and the importance sensitivity of each component in one loop is calculated using the Birnbaum method.

Currently, the research on the sensitivity index calculation based on the derivation operation focuses only on transmission systems. No partial derivation method exists for distribution systems. The reason is that the network structure, the operation mode, the protection practice, the load transfer, and the fault restore process for distribution systems are entirely different from those of transmission systems. Therefore, it is unrealistic to apply the sensitivity calculation method for transmission systems directly to distribution systems.

The analytical formula for the reliability indexes of distribution systems is the premise for the sensitivity calculation, and can be derived using the minimum cut set [12] or the path set [13] methods. But the former non-intersect operation will become tedious with increasing network size. Reference [14] presents a network-equivalent method to analytically calculate the reliability indexes. However, the process of the equivalent operation of the upper and downward components in this method cannot be used to determine the impact of each failure component on the reliability indexes. Currently, there are no partial derivation method targeting reliability sensitivity analysis for distribution systems.

Apart from the lack of an explicit analytical formula for the simple sensitivity analysis for distribution systems, another obstacle is the sensitivity analysis for non-quantifiable network structure factors such as the position of the disconnecting switch [15], the tie line [16], and the circuit breaker. The sensitivity of these factors cannot be obtained using the derivation operation, which increases the difficulty of the explicit sensitivity analysis. The research has already been done on calculating the sensitivity of distribution system. Reference [17] calculates the component sensitivity based on simulation results rather than an analytical formula. Reference [18] calculates the failure rate and repair duration sensitivity analytically, but does not consider the network structure. Reference [19] establishes a Bayesian network model to provide the probability ratio of each component failure event on the whole-system outages without providing the quantified sensitivity magnitude.

To sum up, a systematic analytical method of reliability sensitivity analysis for distribution systems is required, which includes various quantifiable and non-quantifiable factors. Therefore, in this paper, an analytical calculation method for reliability sensitivity based on the fault incidence matrix (FIM) is proposed to explicitly quantify the impact of various influence factors on the reliability of distribution systems. The remainder part of this paper is organized as follows. A brief review of the analytical reliability calculation method based on the FIM is provided in Section II. The sensitivity derivation method for quantifiable and non-quantifiable parameter factors of a distribution system is presented in Section III. In Section IV, an IEEE 6-bus RBTS system is taken as an example to demonstrate the effectiveness of the proposed sensitivity calculation method. Section V concludes the paper.

II. A Brief Review of Analytical Reliability Calculation Method Based on FIM

An explicit expression of a system reliability index is the basis for achieving the analytical deduction of the reliability sensitivity indexes. The method for calculating sensitivity indexes proposed in this paper is based on the analytical reliability calculation model in [1].

A. Definition of FIM

The impact of a component failure on system load points in a distribution system can be classified into three types [20]:

1) The fault leads to the disconnection of all the power supply paths to the load points, and only when the fault component is repaired can a load be restored.

2) The fault leads to the disconnection of all the power supply paths to the load points, but after fault isolation, a load can be restored from the main power supply.

3) The fault leads to the disconnection of all the power supply paths to the load points, and after the fault isolation, a load can be restored from an alternative supply.

In [1], three types of FIM are defined as $\boldsymbol{F}_A$, $\boldsymbol{F}_B$, and $\boldsymbol{F}_C$ corresponding to three impact types. Taking the distribution system in Fig. 1 as an example, considering the branch fault events, three FIMs are shown in Fig. 2.

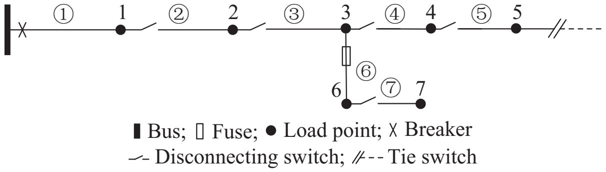


Fig. 1. Schematic diagram of network structure.

$$\boldsymbol{F}_A=\begin{array}{c|ccccccc} & 1 & 2 & 3 & 4 & 5 & 6 & 7\\ \hline ① & 1 & 0 & 0 & 0 & 0 & 0 & 0\\ ② & 0 & 1 & 0 & 0 & 0 & 0 & 0\\ ③ & 0 & 0 & 1 & 0 & 0 & 1 & 1\\ ④ & 0 & 0 & 0 & 1 & 0 & 0 & 0\\ ⑤ & 0 & 0 & 0 & 0 & 1 & 0 & 0\\ ⑥ & 0 & 0 & 0 & 0 & 0 & 1 & 1\\ ⑦ & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array}\quad \boldsymbol{F}_B=\begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7\\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 1 & 0 & 0 & 0 & 0 & 0 & 0\\ 1 & 1 & 0 & 0 & 0 & 0 & 0\\ 1 & 1 & 1 & 0 & 0 & 1 & 1\\ 1 & 1 & 1 & 1 & 0 & 1 & 1\\ 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{array}\quad \boldsymbol{F}_C=\begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7\\ \hline 0 & 1 & 1 & 1 & 1 & 1 & 1\\ 0 & 0 & 1 & 1 & 1 & 1 & 1\\ 0 & 0 & 0 & 1 & 1 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}$$

Fig. 2. Schematic diagram of three types of FIM.

The three FIMs in Fig. 2 describe three types of branch fault influences on load points. For example, if branch ④ fails, the element at row ④ column 4 in $\boldsymbol{F}_A$ is "1", indicating that load point 4 is affected and can be re-powered only when branch ④ is fixed. Similarly, the elements at row ④ columns 1, 2, 3, 6, and 7 in $\boldsymbol{F}_B$ are "1", indicating the load points 1, 2, 3, 6, and 7 can be restored by the main supply after the fault isolation. The element at row ④ column 5 in $\boldsymbol{F}_C$ is "1", indicating that load point 5 can be re-powered by an alternative supply though a tie switch operation after the fault isolation.

B. Analytical Reliability Calculation Model Based on FIM

Based on the FIM, an explicit calculation of the system reliability index can easily be realized. Three indexes, system average interruption frequency index (SAIFI), system average interruption duration index (SAIDI), and expected energy not supplied (EENS) are calculated as:

$$SAIFI=\boldsymbol{\lambda}\left(\boldsymbol{F}_A+\boldsymbol{F}_B+\boldsymbol{F}_C\right)\frac{\boldsymbol{n}^{\mathrm{T}}}{N} \tag{1}$$

$$SAIDI=\left(\boldsymbol{\lambda}\circ\boldsymbol{\mu}\boldsymbol{F}_A+\boldsymbol{\lambda}t_{sw}\boldsymbol{F}_B+\boldsymbol{\lambda}t_{op}\boldsymbol{F}_C\right)\frac{\boldsymbol{n}^{\mathrm{T}}}{N} \tag{2}$$

$$EENS=\left(\boldsymbol{\lambda}\circ\boldsymbol{\mu}\boldsymbol{F}_A+\boldsymbol{\lambda}t_{sw}\boldsymbol{F}_B+\boldsymbol{\lambda}t_{op}\boldsymbol{F}_C\right)\boldsymbol{P}^{\mathrm{T}} \tag{3}$$

where $\boldsymbol{\lambda}$ and $\boldsymbol{\mu}$ are the vectors formed by the branch failure rate and repair duration according to the branch numbers, respectively; $\boldsymbol{n}$ and $\boldsymbol{P}$ are the vectors formed by the number of customers and the load demand according to the load numbers, respectively; N is the total number of customers of the whole system; t_{sw} and t_{op} are the switching times of the disconnecting switches and the tie switches, respectively; and the symbol "∘" represents the Hadamard product operation. The operation rule is the multiplication of the elements in corresponding positions of two matrixes or vectors.

More details about the reliability calculation process of the system can be found in [1].

III. CALCULATION MODEL OF SENSITIVITY INDEX

A. Classification of Influence Factors on System Reliability

The factors that influence the system reliability indexes are classified into two categories:

1) Quantifiable parameter factors such as the failure rate, the repair duration, and the switching time. The sensitivity index of the influence factors can be obtained using partial derivation, which will be described in Section III-B.

2) Non-quantifiable network structure factors such as the installation position of the breaker, the disconnecting switch, and the tie line. The sensitivity index of this kind of influence factors can be obtained using the transformation of the FIMs, which will be described in Section III-C.

B. Sensitivity Index Calculation Method for Quantifiable Parameter Factors

1) Sensitivity Index with Respect to Component Failure Rate

Component failure rates have the impacts on SAIFI, SAIDI, and EENS. Taking the system in Fig. 1 as an example, the sensitivity indexes with respect to the i^{th} branch failure rate λ_i can be calculated using the partial derivation of λ_i as follows:

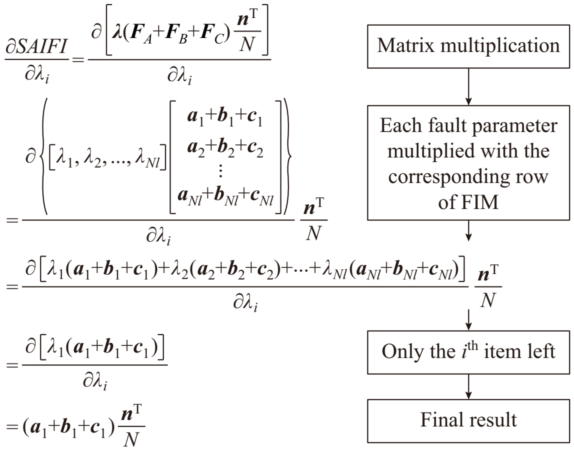
$$\frac{\partial SAIFI}{\partial\lambda_i}=\left(\boldsymbol{a}_i+\boldsymbol{b}_i+\boldsymbol{c}_i\right)\frac{\boldsymbol{n}^{\mathrm{T}}}{N} \tag{4}$$

$$\frac{\partial SAIDI}{\partial\lambda_i}=\left(\mu_i\boldsymbol{a}_i+t_{sw}\boldsymbol{b}_i+t_{op}\boldsymbol{c}_i\right)\frac{\boldsymbol{n}^{\mathrm{T}}}{N} \tag{5}$$

$$\frac{\partial EENS}{\partial\lambda_i}=\left(\mu_i\boldsymbol{a}_i+t_{sw}\boldsymbol{b}_i+t_{op}\boldsymbol{c}_i\right)\boldsymbol{P}^{\mathrm{T}} \tag{6}$$

where $\boldsymbol{a}_i$, $\boldsymbol{b}_i$, $\boldsymbol{c}_i$ are the i^{th} row in $\boldsymbol{F}_A$, $\boldsymbol{F}_B$, and $\boldsymbol{F}_C$, respectively. The specific deduction process for (4) is shown in Fig. 3.

The deduction of the sensitivity of SAIDI and EENS with respect to the failure rate λ_i is similar to the process in Fig. 3. Equations (4)-(6) describe the simple and intuitionistic expressions for the sensitivity index with respect to the failure rate. Thus, repetitive calculation is avoided when the system parameters change.



$$\frac{\partial SAIFI}{\partial\lambda_i}=\frac{\partial\left[\lambda(\boldsymbol{F}_A+\boldsymbol{F}_B+\boldsymbol{F}_C)\frac{\boldsymbol{n}^{\mathrm{T}}}{N}\right]}{\partial\lambda_i}$$

$$=\frac{\partial\left\{\left[\lambda_1,\lambda_2,\ldots,\lambda_{NI}\right]\begin{bmatrix}\boldsymbol{a}_1+\boldsymbol{b}_1+\boldsymbol{c}_1\\ \boldsymbol{a}_2+\boldsymbol{b}_2+\boldsymbol{c}_2\\ \vdots\\ \boldsymbol{a}_{NI}+\boldsymbol{b}_{NI}+\boldsymbol{c}_{NI}\end{bmatrix}\right\}}{\partial\lambda_i}\frac{\boldsymbol{n}^{\mathrm{T}}}{N}$$

$$=\frac{\partial\left[\lambda_1(\boldsymbol{a}_1+\boldsymbol{b}_1+\boldsymbol{c}_1)+\lambda_2(\boldsymbol{a}_2+\boldsymbol{b}_2+\boldsymbol{c}_2)+\ldots+\lambda_{NI}(\boldsymbol{a}_{NI}+\boldsymbol{b}_{NI}+\boldsymbol{c}_{NI})\right]}{\partial\lambda_i}\frac{\boldsymbol{n}^{\mathrm{T}}}{N}$$

$$=\frac{\partial\left[\lambda_1(\boldsymbol{a}_1+\boldsymbol{b}_1+\boldsymbol{c}_1)\right]}{\partial\lambda_i}$$

$$=(\boldsymbol{a}_1+\boldsymbol{b}_1+\boldsymbol{c}_1)\frac{\boldsymbol{n}^{\mathrm{T}}}{N}$$

Fig. 3. Derivation of sensitivity of SAIFI with respect to failure rate parameters.

2) Sensitivity Index with Respect to Component Repair Duration

The change of the component repair duration influences only SAIDI and EENS. Taking the system in Fig. 1 as an example, the sensitivity indexes with respect to the i^{th} branch repair duration μ_i can be calculated using the partial derivation of μ_i as:

$$\frac{\partial SAIDI}{\partial\mu_i}=\lambda_i\boldsymbol{a}_i\frac{\boldsymbol{n}^{\mathrm{T}}}{N} \tag{7}$$

$$\frac{\partial EENS}{\partial\mu_i}=\lambda_i\boldsymbol{a}_i\boldsymbol{P}^{\mathrm{T}} \tag{8}$$

We can see that the sensitivity result of μ_i can be easily calculated using (7) and (8) as long as $\boldsymbol{F}_B$ is obtained.

3) Sensitivity Index with Respect to Operation Time of Disconnecting Switch

The function of the disconnecting switch is to isolate the fault component from the non-fault area and can be re-powered after the isolation. Therefore, the operation time of the disconnecting switch has an influence on the restoration time of the non-fault area. The sensitivity indexes with respect to the operation time of disconnecting switch t_{sw} are calculated as follows:

$$\frac{\partial SAIDI}{\partial t_{sw}}=\boldsymbol{\lambda}\boldsymbol{F}_B\frac{\boldsymbol{n}^{\mathrm{T}}}{N} \tag{9}$$

$$\frac{\partial EENS}{\partial t_{sw}}=\boldsymbol{\lambda}\boldsymbol{F}_B\boldsymbol{P}^{\mathrm{T}} \tag{10}$$

After deriving $\boldsymbol{F}_B$, the sensitivity with respect to the operation time of disconnecting switch can be calculated using (9) and (10), which can help a planner or operator assess whether the operation time is the bottleneck in improving system reliability.

4) Sensitivity Index with Respect to Operation Time of Tie Switch

The function of the tie switch is to transfer some of the load points to other feeders to restore a non-fault area. Therefore, the operation time of the tie switching t_{op} has an influence on the system reliability. The sensitivity results can be obtained using the partial derivative calculation as:

$$\frac{\partial SAIDI}{\partial t_{op}}=\boldsymbol{\lambda F}_C\frac{\boldsymbol{n}^{\mathrm{T}}}{N} \tag{11}$$

$$\frac{\partial EENS}{\partial t_{op}}=\boldsymbol{\lambda F}_C\boldsymbol{P}^{\mathrm{T}} \tag{12}$$

After deriving $\boldsymbol{F}_C$, the sensitivity with respect to the operation time of tie switch can be calculated using (11) and (12).

C. Sensitivity Index Calculation Method for Non-quantifiable Network Structure Factors

For non-quantifiable network structure factors, the sensitivity indexes cannot be obtained using the partial derivation operation. For example, if a new disconnecting switch must be installed in an existing distribution network to maximally enhance the system reliability, the best installation position should be decided from a few candidate positions. Traditional practice involves the re-calculation of system reliability indexes each time when a new switch changes its installation position. The process is time-consuming. In this paper, the sensitivity indexes with respect to non-quantifiable influence factors can be obtained using the transformation of the FIMs. In the following subsections, four non-quantifiable influence factors are considered: the sensitivity of a circuit breaker installation position; the sensitivity of a disconnecting switch installation position; the sensitivity of a tie line access position; and the sensitivity of automation transformation of selected manual switches.

1) Sensitivity of a Circuit Breaker Installation Position

Taking the distribution system in Fig. 4(a) as an example, the best position must be decided for an added circuit breaker to maximally improve the system reliability. Hence, the sensitivity of the circuit breaker installation position should be calculated.

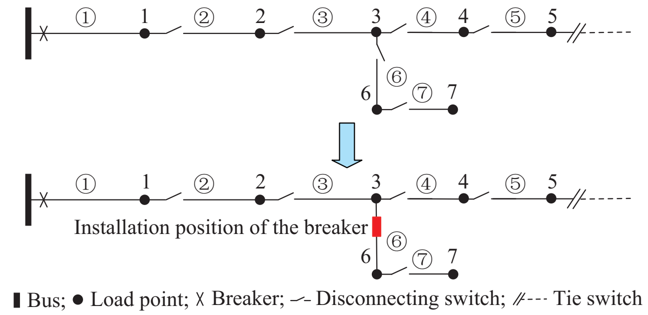


Fig. 4. Installation position of an added circuit breaker.

The installation of the new breaker only changes the elements in $\boldsymbol{F}_B$. The reason is that the load points within a non-fault area will be affected by the fault component without a breaker to isolate the fault component rapidly. This kind of effect belongs to the impact type 2) corresponding to $\boldsymbol{F}_B$. If a new breaker is installed in a certain position, the affected load points in the non-fault area will change, and the elements in $\boldsymbol{F}_B$ change correspondingly. Therefore, as long as $\boldsymbol{F}_B$ is derived considering the new breaker installation, the sensitivity indexes can be calculated easily.

Taking the distribution system in Fig. 4 as an example, the breaker is installed on branch ⑥, $\boldsymbol{F}_B$ and $\boldsymbol{F}_{B'}$ are obtained using the method in [1] before and after the installation, respectively, as shown in Fig. 5.

$$\boldsymbol{F}_B:\ \begin{array}{c|ccccccc} & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \hline ① & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ ② & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ ③ & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ ④ & 1 & 1 & 1 & 0 & 0 & 1 & 1 \\ ⑤ & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ ⑥ & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ ⑦ & 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{array} \qquad \boldsymbol{F}_{B'}:\ \begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{array}$$

Fig. 5. Change of $\boldsymbol{F}_B$ before and after installation of a circuit breaker.

As shown in Fig. 5, the elements in the red solid frame in $\boldsymbol{F}_B$ before the installation change from "1" to "0" after the breaker installation, which indicates that the fault event on branches ⑥ and ⑦ will not influence load points 1-5 because the newly installed breaker will isolate the fault event rapidly. Consequently, the sensitivity indexes with respect to this new breaker installation position at branch ⑥ can be calculated as:

$$\begin{aligned} SAIFI_{br}&=\boldsymbol{\lambda}\left(\boldsymbol{F}_A+\boldsymbol{F}_B+\boldsymbol{F}_C\right)\frac{\boldsymbol{n}^{\mathrm{T}}}{N}-\boldsymbol{\lambda}\left(\boldsymbol{F}_A+\boldsymbol{F}_{B'}+\boldsymbol{F}_C\right)\frac{\boldsymbol{n}^{\mathrm{T}}}{N}=\\ &\boldsymbol{\lambda}\left(\boldsymbol{F}_B-\boldsymbol{F}_{B'}\right)\frac{\boldsymbol{n}^{\mathrm{T}}}{N} \end{aligned} \tag{13}$$

Equation (13) provides the specific deduction process of the sensitivity of the circuit breaker installation position. $\boldsymbol{F}_B$ and $\boldsymbol{F}_{B'}$ represent before and after the installation of the new breaker, respectively. Equation (13) calculates the SAIFI improvement arising from this newly installed breaker. Similarly, SAIDI and EENS sensitivities with respect to the breaker installation position can be calculated as:

$$SAIDI_{br}=\boldsymbol{\lambda}t_{sw}\left(\boldsymbol{F}_B-\boldsymbol{F}_{B'}\right)\frac{\boldsymbol{n}^{\mathrm{T}}}{N} \tag{14}$$

$$EENS_{br}=\boldsymbol{\lambda}t_{sw}\left(\boldsymbol{F}_B-\boldsymbol{F}_{B'}\right)\boldsymbol{P}^{\mathrm{T}} \tag{15}$$

Equations (13)-(15) describe the explicit expression of the sensitivity of breaker installation position, avoiding the calculation process of redundant reliability index.

2) Sensitivity of a Disconnecting Installation Position of Switch

Taking the distribution system in Fig. 6 as an example, the best position must be decided for an added disconnecting switch to maximally improve the system reliability.

The installation of the new disconnecting switch will change the elements of all three FIMs. The reason is that the non-fault area will be interrupted without a switch to isolate the fault event and cannot be re-powered until the fault event is cleared, which belongs to the impact type 1) corresponding to $\boldsymbol{F}_A$. If a new switch is added in a certain position, the non-fault area will be re-powered as soon as the fault is isolated by this switch. Thus, the impact type changes to type 2) or type 3) corresponding to $\boldsymbol{F}_B$ or $\boldsymbol{F}_C$. Therefore, the new switch installation influences the elements of all three FIMs, and the sensitivity results can be calculated accordingly, as long as the three FIMs are derived.

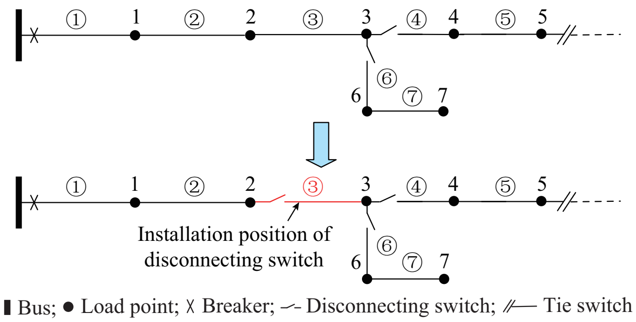


Fig. 6. Position of an added disconnecting switch.

Taking the distribution system in Fig. 6 as an example, the switch is installed at branch ③. The three FIMs, before and after the installation, can be obtained using the method in [1] as shown in Fig. 7.

$$\boldsymbol{F}_A = \begin{array}{c|ccccccc} & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \hline ① & 1 & 1 & 1 & 0 & 0 & 1 & 1 \\ ② & 1 & 1 & 1 & 0 & 0 & 1 & 1 \\ ③ & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ ④ & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ ⑤ & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ ⑥ & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ ⑦ & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{array} \quad \boldsymbol{F}_B = \begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 \end{array} \quad \boldsymbol{F}_C = \begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \hline 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}$$

(a)

$$\boldsymbol{F}_{A'} = \begin{array}{c|ccccccc} & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \hline ① & 1 & 1 & 1 & 0 & 0 & 1 & 1 \\ ② & 1 & 1 & 1 & 0 & 0 & 1 & 1 \\ ③ & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ ④ & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ ⑤ & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ ⑥ & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ ⑦ & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{array} \quad \boldsymbol{F}_{B'} = \begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 \end{array} \quad \boldsymbol{F}_{C'} = \begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \hline 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}$$

(b)

Fig. 7. Change of FIMs. (a) Before installation of a disconnecting switch. (b) After installation of a disconnecting switch.

As shown in Fig. 7(a), before the installation, the elements circled by blue dash-dotted lines in $\boldsymbol{F}_A$ are "1", indicating that the impact of loads 3, 6, and 7 is type 1) with the fault in branches ① and ②. After the installation of the switch on branch ③, these elements are transferred to $\boldsymbol{F}_{C'}$ which indicates the impact type of these load points changes to type 3) because of the isolating function of the newly added switch. Similarly, some "1" elements in $\boldsymbol{F}_A$ are transferred to $\boldsymbol{F}_{B'}$, shown in Fig. 7(b) circled in green dotted lines. After the derivation of the three FIMs, the sensitivity with respect to the switch installation position can be calculated as:

$$SAIDI_{sw} = SAIDI - SAIDI' = \left[\boldsymbol{\lambda}\circ\boldsymbol{\mu}\left(\boldsymbol{F}_A - \boldsymbol{F}_{A'}\right) - \boldsymbol{\lambda} t_{sw}\left(\boldsymbol{F}_B - \boldsymbol{F}_{B'}\right) - \boldsymbol{\lambda} t_{op}\left(\boldsymbol{F}_C - \boldsymbol{F}_{C'}\right)\right]\frac{\boldsymbol{n}^{\mathrm{T}}}{N} \tag{16}$$

$$EENS_{sw} = EENS - EENS' = \left[\boldsymbol{\lambda}\circ\boldsymbol{\mu}\left(\boldsymbol{F}_A - \boldsymbol{F}_{A'}\right) - \boldsymbol{\lambda} t_{sw}\left(\boldsymbol{F}_B - \boldsymbol{F}_{B'}\right) - \boldsymbol{\lambda} t_{op}\left(\boldsymbol{F}_C - \boldsymbol{F}_{C'}\right)\right]\boldsymbol{P}^{\mathrm{T}} \tag{17}$$

In (16) and (17), $\boldsymbol{F}_A$, $\boldsymbol{F}_B$, $\boldsymbol{F}_C$ and $\boldsymbol{F}_{A'}$, $\boldsymbol{F}_{B'}$, $\boldsymbol{F}_{C'}$ represent the three FIMs before and after the installation, respectively. The installation of a disconnecting switch will not influence the SAIFI index because the power outage frequency will not decrease despite the increase of switches. Equations (16) and (17) reveal the function of the disconnecting switch, which is to reduce the outage duration of the interrupted load within the non-fault area. With the help of the sensitivity calculation formulas, valuable guidance can be provided to optimize the position of switch installation and efficiently improve the system reliability.

3) Sensitivity of Access Position of Tie Line

The role of a tie line is to restore the interrupted load points in the non-fault area. Hence, optimizing the layout of a tie line can effectively improve system reliability. Taking the distribution system in Fig. 8 as an example, the best position must be decided for an added tie line to maximally improve the system reliability. The sensitivity of the tie line access position should be calculated.

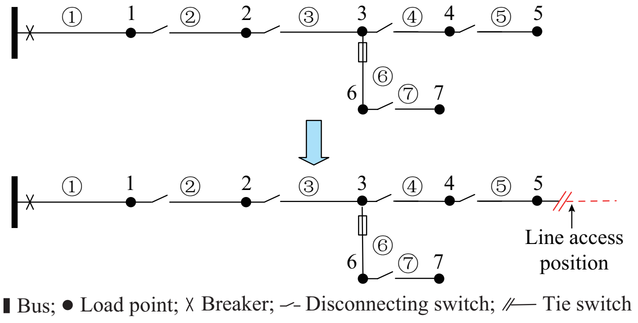


Fig. 8. Position of an added tie line.

The access of a tie line will change the elements in $\boldsymbol{F}_A$ and $\boldsymbol{F}_C$. As shown in Fig. 8, the tie line is accessed at node 5, and the FIMs before and after the tie line access are shown in Fig. 9.

$$\boldsymbol{F}_A = \begin{array}{c|ccccccc} & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \hline ① & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ ② & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ ③ & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ ④ & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ ⑤ & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ ⑥ & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ ⑦ & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array} \Rightarrow \boldsymbol{F}_{A'} = \begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \hline 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array} \quad \boldsymbol{F}_{C'} = \begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \hline 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}$$

Fig. 9. Change of FIMs before and after access of a tie line.

As shown in Fig. 9, some "1" elements in the original $\boldsymbol{F}_A$ (circled by blue dash-dotted lines) are transferred to $\boldsymbol{F}_{C'}$ after a tie line is added, which indicates that these load points can be re-powered by the tie line with some faults instead of waiting for the fault component repair, thus reducing the outage duration. Hence, the sensitivity with respect to this tie line access position can be calculated as:

$$SAIDI_{tie} = \boldsymbol{\lambda} \circ \left(\boldsymbol{\mu} - t_{op}\right) \boldsymbol{F}_{C'} \frac{\boldsymbol{n}^{\mathrm{T}}}{N} \tag{18}$$

$$EENS_{tie} = \boldsymbol{\lambda} \circ \left(\boldsymbol{\mu} - t_{op}\right) \boldsymbol{F}_{C'} \boldsymbol{P}^{\mathrm{T}} \tag{19}$$

These equations describe the function of the tie line, which is to reduce the outage time from the fault repair duration to the load transferring duration. With the help of the sensitivity calculation equations, valuable guidance can be provided to optimize the position of tie line access and efficiently improve the system reliability.

4) Sensitivity of Automation Transformation of Selected Manual Switches

An effective and practical measure to improve system reliability is through the automation transformation of manual switches. After the automation transformation, the switching time will be shortened. Thus, the fault isolation and re-power durations will be greatly shortened for the load points within the non-fault area. Taking the distribution system in Fig. 10 as an example, the sensitivity of the automation transformation selection among all the manual switches will be described in this section.

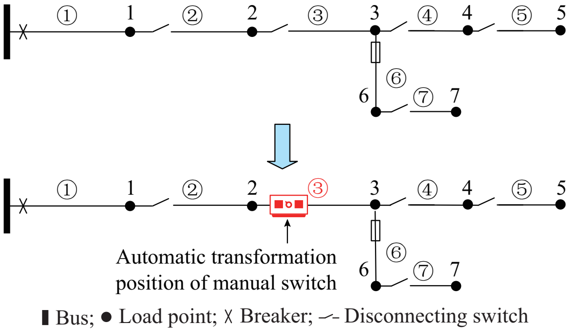


Fig. 10. Position of automatic switch.

The automation transformation will change the elements in $\boldsymbol{F}_B$. As shown in Fig. 10, the manual switch at branch ③ is replaced by an automatic switch. Accordingly, $\boldsymbol{F}_B$ of the original system will be separated into two matrices, $\boldsymbol{F}_{B'}$ and $\boldsymbol{F}_{B''}$, as shown in Fig. 11. The "1" elements in $\boldsymbol{F}_{B''}$ (circled in green dotted lines) indicate the load points that can be re-powered in the automatic switching time. The "1" elements in $\boldsymbol{F}_{B'}$ still represent the load points that can be re-powered in the manual switching time.

$$\begin{array}{c} \boldsymbol{F}_B \\ \begin{array}{c|ccccccc} & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \hline \text{①} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \text{②} & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \text{③} & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ \text{④} & 1 & 1 & 1 & 0 & 0 & 1 & 1 \\ \text{⑤} & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ \text{⑥} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \text{⑦} & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{array} \end{array} \Rightarrow \begin{array}{c} \boldsymbol{F}_{B'} \\ \begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{array} \end{array} \begin{array}{c} \boldsymbol{F}_{B''} \\ \begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \end{array}$$

Fig. 11. Change of FIMs before and after installation of automatic switch.

The matrix $\boldsymbol{F}_{B''}$ reveals the function of the automatic transformation, which is to reduce the restoration duration of the non-fault area. Hence, the sensitivity with respect to the automatic transformation of the manual switches is obtained by the difference between the reliability indexes before and after the automatic transformation.

$$SAIDI_{auto} = \boldsymbol{\lambda} \left(t_{sw} - t_{auto}\right) \boldsymbol{F}_{B''} \frac{\boldsymbol{n}^{\mathrm{T}}}{N} \tag{20}$$

$$EENS_{auto} = \boldsymbol{\lambda} \left(t_{sw} - t_{auto}\right) \boldsymbol{F}_{B''} \boldsymbol{P}^{\mathrm{T}} \tag{21}$$

where t_{auto} represents the automatic switching time.

In summary, the influence factors are classified into two categories: quantifiable parameter factors and non-quantifiable network structure factors. The partial derivation method is used to calculate the sensitivity indexes with respect to the quantifiable parameter factors. The FIM transformation method is used to calculate the sensitivity indexes with respect to non-quantifiable network structure factors.

IV. CASE STUDY

A. Introduction of Case

A practical distribution network in Taiwan Power Company (TPC), China is used to verify the effectiveness of the sensitivity calculation method proposed in Fig. 12. The failure rate of the branches is 0.065 times/year/km [21]. The switch operation time is set as one hour. The load demand for each section of the feeder (F1-F11) is shown in [22].

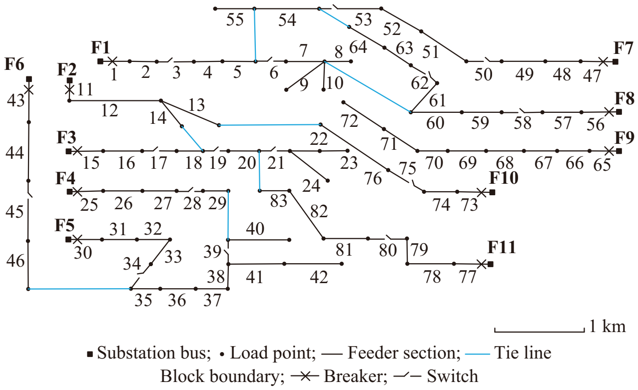


Fig. 12. Diagram of 94-bus distribution network in TPC, China.

B. Sensitivity Sorting of Component Failure Rate

In order to prove the accuracy of our calculation method for reliability sensitivity, the SAIFI sensitivity with respect to $\boldsymbol{\lambda}$ is calculated using both our FIM method and the method in [23]. Table I gives the top five components with the highest SAIFI sensitivity magnitudes with respect to $\boldsymbol{\lambda}$.

The sensitivity indexes calculated by the method in this paper and [23] are the same, proving that the accuracy of our method can be guaranteed. As shown in Table I, the failure rate of the branches 30-40 at F5 have the highest impact on the system SAIFI. The reason is that no breaker is separating these branches. The whole F5 will experience the disturbance if any one of these branches fails. Furthermore, F5 is the longest feeder and has a large number of customers.

Therefore, the sensitivity of the components' failure rate at F5 is the largest. An efficient measure to improve the SAIFI is to reduce the failure rates of these components by updating the equipment or installing breakers.

TABLE I
TOP FIVE SENSITIVITY SORTING

Sensitivity soring	Component	FIM	Ref. [23]
1	Branches 30-40	0.403	0.403
2	Branches 1-10,15-24	0.335	0.335
3	Branches 47-64	0.268	0.268
4	Branches 65-72	0.251	0.251
5	Branches 77-83	0.216	0.216

Tables II and III give the top nine components with the largest SAIDI and EENS sensitivity magnitudes with respect to component failure rate.

TABLE II
TOP NINE COMPONENTS WITH LARGEST SENSITIVITY MAGNITUDE OF SAIDI WITH RESPECT TO COMPONENT FAILURE RATE

Sensitivity sorting	Component	$\partial SAIDI/\partial\lambda$
1	Branches 65-72	2.016
2	Branches 11-14	1.726
3	Branches 30-33	1.534
4	Branches 77-79	1.434
5	Branches 1, 2	1.406
6	Branches 25-27	1.342
7	Branches 47-49	1.325
8	Branches 43, 44	1.163
9	Branches 15, 16	1.089

TABLE III
TOP NINE COMPONENTS WITH LARGEST SENSITIVITY MAGNITUDE OF EENS

Sensitivity sorting	Component	$\partial EENS/\partial\lambda$
1	Branches 11-14	24.735
2	Branches 65-72	23.965
3	Branches 77-79	20.632
4	Branches 1, 2	16.595
5	Branches 25-27	16.298
6	Branches 30-33	15.387
7	Branches 43, 44	14.963
8	Branches 47-49	12.834
9	Branches 80-83	12.587

Table II shows that the components at F2 and F9 have the highest sensitivity to SAIDI and EENS. The reason is that there are no switches along these two feeders. If a fault occurs along these feeders, there is no switch to separate the fault, which leads to an outage of the whole feeder. Therefore, the sensitivity indexes of SAIDI and EENS for these two feeders are the highest, which means that F2 and F9 are the weak links of SAIDI and EENS indexes. An efficient measure to improve SAIDI and EENS indexes is to reduce the failure rates of these components by updating the equipment or installing switches.

C. Sensitivity Sorting of Component Repair Duration

Tables IV and V indicate the components with the highest SAIDI and EENS sensitivity magnitudes with respect to μ.

TABLE IV
TOP NINE COMPONENTS WITH HIGHEST SENSITIVITY MAGNITUDE OF SAIDI WITH RESPECT TO COMPONENT REPAIR DURATION

Sensitivity sorting	Component	$\partial SAIDI/\partial\mu$
1	Branches 65-72	0.659
2	Branches 75, 76	0.657
3	Branches 11-14	0.495
4	Branches 30-33	0.365
5	Branches 77-79	0.328

TABLE V
TOP NINE COMPONENTS WITH HIGHEST SENSITIVITY MAGNITUDE OF EENS WITH RESPECT TO COMPONENT REPAIR DURATION

Sensitivity sorting	Component	$\partial EENS/\partial\mu$
1	Branches 65-72	5.627
2	Branches 11-14	5.394
3	Branches 77-79	3.958
4	Branches 1, 2	2.862
5	Branches 25-27	2.657

These tables show that the components at F9 have the largest sensitivity to SAIDI and EENS. The reason is that no tie line is connected to F9. The outage load points cannot be re-powered until the fault repair is done. Furthermore, F9 has no disconnecting switch, hence the fault range will cover a large non-fault area leading to a high sensitivity magnitude of the component repair duration. An efficient measure to improve the reliability of F9 is to reduce the repair time of the fault components.

D. Sensitivity with Respect to Operation Time of Switches

Table VI shows the sensitivity with respect to the operation time of the disconnecting and tie switches.

TABLE VI
OPERATION TIME SENSITIVITY OF DISCONNECTING SWITCH AND TIE SWITCH

Type	$\partial SAIDI/\partial t$	$\partial EENS/\partial t$
Disconnecting switch	0.252	2.386
Tie switch	0.129	1.164

It can be seen that the operation time of the disconnecting switch has a larger sensitivity on system reliability as there are fourteen disconnecting switches but only eight tie lines between feeders. The tie lines have limited influence on the whole-system reliability indexes. Therefore, reducing the operation time of the disconnecting switches will have a larger effect on reliability improvement.

E. Optimal Installation Position of Disconnecting Switch at F9

Most of the components at F9 have a larger sensitivity magnitude than the components at other feeders. One reason is the lack of a disconnecting switch at F4 to isolate the fault component. And it will lead to a large non-fault area outage. Therefore, installing new disconnecting switches at F4 will be an efficient measure to improve system reliability. The method in Section III-C is used to calculate the switch position sensitivity to find the most effective switch installation positions. The SAIDI and EENS sensitivities of the newly installed switch at candidate position branch 66-72 are calculated as shown in Fig. 13.

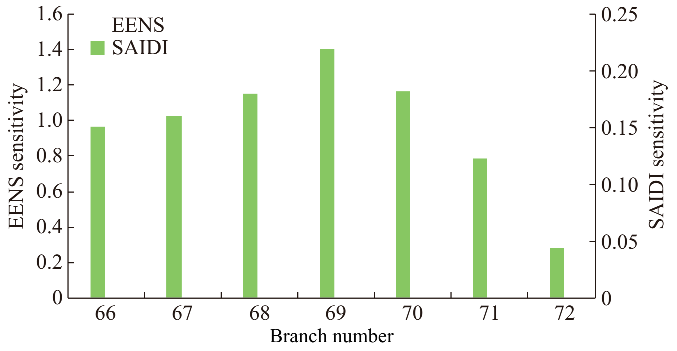


Fig. 13. Sensitivity of installation position of a disconnecting switch.

Figure 13 shows that the optimal installation positions with EENS and SAIDI indexes as the references are different. The reason is that the sensitivity of SAIDI is related to the sum of customer numbers in the system affected by the fault event, but the sensitivity calculation of EENS is related to the sum of power outages in the system. SAIDI and EENS have different concerns about one outage accident. If the focus is placed on the promotion of SAIDI, the best installation location is branch 69. For EENS, the optimal position is branch 71.

F. Optimal Access Position of Tie Line at F9

Adding one tie line to F9 will further enhance the system reliability. The sensitivity of the access position of the tie line is shown in Fig. 14.

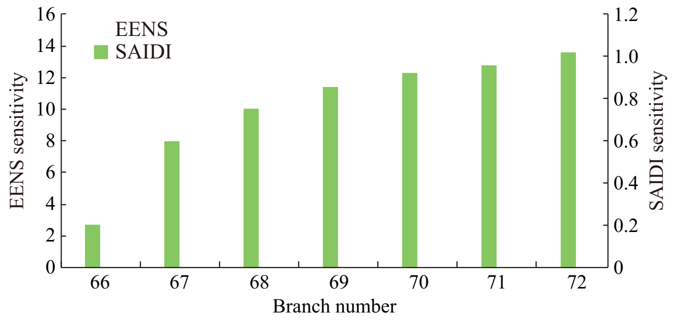


Fig. 14. Sensitivity of access position of a tie switch.

Figure 14 shows that the optimal access position of the tie line is the end of F9, branch 72. The reason is that the load point at the end of a feeder will be affected by the front components and there is quite a number of fault events affecting the endpoints. Therefore, it is more beneficial to access the tie line at the endpoint to transfer a larger range of non-fault areas to other feeders to reduce the non-fault area outage duration. And it will maximally improve the system reliability.

G. Contrast Analysis of Calculation Efficiency

Thanks to the simplification and the simple algebraic operation based on the FIM, the efficiency of the reliability sensitivity calculation is improved. The computation time contrasts among [23], [24] and our FIM method are 5.36 s, 1.27 s and 0.62 s, respectively, which shows that the calculation speed of the FIM method is faster than the methods in [23] and [24]. Both [24] and our method are analytical methods. The method in [23] is a simulation method. Because a repetitive simulation process is required in [23], its calculation efficiency is worse than that of [24] and the FIM method. Our FIM matrix method is much more efficient than that of [24]. The specific proof of the high efficiency of the FIM method is shown in [1].

V. CONCLUSION

This paper sums up the factors that influence the reliability of a distribution network in detail and gives the explicit analytical calculation method for sensitivity index for each type of influence factors. The repetitive calculation of system reliability can be avoided as component reliability parameters vary during the sensitivity analysis. The bottleneck that affects the reliability level of distribution networks can be identified efficiently, and valuable information and guidance can be provided to enhance the reliability for the distribution system using the method proposed in this paper.

An optimal design to enhance the system reliability can also be conducted with the help of the analytical calculation method for sensitivity indexes such as the optimal configuration of the disconnecting switch and tie line described in Sections IV-E and IV-F. In future research, we will focus on further applications of this analytical calculation model for sensitivity in improving the reliability of distribution systems.

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Tianyu Zhang received the Ph. D. degree in electrical engineering from Tianjin University, Tianjin, China, in 2019. He is currently the engineer of State Grid Tianjin Electric Power Company Economic and Technological Research Institute, Tianjin, China. His current research interests include power system planning and reliability analysis.

Chengshan Wang received the Ph.D. degree in electrical engineering from Tianjin University, Tianjin, China, in 1991. He is currently a Professor with the School of Electrical and Information Engineering, Tianjin University. His current research interests include the area of distribution system analysis and planning, distributed generation system and microgrid, and power system security analysis.

Fengzhang Luo received the B.Eng. degree, the M.Eng. degree and the Ph. D. degree from Tianjin University, Tianjin, China, in 2003, 2006 and 2010, respectively. His research interests include (active) distribution system planning and decision-making, artificial intelligence application in power systems.

Peng Li received the B.S. and Ph.D. degrees in electrical engineering from Tianjin University, Tianjin, China, in 2004 and 2010, respectively. He is currently an Associate Professor with the School of Electrical and Information Engineering, Tianjin University. His current research interests include distributed generation system and microgrid, smart distribution system, and transient simulation and analysis.

Liangzhong Yao received the M. Sc and Ph. D. degrees in electrical power system engineering from Tsinghua University, Beijing, China, in 1989 and 1993, respectively. His research interests include renewable energy and smart grid technologies, electric vehicles integration with grid, energy storage, and high voltage direct current and flexible AC transmission systems (FACTS) technologies. He is the Chartered Engineer, the fellow of IET, and a member of CIGRE.