1. (a) The center of mass is given by

$$x_{\text{com}} = \frac{0+0+0+(m)(2.00\text{ m})+(m)(2.00\text{ m})+(m)(2.00\text{ m})}{6m} = 1.00\text{ m}.$$

(b) Similarly, we have

$$y_{\text{com}} = \frac{0+(m)(2.00\text{ m})+(m)(4.00\text{ m})+(m)(4.00\text{ m})+(m)(2.00\text{ m})+0}{6m} = 2.00\text{ m}.$$

(c) Using Eq. 12-14 and noting that the gravitational effects are different at the different locations in this problem, we have

$$x_{\text{cog}} = \frac{\sum_{i=1}^{6} x_i m_i g_i}{\sum_{i=1}^{6} m_i g_i} = \frac{x_1 m_1 g_1 + x_2 m_2 g_2 + x_3 m_3 g_3 + x_4 m_4 g_4 + x_5 m_5 g_5 + x_6 m_6 g_6}{m_1 g_1 + m_2 g_2 + m_3 g_3 + m_4 g_4 + m_5 g_5 + m_6 g_6} = 0.987\text{ m}.$$

(d) Similarly, y_{cog} = [0 + (2.00)(m)(7.80) + (4.00)(m)(7.60) + (4.00)(m)(7.40) + (2.00)(m)(7.60) + 0]/(8.00m + 7.80m + 7.60m + 7.40m + 7.60m + 7.80m) = 1.97 m.

2. The situation is somewhat similar to that depicted for problem 10 (see the figure that accompanies that problem). By analyzing the forces at the "kink" where $\vec{F}$ is exerted, we find (since the acceleration is zero) $2T \sin \theta = F$, where θ is the angle (taken positive) between each segment of the string and its "relaxed" position (when the two segments are collinear). Setting $T = F$ therefore yields $\theta = 30°$. Since $\alpha = 180° - 2\theta$ is the angle between the two segments, then we find $\alpha = 120°$.

3. The object exerts a downward force of magnitude F = 3160 N at the midpoint of the rope, causing a "kink" similar to that shown for problem 10 (see the figure that accompanies that problem). By analyzing the forces at the "kink" where $\vec{F}$ is exerted, we find (since the acceleration is zero) $2T \sin\theta = F$, where θ is the angle (taken positive) between each segment of the string and its "relaxed" position (when the two segments are colinear). In this problem, we have

$$\theta = \tan^{-1}\left(\frac{0.35\,\text{m}}{1.72\,\text{m}}\right) = 11.5°.$$

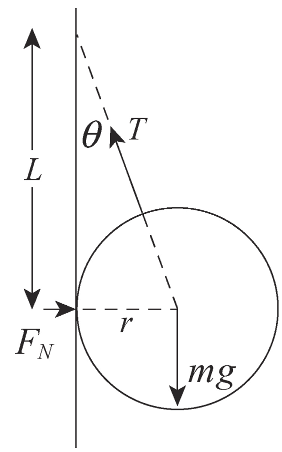
Therefore, $T = F/(2\sin\theta) = 7.92 \times 10^3$ N.

4. From $\vec{\tau} = \vec{r} \times \vec{F}$, we note that persons 1 through 4 exert torques pointing out of the page (relative to the fulcrum), and persons 5 through 8 exert torques pointing into the page.

(a) Among persons 1 through 4, the largest magnitude of torque is (330 N)(3 m) = 990 N·m, due to the weight of person 2.

(b) Among persons 5 through 8, the largest magnitude of torque is (330 N)(3 m) = 990 N·m, due to the weight of person 7.

5. Three forces act on the sphere: the tension force $\vec{T}$ of the rope (acting along the rope), the force of the wall $\vec{F}_N$ (acting horizontally away from the wall), and the force of gravity $m\vec{g}$ (acting downward). Since the sphere is in equilibrium they sum to zero. Let θ be the angle between the rope and the vertical. Then Newton's second law gives



$$\begin{aligned} \text{vertical component :} \quad & T\cos\theta - mg = 0 \\ \text{horizontal component:} \quad & F_N - T\sin\theta = 0. \end{aligned}$$

(a) We solve the first equation for the tension: $T = mg/\cos\theta$. We substitute $\cos\theta = L/\sqrt{L^2+r^2}$ to obtain

$$T = \frac{mg\sqrt{L^2+r^2}}{L} = \frac{(0.85\text{ kg})(9.8\text{ m/s}^2)\sqrt{(0.080\text{ m})^2+(0.042\text{ m})^2}}{0.080\text{ m}} = 9.4\text{ N}.$$

(b) We solve the second equation for the normal force: $F_N = T\sin\theta$. Using $\sin\theta = r/\sqrt{L^2+r^2}$, we obtain

$$F_N = \frac{Tr}{\sqrt{L^2+r^2}} = \frac{mg\sqrt{L^2+r^2}}{L}\frac{r}{\sqrt{L^2+r^2}} = \frac{mgr}{L} = \frac{(0.85\text{ kg})(9.8\text{ m/s}^2)(0.042\text{ m})}{(0.080\text{ m})} = 4.4\text{ N}.$$

6. Our notation is as follows: M = 1360 kg is the mass of the automobile; L = 3.05 m is the horizontal distance between the axles; $\ell = (3.05 - 1.78)$ m $= 1.27$ m is the horizontal distance from the rear axle to the center of mass; F_1 is the force exerted on each front wheel; and, F_2 is the force exerted on each back wheel.

(a) Taking torques about the rear axle, we find

$$F_1 = \frac{Mg\ell}{2L} = \frac{(1360\,\text{kg})(9.80\,\text{m/s}^2)(1.27\,\text{m})}{2(3.05\,\text{m})} = 2.77\times10^3\ \text{N}.$$

(b) Equilibrium of forces leads to $2F_1 + 2F_2 = Mg$, from which we obtain $F_2 = 3.89\times10^3\,\text{N}$.

7. We take the force of the left pedestal to be F_1 at $x = 0$, where the x axis is along the diving board. We take the force of the right pedestal to be F_2 and denote its position as $x = d$. W is the weight of the diver, located at $x = L$. The following two equations result from setting the sum of forces equal to zero (with upwards positive), and the sum of torques (about x_2) equal to zero:

$$F_1 + F_2 - W = 0$$
$$F_1 d + W(L - d) = 0$$

(a) The second equation gives

$$F_1 = -\frac{L-d}{d}W = -\left(\frac{3.0\,\text{m}}{1.5\,\text{m}}\right)(580\,\text{N}) = -1160\,\text{N}$$

which should be rounded off to $F_1 = -1.2\times10^3$ N. Thus, $|F_1| = 1.2\times10^3$ N.

(b) Since F_1 is negative, indicating that this force is downward.

(c) The first equation gives $F_2 = W - F_1 = 580\,\text{N} + 1160\,\text{N} = 1740\,\text{N}$

which should be rounded off to $F_2 = 1.7\times10^3$ N. Thus, $|F_2| = 1.7\times10^3$ N.

(d) The result is positive, indicating that this force is upward.

(e) The force of the diving board on the left pedestal is upward (opposite to the force of the pedestal on the diving board), so this pedestal is being stretched.

(f) The force of the diving board on the right pedestal is downward, so this pedestal is being compressed.

8. Let $\ell_1 = 1.5\,\text{m}$ and $\ell_2 = (5.0 - 1.5)\ \text{m} = 3.5\ \text{m}$. We denote tension in the cable closer to the window as F_1 and that in the other cable as F_2. The force of gravity on the scaffold itself (of magnitude $m_s g$) is at its midpoint, $\ell_3 = 2.5\,\text{m}$ from either end.

(a) Taking torques about the end of the plank farthest from the window washer, we find

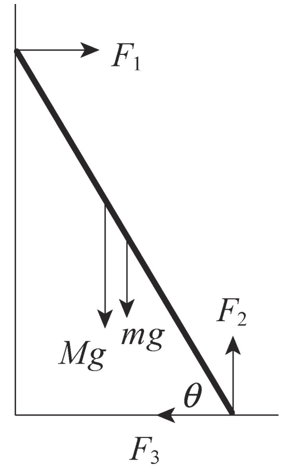
$$F_1 = \frac{m_w g \ell_2 + m_s g \ell_3}{\ell_1 + \ell_2} = \frac{(80\,\text{kg})(9.8\,\text{m/s}^2)(3.5\,\text{m}) + (60\,\text{kg})(9.8\,\text{m/s}^2)(2.5\,\text{m})}{5.0\,\text{m}}$$
$$= 8.4 \times 10^2\ \text{N}.$$

(b) Equilibrium of forces leads to

$$F_1 + F_2 = m_s g + m_w g = (60\,\text{kg} + 80\,\text{kg})(9.8\,\text{m/s}^2) = 1.4 \times 10^3\ \text{N}$$

which (using our result from part (a)) yields $F_2 = 5.3 \times 10^2\,\text{N}$.

9. The forces on the ladder are shown in the diagram on the right. F_1 is the force of the window, horizontal because the window is frictionless. F_2 and F_3 are components of the force of the ground on the ladder. M is the mass of the window cleaner and m is the mass of the ladder.



The force of gravity on the man acts at a point 3.0 m up the ladder and the force of gravity on the ladder acts at the center of the ladder. Let θ be the angle between the ladder and the ground. We use $\cos\theta = d/L$ or $\sin\theta = \sqrt{L^2 - d^2}/L$ to find θ = 60º. Here L is the length of the ladder (5.0 m) and d is the distance from the wall to the foot of the ladder (2.5 m).

(a) Since the ladder is in equilibrium the sum of the torques about its foot (or any other point) vanishes. Let ℓ be the distance from the foot of the ladder to the position of the window cleaner. Then,

$$Mg\ell\cos\theta + mg\left(L/2\right)\cos\theta - F_1 L\sin\theta = 0\,,$$

and

$$F_1 = \frac{(M\ell + mL/2)g\cos\theta}{L\sin\theta} = \frac{[(75\,\text{kg})(3.0\,\text{m}){+}(10\,\text{kg})(2.5\,\text{m})](9.8\,\text{m/s}^2)\cos 60°}{(5.0\,\text{m})\sin 60°}$$
$$= 2.8\times10^2\ \text{N}.$$

This force is outward, away from the wall. The force of the ladder on the window has the same magnitude but is in the opposite direction: it is approximately 280 N, inward.

(b) The sum of the horizontal forces and the sum of the vertical forces also vanish:

$$F_1 - F_3 = 0$$
$$F_2 - Mg - mg = 0$$

The first of these equations gives $F_3 = F_1 = 2.8\times10^2\,\text{N}$ and the second gives

$$F_2 = (M+m)g = (75\,\text{kg} + 10\,\text{kg})(9.8\,\text{m/s}^2) = 8.3\times10^2\,\text{N}$$

The magnitude of the force of the ground on the ladder is given by the square root of the sum of the squares of its components:

$$F = \sqrt{F_2^{\,2} + F_3^{\,2}} = \sqrt{(2.8\times10^2\,\text{N})^2 + (8.3\times10^2\,\text{N})^2} = 8.8\times10^2\,\text{N}.$$

(c) The angle ϕ between the force and the horizontal is given by

$$\tan\phi = F_3/F_2 = 830/280 = 2.94,$$

so ϕ = 71º. The force points to the left and upward, 71º above the horizontal. We note that this force is not directed along the ladder.

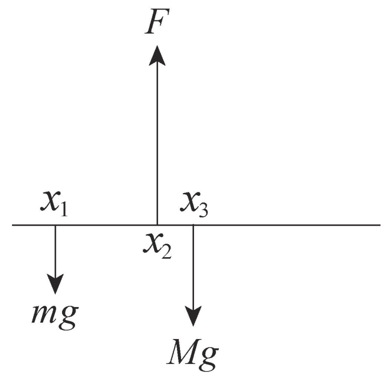
10. The angle of each half of the rope, measured from the dashed line, is

$$\theta = \tan^{-1}\left(\frac{0.30\,\text{m}}{9.0\,\text{m}}\right) = 1.9°.$$

Analyzing forces at the "kink" (where $\vec{F}$ is exerted) we find

$$T = \frac{F}{2\sin\theta} = \frac{550\,\text{N}}{2\sin 1.9°} = 8.3\times10^{3}\ \text{N}.$$

11. The x axis is along the meter stick, with the origin at the zero position on the scale. The forces acting on it are shown on the diagram below. The nickels are at $x = x_1 = 0.120$ m, and m is their total mass. The knife edge is at $x = x_2 = 0.455$ m and exerts force $\vec{F}$. The mass of the meter stick is M, and the force of gravity acts at the center of the stick, $x = x_3 = 0.500$ m. Since the meter stick is in equilibrium, the sum of the torques about x_2 must vanish:



$$Mg(x_3 - x_2) - mg(x_2 - x_1) = 0.$$

Thus,

$$M = \frac{x_2 - x_1}{x_3 - x_2} m = \left(\frac{0.455\,\text{m} - 0.120\,\text{m}}{0.500\,\text{m} - 0.455\,\text{m}} \right) (10.0\text{g}) = 74.4\ \text{g}.$$

12. (a) Analyzing vertical forces where string 1 and string 2 meet, we find

$$T_1 = \frac{w_A}{\cos\phi} = \frac{40\text{N}}{\cos 35°} = 49\text{N}.$$

(b) Looking at the horizontal forces at that point leads to

$$T_2 = T_1 \sin 35° = (49\text{N}) \sin 35° = 28\,\text{N}.$$

(c) We denote the components of T_3 as T_x (rightward) and T_y (upward). Analyzing horizontal forces where string 2 and string 3 meet, we find $T_x = T_2 = 28$ N. From the vertical forces there, we conclude $T_y = w_B$ =50 N. Therefore,

$$T_3 = \sqrt{T_x^2 + T_y^2} = 57\,\text{N}.$$

(d) The angle of string 3 (measured from vertical) is

$$\theta = \tan^{-1}\left(\frac{T_x}{T_y}\right) = \tan^{-1}\left(\frac{28}{50}\right) = 29°.$$

13. (a) Analyzing the horizontal forces (which add to zero) we find $F_h = F_3 = 5.0$ N.

(b) Equilibrium of vertical forces leads to $F_v = F_1 + F_2 = 30$ N.

(c) Computing torques about point O, we obtain

$$F_v d = F_2 b + F_3 a \Rightarrow d = \frac{(10\text{ N})(3.0\,\text{m}) + (5.0\text{ N})(2.0\,\text{m})}{30\text{ N}} = 1.3\,\text{m}.$$

14. The forces exerted horizontally by the obstruction and vertically (upward) by the floor are applied at the bottom front corner C of the crate, as it verges on tipping. The center of the crate, which is where we locate the gravity force of magnitude mg = 500 N, is a horizontal distance $\ell = 0.375\,\text{m}$ from C. The applied force of magnitude F = 350 N is a vertical distance h from C. Taking torques about C, we obtain

$$h = \frac{mg\ell}{F} = \frac{(500\,\text{N})(0.375\,\text{m})}{350\,\text{N}} = 0.536\,\text{m}.$$

15. Setting up equilibrium of torques leads to a simple “level principle” ratio:

$$F_{\perp} = (40\,\text{N})\frac{d}{L} = (40\,\text{N})\frac{2.6\,\text{cm}}{12\,\text{cm}} = 8.7\,\text{N}.$$

16. With pivot at the left end, Eq. 12-9 leads to

$$-m_s g \frac{L}{2} - Mgx + T_R L = 0$$

where m_s is the scaffold's mass (50 kg) and M is the total mass of the paint cans (75 kg). The variable x indicates the center of mass of the paint can collection (as measured from the left end), and T_R is the tension in the right cable (722 N). Thus we obtain $x = 0.702$ m.

17. The (vertical) forces at points *A, B* and *P* are F_A, F_B and F_P, respectively. We note that $F_P = W$ and is upward. Equilibrium of forces and torques (about point *B*) lead to

$$F_A + F_B + W = 0$$
$$bW - aF_A = 0.$$

(a) From the second equation, we find

$$F_A = bW/a = (15/5)W = 3W = 3(900 \text{ N}) = 2.7\times10^3 \text{ N}.$$

(b) The direction is upward since $F_A > 0$.

(c) Using this result in the first equation above, we obtain

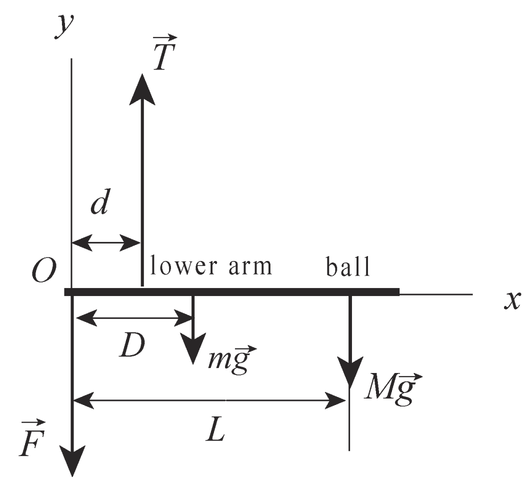
$$F_B = W - F_A = -4W = -4(900 \text{ N}) = -3.6\times10^3 \text{ N},$$

or $|F_B| = 3.6\times10^3 \text{ N}$.

(d) F_B points downward, as indicated by the minus sign.

18. Our system consists of the lower arm holding a bowling ball. As shown in the free-body diagram, the forces on the lower arm consist of $\vec{T}$ from the biceps muscle, $\vec{F}$ from the bone of the upper arm, and the gravitational forces, $m\vec{g}$ and $M\vec{g}$. Since the system is in static equilibrium, the net force acting on the system is zero:

$$0=\sum F_{\text{net},y}=T-F-(m+M)g$$



In addition, the net torque about O must also vanish:

$$0=\sum_{O}\tau_{\text{net}}=(d)(T)+(0)F-(D)(mg)-L(Mg)\,.$$

(a) From the torque equation, we find the force on the lower arms by the biceps muscle to be

$$T=\frac{(mD+ML)g}{d}=\frac{[(1.8\text{ kg})(0.15\text{ m})+(7.2\text{ kg})(0.33\text{ m})](9.8\text{ m/s}^2)}{0.040\text{ m}}$$
$$=648\text{ N}\approx 6.5\times 10^2\text{ N}.$$

(b) Substituting the above result into the force equation, we find F to be

$$F=T-(M+m)g=648\text{ N}-(7.2\text{ kg}+1.8\text{ kg})(9.8\text{ m/s}^2)=560\text{ N}=5.6\times 10^2\text{ N}.$$

19. (a) With the pivot at the hinge, Eq. 12-9 gives $TL\cos\theta - mg\frac{L}{2} = 0$. This leads to $\theta = 78°$. Then the geometric relation $\tan\theta = L/D$ gives $D = 0.64$ m.

(b) A higher (steeper) slope for the cable results in a smaller tension. Thus, making D greater than the value of part (a) should prevent rupture.

20. With pivot at the left end of the lower scaffold, Eq. 12-9 leads to

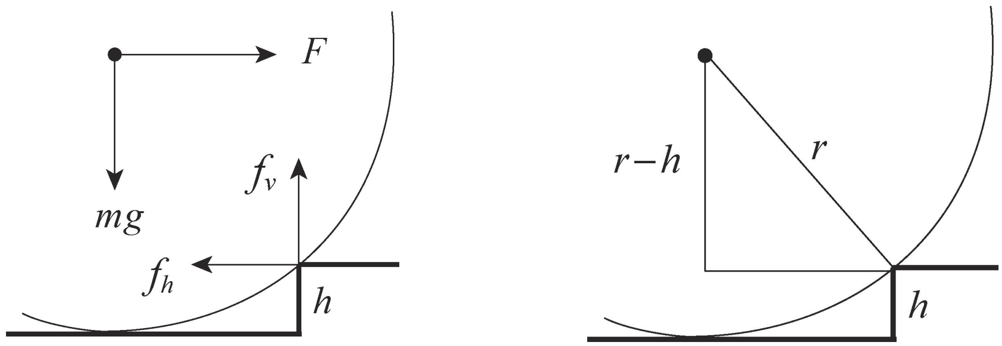
$$-m_2 g \frac{L_2}{2} - mgd + T_R L_2 = 0$$

where m_2 is the lower scaffold's mass (30 kg) and L_2 is the lower scaffold's length (2.00 m). The mass of the package (m = 20 kg) is a distance d = 0.50 m from the pivot, and T_R is the tension in the rope connecting the right end of the lower scaffold to the larger scaffold above it. This equation yields T_R = 196 N. Then Eq. 12-8 determines T_L (the tension in the cable connecting the right end of the lower scaffold to the larger scaffold above it): T_L = 294 N. Next, we analyze the larger scaffold (of length $L_1 = L_2 + 2d$ and mass m_1, given in the problem statement) placing our pivot at its left end and using Eq. 12-9:

$$-m_1 g \frac{L_1}{2} - T_L d - T_R (L_1 - d) + T L_1 = 0 .$$

This yields T = 457 N.

21. We consider the wheel as it leaves the lower floor. The floor no longer exerts a force on the wheel, and the only forces acting are the force F applied horizontally at the axle, the force of gravity mg acting vertically at the center of the wheel, and the force of the step corner, shown as the two components f_h and f_v. If the minimum force is applied the wheel does not accelerate, so both the total force and the total torque acting on it are zero.



We calculate the torque around the step corner. The second diagram indicates that the distance from the line of F to the corner is $r - h$, where r is the radius of the wheel and h is the height of the step.

The distance from the line of mg to the corner is $\sqrt{r^2 + (r-h)^2} = \sqrt{2rh - h^2}$. Thus,

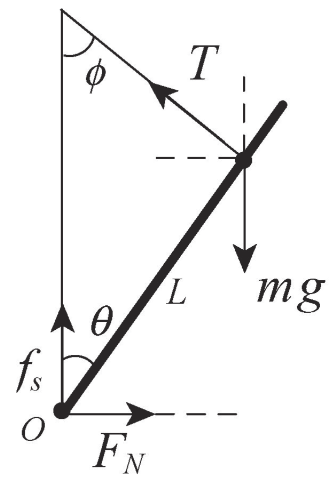
$$F(r-h) - mg\sqrt{2rh-h^2} = 0\,.$$

The solution for F is

$$F = \frac{\sqrt{2rh-h^2}}{r-h}mg = \frac{\sqrt{2(6.00\times10^{-2}\,\text{m})(3.00\times10^{-2}\,\text{m}) - (3.00\times10^{-2}\,\text{m})^2}}{(6.00\times10^{-2}\,\text{m}) - (3.00\times10^{-2}\,\text{m})}(0.800\ \text{kg})(9.80\ \text{m/s}^2)$$

$$= 13.6\ \text{N}.$$

22. As shown in the free-body diagram, the forces on the climber consist of $\vec{T}$ from the rope, normal force $\vec{F}_N$ on her feet, upward static frictional force $\vec{f}_s$ and downward gravitational force $m\vec{g}$. Since the climber is in static equilibrium, the net force acting on her is zero. Applying Newton's second law to the vertical and horizontal directions, we have



$$0 = \sum F_{\text{net},x} = F_N - T\sin\phi$$
$$0 = \sum F_{\text{net},y} = T\cos\phi + f_s - mg\,.$$

In addition, the net torque about O (contact point between her feet and the wall) must also vanish:

$$0 = \sum_O \tau_{\text{net}} = mgL\sin\theta - TL\sin(180° - \theta - \phi)$$

From the torque equation, we obtain $T = mg\sin\theta/\sin(180° - \theta - \phi)$. Substituting the expression into the force equations, and noting that $f_s = \mu_s F_N$, we find the coefficient of static friction to be

$$\mu_s = \frac{f_s}{F_N} = \frac{mg - T\cos\phi}{T\sin\phi} = \frac{mg - mg\sin\theta\cos\phi/\sin(180° - \theta - \phi)}{mg\sin\theta\sin\phi/\sin(180° - \theta - \phi)}$$
$$= \frac{1 - \sin\theta\cos\phi/\sin(180° - \theta - \phi)}{\sin\theta\sin\phi/\sin(180° - \theta - \phi)}$$

With $\theta = 40°$ and $\phi = 30°$, the result is

$$\mu_s = \frac{1 - \sin\theta\cos\phi/\sin(180° - \theta - \phi)}{\sin\theta\sin\phi/\sin(180° - \theta - \phi)} = \frac{1 - \sin 40°\cos 30°/\sin(180° - 40° - 30°)}{\sin 40°\sin 30°/\sin(180° - 40° - 30°)} = 1.19.$$

23. (a) All forces are vertical and all distances are measured along an axis inclined at $\theta =$ 30º. Thus, any trigonometric factor cancels out and the application of torques about the contact point (referred to in the problem) leads to

$$F_{\text{tripcep}} = \frac{(15\,\text{kg})(9.8\,\text{m/s}^2)(35\,\text{cm}) - (2.0\,\text{kg})(9.8\,\text{m/s}^2)(15\,\text{cm})}{2.5\,\text{cm}} = 1.9\times10^3\ \text{N}.$$

(b) The direction is upward since $F_{\text{tricep}} > 0$

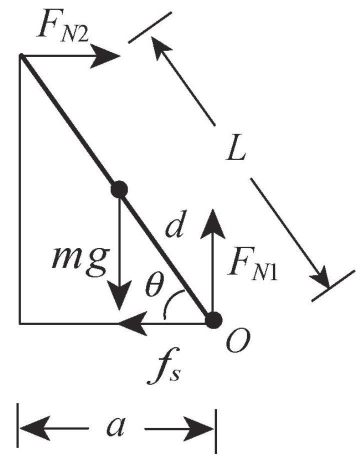
(c) Equilibrium of forces (with upwards positive) leads to

$$F_{\text{tripcep}} + F_{\text{humer}} + (15\,\text{kg})(9.8\,\text{m/s}^2) - (2.0\,\text{kg})(9.8\,\text{m/s}^2) = 0$$

and thus to $F_{\text{humer}} = -2.1\times10^3\,\text{N}$, or $|\, F_{\text{humer}} \,| = 2.1\times10^3\,\text{N}$.

(d) The minus sign implies that F_{humer} points downward.

24. As shown in the free-body diagram, the forces on the climber consist of the normal forces F_{N1} on his hands from the ground and F_{N2} on his feet from the wall, static frictional force f_s and downward gravitational force mg. Since the climber is in static equilibrium, the net force acting on him is zero. Applying Newton's second law to the vertical and horizontal directions, we have



$$0 = \sum F_{\text{net},x} = F_{N2} - f_s$$
$$0 = \sum F_{\text{net},y} = F_{N1} - mg\,.$$

In addition, the net torque about O (contact point between his feet and the wall) must also vanish:

$$0 = \sum_O \tau_{\text{net}} = mgd\cos\theta - F_{N2}L\sin\theta\,.$$

The torque equation gives $F_{N2} = mgd\cos\theta / L\sin\theta = mgd\cot\theta / L$. On the other hand, from the force equation we have $F_{N2} = f_s$ and $F_{N1} = mg$. These expressions can be combined to yield

$$f_s = F_{N2} = F_{N1}\cot\theta\frac{d}{L}\,.$$

On the other hand, the frictional force can also be written as $f_s = \mu_s F_{N1}$, where μ_s is the coefficient of static friction between his feet and the ground. From the above equation and the values given in the problem statement, we find μ_s to be

$$\mu_s = \cot\theta\frac{d}{L} = \frac{a}{\sqrt{L^2 - a^2}}\frac{d}{L} = \frac{0.914\text{ m}}{\sqrt{(2.10\text{ m})^2 - (0.914\text{ m})^2}}\frac{0.940\text{ m}}{2.10\text{ m}} = 0.216\,.$$

25. The beam is in equilibrium: the sum of the forces and the sum of the torques acting on it each vanish. As shown in the figure, the beam makes an angle of 60º with the vertical and the wire makes an angle of 30º with the vertical.

(a) We calculate the torques around the hinge. Their sum is

$$TL \sin 30^\circ - W(L/2) \sin 60^\circ = 0.$$

Here W is the force of gravity acting at the center of the beam, and T is the tension force of the wire. We solve for the tension:

$$T = \frac{W \sin 60^\circ}{2 \sin 30^\circ} = \frac{(222\,\text{N}) \sin 60^\circ}{2 \sin 30^\circ} = 192\,\text{N}.$$

(b) Let F_h be the horizontal component of the force exerted by the hinge and take it to be positive if the force is outward from the wall. Then, the vanishing of the horizontal component of the net force on the beam yields $F_h - T \sin 30^\circ = 0$ or

$$F_h = T \sin 30^\circ = (192.3\,\text{N}) \sin 30^\circ = 96.1\,\text{N}.$$

(c) Let F_v be the vertical component of the force exerted by the hinge and take it to be positive if it is upward. Then, the vanishing of the vertical component of the net force on the beam yields $F_v + T \cos 30^\circ - W = 0$ or

$$F_v = W - T \cos 30^\circ = 222\,\text{N} - (192.3\,\text{N}) \cos 30^\circ = 55.5\,\text{N}.$$

26. (a) The problem asks for the person's pull (his force exerted on the rock) but since we are examining forces and torques *on the person*, we solve for the reaction force F_{N1} (exerted leftward on the hands by the rock). At that point, there is also an upward force of static friction on his hands f_1 which we will take to be at its maximum value $\mu_1 F_{N1}$. We note that equilibrium of horizontal forces requires $F_{N1} = F_{N2}$ (the force exerted leftward on his feet); on this feet there is also an upward static friction force of magnitude $\mu_2 F_{N2}$. Equilibrium of vertical forces gives

$$f_1 + f_2 - mg = 0 \Rightarrow F_{N1} = \frac{mg}{\mu_1 + \mu_2} = 3.4\times10^2\,\mathrm{N}.$$

(b) Computing torques about the point where his feet come in contact with the rock, we find

$$mg\left(d+w\right) - f_1 w - F_{N1} h = 0 \;\Rightarrow\; h = \frac{mg\left(d+w\right) - \mu_1 F_{N1} w}{F_{N1}} = 0.88\ \mathrm{m}.$$

(c) Both intuitively and mathematically (since both coefficients are in the denominator) we see from part (a) that F_{N1} would increase in such a case.

(d) As for part (b), it helps to plug part (a) into part (b) and simplify:

$$h = (d+w)\mu_2 + d\mu_1$$

from which it becomes apparent that h should decrease if the coefficients decrease.

27. (a) We note that the angle between the cable and the strut is

$$\alpha = \theta - \phi = 45^\circ - 30^\circ = 15^\circ.$$

The angle between the strut and any vertical force (like the weights in the problem) is β = 90º – 45º = 45º. Denoting M = 225 kg and m = 45.0 kg, and ℓ as the length of the boom, we compute torques about the hinge and find

$$T = \frac{Mg\ell\sin\beta + mg\left(\frac{\ell}{2}\right)\sin\beta}{\ell\sin\alpha} = \frac{Mg\sin\beta + mg\sin\beta/2}{\sin\alpha}.$$

The unknown length ℓ cancels out and we obtain $T = 6.63 \times 10^3$ N.

(b) Since the cable is at 30º from horizontal, then horizontal equilibrium of forces requires that the horizontal hinge force be

$$F_x = T\cos 30^\circ = 5.74\times 10^3\,\text{N}.$$

(c) And vertical equilibrium of forces gives the vertical hinge force component:

$$F_y = Mg + mg + T\sin 30^\circ = 5.96\times 10^3\,\text{N}.$$

28. (a) The sign is attached in two places: at x_1 = 1.00 m (measured rightward from the hinge) and at x_2 = 3.00 m. We assume the downward force due to the sign's weight is equal at these two attachment points: each being *half* the sign's weight of mg. The angle where the cable comes into contact (also at x_2) is

$$\theta = \tan^{-1}(d_v/d_h) = \tan^{-1}(4.00 \text{ m}/3.00 \text{ m})$$

and the force exerted there is the tension T. Computing torques about the hinge, we find

$$T = \frac{\frac{1}{2}mgx_1 + \frac{1}{2}mgx_2}{x_2 \sin\theta} = \frac{\frac{1}{2}(50.0 \text{ kg})\left(9.8 \text{ m/s}^2\right)\left(1.00 \text{ m}\right) + \frac{1}{2}(50.0 \text{ kg})(9.8\text{m/s}^2)(3.00 \text{ m})}{(3.00 \text{ m})(0.800)}$$
$$= 408 \text{ N}.$$

(b) Equilibrium of horizontal forces requires the horizontal hinge force be

$$F_x = T\cos\theta = 245 \text{ N}.$$

(c) The direction of the horizontal force is rightward.

(d) Equilibrium of vertical forces requires the vertical hinge force be

$$F_y = mg - T\sin\theta = 163 \text{ N}.$$

(e) The direction of the vertical force is upward.

29. The bar is in equilibrium, so the forces and the torques acting on it each sum to zero. Let T_l be the tension force of the left–hand cord, T_r be the tension force of the right–hand cord, and m be the mass of the bar. The equations for equilibrium are:

$$\begin{array}{rr} \text{vertical force components} & T_l\cos\theta + T_r\cos\phi - mg = 0 \\ \text{horizontal force components} & -T_l\sin\theta + T_r\sin\phi = 0 \\ \text{torques} & mgx - T_r L\cos\phi = 0. \end{array}$$

The origin was chosen to be at the left end of the bar for purposes of calculating the torque. The unknown quantities are T_l, T_r, and x. We want to eliminate T_l and T_r, then solve for x. The second equation yields $T_l = T_r \sin\phi/\sin\theta$ and when this is substituted into the first and solved for T_r the result is

$$T_r = \frac{mg\sin\theta}{\sin\phi\cos\theta + \cos\phi\sin\theta}.$$

This expression is substituted into the third equation and the result is solved for x:

$$x = L\frac{\sin\theta\cos\phi}{\sin\phi\cos\theta + \cos\phi\sin\theta} = L\frac{\sin\theta\cos\phi}{\sin(\theta+\phi)}.$$

The last form was obtained using the trigonometric identity $\sin(A + B) = \sin A\cos B + \cos A\sin B$. For the special case of this problem $\theta + \phi = 90^\circ$ and $\sin(\theta + \phi) = 1$. Thus,

$$x = L\sin\theta\cos\phi = (6.10\text{ m})\ \sin 36.9°\cos 53.1° = 2.20\text{ m}.$$

30. (a) Computing torques about point A, we find

$$T_{\max} L \sin\theta = W x_{\max} + W_b \left(\frac{L}{2}\right).$$

We solve for the maximum distance:

$$x_{\max} = \left(\frac{T_{\max}\sin\theta - W_b/2}{W}\right)L = \left(\frac{(500\text{ N})\sin 30.0° - (200\text{ N})/2}{300\text{ N}}\right)(3.00\text{ m}) = 1.50\text{ m}.$$

(b) Equilibrium of horizontal forces gives $F_x = T_{\max}\cos\theta = 433\text{ N}$.

(c) And equilibrium of vertical forces gives $F_y = W + W_b - T_{\max}\sin\theta = 250\text{ N}$.

31. The problem states that each hinge supports half the door's weight, so each vertical hinge force component is $F_y = mg/2 = 1.3 \times 10^2$ N. Computing torques about the top hinge, we find the horizontal hinge force component (at the bottom hinge) is

$$F_h = \frac{(27\,\text{kg})(9.8\,\text{m/s}^2)(0.91\text{ m}/2)}{2.1\,\text{m} - 2(0.30\,\text{m})} = 80\,\text{N}.$$

Equilibrium of horizontal forces demands that the horizontal component of the top hinge force has the same magnitude (though opposite direction).

(a) In unit-vector notation, the force on the door at the top hinge is

$$F_{\text{top}} = (-80\text{ N})\hat{\text{i}} + (1.3\times10^2\,\text{N})\hat{\text{j}}.$$

(b) Similarly, the force on the door at the bottom hinge is

$$F_{\text{bottom}} = (+80\text{ N})\hat{\text{i}} + (1.3\times10^2\,\text{N})\hat{\text{j}}$$

32. (a) Computing torques about the hinge, we find the tension in the wire:

$$TL\sin\theta - Wx = 0 \Rightarrow T = \frac{Wx}{L\sin\theta}.$$

(b) The horizontal component of the tension is $T\cos\theta$, so equilibrium of horizontal forces requires that the horizontal component of the hinge force is

$$F_x = \left(\frac{Wx}{L\sin\theta}\right)\cos\theta = \frac{Wx}{L\tan\theta}.$$

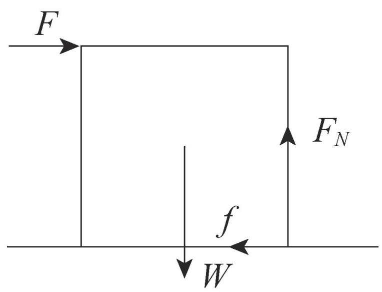
(c) The vertical component of the tension is $T\sin\theta$, so equilibrium of vertical forces requires that the vertical component of the hinge force is

$$F_y = W - \left(\frac{Wx}{L\sin\theta}\right)\sin\theta = W\left(1 - \frac{x}{L}\right).$$

33. We examine the box when it is about to tip. Since it will rotate about the lower right edge, that is where the normal force of the floor is exerted. This force is labeled F_N on the diagram below. The force of friction is denoted by f, the applied force by F, and the force of gravity by W. Note that the force of gravity is applied at the center of the box. When the minimum force is applied the box does not accelerate, so the sum of the horizontal force components vanishes: $F - f = 0$, the sum of the vertical force components vanishes: $F_N - W = 0$, and the sum of the torques vanishes:

$$FL - WL/2 = 0.$$

Here L is the length of a side of the box and the origin was chosen to be at the lower right edge.



(a) From the torque equation, we find

$$F = \frac{W}{2} = \frac{890\,\text{N}}{2} = 445\,\text{N}.$$

(b) The coefficient of static friction must be large enough that the box does not slip. The box is on the verge of slipping if $\mu_s = f/F_N$. According to the equations of equilibrium

$$F_N = W = 890 \text{ N and } f = F = 445 \text{ N},$$

so

$$\mu_s = \frac{445\,\text{N}}{890\,\text{N}} = 0.50.$$

(c) The box can be rolled with a smaller applied force if the force points upward as well as to the right. Let θ be the angle the force makes with the horizontal. The torque equation then becomes

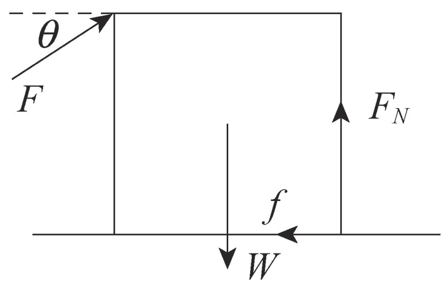
$$FL \cos \theta + FL \sin \theta - WL/2 = 0,$$

with the solution

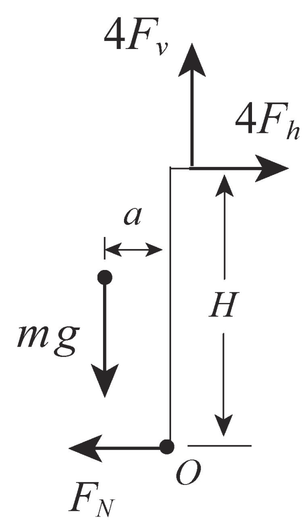
$$F = \frac{W}{2(\cos\theta + \sin\theta)}.$$

We want $\cos\theta + \sin\theta$ to have the largest possible value. This occurs if $\theta = 45°$, a result we can prove by setting the derivative of $\cos\theta + \sin\theta$ equal to zero and solving for θ. The minimum force needed is

$$F = \frac{W}{4\cos 45°} = \frac{890\ \text{N}}{4\cos 45°} = 315\ \text{N}.$$



34. As shown in the free-body diagram, the forces on the climber consist of the normal force from the wall, the vertical component F_v and the horizontal component F_h of the force acting on her four fingertips, and the downward gravitational force mg. Since the climber is in static equilibrium, the net force acting on her is zero. Applying Newton's second law to the vertical and horizontal directions, we have



$$0 = \sum F_{\text{net},x} = 4F_h - F_N$$
$$0 = \sum F_{\text{net},y} = 4F_v - mg\,.$$

In addition, the net torque about O (contact point between her feet and the wall) must also vanish:

$$0 = \sum_O \tau_{\text{net}} = (mg)a - (4F_h)H\,.$$

(a) From the torque equation, we find the horizontal component of the force on her fingertip to be

$$F_h = \frac{mga}{4H} = \frac{(70\text{ kg})(9.8\text{ m/s}^2)(0.20\text{ m})}{4(2.0\text{ m})} \approx 17\text{ N.}$$

(b) From the y-component of the force equation, we obtain

$$F_v = \frac{mg}{4} = \frac{(70\text{ kg})(9.8\text{ m/s}^2)}{4} \approx 1.7\times 10^2\text{ N.}$$

35. (a) With the pivot at the hinge, Eq. 12-9 yields

$$TL\cos\theta - F_a y = 0 \,.$$

This leads to $T = (F_a/\cos\theta)(y/L)$ so that we can interpret $F_a/\cos\theta$ as the slope on the tension graph (which we estimate to be 600 in SI units). Regarding the F_h graph, we use Eq. 12–7 to get

$$F_h = T\cos\theta - F_a = (-F_a)(y/L) - F_a$$

after substituting our previous expression. The result implies that the slope on the F_h graph (which we estimate to be –300) is equal to $-F_a$, or F_a = 300 N and (plugging back in) θ = 60.0°.

(b) As mentioned in the previous part, F_a = 300 N.

36. (a) With $F = ma = -\mu_k mg$ the magnitude of the deceleration is

$$|a| = \mu_k g = (0.40)(9.8 \text{ m/s}^2) = 3.92 \text{ m/s}^2.$$

(b) As hinted in the problem statement, we can use Eq. 12-9, evaluating the torques about the car's center of mass, and bearing in mind that the friction forces are acting horizontally at the bottom of the wheels; the total friction force there is $f_k = \mu_k gm = 3.92m$ (with SI units understood – and m is the car's mass), a vertical distance of 0.75 meter below the center of mass. Thus, torque equilibrium leads to

$$(3.92m)(0.75) + F_{Nr}(2.4) - F_{Nf}(1.8) = 0 .$$

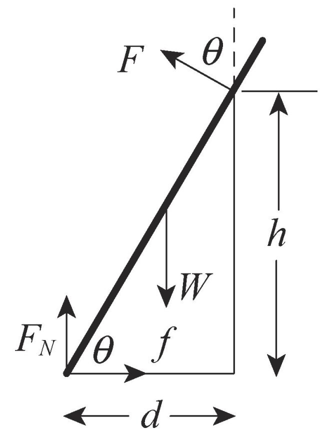
Eq. 12-8 also holds (the acceleration is horizontal, not vertical), so we have $F_{Nr} + F_{Nf} = mg$, which we can solve simultaneously with the above torque equation. The mass is obtained from the car's weight: $m = 11000/9.8$, and we obtain $F_{Nr} = 3929 \approx 4000$ N. Since each involves <u>two</u> wheels then we have (roughly) 2.0×10^3 N on each rear wheel.

(c) From the above equation, we also have $F_{Nf} = 7071 \approx 7000$ N, or 3.5×10^3 N on each front wheel, as the values of the individual normal forces.

(d) Eq. 6-2 directly yields (approximately) 7.9×10^2 N of friction on each rear wheel,

(e) Similarly, Eq. 6-2 yields 1.4×10^3 N on each front wheel.

37. The free-body diagram on the right shows the forces acting on the plank. Since the roller is frictionless the force it exerts is normal to the plank and makes the angle θ with the vertical. Its magnitude is designated F. W is the force of gravity; this force acts at the center of the plank, a distance $L/2$ from the point where the plank touches the floor. F_N is the normal force of the floor and f is the force of friction. The distance from the foot of the plank to the wall is denoted by d. This quantity is not given directly but it can be computed using $d = h/\tan\theta$.



The equations of equilibrium are:

horizontal force components	$F\sin\theta - f = 0$
vertical force components	$F\cos\theta - W + F_N = 0$
torques	$F_N d - fh - W\left(d - \frac{L}{2}\cos\theta\right) = 0.$

The point of contact between the plank and the roller was used as the origin for writing the torque equation.

When $\theta = 70°$ the plank just begins to slip and $f = \mu_s F_N$, where μ_s is the coefficient of static friction. We want to use the equations of equilibrium to compute F_N and f for $\theta = 70°$, then use $\mu_s = f/F_N$ to compute the coefficient of friction.

The second equation gives $F = (W - F_N)/\cos\,\theta$ and this is substituted into the first to obtain

$$f = (W - F_N)\sin\theta/\cos\theta = (W - F_N)\tan\theta.$$

This is substituted into the third equation and the result is solved for F_N:

$$F_N = \frac{d - (L/2)\cos\theta + h\tan\theta}{d + h\tan\theta}W = \frac{h(1+\tan^2\theta) - (L/2)\sin\theta}{h(1+\tan^2\theta)}W,$$

where we have use $d = h/\tan\theta$ and multiplied both numerator and denominator by $\tan\,\theta$. We use the trigonometric identity $1 + \tan^2\theta = 1/\cos^2\theta$ and multiply both numerator and denominator by $\cos^2\theta$ to obtain

$$F_N = W\left(1 - \frac{L}{2h}\cos^2\theta\sin\theta\right).$$

Now we use this expression for F_N in $f = (W - F_N)\tan\,\theta$ to find the friction:

$$f = \frac{WL}{2h}\sin^2\theta\cos\theta.$$

We substitute these expressions for f and F_N into $\mu_s = f/F_N$ and obtain

$$\mu_s = \frac{L\sin^2\theta\cos\theta}{2h - L\sin\theta\cos^2\theta}.$$

Evaluating this expression for $\theta = 70°$, we obtain

$$\mu_s = \frac{(6.1\,\text{m})\sin^2 70°\cos 70°}{2(3.05\,\text{m}) - (6.1\,\text{m})\sin 70°\cos^2 70°} = 0.34.$$

38. The phrase "loosely bolted" means that there is no torque exerted by the bolt at that point (where A connects with B). The force exerted on A at the hinge has x and y components F_x and F_y. The force exerted on A at the bolt has components G_x and G_y and those exerted on B are simply $-G_x$ and $-G_y$ by Newton's third law. The force exerted on B at its hinge has components H_x and H_y. If a horizontal force is positive, it points rightward, and if a vertical force is positive it points upward.

(a) We consider the combined A∪B system, which has a total weight of Mg where M = 122 kg and the line of action of that downward force of gravity is x = 1.20 m from the wall. The vertical distance between the hinges is y = 1.80 m. We compute torques about the bottom hinge and find

$$F_x = -\frac{Mgx}{y} = -797\,\text{N}.$$

If we examine the forces on A alone and compute torques about the bolt, we instead find

$$F_y = \frac{m_A gx}{\ell} = 265\,\text{N}$$

where m_A = 54.0 kg and ℓ = 2.40 m (the length of beam A). Thus, in unit-vector notation, we have

$$\vec{F} = F_x\hat{\text{i}} + F_y\hat{\text{j}} = (-797\text{ N})\hat{\text{i}} + (265\text{ N})\hat{\text{j}}.$$

(b) Equilibrium of horizontal and vertical forces on beam A readily yields $G_x = -F_x$ = 797 N and $G_y = m_A g - F_y$ = 265 N. In unit-vector notation, we have

$$\vec{G} = G_x\hat{\text{i}} + G_y\hat{\text{j}} = (+797\text{ N})\hat{\text{i}} + (265\text{ N})\hat{\text{j}}$$

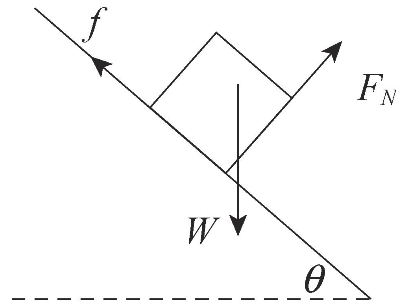
(c) Considering again the combined A∪B system, equilibrium of horizontal and vertical forces readily yields $H_x = -F_x$ = 797 N and $H_y = Mg - F_y$ = 931 N. In unit-vector notation, we have

$$\vec{H} = H_x\hat{\text{i}} + H_y\hat{\text{j}} = (+797\text{ N})\hat{\text{i}} + (931\text{ N})\hat{\text{j}}$$

(d) As mentioned above, Newton's third law (and the results from part (b)) immediately provide $-G_x = -797$ N and $-G_y = -265$ N for the force components acting on B at the bolt. In unit-vector notation, we have

$$-\vec{G} = -G_x\hat{\text{i}} - G_y\hat{\text{j}} = (-797\text{ N})\hat{\text{i}} - (265\text{ N})\hat{\text{j}}$$

39. The force diagram shown below depicts the situation just before the crate tips, when the normal force acts at the front edge. However, it may also be used to calculate the angle for which the crate begins to slide. W is the force of gravity on the crate, F_N is the normal force of the plane on the crate, and f is the force of friction. We take the x axis to be down the plane and the y axis to be in the direction of the normal force. We assume the acceleration is zero but the crate is on the verge of sliding.



(a) The x and y components of Newton's second law are

$$W \sin\theta - f = 0 \text{ and } F_N - W\cos\theta = 0$$

respectively. The y equation gives $F_N = W \cos\theta$. Since the crate is about to slide

$$f = \mu_s F_N = \mu_s W \cos\theta,$$

where μ_s is the coefficient of static friction. We substitute into the x equation and find

$$W \sin\theta - \mu_s W \cos\theta = 0 \Rightarrow \tan\theta = \mu_s.$$

This leads to $\theta = \tan^{-1}\mu_s = \tan^{-1} 0.60 = 31.0°$.

In developing an expression for the total torque about the center of mass when the crate is about to tip, we find that the normal force and the force of friction act at the front edge. The torque associated with the force of friction tends to turn the crate clockwise and has magnitude fh, where h is the perpendicular distance from the bottom of the crate to the center of gravity. The torque associated with the normal force tends to turn the crate counterclockwise and has magnitude $F_N\ell/2$, where ℓ is the length of an edge. Since the total torque vanishes, $fh = F_N\ell/2$. When the crate is about to tip, the acceleration of the center of gravity vanishes, so $f = W \sin\theta$ and $F_N = W \cos\theta$. Substituting these expressions into the torque equation, we obtain

$$\theta = \tan^{-1}\frac{\ell}{2h} = \tan^{-1}\frac{1.2\,\text{m}}{2(0.90\,\text{m})} = 33.7°.$$

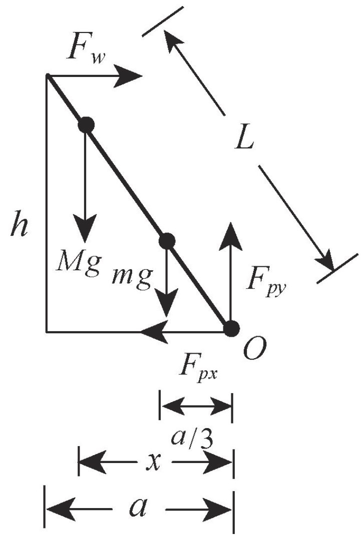
As θ is increased from zero the crate slides before it tips.

(b) It starts to slide when $\theta = 31°$.

(c) The crate begins to slide when $\theta = \tan^{-1}\mu_s = \tan^{-1} 0.70 = 35.0°$ and begins to tip when $\theta = 33.7°$. Thus, it tips first as the angle is increased.

(d) Tipping begins at $\theta = 33.7° \approx 34°$.

40. Let x be the horizontal distance between the firefighter and the origin O (see figure) that makes the ladder on the verge of sliding. The forces on the firefighter + ladder system consist of the horizontal force F_w from the wall, the vertical component F_{py} and the horizontal component F_{px} of the force $\vec{F}_p$ on the ladder from the pavement, and the downward gravitational forces Mg and mg, where M and m are the masses of the firefighter and the ladder, respectively. Since the system is in static equilibrium, the net force acting on the system is zero. Applying Newton's second law to the vertical and horizontal directions, we have



$$0 = \sum F_{\text{net},x} = F_w - F_{px}$$
$$0 = \sum F_{\text{net},y} = F_{py} - (M+m)g \,.$$

Since the ladder is on the verge of sliding, $F_{px} = \mu_s F_{py}$. Therefore, we have

$$F_w = F_{px} = \mu_s F_{py} = \mu_s (M+m)g \,.$$

In addition, the net torque about O (contact point between the ladder and the wall) must also vanish:

$$0 = \sum_O \tau_{\text{net}} = -h(F_w) + x(Mg) + \frac{a}{3}(mg) = 0 \,.$$

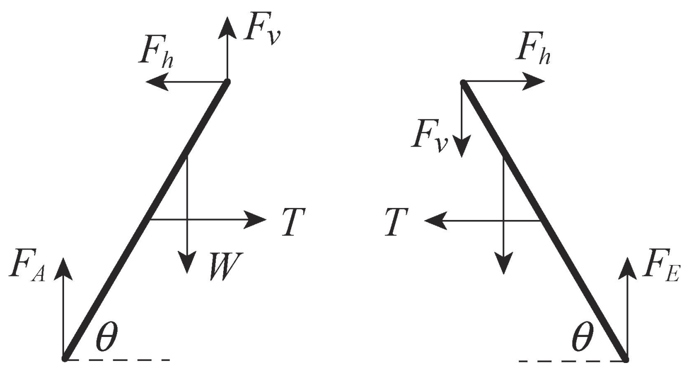
Solving for x, we obtain

$$x = \frac{hF_w - (a/3)mg}{Mg} = \frac{h\mu_s(M+m)g - (a/3)mg}{Mg} = \frac{h\mu_s(M+m) - (a/3)m}{M}$$

Substituting the values given in the problem statement (with $a = \sqrt{L^2 - h^2} = 7.58$ m), the fraction of ladder climbed is

$$\frac{x}{a} = \frac{h\mu_s(M+m) - (a/3)m}{Ma} = \frac{(9.3\text{ m})(0.53)(72\text{ kg} + 45\text{ kg}) - (7.58\text{ m}/3)(45\text{ kg})}{(72\text{ kg})(7.58\text{ m})}$$
$$= 0.848 \approx 85\%.$$

41. The diagrams below show the forces on the two sides of the ladder, separated. F_A and F_E are the forces of the floor on the two feet, T is the tension force of the tie rod, W is the force of the man (equal to his weight), F_h is the horizontal component of the force exerted by one side of the ladder on the other, and F_v is the vertical component of that force. Note that the forces exerted by the floor are normal to the floor since the floor is frictionless. Also note that the force of the left side on the right and the force of the right side on the left are equal in magnitude and opposite in direction.



Since the ladder is in equilibrium, the vertical components of the forces on the left side of the ladder must sum to zero: $F_v + F_A - W = 0$. The horizontal components must sum to zero: $T - F_h = 0$. The torques must also sum to zero. We take the origin to be at the hinge and let L be the length of a ladder side. Then

$$F_A L \cos\theta - W(L/4)\cos\theta - T(L/2)\sin\theta = 0.$$

Here we recognize that the man is one–fourth the length of the ladder side from the top and the tie rod is at the midpoint of the side.

The analogous equations for the right side are $F_E - F_v = 0$, $F_h - T = 0$, and $F_E L \cos\theta - T(L/2)\sin\theta = 0$.

There are 5 different equations:

$$\begin{aligned} F_v + F_A - W &= 0, \\ T - F_h &= 0 \\ F_A L\cos\theta - W(L/4)\cos\theta - T(L/2)\sin\theta &= 0 \\ F_E - F_v &= 0 \\ F_E L\cos\theta - T(L/2)\sin\theta &= 0. \end{aligned}$$

The unknown quantities are F_A, F_E, F_v, F_h, and T.

(a) First we solve for T by systematically eliminating the other unknowns. The first equation gives $F_A = W - F_v$ and the fourth gives $F_v = F_E$. We use these to substitute into the remaining three equations to obtain

$$\begin{aligned} T - F_h &= 0 \\ WL\cos\theta - F_E L\cos\theta - W(L/4)\cos\theta - T(L/2)\sin\theta &= 0 \\ F_E L\cos\theta - T(L/2)\sin\theta &= 0. \end{aligned}$$

The last of these gives $F_E = T\sin\theta/2\cos\theta = (T/2)\tan\theta$. We substitute this expression into the second equation and solve for T. The result is

$$T = \frac{3W}{4\tan\theta}.$$

To find $\tan\theta$, we consider the right triangle formed by the upper half of one side of the ladder, half the tie rod, and the vertical line from the hinge to the tie rod. The lower side of the triangle has a length of 0.381 m, the hypotenuse has a length of 1.22 m, and the vertical side has a length of $\sqrt{(1.22\,\text{m})^2 - (0.381\,\text{m})^2} = 1.16\,\text{m}$. This means

$$\tan\theta = (1.16\text{m})/(0.381\text{m}) = 3.04.$$

Thus,

$$T = \frac{3(854\,\text{N})}{4(3.04)} = 211\,\text{N}.$$

(b) We now solve for F_A. Since $F_v = F_E$ and $F_E = T\sin\theta/2\cos\theta$, $F_v = 3W/8$. We substitute this into $F_v + F_A - W = 0$ and solve for F_A. We find

$$F_A = W - F_v = W - 3W/8 = 5W/8 = 5(884\,\text{N})/8 = 534\,\text{N}.$$

(c) We have already obtained an expression for F_E: $F_E = 3W/8$. Evaluating it, we get $F_E =$ 320 N.

42. (a) Eq. 12-9 leads to

$$TL\sin\theta - m_p g x - m_b g\left(\tfrac{L}{2}\right) = 0 \ .$$

This can be written in the form of a straight line (in the graph) with

$$T = (\text{“slope”})\,\tfrac{x}{L} + \text{“}y\text{-intercept”},$$

where “slope” = $m_p g/\sin\theta$ and “y-intercept” = $m_b g/2\sin\theta$. The graph suggests that the slope (in SI units) is 200 and the y-intercept is 500. These facts, combined with the given $m_p + m_b = 61.2$ kg datum, lead to the conclusion:

$$\sin\theta = 61.22g/1200 \Rightarrow \theta = 30.0^{\circ}.$$

(b) It also follows that $m_p = 51.0$ kg.

(c) Similarly, $m_b = 10.2$ kg.

43. (a) The shear stress is given by F/A, where F is the magnitude of the force applied parallel to one face of the aluminum rod and A is the cross–sectional area of the rod. In this case F is the weight of the object hung on the end: $F = mg$, where m is the mass of the object. If r is the radius of the rod then $A = \pi r^2$. Thus, the shear stress is

$$\frac{F}{A} = \frac{mg}{\pi r^2} = \frac{(1200\,\text{kg})(9.8\,\text{m/s}^2)}{\pi(0.024\,\text{m})^2} = 6.5\times 10^6\,\text{N/m}^2.$$

(b) The shear modulus G is given by

$$G = \frac{F / A}{\Delta x / L}$$

where L is the protrusion of the rod and Δx is its vertical deflection at its end. Thus,

$$\Delta x = \frac{(F / A)L}{G} = \frac{(6.5\times 10^6\ \text{N/m}^2)(0.053\,\text{m})}{3.0\times 10^{10}\ \text{N/m}^2} = 1.1\times 10^{-5}\ \text{m}.$$

44. (a) The Young's modulus is given by

$$E = \frac{\text{stress}}{\text{strain}} = \text{slope of the stress-strain curve} = \frac{150\times10^{6}\,\text{N/m}^2}{0.002} = 7.5\times10^{10}\,\text{N/m}^2.$$

(b) Since the linear range of the curve extends to about 2.9×10^8 N/m^2, this is approximately the yield strength for the material.

45. (a) Let F_A and F_B be the forces exerted by the wires on the log and let m be the mass of the log. Since the log is in equilibrium $F_A + F_B - mg = 0$. Information given about the stretching of the wires allows us to find a relationship between F_A and F_B. If wire A originally had a length L_A and stretches by ΔL_A, then $\Delta L_A = F_A L_A / AE$, where A is the cross–sectional area of the wire and E is Young's modulus for steel (200×10^9 N/m^2). Similarly, $\Delta L_B = F_B L_B / AE$. If ℓ is the amount by which B was originally longer than A then, since they have the same length after the log is attached, $\Delta L_A = \Delta L_B + \ell$. This means

$$\frac{F_A L_A}{AE} = \frac{F_B L_B}{AE} + \ell.$$

We solve for F_B:

$$F_B = \frac{F_A L_A}{L_B} - \frac{AE\ell}{L_B}.$$

We substitute into $F_A + F_B - mg = 0$ and obtain

$$F_A = \frac{mgL_B + AE\ell}{L_A + L_B}.$$

The cross–sectional area of a wire is

$$A = \pi r^2 = \pi\left(1.20\times10^{-3}\,\text{m}\right)^2 = 4.52\times10^{-6}\,\text{m}^2.$$

Both L_A and L_B may be taken to be 2.50 m without loss of significance. Thus

$$F_A = \frac{(103\,\text{kg})(9.8\,\text{m/s}^2)(2.50\,\text{m}) + (4.52\times10^{-6}\,\text{m}^2)(200\times10^9\,\text{N/m}^2)(2.0\times10^{-3}\,\text{m})}{2.50\,\text{m} + 2.50\,\text{m}}$$
$$= 866\,\text{N}.$$

(b) From the condition $F_A + F_B - mg = 0$, we obtain

$$F_B = mg - F_A = (103\,\text{kg})(9.8\,\text{m/s}^2) - 866\,\text{N} = 143\,\text{N}.$$

(c) The net torque must also vanish. We place the origin on the surface of the log at a point directly above the center of mass. The force of gravity does not exert a torque about this point. Then, the torque equation becomes $F_A d_A - F_B d_B = 0$, which leads to

$$\frac{d_A}{d_B} = \frac{F_B}{F_A} = \frac{143\,\text{N}}{866\,\text{N}} = 0.165.$$

46. Since the force is (stress × area) and the displacement is (strain × length), we can write the work integral (eq. 7-32) as

$$W = \int F dx \;=\; \int (\text{stress})\, A\, (\text{differential strain}) L \;= AL \int (\text{stress})\, (\text{differential strain})$$

which means the work is (wire-area) × (wire-length) × (graph-area-under-curve). Since the area of a triangle (see the graph in the problem statement) is $\frac{1}{2}$(base)(height) then we determine the work done to be

$$W = (2.00 \times 10^{-6}\ \text{m}^2)(0.800\ \text{m})\left(\tfrac{1}{2}\right)(1.0 \times 10^{-3})(7.0 \times 10^{7}\ \text{N/m}^2) = 0.0560\ \text{J}\,.$$

47. (a) Since the brick is now horizontal and the cylinders were initially the same length ℓ, then both have been compressed an equal amount $\Delta\ell$. Thus,

$$\frac{\Delta\ell}{\ell} = \frac{FA}{A_A E_A} \quad \text{and} \quad \frac{\Delta\ell}{\ell} = \frac{F_B}{A_B E_B}$$

which leads to

$$\frac{F_A}{F_B} = \frac{A_A E_A}{A_B E_B} = \frac{(2A_B)(2E_B)}{A_B E_B} = 4.$$

When we combine this ratio with the equation $F_A + F_B = W$, we find $F_A/W = 4/5 = 0.80$.

(b) This also leads to the result $F_B/W = 1/5 = 0.20$.

(c) Computing torques about the center of mass, we find $F_A d_A = F_B d_B$ which leads to

$$\frac{d_A}{d_B} = \frac{F_B}{F_A} = \frac{1}{4} = 0.25.$$

48. Since the force is (stress × area) and the displacement is (strain × length), we can write the work integral (eq. 7-32) as

$$W = \int F dx \;=\; \int (\text{stress})\, A\; (\text{differential strain}) L \;=\; AL \int (\text{stress})\, (\text{differential strain})$$

which means the work is (thread cross-sectional area) × (thread length) × (graph-area-under-curve). The area under the curve is

$$\begin{aligned}\text{graph area} &= \frac{1}{2}as_1 + \frac{1}{2}(a+b)(s_2 - s_1) + \frac{1}{2}(b+c)(s_3 - s_2) = \frac{1}{2}\left[as_2 + b(s_3 - s_1) + c(s_3 - s_2)\right] \\ &= \frac{1}{2}\left[(0.12\times 10^9 \text{ N/m}^2)(1.4) + (0.30\times 10^9 \text{ N/m}^2)(1.0) + (0.80\times 10^9 \text{ N/m}^2)(0.60)\right] \\ &= 4.74\times 10^8 \text{ N/m}^2.\end{aligned}$$

(a) The kinetic energy that would put the thread on the verge of breaking is simply equal to W:

$$K = W = AL(\text{graph area}) = (8.0\times 10^{-12} \text{ m}^2)(8.0\times 10^{-3} \text{ m})(4.74\times 10^8 \text{ N/m}^2) = 3.03\times 10^{-5} \text{ J}.$$

(b) The kinetic energy of the fruit fly of mass 6.00 mg and speed 1.70 m/s is

$$K_f = \frac{1}{2}m_f v_f^2 = \frac{1}{2}(6.00\times 10^{-6} \text{ kg})(1.70 \text{ m/s})^2 = 8.67\times 10^{-6} \text{ J}.$$

(c) Since $K_f < W$, the fruit fly will not be able to break the thread.

(d) The kinetic energy of a bumble bee of mass 0.388 g and speed 0.420 m/s is

$$K_b = \frac{1}{2}m_b v_b^2 = \frac{1}{2}(3.99\times 10^{-4} \text{ kg})0.420 \text{ m/s})^2 = 3.42\times 10^{-5} \text{ J}.$$

(e) On the other hand, since $K_b > W$, the bumble bee will be able to break the thread.

49. The flat roof (as seen from the air) has area $A = 150 \text{ m} \times 5.8 \text{ m} = 870 \text{ m}^2$. The volume of material directly above the tunnel (which is at depth $d = 60$ m) is therefore

$$V = A \times d = (870 \text{ m}^2) \times (60\text{m}) = 52200 \text{ m}^3.$$

Since the density is $\rho = 2.8 \text{ g/cm}^3 = 2800 \text{ kg/m}^3$, we find the mass of material supported by the steel columns to be $m = \rho V = 1.46 \times 10^8 \text{ m}^3$.

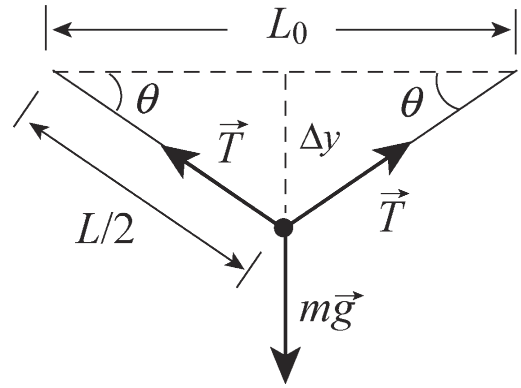
(a) The weight of the material supported by the columns is $mg = 1.4 \times 10^9$ N.

(b) The number of columns needed is

$$n = \frac{1.43 \times 10^9 \text{ N}}{\frac{1}{2}(400 \times 10^6 \text{ N/m}^2)(960 \times 10^{-4} \text{ m}^2)} = 75.$$

50. On the verge of breaking, the length of the thread is

$$L = L_0 + \Delta L = L_0(1 + \Delta L / L_0) = L_0(1+2) = 3L_0 ,$$



where $L_0 = 0.020$ m is the original length, and $\text{strain} = \Delta L / L_0 = 2$, as given in the problem. The free-body diagram of the system is shown on the right. The condition for equilibrium is

$$mg = 2T \sin\theta$$

where m is the mass of the insect and $T = A(\text{stress})$. Since the volume of the thread remains constant is it is being stretched, we have $V = A_0 L_0 = AL$, or $A = A_0(L_0 / L) = A_0 / 3$. The vertical distance Δy is

$$\Delta y = \sqrt{(L/2)^2 - (L0/2)^2} = \sqrt{\frac{9L_0^2}{4} - \frac{L_0^2}{4}} = \sqrt{2}L_0 .$$

Thus, the mass of the insect is

$$\begin{aligned} m &= \frac{2T\sin\theta}{g} = \frac{2(A_0/3)(\text{stress})\sin\theta}{g} = \frac{2A_0(\text{stress})}{3g}\frac{\Delta y}{3L_0/2} = \frac{4\sqrt{2}A_0(\text{stress})}{9g} \\ &= \frac{4\sqrt{2}(8.00\times10^{-12}\ \text{m}^2)(8.20\times10^{8}\ \text{N/m}^2)}{9(9.8\ \text{m/s}^2)} \\ &= 4.21\times10^{-4}\ \text{kg} \end{aligned}$$

or 0.421 g.

51. Let the forces that compress stoppers A and B be F_A and F_B, respectively. Then equilibrium of torques about the axle requires $FR = r_AF_A + r_BF_B$. If the stoppers are compressed by amounts $|\Delta y_A|$ and $|\Delta y_B|$ respectively, when the rod rotates a (presumably small) angle θ (in radians), then $|\Delta y_A| = r_A\theta$ and $|\Delta y_B| = r_B\theta$.

Furthermore, if their "spring constants" k are identical, then $k = |F/\Delta y|$ leads to the condition $F_A/r_A = F_B/r_B$ which provides us with enough information to solve.

(a) Simultaneous solution of the two conditions leads to

$$F_A = \frac{Rr_A}{r_A^2 + r_B^2}F = \frac{(5.0\text{ cm})(7.0\text{ cm})}{(7.0\text{ cm})^2 + (4.0\text{ cm})^2}(220\text{ N}) = 118\text{ N} \approx 1.2\times10^2\text{ N}.$$

(b) It also yields

$$F_B = \frac{Rr_B}{r_A^2 + r_B^2}F = \frac{(5.0\text{ cm})(4.0\text{ cm})}{(7.0\text{ cm})^2 + (4.0\text{ cm})^2}(220\text{ N}) = 68\text{ N}.$$

52. (a) With pivot at the hinge (at the left end), Eq. 12-9 gives

$$-mgx - Mg\frac{L}{2} + F_h h = 0$$

where m is the man's mass and M is that of the ramp; F_h is the leftward push of the right wall onto the right edge of the ramp. This equation can be written to be of the form (for a straight line in a graph)

$$F_h = (\text{"slope"})x + (\text{"y-intercept"}),$$

where the "slope" is mg/h and the "y-intercept" is $MgD/2h$. Since h = 0.480 m and D = 4.00 m, and the graph seems to intercept the vertical axis at 20 kN, then we find M = 500 kg.

(b) Since the "slope" (estimated from the graph) is (5000 N)/(4 m), then the man's mass must be m = 62.5 kg.

53. With the x axis parallel to the incline (positive uphill), then

$$\sum F_x = 0 \quad \Rightarrow \quad T\cos 25° - mg\sin 45° \;=\; 0.$$

Therefore, $T = 76$ N.

54. The beam has a mass M = 40.0 kg and a length L = 0.800 m. The mass of the package of tamale is m = 10.0 kg.

(a) Since the system is in static equilibrium, the normal force on the beam from roller A is equal to half of the weight of the beam:

$$F_A = Mg/2 = (40.0 \text{ kg})(9.80 \text{ m/s}^2)/2 = 196 \text{ N}.$$

(b) The normal force on the beam from roller B is equal to half of the weight of the beam plus the weight of the tamale:

$$F_B = Mg/2 + mg = (40.0 \text{ kg})(9.80 \text{ m/s}^2)/2 + (10.0 \text{ kg})(9.80 \text{ m/s}^2) = 294 \text{ N}.$$

(c) When the right-hand end of the beam is centered over roller B, the normal force on the beam from roller A is equal to the weight of the beam plus half of the weight of the tamale:

$$F_A = Mg + mg/2 = (40.0 \text{ kg})(9.8 \text{ m/s}^2) + (10.0 \text{ kg})(9.80 \text{ m/s}^2)/2 = 441 \text{ N}.$$

(d) Similarly, the normal force on the beam from roller B is equal to half of the weight of the tamale:

$$F_B = mg/2 = (10.0 \text{ kg})(9.80 \text{ m/s}^2)/2 = 49.0 \text{ N}.$$

(e) We choose the rotational axis to pass through roller B. When the beam is on the verge of losing contact with roller A, the net torque is zero. The balancing equation may be written as

$$mgx = Mg(L/4 - x) \quad \Rightarrow \quad x = \frac{L}{4}\frac{M}{M+m}.$$

Substituting the values given, we obtain x = 0.160 m.

55. (a) The forces acting on bucket are the force of gravity, down, and the tension force of cable A, up. Since the bucket is in equilibrium and its weight is

$$W_B = m_B g = (817\,\text{kg})(9.80\,\text{m/s}^2) = 8.01\times10^3\,\text{N},$$

the tension force of cable A is $T_A = 8.01\times10^3\,\text{N}$.

(b) We use the coordinates axes defined in the diagram. Cable A makes an angle of θ_2 = 66.0º with the negative y axis, cable B makes an angle of 27.0º with the positive y axis, and cable C is along the x axis. The y components of the forces must sum to zero since the knot is in equilibrium. This means $T_B \cos 27.0° - T_A \cos 66.0° = 0$ and

$$T_B = \frac{\cos 66.0°}{\cos 27.0°} T_A = \left(\frac{\cos 66.0°}{\cos 27.0°}\right)(8.01\times10^3\,\text{N}) = 3.65\times10^3\text{N}.$$

(c) The x components must also sum to zero. This means

$$T_C + T_B \sin 27.0° - T_A \sin 66.0° = 0$$

Which yields

$$\begin{aligned} T_C &= T_A \sin 66.0° - T_B \sin 27.0° = (8.01\times10^3\,\text{N})\sin 66.0° - (3.65\times10^3\,\text{N})\sin 27.0° \\ &= 5.66\times10^3\,\text{N}. \end{aligned}$$

56. (a) Eq. 12-8 leads to $T_1 \sin 40^\circ + T_2 \sin\theta = mg$. Also, Eq. 12-7 leads to

$$T_1 \cos 40^\circ - T_2 \cos\theta = 0.$$

Combining these gives the expression

$$T_2 = \frac{mg}{\cos\theta \tan 40^\circ + \sin\theta}.$$

To minimize this, we can plot it or set its derivative equal to zero. In either case, we find that it is at its minimum at $\theta = 50^\circ$.

(b) At $\theta = 50^\circ$, we find $T_2 = 0.77mg$.

57. The cable that goes around the lowest pulley is cable 1 and has tension $T_1 = F$. That pulley is supported by the cable 2 (so $T_2 = 2T_1 = 2F$) and goes around the middle pulley. The middle pulley is supported by cable 3 (so $T_3 = 2T_2 = 4F$) and goes around the top pulley. The top pulley is supported by the upper cable with tension T, so $T = 2T_3 = 8F$. Three cables are supporting the block (which has mass $m = 6.40$ kg):

$$T_1 + T_2 + T_3 = mg \Rightarrow F = \frac{mg}{7} = 8.96\,\text{N}.$$

Therefore, $T = 8(8.96\text{ N}) = 71.7$ N.

58. Since all surfaces are frictionless, the contact force $\vec{F}$ exerted by the lower sphere on the upper one is along that 45° line, and the forces exerted by walls and floors are "normal" (perpendicular to the wall and floor surfaces, respectively). Equilibrium of forces on the top sphere leads to the two conditions

$$F_{\text{wall}} = F\cos 45° \quad \text{and} \quad F\sin 45° = mg.$$

And (using Newton's third law) equilibrium of forces on the bottom sphere leads to the two conditions

$$F'_{\text{wall}} = F\cos 45° \quad \text{and} \quad F'_{\text{floor}} = F\sin 45° + mg.$$

(a) Solving the above equations, we find $F'_{\text{floor}} = 2mg$.

(b) We obtain for the left side of the container, $F'_{\text{wall}} = mg$.

(c) We obtain for the right side of the container, $F_{\text{wall}} = mg$.

(d) We get $F = mg/\sin 45° = \sqrt{2}mg$.

59. (a) The center of mass of the top brick cannot be further (to the right) with respect to the brick below it (brick 2) than $L/2$; otherwise, its center of gravity is past any point of support and it will fall. So $a_1 = L/2$ in the maximum case.

(b) With brick 1 (the top brick) in the maximum situation, then the combined center of mass of brick 1 and brick 2 is halfway between the middle of brick 2 and its right edge. That point (the combined com) must be supported, so in the maximum case, it is just above the right edge of brick 3. Thus, $a_2 = L/4$.

(c) Now the total center of mass of bricks 1, 2 and 3 is one–third of the way between the middle of brick 3 and its right edge, as shown by this calculation:

$$x_{\text{com}} = \frac{2m(0) + m(-L/2)}{3m} = -\frac{L}{6}$$

where the origin is at the right edge of brick 3. This point is above the right edge of brick 4 in the maximum case, so $a_3 = L/6$.

(d) A similar calculation

$$x'_{\text{com}} = \frac{3m(0) + m(-L/2)}{4m} = -\frac{L}{8}$$

shows that $a_4 = L/8$.

(e) We find $h = \sum_{i=1}^{4} a_i = 25L/24$.

60. (a) If L (= 1500 cm) is the unstretched length of the rope and $\Delta L = 2.8\,\text{cm}$ is the amount it stretches then the strain is

$$\Delta L / L = (2.8\,\text{cm})/(1500\,\text{cm}) = 1.9\times10^{-3}.$$

(b) The stress is given by F/A where F is the stretching force applied to one end of the rope and A is the cross–sectional area of the rope. Here F is the force of gravity on the rock climber. If m is the mass of the rock climber then $F = mg$. If r is the radius of the rope then $A = \pi r^2$. Thus the stress is

$$\frac{F}{A} = \frac{mg}{\pi r^2} = \frac{(95\,\text{kg})(9.8\,\text{m/s}^2)}{\pi(4.8\times10^{-3}\ \text{m})^2} = 1.3\times10^7\ \text{N/m}^2.$$

(c) Young's modulus is the stress divided by the strain:

$$E = (1.3\times10^7\ \text{N/m}^2)/(1.9\times10^{-3}) = 6.9\times10^9\ \text{N/m}^2.$$

61. We denote the mass of the slab as m, its density as ρ, and volume as $V = LTW$. The angle of inclination is $\theta = 26°$.

(a) The component of the weight of the slab along the incline is

$$\begin{aligned} F_1 &= mg\sin\theta = \rho Vg\sin\theta \\ &= (3.2\times10^3\ \text{kg/m}^3)(43\ \text{m})(2.5\ \text{m})(12\ \text{m})(9.8\ \text{m/s}^2)\sin 26° \approx 1.8\times10^7\ \text{N}. \end{aligned}$$

(b) The static force of friction is

$$\begin{aligned} f_s &= \mu_s F_N = \mu_s mg\cos\theta = \mu_s \rho Vg\cos\theta \\ &= (0.39)(3.2\times10^3\ \text{kg/m}^3)(43\ \text{m})(2.5\ \text{m})(12\ \text{m})(9.8\ \text{m/s}^2)\cos 26° \approx 1.4\times10^7\ \text{N}. \end{aligned}$$

(c) The minimum force needed from the bolts to stabilize the slab is

$$F_2 = F_1 - f_s = 1.77\times10^7\ \text{N} - 1.42\times10^7\ \text{N} = 3.5\times10^6\ \text{N}.$$

If the minimum number of bolts needed is n, then $F_2 / nA \le 3.6\times10^8\ \text{N/m}^2$, or

$$n \ge \frac{3.5\times10^6\ \text{N}}{(3.6\times10^8\ \text{N/m}^2)(6.4\times10^{-4}\ \text{m}^2)} = 15.2$$

Thus 16 bolts are needed.

62. The notation and coordinates are as shown in Fig. 12-6 in the textbook. Here, the ladder's center of mass is halfway up the ladder (unlike in the textbook figure). Also, we label the x and y forces at the ground f_s and F_N, respectively. Now, balancing forces, we have

$$\sum F_x = 0 \quad \Rightarrow \quad f_s = F_w$$
$$\sum F_y = 0 \quad \Rightarrow \quad F_N = mg$$

Since $f_s = f_{s,\,\max}$, we divide the equations to obtain

$$\frac{f_{s,\max}}{F_N} = \mu_s = \frac{F_w}{mg} \quad .$$

Now, from $\Sigma\, \tau_z = 0$ (with axis at the ground) we have $mg(a/2) - F_w h = 0$. But from the Pythagorean theorem, $h = \sqrt{L^2 - a^2}$, where L = length of ladder. Therefore,

$$\frac{F_w}{mg} = \frac{a/2}{h} = \frac{a}{2\sqrt{L^2 - a^2}} \quad .$$

In this way, we find

$$\mu_s = \frac{a}{2\sqrt{L^2 - a^2}} \quad \Rightarrow \quad a = \frac{2\mu_s L}{\sqrt{1 + 4\mu_s^2}} = 3.4 \text{ m.}$$

63. Analyzing forces at the knot (particularly helpful is a graphical view of the vector right–triangle with horizontal "side" equal to the static friction force f_s and vertical "side" equal to the weight $m_B g$ of block B), we find $f_s = m_B g \tan\theta$ where $\theta = 30°$. For f_s to be at its maximum value, then it must equal $\mu_s m_A g$ where the weight of block A is $m_A g = (10\ \text{kg})(9.8\ \text{m/s}^2)$. Therefore,

$$\mu_s m_A g = m_B g \tan\theta \Rightarrow \mu_s = \frac{5.0}{10}\tan 30° = 0.29.$$

64. To support a load of $W = mg = (670 \text{ kg})(9.8 \text{ m/s}^2) = 6566$ N, the steel cable must stretch an amount proportional to its "free" length:

$$\Delta L = \left(\frac{W}{AY}\right)L \quad \text{where } A = \pi r^2$$

and $r = 0.0125$ m.

(a) If $L = 12$ m, then $\Delta L = \left(\dfrac{6566 \text{ N}}{\pi(0.0125 \text{ m})^2\,(2.0\times10^{11} \text{ N/m}^2)}\right)(12 \text{ m}) = 8.0\times10^{-4}\,\text{m}.$

(b) Similarly, when $L = 350$ m, we find $\Delta L = 0.023\,\text{m}$.

65. With the pivot at the hinge, Eq. 12-9 leads to

$$-mg\sin\theta_1\frac{L}{2} + TL\sin(180° - \theta_1 - \theta_2) = 0\ .$$

where $\theta_1 = 60°$ and $T = mg/2$. This yields $\theta_2 = 60°$.

66. (a) Setting up equilibrium of torques leads to

$$F_{\text{far end}} L = (73\,\text{kg})(9.8\,\text{m/s}^2)\frac{L}{4} + (2700\,\text{N})\frac{L}{2}$$

which yields $F_{\text{far end}} = 1.5 \times 10^3$ N.

(b) Then, equilibrium of vertical forces provides

$$F_{\text{near end}} = (73)(9.8) + 2700 - F_{\text{far end}} = 1.9 \times 10^3\,\text{N}.$$

67. (a) and (b) With $+x$ rightward and $+y$ upward (we assume the adult is pulling with force $\vec{P}$ to the right), we have

$$\sum F_y = 0 \quad \Rightarrow \quad W = T\cos\theta = 270 \text{ N}$$
$$\sum F_x = 0 \quad \Rightarrow \quad P = T\sin\theta = 72 \text{ N}$$

where $\theta = 15°$.

(c) Dividing the above equations leads to

$$\frac{P}{W} = \tan\theta \; .$$

Thus, with $W = 270$ N and $P = 93$ N, we find $\theta = 19°$.

68. We denote the tension in the upper left string (bc) as T' and the tension in the lower right string (ab) as T. The supported weight is Mg = 19.6 N. The force equilibrium conditions lead to

$$T'\cos 60° = T\cos 20° \qquad \text{horizontal forces}$$
$$T'\sin 60° = W + T\sin 20° \qquad \text{vertical forces.}$$

(a) We solve the above simultaneous equations and find

$$T = \frac{W}{\tan 60°\cos 20° - \sin 20°} = 15\text{N}.$$

(b) Also, we obtain $T' = T\cos 20° / \cos 60° = 29$ N.

69. (a) Because of Eq. 12-3, we can write

$$\vec{T} + (m_B\, g \;\angle -90^{\circ}) + (m_A\, g \;\angle -150^{\circ}) = 0\,.$$

Solving the equation, we obtain $\vec{T} = (106.34 \;\angle\, 63.963^{\circ})$. Thus, the magnitude of the tension in the upper cord is 106 N,

(b) and its angle (measured ccw from the $+x$ axis) is 64.0°.

70. (a) The angle between the beam and the floor is

$$\sin^{-1}(d/L) = \sin^{-1}(1.5/2.5) = 37°,$$

so that the angle between the beam and the weight vector $\vec{W}$ of the beam is 53°. With L = 2.5 m being the length of beam, and choosing the axis of rotation to be at the base,

$$\sum \tau_z = 0 \Rightarrow PL - W\left(\frac{L}{2}\right)\sin 53° = 0$$

Thus, $P = \frac{1}{2} W \sin 53° = 200$ N.

(b) Note that

$$\vec{P} + \vec{W} = (200 \angle 90°) + (500 \angle -127°) = (360 \angle -146°)$$

using magnitude-angle notation (with angles measured relative to the beam, where "uphill" along the beam would correspond to 0°) with the unit Newton understood. The "net force of the floor" $\vec{F_f}$ is equal and opposite to this (so that the total net force on the beam is zero), so that $|\vec{F_f}|$ = 360 N and is directed 34° counterclockwise from the beam.

(c) Converting that angle to one measured from true horizontal, we have θ = 34° + 37° = 71°. Thus, $f_s = F_f \cos\theta$ and $F_N = F_f \sin\theta$. Since $f_s = f_{s,\max}$, we divide the equations to obtain

$$\frac{F_N}{f_{s,\max}} = \frac{1}{\mu_s} = \tan\theta \ .$$

Therefore, $\mu_s = 0.35$.

71. The cube has side length l and volume $V = l^3$. We use $p = B\Delta V / V$ for the pressure p. We note that

$$\frac{\Delta V}{V} = \frac{\Delta l^3}{l^3} = \frac{(l+\Delta l)^3 - l^3}{l^3} \approx \frac{3l^2 \Delta l}{l^3} = 3\frac{\Delta l}{l}.$$

Thus, the pressure required is

$$p = \frac{3B\Delta l}{l} = \frac{3(1.4\times10^{11}\ \text{N/m}^2)(85.5\,\text{cm} - 85.0\,\text{cm})}{85.5\,\text{cm}} = 2.4\times10^{9}\ \text{N/m}^2.$$

72. Adopting the usual convention that torques that would produce counterclockwise rotation are positive, we have (with axis at the hinge)

$$\sum \tau_z = 0 \Rightarrow TL\sin 60^\circ - Mg\left(\frac{L}{2}\right) = 0$$

where L = 5.0 m and M = 53 kg. Thus, T = 300 N. Now (with F_p for the force of the hinge)

$$\sum F_x = 0 \Rightarrow F_{px} = -T\cos\theta = -150\text{N}$$
$$\sum F_y = 0 \Rightarrow F_{py} = Mg - T\sin\theta = 260\text{N}$$

where θ = 60°. Therefore, $\vec{F}_p = (-1.5\times10^2\ \text{N})\hat{\text{i}} + (2.6\times10^2\ \text{N})\hat{\text{j}}$.

73. (a) Choosing an axis through the hinge, perpendicular to the plane of the figure and taking torques that would cause counterclockwise rotation as positive, we require the net torque to vanish:

$$FL\sin 90° - Th\sin 65° = 0$$

where the length of the beam is L = 3.2 m and the height at which the cable attaches is h = 2.0 m. Note that the weight of the beam does not enter this equation since its line of action is directed towards the hinge. With F = 50 N, the above equation yields T = 88 N.

(b) To find the components of $\vec{F}_p$ we balance the forces:

$$\sum F_x = 0 \;\Rightarrow\; F_{px} = T\cos 25° - F$$
$$\sum F_y = 0 \;\Rightarrow\; F_{py} = T\sin 25° + W$$

where W is the weight of the beam (60 N). Thus, we find that the hinge force components are F_{px} = 30 N rightward and F_{py} = 97 N upward. In unit-vector notation, $\vec{F}_p = (30\text{ N})\hat{\text{i}} + (97\text{ N})\hat{\text{j}}$.

74. (a) Computing the torques about the hinge, we have $TL\sin 40° = W\frac{L}{2}\sin 50°$ where the length of the beam is L = 12 m and the tension is T = 400 N. Therefore, the weight is $W = 671$ N, which means that the gravitational force on the beam is $\vec{F}_w = (-671\text{ N})\hat{\text{j}}$.

(b) Equilibrium of horizontal and vertical forces yields, respectively,

$$F_{\text{hinge}\,x} = T = 400\text{ N}$$
$$F_{\text{hinge}\,y} = W = 671\text{ N}$$

where the hinge force components are rightward (for x) and upward (for y). In unit-vector notation, we have $\vec{F}_{\text{hinge}} = (400\text{ N})\hat{\text{i}} + (671\text{ N})\hat{\text{j}}$

75. We locate the origin of the x axis at the edge of the table and choose rightwards positive. The criterion (in part (a)) is that the center of mass of the block above another must be no further than the edge of the one below; the criterion in part (b) is more subtle and is discussed below. Since the edge of the table corresponds to $x = 0$ then the total center of mass of the blocks must be zero.

(a) We treat this as three items: one on the upper left (composed of two bricks, one directly on top of the other) of mass $2m$ whose center is above the left edge of the bottom brick; a single brick at the upper right of mass m which necessarily has its center over the right edge of the bottom brick (so $a_1 = L/2$ trivially); and, the bottom brick of mass m. The total center of mass is

$$\frac{(2m)(a_2 - L) + ma_2 + m(a_2 - L/2)}{4m} = 0$$

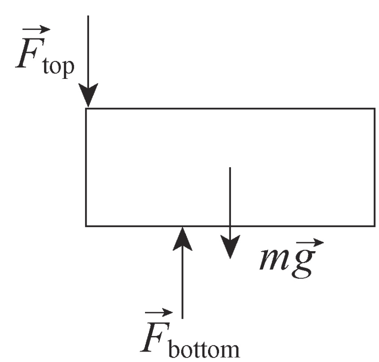
which leads to $a_2 = 5L/8$. Consequently, $h = a_2 + a_1 = 9L/8$.

(b) We have four bricks (each of mass m) where the center of mass of the top and the center of mass of the bottom one have the same value $x_{cm} = b_2 - L/2$. The middle layer consists of two bricks, and we note that it is possible for each of their centers of mass to be beyond the respective edges of the bottom one! This is due to the fact that the top brick is exerting downward forces (each equal to half its weight) on the middle blocks — and in the extreme case, this may be thought of as a pair of concentrated forces exerted at the innermost edges of the middle bricks. Also, in the extreme case, the support force (upward) exerted on a middle block (by the bottom one) may be thought of as a concentrated force located at the edge of the bottom block (which is the point about which we compute torques, in the following).

If (as indicated in our sketch, where $\vec{F}_{\text{top}}$ has magnitude $mg/2$) we consider equilibrium of torques on the rightmost brick, we obtain

$$mg\left(b_1 - \frac{1}{2}L\right) = \frac{mg}{2}(L - b_1)$$

which leads to $b_1 = 2L/3$. Once we conclude from symmetry that $b_2 = L/2$ then we also arrive at $h = b_2 + b_1 = 7L/6$.



76. One arm of the balance has length ℓ_1 and the other has length ℓ_2. The two cases described in the problem are expressed (in terms of torque equilibrium) as

$$m_1\ell_1 = m\ell_2 \quad \text{and} \quad m\ell_1 = m_2\ell_2.$$

We divide equations and solve for the unknown mass: $m = \sqrt{m_1 m_2}$.

77. Since GA exerts a leftward force T at the corner A, then (by equilibrium of horizontal forces at that point) the force F_{diag} in CA must be pulling with magnitude

$$F_{\text{diag}} = \frac{T}{\sin 45°} = T\sqrt{2}.$$

This analysis applies equally well to the force in DB. And these diagonal bars are pulling on the bottom horizontal bar exactly as they do to the top bar, so the bottom bar CD is the "mirror image" of the top one (it is also under tension T). Since the figure is symmetrical (except for the presence of the turnbuckle) under 90° rotations, we conclude that the side bars (DA and BC) also are under tension T (a conclusion that also follows from considering the vertical components of the pull exerted at the corners by the diagonal bars).

(a) Bars that are in tension are BC, CD and DA.

(b) The magnitude of the forces causing tension is $T = 535$ N .

(c) The magnitude of the forces causing compression on CA and DB is

$$F_{\text{diag}} = \sqrt{2}T = (1.41)535 \text{ N} = 757 \text{ N} .$$

78. (a) For computing torques, we choose the axis to be at support 2 and consider torques which encourage counterclockwise rotation to be positive. Let m = mass of gymnast and M = mass of beam. Thus, equilibrium of torques leads to

$$Mg(1.96\,\mathrm{m}) - mg(0.54\,\mathrm{m}) - F_1(3.92\,\mathrm{m}) = 0.$$

Therefore, the upward force at support 1 is F_1 = 1163 N (quoting more figures than are significant — but with an eye toward using this result in the remaining calculation). In unit-vector notation, we have $\vec{F}_1 \approx (1.16\times10^3\ \mathrm{N})\hat{\mathrm{j}}$.

(b) Balancing forces in the vertical direction, we have $F_1 + F_2 - Mg - mg = 0$, so that the upward force at support 2 is F_2 = 1.74×10^3 N. In unit-vector notation, we have $\vec{F}_2 \approx (1.74\times10^3\ \mathrm{N})\hat{\mathrm{j}}$.

79. (a) Let d = 0.00600 m. In order to achieve the same final lengths, wires 1 and 3 must stretch an amount d more than wire 2 stretches:

$$\Delta L_1 = \Delta L_3 = \Delta L_2 + d \,.$$

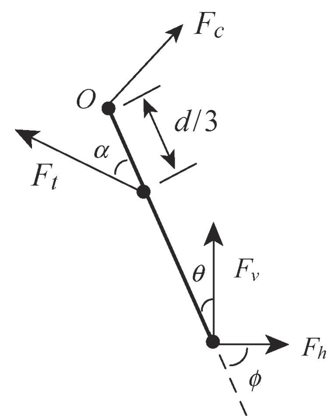
Combining this with Eq. 12-23 we obtain

$$F_1 = F_3 = F_2 + \frac{dAE}{L} \,.$$

Now, Eq. 12-8 produces $F_1 + F_3 + F_2 - mg = 0$. Combining this with the previous relation (and using Table 12-1) leads to F_1= 1380 N $\approx 1.38\times10^3$ N .

(b) Similarly, F_2 = 180 N.

80. Our system is the second finger bone. Since the system is in static equilibrium, the net force acting on it is zero. In addition, the torque about any point must be zero. We set up the torque equation about point O where $\vec{F}_c$ act:



$$0=\sum_{O}\tau_{\text{net}}=-\left(\frac{d}{3}\right)F_t\sin\alpha+(d)F_v\sin\theta+(d)F_h\sin\phi\,.$$

Solving for F_t and substituting the values given, we obtain

$$F_t=\frac{3(F_v\sin\theta+F_h\sin\phi)}{\sin\alpha}=\frac{3[(162.4\text{ N})\sin 10^\circ+(13.4\text{ N})\sin 80^\circ]}{\sin 45^\circ}=175.6\text{ N}$$
$$\approx 1.8\times 10^2\text{ N}.$$

81. When it is about to move, we are still able to apply the equilibrium conditions, but (to obtain the critical condition) we set static friction equal to its maximum value and picture the normal force $\vec{F}_N$ as a concentrated force (upward) at the bottom corner of the cube, directly below the point O where P is being applied. Thus, the line of action of $\vec{F}_N$ passes through point O and exerts no torque about O (of course, a similar observation applied to the pull P). Since $F_N = mg$ in this problem, we have $f_{s\max} = \mu mg$ applied a distance h away from O. And the line of action of force of gravity (of magnitude mg), which is best pictured as a concentrated force at the center of the cube, is a distance $L/2$ away from O. Therefore, equilibrium of torques about O produces

$$\mu mgh = mg\left(\frac{L}{2}\right) \Rightarrow \mu = \frac{L}{2h} = \frac{(8.0 \text{ cm})}{2(7.0 \text{ cm})} = 0.57$$

for the critical condition we have been considering. We now interpret this in terms of a range of values for μ.

(a) For it to slide but not tip, a value of μ *less* than that derived above is needed, since then — static friction will be exceeded for a smaller value of P, before the pull is strong enough to cause it to tip. Thus, $\mu < L/2h = 0.57$ is required.

(b) And for it to tip but not slide, we need μ *greater* than that derived above is needed, since now — static friction will not be exceeded even for the value of P which makes the cube rotate about its front lower corner. That is, we need to have $\mu > L/2h = 0.57$ in this case.

82. The assumption stated in the problem (that the density does not change) is not meant to be realistic; those who are familiar with Poisson's ratio (and other topics related to the strengths of materials) might wish to think of this problem as treating a fictitious material (which happens to have the same value of E as aluminum, given in Table 12-1) whose density does not significantly change during stretching. Since the mass does not change, either, then the constant-density assumption implies the volume (which is the circular area times its length) stays the same:

$$(\pi r^2 L)_{\text{new}} = (\pi r^2 L)_{\text{old}} \quad \Rightarrow \quad \Delta L = L\left[(1000/999.9)^2 - 1\right].$$

Now, Eq. 12-23 gives

$$F = \pi r^2 E\, \Delta L/L \;=\; \pi r^2 (7.0 \times 10^9\ \text{N/m}^2)\left[(1000/999.9)^2 - 1\right].$$

Using either the new or old value for r gives the answer $F = 44$ N.

83. Where the crosspiece comes into contact with the beam, there is an upward force of $2F$ (where F is the upward force exerted by each man). By equilibrium of vertical forces, $W = 3F$ where W is the weight of the beam. If the beam is uniform, its center of gravity is a distance $L/2$ from the man in front, so that computing torques about the front end leads to

$$W\frac{L}{2} = 2Fx = 2\left(\frac{W}{3}\right)x$$

which yields $x = 3L/4$ for the distance from the crosspiece to the front end. It is therefore a distance $L/4$ from the rear end (the "free" end).

84. (a) Setting up equilibrium of torques leads to a simple “level principle” ratio:

$$F_{\text{catch}} = (11\,\text{kg})\,(9.8\,\text{m/s}^2)\,\frac{(91/2-10)\,\text{cm}}{91\,\text{cm}} = 42\,\text{N}.$$

(b) Then, equilibrium of vertical forces provides

$$F_{\text{hinge}} = (11\,\text{kg})\,(9.8\,\text{m/s}^2) - F_{\text{catch}} = 66\,\text{N}.$$

85. We choose an axis through the top (where the ladder comes into contact with the wall), perpendicular to the plane of the figure and take torques that would cause counterclockwise rotation as positive. Note that the line of action of the applied force $\vec{F}$ intersects the wall at a height of $(8.0\text{ m})/5 = 1.6\,\text{m}$; in other words, the *moment arm* for the applied force (in terms of where we have chosen the axis) is $r_\perp = (4/5)(8.0\text{ m}) = 6.4\,\text{m}$. The moment arm for the weight is half the horizontal distance from the wall to the base of the ladder; this works out to be $\sqrt{(10\text{ m})^2 - (8\text{ m})^2}\,/2 = 3.0\,\text{m}$. Similarly, the moment arms for the x and y components of the force at the ground $\left(\vec{F}_g\right)$ are 8.0 m and 6.0 m, respectively. Thus, with lengths in meters, we have

$$\sum \tau_z = F(6.4\text{ m}) + W(3.0\text{ m}) + F_{gx}(8.0\text{ m}) - F_{gy}(6.0\text{ m}) = 0.$$

In addition, from balancing the vertical forces we find that $W = F_{gy}$ (keeping in mind that the wall has no friction). Therefore, the above equation can be written as

$$\sum \tau_z = F(6.4\text{ m}) + W(3.0\text{ m}) + F_{gx}(8.0\text{ m}) - W(6.0\text{ m}) = 0.$$

(a) With F = 50 N and W = 200 N, the above equation yields F_{gx} = 35 N. Thus, in unit vector notation we obtain

$$\vec{F}_g = (35\text{ N})\hat{\text{i}} + (200\text{ N})\hat{\text{j}}.$$

(b) With F = 150 N and W = 200 N, the above equation yields F_{gx} = –45 N. Therefore, in unit vector notation we obtain

$$\vec{F}_g = (-45\text{ N})\hat{\text{i}} + (200\text{ N})\hat{\text{j}}.$$

(c) Note that the phrase "start to move towards the wall" implies that the friction force is pointed away from the wall (in the $-\hat{\text{i}}$ direction). Now, if $f = -F_{gx}$ and $F_N = F_{gy}$ = 200 N are related by the (maximum) static friction relation ($f = f_{s,\max} = \mu_s F_N$) with μ_s = 0.38, then we find F_{gx} = –76 N. Returning this to the above equation, we obtain

$$F = \frac{(200\,\text{N})(3.0\,\text{m}) + (76\,\text{N})(8.0\,\text{m})}{6.4\,\text{m}} = 1.9\times10^2\text{ N}.$$

86. The force F exerted on the beam is F = 7900 N, as computed in the Sample Problem. Let $F/A = S_u/6$, where $S_u = 50\times10^6\ \text{N/m}^2$ is the ultimate strength (see Table 12-1), then

$$A = \frac{6F}{S_u} = \frac{6(7900\ \text{N})}{50\times10^6\ \text{N/m}^2} = 9.5\times10^{-4}\,\text{m}^2.$$

Thus the thickness is $\sqrt{A} = \sqrt{9.5\times10^{-4}\ \text{m}^2} = 0.031\,\text{m}$.