1. The problem asks us to assume v_{com} and ω are constant. For consistency of units, we write

$$v_{\text{com}} = (85\,\text{mi/h})\left(\frac{5280\,\text{ft/mi}}{60\,\text{min/h}}\right) = 7480\,\text{ft/min} .$$

Thus, with $\Delta x = 60\,\text{ft}$, the time of flight is

$$t = \Delta x / v_{\text{com}} = (60\ \text{ft})/(7480\ \text{ft/min}) = 0.00802\,\text{min} .$$

During that time, the angular displacement of a point on the ball's surface is

$$\theta = \omega t = (1800\,\text{rev/min})(0.00802\,\text{min}) \approx 14\,\text{rev} .$$

2. (a) The second hand of the smoothly running watch turns through 2π radians during 60 s . Thus,

$$\omega = \frac{2\pi}{60} = 0.105 \text{ rad/s}.$$

(b) The minute hand of the smoothly running watch turns through 2π radians during 3600 s . Thus,

$$\omega = \frac{2\pi}{3600} = 1.75 \times 10^{-3} \text{ rad / s}.$$

(c) The hour hand of the smoothly running 12-hour watch turns through 2π radians during 43200 s. Thus,

$$\omega = \frac{2\pi}{43200} = 1.45 \times 10^{-4} \text{ rad / s}.$$

3. Applying Eq. 2-15 to the vertical axis (with $+y$ downward) we obtain the free-fall time:

$$\Delta y = v_{0y} t + \frac{1}{2} g t^2 \Rightarrow t = \sqrt{\frac{2(10\ \text{m})}{9.8\ \text{m/s}^2}} = 1.4\ \text{s}.$$

Thus, by Eq. 10-5, the magnitude of the average angular velocity is

$$\omega_{\text{avg}} = \frac{(2.5\ \text{rev})(2\pi\ \text{rad/rev})}{1.4\ \text{s}} = 11\ \text{rad/s}.$$

4. If we make the units explicit, the function is

$$\theta = (4.0 \text{ rad / s})t - (3.0 \text{ rad / s}^2)t^2 + (1.0 \text{ rad / s}^3)t^3$$

but generally we will proceed as shown in the problem—letting these units be understood. Also, in our manipulations we will generally not display the coefficients with their proper number of significant figures.

(a) Eq. 10-6 leads to

$$\omega = \frac{d}{dt}\left(4t - 3t^2 + t^3\right) = 4 - 6t + 3t^2.$$

Evaluating this at $t = 2$ s yields $\omega_2 = 4.0$ rad/s.

(b) Evaluating the expression in part (a) at $t = 4$ s gives $\omega_4 = 28$ rad/s.

(c) Consequently, Eq. 10-7 gives

$$\alpha_{\text{avg}} = \frac{\omega_4 - \omega_2}{4 - 2} = 12 \text{ rad / s}^2.$$

(d) And Eq. 10-8 gives

$$\alpha = \frac{d\omega}{dt} = \frac{d}{dt}\left(4 - 6t + 3t^2\right) = -6 + 6t.$$

Evaluating this at $t = 2$ s produces $\alpha_2 = 6.0$ rad/s^2.

(e) Evaluating the expression in part (d) at $t = 4$ s yields $\alpha_4 = 18$ rad/s^2. We note that our answer for α_{avg} does turn out to be the arithmetic average of α_2 and α_4 but point out that this will not always be the case.

5. The falling is the type of constant-acceleration motion you had in Chapter 2. The time it takes for the buttered toast to hit the floor is

$$\Delta t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2(0.76 \text{ m})}{9.8 \text{ m/s}^2}} = 0.394 \text{ s}.$$

(a) The smallest angle turned for the toast to land butter-side down is $\Delta\theta_{\text{min}} = 0.25 \text{ rev} = \pi/2 \text{ rad}.$ This corresponds to an angular speed of

$$\omega_{\text{min}} = \frac{\Delta\theta_{\text{min}}}{\Delta t} = \frac{\pi/2 \text{ rad}}{0.394 \text{ s}} = 4.0 \text{ rad/s}.$$

(b) The largest angle (less than 1 revolution) turned for the toast to land butter-side down is $\Delta\theta_{\text{max}} = 0.75 \text{ rev} = 3\pi/2 \text{ rad}.$ This corresponds to an angular speed of

$$\omega_{\text{max}} = \frac{\Delta\theta_{\text{max}}}{\Delta t} = \frac{3\pi/2 \text{ rad}}{0.394 \text{ s}} = 12.0 \text{ rad/s}.$$

6. If we make the units explicit, the function is

$$\theta = 2.0 \text{ rad} + \left(4.0 \text{ rad/s}^2\right)t^2 + \left(2.0 \text{ rad/s}^3\right)t^3$$

but in some places we will proceed as indicated in the problem—by letting these units be understood.

(a) We evaluate the function θ at $t = 0$ to obtain $\theta_0 = 2.0$ rad.

(b) The angular velocity as a function of time is given by Eq. 10-6:

$$\omega = \frac{d\theta}{dt} = \left(8.0 \text{ rad/s}^2\right)t + \left(6.0 \text{ rad/s}^3\right)t^2$$

which we evaluate at $t = 0$ to obtain $\omega_0 = 0$.

(c) For $t = 4.0$ s, the function found in the previous part is

$$\omega_4 = (8.0)(4.0) + (6.0)(4.0)^2 = 128 \text{ rad/s}.$$

If we round this to two figures, we obtain $\omega_4 \approx 1.3\times10^2$ rad/s.

(d) The angular acceleration as a function of time is given by Eq. 10-8:

$$\alpha = \frac{d\omega}{dt} = 8.0 \text{ rad/s}^2 + \left(12 \text{ rad/s}^3\right)t$$

which yields $\alpha_2 = 8.0 + (12)(2.0) = 32 \text{ rad/s}^2$ at $t = 2.0$ s.

(e) The angular acceleration, given by the function obtained in the previous part, depends on time; it is not constant.

7. (a) To avoid touching the spokes, the arrow must go through the wheel in not more than

$$\Delta t = \frac{1/8 \text{ rev}}{2.5 \text{ rev/s}} = 0.050 \text{ s}.$$

The minimum speed of the arrow is then $v_{\min} = \dfrac{20 \text{ cm}}{0.050 \text{ s}} = 400 \text{ cm/s} = 4.0 \text{ m/s}.$

(b) No—there is no dependence on radial position in the above computation.

8. (a) We integrate (with respect to time) the $\alpha = 6.0t^4 - 4.0t^2$ expression, taking into account that the initial angular velocity is 2.0 rad/s. The result is

$$\omega = 1.2\,t^5 - 1.33\,t^3 + 2.0.$$

(b) Integrating again (and keeping in mind that $\theta_o = 1$) we get

$$\theta = 0.20t^6 - 0.33\,t^4 + 2.0\,t + 1.0\ .$$

9. We assume the sense of initial rotation is positive. Then, with $\omega_0 = +120$ rad/s and $\omega = 0$ (since it stops at time t), our angular acceleration (''deceleration'') will be negative-valued: $\alpha = -4.0\ \text{rad/s}^2$.

(a) We apply Eq. 10-12 to obtain t.

$$\omega = \omega_0 + \alpha t \quad \Rightarrow \quad t = \frac{0 - 120\ \text{rad/s}}{-4.0\ \text{rad/s}^2} = 30\ \text{s}.$$

(b) And Eq. 10-15 gives

$$\theta = \frac{1}{2}(\omega_0 + \omega)\,t = \frac{1}{2}(120\ \text{rad/s} + 0)(30\ \text{s}) = 1.8\times10^3\ \text{rad}.$$

Alternatively, Eq. 10-14 could be used if it is desired to only use the given information (as opposed to using the result from part (a)) in obtaining θ. If using the result of part (a) is acceptable, then any angular equation in Table 10-1 (except Eq. 10-12) can be used to find θ.

10. (a) We assume the sense of rotation is positive. Applying Eq. 10-12, we obtain

$$\omega = \omega_0 + \alpha t \quad \Rightarrow \quad \alpha = \frac{(3000 - 1200)\ \text{rev/min}}{(12/60)\ \text{min}} = 9.0 \times 10^3\ \text{rev/min}^2.$$

(b) And Eq. 10-15 gives

$$\theta = \frac{1}{2}(\omega_0 + \omega)t = \frac{1}{2}(1200\ \text{rev/min} + 3000\ \text{rev/min})\left(\frac{12}{60}\ \text{min}\right) = 4.2 \times 10^2\ \text{rev}.$$

11. (a) With $\omega = 0$ and $\alpha = -4.2$ rad/s^2, Eq. 10-12 yields $t = -\omega_o/\alpha = 3.00$ s.

(b) Eq. 10-4 gives $\theta - \theta_o = -\omega_o^2 / 2\alpha = 18.9$ rad.

12. We assume the sense of rotation is positive, which (since it starts from rest) means all quantities (angular displacements, accelerations, etc.) are positive-valued.

(a) The angular acceleration satisfies Eq. 10-13:

$$25 \text{ rad} = \frac{1}{2}\alpha(5.0 \text{ s})^2 \quad \Rightarrow \quad \alpha = 2.0 \text{ rad/s}^2.$$

(b) The average angular velocity is given by Eq. 10-5:

$$\omega_{\text{avg}} = \frac{\Delta\theta}{\Delta t} = \frac{25 \text{ rad}}{5.0 \text{ s}} = 5.0 \text{ rad / s}.$$

(c) Using Eq. 10-12, the instantaneous angular velocity at t = 5.0 s is

$$\omega = \left(2.0 \text{ rad/s}^2\right)(5.0 \text{ s}) = 10 \text{ rad/s} .$$

(d) According to Eq. 10-13, the angular displacement at t = 10 s is

$$\theta = \omega_0 + \frac{1}{2}\alpha t^2 = 0 + \frac{1}{2}(2.0 \text{ rad/s}^2)(10 \text{ s})^2 = 100 \text{ rad}.$$

Thus, the displacement between t = 5 s and t = 10 s is $\Delta\theta$= 100 rad – 25 rad = 75 rad.

13. We take $t = 0$ at the start of the interval and take the sense of rotation as positive. Then at the end of the $t = 4.0$ s interval, the angular displacement is $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$. We solve for the angular velocity at the start of the interval:

$$\omega_0 = \frac{\theta - \frac{1}{2}\alpha t^2}{t} = \frac{120 \text{ rad} - \frac{1}{2}\left(3.0 \text{ rad/s}^2\right)\left(4.0 \text{ s}\right)^2}{4.0 \text{ s}} = 24 \text{ rad/s}.$$

We now use $\omega = \omega_0 + \alpha t$ (Eq. 10-12) to find the time when the wheel is at rest:

$$t = -\frac{\omega_0}{\alpha} = -\frac{24 \text{ rad / s}}{3.0 \text{ rad / s}^2} = -8.0 \text{ s}.$$

That is, the wheel started from rest 8.0 s before the start of the described 4.0 s interval.

14. (a) Eq. 10-13 gives

$$\theta - \theta_o = \omega_o t + \tfrac{1}{2}\alpha t^2 = 0 + \tfrac{1}{2}(1.5 \text{ rad/s}^2)\, t_1^{\,2}$$

where $\theta - \theta_o = (2 \text{ rev})(2\pi \text{ rad/rev})$. Therefore, $t_1 = 4.09$ s.

(b) We can find the time to go through a full 4 rev (using the same equation to solve for a new time t_2) and then subtract the result of part (a) for t_1 in order to find this answer.

$$(4 \text{ rev})(2\pi \text{ rad/rev}) = 0 + \tfrac{1}{2}(1.5 \text{ rad/s}^2)\, t_2^{\,2} \quad \Rightarrow \quad t_2 = 5.789 \text{ s}.$$

Thus, the answer is 5.789 s – 4.093 s ≈ 1.70 s.

15. The problem has (implicitly) specified the positive sense of rotation. The angular acceleration of magnitude 0.25 rad/s^2 in the negative direction is assumed to be constant over a large time interval, including negative values (for t).

(a) We specify θ_{max} with the condition ω = 0 (this is when the wheel reverses from positive rotation to rotation in the negative direction). We obtain θ_{max} using Eq. 10-14:

$$\theta_{\text{max}} = -\frac{\omega_o^2}{2\alpha} = -\frac{(4.7 \text{ rad/s})^2}{2(-0.25 \text{ rad/s}^2)} = 44 \text{ rad.}$$

(b) We find values for t_1 when the angular displacement (relative to its orientation at t = 0) is θ_1 = 22 rad (or 22.09 rad if we wish to keep track of accurate values in all intermediate steps and only round off on the final answers). Using Eq. 10-13 and the quadratic formula, we have

$$\theta_1 = \omega_o t_1 + \frac{1}{2}\alpha t_1^2 \Rightarrow t_1 = \frac{-\omega_o \pm \sqrt{\omega_o^2 + 2\theta_1\alpha}}{\alpha}$$

which yields the two roots 5.5 s and 32 s. Thus, the first time the reference line will be at θ_1 = 22 rad is t = 5.5 s.

(c) The second time the reference line will be at θ_1 = 22 rad is t = 32 s.

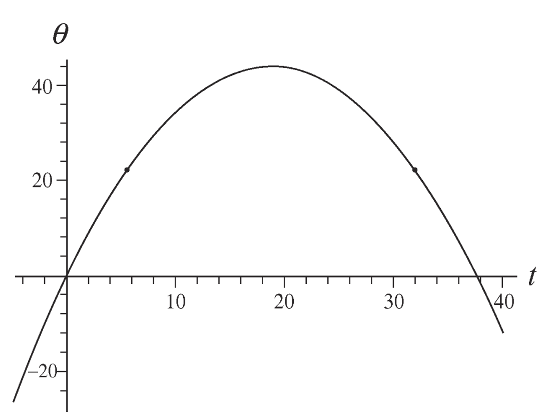
(d) We find values for t_2 when the angular displacement (relative to its orientation at t = 0) is θ_2 = –10.5 rad. Using Eq. 10-13 and the quadratic formula, we have

$$\theta_2 = \omega_o t_2 + \frac{1}{2}\alpha t_2^2 \Rightarrow t_2 = \frac{-\omega_o \pm \sqrt{\omega_o^2 + 2\theta_2\alpha}}{\alpha}$$

which yields the two roots –2.1 s and 40 s. Thus, at t = –2.1 s the reference line will be at θ_2 = –10.5 rad.

(e) At t = 40 s the reference line will be at θ_2 = –10.5 rad.

(f) With radians and seconds understood, the graph of θ versus t is shown below (with the points found in the previous parts indicated as small circles).



16. The wheel starts turning from rest ($\omega_0 = 0$) at $t = 0$, and accelerates uniformly at $\alpha > 0$, which makes our choice for positive sense of rotation. At t_1 its angular velocity is $\omega_1 = +10$ rev/s, and at t_2 its angular velocity is $\omega_2 = +15$ rev/s. Between t_1 and t_2 it turns through $\Delta\theta = 60$ rev, where $t_2 - t_1 = \Delta t$.

(a) We find α using Eq. 10-14:

$$\omega_2^2 = \omega_1^2 + 2\alpha\Delta\theta \quad \Rightarrow \quad \alpha = \frac{(15 \text{ rev/s})^2 - (10 \text{ rev/s})^2}{2(60 \text{ rev})} = 1.04 \text{ rev/s}^2$$

which we round off to 1.0 rev/s^2.

(b) We find Δt using Eq. 10-15:

$$\Delta\theta = \frac{1}{2}\left(\omega_1 + \omega_2\right)\Delta t \Rightarrow \Delta t = \frac{2(60 \text{ rev})}{10 \text{ rev/s} + 15 \text{ rev/s}} = 4.8 \text{ s.}$$

(c) We obtain t_1 using Eq. 10-12: $\omega_1 = \omega_0 + \alpha t_1 \Rightarrow t_1 = \dfrac{10 \text{ rev/s}}{1.04 \text{ rev/s}^2} = 9.6 \text{ s.}$

(d) Any equation in Table 10-1 involving θ can be used to find θ_1 (the angular displacement during $0 \le t \le t_1$); we select Eq. 10-14.

$$\omega_1^2 = \omega_0^2 + 2\alpha\theta_1 \Rightarrow \theta_1 = \frac{(10 \text{ rev/s})^2}{2(1.04 \text{ rev/s}^2)} = 48 \text{ rev.}$$

17. The wheel has angular velocity ω_0 = +1.5 rad/s = +0.239 rev/s at t = 0, and has constant value of angular acceleration $\alpha < 0$, which indicates our choice for positive sense of rotation. At t_1 its angular displacement (relative to its orientation at t = 0) is θ_1 = +20 rev, and at t_2 its angular displacement is θ_2 = +40 rev and its angular velocity is $\omega_2 = 0$.

(a) We obtain t_2 using Eq. 10-15:

$$\theta_2 = \frac{1}{2}(\omega_0 + \omega_2)t_2 \;\Rightarrow\; t_2 = \frac{2(40 \text{ rev})}{0.239 \text{ rev/s}} = 335 \text{ s}$$

which we round off to $t_2 \approx 3.4 \times 10^2$ s.

(b) Any equation in Table 10-1 involving α can be used to find the angular acceleration; we select Eq. 10-16.

$$\theta_2 = \omega_2 t_2 - \frac{1}{2}\alpha t_2^2 \;\Rightarrow\; \alpha = -\frac{2(40 \text{ rev})}{(335 \text{ s})^2} = -7.12 \times 10^{-4} \text{ rev/s}^2$$

which we convert to $\alpha = -4.5 \times 10^{-3}$ rad/s^2.

(c) Using $\theta_1 = \omega_0 t_1 + \frac{1}{2}\alpha t_1^2$ (Eq. 10-13) and the quadratic formula, we have

$$t_1 = \frac{-\omega_0 \pm \sqrt{\omega_0^2 + 2\theta_1\alpha}}{\alpha} = \frac{-(0.239 \text{ rev/s}) \pm \sqrt{(0.239 \text{ rev/s})^2 + 2(20 \text{ rev})(-7.12 \times 10^{-4} \text{ rev/s}^2)}}{-7.12 \times 10^{-4} \text{ rev/s}^2}$$

which yields two positive roots: 98 s and 572 s. Since the question makes sense only if $t_1 < t_2$ we conclude the correct result is t_1 = 98 s.

18. Converting $33\frac{1}{3}$ rev/min to radians-per-second, we get $\omega = 3.49$ rad/s. Combining $v = \omega r$ (Eq. 10-18) with $\Delta t = d/v$ where Δt is the time between bumps (a distance d apart), we arrive at the rate of striking bumps:

$$\frac{1}{\Delta t} = \frac{\omega r}{d} \approx 199/\mathrm{s}\,.$$

19. We assume the given rate of 1.2×10^{-3} m/y is the linear speed of the top; it is also possible to interpret it as just the horizontal component of the linear speed but the difference between these interpretations is arguably negligible. Thus, Eq. 10-18 leads to

$$\omega = \frac{1.2 \times 10^{-3} \text{ m/y}}{55 \text{ m}} = 2.18 \times 10^{-5} \text{ rad/y}$$

which we convert (since there are about 3.16×10^{7} s in a year) to $\omega = 6.9 \times 10^{-13}$ rad/s.

20. (a) Using Eq. 10-6, the angular velocity at $t = 5.0$s is

$$\omega = \left.\frac{d\theta}{dt}\right|_{t=5.0} = \left.\frac{d}{dt}\left(0.30t^2\right)\right|_{t=5.0} = 2(0.30)(5.0) = 3.0 \text{ rad / s.}$$

(b) Eq. 10-18 gives the linear speed at $t = 5.0$s: $v = \omega r = (3.0 \text{ rad/s})(10 \text{ m}) = 30 \text{ m/s}$.

(c) The angular acceleration is, from Eq. 10-8,

$$\alpha = \frac{d\omega}{dt} = \frac{d}{dt}(0.60t) = 0.60 \text{ rad / s}^2.$$

Then, the tangential acceleration at $t = 5.0$s is, using Eq. 10-22,

$$a_t = r\alpha = (10 \text{ m})\left(0.60 \text{ rad / s}^2\right) = 6.0 \text{ m / s}^2.$$

(d) The radial (centripetal) acceleration is given by Eq. 10-23:

$$a_r = \omega^2 r = \left(3.0 \text{ rad / s}\right)^2 \left(10 \text{ m}\right) = 90 \text{ m / s}^2.$$

21. (a) We obtain

$$\omega = \frac{(200 \text{ rev/min})(2\pi \text{ rad/rev})}{60 \text{ s/min}} = 20.9 \text{ rad/s}.$$

(b) With $r = 1.20/2 = 0.60$ m, Eq. 10-18 gives $v = r\omega = (0.60 \text{ m})(20.9 \text{ rad/s}) = 12.5 \text{ m/s}$.

(c) With $t = 1$ min, $\omega = 1000$ rev/min and $\omega_0 = 200$ rev/min, Eq. 10-12 gives

$$\alpha = \frac{\omega - \omega_o}{t} = 800 \text{ rev/min}^2.$$

(d) With the same values used in part (c), Eq. 10-15 becomes

$$\theta = \frac{1}{2}(\omega_o + \omega)t = \frac{1}{2}(200 \text{ rev/min} + 1000 \text{ rev/min})(1.0 \text{ min}) = 600 \text{ rev}.$$

22. First, we convert the angular velocity: ω = (2000 rev/min)(2π/60) = 209 rad/s. Also, we convert the plane's speed to SI units: (480)(1000/3600) = 133 m/s. We use Eq. 10-18 in part (a) and (implicitly) Eq. 4-39 in part (b).

(a) The speed of the tip as seen by the pilot is $v_t = \omega r = (209\,\text{rad/s})(1.5\,\text{m}) = 314\,\text{m/s}$, which (since the radius is given to only two significant figures) we write as $v_t = 3.1\times10^2\ \text{m/s}$.

(b) The plane's velocity $\vec{v}_p$ and the velocity of the tip $\vec{v}_t$ (found in the plane's frame of reference), in any of the tip's positions, must be perpendicular to each other. Thus, the speed as seen by an observer on the ground is

$$v = \sqrt{v_p^2 + v_t^2} = \sqrt{(133\,\text{m/s})^2 + (314\,\text{m/s})^2} = 3.4\times10^2\ \text{m/s}\,.$$

23. (a) Converting from hours to seconds, we find the angular velocity (assuming it is positive) from Eq. 10-18:

$$\omega = \frac{v}{r} = \frac{\left(2.90\times10^{4}\ \text{km/h}\right)\left(1.000\ \text{h}/3600\ \text{s}\right)}{3.22\times10^{3}\ \text{km}} = 2.50\times10^{-3}\ \text{rad/s}.$$

(b) The radial (or centripetal) acceleration is computed according to Eq. 10-23:

$$a_r = \omega^2 r = \left(2.50\times10^{-3}\ \text{rad/s}\right)^2 \left(3.22\times10^{6}\ \text{m}\right) = 20.2\ \text{m/s}^2.$$

(c) Assuming the angular velocity is constant, then the angular acceleration and the tangential acceleration vanish, since

$$\alpha = \frac{d\omega}{dt} = 0 \ \text{ and } \ a_t = r\alpha = 0.$$

24. The function $\theta = \xi e^{\beta t}$ where ξ = 0.40 rad and β = 2 s^{-1} is describing the angular coordinate of a line (which is marked in such a way that all points on it have the same value of angle at a given time) on the object. Taking derivatives with respect to time leads to $\frac{d\theta}{dt} = \xi\beta e^{\beta t}$ and $\frac{d^2\theta}{dt^2} = \xi\beta^2 e^{\beta t}$.

(a) Using Eq. 10-22, we have $a_t = \alpha r = \dfrac{d^2\theta}{dt^2} r = 6.4\ \text{cm}/\text{s}^2$.

(b) Using Eq. 10-23, we get $a_r = \omega^2 r = \left(\dfrac{d\theta}{dt}\right)^2 r = 2.6\ \text{cm}/\text{s}^2$.

25. (a) The upper limit for centripetal acceleration (same as the radial acceleration – see Eq. 10-23) places an upper limit of the rate of spin (the angular velocity ω) by considering a point at the rim (r = 0.25 m). Thus, $\omega_{max} = \sqrt{a/r}$ = 40 rad/s. Now we apply Eq. 10-15 to first half of the motion (where $\omega_o = 0$):

$$\theta - \theta_o = \tfrac{1}{2}(\omega_o + \omega)t \;\Rightarrow\; 400 \text{ rad} = \tfrac{1}{2}(0 + 40 \text{ rad/s})t$$

which leads to t = 20 s. The second half of the motion takes the same amount of time (the process is essentially the reverse of the first); the total time is therefore 40 s.

(b) Considering the first half of the motion again, Eq. 10-11 leads to

$$\omega = \omega_o + \alpha t \;\Rightarrow\; \alpha = \frac{40 \text{ rad/s}}{20 \text{ s}} = 2.0 \text{ rad/s}^2 .$$

26. (a) The tangential acceleration, using Eq. 10-22, is

$$a_t = \alpha r = \left(14.2 \text{ rad / s}^2\right)(2.83 \text{ cm}) = 40.2 \text{ cm / s}^2.$$

(b) In rad/s, the angular velocity is ω= (2760)(2π/60) = 289 rad/s, so

$$a_r = \omega^2 r = (289 \text{ rad / s})^2 (0.0283 \text{ m}) = 2.36\times10^3 \text{ m / s}^2.$$

(c) The angular displacement is, using Eq. 10-14,

$$\theta = \frac{\omega^2}{2\alpha} = \frac{(289 \text{ rad/s})^2}{2(14.2 \text{ rad/s}^2)} = 2.94\times10^3 \text{ rad}.$$

Then, using Eq. 10-1, the distance traveled is

$$s = r\theta = (0.0283 \text{ m})(2.94\times10^3 \text{ rad}) = 83.2 \text{ m}.$$

27. (a) In the time light takes to go from the wheel to the mirror and back again, the wheel turns through an angle of $\theta = 2\pi/500 = 1.26 \times 10^{-2}$ rad. That time is

$$t = \frac{2\ell}{c} = \frac{2(500\text{ m})}{2.998 \times 10^8\text{ m/s}} = 3.34 \times 10^{-6}\text{ s}$$

so the angular velocity of the wheel is

$$\omega = \frac{\theta}{t} = \frac{1.26 \times 10^{-2}\text{ rad}}{3.34 \times 10^{-6}\text{ s}} = 3.8 \times 10^3\text{ rad/s.}$$

(b) If r is the radius of the wheel, the linear speed of a point on its rim is

$$v = \omega r = \left(3.8 \times 10^3\text{ rad/s}\right)\left(0.050\text{ m}\right) = 1.9 \times 10^2\text{ m/s.}$$

28. (a) The angular acceleration is

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{0 - 150 \text{ rev/min}}{(2.2 \text{ h})(60 \text{ min/1h})} = -1.14 \text{ rev/min}^2.$$

(b) Using Eq. 10-13 with $t = (2.2)(60) = 132$ min, the number of revolutions is

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2 = (150 \text{ rev/min})(132 \text{ min}) + \frac{1}{2}\left(-1.14 \text{ rev/min}^2\right)(132 \text{ min})^2 = 9.9\times10^3 \text{ rev}.$$

(c) With $r = 500$ mm, the tangential acceleration is

$$a_t = \alpha r = \left(-1.14 \text{ rev/min}^2\right)\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right)^2 (500 \text{ mm})$$

which yields $a_t = -0.99$ mm/s^2.

(d) The angular speed of the flywheel is

$$\omega = (75 \text{ rev/min})(2\pi \text{ rad/rev})(1 \text{ min/ } 60 \text{ s}) = 7.85 \text{ rad/s}.$$

With $r = 0.50$ m, the radial (or centripetal) acceleration is given by Eq. 10-23:

$$a_r = \omega^2 r = (7.85 \text{ rad/s})^2 (0.50 \text{ m}) \approx 31 \text{ m/s}^2$$

which is much bigger than a_t. Consequently, the magnitude of the acceleration is

$$|\vec{a}| = \sqrt{a_r^2 + a_t^2} \approx a_r = 31 \text{ m/s}^2.$$

29. (a) Earth makes one rotation per day and 1 *d* is (24 h) (3600 s/h) = 8.64×10^4 s, so the angular speed of Earth is

$$\omega = \frac{2\pi\,\text{rad}}{8.64\times10^4\ \text{s}} = 7.3\times10^{-5}\ \text{rad/s}.$$

(b) We use $v = \omega r$, where r is the radius of its orbit. A point on Earth at a latitude of 40° moves along a circular path of radius $r = R \cos 40°$, where R is the radius of Earth (6.4×10^6 m). Therefore, its speed is

$$v = \omega(R\cos 40°) = (7.3\times10^{-5}\ \text{rad/s})(6.4\times10^6\ \text{m})\cos 40° = 3.5\times10^2\ \text{m/s}.$$

(c) At the equator (and all other points on Earth) the value of ω is the same (7.3×10^{-5} rad/s).

(d) The latitude is 0° and the speed is

$$v = \omega R = (7.3\times10^{-5}\ \text{rad/s})(6.4\times10^6\ \text{m}) = 4.6\times10^2\ \text{m/s}.$$

30. Since the belt does not slip, a point on the rim of wheel C has the same tangential acceleration as a point on the rim of wheel A. This means that $\alpha_A r_A = \alpha_C r_C$, where α_A is the angular acceleration of wheel A and α_C is the angular acceleration of wheel C. Thus,

$$\alpha_C = \left(\frac{r_A}{r_C}\right)\alpha_C = \left(\frac{10 \text{ cm}}{25 \text{ cm}}\right)(1.6 \text{ rad/s}^2) = 0.64 \text{ rad/s}^2.$$

Since the angular speed of wheel C is given by $\omega_C = \alpha_C t$, the time for it to reach an angular speed of ω= 100 rev/min = 10.5 rad/s starting from rest is

$$t = \frac{\omega_C}{\alpha_C} = \frac{10.5 \text{ rad/s}}{0.64 \text{ rad/s}^2} = 16 \text{ s}.$$

31. (a) The angular speed in rad/s is

$$\omega = \left(33\frac{1}{3}\,\text{rev/min}\right)\left(\frac{2\pi\,\text{rad/rev}}{60\ \text{s/min}}\right) = 3.49\ \text{rad/s}.$$

Consequently, the radial (centripetal) acceleration is (using Eq. 10-23)

$$a = \omega^2 r = (3.49\ \text{rad/s})^2 (6.0\times10^{-2}\ \text{m}) = 0.73\ \text{m/s}^2.$$

(b) Using Ch. 6 methods, we have $ma = f_s \le f_{s,\max} = \mu_s mg$, which is used to obtain the (minimum allowable) coefficient of friction:

$$\mu_{s,\min} = \frac{a}{g} = \frac{0.73}{9.8} = 0.075.$$

(c) The radial acceleration of the object is $a_r = \omega^2 r$, while the tangential acceleration is $a_t = \alpha r$. Thus,

$$|\vec{a}| = \sqrt{a_r^2 + a_t^2} = \sqrt{(\omega^2 r)^2 + (\alpha r)^2} = r\sqrt{\omega^4 + \alpha^2}.$$

If the object is not to slip at any time, we require

$$f_{s,\max} = \mu_s mg = ma_{\max} = mr\sqrt{\omega_{\max}^4 + \alpha^2}.$$

Thus, since $\alpha = \omega/t$ (from Eq. 10-12), we find

$$\mu_{s,\min} = \frac{r\sqrt{\omega_{\max}^4 + \alpha^2}}{g} = \frac{r\sqrt{\omega_{\max}^4 + (\omega_{\max}/t)^2}}{g} = \frac{(0.060)\sqrt{3.49^4 + (3.4/0.25)^2}}{9.8} = 0.11.$$

32. (a) A complete revolution is an angular displacement of $\Delta\theta = 2\pi$ rad, so the angular velocity in rad/s is given by $\omega = \Delta\theta/T = 2\pi/T$. The angular acceleration is given by

$$\alpha = \frac{d\omega}{dt} = -\frac{2\pi}{T^2}\frac{dT}{dt}.$$

For the pulsar described in the problem, we have

$$\frac{dT}{dt} = \frac{1.26\times10^{-5}\ \text{s/y}}{3.16\times10^{7}\ \text{s/y}} = 4.00\times10^{-13}.$$

Therefore,

$$\alpha = -\left(\frac{2\pi}{(0.033\ \text{s})^2}\right)(4.00\times10^{-13}) = -2.3\times10^{-9}\ \text{rad/s}^2.$$

The negative sign indicates that the angular acceleration is opposite the angular velocity and the pulsar is slowing down.

(b) We solve $\omega = \omega_0 + \alpha t$ for the time t when $\omega = 0$:

$$t = -\frac{\omega_0}{\alpha} = -\frac{2\pi}{\alpha T} = -\frac{2\pi}{(-2.3\times10^{-9}\ \text{rad/s}^2)(0.033\ \text{s})} = 8.3\times10^{10}\ \text{s} \approx 2.6\times10^{3}\ \text{ years}$$

(c) The pulsar was born 1992–1054 = 938 years ago. This is equivalent to (938 y)(3.16×10^7 s/y) = 2.96×10^{10} s. Its angular velocity at that time was

$$\omega = \omega_0 + \alpha t + \frac{2\pi}{T} + \alpha t = \frac{2\pi}{0.033\ \text{s}} + (-2.3\times10^{-9}\ \text{rad/s}^2)(-2.96\times10^{10}\ \text{s}) = 258\ \text{rad/s}.$$

Its period was

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{258\ \text{rad/s}} = 2.4\times10^{-2}\ \text{s}.$$

33. The kinetic energy (in J) is given by $K = \frac{1}{2} I\omega^2$, where I is the rotational inertia (in $\text{kg}\cdot\text{m}^2$) and ω is the angular velocity (in rad/s). We have

$$\omega = \frac{(602 \text{ rev/min})(2\pi \text{ rad/rev})}{60 \text{ s/min}} = 63.0 \text{ rad/s}.$$

Consequently, the rotational inertia is

$$I = \frac{2K}{\omega^2} = \frac{2(24400 \text{ J})}{(63.0 \text{ rad/s})^2} = 12.3 \text{ kg}\cdot\text{m}^2.$$

34. (a) Eq. 10-12 implies that the angular acceleration α should be the slope of the ω vs t graph. Thus, $\alpha = 9/6 = 1.5$ rad/s^2.

(b) By Eq. 10-34, K is proportional to ω^2. Since the angular velocity at $t = 0$ is –2 rad/s (and this value squared is 4) and the angular velocity at $t = 4$ s is 4 rad/s (and this value squared is 16), then the ratio of the corresponding kinetic energies must be

$$\frac{K_o}{K_4} = \frac{4}{16} \Rightarrow K_o = \tfrac{1}{4} K_4 = 0.40 \text{ J} .$$

35. We use the parallel axis theorem: $I = I_{com} + Mh^2$, where I_{com} is the rotational inertia about the center of mass (see Table 10-2(d)), M is the mass, and h is the distance between the center of mass and the chosen rotation axis. The center of mass is at the center of the meter stick, which implies $h = 0.50$ m – 0.20 m = 0.30 m. We find

$$I_{\text{com}} = \frac{1}{12} ML^2 = \frac{1}{12}(0.56 \text{ kg})(1.0 \text{ m})^2 = 4.67 \times 10^{-2} \text{ kg} \cdot \text{m}^2.$$

Consequently, the parallel axis theorem yields

$$I = 4.67 \times 10^{-2} \text{ kg} \cdot \text{m}^2 + (0.56 \text{ kg})(0.30 \text{ m})^2 = 9.7 \times 10^{-2} \text{ kg} \cdot \text{m}^2.$$

36. (a) Eq. 10-33 gives

$$I_{\text{total}} = md^2 + m(2d)^2 + m(3d)^2 = 14\ md^2.$$

If the innermost one is removed then we would only obtain $m(2d)^2 + m(3d)^2 = 13\ md^2$. The percentage difference between these is $(13 - 14)/14 = 0.0714 \approx 7.1\%$.

(b) If, instead, the outermost particle is removed, we would have $md^2 + m(2d)^2 = 5\ md^2$. The percentage difference in this case is $0.643 \approx 64\%$.

37. Since the rotational inertia of a cylinder is $I = \frac{1}{2} MR^2$ (Table 10-2(c)), its rotational kinetic energy is

$$K = \frac{1}{2} I\omega^2 = \frac{1}{4} MR^2\omega^2 .$$

(a) For the smaller cylinder, we have $K = \frac{1}{4}(1.25)(0.25)^2(235)^2 = 1.1\times10^3$ J.

(b) For the larger cylinder, we obtain $K = \frac{1}{4}(1.25)(0.75)^2(235)^2 = 9.7\times10^3$ J.

38. The parallel axis theorem (Eq. 10-36) shows that I increases with h. The phrase "out to the edge of the disk" (in the problem statement) implies that the maximum h in the graph is, in fact, the radius R of the disk. Thus, $R = 0.20$ m. Now we can examine, say, the $h = 0$ datum and use the formula for I_{com} (see Table 10-2(c)) for a solid disk, or (which might be a little better, since this is independent of whether it is really a solid disk) we can the difference between the $h = 0$ datum and the $h = h_{max} = R$ datum and relate that difference to the parallel axis theorem (thus the difference is $M(h_{max})^2 = 0.10$ kg·m^2). In either case, we arrive at $M = 2.5$ kg.

39. The particles are treated "point-like" in the sense that Eq. 10-33 yields their rotational inertia, and the rotational inertia for the rods is figured using Table 10-2(e) and the parallel-axis theorem (Eq. 10-36).

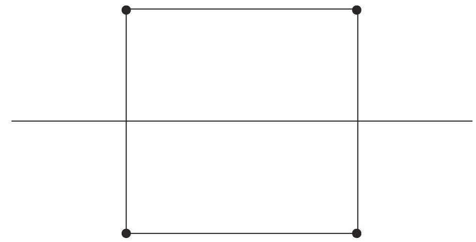
(a) With subscript 1 standing for the rod nearest the axis and 4 for the particle farthest from it, we have

$$
\begin{aligned}
I &= I_1 + I_2 + I_3 + I_4 = \left(\frac{1}{12}Md^2 + M\left(\frac{1}{2}d\right)^2\right) + md^2 + \left(\frac{1}{12}Md^2 + M\left(\frac{3}{2}d\right)^2\right) + m(2d)^2 \\
&= \frac{8}{3}Md^2 + 5md^2 = \frac{8}{3}(1.2\text{ kg})(0.056\text{ m})^2 + 5(0.85\text{ kg})(0.056\text{ m})^2 \\
&= 0.023\text{ kg}\cdot\text{m}^2.
\end{aligned}
$$

(b) Using Eq. 10-34, we have

$$
\begin{aligned}
K &= \frac{1}{2}I\omega^2 = \left(\frac{4}{3}M + \frac{5}{2}m\right)d^2\omega^2 = \left[\frac{4}{3}(1.2\text{ kg}) + \frac{5}{2}(0.85\text{ kg})\right](0.056\text{ m})^2(0.30\text{ rad/s})^2 \\
&= 1.1\times10^{-3}\text{ J}.
\end{aligned}
$$

40. (a) We show the figure with its axis of rotation (the thin horizontal line).



We note that each mass is r = 1.0 m from the axis. Therefore, using Eq. 10-26, we obtain

$$I = \sum m_i r_i^2 = 4\ (0.50\ \text{kg})\ (1.0\ \text{m})^2 = 2.0\ \text{kg}\cdot\text{m}^2.$$

(b) In this case, the two masses nearest the axis are r = 1.0 m away from it, but the two furthest from the axis are $r = \sqrt{(1.0\ \text{m})^2 + (2.0\ \text{m})^2}$ from it. Here, then, Eq. 10-33 leads to

$$I = \sum m_i r_i^2 = 2(0.50\ \text{kg})\left(1.0\ \text{m}^2\right) + 2(0.50\ \text{kg})\left(5.0\ \text{m}^2\right) = 6.0\ \text{kg}\cdot\text{m}^2.$$

(c) Now, two masses are on the axis (with r = 0) and the other two are a distance $r = \sqrt{(1.0\ \text{m})^2 + (1.0\ \text{m})^2}$ away. Now we obtain $I = 2.0\ \text{kg}\cdot\text{m}^2$.

41. We use the parallel-axis theorem. According to Table 10-2(i), the rotational inertia of a uniform slab about an axis through the center and perpendicular to the large faces is given by

$$I_{\text{com}} = \frac{M}{12}\left(a^2 + b^2\right).$$

A parallel axis through the corner is a distance $h = \sqrt{\left(a/2\right)^2 + \left(b/2\right)^2}$ from the center. Therefore,

$$\begin{aligned} I &= I_{\text{com}} + Mh^2 = \frac{M}{12}\left(a^2 + b^2\right) + \frac{M}{4}\left(a^2 + b^2\right) = \frac{M}{3}\left(a^2 + b^2\right) \\ &= \frac{0.172\ \text{kg}}{3}[(0.035\ \text{m})^2 + (0.084\ \text{m})^2] \\ &= 4.7 \times 10^{-4}\ \text{kg} \cdot \text{m}^2. \end{aligned}$$

42. (a) Consider three of the disks (starting with the one at point O): ⊕OO . The first one (the one at point O – shown here with the plus sign inside) has rotational inertial (see item (c) in Table 10-2) $I = \frac{1}{2}mR^2$. The next one (using the parallel-axis theorem) has

$$I = \tfrac{1}{2}mR^2 + mh^2$$

where $h = 2R$. The third one has $I = \frac{1}{2}mR^2 + m(4R)^2$. If we had considered five of the disks OO⊕OO with the one at O in the middle, then the total rotational inertia is

$$I = 5(\tfrac{1}{2}mR^2) + 2(m(2R)^2 + m(4R)^2).$$

The pattern is now clear and we can write down the total I for the collection of fifteen disks:

$$I = 15(\tfrac{1}{2}mR^2) + 2(m(2R)^2 + m(4R)^2 + m(6R)^2 + \ldots + m(14R)^2) = \tfrac{2255}{2}mR^2.$$

The generalization to N disks (where N is assumed to be an odd number) is

$$I = \tfrac{1}{6}(2N^2 + 1)NmR^2.$$

In terms of the total mass ($m = M/15$) and the total length ($R = L/30$), we obtain

$$I = 0.083519ML^2 \approx (0.08352)(0.1000\text{ kg})(1.0000\text{ m})^2 = 8.352 \times 10^{-3}\text{ kg}\cdot\text{m}^2.$$

(b) Comparing to the formula (e) in Table 10-2 (which gives roughly $I = 0.08333\ ML^2$), we find our answer to part (a) is 0.22% lower.

43. (a) Using Table 10-2(c) and Eq. 10-34, the rotational kinetic energy is

$$K = \frac{1}{2} I\omega^2 = \frac{1}{2}\left(\frac{1}{2} MR^2\right)\omega^2 = \frac{1}{4}(500\,\text{kg})(200\pi \text{ rad/s})^2(1.0\,\text{m})^2 = 4.9\times 10^7 \text{ J}.$$

(b) We solve $P = K/t$ (where P is the average power) for the operating time t.

$$t = \frac{K}{P} = \frac{4.9\times 10^7\,\text{J}}{8.0\times 10^3 \text{ W}} = 6.2\times 10^3 \text{ s}$$

which we rewrite as $t \approx 1.0\times 10^2$ min.

44. (a) We apply Eq. 10-33:

$$I_x = \sum_{i=1}^{4} m_i y_i^2 = \left[50(2.0)^2 + (25)(4.0)^2 + 25(-3.0)^2 + 30(4.0)^2\right] \text{g}\cdot\text{cm}^2 = 1.3\times10^3 \ \text{g}\cdot\text{cm}^2.$$

(b) For rotation about the y axis we obtain

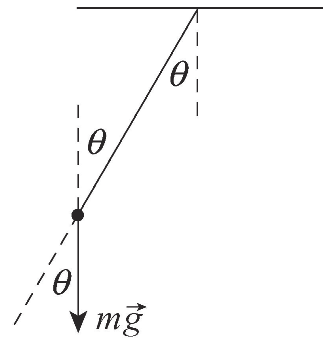
$$I_y = \sum_{i=1}^{4} m_i x_i^2 = 50(2.0)^2 + (25)\ (0)^2 + 25\ (3.0)^2 + 30(2.0)^2 = 5.5\times10^2 \ \text{g}\cdot\text{cm}^2.$$

(c) And about the z axis, we find (using the fact that the distance from the z axis is $\sqrt{x^2+y^2}$)

$$I_z = \sum_{i=1}^{4} m_i\left(x_i^2 + y_i^2\right) = I_x + I_y = 1.3\times10^3 + 5.5\times10^2 = 1.9\times10^2 \ \text{g}\cdot\text{cm}^2.$$

(d) Clearly, the answer to part (c) is $A + B$.

45. Two forces act on the ball, the force of the rod and the force of gravity. No torque about the pivot point is associated with the force of the rod since that force is along the line from the pivot point to the ball. As can be seen from the diagram, the component of the force of gravity that is perpendicular to the rod is mg sin θ. If ℓ is the length of the rod, then the torque associated with this force has magnitude



$$\tau = mg\ell \sin\theta = (0.75)(9.8)(1.25)\sin 30° = 4.6 \text{ N}\cdot\text{m}.$$

For the position shown, the torque is counter-clockwise.

46. We compute the torques using $\tau = rF \sin \phi$.

(a) For $\phi = 30°$, $\tau_a = (0.152\text{ m})(111\text{ N})\sin 30° = 8.4\text{ N}\cdot\text{m}$.

(b) For $\phi = 90°$, $\tau_b = (0.152\text{ m})(111\text{ N})\sin 90° = 17\text{ N}\cdot\text{m}$.

(c) For $\phi = 180°$, $\tau_c = (0.152\text{ m})(111\text{N})\sin 180° = 0$.

47. We take a torque that tends to cause a counterclockwise rotation from rest to be positive and a torque tending to cause a clockwise rotation to be negative. Thus, a positive torque of magnitude $r_1 F_1 \sin \theta_1$ is associated with $\vec{F}_1$ and a negative torque of magnitude $r_2F_2 \sin \theta_2$ is associated with $\vec{F}_2$. The net torque is consequently

$$\tau = r_1F_1 \sin\theta_1 - r_2F_2 \sin\theta_2 .$$

Substituting the given values, we obtain

$$\tau = (1.30 \text{ m})(4.20 \text{ N})\sin 75° - (2.15 \text{ m})(4.90 \text{ N})\sin 60° = -3.85 \text{ N}\cdot\text{m}.$$

48. The net torque is

$$
\begin{aligned}
\tau &= \tau_A + \tau_B + \tau_C = F_A r_A \sin\phi_A - F_B r_B \sin\phi_B + F_C r_C \sin\phi_C \\
&= (10)(8.0)\sin 135^\circ - (16)(4.0)\sin 90^\circ + (19)(3.0)\sin 160^\circ \\
&= 12\ \mathrm{N \cdot m}.
\end{aligned}
$$

49. (a) We use the kinematic equation $\omega = \omega_0 + \alpha t$, where ω_0 is the initial angular velocity, ω is the final angular velocity, α is the angular acceleration, and t is the time. This gives

$$\alpha = \frac{\omega - \omega_0}{t} = \frac{6.20 \text{ rad/s}}{220 \times 10^{-3} \text{ s}} = 28.2 \text{ rad/s}^2.$$

(b) If I is the rotational inertia of the diver, then the magnitude of the torque acting on her is

$$\tau = I\alpha = (12.0 \text{ kg} \cdot \text{m}^2)(28.2 \text{ rad/s}^2) = 3.38 \times 10^2 \text{ N} \cdot \text{m}.$$

50. The rotational inertia is found from Eq. 10-45.

$$I = \frac{\tau}{\alpha} = \frac{32.0}{25.0} = 1.28 \text{ kg} \cdot \text{m}^2$$

51. Combining Eq. 10-45 ($\tau_{net} = I\,\alpha$) with Eq. 10-38 gives $RF_2 - RF_1 = I\alpha$, where $\alpha = \omega/t$ by Eq. 10-12 (with $\omega_0 = 0$). Using item (c) in Table 10-2 and solving for F_2 we find

$$F_2 = \frac{MR\omega}{2t} + F_1 = \frac{(0.02)(0.02)(250)}{2(1.25)} + 0.1 = 0.140 \text{ N}.$$

52. With counterclockwise positive, the angular acceleration α for both masses satisfies $\tau = mgL_1 - mgL_2 = I\alpha = \left(mL_1^2 + mL_2^2\right)\alpha$, by combining Eq. 10-45 with Eq. 10-39 and Eq. 10-33. Therefore, using SI units,

$$\alpha = \frac{g\left(L_1 - L_2\right)}{L_1^2 + L_2^2} = \frac{\left(9.8 \text{ m/s}^2\right)\left(0.20 \text{ m} - 0.80 \text{ m}\right)}{(0.20 \text{ m})^2 + (0.80 \text{ m})^2} = -\, 8.65 \text{ rad/s}^2$$

where the negative sign indicates the system starts turning in the clockwise sense. The magnitude of the acceleration vector involves no radial component (yet) since it is evaluated at $t = 0$ when the instantaneous velocity is zero. Thus, for the two masses, we apply Eq. 10-22:

(a) $\left|\vec{a}_1\right| = |\alpha| L_1 = \left(8.65 \text{ rad/s}^2\right)\left(0.20 \text{ m}\right) = 1.7 \text{ m/s}.$

(b) $\left|\vec{a}_2\right| = |\alpha| L_2 = \left(8.65 \text{ rad/s}^2\right)\left(0.80 \text{ m}\right) = 6.9 \text{ m/s}^2.$

53. Combining Eq. 10-34 and Eq. 10-45, we have $RF = I\alpha$, where α is given by ω/t (according to Eq. 10-12, since $\omega_o = 0$ in this case). We also use the fact that

$$I = I_{\text{plate}} + I_{\text{disk}}$$

where $I_{\text{disk}} = \frac{1}{2}MR^2$ (item (c) in Table 10-2). Therefore,

$$I_{\text{plate}} = \frac{RFt}{\omega} - \frac{1}{2}MR^2 = 2.51 \times 10^{-4}\ \text{kg}\cdot\text{m}^2.$$

54. According to the sign conventions used in the book, the magnitude of the net torque exerted on the cylinder of mass m and radius R is

$$\tau_{\text{net}} = F_1R - F_2R - F_3r = (6.0\text{ N})(0.12\text{ m}) - (4.0\text{ N})(0.12\text{ m}) - (2.0\text{ N})(0.050\text{ m}) = 71\text{ N}\cdot\text{m}.$$

(a) The resulting angular acceleration of the cylinder (with $I = \frac{1}{2}MR^2$ according to Table 10-2(c)) is

$$\alpha = \frac{\tau_{\text{net}}}{I} = \frac{71\text{N}\cdot\text{m}}{\frac{1}{2}(2.0\text{kg})(0.12\text{ m})^2} = 9.7\text{rad/s}^2.$$

(b) The direction is counterclockwise (which is the positive sense of rotation).

55. (a) We use constant acceleration kinematics. If down is taken to be positive and a is the acceleration of the heavier block m_2, then its coordinate is given by $y = \frac{1}{2}at^2$, so

$$a = \frac{2y}{t^2} = \frac{2(0.750\text{ m})}{(5.00\text{ s})^2} = 6.00\times10^{-2}\text{ m/s}^2.$$

Block 1 has an acceleration of 6.00×10^{-2} m/s^2 upward.

(b) Newton's second law for block 2 is $m_2 g - T_2 = m_2 a$, where m_2 is its mass and T_2 is the tension force on the block. Thus,

$$T_2 = m_2(g-a) = (0.500\text{ kg})\left(9.8\text{ m/s}^2 - 6.00\times10^{-2}\text{ m/s}^2\right) = 4.87\text{ N}.$$

(c) Newton's second law for block 1 is $m_1 g - T_1 = -m_1 a,$ where T_1 is the tension force on the block. Thus,

$$T_1 = m_1(g+a) = (0.460\text{ kg})\left(9.8\text{ m/s}^2 + 6.00\times10^{-2}\text{ m/s}^2\right) = 4.54\text{ N}.$$

(d) Since the cord does not slip on the pulley, the tangential acceleration of a point on the rim of the pulley must be the same as the acceleration of the blocks, so

$$\alpha = \frac{a}{R} = \frac{6.00\times10^{-2}\text{ m/s}^2}{5.00\times10^{-2}\text{ m}} = 1.20\text{ rad/s}^2.$$

(e) The net torque acting on the pulley is $\tau = (T_2 - T_1)R$. Equating this to $I\alpha$ we solve for the rotational inertia:

$$I = \frac{(T_2 - T_1)R}{\alpha} = \frac{(4.87\text{ N} - 4.54\text{ N})\left(5.00\times10^{-2}\text{ m}\right)}{1.20\text{ rad/s}^2} = 1.38\times10^{-2}\text{ kg}\cdot\text{m}^2.$$

56. (a) In this case, the force is mg = (70 kg)(9.8 m/s^2), and the "lever arm" (the perpendicular distance from point O to the line of action of the force) is 0.28 m. Thus, the torque (in absolute value) is (70 kg)(9.8 m/s^2)(0.28 m). Since the moment-of-inertia is I = 65 kg·m^2, then Eq. 10-45 gives $|\alpha| = 2.955 \approx 3.0$ rad/s^2.

(b) Now we have another contribution (1.4 m × 300 N) to the net torque, so

$$|\tau_{net}| = (70\text{ kg})(9.8\text{ m/s}^2)(0.28\text{ m}) + (1.4\text{ m})(300\text{ N}) = (65\text{ kg}\cdot\text{m}^2)\,|\alpha|$$

which leads to $|\alpha| = 9.4$ rad/s^2.

57. Since the force acts tangentially at r = 0.10 m, the angular acceleration (presumed positive) is

$$\alpha = \frac{\tau}{I} = \frac{Fr}{I} = \frac{\left(0.5t + 0.3t^2\right)\left(0.10\right)}{1.0 \times 10^{-3}} = 50t + 30t^2$$

in SI units (rad/s^2).

(a) At t = 3 s, the above expression becomes $\alpha = 4.2 \times 10^2\ \text{rad/s}^2$.

(b) We integrate the above expression, noting that $\omega_0 = 0$, to obtain the angular speed at t = 3 s:

$$\omega = \int_0^3 \alpha\, dt = \left(25t^2 + 10t^3\right)\Big|_0^3 = 5.0 \times 10^2\ \text{rad/s}.$$

58. (a) We apply Eq. 10-34:

$$K=\frac{1}{2}I\omega^2=\frac{1}{2}\left(\frac{1}{3}mL^2\right)\omega^2=\frac{1}{6}mL^2\omega^2=\frac{1}{6}(0.42\text{ kg})(0.75\text{ m})^2(4.0\text{ rad/s})^2=0.63\text{ J}.$$

(b) Simple conservation of mechanical energy leads to $K = mgh$. Consequently, the center of mass rises by

$$h=\frac{K}{mg}=\frac{mL^2\omega^2}{6mg}=\frac{L^2\omega^2}{6g}=\frac{(0.75\text{ m})^2(4.0\text{ rad/s})^2}{6(9.8\text{ m/s}^2)}=0.153\text{ m}\approx 0.15\text{ m}.$$

59. The initial angular speed is ω = (280 rev/min)(2π/60) = 29.3 rad/s.

(a) Since the rotational inertia is (Table 10-2(a)) $I = (32\text{ kg})(1.2\text{ m})^2 = 46.1\text{ kg}\cdot\text{m}^2$, the work done is

$$W = \Delta K = 0 - \frac{1}{2}I\omega^2 = -\frac{1}{2}(46.1\text{ kg}\cdot\text{m}^2)(29.3\text{ rad/s})^2$$

which yields $|W| = 1.98 \times 10^4$ J.

(b) The average power (in absolute value) is therefore

$$|P| = \frac{|W|}{\Delta t} = \frac{19.8\times 10^3}{15} = 1.32\times 10^3\ \text{W}.$$

60. (a) The speed of v of the mass m after it has descended d = 50 cm is given by $v^2 = 2ad$ (Eq. 2-16). Thus, using g = 980 cm/s^2, we have

$$v = \sqrt{2ad} = \sqrt{\frac{2(2mg)d}{M+2m}} = \sqrt{\frac{4(50)(980)(50)}{400+2(50)}} = 1.4\times10^2 \text{ cm/s}.$$

(b) The answer is still 1.4×10^2 cm/s = 1.4 m/s, since it is independent of R.

61. With $\omega = (1800)(2\pi/60) = 188.5$ rad/s, we apply Eq. 10-55:

$$P = \tau\omega \quad \Rightarrow \quad \tau = \frac{74600 \text{ W}}{188.5 \text{ rad/s}} = 396 \text{ N} \cdot \text{m} .$$

62. (a) We use the parallel-axis theorem to find the rotational inertia:

$$I = I_{\text{com}} + Mh^2 = \frac{1}{2}MR^2 + Mh^2 = \frac{1}{2}(20\text{ kg})(0.10\text{ m})^2 + (20\text{ kg})(0.50\text{ m})^2 = 0.15\text{ kg}\cdot\text{m}^2.$$

(b) Conservation of energy requires that $Mgh = \frac{1}{2}I\omega^2$, where ω is the angular speed of the cylinder as it passes through the lowest position. Therefore,

$$\omega = \sqrt{\frac{2Mgh}{I}} = \sqrt{\frac{2(20\text{ kg})(9.8\text{ m/s}^2)(0.050\text{ m})}{0.15\text{ kg}\cdot\text{m}^2}} = 11\text{ rad/s}.$$

63. We use ℓ to denote the length of the stick. Since its center of mass is $\ell/2$ from either end, its initial potential energy is $\frac{1}{2}mg\ell$, where m is its mass. Its initial kinetic energy is zero. Its final potential energy is zero, and its final kinetic energy is $\frac{1}{2}I\omega^2$, where I is its rotational inertia about an axis passing through one end of the stick and ω is the angular velocity just before it hits the floor. Conservation of energy yields

$$\frac{1}{2}mg\ell = \frac{1}{2}I\omega^2 \Rightarrow \omega = \sqrt{\frac{mg\ell}{I}}.$$

The free end of the stick is a distance ℓ from the rotation axis, so its speed as it hits the floor is (from Eq. 10-18)

$$v = \omega\ell = \sqrt{\frac{mg\ell^3}{I}}.$$

Using Table 10-2 and the parallel-axis theorem, the rotational inertial is $I = \frac{1}{3}m\ell^2$, so

$$v = \sqrt{3g\ell} = \sqrt{3\left(9.8\ \mathrm{m/s^2}\right)\left(1.00\ \mathrm{m}\right)} = 5.42\ \mathrm{m/s}.$$

64. (a) Eq. 10-33 gives

$$I_{\text{total}} = md^2 + m(2d)^2 + m(3d)^2 = 14\, md^2,$$

where $d = 0.020$ m and $m = 0.010$ kg. The work done is $W = \Delta K = \frac{1}{2} I\omega_f^{\,2} - \frac{1}{2} I\omega_i^2$, where $\omega_f = 20$ rad/s and $\omega_i = 0$. This gives $W = 11.2$ mJ.

(b) Now, $\omega_f = 40$ rad/s and $\omega_i = 20$ rad/s, and we get $W = 33.6$ mJ.

(c) In this case, $\omega_f = 60$ rad/s and $\omega_i = 40$ rad/s. This gives $W = 56.0$ mJ.

(d) Eq. 10-34 indicates that the slope should be $\frac{1}{2} I$. Therefore, it should be

$$7md^2 = 2.80 \times 10^{-5}\ \text{J}\cdot\text{s}^2/\ \text{rad}^2.$$

65. Using the parallel axis theorem and items (*e*) and (*h*) in Table 10-2, the rotational inertia is

$$I = \tfrac{1}{12}mL^2 + m(L/2)^2 + \tfrac{1}{2}mR^2 + m(R + L)^2 = 10.83mR^2 ,$$

where $L = 2R$ has been used. If we take the base of the rod to be at the coordinate origin ($x = 0$, $y = 0$) then the center of mass is at

$$y = \frac{mL/2 + m(L + R)}{m + m} = 2R .$$

Comparing the position shown in the textbook figure to its upside down (inverted) position shows that the change in center of mass position (in absolute value) is $|\Delta y| = 4R$. The corresponding loss in gravitational potential energy is converted into kinetic energy. Thus,

$$K = (2m)g(4R) \quad \Rightarrow \quad \omega = 9.82 \text{ rad/s} .$$

where Eq. 10-34 has been used.

66. From Table 10-2, the rotational inertia of the spherical shell is $2MR^2/3$, so the kinetic energy (after the object has descended distance h) is

$$K = \frac{1}{2}\left(\frac{2}{3}MR^2\right)\omega_{\text{sphere}}^2 + \frac{1}{2}I\omega_{\text{pulley}}^2 + \frac{1}{2}mv^2.$$

Since it started from rest, then this energy must be equal (in the absence of friction) to the potential energy mgh with which the system started. We substitute v/r for the pulley's angular speed and v/R for that of the sphere and solve for v.

$$v = \sqrt{\frac{mgh}{\frac{1}{2}m + \frac{1}{2}\frac{I}{r^2} + \frac{M}{3}}} = \sqrt{\frac{2gh}{1 + (I/mr^2) + (2M/3m)}}$$

$$= \sqrt{\frac{2(9.8)(0.82)}{1 + 3.0\times10^{-3}/((0.60)(0.050)^2) + 2(4.5)/3(0.60)}} = 1.4 \text{ m/s}$$

67. (a) We use conservation of mechanical energy to find an expression for ω^2 as a function of the angle θ that the chimney makes with the vertical. The potential energy of the chimney is given by $U = Mgh$, where M is its mass and h is the altitude of its center of mass above the ground. When the chimney makes the angle θ with the vertical, $h = (H/2)\cos\theta$. Initially the potential energy is $U_i = Mg(H/2)$ and the kinetic energy is zero. The kinetic energy is $\frac{1}{2}I\omega^2$ when the chimney makes the angle θ with the vertical, where I is its rotational inertia about its bottom edge. Conservation of energy then leads to

$$MgH/2 = Mg(H/2)\cos\theta + \frac{1}{2}I\omega^2 \Rightarrow \omega^2 = (MgH/I)(1-\cos\theta).$$

The rotational inertia of the chimney about its base is $I = MH^2/3$ (found using Table 10-2(e) with the parallel axis theorem). Thus

$$\omega = \sqrt{\frac{3g}{H}(1-\cos\theta)} = \sqrt{\frac{3(9.80\text{ m/s}^2)}{55.0\text{ m}}(1-\cos 35.0°)} = 0.311\text{ rad/s}.$$

(b) The radial component of the acceleration of the chimney top is given by $a_r = H\omega^2$, so

$$a_r = 3g(1-\cos\theta) = 3(9.80\text{ m/s}^2)(1-\cos 35.0°) = 5.32\text{ m/s}^2.$$

(c) The tangential component of the acceleration of the chimney top is given by $a_t = H\alpha$, where α is the angular acceleration. We are unable to use Table 10-1 since the acceleration is not uniform. Hence, we differentiate

$$\omega^2 = (3g/H)(1-\cos\theta)$$

with respect to time, replacing $d\omega/dt$ with α, and $d\theta/dt$ with ω, and obtain

$$\frac{d\omega^2}{dt} = 2\omega\alpha = (3g/H)\omega\sin\theta \Rightarrow \alpha = (3g/2H)\sin\theta.$$

Consequently,

$$a_t = H\alpha = \frac{3g}{2}\sin\theta = \frac{3(9.80\text{ m/s}^2)}{2}\sin 35.0° = 8.43\text{ m/s}^2.$$

(d) The angle θ at which $a_t = g$ is the solution to $\frac{3g}{2}\sin\theta = g$. Thus, $\sin\theta = 2/3$ and we obtain $\theta = 41.8°$.

68. The rotational inertia of the passengers is (to a good approximation) given by Eq. 10-53: $I = \sum mR^2 = NmR^2$ where N is the number of people and m is the (estimated) mass per person. We apply Eq. 10-52:

$$W = \frac{1}{2} I\omega^2 = \frac{1}{2} NmR^2\omega^2$$

where R = 38 m and $N = 36 \times 60 = 2160$ persons. The rotation rate is constant so that $\omega = \theta/t$ which leads to $\omega = 2\pi/120 = 0.052$ rad/s. The mass (in kg) of the average person is probably in the range $50 \le m \le 100$, so the work should be in the range

$$\frac{1}{2}(2160)(50)(38)^2(0.052)^2 \le W \le \frac{1}{2}(2160)(100)(38)^2(0.052)^2$$
$$2\times 10^5 \text{ J} \le W \le 4\times 10^5 \text{ J}.$$

69. We choose positive coordinate directions (different choices for each item) so that each is accelerating positively, which will allow us to set $a_2 = a_1 = R\alpha$ (for simplicity, we denote this as a). Thus, we choose rightward positive for $m_2 = M$ (the block on the table), downward positive for $m_1 = M$ (the block at the end of the string) and (somewhat unconventionally) clockwise for positive sense of disk rotation. This means that we interpret θ given in the problem as a positive-valued quantity. Applying Newton's second law to m_1, m_2 and (in the form of Eq. 10-45) to M, respectively, we arrive at the following three equations (where we allow for the possibility of friction f_2 acting on m_2).

$$\begin{aligned} m_1 g - T_1 &= m_1 a_1 \\ T_2 - f_2 &= m_2 a_2 \\ T_1 R - T_2 R &= I\alpha \end{aligned}$$

(a) From Eq. 10-13 (with $\omega_0 = 0$) we find

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2 \;\Rightarrow\; \alpha = \frac{2\theta}{t^2} = \frac{2(1.30\text{ rad})}{(0.0910\text{ s})^2} = 314\text{ rad/s}^2.$$

(b) From the fact that $a = R\alpha$ (noted above), we obtain

$$a = \frac{2R\theta}{t^2} = \frac{2(0.024\text{ m})(1.30\text{ rad})}{(0.0910\text{ s})^2} = 7.54\text{ m/s}^2.$$

(c) From the first of the above equations, we find

$$T_1 = m_1(g - a_1) = M\left(g - \frac{2R\theta}{t^2}\right) = (6.20\text{ kg})\left(9.80\text{ m/s}^2 - \frac{2(0.024\text{ m})(1.30\text{ rad})}{(0.0910\text{ s})^2}\right)$$
$$= 14.0\text{ N}.$$

(d) From the last of the above equations, we obtain the second tension:

$$T_2 = T_1 - \frac{I\alpha}{R} = M\left(g - \frac{2R\theta}{t^2}\right) - \frac{2I\theta}{Rt^2} = 14.0\text{ N} - \frac{(7.40\times10^{-4}\text{ kg}\cdot\text{m}^2)(314\text{ rad/s}^2)}{0.024\text{ m}}$$
$$= 4.36\text{ N}.$$

70. In the calculation below, M_1 and M_2 are the ring masses, R_{1i} and R_{2i} are their inner radii, and R_{1o} and R_{2o} are their outer radii. Referring to item (b) in Table 10-2, we compute

$$I = \tfrac{1}{2} M_1 (R_{1i}^{\,2} + R_{1o}^{\,2}) + \tfrac{1}{2} M_2 (R_{2i}^{\,2} + R_{2o}^{\,2}) \;\; = 0.00346 \text{ kg·m}^2 .$$

Thus, with Eq. 10-38 ($\tau = rF$ where $r = R_{2o}$) and $\tau = I\alpha$ (Eq. 10-45), we find

$$\alpha = \frac{(0.140)(12.0)}{0.00346} = 485 \text{ rad/s}^2 .$$

Then Eq. 10-12 gives $\omega = \alpha t = 146$ rad/s.

71. The volume of each disk is $\pi r^2 h$ where we are using h to denote the thickness (which equals 0.00500 m). If we use R (which equals 0.0400 m) for the radius of the larger disk and r (which equals 0.0200 m) for the radius of the smaller one, then the mass of each is $m = \rho\pi r^2 h$ and $M = \rho\pi R^2 h$ where $\rho = 1400$ kg/m^3 is the given density. We now use the parallel axis theorem as well as item (c) in Table 10-2 to obtain the rotation inertia of the two-disk assembly:

$$I = \tfrac{1}{2}MR^2 + \tfrac{1}{2}mr^2 + m(r+R)^2 = \rho\pi h[\tfrac{1}{2}R^4 + \tfrac{1}{2}r^4 + r^2(r+R)^2] = 6.16 \times 10^{-5}\ \text{kg}{\cdot}\text{m}^2.$$

72. (a) The longitudinal separation between Helsinki and the explosion site is $\Delta\theta = 102° - 25° = 77°$. The spin of the earth is constant at

$$\omega = \frac{1 \text{ rev}}{1 \text{ day}} = \frac{360°}{24 \text{ h}}$$

so that an angular displacement of $\Delta\theta$ corresponds to a time interval of

$$\Delta t = (77°)\left(\frac{24 \text{ h}}{360°}\right) = 5.1 \text{ h}.$$

(b) Now $\Delta\theta = 102° - (-20°) = 122°$ so the required time shift would be

$$\Delta t = (122°)\left(\frac{24 \text{ h}}{360°}\right) = 8.1 \text{ h}.$$

73. We choose positive coordinate directions (different choices for each item) so that each is accelerating positively, which will allow us to set $a_1 = a_2 = R\alpha$ (for simplicity, we denote this as a). Thus, we choose upward positive for m_1, downward positive for m_2 and (somewhat unconventionally) clockwise for positive sense of disk rotation. Applying Newton's second law to $m_1 m_2$ and (in the form of Eq. 10-45) to M, respectively, we arrive at the following three equations.

$$\begin{aligned} T_1 - m_1 g &= m_1 a_1 \\ m_2 g - T_2 &= m_2 a_2 \\ T_2 R - T_1 R &= I\alpha \end{aligned}$$

(a) The rotational inertia of the disk is $I = \frac{1}{2} MR^2$ (Table 10-2(c)), so we divide the third equation (above) by R, add them all, and use the earlier equality among accelerations — to obtain:

$$m_2 g - m_1 g = \left(m_1 + m_2 + \frac{1}{2} M \right) a$$

which yields $a = \frac{4}{25} g = 1.57 \text{ m/s}^2$.

(b) Plugging back in to the first equation, we find

$$T_1 = \frac{29}{25} m_1 g = 4.55 \text{ N}$$

where it is important in this step to have the mass in SI units: $m_1 = 0.40$ kg.

(c) Similarly, with $m_2 = 0.60$ kg, we find

$$T_2 = \frac{5}{6} m_2 g = 4.94 \text{ N}.$$

74. (a) Constant angular acceleration kinematics can be used to compute the angular acceleration α. If ω_0 is the initial angular velocity and t is the time to come to rest, then

$$0 = \omega_0 + \alpha t \Rightarrow \alpha = -\frac{\omega_0}{t}$$

which yields –39/32 = –1.2 rev/s or (multiplying by 2π) –7.66 rad/s^2 for the value of α.

(b) We use $\tau = I\alpha$, where τ is the torque and I is the rotational inertia. The contribution of the rod to I is $M\ell^2/12$ (Table 10-2(e)), where M is its mass and ℓ is its length. The contribution of each ball is $m(\ell/2)^2$, where m is the mass of a ball. The total rotational inertia is

$$I = \frac{M\ell^2}{12} + 2\frac{m\ell^2}{4} = \frac{(6.40 \text{ kg})(1.20 \text{ m})^2}{12} + \frac{(1.06 \text{ kg})(1.20 \text{ m})^2}{2}$$

which yields $I = 1.53 \text{ kg} \cdot \text{m}^2$. The torque, therefore, is

$$\tau = (1.53 \text{ kg} \cdot \text{m}^2)(-7.66 \text{ rad/s}^2) = -11.7 \text{ N} \cdot \text{m}.$$

(c) Since the system comes to rest the mechanical energy that is converted to thermal energy is simply the initial kinetic energy

$$K_i = \frac{1}{2} I\omega_0^2 = \frac{1}{2}(1.53 \text{ kg} \cdot \text{m}^2)((2\pi)(39) \text{ rad/s})^2 = 4.59 \times 10^4 \text{ J}.$$

(d) We apply Eq. 10-13:

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2 = ((2\pi)(39) \text{ rad/s})(32.0 \text{ s}) + \frac{1}{2}(-7.66 \text{ rad/s}^2)(32.0 \text{ s})^2$$

which yields 3920 rad or (dividing by 2π) 624 rev for the value of angular displacement θ.

(e) Only the mechanical energy that is converted to thermal energy can still be computed without additional information. It is 4.59×10^4 J no matter how τ varies with time, as long as the system comes to rest.

75. The *Hint* given in the problem would make the computation in part (a) very straightforward (without doing the integration as we show here), but we present this further level of detail in case that hint is not obvious or — simply — in case one wishes to see how the calculus supports our intuition.

(a) The (centripetal) force exerted on an infinitesimal portion of the blade with mass dm located a distance r from the rotational axis is (Newton's second law) $dF = (dm)\omega^2 r$, where dm can be written as $(M/L)dr$ and the angular speed is

$$\omega = (320)(2\pi/60) = 33.5 \text{ rad/s}.$$

Thus for the entire blade of mass M and length L the total force is given by

$$F = \int dF = \int \omega^2 r\,dm = \frac{M}{L}\int_0^L \omega^2 r\,dr = \frac{M\omega^2 L}{2} = \frac{(110\,\text{kg})\,(33.5 \text{ rad/s})^2\,(7.80\,\text{m})}{2}$$
$$= 4.81\times 10^5\,\text{N}.$$

(b) About its center of mass, the blade has $I = ML^2/12$ according to Table 10-2(e), and using the parallel-axis theorem to "move" the axis of rotation to its end-point, we find the rotational inertia becomes $I = ML^2/3$. Using Eq. 10-45, the torque (assumed constant) is

$$\tau = I\alpha = \left(\frac{1}{3}ML^2\right)\left(\frac{\Delta\omega}{\Delta t}\right) = \frac{1}{3}(110\,\text{kg})(7.8\text{ m})^2\left(\frac{33.5\,\text{rad/s}}{6.7\,\text{s}}\right) = 1.12\times 10^4\,\text{N}\cdot\text{m}.$$

(c) Using Eq. 10-52, the work done is

$$W = \Delta K = \frac{1}{2}I\omega^2 - 0 = \frac{1}{2}\left(\frac{1}{3}ML^2\right)\omega^2 = \frac{1}{6}(110\,\text{kg})(7.80\,\text{m})^2(33.5\,\text{rad/s})^2 = 1.25\times 10^6\text{ J}.$$

76. The wheel starts turning from rest ($\omega_0 = 0$) at $t = 0$, and accelerates uniformly at $\alpha = 2.00 \text{ rad/s}^2$ .Between t_1 and t_2 the wheel turns through $\Delta\theta = 90.0$ rad, where $t_2 - t_1 = \Delta t = 3.00$ s. We solve (b) first.

(b) We use Eq. 10-13 (with a slight change in notation) to describe the motion for $t_1 \le t \le t_2$:

$$\Delta\theta = \omega_1 \Delta t + \frac{1}{2}\alpha(\Delta t)^2 \Rightarrow \omega_1 = \frac{\Delta\theta}{\Delta t} - \frac{\alpha\Delta t}{2}$$

which we plug into Eq. 10-12, set up to describe the motion during $0 \le t \le t_1$:

$$\omega_1 = \omega_0 + \alpha t_1 \;\Rightarrow\; \frac{\Delta\theta}{\Delta t} - \frac{\alpha\Delta t}{2} = \alpha t_1 \;\Rightarrow\; \frac{90.0}{3.00} - \frac{(2.00)(3.00)}{2} = (2.00)\,t_1$$

yielding $t_1 = 13.5$ s.

(a) Plugging into our expression for ω_1 (in previous part) we obtain

$$\omega_1 = \frac{\Delta\theta}{\Delta t} - \frac{\alpha\Delta t}{2} = \frac{90.0}{3.00} - \frac{(2.00)(3.00)}{2} = 27.0 \text{ rad / s}.$$

77. To get the time to reach the maximum height, we use Eq. 4-23, setting the left-hand side to zero. Thus, we find

$$t = \frac{(60 \text{ m/s})\sin(20^\circ)}{9.8 \text{ m/s}^2} = 2.094 \text{ s}.$$

Then (assuming $\alpha = 0$) Eq. 10-13 gives

$$\theta - \theta_0 = \omega_0 t = (90 \text{ rad/s})(2.094 \text{ s}) = 188 \text{ rad},$$

which is equivalent to roughly 30 rev.

78. We choose ± directions such that the initial angular velocity is $\omega_0 = -317$ rad/s and the values for α, τ and F are positive.

(a) Combining Eq. 10-12 with Eq. 10-45 and Table 10-2(f) (and using the fact that $\omega = 0$) we arrive at the expression

$$\tau = \left(\frac{2}{5}MR^2\right)\left(-\frac{\omega_0}{t}\right) = -\frac{2}{5}\frac{MR^2\omega_0}{t}.$$

With $t = 15.5$ s, $R = 0.226$ m and $M = 1.65$ kg, we obtain $\tau = 0.689$ N · m.

(b) From Eq. 10-40, we find $F = \tau / R = 3.05$ N.

(c) Using again the expression found in part (a), but this time with $R = 0.854$ m, we get $\tau = 9.84\ \text{N}\cdot\text{m}$.

(d) Now, $F = \tau / R = 11.5$ N.

79. The center of mass is initially at height $h = \frac{L}{2}\sin 40°$ when the system is released (where L = 2.0 m). The corresponding potential energy Mgh (where M = 1.5 kg) becomes rotational kinetic energy $\frac{1}{2}I\omega^2$ as it passes the horizontal position (where I is the rotational inertia about the pin). Using Table 10-2 (e) and the parallel axis theorem, we find

$$I = \tfrac{1}{12}ML^2 + M(L/2)^2 = \tfrac{1}{3}ML^2.$$

Therefore,

$$Mg\frac{L}{2}\sin 40° = \frac{1}{2}\left(\frac{1}{3}ML^2\right)\omega^2 \quad \Rightarrow \quad \omega = \sqrt{\frac{3g\sin 40°}{L}} = 3.1 \text{ rad/s}.$$

80. (a) Eq. 10-12 leads to $\alpha = -\omega_o / t = -(25.0 \text{ rad/s})/(20.0 \text{ s}) = -1.25 \text{ rad/s}^2$.

(b) Eq. 10-15 leads to $\theta = \frac{1}{2}\omega_o t = \frac{1}{2}(25.0 \text{ rad/s})(20.0 \text{ s}) = 250 \text{ rad}$.

(c) Dividing the previous result by 2π we obtain θ= 39.8 rev.

81. (a) With $r = 0.780$ m, the rotational inertia is

$$I = Mr^2 = (1.30 \text{ kg})(0.780 \text{ m})^2 = 0.791 \text{ kg} \cdot \text{m}^2.$$

(b) The torque that must be applied to counteract the effect of the drag is

$$\tau = rf = (0.780 \text{ m})(2.30 \times 10^{-2} \text{ N}) = 1.79 \times 10^{-2} \text{ N} \cdot \text{m}.$$

82. The motion consists of two stages. The first, the interval $0 \le t \le 20$ s, consists of constant angular acceleration given by

$$\alpha = \frac{5.0 \text{ rad/s}}{2.0 \text{ s}} = 2.5 \text{ rad/s}^2 .$$

The second stage, $20 < t \le 40$ s, consists of constant angular velocity $\omega = \Delta\theta / \Delta t$. Analyzing the first stage, we find

$$\theta_1 = \frac{1}{2}\alpha t^2 \bigg|_{t=20} = 500 \text{ rad}, \quad \omega = \alpha t\big|_{t=20} = 50 \text{ rad/s}.$$

Analyzing the second stage, we obtain

$$\theta_2 = \theta_1 + \omega\Delta t = 500 \text{ rad} + (50 \text{ rad/s})(20 \text{ s}) = 1.5\times10^3 \text{ rad}.$$

83. The magnitude of torque is the product of the force magnitude and the distance from the pivot to the line of action of the force. In our case, it is the gravitational force that passes through the walker's center of mass. Thus,

$$\tau = I\alpha = rF = rmg.$$

(a) Without the pole, with $I = 15\ \text{kg}\cdot\text{m}^2$, the angular acceleration is

$$\alpha = \frac{rF}{I} = \frac{rmg}{I} = \frac{(0.050\ \text{m})(70\ \text{kg})(9.8\ \text{m/s}^2)}{15\ \text{kg}\cdot\text{m}^2} = 2.3\ \text{rad/s}^2.$$

(b) When the walker carries a pole, the torque due to the gravitational force through the pole's center of mass opposes the torque due to the gravitational force that passes through the walker's center of mass. Therefore,

$$\tau_{\text{net}} = \sum_i r_i F_i = (0.050\ \text{m})(70\ \text{kg})(9.8\ \text{m/s}^2) - (0.10\ \text{m})(14\ \text{kg})(9.8\ \text{m/s}^2) = 20.58\ \text{N}\cdot\text{m},$$

and the resulting angular acceleration is

$$\alpha = \frac{\tau_{\text{net}}}{I} = \frac{20.58\ \text{N}\cdot\text{m}}{15\ \text{kg}\cdot\text{m}^2} \approx 1.4\ \text{rad/s}^2.$$

84. The angular displacements of disks A and B can be written as:

$$\theta_A = \omega_A t, \quad \theta_B = \frac{1}{2}\alpha_B t^2.$$

(a) The time when $\theta_A = \theta_B$ is given by

$$\omega_A t = \frac{1}{2}\alpha_B t^2 \quad \Rightarrow \quad t = \frac{2\omega_A}{\alpha_B} = \frac{2(9.5 \text{ rad/s})}{(2.2 \text{ rad/s}^2)} = 8.6 \text{ s}.$$

(b) The difference in the angular displacement is

$$\Delta\theta = \theta_A - \theta_B = \omega_A t - \frac{1}{2}\alpha_B t^2 = 9.5t - 1.1t^2.$$

For their reference lines to align momentarily, we only require $\Delta\theta = 2\pi N$, where N is an integer. The quadratic equation can be readily solve to yield

$$t_N = \frac{9.5 \pm \sqrt{(9.5)^2 - 4(1.1)(2\pi N)}}{2(1.1)} = \frac{9.5 \pm \sqrt{90.25 - 27.6N}}{2.2}.$$

The solution $t_0 = 8.63$ s (taking the positive root) coincides with the result obtained in (a), while $t_0 = 0$ (taking the negative root) is the moment when both disks begin to rotate. In fact, two solutions exist for N = 0, 1, 2, and 3.

85. Eq. 10-40 leads to $\tau = mgr$ = (70 kg) (9.8 m/s^2) (0.20 m) = 1.4×10^2 N·m.

86. (a) Using Eq. 10-15, we have 60.0 rad $= \frac{1}{2}(\omega_1 + \omega_2)(6.00\ \text{s})$. With ω_2 = 15.0 rad/s, then ω_1 = 5.00 rad/s.

(b) Eq. 10-12 gives α = (15.0 rad/s – 5.0 rad/s)/(6.00 s) = 1.67 rad/s^2.

(c) Interpreting ω now as ω_1 and θ as θ_1 = 10.0 rad (and $\omega_0 = 0$) Eq. 10-14 leads to

$$\theta_o = -\frac{\omega_1^2}{2\alpha} + \theta_1 = 2.50\ \text{rad}\ .$$

87. With rightward positive for the block and clockwise negative for the wheel (as is conventional), then we note that the tangential acceleration of the wheel is of opposite sign from the block's acceleration (which we simply denote as a); that is, $a_t = -a$. Applying Newton's second law to the block leads to $P - T = ma$, where $m = 2.0$ kg. Applying Newton's second law (for rotation) to the wheel leads to $-TR = I\alpha$, where $I = 0.050 \text{ kg} \cdot \text{m}^2$.

Noting that $R\alpha = a_t = -a$, we multiply this equation by R and obtain

$$-TR^2 = -Ia \Rightarrow T = a\frac{I}{R^2}.$$

Adding this to the above equation (for the block) leads to $P = (m + I/R^2)a$.
Thus, $a = 0.92 \text{ m/s}^2$ and therefore $\alpha = -4.6 \text{ rad/s}^2$ (or $|\alpha| = 4.6 \text{ rad/s}^2$), where the negative sign in α should not be mistaken for a deceleration (it simply indicates the clockwise sense to the motion).

88. (a) The time for one revolution is the circumference of the orbit divided by the speed v of the Sun: $T = 2\pi R/v$, where R is the radius of the orbit. We convert the radius:

$$R = \left(2.3\times10^{4}\ \text{ly}\right)\left(9.46\times10^{12}\ \text{km/ly}\right) = 2.18\times10^{17}\ \text{km}$$

where the ly $\leftrightarrow$ km conversion can be found in Appendix D or figured "from basics" (knowing the speed of light). Therefore, we obtain

$$T = \frac{2\pi\left(2.18\times10^{17}\ \text{km}\right)}{250\ \text{km/s}} = 5.5\times10^{15}\ \text{s}.$$

(b) The number of revolutions N is the total time t divided by the time T for one revolution; that is, $N = t/T$. We convert the total time from years to seconds and obtain

$$N = \frac{\left(4.5\times10^{9}\ \text{y}\right)\left(3.16\times10^{7}\ \text{s/y}\right)}{5.5\times10^{15}\ \text{s}} = 26.$$

89. We assume the sense of initial rotation is positive. Then, with $\omega_0 > 0$ and $\omega = 0$ (since it stops at time t), our angular acceleration is negative-valued.

(a) The angular acceleration is constant, so we can apply Eq. 10-12 ($\omega = \omega_0 + \alpha t$). To obtain the requested units, we have $t = 30/60 = 0.50$ min. Thus,

$$\alpha = -\frac{33.33 \text{ rev/min}}{0.50 \text{min}} = -66.7 \text{ rev/min}^2 \approx -67 \text{ rev/min}^2.$$

(b) We use Eq. 10-13:

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2 = (33.33 \text{ rev/min})(0.50 \text{ min}) + \frac{1}{2}(-66.7 \text{rev/min}^2)(0.50 \text{ min})^2 = 8.3 \text{ rev}.$$

90. We use conservation of mechanical energy. The center of mass is at the midpoint of the cross bar of the **H** and it drops by $L/2$, where L is the length of any one of the rods. The gravitational potential energy decreases by $MgL/2$, where M is the mass of the body. The initial kinetic energy is zero and the final kinetic energy may be written $\frac{1}{2}I\omega^2$, where I is the rotational inertia of the body and ω is its angular velocity when it is vertical. Thus,

$$0 = -MgL/2 + \frac{1}{2}I\omega^2 \Rightarrow \omega = \sqrt{MgL/I}.$$

Since the rods are thin the one along the axis of rotation does not contribute to the rotational inertia. All points on the other leg are the same distance from the axis of rotation, so that leg contributes $(M/3)L^2$, where $M/3$ is its mass. The cross bar is a rod that rotates around one end, so its contribution is $(M/3)L^2/3 = ML^2/9$. The total rotational inertia is

$$I = (ML^2/3) + (ML^2/9) = 4ML^2/9.$$

Consequently, the angular velocity is

$$\omega = \sqrt{\frac{MgL}{I}} = \sqrt{\frac{MgL}{4ML^2/9}} = \sqrt{\frac{9g}{4L}} = \sqrt{\frac{9(9.800 \text{ m/s}^2)}{4(0.600 \text{ m})}} = 6.06 \text{ rad/s}.$$

91. (a) According to Table 10-2, the rotational inertia formulas for the cylinder (radius *R*) and the hoop (radius *r*) are given by

$$I_C = \frac{1}{2} MR^2 \;\text{ and }\; I_H = Mr^2 .$$

Since the two bodies have the same mass, then they will have the same rotational inertia if

$$R^2 / 2 = R_H^2 \quad \rightarrow \quad R_H = R / \sqrt{2} .$$

(b) We require the rotational inertia to be written as $I = Mk^2$, where *M* is the mass of the given body and *k* is the radius of the "equivalent hoop." It follows directly that $k = \sqrt{I / M}$.

92. (a) We use $\tau = I\alpha$, where τ is the net torque acting on the shell, I is the rotational inertia of the shell, and α is its angular acceleration. Therefore,

$$I = \frac{\tau}{\alpha} = \frac{960\ \text{N}\cdot\text{m}}{6.20\ \text{rad/s}^2} = 155\ \text{kg}\cdot\text{m}^2.$$

(b) The rotational inertia of the shell is given by $I = (2/3)\, MR^2$ (see Table 10-2 of the text). This implies

$$M = \frac{3I}{2R^2} = \frac{3\left(155\ \text{kg}\cdot\text{m}^2\right)}{2\left(1.90\ \text{m}\right)^2} = 64.4\ \text{kg}.$$

93. We choose positive coordinate directions so that each is accelerating positively, which will allow us to set $a_{\text{box}} = R\alpha$ (for simplicity, we denote this as a). Thus, we choose downhill positive for the m = 2.0 kg box and (as is conventional) counterclockwise for positive sense of wheel rotation. Applying Newton's second law to the box and (in the form of Eq. 10-45) to the wheel, respectively, we arrive at the following two equations (using θ as the incline angle 20°, not as the angular displacement of the wheel).

$$mg\sin\theta - T = ma$$
$$TR = I\alpha$$

Since the problem gives a = 2.0 m/s^2, the first equation gives the tension $T = m\,(g\sin\theta - a)$ = 2.7 N. Plugging this and R = 0.20 m into the second equation (along with the fact that $\alpha = a/R$) we find the rotational inertia

$$I = TR^2/a = 0.054\ \text{kg}\cdot\text{m}^2.$$

94. Analyzing the forces tending to drag the M = 5124 kg stone down the oak beam, we find

$$F = Mg\left(\sin\theta + \mu_s \cos\theta\right)$$

where μ_s = 0.22 (static friction is assumed to be at its maximum value) and the incline angle θ for the oak beam is $\sin^{-1}\left(3.9/10\right) = 23°$ (but the incline angle for the spruce log is the complement of that). We note that the component of the weight of the workers (N of them) which is perpendicular to the spruce log is $Nmg\cos(90° - \theta) = Nmg\sin\theta$, where m = 85 kg. The corresponding torque is therefore $Nmg\ell\sin\theta$ where $\ell = 4.5 - 0.7 = 3.8$ m. This must (at least) equal the magnitude of torque due to F, so with r = 0.7 m, we have

$$Mgr\left(\sin\theta + \mu_s \cos\theta\right) = Ngm\ell\sin\theta.$$

This expression yields $N \approx 17$ for the number of workers.

95. The centripetal acceleration at a point P which is r away from the axis of rotation is given by Eq. 10-23: $a = v^2/r = \omega^2 r$, where $v = \omega r$, with $\omega = 2000$ rev/min $\approx$ 209.4 rad/s.

(a) If points A and P are at a radial distance r_A=1.50 m and r = 0.150 m from the axis, the difference in their acceleration is

$$\Delta a = a_A - a = \omega^2 (r_A - r) = (209.4 \text{ rad/s})^2 (1.50 \text{ m} - 0.150 \text{ m}) \approx 5.92 \times 10^4 \text{ m/s}^2$$

(b) The slope is given by $a/r = \omega^2 = 4.39 \times 10^4 \text{ /s}^2$.

96. Let T be the tension on the rope. From Newton's second law, we have

$$T - mg = ma \quad \Rightarrow \quad T = m(g + a).$$

Since the box has an upward acceleration a = 0.80 m/s^2, the tension is given by

$$T = (30\ \text{kg})(9.8\ \text{m/s}^2 + 0.8\ \text{m/s}^2) = 318\ \text{N}.$$

The rotation of the device is described by $F_{\text{app}}R - Tr = I\alpha = Ia/r$. The moment of inertia can then be obtained as

$$I = \frac{r(F_{\text{app}}R - Tr)}{a} = \frac{(0.20\ \text{m})[(140\ \text{N})(0.50\ \text{m}) - (318\ \text{N})(0.20\ \text{m})]}{0.80\ \text{m/s}^2} = 1.6\ \text{kg}\cdot\text{m}^2$$

97. The distances from P to the particles are as follows:

$$r_1 = a \text{ for } m_1 = 2M(\text{lower left})$$
$$r_2 = \sqrt{b^2 - a^2} \text{ for } m_2 = M(\text{top})$$
$$r_3 = a \text{ for } m_1 = 2M(\text{lower right})$$

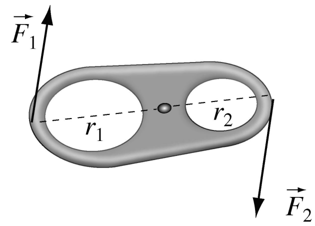
The rotational inertia of the system about P is

$$I = \sum_{i=1}^{3} m_i r_i^2 = \left(3a^2 + b^2\right)M$$

which yields $I = 0.208 \text{ kg} \cdot \text{m}^2$ for M = 0.40 kg, a = 0.30 m and b = 0.50 m. Applying Eq. 10-52, we find

$$W = \frac{1}{2} I\omega^2 = \frac{1}{2}\left(0.208 \text{ kg} \cdot \text{m}^2\right)\left(5.0 \text{ rad/s}\right)^2 = 2.6 \text{ J}.$$

98. In the figure below, we show a pull tab of a beverage can. Since the tab is pivoted, when pulling on one end upward with a force $\vec{F}_1$, a force $\vec{F}_2$ will be exerted on the other end. The torque produced by $\vec{F}_1$ must be balanced by the torque produced by $\vec{F}_2$ so that the tab does not rotate.



The two forces are related by

$$r_1 F_1 = r_2 F_2$$

where $r_1 \approx 1.8 \text{ cm}$ and $r_2 \approx 0.73 \text{ cm}$. Thus, if $F_1 = 10$ N,

$$F_2 = \left(\frac{r_1}{r_2}\right) F_1 \approx \left(\frac{1.8 \text{ cm}}{0.73 \text{ cm}}\right)(10 \text{ N}) \approx 25 \text{ N}.$$

99. (a) We apply Eq. 10-18, using the subscript J for the Jeep.

$$\omega = \frac{v_J}{r_J} = \frac{114 \text{ km/h}}{0.100 \text{ km}}$$

which yields 1140 rad/h or (dividing by 3600) 0.32 rad/s for the value of the angular speed ω.

(b) Since the cheetah has the same angular speed, we again apply Eq. 10-18, using the subscript c for the cheetah.

$$v_c = r_c\omega = (92\,\text{m})\,(1140 \text{ rad/h}) = 1.048 \times 10^5 \text{ m/h} \approx 1.0\times10^2 \text{ km/h}$$

for the cheetah's speed.

100. Using Eq. 10-7 and Eq. 10-18, the average angular acceleration is

$$\alpha_{\text{avg}} = \frac{\Delta\omega}{\Delta t} = \frac{\Delta v}{r\Delta t} = \frac{25-12}{(0.75/2)(6.2)} = 5.6\,\text{rad/s}^2 .$$

101. We make use of Table 10-2(e) and the parallel-axis theorem in Eq. 10-36.

(a) The moment of inertia is

$$I = \frac{1}{12}ML^2 + Mh^2 = \frac{1}{12}(3.0 \text{ kg})(4.0 \text{ m})^2 + (3.0 \text{ kg})(1.0 \text{ m})^2 = 7.0 \text{ kg}\cdot\text{m}^2.$$

(b) The rotational kinetic energy is

$$K_{\text{rot}} = \frac{1}{2}I\omega^2 \quad\Rightarrow\quad \omega = \sqrt{\frac{2K_{\text{rot}}}{I}} = \sqrt{\frac{2(20 \text{ J})}{7 \text{ kg}\cdot\text{m}^2}} = 2.4 \text{ rad/s}$$

The linear speed of the end B is given by $v_B = \omega r_{AB} = (2.4 \text{ rad/s})(3.00 \text{ m}) = 7.2 \text{ m/s}$, where r_{AB} is the distance between A and B.

(c) The maximum angle θ is attained when all the rotational kinetic energy is transformed into potential energy. Moving from the vertical position ($\theta = 0$) to the maximum angle θ, the center of mass is elevated by $\Delta y = d_{AC}(1-\cos\theta)$, where d_{AC} = 1.00 m is the distance between A and the center of mass of the rod. Thus, the change in potential energy is

$$\Delta U = mg\Delta y = mgd_{AC}(1-\cos\theta) \quad\Rightarrow\quad 20 \text{ J} = (3.0 \text{ kg})(9.8 \text{ m/s}^2)(1.0 \text{ m})(1-\cos\theta)$$

which yields $\cos\theta = 0.32$, or $\theta \approx 71°$.

102. (a) The linear speed at $t = 15.0$ s is

$$v = a_t t = \left(0.500\,\text{m/s}^2\right)\left(15.0\,\text{s}\right) = 7.50\,\text{m/s}\ .$$

The radial (centripetal) acceleration at that moment is

$$a_r = \frac{v^2}{r} = \frac{\left(7.50\,\text{m/s}\right)^2}{30.0\,\text{m}} = 1.875\,\text{m/s}^2\ .$$

Thus, the net acceleration has magnitude:

$$a = \sqrt{a_t^2 + a_r^2} = \sqrt{\left(0.500\,\text{m/s}^2\right)^2 + \left(1.875\,\text{m/s}^2\right)^2} = 1.94\,\text{m/s}^2\ .$$

(b) We note that $\vec{a}_t \,||\, \vec{v}$. Therefore, the angle between $\vec{v}$ and $\vec{a}$ is

$$\tan^{-1}\left(\frac{a_r}{a_t}\right) = \tan^{-1}\left(\frac{1.875}{0.5}\right) = 75.1°$$

so that the vector is pointing more toward the center of the track than in the direction of motion.

103. (a) Using Eq. 10-1, the angular displacement is

$$\theta = \frac{5.6\,\text{m}}{8.0\times 10^{-2}\ \text{m}} = 1.4\times 10^{2}\ \text{rad}\ .$$

(b) We use $\theta = \frac{1}{2}\alpha t^2$ (Eq. 10-13) to obtain *t*:

$$t = \sqrt{\frac{2\theta}{\alpha}} = \sqrt{\frac{2\left(1.4\times 10^{2}\ \text{rad}\right)}{1.5\,\text{rad/s}^2}} = 14\,\text{s}\ .$$

104. We apply Eq. 10-12 twice, assuming the sense of rotation is positive. We have $\omega > 0$ and $\alpha < 0$. Since the angular velocity at t = 1 min is ω_1 = (0.90)(250) = 225 rev/min, we have

$$\omega_1 = \omega_0 + \alpha t \Rightarrow a = \frac{225 - 250}{1} = -25 \text{ rev / min}^2.$$

Next, between t = 1 min and t = 2 min we have the interval Δt = 1 min. Consequently, the angular velocity at t = 2 min is

$$\omega_2 = \omega_1 + \alpha \Delta t = 225 + (-25)(1) = 200 \text{ rev / min} .$$

105. (a) Using Table 10-2(c), the rotational inertia is

$$I = \frac{1}{2}mR^2 = \frac{1}{2}(1210\ \text{kg})\left(\frac{1.21\ \text{m}}{2}\right)^2 = 221\ \text{kg}\cdot\text{m}^2.$$

(b) The rotational kinetic energy is, by Eq. 10-34,

$$K = \frac{1}{2}I\omega^2 = \frac{1}{2}(2.21\times10^2\ \text{kg}\cdot\text{m}^2)[(1.52\ \text{rev/s})(2\pi\,\text{rad/rev})]^2 = 1.10\times10^4\ \text{J}.$$

106. (a) We obtain

$$\omega = \frac{(33.33 \text{ rev/min})\ (2\pi \text{ rad/rev})}{60 \text{ s/min}} = 3.5 \text{ rad/s}.$$

(b) Using Eq. 10-18, we have $v = r\omega = (15)(3.49) = 52$ cm/s.

(c) Similarly, when r = 7.4 cm we find $v = r\omega$ = 26 cm/s. The goal of this exercise is to observe what is and is not the same at different locations on a body in rotational motion (ω is the same, v is not), as well as to emphasize the importance of radians when working with equations such as Eq. 10-18.

107. With v = 50(1000/3600) = 13.9 m/s, Eq. 10-18 leads to

$$\omega = \frac{v}{r} = \frac{13.9}{110} = 0.13 \text{ rad / s.}$$

108. (a) The angular speed ω associated with Earth's spin is $\omega = 2\pi/T$, where T = 86400s (one day). Thus,

$$\omega = \frac{2\pi}{86400\ \text{s}} = 7.3\times10^{-5}\ \text{rad/s}$$

and the angular acceleration α required to accelerate the Earth from rest to ω in one day is $\alpha = \omega/T$. The torque needed is then

$$\tau = I\alpha = \frac{I\omega}{T} = \frac{(9.7\times10^{37}\ \text{kg}\cdot\text{m}^2)(7.3\times10^{-5}\ \text{rad/s})}{86400\,\text{s}} = 8.2\times10^{28}\ \text{N}\cdot\text{m}$$

where we used

$$I = \frac{2}{5}M\,R^2 = \frac{2}{5}\left(5.98\times10^{24}\ \text{kg}\right)\left(6.37\times10^{6}\ \text{m}\right)^2 = 9.7\times10^{37}\ \text{kg}\cdot\text{m}^2$$

for Earth's rotational inertia.

(b) Using the values from part (a), the kinetic energy of the Earth associated with its rotation about its own axis is $K = \frac{1}{2}I\omega^2 = 2.6\times10^{29}\text{J}$. This is how much energy would need to be supplied to bring it (starting from rest) to the current angular speed.

(c) The associated power is

$$P = \frac{K}{T} = \frac{2.57\times10^{29}\text{J}}{86400\ \text{s}} = 3.0\times10^{24}\ \text{W}.$$

109. The translational kinetic energy of the molecule is

$$K_t = \frac{1}{2}mv^2 = \frac{1}{2}(5.30\times10^{-26}\ \text{kg})(500\ \text{m/s})^2 = 6.63\times10^{-21}\ \text{J}.$$

With $I = 1.94\times10^{-46}\ \text{kg}\cdot\text{m}^2$, we employ Eq. 10-34:

$$K_r = \frac{2}{3}K_t \quad\Rightarrow\quad \frac{1}{2}I\omega^2 = \frac{2}{3}(6.63\times10^{-21}\ \text{J})$$

which leads to $\omega = 6.75\times10^{12}$ rad/s.

110. (a) The rotational inertia relative to the specified axis is

$$I = \sum m_i r_i^2 = (2M)L^2 + (2M)L^2 + M(2L)^2$$

which is found to be I = 4.6 kg $\cdot$ m^2. Then, with ω = 1.2 rad/s, we obtain the kinetic energy from Eq. 10-34:

$$K = \frac{1}{2} I\omega^2 = 3.3 \text{ J}.$$

(b) In this case the axis of rotation would appear as a standard y axis with origin at P. Each of the 2M balls are a distance of $r = L \cos 30°$ from that axis. Thus, the rotational inertia in this case is

$$I = \sum m_i r_i^2 = (2M)r^2 + (2M)r^2 + M(2L)^2$$

which is found to be I = 4.0 kg $\cdot$ m^2. Again, from Eq. 10-34 we obtain the kinetic energy

$$K = \frac{1}{2} I\omega^2 = 2.9 \text{ J}.$$

111. (a) The linear speed of a point on belt 1 is

$$v_1 = r_A\omega_A = (15\text{ cm})(10\text{ rad/s}) = 1.5\times10^2\text{ cm/s}.$$

(b) The angular speed of pulley B is

$$r_B\omega_B = r_A\omega_A \quad\Rightarrow\quad \omega_B = \frac{r_A\omega_A}{r_B} = \left(\frac{15\text{ cm}}{10\text{ cm}}\right)(10\text{ rad/s}) = 15\text{ rad/s}.$$

(c) Since the two pulleys are rigidly attached to each other, the angular speed of pulley B' is the same as that of pulley B, i.e., $\omega_B' = 15\text{ rad/s}$.

(d) The linear speed of a point on belt 2 is

$$v_2 = r_{B'}\omega_B' = (5\text{ cm})(15\text{ rad/s}) = 75\text{ cm/s}.$$

(e) The angular speed of pulley C is

$$r_C\omega_C = r_{B'}\omega_B' \quad\Rightarrow\quad \omega_C = \frac{r_{B'}\omega_B'}{r_C} = \left(\frac{5\text{ cm}}{25\text{ cm}}\right)(15\text{ rad/s}) = 3.0\text{ rad/s}$$

112. (a) The particle at A has $r = 0$ with respect to the axis of rotation. The particle at B is $r = L = 0.50$ m from the axis; similarly for the particle directly above A in the figure. The particle diagonally opposite A is a distance $r = \sqrt{2}L = 0.71\text{ m}$ from the axis. Therefore,

$$I = \sum m_i r_i^2 = 2mL^2 + m\left(\sqrt{2}L\right)^2 = 0.20\text{ kg}\cdot\text{m}^2.$$

(b) One imagines rotating the figure (about point A) clockwise by 90° and noting that the center of mass has fallen a distance equal to L as a result. If we let our reference position for gravitational potential be the height of the center of mass at the instant AB swings through vertical orientation, then

$$K_0 + U_0 = K + U \quad \Rightarrow \quad 0 + (4m)gh_0 = K + 0.$$

Since $h_0 = L = 0.50$ m, we find $K = 3.9$ J. Then, using Eq. 10-34, we obtain

$$K = \frac{1}{2} I_A \omega^2 \Rightarrow \omega = 6.3\text{ rad/s}.$$

113. Using Eq. 10-12, we have

$$\omega = \omega_0 + \alpha t \Rightarrow \alpha = \frac{2.6\ \text{rad/s} - 8.0\ \text{rad/s}}{3.0\ \text{s}} = -1.8\ \text{rad/s}^2.$$

Using this value in Eq. 10-14 leads to

$$\omega^2 = \omega_0^2 + 2\alpha\theta \ \Rightarrow \ \theta = \frac{0 - (8.0\ \text{rad/s})^2}{2\left(-1.8\ \text{rad/s}^2\right)} = 18\ \text{rad}.$$

114. We make use of Table 10-2(e) as well as the parallel-axis theorem, Eq. 10-34, where needed. We use ℓ (as a subscript) to refer to the long rod and *s* to refer to the short rod.

(a) The rotational inertia is

$$I = I_s + I_\ell = \frac{1}{12} m_s L_s^2 + \frac{1}{3} m_\ell L_\ell^2 = 0.019 \text{ kg}\cdot\text{m}^2.$$

(b) We note that the center of the short rod is a distance of $h = 0.25$ m from the axis. The rotational inertia is

$$I = I_s + I_\ell = \frac{1}{12} m_s L_s^2 + m_s h^2 + \frac{1}{12} m_\ell L_\ell^2$$

which again yields $I = 0.019 \text{ kg}\cdot\text{m}^2$.

115. We employ energy methods in this solution; thus, considerations of positive versus negative sense (regarding the rotation of the wheel) are not relevant.

(a) The speed of the box is related to the angular speed of the wheel by $v = R\omega$, so that

$$K_{\text{box}} = \frac{1}{2} m_{\text{box}} v^2 \Rightarrow v = \sqrt{\frac{2K_{\text{box}}}{m_{\text{box}}}} = 1.41 \text{ m/s}$$

implies that the angular speed is $\omega = 1.41/0.20 = 0.71$ rad/s. Thus, the kinetic energy of rotation is $\frac{1}{2} I\omega^2 = 10.0$ J.

(b) Since it was released from rest at what we will consider to be the reference position for gravitational potential, then (with SI units understood) energy conservation requires

$$K_0 + U_0 = K + U \quad \Rightarrow \quad 0 + 0 = (6.0 + 10.0) + m_{\text{box}} g(-h).$$

Therefore, $h = 16.0/58.8 = 0.27$ m.

116. (a) One particle is on the axis, so $r = 0$ for it. For each of the others, the distance from the axis is

$$r = (0.60 \text{ m}) \sin 60° = 0.52 \text{ m}.$$

Therefore, the rotational inertia is $I = \sum m_i r_i^2 = 0.27 \, \text{kg} \cdot \text{m}^2$.

(b) The two particles that are nearest the axis are each a distance of $r = 0.30$ m from it. The particle "opposite" from that side is a distance $r = (0.60 \text{ m}) \sin 60° = 0.52$ m from the axis. Thus, the rotational inertia is

$$I = \sum m_i r_i^2 = 0.22 \, \text{kg} \cdot \text{m}^2.$$

(c) The distance from the axis for each of the particles is $r = \frac{1}{2}(0.60 \, \text{m}) \sin 60°$. The rotational inertia is

$$I = 3(0.50 \, \text{kg})(0.26 \, \text{m})^2 = 0.10 \, \text{kg} \cdot \text{m}^2.$$