

1. Let  $T_L$  be the temperature and  $p_L$  be the pressure in the left-hand thermometer. Similarly, let  $T_R$  be the temperature and  $p_R$  be the pressure in the right-hand thermometer. According to the problem statement, the pressure is the same in the two thermometers when they are both at the triple point of water. We take this pressure to be  $p_3$ . Writing Eq. 18-5 for each thermometer,

$$T_L = (273.16 \text{ K}) \left( \frac{p_L}{p_3} \right) \quad \text{and} \quad T_R = (273.16 \text{ K}) \left( \frac{p_R}{p_3} \right),$$

we subtract the second equation from the first to obtain

$$T_L - T_R = (273.16 \text{ K}) \left( \frac{p_L - p_R}{p_3} \right).$$

First, we take  $T_L = 373.125 \text{ K}$  (the boiling point of water) and  $T_R = 273.16 \text{ K}$  (the triple point of water). Then,  $p_L - p_R = 120 \text{ torr}$ . We solve

$$373.125 \text{ K} - 273.16 \text{ K} = (273.16 \text{ K}) \left( \frac{120 \text{ torr}}{p_3} \right)$$

for  $p_3$ . The result is  $p_3 = 328 \text{ torr}$ . Now, we let  $T_L = 273.16 \text{ K}$  (the triple point of water) and  $T_R$  be the unknown temperature. The pressure difference is  $p_L - p_R = 90.0 \text{ torr}$ . Solving the equation

$$273.16 \text{ K} - T_R = (273.16 \text{ K}) \left( \frac{90.0 \text{ torr}}{328 \text{ torr}} \right)$$

for the unknown temperature, we obtain  $T_R = 348 \text{ K}$ .

2. We take  $p_3$  to be 80 kPa for both thermometers. According to Fig. 18-6, the nitrogen thermometer gives 373.35 K for the boiling point of water. Use Eq. 18-5 to compute the pressure:

$$p_N = \frac{T}{273.16 \text{ K}} p_3 = \left( \frac{373.35 \text{ K}}{273.16 \text{ K}} \right) (80 \text{ kPa}) = 109.343 \text{ kPa}.$$

The hydrogen thermometer gives 373.16 K for the boiling point of water and

$$p_H = \left( \frac{373.16 \text{ K}}{273.16 \text{ K}} \right) (80 \text{ kPa}) = 109.287 \text{ kPa}.$$

(a) The difference is  $p_N - p_H = 0.056 \text{ kPa} \approx 0.06 \text{ kPa}$ .

(b) The pressure in the nitrogen thermometer is higher than the pressure in the hydrogen thermometer.

3. From Eq. 18-6, we see that the limiting value of the pressure ratio is the same as the absolute temperature ratio:  $(373.15 \text{ K})/(273.16 \text{ K}) = 1.366$ .

4. (a) Let the reading on the Celsius scale be  $x$  and the reading on the Fahrenheit scale be  $y$ . Then  $y = \frac{9}{5}x + 32$ . For  $x = -71^\circ\text{C}$ , this gives  $y = -96^\circ\text{F}$ .

(b) The relationship between  $y$  and  $x$  may be inverted to yield  $x = \frac{5}{9}(y - 32)$ . Thus, for  $y = 134$  we find  $x \approx 56.7$  on the Celsius scale.

5. (a) Let the reading on the Celsius scale be  $x$  and the reading on the Fahrenheit scale be  $y$ . Then  $y = \frac{9}{5}x + 32$ . If we require  $y = 2x$ , then we have

$$2x = \frac{9}{5}x + 32 \Rightarrow x = (5)(32) = 160^\circ\text{C}$$

which yields  $y = 2x = 320^\circ\text{F}$ .

(b) In this case, we require  $y = \frac{1}{2}x$  and find

$$\frac{1}{2}x = \frac{9}{5}x + 32 \Rightarrow x = -\frac{(10)(32)}{13} \approx -24.6^\circ\text{C}$$

which yields  $y = x/2 = -12.3^\circ\text{F}$ .

6. We assume scales X and Y are linearly related in the sense that reading  $x$  is related to reading  $y$  by a linear relationship  $y = mx + b$ . We determine the constants  $m$  and  $b$  by solving the simultaneous equations:

$$-70.00 = m(-125.0) + b$$

$$-30.00 = m(375.0) + b$$

which yield the solutions  $m = 40.00/500.0 = 8.000 \times 10^{-2}$  and  $b = -60.00$ . With these values, we find  $x$  for  $y = 50.00$ :

$$x = \frac{y - b}{m} = \frac{50.00 + 60.00}{0.08000} = 1375^\circ X.$$

7. We assume scale X is a linear scale in the sense that if its reading is  $x$  then it is related to a reading  $y$  on the Kelvin scale by a linear relationship  $y = mx + b$ . We determine the constants  $m$  and  $b$  by solving the simultaneous equations:

$$373.15 = m(-53.5) + b$$

$$273.15 = m(-170) + b$$

which yield the solutions  $m = 100/(170 - 53.5) = 0.858$  and  $b = 419$ . With these values, we find  $x$  for  $y = 340$ :

$$x = \frac{y - b}{m} = \frac{340 - 419}{0.858} = -92.1^\circ X.$$

8. The change in length for the aluminum pole is

$$\Delta \ell = \ell_0 \alpha_{Al} \Delta T = (33 \text{ m})(23 \times 10^{-6} / \text{C}^\circ)(15 \text{ }^\circ\text{C}) = 0.011 \text{ m}.$$

9. Since a volume is the product of three lengths, the change in volume due to a temperature change  $\Delta T$  is given by  $\Delta V = 3\alpha V \Delta T$ , where  $V$  is the original volume and  $\alpha$  is the coefficient of linear expansion. See Eq. 18-11. Since  $V = (4\pi/3)R^3$ , where  $R$  is the original radius of the sphere, then

$$\Delta V = 3\alpha \left( \frac{4\pi}{3} R^3 \right) \Delta T = (23 \times 10^{-6} / \text{C}^\circ) (4\pi) (10 \text{ cm})^3 (100^\circ \text{C}) = 29 \text{ cm}^3.$$

The value for the coefficient of linear expansion is found in Table 18-2.

10. (a) The coefficient of linear expansion  $\alpha$  for the alloy is

$$\alpha = \frac{\Delta L}{L\Delta T} = \frac{10.015 \text{ cm} - 10.000 \text{ cm}}{(10.01 \text{ cm})(100^\circ\text{C} - 20.000^\circ\text{C})} = 1.88 \times 10^{-5} / \text{C}^\circ.$$

Thus, from  $100^\circ\text{C}$  to  $0^\circ\text{C}$  we have

$$\Delta L = L\alpha\Delta T = (10.015 \text{ cm})(1.88 \times 10^{-5} / \text{C}^\circ)(0^\circ\text{C} - 100^\circ\text{C}) = -1.88 \times 10^{-2} \text{ cm}.$$

The length at  $0^\circ\text{C}$  is therefore  $L' = L + \Delta L = (10.015 \text{ cm} - 0.0188 \text{ cm}) = 9.996 \text{ cm}$ .

(b) Let the temperature be  $T_x$ . Then from  $20^\circ\text{C}$  to  $T_x$  we have

$$\Delta L = 10.009 \text{ cm} - 10.000 \text{ cm} = \alpha L\Delta T = (1.88 \times 10^{-5} / \text{C}^\circ)(10.000 \text{ cm})\Delta T,$$

giving  $\Delta T = 48^\circ\text{C}$ . Thus,  $T_x = (20^\circ\text{C} + 48^\circ\text{C}) = 68^\circ\text{C}$ .

11. The new diameter is

$$D = D_0(1 + \alpha_{A1}\Delta T) = (2.725 \text{ cm})[1 + (23 \times 10^{-6} / \text{C}^\circ)(100.0^\circ\text{C} - 0.000^\circ\text{C})] = 2.731 \text{ cm}.$$

12. The increase in the surface area of the brass cube (which has six faces), which had side length is  $L$  at  $20^\circ$ , is

$$\begin{aligned}\Delta A &= 6(L + \Delta L)^2 - 6L^2 \approx 12L\Delta L = 12\alpha_b L^2 \Delta T = 12 (19 \times 10^{-6} / \text{C}^\circ) (30 \text{ cm})^2 (75^\circ \text{C} - 20^\circ \text{C}) \\ &= 11 \text{ cm}^2.\end{aligned}$$

13. The volume at 30°C is given by

$$\begin{aligned} V' &= V(1 + \beta\Delta T) = V(1 + 3\alpha\Delta T) = (50.00 \text{ cm}^3)[1 + 3(29.00 \times 10^{-6} / \text{C}^\circ)(30.00^\circ\text{C} - 60.00^\circ\text{C})] \\ &= 49.87 \text{ cm}^3 \end{aligned}$$

where we have used  $\beta = 3\alpha$ .

14. (a) We use  $\rho = m/V$  and

$$\Delta\rho = \Delta(m/V) = m\Delta(1/V) \approx -m\Delta V/V^2 = -\rho(\Delta V/V) = -3\rho(\Delta L/L).$$

The percent change in density is

$$\frac{\Delta\rho}{\rho} = -3\frac{\Delta L}{L} = -3(0.23\%) = -0.69\%.$$

(b) Since  $\alpha = \Delta L/(L\Delta T) = (0.23 \times 10^{-2}) / (100^\circ\text{C} - 0.0^\circ\text{C}) = 23 \times 10^{-6} / \text{C}^\circ$ , the metal is aluminum (using Table 18-2).

15. If  $V_c$  is the original volume of the cup,  $\alpha_a$  is the coefficient of linear expansion of aluminum, and  $\Delta T$  is the temperature increase, then the change in the volume of the cup is  $\Delta V_c = 3\alpha_a V_c \Delta T$ . See Eq. 18-11. If  $\beta$  is the coefficient of volume expansion for glycerin then the change in the volume of glycerin is  $\Delta V_g = \beta V_c \Delta T$ . Note that the original volume of glycerin is the same as the original volume of the cup. The volume of glycerin that spills is

$$\begin{aligned}\Delta V_g - \Delta V_c &= (\beta - 3\alpha_a) V_c \Delta T = [(5.1 \times 10^{-4} / \text{C}^\circ) - 3(23 \times 10^{-6} / \text{C}^\circ)] (100 \text{ cm}^3) (6.0 \text{ }^\circ\text{C}) \\ &= 0.26 \text{ cm}^3.\end{aligned}$$

16. The change in length for the section of the steel ruler between its 20.05 cm mark and 20.11 cm mark is

$$\Delta L_s = L_s \alpha_s \Delta T = (20.11 \text{ cm})(11 \times 10^{-6} / \text{C}^\circ)(270^\circ\text{C} - 20^\circ\text{C}) = 0.055 \text{ cm}.$$

Thus, the actual change in length for the rod is

$$\Delta L = (20.11 \text{ cm} - 20.05 \text{ cm}) + 0.055 \text{ cm} = 0.115 \text{ cm}.$$

The coefficient of thermal expansion for the material of which the rod is made is then

$$\alpha = \frac{\Delta L}{\Delta T} = \frac{0.115 \text{ cm}}{270^\circ\text{C} - 20^\circ\text{C}} = 23 \times 10^{-6} / \text{C}^\circ.$$

17. After the change in temperature the diameter of the steel rod is  $D_s = D_{s0} + \alpha_s D_{s0} \Delta T$  and the diameter of the brass ring is  $D_b = D_{b0} + \alpha_b D_{b0} \Delta T$ , where  $D_{s0}$  and  $D_{b0}$  are the original diameters,  $\alpha_s$  and  $\alpha_b$  are the coefficients of linear expansion, and  $\Delta T$  is the change in temperature. The rod just fits through the ring if  $D_s = D_b$ . This means

$$D_{s0} + \alpha_s D_{s0} \Delta T = D_{b0} + \alpha_b D_{b0} \Delta T.$$

Therefore,

$$\begin{aligned} \Delta T &= \frac{D_{s0} - D_{b0}}{\alpha_b D_{b0} - \alpha_s D_{s0}} = \frac{3.000 \text{ cm} - 2.992 \text{ cm}}{(19.00 \times 10^{-6} / \text{C}^\circ)(2.992 \text{ cm}) - (11.00 \times 10^{-6} / \text{C}^\circ)(3.000 \text{ cm})} \\ &= 335.0^\circ\text{C}. \end{aligned}$$

The temperature is  $T = (25.00^\circ\text{C} + 335.0^\circ\text{C}) = 360.0^\circ\text{C}$ .

18. (a) Since  $A = \pi D^2/4$ , we have the differential  $dA = 2(\pi D/4)dD$ . Dividing the latter relation by the former, we obtain  $dA/A = 2 dD/D$ . In terms of  $\Delta$ 's, this reads

$$\frac{\Delta A}{A} = 2 \frac{\Delta D}{D} \quad \text{for} \quad \frac{\Delta D}{D} \ll 1.$$

We can think of the factor of 2 as being due to the fact that area is a two-dimensional quantity. Therefore, the area increases by  $2(0.18\%) = 0.36\%$ .

(b) Assuming that all dimensions are allowed to freely expand, then the thickness increases by 0.18%.

(c) The volume (a three-dimensional quantity) increases by  $3(0.18\%) = 0.54\%$ .

(d) The mass does not change.

(e) The coefficient of linear expansion is

$$\alpha = \frac{\Delta D}{D\Delta T} = \frac{0.18 \times 10^{-2}}{100^\circ\text{C}} = 1.8 \times 10^{-5} / \text{C}^\circ.$$

19. The initial volume  $V_0$  of the liquid is  $h_0 A_0$  where  $A_0$  is the initial cross-section area and  $h_0 = 0.64$  m. Its final volume is  $V = hA$  where  $h - h_0$  is what we wish to compute. Now, the area expands according to how the glass expands, which we analyze as follows: Using  $A = \pi r^2$ , we obtain

$$dA = 2\pi r dr = 2\pi r (r\alpha dT) = 2\alpha(\pi r^2)dT = 2\alpha A dT.$$

Therefore, the height is

$$h = \frac{V}{A} = \frac{V_0 (1 + \beta_{\text{liquid}} \Delta T)}{A_0 (1 + 2\alpha_{\text{glass}} \Delta T)}.$$

Thus, with  $V_0/A_0 = h_0$  we obtain

$$h - h_0 = h_0 \left( \frac{1 + \beta_{\text{liquid}} \Delta T}{1 + 2\alpha_{\text{glass}} \Delta T} - 1 \right) = (0.64) \left( \frac{1 + (4 \times 10^{-5})(10^\circ)}{1 + 2(1 \times 10^{-5})(10^\circ)} \right) = 1.3 \times 10^{-4} \text{ m}.$$

20. We divide Eq. 18-9 by the time increment  $\Delta t$  and equate it to the (constant) speed  $v = 100 \times 10^{-9} \text{ m/s}$ .

$$v = \alpha L_0 \frac{\Delta T}{\Delta t}$$

where  $L_0 = 0.0200 \text{ m}$  and  $\alpha = 23 \times 10^{-6}/\text{C}^\circ$ . Thus, we obtain

$$\frac{\Delta T}{\Delta t} = 0.217 \frac{\text{C}^\circ}{\text{s}} = 0.217 \frac{\text{K}}{\text{s}}.$$

21. Consider half the bar. Its original length is  $\ell_0 = L_0/2$  and its length after the temperature increase is  $\ell = \ell_0 + \alpha\ell_0\Delta T$ . The old position of the half-bar, its new position, and the distance  $x$  that one end is displaced form a right triangle, with a hypotenuse of length  $\ell$ , one side of length  $\ell_0$ , and the other side of length  $x$ . The Pythagorean theorem yields

$$x^2 = \ell^2 - \ell_0^2 = \ell_0^2(1 + \alpha\Delta T)^2 - \ell_0^2.$$

Since the change in length is small we may approximate  $(1 + \alpha\Delta T)^2$  by  $1 + 2\alpha\Delta T$ , where the small term  $(\alpha\Delta T)^2$  was neglected. Then,

$$x^2 = \ell_0^2 + 2\ell_0^2\alpha\Delta T - \ell_0^2 = 2\ell_0^2\alpha\Delta T$$

and

$$x = \ell_0\sqrt{2\alpha\Delta T} = \frac{3.77 \text{ m}}{2}\sqrt{2(25\times 10^{-6}/\text{C}^\circ)(32^\circ\text{C})} = 7.5\times 10^{-2} \text{ m}.$$

22. (a) The specific heat is given by  $c = Q/m(T_f - T_i)$ , where  $Q$  is the heat added,  $m$  is the mass of the sample,  $T_i$  is the initial temperature, and  $T_f$  is the final temperature. Thus, recalling that a change in Celsius degrees is equal to the corresponding change on the Kelvin scale,

$$c = \frac{314\text{ J}}{(30.0 \times 10^{-3}\text{ kg})(45.0^\circ\text{C} - 25.0^\circ\text{C})} = 523\text{ J/kg} \cdot \text{K}.$$

(b) The molar specific heat is given by

$$c_m = \frac{Q}{N(T_f - T_i)} = \frac{314\text{ J}}{(0.600\text{ mol})(45.0^\circ\text{C} - 25.0^\circ\text{C})} = 26.2\text{ J/mol} \cdot \text{K}.$$

(c) If  $N$  is the number of moles of the substance and  $M$  is the mass per mole, then  $m = NM$ , so

$$N = \frac{m}{M} = \frac{30.0 \times 10^{-3}\text{ kg}}{50 \times 10^{-3}\text{ kg/mol}} = 0.600\text{ mol}.$$

23. We use  $Q = cm\Delta T$ . The textbook notes that a nutritionist's "Calorie" is equivalent to 1000 cal. The mass  $m$  of the water that must be consumed is

$$m = \frac{Q}{c\Delta T} = \frac{3500 \times 10^3 \text{ cal}}{(1 \text{ g/cal} \cdot \text{C}^\circ)(37.0^\circ\text{C} - 0.0^\circ\text{C})} = 94.6 \times 10^4 \text{ g},$$

which is equivalent to  $9.46 \times 10^4 \text{ g}/(1000 \text{ g/liter}) = 94.6$  liters of water. This is certainly too much to drink in a single day!

24. The amount of water  $m$  that is frozen is

$$m = \frac{Q}{L_F} = \frac{50.2 \text{ kJ}}{333 \text{ kJ/kg}} = 0.151 \text{ kg} = 151 \text{ g}.$$

Therefore the amount of water which remains unfrozen is  $260 \text{ g} - 151 \text{ g} = 109 \text{ g}$ .

25. The melting point of silver is 1235 K, so the temperature of the silver must first be raised from 15.0° C (= 288 K) to 1235 K. This requires heat

$$Q = cm(T_f - T_i) = (236 \text{ J/kg} \cdot \text{K})(0.130 \text{ kg})(1235^\circ\text{C} - 288^\circ\text{C}) = 2.91 \times 10^4 \text{ J}.$$

Now the silver at its melting point must be melted. If  $L_F$  is the heat of fusion for silver this requires

$$Q = mL_F = (0.130 \text{ kg})(105 \times 10^3 \text{ J/kg}) = 1.36 \times 10^4 \text{ J}.$$

The total heat required is  $(2.91 \times 10^4 \text{ J} + 1.36 \times 10^4 \text{ J}) = 4.27 \times 10^4 \text{ J}$ .

26. (a) The water (of mass  $m$ ) releases energy in two steps, first by lowering its temperature from  $20^\circ\text{C}$  to  $0^\circ\text{C}$ , and then by freezing into ice. Thus the total energy transferred from the water to the surroundings is

$$Q = c_w m \Delta T + L_F m = (4190 \text{ J/kg} \cdot \text{K})(125 \text{ kg})(20^\circ\text{C}) + (333 \text{ kJ/kg})(125 \text{ kg}) = 5.2 \times 10^7 \text{ J}.$$

(b) Before all the water freezes, the lowest temperature possible is  $0^\circ\text{C}$ , below which the water must have already turned into ice.

27. The mass  $m = 0.100$  kg of water, with specific heat  $c = 4190$  J/kg·K, is raised from an initial temperature  $T_i = 23^\circ\text{C}$  to its boiling point  $T_f = 100^\circ\text{C}$ . The heat input is given by  $Q = cm(T_f - T_i)$ . This must be the power output of the heater  $P$  multiplied by the time  $t$ ;  $Q = Pt$ . Thus,

$$t = \frac{Q}{P} = \frac{cm(T_f - T_i)}{P} = \frac{(4190 \text{ J/kg} \cdot \text{K})(0.100 \text{ kg})(100^\circ\text{C} - 23^\circ\text{C})}{200 \text{ J/s}} = 160 \text{ s}.$$

28. The work the man has to do to climb to the top of Mt. Everest is given by

$$W = mgy = (73.0 \text{ kg})(9.80 \text{ m/s}^2)(8840 \text{ m}) = 6.32 \times 10^6 \text{ J.}$$

Thus, the amount of butter needed is

$$m = \frac{(6.32 \times 10^6 \text{ J}) \left( \frac{1.00 \text{ cal}}{4.186 \text{ J}} \right)}{6000 \text{ cal/g}} \approx 250 \text{ g.}$$

29. Let the mass of the steam be  $m_s$  and that of the ice be  $m_i$ . Then

$$L_F m_c + c_w m_c (T_f - 0.0^\circ\text{C}) = m_s L_s + m_s c_w (100^\circ\text{C} - T_f),$$

where  $T_f = 50^\circ\text{C}$  is the final temperature. We solve for  $m_s$ :

$$\begin{aligned} m_s &= \frac{L_F m_c + c_w m_c (T_f - 0.0^\circ\text{C})}{L_s + c_w (100^\circ\text{C} - T_f)} = \frac{(79.7 \text{ cal/g})(150 \text{ g}) + (1 \text{ cal/g}\cdot^\circ\text{C})(150 \text{ g})(50^\circ\text{C} - 0.0^\circ\text{C})}{539 \text{ cal/g} + (1 \text{ cal/g}\cdot^\circ\text{C})(100^\circ\text{C} - 50^\circ\text{C})} \\ &= 33 \text{ g}. \end{aligned}$$

30. (a) Using Eq. 18-17, the heat transferred to the water is

$$Q_w = c_w m_w \Delta T + L_v m_s = (1 \text{ cal/g} \cdot \text{C}^\circ)(220 \text{ g})(100^\circ\text{C} - 20.0^\circ\text{C}) + (539 \text{ cal/g})(5.00 \text{ g}) \\ = 20.3 \text{ kcal.}$$

(b) The heat transferred to the bowl is

$$Q_b = c_b m_b \Delta T = (0.0923 \text{ cal/g} \cdot \text{C}^\circ)(150 \text{ g})(100^\circ\text{C} - 20.0^\circ\text{C}) = 1.11 \text{ kcal.}$$

(c) If the original temperature of the cylinder be  $T_i$ , then  $Q_w + Q_b = c_c m_c (T_i - T_f)$ , which leads to

$$T_i = \frac{Q_w + Q_b}{c_c m_c} + T_f = \frac{20.3 \text{ kcal} + 1.11 \text{ kcal}}{(0.0923 \text{ cal/g} \cdot \text{C}^\circ)(300 \text{ g})} + 100^\circ\text{C} = 873^\circ\text{C.}$$

31. We note from Eq. 18-12 that  $1 \text{ Btu} = 252 \text{ cal}$ . The heat relates to the power, and to the temperature change, through  $Q = Pt = cm\Delta T$ . Therefore, the time  $t$  required is

$$t = \frac{cm\Delta T}{P} = \frac{(1000 \text{ cal/kg}\cdot\text{C}^\circ)(40 \text{ gal})(1000 \text{ kg}/264 \text{ gal})(100^\circ\text{F} - 70^\circ\text{F})(5^\circ\text{C}/9^\circ\text{F})}{(2.0 \times 10^5 \text{ Btu/h})(252.0 \text{ cal/Btu})(1 \text{ h}/60 \text{ min})}$$
$$= 3.0 \text{ min.}$$

The metric version proceeds similarly:

$$t = \frac{c\rho V\Delta T}{P} = \frac{(4190 \text{ J/kg}\cdot\text{C}^\circ)(1000 \text{ kg/m}^3)(150 \text{ L})(1 \text{ m}^3/1000 \text{ L})(38^\circ\text{C} - 21^\circ\text{C})}{(59000 \text{ J/s})(60 \text{ s}/1 \text{ min})}$$
$$= 3.0 \text{ min.}$$

32. We note that the heat capacity of sample  $B$  is given by the reciprocal of the slope of the line in Figure 18-32(b) (compare with Eq. 18-14). Since the reciprocal of that slope is  $16/4 = 4 \text{ kJ/kg}\cdot\text{C}^\circ$ , then  $c_B = 4000 \text{ J/kg}\cdot\text{C}^\circ = 4000 \text{ J/kg}\cdot\text{K}$  (since a change in Celsius is equivalent to a change in Kelvins). Now, following the same procedure as shown in Sample Problem 18-4, we find

$$c_A m_A (T_f - T_A) + c_B m_B (T_f - T_B) = 0$$

$$c_A (5.0 \text{ kg})(40^\circ\text{C} - 100^\circ\text{C}) + (4000 \text{ J/kg}\cdot\text{C}^\circ)(1.5 \text{ kg})(40^\circ\text{C} - 20^\circ\text{C}) = 0$$

which leads to  $c_A = 4.0 \times 10^2 \text{ J/kg}\cdot\text{K}$ .

33. The power consumed by the system is

$$P = \left( \frac{1}{20\%} \right) \frac{cm\Delta T}{t} = \left( \frac{1}{20\%} \right) \frac{(4.18 \text{ J/g} \cdot ^\circ\text{C})(200 \times 10^3 \text{ cm}^3)(1 \text{ g/cm}^3)(40^\circ\text{C} - 20^\circ\text{C})}{(1.0 \text{ h})(3600 \text{ s/h})}$$
$$= 2.3 \times 10^4 \text{ W.}$$

The area needed is then  $A = \frac{2.3 \times 10^4 \text{ W}}{700 \text{ W/m}^2} = 33 \text{ m}^2$ .

34. While the sample is in its liquid phase, its temperature change (in absolute values) is  $|\Delta T| = 30^\circ\text{C}$ . Thus, with  $m = 0.40\text{ kg}$ , the absolute value of Eq. 18-14 leads to

$$|Q| = c m |\Delta T| = (3000\text{ J/kg}\cdot^\circ\text{C})(0.40\text{ kg})(30^\circ\text{C}) = 36000\text{ J}.$$

The rate (which is constant) is

$$P = |Q|/t = (36000\text{ J})/(40\text{ min}) = 900\text{ J/min},$$

which is equivalent to 15 Watts.

(a) During the next 30 minutes, a phase change occurs which is described by Eq. 18-16:

$$|Q| = P t = (900\text{ J/min})(30\text{ min}) = 27000\text{ J} = L m.$$

Thus, with  $m = 0.40\text{ kg}$ , we find  $L = 67500\text{ J/kg} \approx 68\text{ kJ/kg}$ .

(b) During the final 20 minutes, the sample is solid and undergoes a temperature change (in absolute values) of  $|\Delta T| = 20^\circ\text{C}$ . Now, the absolute value of Eq. 18-14 leads to

$$c = \frac{|Q|}{m |\Delta T|} = \frac{P t}{m |\Delta T|} = \frac{(900)(20)}{(0.40)(20)} = 2250 \frac{\text{J}}{\text{kg}\cdot^\circ\text{C}} \approx 2.3 \frac{\text{kJ}}{\text{kg}\cdot^\circ\text{C}}.$$

35. We denote the ice with subscript  $I$  and the coffee with  $c$ , respectively. Let the final temperature be  $T_f$ . The heat absorbed by the ice is

$$Q_I = \lambda_F m_I + m_I c_w (T_f - 0^\circ\text{C}),$$

and the heat given away by the coffee is  $|Q_c| = m_w c_w (T_I - T_f)$ . Setting  $Q_I = |Q_c|$ , we solve for  $T_f$ :

$$\begin{aligned} T_f &= \frac{m_w c_w T_I - \lambda_F m_I}{(m_I + m_c) c_w} = \frac{(130\text{ g})(4190\text{ J/kg}\cdot\text{C}^\circ) (80.0^\circ\text{C}) - (333 \times 10^3\text{ J/g})(12.0\text{ g})}{(12.0\text{ g} + 130\text{ g})(4190\text{ J/kg}\cdot\text{C}^\circ)} \\ &= 66.5^\circ\text{C}. \end{aligned}$$

Note that we work in Celsius temperature, which poses no difficulty for the  $\text{J/kg}\cdot\text{K}$  values of specific heat capacity (see Table 18-3) since a change of Kelvin temperature is numerically equal to the corresponding change on the Celsius scale. Therefore, the temperature of the coffee will cool by  $|\Delta T| = 80.0^\circ\text{C} - 66.5^\circ\text{C} = 13.5^\circ\text{C}$ .

36. (a) Eq. 18-14 (in absolute value) gives

$$|Q| = (4190 \text{ J/kg}\cdot^{\circ}\text{C})(0.530 \text{ kg})(40^{\circ}\text{C}) = 88828 \text{ J}.$$

Since  $\frac{dQ}{dt}$  is assumed constant (we will call it  $P$ ) then we have

$$P = \frac{88828 \text{ J}}{40 \text{ min}} = \frac{88828 \text{ J}}{2400 \text{ s}} = 37 \text{ W}.$$

(b) During that same time (used in part (a)) the ice warms by  $20^{\circ}\text{C}$ . Using Table 18-3 and Eq. 18-14 again we have

$$m_{\text{ice}} = \frac{Q}{c_{\text{ice}} \Delta T} = \frac{88828}{(2220)(20^{\circ})} = 2.0 \text{ kg}.$$

(c) To find the ice produced (by freezing the water that has already reached  $0^{\circ}\text{C}$  – so we concerned with the  $40 \text{ min} < t < 60 \text{ min}$  time span), we use Table 18-4 and Eq. 18-16:

$$m_{\text{water becoming ice}} = \frac{Q_{20 \text{ min}}}{L_F} = \frac{44414}{333000} = 0.13 \text{ kg}.$$

37. To accomplish the phase change at  $78^{\circ}\text{C}$ ,

$$Q = L_V m = (879 \text{ kJ/kg}) (0.510 \text{ kg}) = 448.29 \text{ kJ}$$

must be removed. To cool the liquid to  $-114^{\circ}\text{C}$ ,

$$Q = cm|\Delta T| = (2.43 \text{ kJ/kg}\cdot\text{K}) (0.510 \text{ kg}) (192 \text{ K}) = 237.95 \text{ kJ},$$

must be removed. Finally, to accomplish the phase change at  $-114^{\circ}\text{C}$ ,

$$Q = L_F m = (109 \text{ kJ/kg}) (0.510 \text{ kg}) = 55.59 \text{ kJ}$$

must be removed. The grand total of heat removed is therefore  $(448.29 + 237.95 + 55.59)$  kJ = 742 kJ.

38. The heat needed is found by integrating the heat capacity:

$$\begin{aligned} Q &= \int_{T_i}^{T_f} cm \, dT = m \int_{T_i}^{T_f} c \, dT = (2.09) \int_{5.0^\circ\text{C}}^{15.0^\circ\text{C}} (0.20 + 0.14T + 0.023T^2) \, dT \\ &= (2.0)(0.20T + 0.070T^2 + 0.00767T^3) \Big|_{5.0}^{15.0} \text{ (cal)} \\ &= 82 \text{ cal.} \end{aligned}$$

39. We compute with Celsius temperature, which poses no difficulty for the J/kg·K values of specific heat capacity (see Table 18-3) since a change of Kelvin temperature is numerically equal to the corresponding change on the Celsius scale. If the equilibrium temperature is  $T_f$  then the energy absorbed as heat by the ice is

$$Q_I = L_F m_I + c_w m_I (T_f - 0^\circ\text{C}),$$

while the energy transferred as heat from the water is  $Q_w = c_w m_w (T_f - T_i)$ . The system is insulated, so  $Q_w + Q_I = 0$ , and we solve for  $T_f$ :

$$T_f = \frac{c_w m_w T_i - L_F m_I}{(m_I + m_w) c_w}.$$

(a) Now  $T_i = 90^\circ\text{C}$  so

$$T_f = \frac{(4190 \text{ J/kg} \cdot \text{C}^\circ)(0.500 \text{ kg})(90^\circ\text{C}) - (333 \times 10^3 \text{ J/kg})(0.500 \text{ kg})}{(0.500 \text{ kg} + 0.500 \text{ kg})(4190 \text{ J/kg} \cdot \text{C}^\circ)} = 5.3^\circ\text{C}.$$

(b) Since no ice has remained at  $T_f = 5.3^\circ\text{C}$ , we have  $m_f = 0$ .

(c) If we were to use the formula above with  $T_i = 70^\circ\text{C}$ , we would get  $T_f < 0$ , which is impossible. In fact, not all the ice has melted in this case and the equilibrium temperature is  $T_f = 0^\circ\text{C}$ .

(d) The amount of ice that melts is given by

$$m'_I = \frac{c_w m_w (T_i - 0^\circ\text{C})}{L_F} = \frac{(4190 \text{ J/kg} \cdot \text{C}^\circ)(0.500 \text{ kg})(70^\circ\text{C})}{333 \times 10^3 \text{ J/kg}} = 0.440 \text{ kg}.$$

Therefore, the amount of (solid) ice remaining is  $m_f = m_I - m'_I = 500 \text{ g} - 440 \text{ g} = 60.0 \text{ g}$ , and (as mentioned) we have  $T_f = 0^\circ\text{C}$  (because the system is an ice-water mixture in thermal equilibrium).

40. (a) Using Eq. 18-32, we find the rate of energy conducted upward to be

$$P_{\text{cond}} = \frac{Q}{t} = kA \frac{T_H - T_C}{L} = (0.400 \text{ W/m} \cdot ^\circ\text{C})A \frac{5.0 \text{ }^\circ\text{C}}{0.12 \text{ m}} = (16.7A) \text{ W}.$$

Recall that a change in Celsius temperature is numerically equivalent to a change on the Kelvin scale.

(b) The heat of fusion in this process is  $Q = L_F m$ , where  $L_F = 3.33 \times 10^5 \text{ J/kg}$ . Differentiating the expression with respect to  $t$  and equating the result with  $P_{\text{cond}}$ , we have

$$P_{\text{cond}} = \frac{dQ}{dt} = L_F \frac{dm}{dt}.$$

Thus, the rate of mass converted from liquid to ice is

$$\frac{dm}{dt} = \frac{P_{\text{cond}}}{L_F} = \frac{16.7A \text{ W}}{3.33 \times 10^5 \text{ J/kg}} = (5.02 \times 10^{-5} A) \text{ kg/s}.$$

(c) Since  $m = \rho V = \rho Ah$ , differentiating both sides of the expression gives

$$\frac{dm}{dt} = \frac{d}{dt}(\rho Ah) = \rho A \frac{dh}{dt}.$$

Thus, the rate of change of the icicle length is

$$\frac{dh}{dt} = \frac{1}{\rho A} \frac{dm}{dt} = \frac{5.02 \times 10^{-5} \text{ kg/m}^2 \cdot \text{s}}{1000 \text{ kg/m}^3} = 5.02 \times 10^{-8} \text{ m/s}$$

41. (a) We work in Celsius temperature, which poses no difficulty for the J/kg·K values of specific heat capacity (see Table 18-3) since a change of Kelvin temperature is numerically equal to the corresponding change on the Celsius scale. There are three possibilities:

- None of the ice melts and the water-ice system reaches thermal equilibrium at a temperature that is at or below the melting point of ice.
- The system reaches thermal equilibrium at the melting point of ice, with some of the ice melted.
- All of the ice melts and the system reaches thermal equilibrium at a temperature at or above the melting point of ice.

First, suppose that no ice melts. The temperature of the water decreases from  $T_{wi} = 25^\circ\text{C}$  to some final temperature  $T_f$  and the temperature of the ice increases from  $T_{li} = -15^\circ\text{C}$  to  $T_f$ . If  $m_w$  is the mass of the water and  $c_w$  is its specific heat then the water rejects heat

$$|Q| = c_w m_w (T_{wi} - T_f).$$

If  $m_I$  is the mass of the ice and  $c_I$  is its specific heat then the ice absorbs heat

$$Q = c_I m_I (T_f - T_{li}).$$

Since no energy is lost to the environment, these two heats (in absolute value) must be the same. Consequently,

$$c_w m_w (T_{wi} - T_f) = c_I m_I (T_f - T_{li}).$$

The solution for the equilibrium temperature is

$$\begin{aligned} T_f &= \frac{c_w m_w T_{wi} + c_I m_I T_{li}}{c_w m_w + c_I m_I} \\ &= \frac{(4190 \text{ J/kg} \cdot \text{K})(0.200 \text{ kg})(25^\circ\text{C}) + (2220 \text{ J/kg} \cdot \text{K})(0.100 \text{ kg})(-15^\circ\text{C})}{(4190 \text{ J/kg} \cdot \text{K})(0.200 \text{ kg}) + (2220 \text{ J/kg} \cdot \text{K})(0.100 \text{ kg})} \\ &= 16.6^\circ\text{C}. \end{aligned}$$

This is above the melting point of ice, which invalidates our assumption that no ice has melted. That is, the calculation just completed does not take into account the melting of the ice and is in error. Consequently, we start with a new assumption: that the water and ice reach thermal equilibrium at  $T_f = 0^\circ\text{C}$ , with mass  $m$  ( $< m_I$ ) of the ice melted. The magnitude of the heat rejected by the water is

$$|Q| = c_w m_w T_{wi},$$

and the heat absorbed by the ice is

$$Q = c_I m_I (0 - T_{ii}) + mL_F,$$

where  $L_F$  is the heat of fusion for water. The first term is the energy required to warm all the ice from its initial temperature to  $0^\circ\text{C}$  and the second term is the energy required to melt mass  $m$  of the ice. The two heats are equal, so

$$c_W m_W T_{Wi} = -c_I m_I T_{ii} + mL_F.$$

This equation can be solved for the mass  $m$  of ice melted:

$$\begin{aligned} m &= \frac{c_W m_W T_{Wi} + c_I m_I T_{ii}}{L_F} \\ &= \frac{(4190 \text{ J/kg} \cdot \text{K})(0.200 \text{ kg})(25^\circ\text{C}) + (2220 \text{ J/kg} \cdot \text{K})(0.100 \text{ kg})(-15^\circ\text{C})}{333 \times 10^3 \text{ J/kg}} \\ &= 5.3 \times 10^{-2} \text{ kg} = 53 \text{ g}. \end{aligned}$$

Since the total mass of ice present initially was 100 g, there *is* enough ice to bring the water temperature down to  $0^\circ\text{C}$ . This is then the solution: the ice and water reach thermal equilibrium at a temperature of  $0^\circ\text{C}$  with 53 g of ice melted.

(b) Now there is less than 53 g of ice present initially. All the ice melts and the final temperature is above the melting point of ice. The heat rejected by the water is

$$|Q| = c_W m_W (T_{Wi} - T_f)$$

and the heat absorbed by the ice and the water it becomes when it melts is

$$Q = c_I m_I (0 - T_{ii}) + c_W m_I (T_f - 0) + m_I L_F.$$

The first term is the energy required to raise the temperature of the ice to  $0^\circ\text{C}$ , the second term is the energy required to raise the temperature of the melted ice from  $0^\circ\text{C}$  to  $T_f$ , and the third term is the energy required to melt all the ice. Since the two heats are equal,

$$c_W m_W (T_{Wi} - T_f) = c_I m_I (-T_{ii}) + c_W m_I T_f + m_I L_F.$$

The solution for  $T_f$  is

$$T_f = \frac{c_W m_W T_{Wi} + c_I m_I T_{ii} - m_I L_F}{c_W (m_W + m_I)}.$$

Inserting the given values, we obtain  $T_f = 2.5^\circ\text{C}$ .

42. If the ring diameter at  $0.000^\circ\text{C}$  is  $D_{r0}$  then its diameter when the ring and sphere are in thermal equilibrium is

$$D_r = D_{r0} (1 + \alpha_c T_f),$$

where  $T_f$  is the final temperature and  $\alpha_c$  is the coefficient of linear expansion for copper. Similarly, if the sphere diameter at  $T_i$  ( $= 100.0^\circ\text{C}$ ) is  $D_{s0}$  then its diameter at the final temperature is

$$D_s = D_{s0} [1 + \alpha_a (T_f - T_i)],$$

where  $\alpha_a$  is the coefficient of linear expansion for aluminum. At equilibrium the two diameters are equal, so

$$D_{r0}(1 + \alpha_c T_f) = D_{s0}[1 + \alpha_a (T_f - T_i)].$$

The solution for the final temperature is

$$\begin{aligned} T_f &= \frac{D_{r0} - D_{s0} + D_{s0}\alpha_a T_i}{D_{s0}\alpha_a - D_{r0}\alpha_c} \\ &= \frac{2.54000 \text{ cm} - 2.54508 \text{ cm} + (2.54508 \text{ cm})(23 \times 10^{-6}/^\circ\text{C})(100.0^\circ\text{C})}{(2.54508 \text{ cm})(23 \times 10^{-6} / ^\circ\text{C}) - (2.54000 \text{ cm})(17 \times 10^{-6} / ^\circ\text{C})} \\ &= 50.38^\circ\text{C}. \end{aligned}$$

The expansion coefficients are from Table 18-2 of the text. Since the initial temperature of the ring is  $0^\circ\text{C}$ , the heat it absorbs is  $Q = c_c m_r T_f$ , where  $c_c$  is the specific heat of copper and  $m_r$  is the mass of the ring. The heat rejected up by the sphere is

$$|Q| = c_a m_s (T_i - T_f)$$

where  $c_a$  is the specific heat of aluminum and  $m_s$  is the mass of the sphere. Since these two heats are equal,

$$c_c m_r T_f = c_a m_s (T_i - T_f),$$

we use specific heat capacities from the textbook to obtain

$$m_s = \frac{c_c m_r T_f}{c_a (T_i - T_f)} = \frac{(386 \text{ J/kg} \cdot \text{K})(0.0200 \text{ kg})(50.38^\circ\text{C})}{(900 \text{ J/kg} \cdot \text{K})(100^\circ\text{C} - 50.38^\circ\text{C})} = 8.71 \times 10^{-3} \text{ kg}.$$

43. Over a cycle, the internal energy is the same at the beginning and end, so the heat  $Q$  absorbed equals the work done:  $Q = W$ . Over the portion of the cycle from  $A$  to  $B$  the pressure  $p$  is a linear function of the volume  $V$  and we may write

$$p = \frac{10}{3} \text{ Pa} + \left( \frac{20}{3} \text{ Pa/m}^3 \right) V,$$

where the coefficients were chosen so that  $p = 10 \text{ Pa}$  when  $V = 1.0 \text{ m}^3$  and  $p = 30 \text{ Pa}$  when  $V = 4.0 \text{ m}^3$ . The work done by the gas during this portion of the cycle is

$$\begin{aligned} W_{AB} &= \int_1^4 p dV = \int_1^4 \left( \frac{10}{3} + \frac{20}{3} V \right) dV = \left( \frac{10}{3} V + \frac{10}{3} V^2 \right) \Big|_1^4 \\ &= \left( \frac{40}{3} + \frac{160}{3} - \frac{10}{3} - \frac{10}{3} \right) \text{ J} = 60 \text{ J}. \end{aligned}$$

The  $BC$  portion of the cycle is at constant pressure and the work done by the gas is

$$W_{BC} = p \Delta V = (30 \text{ Pa})(1.0 \text{ m}^3 - 4.0 \text{ m}^3) = -90 \text{ J}.$$

The  $CA$  portion of the cycle is at constant volume, so no work is done. The total work done by the gas is

$$W = W_{AB} + W_{BC} + W_{CA} = 60 \text{ J} - 90 \text{ J} + 0 = -30 \text{ J}$$

and the total heat absorbed is  $Q = W = -30 \text{ J}$ . This means the gas loses  $30 \text{ J}$  of energy in the form of heat.

44. (a) Since work is done *on* the system (perhaps to compress it) we write  $W = -200 \text{ J}$ .

(b) Since heat leaves the system, we have  $Q = -70.0 \text{ cal} = -293 \text{ J}$ .

(c) The change in internal energy is  $\Delta E_{\text{int}} = Q - W = -293 \text{ J} - (-200 \text{ J}) = -93 \text{ J}$ .

45. (a) One part of path  $A$  represents a constant pressure process. The volume changes from  $1.0 \text{ m}^3$  to  $4.0 \text{ m}^3$  while the pressure remains at  $40 \text{ Pa}$ . The work done is

$$W_A = p\Delta V = (40 \text{ Pa})(4.0 \text{ m}^3 - 1.0 \text{ m}^3) = 1.2 \times 10^2 \text{ J}.$$

(b) The other part of the path represents a constant volume process. No work is done during this process. The total work done over the entire path is  $120 \text{ J}$ . To find the work done over path  $B$  we need to know the pressure as a function of volume. Then, we can evaluate the integral  $W = \int p \, dV$ . According to the graph, the pressure is a linear function of the volume, so we may write  $p = a + bV$ , where  $a$  and  $b$  are constants. In order for the pressure to be  $40 \text{ Pa}$  when the volume is  $1.0 \text{ m}^3$  and  $10 \text{ Pa}$  when the volume is  $4.00 \text{ m}^3$  the values of the constants must be  $a = 50 \text{ Pa}$  and  $b = -10 \text{ Pa/m}^3$ . Thus,

$$p = 50 \text{ Pa} - (10 \text{ Pa/m}^3)V$$

and

$$W_B = \int_1^4 p \, dV = \int_1^4 (50 - 10V) \, dV = (50V - 5V^2) \Big|_1^4 = 200 \text{ J} - 50 \text{ J} - 80 \text{ J} + 5.0 \text{ J} = 75 \text{ J}.$$

(c) One part of path  $C$  represents a constant pressure process in which the volume changes from  $1.0 \text{ m}^3$  to  $4.0 \text{ m}^3$  while  $p$  remains at  $10 \text{ Pa}$ . The work done is

$$W_C = p\Delta V = (10 \text{ Pa})(4.0 \text{ m}^3 - 1.0 \text{ m}^3) = 30 \text{ J}.$$

The other part of the process is at constant volume and no work is done. The total work is  $30 \text{ J}$ . We note that the work is different for different paths.

46. During process  $A \rightarrow B$ , the system is expanding, doing work on its environment, so  $W > 0$ , and since  $\Delta E_{\text{int}} > 0$  is given then  $Q = W + \Delta E_{\text{int}}$  must also be positive.

(a)  $Q > 0$ .

(b)  $W > 0$ .

During process  $B \rightarrow C$ , the system is neither expanding nor contracting. Thus,

(c)  $W = 0$ .

(d) The sign of  $\Delta E_{\text{int}}$  must be the same (by the first law of thermodynamics) as that of  $Q$  which is given as positive. Thus,  $\Delta E_{\text{int}} > 0$ .

During process  $C \rightarrow A$ , the system is contracting. The environment is doing work on the system, which implies  $W < 0$ . Also,  $\Delta E_{\text{int}} < 0$  because  $\sum \Delta E_{\text{int}} = 0$  (for the whole cycle) and the other values of  $\Delta E_{\text{int}}$  (for the other processes) were positive. Therefore,  $Q = W + \Delta E_{\text{int}}$  must also be negative.

(e)  $Q < 0$ .

(f)  $W < 0$ .

(g)  $\Delta E_{\text{int}} < 0$ .

(h) The area of a triangle is  $\frac{1}{2}$  (base)(height). Applying this to the figure, we find  $|W_{\text{net}}| = \frac{1}{2}(2.0 \text{ m}^3)(20 \text{ Pa}) = 20 \text{ J}$ . Since process  $C \rightarrow A$  involves larger negative work (it occurs at higher average pressure) than the positive work done during process  $A \rightarrow B$ , then the net work done during the cycle must be negative. The answer is therefore  $W_{\text{net}} = -20 \text{ J}$ .

47. We note that there is no work done in the process going from  $d$  to  $a$ , so  $Q_{da} = \Delta E_{\text{int } da} = 80 \text{ J}$ . Also, since the total change in internal energy around the cycle is zero, then

$$\Delta E_{\text{int } ac} + \Delta E_{\text{int } cd} + \Delta E_{\text{int } da} = 0$$

$$-200 \text{ J} + \Delta E_{\text{int } cd} + 80 \text{ J} = 0$$

which yields  $\Delta E_{\text{int } cd} = 120 \text{ J}$ . Thus, applying the first law of thermodynamics to the  $c$  to  $d$  process gives the work done as

$$W_{cd} = Q_{cd} - \Delta E_{\text{int } cd} = 180 \text{ J} - 120 \text{ J} = 60 \text{ J}.$$

48. (a) We note that process  $a$  to  $b$  is an expansion, so  $W > 0$  for it. Thus,  $W_{ab} = +5.0$  J. We are told that the change in internal energy during that process is  $+3.0$  J, so application of the first law of thermodynamics for that process immediately yields  $Q_{ab} = +8.0$  J.

(b) The net work ( $+1.2$  J) is the same as the net heat ( $Q_{ab} + Q_{bc} + Q_{ca}$ ), and we are told that  $Q_{ca} = +2.5$  J. Thus we readily find  $Q_{bc} = (1.2 - 8.0 - 2.5)$  J =  $-9.3$  J.

49. (a) The change in internal energy  $\Delta E_{\text{int}}$  is the same for path  $iaf$  and path  $ibf$ . According to the first law of thermodynamics,  $\Delta E_{\text{int}} = Q - W$ , where  $Q$  is the heat absorbed and  $W$  is the work done by the system. Along  $iaf$

$$\Delta E_{\text{int}} = Q - W = 50 \text{ cal} - 20 \text{ cal} = 30 \text{ cal}.$$

Along  $ibf$ ,

$$W = Q - \Delta E_{\text{int}} = 36 \text{ cal} - 30 \text{ cal} = 6.0 \text{ cal}.$$

(b) Since the curved path is traversed from  $f$  to  $i$  the change in internal energy is  $-30$  cal and  $Q = \Delta E_{\text{int}} + W = -30 \text{ cal} - 13 \text{ cal} = -43 \text{ cal}$ .

(c) Let  $\Delta E_{\text{int}} = E_{\text{int}, f} - E_{\text{int}, i}$ . Then,  $E_{\text{int}, f} = \Delta E_{\text{int}} + E_{\text{int}, i} = 30 \text{ cal} + 10 \text{ cal} = 40 \text{ cal}$ .

(d) The work  $W_{bf}$  for the path  $bf$  is zero, so  $Q_{bf} = E_{\text{int}, f} - E_{\text{int}, b} = 40 \text{ cal} - 22 \text{ cal} = 18 \text{ cal}$ .

(e) For the path  $ibf$ ,  $Q = 36$  cal so  $Q_{ib} = Q - Q_{bf} = 36 \text{ cal} - 18 \text{ cal} = 18 \text{ cal}$ .

50. Since the process is a complete cycle (beginning and ending in the same thermodynamic state) the change in the internal energy is zero and the heat absorbed by the gas is equal to the work done by the gas:  $Q = W$ . In terms of the contributions of the individual parts of the cycle  $Q_{AB} + Q_{BC} + Q_{CA} = W$  and

$$Q_{CA} = W - Q_{AB} - Q_{BC} = +15.0 \text{ J} - 20.0 \text{ J} - 0 = -5.0 \text{ J}.$$

This means 5.0 J of energy leaves the gas in the form of heat.

51. The rate of heat flow is given by

$$P_{\text{cond}} = kA \frac{T_H - T_C}{L},$$

where  $k$  is the thermal conductivity of copper (401 W/m·K),  $A$  is the cross-sectional area (in a plane perpendicular to the flow),  $L$  is the distance along the direction of flow between the points where the temperature is  $T_H$  and  $T_C$ . Thus,

$$P_{\text{cond}} = \frac{(401 \text{ W/m} \cdot \text{K})(90.0 \times 10^{-4} \text{ m}^2)(125^\circ\text{C} - 10.0^\circ\text{C})}{0.250 \text{ m}} = 1.66 \times 10^3 \text{ J/s}.$$

The thermal conductivity is found in Table 18-6 of the text. Recall that a change in Kelvin temperature is numerically equivalent to a change on the Celsius scale.

52. (a) We estimate the surface area of the average human body to be about  $2 \text{ m}^2$  and the skin temperature to be about  $300 \text{ K}$  (somewhat less than the internal temperature of  $310 \text{ K}$ ). Then from Eq. 18-37

$$P_r = \sigma \epsilon A T^4 \approx (5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)(0.9)(2.0 \text{ m}^2)(300 \text{ K})^4 = 8 \times 10^2 \text{ W}.$$

(b) The energy lost is given by

$$\Delta E = P_r \Delta t = (8 \times 10^2 \text{ W})(30 \text{ s}) = 2 \times 10^4 \text{ J}.$$

53. (a) Recalling that a change in Kelvin temperature is numerically equivalent to a change on the Celsius scale, we find that the rate of heat conduction is

$$P_{\text{cond}} = \frac{kA(T_H - T_C)}{L} = \frac{(401 \text{ W/m} \cdot \text{K})(4.8 \times 10^{-4} \text{ m}^2)(100^\circ\text{C})}{1.2 \text{ m}} = 16 \text{ J/s.}$$

(b) Using Table 18-4, the rate at which ice melts is

$$\left| \frac{dm}{dt} \right| = \frac{P_{\text{cond}}}{L_F} = \frac{16 \text{ J/s}}{333 \text{ J/g}} = 0.048 \text{ g/s.}$$

54. We refer to the polyurethane foam with subscript  $p$  and silver with subscript  $s$ . We use Eq. 18–32 to find  $L = kR$ .

(a) From Table 18-6 we find  $k_p = 0.024 \text{ W/m}\cdot\text{K}$  so

$$\begin{aligned} L_p &= k_p R_p \\ &= (0.024 \text{ W/m}\cdot\text{K})(30 \text{ ft}^2 \cdot \text{F}^\circ \cdot \text{h/Btu})(1 \text{ m}/3.281 \text{ ft})^2 (5\text{C}^\circ/9\text{F}^\circ)(3600 \text{ s/h})(1 \text{ Btu}/1055 \text{ J}) \\ &= 0.13 \text{ m}. \end{aligned}$$

(b) For silver  $k_s = 428 \text{ W/m}\cdot\text{K}$ , so

$$L_s = k_s R_s = \left( \frac{k_s R_s}{k_p R_p} \right) L_p = \left[ \frac{428(30)}{0.024(30)} \right] (0.13 \text{ m}) = 2.3 \times 10^3 \text{ m}.$$

55. We use Eqs. 18-38 through 18-40. Note that the surface area of the sphere is given by  $A = 4\pi r^2$ , where  $r = 0.500$  m is the radius.

(a) The temperature of the sphere is  $T = (273.15 + 27.00)$  K = 300.15 K. Thus

$$\begin{aligned} P_r &= \sigma \epsilon A T^4 = (5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)(0.850)(4\pi)(0.500 \text{ m})^2 (300.15 \text{ K})^4 \\ &= 1.23 \times 10^3 \text{ W}. \end{aligned}$$

(b) Now,  $T_{\text{env}} = 273.15 + 77.00 = 350.15$  K so

$$P_a = \sigma \epsilon A T_{\text{env}}^4 = (5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)(0.850)(4\pi)(0.500 \text{ m})^2 (350.15 \text{ K})^4 = 2.28 \times 10^3 \text{ W}.$$

(c) From Eq. 18-40, we have

$$P_n = P_a - P_r = 2.28 \times 10^3 \text{ W} - 1.23 \times 10^3 \text{ W} = 1.05 \times 10^3 \text{ W}.$$

56. (a) The surface area of the cylinder is given by

$$A_1 = 2\pi r_1^2 + 2\pi r_1 h_1 = 2\pi(2.5 \times 10^{-2} \text{ m})^2 + 2\pi(2.5 \times 10^{-2} \text{ m})(5.0 \times 10^{-2} \text{ m}) = 1.18 \times 10^{-2} \text{ m}^2,$$

its temperature is  $T_1 = 273 + 30 = 303 \text{ K}$ , and the temperature of the environment is  $T_{\text{env}} = 273 + 50 = 323 \text{ K}$ . From Eq. 18-39 we have

$$P_1 = \sigma \varepsilon A_1 (T_{\text{env}}^4 - T^4) = (0.85)(1.18 \times 10^{-2} \text{ m}^2)((323 \text{ K})^4 - (303 \text{ K})^4) = 1.4 \text{ W}.$$

(b) Let the new height of the cylinder be  $h_2$ . Since the volume  $V$  of the cylinder is fixed, we must have  $V = \pi r_1^2 h_1 = \pi r_2^2 h_2$ . We solve for  $h_2$ :

$$h_2 = \left(\frac{r_1}{r_2}\right)^2 h_1 = \left(\frac{2.5 \text{ cm}}{0.50 \text{ cm}}\right)^2 (5.0 \text{ cm}) = 125 \text{ cm} = 1.25 \text{ m}.$$

The corresponding new surface area  $A_2$  of the cylinder is

$$A_2 = 2\pi r_2^2 + 2\pi r_2 h_2 = 2\pi(0.50 \times 10^{-2} \text{ m})^2 + 2\pi(0.50 \times 10^{-2} \text{ m})(1.25 \text{ m}) = 3.94 \times 10^{-2} \text{ m}^2.$$

Consequently,

$$\frac{P_2}{P_1} = \frac{A_2}{A_1} = \frac{3.94 \times 10^{-2} \text{ m}^2}{1.18 \times 10^{-2} \text{ m}^2} = 3.3.$$

57. We use  $P_{\text{cond}} = kA\Delta T/L \propto A/L$ . Comparing cases (a) and (b) in Figure 18–44, we have

$$P_{\text{cond } b} = \left( \frac{A_b L_a}{A_a L_b} \right) P_{\text{cond } a} = 4P_{\text{cond } a}.$$

Consequently, it would take  $2.0 \text{ min}/4 = 0.50 \text{ min}$  for the same amount of heat to be conducted through the rods welded as shown in Fig. 18-44(b).

58. (a) As in Sample Problem 18-6, we take the rate of conductive heat transfer through each layer to be the same. Thus, the rate of heat transfer across the entire wall  $P_w$  is equal to the rate across layer 2 ( $P_2$ ). Using Eq. 18-37 and canceling out the common factor of area  $A$ , we obtain

$$\frac{T_H - T_c}{(L_1/k_1 + L_2/k_2 + L_3/k_3)} = \frac{\Delta T_2}{(L_2/k_2)} \Rightarrow \frac{45 \text{ C}^\circ}{(1 + 7/9 + 35/80)} = \frac{\Delta T_2}{(7/9)}$$

which leads to  $\Delta T_2 = 15.8 \text{ }^\circ\text{C}$ .

(b) We expect (and this is supported by the result in the next part) that greater conductivity should mean a larger rate of conductive heat transfer.

(c) Repeating the calculation above with the new value for  $k_2$ , we have

$$\frac{45 \text{ C}^\circ}{(1 + 7/11 + 35/80)} = \frac{\Delta T_2}{(7/11)}$$

which leads to  $\Delta T_2 = 13.8 \text{ }^\circ\text{C}$ . This is less than our part (a) result which implies that the temperature gradients across layers 1 and 3 (the ones where the parameters did not change) are greater than in part (a); those larger temperature gradients lead to larger conductive heat currents (which is basically a statement of “Ohm’s law as applied to heat conduction”).

59. (a) We use

$$P_{\text{cond}} = kA \frac{T_H - T_C}{L}$$

with the conductivity of glass given in Table 18-6 as  $1.0 \text{ W/m}\cdot\text{K}$ . We choose to use the Celsius scale for the temperature: a temperature difference of

$$T_H - T_C = 72^\circ\text{F} - (-20^\circ\text{F}) = 92^\circ\text{F}$$

is equivalent to  $\frac{5}{9}(92) = 51.1^\circ\text{C}$ . This, in turn, is equal to  $51.1 \text{ K}$  since a change in Kelvin temperature is entirely equivalent to a Celsius change. Thus,

$$\frac{P_{\text{cond}}}{A} = k \frac{T_H - T_C}{L} = (1.0 \text{ W/m}\cdot\text{K}) \left( \frac{51.1^\circ\text{C}}{3.0 \times 10^{-3} \text{ m}} \right) = 1.7 \times 10^4 \text{ W/m}^2.$$

(b) The energy now passes in succession through 3 layers, one of air and two of glass. The heat transfer rate  $P$  is the same in each layer and is given by

$$P_{\text{cond}} = \frac{A(T_H - T_C)}{\sum L/k}$$

where the sum in the denominator is over the layers. If  $L_g$  is the thickness of a glass layer,  $L_a$  is the thickness of the air layer,  $k_g$  is the thermal conductivity of glass, and  $k_a$  is the thermal conductivity of air, then the denominator is

$$\sum \frac{L}{k} = \frac{2L_g}{k_g} + \frac{L_a}{k_a} = \frac{2L_g k_a + L_a k_g}{k_a k_g}.$$

Therefore, the heat conducted per unit area occurs at the following rate:

$$\begin{aligned} \frac{P_{\text{cond}}}{A} &= \frac{(T_H - T_C) k_a k_g}{2L_g k_a + L_a k_g} = \frac{(51.1^\circ\text{C})(0.026 \text{ W/m}\cdot\text{K})(1.0 \text{ W/m}\cdot\text{K})}{2(3.0 \times 10^{-3} \text{ m})(0.026 \text{ W/m}\cdot\text{K}) + (0.075 \text{ m})(1.0 \text{ W/m}\cdot\text{K})} \\ &= 18 \text{ W/m}^2. \end{aligned}$$

60. The surface area of the ball is  $A = 4\pi R^2 = 4\pi(0.020 \text{ m})^2 = 5.03 \times 10^{-3} \text{ m}^2$ . Using Eq. 18-37 with  $T_i = 35 + 273 = 308 \text{ K}$  and  $T_f = 47 + 273 = 320 \text{ K}$ , the power required to maintain the temperature is

$$P_r = \sigma \epsilon A (T_f^4 - T_i^4) \approx (5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)(0.80)(5.03 \times 10^{-3} \text{ m}^2) [(320 \text{ K})^4 - (308 \text{ K})^4] \\ = 0.34 \text{ W}.$$

Thus, the heat each bee must produce during the 20-minutes interval is

$$\frac{Q}{N} = \frac{P_r t}{N} = \frac{(0.34 \text{ W})(20 \text{ min})(60 \text{ s/min})}{500} = 0.81 \text{ J}.$$

61. We divide both sides of Eq. 18-32 by area  $A$ , which gives us the (uniform) rate of heat conduction per unit area:

$$\frac{P_{\text{cond}}}{A} = k_1 \frac{T_H - T_1}{L_1} = k_4 \frac{T - T_C}{L_4}$$

where  $T_H = 30^\circ\text{C}$ ,  $T_1 = 25^\circ\text{C}$  and  $T_C = -10^\circ\text{C}$ . We solve for the unknown  $T$ .

$$T = T_C + \frac{k_1 L_4}{k_4 L_1} (T_H - T_1) = -4.2^\circ\text{C}.$$

62. (a) For each individual penguin, the surface area that radiates is the sum of the top surface area and the sides:

$$A_r = a + 2\pi rh = a + 2\pi \sqrt{\frac{a}{\pi}} h = a + 2h\sqrt{\pi a},$$

where we have used  $r = \sqrt{a/\pi}$  (from  $a = \pi r^2$ ) for the radius of the cylinder. For the huddled cylinder, the radius is  $r' = \sqrt{Na/\pi}$  (since  $Na = \pi r'^2$ ), and the total surface area is

$$A_h = Na + 2\pi r'h = Na + 2\pi \sqrt{\frac{Na}{\pi}} h = Na + 2h\sqrt{N\pi a}.$$

Since the power radiated is proportional to the surface area, we have

$$\frac{P_h}{NP_r} = \frac{A_h}{NA_r} = \frac{Na + 2h\sqrt{N\pi a}}{N(a + 2h\sqrt{\pi a})} = \frac{1 + 2h\sqrt{\pi/Na}}{1 + 2h\sqrt{\pi/a}}.$$

With  $N = 1000$ ,  $a = 0.34 \text{ m}^2$  and  $h = 1.1 \text{ m}$ , the ratio is

$$\frac{P_h}{NP_r} = \frac{1 + 2h\sqrt{\pi/Na}}{1 + 2h\sqrt{\pi/a}} = \frac{1 + 2(1.1 \text{ m})\sqrt{\pi/(1000 \cdot 0.34 \text{ m}^2)}}{1 + 2(1.1 \text{ m})\sqrt{\pi/(0.34 \text{ m}^2)}} = 0.16.$$

(b) The total radiation loss is reduced by  $1.00 - 0.16 = 0.84$ , or 84%.

63. We assume (although this should be viewed as a “controversial” assumption) that the top surface of the ice is at  $T_C = -5.0^\circ\text{C}$ . Less controversial are the assumptions that the bottom of the body of water is at  $T_H = 4.0^\circ\text{C}$  and the interface between the ice and the water is at  $T_X = 0.0^\circ\text{C}$ . The primary mechanism for the heat transfer through the total distance  $L = 1.4\text{ m}$  is assumed to be conduction, and we use Eq. 18-34:

$$\frac{k_{\text{water}}A(T_H - T_X)}{L - L_{\text{ice}}} = \frac{k_{\text{ice}}A(T_X - T_C)}{L_{\text{ice}}} \Rightarrow \frac{(0.12)A(4.0^\circ - 0.0^\circ)}{1.4 - L_{\text{ice}}} = \frac{(0.40)A(0.0^\circ + 5.0^\circ)}{L_{\text{ice}}}.$$

We cancel the area  $A$  and solve for thickness of the ice layer:  $L_{\text{ice}} = 1.1\text{ m}$ .

64. (a) Using Eq. 18-32, the rate of energy flow through the surface is

$$P_{\text{cond}} = \frac{kA(T_s - T_w)}{L} = (0.026 \text{ W/m} \cdot \text{K})(4.00 \times 10^{-6} \text{ m}^2) \frac{300^\circ\text{C} - 100^\circ\text{C}}{1.0 \times 10^{-4} \text{ m}} = 0.208 \text{ W} \approx 0.21 \text{ W}.$$

(Recall that a change in Celsius temperature is numerically equivalent to a change on the Kelvin scale.)

(b) With  $P_{\text{cond}}t = L_V m = L_V(\rho V) = L_V(\rho Ah)$ , the drop will last a duration of

$$t = \frac{L_V \rho Ah}{P_{\text{cond}}} = \frac{(2.256 \times 10^6 \text{ J/kg})(1000 \text{ kg/m}^3)(4.00 \times 10^{-6} \text{ m}^2)(1.50 \times 10^{-3} \text{ m})}{0.208 \text{ W}} = 65 \text{ s}.$$

65. Let  $h$  be the thickness of the slab and  $A$  be its area. Then, the rate of heat flow through the slab is

$$P_{\text{cond}} = \frac{kA(T_H - T_C)}{h}$$

where  $k$  is the thermal conductivity of ice,  $T_H$  is the temperature of the water ( $0^\circ\text{C}$ ), and  $T_C$  is the temperature of the air above the ice ( $-10^\circ\text{C}$ ). The heat leaving the water freezes it, the heat required to freeze mass  $m$  of water being  $Q = L_F m$ , where  $L_F$  is the heat of fusion for water. Differentiate with respect to time and recognize that  $dQ/dt = P_{\text{cond}}$  to obtain

$$P_{\text{cond}} = L_F \frac{dm}{dt}$$

Now, the mass of the ice is given by  $m = \rho Ah$ , where  $\rho$  is the density of ice and  $h$  is the thickness of the ice slab, so  $dm/dt = \rho A(dh/dt)$  and

$$P_{\text{cond}} = L_F \rho A \frac{dh}{dt}$$

We equate the two expressions for  $P_{\text{cond}}$  and solve for  $dh/dt$ :

$$\frac{dh}{dt} = \frac{k(T_H - T_C)}{L_F \rho h}$$

Since  $1 \text{ cal} = 4.186 \text{ J}$  and  $1 \text{ cm} = 1 \times 10^{-2} \text{ m}$ , the thermal conductivity of ice has the SI value

$$k = (0.0040 \text{ cal/s}\cdot\text{cm}\cdot\text{K}) (4.186 \text{ J/cal}) / (1 \times 10^{-2} \text{ m/cm}) = 1.674 \text{ W/m}\cdot\text{K}.$$

The density of ice is  $\rho = 0.92 \text{ g/cm}^3 = 0.92 \times 10^3 \text{ kg/m}^3$ . Thus,

$$\frac{dh}{dt} = \frac{(1.674 \text{ W/m}\cdot\text{K})(0^\circ\text{C} + 10^\circ\text{C})}{(333 \times 10^3 \text{ J/kg})(0.92 \times 10^3 \text{ kg/m}^3)(0.050 \text{ m})} = 1.1 \times 10^{-6} \text{ m/s} = 0.40 \text{ cm/h}.$$

66. The condition that the energy lost by the beverage can due to evaporation equals the energy gained via radiation exchange implies

$$L_v \frac{dm}{dt} = P_{\text{rad}} = \sigma \varepsilon A (T_{\text{env}}^4 - T^4).$$

The total area of the top and side surfaces of the can is

$$A = \pi r^2 + 2\pi r h = \pi(0.022 \text{ m})^2 + 2\pi(0.022 \text{ m})(0.10 \text{ m}) = 1.53 \times 10^{-2} \text{ m}^2.$$

With  $T_{\text{env}} = 32^\circ\text{C} = 305 \text{ K}$ ,  $T = 15^\circ\text{C} = 288 \text{ K}$  and  $\varepsilon = 1$ , the rate of water mass loss is

$$\begin{aligned} \frac{dm}{dt} &= \frac{\sigma \varepsilon A}{L_v} (T_{\text{env}}^4 - T^4) = \frac{(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)(1.0)(1.53 \times 10^{-2} \text{ m}^2)}{2.256 \times 10^6 \text{ J/kg}} [(305 \text{ K})^4 - (288 \text{ K})^4] \\ &= 6.82 \times 10^{-7} \text{ kg/s} \approx 0.68 \text{ mg/s}. \end{aligned}$$

67. We denote the total mass  $M$  and the melted mass  $m$ . The problem tells us that  $\text{Work}/M = p/\rho$ , and that all the work is assumed to contribute to the phase change  $Q = Lm$  where  $L = 150 \times 10^3$  J/kg. Thus,

$$\frac{p}{\rho} M = Lm \Rightarrow m = \frac{5.5 \times 10^6}{1200} \frac{M}{150 \times 10^3}$$

which yields  $m = 0.0306M$ . Dividing this by 0.30  $M$  (the mass of the fats, which we are told is equal to 30% of the total mass), leads to a percentage  $0.0306/0.30 = 10\%$ .

68. As is shown in the textbook for Sample Problem 18-4, we can express the final temperature in the following way:

$$T_f = \frac{m_A c_A T_A + m_B c_B T_B}{m_A c_A + m_B c_B} = \frac{c_A T_A + c_B T_B}{c_A + c_B}$$

where the last equality is made possible by the fact that  $m_A = m_B$ . Thus, in a graph of  $T_f$  versus  $T_A$ , the “slope” must be  $c_A / (c_A + c_B)$ , and the “y intercept” is  $c_B / (c_A + c_B) T_B$ . From the observation that the “slope” is equal to  $2/5$  we can determine, then, not only the ratio of the heat capacities but also the coefficient of  $T_B$  in the “y intercept”; that is,

$$c_B / (c_A + c_B) T_B = (1 - \text{“slope”}) T_B.$$

(a) We observe that the “y intercept” is 150 K, so

$$T_B = 150 / (1 - \text{“slope”}) = 150 / (3/5)$$

which yields  $T_B = 2.5 \times 10^2$  K.

(b) As noted already,  $c_A / (c_A + c_B) = \frac{2}{5}$ , so  $5 c_A = 2 c_A + 2 c_B$ , which leads to  $c_B / c_A = \frac{3}{2} = 1.5$ .

69. The graph shows that the absolute value of the temperature change is  $|\Delta T| = 25\text{ }^\circ\text{C}$ . Since a Watt is a Joule per second, we reason that the energy removed is

$$|Q| = (2.81\text{ J/s})(20\text{ min})(60\text{ s/min}) = 3372\text{ J} .$$

Thus, with  $m = 0.30\text{ kg}$ , the absolute value of Eq. 18-14 leads to

$$c = \frac{|Q|}{m |\Delta T|} = 4.5 \times 10^2\text{ J/kg}\cdot\text{K} .$$

70. Let  $m_w = 14 \text{ kg}$ ,  $m_c = 3.6 \text{ kg}$ ,  $m_m = 1.8 \text{ kg}$ ,  $T_{i1} = 180^\circ\text{C}$ ,  $T_{i2} = 16.0^\circ\text{C}$ , and  $T_f = 18.0^\circ\text{C}$ . The specific heat  $c_m$  of the metal then satisfies

$$(m_w c_w + m_c c_m)(T_f - T_{i2}) + m_m c_m (T_f - T_{i1}) = 0$$

which we solve for  $c_m$ :

$$\begin{aligned} c_m &= \frac{m_w c_w (T_{i2} - T_f)}{m_c (T_f - T_{i2}) + m_m (T_f - T_{i1})} = \frac{(14 \text{ kg})(4.18 \text{ kJ/kg} \cdot \text{K})(16.0^\circ\text{C} - 18.0^\circ\text{C})}{(3.6 \text{ kg})(18.0^\circ\text{C} - 16.0^\circ\text{C}) + (1.8 \text{ kg})(18.0^\circ\text{C} - 180^\circ\text{C})} \\ &= 0.41 \text{ kJ/kg} \cdot \text{C}^\circ = 0.41 \text{ kJ/kg} \cdot \text{K}. \end{aligned}$$

71. Its initial volume is  $5^3 = 125 \text{ cm}^3$ , and using Table 18-2, Eq. 18-10 and Eq. 18-11, we find

$$\Delta V = (125 \text{ m}^3) (3 \times 23 \times 10^{-6} / \text{C}^\circ) (50.0 \text{ C}^\circ) = 0.432 \text{ cm}^3.$$

72. (a) We denote  $T_H = 100^\circ\text{C}$ ,  $T_C = 0^\circ\text{C}$ , the temperature of the copper-aluminum junction by  $T_1$ . and that of the aluminum-brass junction by  $T_2$ . Then,

$$P_{\text{cond}} = \frac{k_c A}{L}(T_H - T_1) = \frac{k_a A}{L}(T_1 - T_2) = \frac{k_b A}{L}(T_2 - T_C).$$

We solve for  $T_1$  and  $T_2$  to obtain

$$T_1 = T_H + \frac{T_C - T_H}{1 + k_c(k_a + k_b)/k_a k_b} = 100^\circ\text{C} + \frac{0.00^\circ\text{C} - 100^\circ\text{C}}{1 + 401(235 + 109)/[(235)(109)]} = 84.3^\circ\text{C}$$

(b) and

$$\begin{aligned} T_2 &= T_C + \frac{T_H - T_C}{1 + k_b(k_c + k_a)/k_c k_a} = 0.00^\circ\text{C} + \frac{100^\circ\text{C} - 0.00^\circ\text{C}}{1 + 109(235 + 401)/[(235)(401)]} \\ &= 57.6^\circ\text{C}. \end{aligned}$$

73. The work (the “area under the curve”) for process 1 is  $4p_iV_i$ , so that

$$U_b - U_a = Q_1 - W_1 = 6p_iV_i$$

by the First Law of Thermodynamics.

(a) Path 2 involves more work than path 1 (note the triangle in the figure of area  $\frac{1}{2}(4V_i)(p_i/2) = p_iV_i$ ). With  $W_2 = 4p_iV_i + p_iV_i = 5p_iV_i$ , we obtain

$$Q_2 = W_2 + U_b - U_a = 5p_iV_i + 6p_iV_i = 11p_iV_i.$$

(b) Path 3 starts at  $a$  and ends at  $b$  so that  $\Delta U = U_b - U_a = 6p_iV_i$ .

74. We use  $P_{\text{cond}} = kA(T_H - T_C)/L$ . The temperature  $T_H$  at a depth of 35.0 km is

$$T_H = \frac{P_{\text{cond}}L}{kA} + T_C = \frac{(54.0 \times 10^{-3} \text{ W/m}^2)(35.0 \times 10^3 \text{ m})}{2.50 \text{ W/m} \cdot \text{K}} + 10.0^\circ\text{C} = 766^\circ\text{C}.$$

75. The volume of the disk (thought of as a short cylinder) is  $\pi r^2 L$  where  $L = 0.50$  cm is its thickness and  $r = 8.0$  cm is its radius. Eq. 18-10, Eq. 18-11 and Table 18-2 (which gives  $\alpha = 3.2 \times 10^{-6}/\text{C}^\circ$ ) lead to

$$\Delta V = (\pi r^2 L)(3\alpha)(60^\circ\text{C} - 10^\circ\text{C}) = 4.83 \times 10^{-2} \text{ cm}^3.$$

76. We use  $Q = cm\Delta T$  and  $m = \rho V$ . The volume of water needed is

$$V = \frac{m}{\rho} = \frac{Q}{\rho C \Delta T} = \frac{(1.00 \times 10^6 \text{ kcal/day})(5 \text{ days})}{(1.00 \times 10^3 \text{ kg/m}^3)(1.00 \text{ kcal/kg})(50.0^\circ\text{C} - 22.0^\circ\text{C})} = 35.7 \text{ m}^3.$$

77. We have  $W = \int p \, dV$  (Eq. 18-24). Therefore,

$$W = a \int V^2 dV = \frac{a}{3} (V_f^3 - V_i^3) = 23 \text{ J.}$$

78. (a) The rate of heat flow is

$$P_{\text{cond}} = \frac{kA(T_H - T_C)}{L} = \frac{(0.040 \text{ W/m} \cdot \text{K})(1.8 \text{ m}^2)(33^\circ\text{C} - 1.0^\circ\text{C})}{1.0 \times 10^{-2} \text{ m}} = 2.3 \times 10^2 \text{ J/s}.$$

(b) The new rate of heat flow is

$$P'_{\text{cond}} = \frac{k'P_{\text{cond}}}{k} = \frac{(0.60 \text{ W/m} \cdot \text{K})(230 \text{ J/s})}{0.040 \text{ W/m} \cdot \text{K}} = 3.5 \times 10^3 \text{ J/s},$$

which is about 15 times as fast as the original heat flow.

79. We note that there is no work done in process  $c \rightarrow b$ , since there is no change of volume. We also note that the *magnitude* of work done in process  $b \rightarrow c$  is given, but not its sign (which we identify as negative as a result of the discussion in §18-8). The total (or *net*) heat transfer is  $Q_{\text{net}} = [(-40) + (-130) + (+400)] \text{ J} = 230 \text{ J}$ . By the First Law of Thermodynamics (or, equivalently, conservation of energy), we have

$$\begin{aligned} Q_{\text{net}} &= W_{\text{net}} \\ 230 \text{ J} &= W_{a \rightarrow c} + W_{c \rightarrow b} + W_{b \rightarrow a} \\ &= W_{a \rightarrow c} + 0 + (-80 \text{ J}) \end{aligned}$$

Therefore,  $W_{a \rightarrow c} = 3.1 \times 10^2 \text{ J}$ .

80. If the window is  $L_1$  high and  $L_2$  wide at the lower temperature and  $L_1 + \Delta L_1$  high and  $L_2 + \Delta L_2$  wide at the higher temperature then its area changes from  $A_1 = L_1 L_2$  to

$$A_2 = (L_1 + \Delta L_1)(L_2 + \Delta L_2) \approx L_1 L_2 + L_1 \Delta L_2 + L_2 \Delta L_1$$

where the term  $\Delta L_1 \Delta L_2$  has been omitted because it is much smaller than the other terms, if the changes in the lengths are small. Consequently, the change in area is

$$\Delta A = A_2 - A_1 = L_1 \Delta L_2 + L_2 \Delta L_1.$$

If  $\Delta T$  is the change in temperature then  $\Delta L_1 = \alpha L_1 \Delta T$  and  $\Delta L_2 = \alpha L_2 \Delta T$ , where  $\alpha$  is the coefficient of linear expansion. Thus

$$\begin{aligned} \Delta A &= \alpha(L_1 L_2 + L_1 L_2) \Delta T = 2\alpha L_1 L_2 \Delta T \\ &= 2(9 \times 10^{-6} / \text{C}^\circ)(30 \text{ cm})(20 \text{ cm})(30^\circ \text{C}) \\ &= 0.32 \text{ cm}^2. \end{aligned}$$

81. Following the method of Sample Problem 18-4 (particularly its third Key Idea), we have

$$(900 \frac{\text{J}}{\text{kg}\cdot\text{C}^\circ})(2.50 \text{ kg})(T_f - 92.0^\circ\text{C}) + (4190 \frac{\text{J}}{\text{kg}\cdot\text{C}^\circ})(8.00 \text{ kg})(T_f - 5.0^\circ\text{C}) = 0$$

where Table 18-3 has been used. Thus we find  $T_f = 10.5^\circ\text{C}$ .

82. We use  $Q = -\lambda_F m_{ice} = W + \Delta E_{\text{int}}$ . In this case  $\Delta E_{\text{int}} = 0$ . Since  $\Delta T = 0$  for the ideal gas, then the work done on the gas is

$$W' = -W = \lambda_F m_i = (333 \text{ J/g})(100 \text{ g}) = 33.3 \text{ kJ}.$$

83. This is similar to Sample Problem 18-3. An important difference with part (b) of that sample problem is that, in this case, the final state of the H<sub>2</sub>O is *all liquid* at  $T_f > 0$ . As discussed in part (a) of that sample problem, there are three steps to the total process:

$$Q = m [c_{\text{ice}}(0 \text{ C}^\circ - (-150 \text{ C}^\circ)) + L_F + c_{\text{liquid}}(T_f - 0 \text{ C}^\circ)]$$

Thus,

$$T_f = \frac{Q/m - (c_{\text{ice}}(150^\circ) + L_F)}{c_{\text{liquid}}} = 79.5^\circ\text{C} .$$

84. We take absolute values of Eq. 18-9 and Eq. 12-25:

$$|\Delta L| = L\alpha |\Delta T| \quad \text{and} \quad \left| \frac{F}{A} \right| = E \left| \frac{\Delta L}{L} \right|.$$

The ultimate strength for steel is  $(F/A)_{\text{rupture}} = S_u = 400 \times 10^6 \text{ N/m}^2$  from Table 12-1. Combining the above equations (eliminating the ratio  $\Delta L/L$ ), we find the rod will rupture if the temperature change exceeds

$$|\Delta T| = \frac{S_u}{E\alpha} = \frac{400 \times 10^6 \text{ N/m}^2}{(200 \times 10^9 \text{ N/m}^2)(11 \times 10^{-6} / \text{C}^\circ)} = 182^\circ\text{C}.$$

Since we are dealing with a temperature decrease, then, the temperature at which the rod will rupture is  $T = 25.0^\circ\text{C} - 182^\circ\text{C} = -157^\circ\text{C}$ .

85. The problem asks for 0.5% of  $E$ , where  $E = Pt$  with  $t = 120$  s and  $P$  given by Eq. 18-38. Therefore, with  $A = 4\pi r^2 = 5.0 \times 10^{-3} \text{ m}^2$ , we obtain

$$(0.005)Pt = (0.005)\sigma\epsilon AT^4t = 8.6 \text{ J}.$$

86. From the law of cosines, with  $\phi = 59.95^\circ$ , we have

$$L_{\text{Invar}}^2 = L_{\text{alum}}^2 + L_{\text{steel}}^2 - 2L_{\text{alum}}L_{\text{steel}} \cos \phi$$

Plugging in  $L = L_0 (1 + \alpha\Delta T)$ , dividing by  $L_0$  (which is the same for all sides) and ignoring terms of order  $(\Delta T)^2$  or higher, we obtain

$$1 + 2\alpha_{\text{Invar}}\Delta T = 2 + 2(\alpha_{\text{alum}} + \alpha_{\text{steel}})\Delta T - 2(1 + (\alpha_{\text{alum}} + \alpha_{\text{steel}})\Delta T)\cos \phi .$$

This is rearranged to yield

$$\Delta T = \frac{\cos \phi - 1/2}{(\alpha_{\text{alum}} + \alpha_{\text{steel}})(1 - \cos \phi) - \alpha_{\text{Invar}}} = \approx 46^\circ\text{C} ,$$

so that the final temperature is  $T = 20.0^\circ + \Delta T = 66^\circ\text{C}$ . Essentially the same argument, but arguably more elegant, can be made in terms of the differential of the above cosine law expression.

87. We assume the ice is at 0°C to begin with, so that the only heat needed for melting is that described by Eq. 18-16 (which requires information from Table 18-4). Thus,

$$Q = Lm = (333 \text{ J/g})(1.00 \text{ g}) = 333 \text{ J} .$$

88. Let the initial water temperature be  $T_{wi}$  and the initial thermometer temperature be  $T_{ti}$ . Then, the heat absorbed by the thermometer is equal (in magnitude) to the heat lost by the water:

$$c_t m_t (T_f - T_{ti}) = c_w m_w (T_{wi} - T_f).$$

We solve for the initial temperature of the water:

$$\begin{aligned} T_{wi} &= \frac{c_t m_t (T_f - T_{ti})}{c_w m_w} + T_f = \frac{(0.0550 \text{ kg})(0.837 \text{ kJ/kg} \cdot \text{K})(44.4 - 15.0) \text{ K}}{(4.18 \text{ kJ/kg} \cdot \text{C}^\circ)(0.300 \text{ kg})} + 44.4^\circ \text{C} \\ &= 45.5^\circ \text{C}. \end{aligned}$$

89. For a cylinder of height  $h$ , the surface area is  $A_c = 2\pi rh$ , and the area of a sphere is  $A_o = 4\pi R^2$ . The net radiative heat transfer is given by Eq. 18-40.

(a) We estimate the surface area  $A$  of the body as that of a cylinder of height 1.8 m and radius  $r = 0.15$  m plus that of a sphere of radius  $R = 0.10$  m. Thus, we have  $A \approx A_c + A_o = 1.8 \text{ m}^2$ . The emissivity  $\varepsilon = 0.80$  is given in the problem, and the Stefan-Boltzmann constant is found in §18-11:  $\sigma = 5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4$ . The “environment” temperature is  $T_{\text{env}} = 303 \text{ K}$ , and the skin temperature is  $T = \frac{5}{9}(102 - 32) + 273 = 312 \text{ K}$ . Therefore,

$$P_{\text{net}} = \sigma \varepsilon A (T_{\text{env}}^4 - T^4) = -86 \text{ W}.$$

The corresponding sign convention is discussed in the textbook immediately after Eq. 18-40. We conclude that heat is being lost by the body at a rate of roughly 90 W.

(b) Half the body surface area is roughly  $A = 1.8/2 = 0.9 \text{ m}^2$ . Now, with  $T_{\text{env}} = 248 \text{ K}$ , we find

$$|P_{\text{net}}| = |\sigma \varepsilon A (T_{\text{env}}^4 - T^4)| \approx 2.3 \times 10^2 \text{ W}.$$

(c) Finally, with  $T_{\text{env}} = 193 \text{ K}$  (and still with  $A = 0.9 \text{ m}^2$ ) we obtain  $|P_{\text{net}}| = 3.3 \times 10^2 \text{ W}$ .

90. One method is to simply compute the change in length in each edge ( $x_0 = 0.200$  m and  $y_0 = 0.300$  m) from Eq. 18-9 ( $\Delta x = 3.6 \times 10^{-5}$  m and  $\Delta y = 5.4 \times 10^{-5}$  m) and then compute the area change:

$$A - A_0 = (x_0 + \Delta x)(y_0 + \Delta y) - x_0 y_0 = 2.16 \times 10^{-5} \text{ m}^2.$$

Another (though related) method uses  $\Delta A = 2\alpha A_0 \Delta T$  (valid for  $\Delta A/A \ll 1$ ) which can be derived by taking the differential of  $A = xy$  and replacing  $d$ 's with  $\Delta$ 's.

91. (a) Let the number of weight lift repetitions be  $N$ . Then  $Nmgh = Q$ , or (using Eq. 18-12 and the discussion preceding it)

$$N = \frac{Q}{mgh} = \frac{(3500 \text{ Cal})(4186 \text{ J/Cal})}{(80.0 \text{ kg})(9.80 \text{ m/s}^2)(1.00 \text{ m})} \approx 1.87 \times 10^4.$$

(b) The time required is

$$t = (18700)(2.00 \text{ s}) \left( \frac{1.00 \text{ h}}{3600 \text{ s}} \right) = 10.4 \text{ h}.$$

92. The heat needed is

$$\begin{aligned} Q &= (10\%)mL_F = \left(\frac{1}{10}\right)(200,000 \text{ metric tons}) (1000 \text{ kg / metric ton}) (333 \text{ kJ/kg}) \\ &= 6.7 \times 10^{12} \text{ J.} \end{aligned}$$

93. The net work may be computed as a sum of works (for the individual processes involved) or as the “area” (with appropriate  $\pm$  sign) inside the figure (representing the cycle). In this solution, we take the former approach (sum over the processes) and will need the following fact related to processes represented in  $pV$  diagrams:

$$\text{for straight line } \text{Work} = \frac{P_i + P_f}{2} \Delta V$$

which is easily verified using the definition Eq. 18-25. The cycle represented by the “triangle”  $BC$  consists of three processes:

- “tilted” straight line from  $(1.0 \text{ m}^3, 40 \text{ Pa})$  to  $(4.0 \text{ m}^3, 10 \text{ Pa})$ , with

$$\text{Work} = \frac{40 \text{ Pa} + 10 \text{ Pa}}{2} (4.0 \text{ m}^3 - 1.0 \text{ m}^3) = 75 \text{ J}$$

- horizontal line from  $(4.0 \text{ m}^3, 10 \text{ Pa})$  to  $(1.0 \text{ m}^3, 10 \text{ Pa})$ , with

$$\text{Work} = (10 \text{ Pa})(1.0 \text{ m}^3 - 4.0 \text{ m}^3) = -30 \text{ J}$$

- vertical line from  $(1.0 \text{ m}^3, 10 \text{ Pa})$  to  $(1.0 \text{ m}^3, 40 \text{ Pa})$ , with

$$\text{Work} = \frac{10 \text{ Pa} + 40 \text{ Pa}}{2} (1.0 \text{ m}^3 - 1.0 \text{ m}^3) = 0$$

(a) and (b) Thus, the total work during the  $BC$  cycle is  $(75 - 30) \text{ J} = 45 \text{ J}$ . During the  $BA$  cycle, the “tilted” part is the same as before, and the main difference is that the horizontal portion is at higher pressure, with  $\text{Work} = (40 \text{ Pa})(-3.0 \text{ m}^3) = -120 \text{ J}$ . Therefore, the total work during the  $BA$  cycle is  $(75 - 120) \text{ J} = -45 \text{ J}$ .

94. For isotropic materials, the coefficient of linear expansion  $\alpha$  is related to that for volume expansion by  $\alpha = \frac{1}{3}\beta$  (Eq. 18-11). The radius of Earth may be found in the Appendix. With these assumptions, the radius of the Earth should have increased by approximately

$$\Delta R_E = R_E \alpha \Delta T = (6.4 \times 10^3 \text{ km}) \left( \frac{1}{3} \right) (3.0 \times 10^{-5} / \text{K}) (3000 \text{ K} - 300 \text{ K}) = 1.7 \times 10^2 \text{ km}.$$

95. (a) Regarding part (a), it is important to recognize that the problem is asking for the total work done during the two-step “path”:  $a \rightarrow b$  followed by  $b \rightarrow c$ . During the latter part of this “path” there is no volume change and consequently no work done. Thus, the answer to part (b) is also the answer to part (a). Since  $\Delta U$  for process  $c \rightarrow a$  is  $-160$  J, then  $U_c - U_a = 160$  J. Therefore, using the First Law of Thermodynamics, we have

$$\begin{aligned} 160 &= U_c - U_b + U_b - U_a \\ &= Q_{b \rightarrow c} - W_{b \rightarrow c} + Q_{a \rightarrow b} - W_{a \rightarrow b} \\ &= 40 - 0 + 200 - W_{a \rightarrow b} \end{aligned}$$

Therefore,  $W_{a \rightarrow b \rightarrow c} = W_{a \rightarrow b} = 80$  J.

(b)  $W_{a \rightarrow b} = 80$  J.

96. Since the combination " $p_1V_1$ " appears frequently in this derivation we denote it as " $x$ ". Thus for process 1, the heat transferred is  $Q_1 = 5x = \Delta E_{\text{int } 1} + W_1$ , and for path 2 (which consists of two steps, one at constant volume followed by an expansion accompanied by a linear pressure decrease) it is  $Q_2 = 5.5x = \Delta E_{\text{int } 2} + W_2$ . If we subtract these two expressions and make use of the fact that internal energy is state function (and thus has the same value for path 1 as for path 2) then we have

$$5.5x - 5x = W_2 - W_1 = \text{"area" inside the triangle} = \frac{1}{2} (2 V_1)(p_2 - p_1) .$$

Thus, dividing both sides by  $x (= p_1V_1)$ , we find

$$0.5 = \frac{p_2}{p_1} - 1$$

which leads immediately to the result:  $p_2/p_1 = 1.5$ .

97. The cube has six faces, each of which has an area of  $(6.0 \times 10^{-6} \text{ m})^2$ . Using Kelvin temperatures and Eq. 18-40, we obtain

$$\begin{aligned} P_{\text{net}} &= \sigma \epsilon A (T_{\text{env}}^4 - T^4) \\ &= \left( 5.67 \times 10^{-8} \frac{\text{W}}{\text{m}^2 \cdot \text{K}^4} \right) (0.75) (2.16 \times 10^{-10} \text{ m}^2) ((123.15 \text{ K})^4 - (173.15 \text{ K})^4) \\ &= -6.1 \times 10^{-9} \text{ W}. \end{aligned}$$

98. We denote the density of the liquid as  $\rho$ , the rate of liquid flowing in the calorimeter as  $\mu$ , the specific heat of the liquid as  $c$ , the rate of heat flow as  $P$ , and the temperature change as  $\Delta T$ . Consider a time duration  $dt$ , during this time interval, the amount of liquid being heated is  $dm = \mu\rho dt$ . The energy required for the heating is

$$dQ = Pdt = c(dm) \Delta T = c\mu\Delta Tdt.$$

Thus,

$$\begin{aligned} c &= \frac{P}{\rho\mu\Delta T} = \frac{250 \text{ W}}{(8.0 \times 10^{-6} \text{ m}^3/\text{s})(0.85 \times 10^3 \text{ kg/m}^3)(15^\circ\text{C})} \\ &= 2.5 \times 10^3 \text{ J/kg} \cdot \text{C}^\circ = 2.5 \times 10^3 \text{ J/kg} \cdot \text{K}. \end{aligned}$$

99. Consider the object of mass  $m_1$  falling through a distance  $h$ . The loss of its mechanical energy is  $\Delta E = m_1gh$ . This amount of energy is then used to heat up the temperature of water of mass  $m_2$ :  $\Delta E = m_1gh = Q = m_2c\Delta T$ . Thus, the maximum possible rise in water temperature is

$$\Delta T = \frac{m_1gh}{m_2c} = \frac{(6.00 \text{ kg})(9.8 \text{ m/s}^2)(50.0 \text{ m})}{(0.600 \text{ kg})(4190 \text{ J/kg} \cdot \text{C}^\circ)} = 1.17^\circ\text{C}.$$