

1. The initial position vector  $\vec{r}_0$  satisfies  $\vec{r} - \vec{r}_0 = \Delta\vec{r}$ , which results in

$$\vec{r}_0 = \vec{r} - \Delta\vec{r} = (3.0\hat{j} - 4.0\hat{k})\text{m} - (2.0\hat{i} - 3.0\hat{j} + 6.0\hat{k})\text{m} = (-2.0\text{ m})\hat{i} + (6.0\text{ m})\hat{j} + (-10\text{ m})\hat{k}.$$

2. (a) The position vector, according to Eq. 4-1, is  $\vec{r} = (-5.0 \text{ m}) \hat{i} + (8.0 \text{ m}) \hat{j}$ .

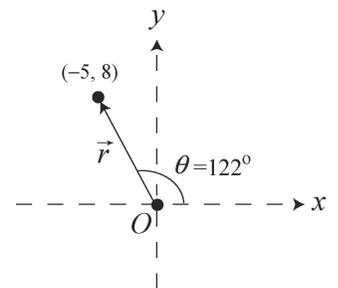
(b) The magnitude is  $|\vec{r}| = \sqrt{x^2 + y^2 + z^2} = \sqrt{(-5.0 \text{ m})^2 + (8.0 \text{ m})^2 + (0 \text{ m})^2} = 9.4 \text{ m}$ .

(c) Many calculators have polar  $\leftrightarrow$  rectangular conversion capabilities which make this computation more efficient than what is shown below. Noting that the vector lies in the  $xy$  plane and using Eq. 3-6, we obtain:

$$\theta = \tan^{-1}\left(\frac{8.0 \text{ m}}{-5.0 \text{ m}}\right) = -58^\circ \text{ or } 122^\circ$$

where the latter possibility ( $122^\circ$  measured counterclockwise from the  $+x$  direction) is chosen since the signs of the components imply the vector is in the second quadrant.

(d) The sketch is shown on the right. The vector is  $122^\circ$  counterclockwise from the  $+x$  direction.



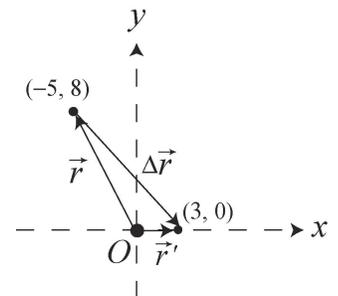
(e) The displacement is  $\Delta\vec{r} = \vec{r}' - \vec{r}$  where  $\vec{r}$  is given in part (a) and  $\vec{r}' = (3.0 \text{ m})\hat{i}$ . Therefore,  $\Delta\vec{r} = (8.0 \text{ m})\hat{i} - (8.0 \text{ m})\hat{j}$ .

(f) The magnitude of the displacement is  $|\Delta\vec{r}| = \sqrt{(8.0 \text{ m})^2 + (-8.0 \text{ m})^2} = 11 \text{ m}$ .

(g) The angle for the displacement, using Eq. 3-6, is

$$\tan^{-1}\left(\frac{8.0 \text{ m}}{-8.0 \text{ m}}\right) = -45^\circ \text{ or } 135^\circ$$

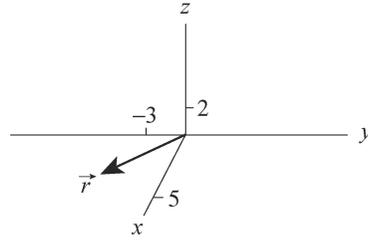
where we choose the former possibility ( $-45^\circ$ , or  $45^\circ$  measured *clockwise* from  $+x$ ) since the signs of the components imply the vector is in the fourth quadrant. A sketch of  $\Delta\vec{r}$  is shown on the right.



3. (a) The magnitude of  $\vec{r}$  is

$$|\vec{r}| = \sqrt{(5.0 \text{ m})^2 + (-3.0 \text{ m})^2 + (2.0 \text{ m})^2} = 6.2 \text{ m}.$$

(b) A sketch is shown. The coordinate values are in meters.



4. We choose a coordinate system with origin at the clock center and  $+x$  rightward (towards the “3:00” position) and  $+y$  upward (towards “12:00”).

(a) In unit-vector notation, we have  $\vec{r}_1 = (10 \text{ cm})\hat{i}$  and  $\vec{r}_2 = (-10 \text{ cm})\hat{j}$ . Thus, Eq. 4-2 gives

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1 = (-10 \text{ cm})\hat{i} + (-10 \text{ cm})\hat{j}.$$

and the magnitude is given by  $|\Delta\vec{r}| = \sqrt{(-10 \text{ cm})^2 + (-10 \text{ cm})^2} = 14 \text{ cm}$ .

(b) Using Eq. 3-6, the angle is

$$\theta = \tan^{-1}\left(\frac{-10 \text{ cm}}{-10 \text{ cm}}\right) = 45^\circ \text{ or } -135^\circ.$$

We choose  $-135^\circ$  since the desired angle is in the third quadrant. In terms of the magnitude-angle notation, one may write

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1 = (-10 \text{ cm})\hat{i} + (-10 \text{ cm})\hat{j} \rightarrow (14 \text{ cm} \angle -135^\circ).$$

(c) In this case, we have  $\vec{r}_1 = (-10 \text{ cm})\hat{j}$  and  $\vec{r}_2 = (10 \text{ cm})\hat{j}$ , and  $\Delta\vec{r} = (20 \text{ cm})\hat{j}$ . Thus,  $|\Delta\vec{r}| = 20 \text{ cm}$ .

(d) Using Eq. 3-6, the angle is given by

$$\theta = \tan^{-1}\left(\frac{20 \text{ cm}}{0 \text{ cm}}\right) = 90^\circ.$$

(e) In a full-hour sweep, the hand returns to its starting position, and the displacement is zero.

(f) The corresponding angle for a full-hour sweep is also zero.

5. Using Eq. 4-3 and Eq. 4-8, we have

$$\vec{v}_{\text{avg}} = \frac{(-2.0\hat{i} + 8.0\hat{j} - 2.0\hat{k}) \text{ m} - (5.0\hat{i} - 6.0\hat{j} + 2.0\hat{k}) \text{ m}}{10 \text{ s}} = (-0.70\hat{i} + 1.40\hat{j} - 0.40\hat{k}) \text{ m/s}.$$

6. To emphasize the fact that the velocity is a function of time, we adopt the notation  $v(t)$  for  $dx/dt$ .

(a) Eq. 4-10 leads to

$$v(t) = \frac{d}{dt} (3.00t\hat{i} - 4.00t^2\hat{j} + 2.00\hat{k}) = (3.00 \text{ m/s})\hat{i} - (8.00t \text{ m/s})\hat{j}$$

(b) Evaluating this result at  $t = 2.00$  s produces  $\vec{v} = (3.00\hat{i} - 16.0\hat{j})$  m/s.

(c) The speed at  $t = 2.00$  s is  $v = |\vec{v}| = \sqrt{(3.00 \text{ m/s})^2 + (-16.0 \text{ m/s})^2} = 16.3 \text{ m/s}$ .

(d) The angle of  $\vec{v}$  at that moment is

$$\tan^{-1} \left( \frac{-16.0 \text{ m/s}}{3.00 \text{ m/s}} \right) = -79.4^\circ \text{ or } 101^\circ$$

where we choose the first possibility ( $79.4^\circ$  measured *clockwise* from the  $+x$  direction, or  $281^\circ$  counterclockwise from  $+x$ ) since the signs of the components imply the vector is in the fourth quadrant.

7. The average velocity is given by Eq. 4-8. The total displacement  $\Delta\vec{r}$  is the sum of three displacements, each result of a (constant) velocity during a given time. We use a coordinate system with +x East and +y North.

(a) In unit-vector notation, the first displacement is given by

$$\Delta\vec{r}_1 = \left(60.0 \frac{\text{km}}{\text{h}}\right) \left(\frac{40.0 \text{ min}}{60 \text{ min/h}}\right) \hat{i} = (40.0 \text{ km})\hat{i}.$$

The second displacement has a magnitude of  $(60.0 \frac{\text{km}}{\text{h}}) \cdot (\frac{20.0 \text{ min}}{60 \text{ min/h}}) = 20.0 \text{ km}$ , and its direction is  $40^\circ$  north of east. Therefore,

$$\Delta\vec{r}_2 = (20.0 \text{ km}) \cos(40.0^\circ) \hat{i} + (20.0 \text{ km}) \sin(40.0^\circ) \hat{j} = (15.3 \text{ km}) \hat{i} + (12.9 \text{ km}) \hat{j}.$$

And the third displacement is

$$\Delta\vec{r}_3 = -\left(60.0 \frac{\text{km}}{\text{h}}\right) \left(\frac{50.0 \text{ min}}{60 \text{ min/h}}\right) \hat{i} = (-50.0 \text{ km})\hat{i}.$$

The total displacement is

$$\begin{aligned} \Delta\vec{r} &= \Delta\vec{r}_1 + \Delta\vec{r}_2 + \Delta\vec{r}_3 = (40.0 \text{ km})\hat{i} + (15.3 \text{ km})\hat{i} + (12.9 \text{ km})\hat{j} - (50.0 \text{ km})\hat{i} \\ &= (5.30 \text{ km}) \hat{i} + (12.9 \text{ km}) \hat{j}. \end{aligned}$$

The time for the trip is  $(40.0 + 20.0 + 50.0) \text{ min} = 110 \text{ min}$ , which is equivalent to 1.83 h. Eq. 4-8 then yields

$$\vec{v}_{\text{avg}} = \left(\frac{5.30 \text{ km}}{1.83 \text{ h}}\right) \hat{i} + \left(\frac{12.9 \text{ km}}{1.83 \text{ h}}\right) \hat{j} = (2.90 \text{ km/h})\hat{i} + (7.01 \text{ km/h})\hat{j}.$$

The magnitude is

$$|\vec{v}_{\text{avg}}| = \sqrt{(2.90 \text{ km/h})^2 + (7.01 \text{ km/h})^2} = 7.59 \text{ km/h}.$$

(b) The angle is given by

$$\theta = \tan^{-1}\left(\frac{7.01 \text{ km/h}}{2.90 \text{ km/h}}\right) = 67.5^\circ \text{ (north of east),}$$

or  $22.5^\circ$  east of due north.

8. Our coordinate system has  $\hat{i}$  pointed east and  $\hat{j}$  pointed north. The first displacement is  $\vec{r}_{AB} = (483 \text{ km})\hat{i}$  and the second is  $\vec{r}_{BC} = (-966 \text{ km})\hat{j}$ .

(a) The net displacement is

$$\vec{r}_{AC} = \vec{r}_{AB} + \vec{r}_{BC} = (483 \text{ km})\hat{i} - (966 \text{ km})\hat{j}$$

which yields  $|\vec{r}_{AC}| = \sqrt{(483 \text{ km})^2 + (-966 \text{ km})^2} = 1.08 \times 10^3 \text{ km}$ .

(b) The angle is given by

$$\theta = \tan^{-1} \left( \frac{-966 \text{ km}}{483 \text{ km}} \right) = -63.4^\circ.$$

We observe that the angle can be alternatively expressed as  $63.4^\circ$  south of east, or  $26.6^\circ$  east of south.

(c) Dividing the magnitude of  $\vec{r}_{AC}$  by the total time (2.25 h) gives

$$\vec{v}_{\text{avg}} = \frac{(483 \text{ km})\hat{i} - (966 \text{ km})\hat{j}}{2.25 \text{ h}} = (215 \text{ km/h})\hat{i} - (429 \text{ km/h})\hat{j}.$$

with a magnitude  $|\vec{v}_{\text{avg}}| = \sqrt{(215 \text{ km/h})^2 + (-429 \text{ km/h})^2} = 480 \text{ km/h}$ .

(d) The direction of  $\vec{v}_{\text{avg}}$  is  $26.6^\circ$  east of south, same as in part (b). In magnitude-angle notation, we would have  $\vec{v}_{\text{avg}} = (480 \text{ km/h} \angle -63.4^\circ)$ .

(e) Assuming the  $AB$  trip was a straight one, and similarly for the  $BC$  trip, then  $|\vec{r}_{AB}|$  is the distance traveled during the  $AB$  trip, and  $|\vec{r}_{BC}|$  is the distance traveled during the  $BC$  trip. Since the average speed is the total distance divided by the total time, it equals

$$\frac{483 \text{ km} + 966 \text{ km}}{2.25 \text{ h}} = 644 \text{ km/h}.$$

9. The  $(x,y)$  coordinates (in meters) of the points are  $A = (15, -15)$ ,  $B = (30, -45)$ ,  $C = (20, -15)$ , and  $D = (45, 45)$ . The respective times are  $t_A = 0$ ,  $t_B = 300$  s,  $t_C = 600$  s, and  $t_D = 900$  s. Average velocity is defined by Eq. 4-8. Each displacement  $\Delta\vec{r}$  is understood to originate at point  $A$ .

(a) The average velocity having the least magnitude ( $5.0$  m/600 s) is for the displacement ending at point  $C$ :  $|\vec{v}_{avg}| = 0.0083$  m/s.

(b) The direction of  $\vec{v}_{avg}$  is  $0^\circ$  (measured counterclockwise from the  $+x$  axis).

(c) The average velocity having the greatest magnitude ( $\sqrt{(15\text{ m})^2 + (30\text{ m})^2} / 300$  s) is for the displacement ending at point  $B$ :  $|\vec{v}_{avg}| = 0.11$  m/s.

(d) The direction of  $\vec{v}_{avg}$  is  $297^\circ$  (counterclockwise from  $+x$ ) or  $-63^\circ$  (which is equivalent to measuring  $63^\circ$  clockwise from the  $+x$  axis).

10. We differentiate  $\vec{r} = 5.00t \hat{i} + (et + ft^2) \hat{j}$ .

(a) The particle's motion is indicated by the derivative of  $\vec{r}$ :  $\vec{v} = 5.00 \hat{i} + (e + 2ft) \hat{j}$ .  
The angle of its direction of motion is consequently

$$\theta = \tan^{-1}(v_y/v_x) = \tan^{-1}[(e + 2ft)/5.00].$$

The graph indicates  $\theta_0 = 35.0^\circ$  which determines the parameter  $e$ :

$$e = (5.00 \text{ m/s}) \tan(35.0^\circ) = 3.50 \text{ m/s}.$$

(b) We note (from the graph) that  $\theta = 0$  when  $t = 14.0$  s. Thus,  $e + 2ft = 0$  at that time.  
This determines the parameter  $f$ :

$$f = \frac{-e}{2t} = \frac{-3.5 \text{ m/s}}{2(14.0 \text{ s})} = -0.125 \text{ m/s}^2.$$

11. We apply Eq. 4-10 and Eq. 4-16.

(a) Taking the derivative of the position vector with respect to time, we have, in SI units (m/s),

$$\vec{v} = \frac{d}{dt}(\hat{i} + 4t^2 \hat{j} + t \hat{k}) = 8t \hat{j} + \hat{k} .$$

(b) Taking another derivative with respect to time leads to, in SI units (m/s<sup>2</sup>),

$$\vec{a} = \frac{d}{dt} (8t \hat{j} + \hat{k}) = 8 \hat{j} .$$

12. We use Eq. 4-15 with  $\vec{v}_1$  designating the initial velocity and  $\vec{v}_2$  designating the later one.

(a) The average acceleration during the  $\Delta t = 4$  s interval is

$$\vec{a}_{\text{avg}} = \frac{(-2.0\hat{i} - 2.0\hat{j} + 5.0\hat{k}) \text{ m/s} - (4.0\hat{i} - 22\hat{j} + 3.0\hat{k}) \text{ m/s}}{4 \text{ s}} = (-1.5 \text{ m/s}^2)\hat{i} + (0.5 \text{ m/s}^2)\hat{k}.$$

(b) The magnitude of  $\vec{a}_{\text{avg}}$  is  $\sqrt{(-1.5 \text{ m/s}^2)^2 + (0.5 \text{ m/s}^2)^2} = 1.6 \text{ m/s}^2$ .

(c) Its angle in the  $xz$  plane (measured from the  $+x$  axis) is one of these possibilities:

$$\tan^{-1}\left(\frac{0.5 \text{ m/s}^2}{-1.5 \text{ m/s}^2}\right) = -18^\circ \text{ or } 162^\circ$$

where we settle on the second choice since the signs of its components imply that it is in the second quadrant.

13. In parts (b) and (c), we use Eq. 4-10 and Eq. 4-16. For part (d), we find the direction of the velocity computed in part (b), since that represents the asked-for tangent line.

(a) Plugging into the given expression, we obtain

$$\vec{r}\Big|_{t=2.00} = [2.00(8) - 5.00(2)]\hat{i} + [6.00 - 7.00(16)]\hat{j} = (6.00\hat{i} - 106\hat{j}) \text{ m}$$

(b) Taking the derivative of the given expression produces

$$\vec{v}(t) = (6.00t^2 - 5.00)\hat{i} - 28.0t^3\hat{j}$$

where we have written  $v(t)$  to emphasize its dependence on time. This becomes, at  $t = 2.00$  s,  $\vec{v} = (19.0\hat{i} - 224\hat{j})$  m/s.

(c) Differentiating the  $\vec{v}(t)$  found above, with respect to  $t$  produces  $12.0t\hat{i} - 84.0t^2\hat{j}$ , which yields  $\vec{a} = (24.0\hat{i} - 336\hat{j})$  m/s<sup>2</sup> at  $t = 2.00$  s.

(d) The angle of  $\vec{v}$ , measured from  $+x$ , is either

$$\tan^{-1}\left(\frac{-224 \text{ m/s}}{19.0 \text{ m/s}}\right) = -85.2^\circ \text{ or } 94.8^\circ$$

where we settle on the first choice ( $-85.2^\circ$ , which is equivalent to  $275^\circ$  measured counterclockwise from the  $+x$  axis) since the signs of its components imply that it is in the fourth quadrant.

14. We adopt a coordinate system with  $\hat{i}$  pointed east and  $\hat{j}$  pointed north; the coordinate origin is the flagpole. We “translate” the given information into unit-vector notation as follows:

$$\begin{aligned}\vec{r}_0 &= (40.0 \text{ m})\hat{i} & \text{and} & & \vec{v}_0 &= (-10.0 \text{ m/s})\hat{j} \\ \vec{r} &= (40.0 \text{ m})\hat{j} & \text{and} & & \vec{v} &= (10.0 \text{ m/s})\hat{i}.\end{aligned}$$

(a) Using Eq. 4-2, the displacement  $\Delta\vec{r}$  is

$$\Delta\vec{r} = \vec{r} - \vec{r}_0 = (-40.0 \text{ m})\hat{i} + (40.0 \text{ m})\hat{j}.$$

with a magnitude  $|\Delta\vec{r}| = \sqrt{(-40.0 \text{ m})^2 + (40.0 \text{ m})^2} = 56.6 \text{ m}$ .

(b) The direction of  $\Delta\vec{r}$  is

$$\theta = \tan^{-1}\left(\frac{\Delta y}{\Delta x}\right) = \tan^{-1}\left(\frac{40.0 \text{ m}}{-40.0 \text{ m}}\right) = -45.0^\circ \text{ or } 135^\circ.$$

Since the desired angle is in the second quadrant, we pick  $135^\circ$  ( $45^\circ$  north of due west). Note that the displacement can be written as  $\Delta\vec{r} = \vec{r} - \vec{r}_0 = (56.6 \angle 135^\circ)$  in terms of the magnitude-angle notation.

(c) The magnitude of  $\vec{v}_{\text{avg}}$  is simply the magnitude of the displacement divided by the time ( $\Delta t = 30.0 \text{ s}$ ). Thus, the average velocity has magnitude  $(56.6 \text{ m})/(30.0 \text{ s}) = 1.89 \text{ m/s}$ .

(d) Eq. 4-8 shows that  $\vec{v}_{\text{avg}}$  points in the same direction as  $\Delta\vec{r}$ , i.e,  $135^\circ$  ( $45^\circ$  north of due west).

(e) Using Eq. 4-15, we have

$$\vec{a}_{\text{avg}} = \frac{\vec{v} - \vec{v}_0}{\Delta t} = (0.333 \text{ m/s}^2)\hat{i} + (0.333 \text{ m/s}^2)\hat{j}.$$

The magnitude of the average acceleration vector is therefore equal to  $|\vec{a}_{\text{avg}}| = \sqrt{(0.333 \text{ m/s}^2)^2 + (0.333 \text{ m/s}^2)^2} = 0.471 \text{ m/s}^2$ .

(f) The direction of  $\vec{a}_{\text{avg}}$  is

$$\theta = \tan^{-1}\left(\frac{0.333 \text{ m/s}^2}{0.333 \text{ m/s}^2}\right) = 45^\circ \text{ or } -135^\circ.$$

Since the desired angle is now in the first quadrant, we choose  $45^\circ$ , and  $\vec{a}_{\text{avg}}$  points north of due east.

15. We find  $t$  by applying Eq. 2-11 to motion along the  $y$  axis (with  $v_y = 0$  characterizing  $y = y_{\max}$ ):

$$0 = (12 \text{ m/s}) + (-2.0 \text{ m/s}^2)t \Rightarrow t = 6.0 \text{ s}.$$

Then, Eq. 2-11 applies to motion along the  $x$  axis to determine the answer:

$$v_x = (8.0 \text{ m/s}) + (4.0 \text{ m/s}^2)(6.0 \text{ s}) = 32 \text{ m/s}.$$

Therefore, the velocity of the cart, when it reaches  $y = y_{\max}$ , is  $(32 \text{ m/s})\hat{i}$ .

16. We find  $t$  by solving  $\Delta x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$ :

$$12.0 \text{ m} = 0 + (4.00 \text{ m/s})t + \frac{1}{2}(5.00 \text{ m/s}^2)t^2$$

where we have used  $\Delta x = 12.0 \text{ m}$ ,  $v_x = 4.00 \text{ m/s}$ , and  $a_x = 5.00 \text{ m/s}^2$ . We use the quadratic formula and find  $t = 1.53 \text{ s}$ . Then, Eq. 2-11 (actually, its analog in two dimensions) applies with this value of  $t$ . Therefore, its velocity (when  $\Delta x = 12.00 \text{ m}$ ) is

$$\begin{aligned}\vec{v} &= \vec{v}_0 + \vec{a}t = (4.00 \text{ m/s})\hat{i} + (5.00 \text{ m/s}^2)(1.53 \text{ s})\hat{i} + (7.00 \text{ m/s}^2)(1.53 \text{ s})\hat{j} \\ &= (11.7 \text{ m/s})\hat{i} + (10.7 \text{ m/s})\hat{j}.\end{aligned}$$

Thus, the magnitude of  $\vec{v}$  is  $|\vec{v}| = \sqrt{(11.7 \text{ m/s})^2 + (10.7 \text{ m/s})^2} = 15.8 \text{ m/s}$ .

(b) The angle of  $\vec{v}$ , measured from  $+x$ , is

$$\tan^{-1}\left(\frac{10.7 \text{ m/s}}{11.7 \text{ m/s}}\right) = 42.6^\circ.$$

17. Constant acceleration in both directions ( $x$  and  $y$ ) allows us to use Table 2-1 for the motion along each direction. This can be handled individually (for  $\Delta x$  and  $\Delta y$ ) or together with the unit-vector notation (for  $\Delta r$ ). Where units are not shown, SI units are to be understood.

(a) The velocity of the particle at any time  $t$  is given by  $\vec{v} = \vec{v}_0 + \vec{a}t$ , where  $\vec{v}_0$  is the initial velocity and  $\vec{a}$  is the (constant) acceleration. The  $x$  component is  $v_x = v_{0x} + a_x t = 3.00 - 1.00t$ , and the  $y$  component is

$$v_y = v_{0y} + a_y t = -0.500t$$

since  $v_{0y} = 0$ . When the particle reaches its maximum  $x$  coordinate at  $t = t_m$ , we must have  $v_x = 0$ . Therefore,  $3.00 - 1.00t_m = 0$  or  $t_m = 3.00$  s. The  $y$  component of the velocity at this time is

$$v_y = 0 - 0.500(3.00) = -1.50 \text{ m/s};$$

this is the only nonzero component of  $\vec{v}$  at  $t_m$ .

(b) Since it started at the origin, the coordinates of the particle at any time  $t$  are given by  $\vec{r} = \vec{v}_0 t + \frac{1}{2} \vec{a} t^2$ . At  $t = t_m$  this becomes

$$\vec{r} = (3.00\hat{i})(3.00) + \frac{1}{2}(-1.00\hat{i} - 0.50\hat{j})(3.00)^2 = (4.50\hat{i} - 2.25\hat{j}) \text{ m}.$$

18. We make use of Eq. 4-16.

(a) The acceleration as a function of time is

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt} \left( (6.0t - 4.0t^2)\hat{i} + 8.0\hat{j} \right) = (6.0 - 8.0t)\hat{i}$$

in SI units. Specifically, we find the acceleration vector at  $t = 3.0$  s to be  $(6.0 - 8.0(3.0))\hat{i} = (-18 \text{ m/s}^2)\hat{i}$ .

(b) The equation is  $\vec{a} = (6.0 - 8.0t)\hat{i} = 0$ ; we find  $t = 0.75$  s.

(c) Since the  $y$  component of the velocity,  $v_y = 8.0$  m/s, is never zero, the velocity cannot vanish.

(d) Since speed is the magnitude of the velocity, we have

$$v = |\vec{v}| = \sqrt{(6.0t - 4.0t^2)^2 + (8.0)^2} = 10$$

in SI units (m/s). To solve for  $t$ , we first square both sides of the above equation, followed by some rearrangement:

$$(6.0t - 4.0t^2)^2 + 64 = 100 \Rightarrow (6.0t - 4.0t^2)^2 = 36$$

Taking the square root of the new expression and making further simplification lead to

$$6.0t - 4.0t^2 = \pm 6.0 \Rightarrow 4.0t^2 - 6.0t \pm 6.0 = 0$$

Finally, using the quadratic formula, we obtain

$$t = \frac{6.0 \pm \sqrt{36 - 4(4.0)(\pm 6.0)}}{2(4.0)}$$

where the requirement of a real positive result leads to the unique answer:  $t = 2.2$  s.

19. We make use of Eq. 4-16 and Eq. 4-10.

Using  $\vec{a} = 3t\hat{i} + 4t\hat{j}$ , we have (in m/s)

$$\vec{v}(t) = \vec{v}_0 + \int_0^t \vec{a} dt = (5.00\hat{i} + 2.00\hat{j}) + \int_0^t (3t\hat{i} + 4t\hat{j}) dt = (5.00 + 3t^2/2)\hat{i} + (2.00 + 2t^2)\hat{j}$$

Integrating using Eq. 4-10 then yields (in metres)

$$\begin{aligned}\vec{r}(t) &= \vec{r}_0 + \int_0^t \vec{v} dt = (20.0\hat{i} + 40.0\hat{j}) + \int_0^t [(5.00 + 3t^2/2)\hat{i} + (2.00 + 2t^2)\hat{j}] dt \\ &= (20.0\hat{i} + 40.0\hat{j}) + (5.00t + t^3/2)\hat{i} + (2.00t + 2t^3/3)\hat{j} \\ &= (20.0 + 5.00t + t^3/2)\hat{i} + (40.0 + 2.00t + 2t^3/3)\hat{j}\end{aligned}$$

(a) At  $t = 4.00$  s, we have  $\vec{r}(t = 4.00 \text{ s}) = (72.0 \text{ m})\hat{i} + (90.7 \text{ m})\hat{j}$ .

(b)  $\vec{v}(t = 4.00 \text{ s}) = (29.0 \text{ m/s})\hat{i} + (34.0 \text{ m/s})\hat{j}$ . Thus, the angle between the direction of travel and  $+x$ , measured counterclockwise, is  $\theta = \tan^{-1}[(34.0 \text{ m/s})/(29.0 \text{ m/s})] = 49.5^\circ$ .

20. The acceleration is constant so that use of Table 2-1 (for both the  $x$  and  $y$  motions) is permitted. Where units are not shown, SI units are to be understood. Collision between particles  $A$  and  $B$  requires two things. First, the  $y$  motion of  $B$  must satisfy (using Eq. 2-15 and noting that  $\theta$  is measured from the  $y$  axis)

$$y = \frac{1}{2} a_y t^2 \Rightarrow 30 \text{ m} = \frac{1}{2} [(0.40 \text{ m/s}^2) \cos \theta] t^2.$$

Second, the  $x$  motions of  $A$  and  $B$  must coincide:

$$vt = \frac{1}{2} a_x t^2 \Rightarrow (3.0 \text{ m/s})t = \frac{1}{2} [(0.40 \text{ m/s}^2) \sin \theta] t^2.$$

We eliminate a factor of  $t$  in the last relationship and formally solve for time:

$$t = \frac{2v}{a_x} = \frac{2(3.0 \text{ m/s})}{(0.40 \text{ m/s}^2) \sin \theta}.$$

This is then plugged into the previous equation to produce

$$30 \text{ m} = \frac{1}{2} [(0.40 \text{ m/s}^2) \cos \theta] \left( \frac{2(3.0 \text{ m/s})}{(0.40 \text{ m/s}^2) \sin \theta} \right)^2$$

which, with the use of  $\sin^2 \theta = 1 - \cos^2 \theta$ , simplifies to

$$30 = \frac{9.0}{0.20} \frac{\cos \theta}{1 - \cos^2 \theta} \Rightarrow 1 - \cos^2 \theta = \frac{9.0}{(0.20)(30)} \cos \theta.$$

We use the quadratic formula (choosing the positive root) to solve for  $\cos \theta$ :

$$\cos \theta = \frac{-1.5 + \sqrt{1.5^2 - 4(1.0)(-1.0)}}{2} = \frac{1}{2}$$

which yields  $\theta = \cos^{-1} \left( \frac{1}{2} \right) = 60^\circ$ .

21. (a) From Eq. 4-22 (with  $\theta_0 = 0$ ), the time of flight is

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2(45.0 \text{ m})}{9.80 \text{ m/s}^2}} = 3.03 \text{ s}.$$

(b) The horizontal distance traveled is given by Eq. 4-21:

$$\Delta x = v_0 t = (250 \text{ m/s})(3.03 \text{ s}) = 758 \text{ m}.$$

(c) And from Eq. 4-23, we find

$$|v_y| = gt = (9.80 \text{ m/s}^2)(3.03 \text{ s}) = 29.7 \text{ m/s}.$$

22. We use Eq. 4-26

$$R_{\max} = \left( \frac{v_0^2}{g} \sin 2\theta_0 \right)_{\max} = \frac{v_0^2}{g} = \frac{(9.50\text{m/s})^2}{9.80\text{m/s}^2} = 9.209 \text{ m} \approx 9.21\text{m}$$

to compare with Powell's long jump; the difference from  $R_{\max}$  is only  $\Delta R = (9.21\text{m} - 8.95\text{m}) = 0.259 \text{ m}$ .

23. Using Eq. (4-26), the take-off speed of the jumper is

$$v_0 = \sqrt{\frac{gR}{\sin 2\theta_0}} = \sqrt{\frac{(9.80 \text{ m/s}^2)(77.0 \text{ m})}{\sin 2(12.0^\circ)}} = 43.1 \text{ m/s}$$

24. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable.

(a) With the origin at the initial point (edge of table), the  $y$  coordinate of the ball is given by  $y = -\frac{1}{2}gt^2$ . If  $t$  is the time of flight and  $y = -1.20$  m indicates the level at which the ball hits the floor, then

$$t = \sqrt{\frac{2(-1.20 \text{ m})}{-9.80 \text{ m/s}^2}} = 0.495 \text{ s}.$$

(b) The initial (horizontal) velocity of the ball is  $\vec{v} = v_0 \hat{i}$ . Since  $x = 1.52$  m is the horizontal position of its impact point with the floor, we have  $x = v_0 t$ . Thus,

$$v_0 = \frac{x}{t} = \frac{1.52 \text{ m}}{0.495 \text{ s}} = 3.07 \text{ m/s}.$$

25. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The initial velocity is horizontal so that  $v_{0,y} = 0$  and  $v_{0,x} = v_0 = 10 \text{ m/s}$ .

(a) With the origin at the initial point (where the dart leaves the thrower's hand), the  $y$  coordinate of the dart is given by  $y = -\frac{1}{2}gt^2$ , so that with  $y = -PQ$  we have  $PQ = \frac{1}{2}(9.8 \text{ m/s}^2)(0.19 \text{ s})^2 = 0.18 \text{ m}$ .

(b) From  $x = v_0t$  we obtain  $x = (10 \text{ m/s})(0.19 \text{ s}) = 1.9 \text{ m}$ .

26. (a) Using the same coordinate system assumed in Eq. 4-22, we solve for  $y = h$ :

$$h = y_0 + v_0 \sin \theta_0 t - \frac{1}{2} g t^2$$

which yields  $h = 51.8$  m for  $y_0 = 0$ ,  $v_0 = 42.0$  m/s,  $\theta_0 = 60.0^\circ$  and  $t = 5.50$  s.

(b) The horizontal motion is steady, so  $v_x = v_{0x} = v_0 \cos \theta_0$ , but the vertical component of velocity varies according to Eq. 4-23. Thus, the speed at impact is

$$v = \sqrt{(v_0 \cos \theta_0)^2 + (v_0 \sin \theta_0 - g t)^2} = 27.4 \text{ m/s.}$$

(c) We use Eq. 4-24 with  $v_y = 0$  and  $y = H$ :

$$H = \frac{(v_0 \sin \theta_0)^2}{2g} = 67.5 \text{ m.}$$

27. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is at ground level directly below the release point. We write  $\theta_0 = -30.0^\circ$  since the angle shown in the figure is measured clockwise from horizontal. We note that the initial speed of the decoy is the plane's speed at the moment of release:  $v_0 = 290 \text{ km/h}$ , which we convert to SI units:  $(290)(1000/3600) = 80.6 \text{ m/s}$ .

(a) We use Eq. 4-12 to solve for the time:

$$\Delta x = (v_0 \cos \theta_0) t \Rightarrow t = \frac{700 \text{ m}}{(80.6 \text{ m/s}) \cos(-30.0^\circ)} = 10.0 \text{ s}.$$

(b) And we use Eq. 4-22 to solve for the initial height  $y_0$ :

$$y - y_0 = (v_0 \sin \theta_0) t - \frac{1}{2} g t^2 \Rightarrow 0 - y_0 = (-40.3 \text{ m/s})(10.0 \text{ s}) - \frac{1}{2} (9.80 \text{ m/s}^2)(10.0 \text{ s})^2$$

which yields  $y_0 = 897 \text{ m}$ .

28. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is throwing point (the stone's initial position). The  $x$  component of its initial velocity is given by  $v_{0,x} = v_0 \cos \theta_0$  and the  $y$  component is given by  $v_{0,y} = v_0 \sin \theta_0$ , where  $v_0 = 20 \text{ m/s}$  is the initial speed and  $\theta_0 = 40.0^\circ$  is the launch angle.

(a) At  $t = 1.10 \text{ s}$ , its  $x$  coordinate is

$$x = v_0 t \cos \theta_0 = (20.0 \text{ m/s})(1.10 \text{ s}) \cos 40.0^\circ = 16.9 \text{ m}$$

(b) Its  $y$  coordinate at that instant is

$$y = v_0 t \sin \theta_0 - \frac{1}{2} g t^2 = (20.0 \text{ m/s})(1.10 \text{ s}) \sin 40.0^\circ - \frac{1}{2} (9.80 \text{ m/s}^2)(1.10 \text{ s})^2 = 8.21 \text{ m}.$$

(c) At  $t' = 1.80 \text{ s}$ , its  $x$  coordinate is  $x = (20.0 \text{ m/s})(1.80 \text{ s}) \cos 40.0^\circ = 27.6 \text{ m}$ .

(d) Its  $y$  coordinate at  $t'$  is

$$y = (20.0 \text{ m/s})(1.80 \text{ s}) \sin 40.0^\circ - \frac{1}{2} (9.80 \text{ m/s}^2) (1.80 \text{ s})^2 = 7.26 \text{ m}.$$

(e) The stone hits the ground earlier than  $t = 5.0 \text{ s}$ . To find the time when it hits the ground solve  $y = v_0 t \sin \theta_0 - \frac{1}{2} g t^2 = 0$  for  $t$ . We find

$$t = \frac{2v_0}{g} \sin \theta_0 = \frac{2(20.0 \text{ m/s})}{9.8 \text{ m/s}^2} \sin 40^\circ = 2.62 \text{ s}.$$

Its  $x$  coordinate on landing is

$$x = v_0 t \cos \theta_0 = (20.0 \text{ m/s})(2.62 \text{ s}) \cos 40^\circ = 40.2 \text{ m}.$$

(f) Assuming it stays where it lands, its vertical component at  $t = 5.00 \text{ s}$  is  $y = 0$ .

29. The initial velocity has no vertical component — only an  $x$  component equal to  $+2.00$  m/s. Also,  $y_0 = +10.0$  m if the water surface is established as  $y = 0$ .

(a)  $x - x_0 = v_x t$  readily yields  $x - x_0 = 1.60$  m.

(b) Using  $y - y_0 = v_{0y} t - \frac{1}{2} g t^2$ , we obtain  $y = 6.86$  m when  $t = 0.800$  s and  $v_{0y} = 0$ .

(c) Using the fact that  $y = 0$  and  $y_0 = 10.0$ , the equation  $y - y_0 = v_{0y} t - \frac{1}{2} g t^2$  leads to

$$t = \sqrt{2(10.0 \text{ m})/9.80 \text{ m/s}^2} = 1.43 \text{ s}.$$

During this time, the  $x$ -displacement of the diver is  $x - x_0 = (2.00 \text{ m/s})(1.43 \text{ s}) = 2.86$  m.

30. (a) Since the  $y$ -component of the velocity of the stone at the top of its path is zero, its speed is

$$v = \sqrt{v_x^2 + v_y^2} = v_x = v_0 \cos \theta_0 = (28.0 \text{ m/s}) \cos 40.0^\circ = 21.4 \text{ m/s}.$$

(b) Using the fact that  $v_y = 0$  at the maximum height  $y_{\max}$ , the amount of time it takes for the stone to reach  $y_{\max}$  is given by Eq. 4-23:

$$0 = v_y = v_0 \sin \theta_0 - gt \Rightarrow t = \frac{v_0 \sin \theta_0}{g}.$$

Substituting the above expression into Eq. 4-22, we find the maximum height to be

$$y_{\max} = (v_0 \sin \theta_0) t - \frac{1}{2} g t^2 = v_0 \sin \theta_0 \left( \frac{v_0 \sin \theta_0}{g} \right) - \frac{1}{2} g \left( \frac{v_0 \sin \theta_0}{g} \right)^2 = \frac{v_0^2 \sin^2 \theta_0}{2g}.$$

To find the time the stone descends to  $y = y_{\max} / 2$ , we solve the quadratic equation given in Eq. 4-22:

$$y = \frac{1}{2} y_{\max} = \frac{v_0^2 \sin^2 \theta_0}{4g} = (v_0 \sin \theta_0) t - \frac{1}{2} g t^2 \Rightarrow t_{\pm} = \frac{(2 \pm \sqrt{2}) v_0 \sin \theta_0}{2g}.$$

Choosing  $t = t_+$  (for descending), we have

$$v_x = v_0 \cos \theta_0 = (28.0 \text{ m/s}) \cos 40.0^\circ = 21.4 \text{ m/s}$$

$$v_y = v_0 \sin \theta_0 - g \frac{(2 + \sqrt{2}) v_0 \sin \theta_0}{2g} = -\frac{\sqrt{2}}{2} v_0 \sin \theta_0 = -\frac{\sqrt{2}}{2} (28.0 \text{ m/s}) \sin 40.0^\circ = -12.7 \text{ m/s}$$

Thus, the speed of the stone when  $y = y_{\max} / 2$  is

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(21.4 \text{ m/s})^2 + (-12.7 \text{ m/s})^2} = 24.9 \text{ m/s}.$$

(c) The percentage difference is

$$\frac{24.9 \text{ m/s} - 21.4 \text{ m/s}}{21.4 \text{ m/s}} = 0.163 = 16.3\%.$$

31. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is at ground level directly below the release point. We write  $\theta_0 = -37.0^\circ$  for the angle measured from  $+x$ , since the angle given in the problem is measured from the  $-y$  direction. We note that the initial speed of the projectile is the plane's speed at the moment of release.

(a) We use Eq. 4-22 to find  $v_0$ :

$$y - y_0 = (v_0 \sin \theta_0) t - \frac{1}{2} g t^2 \Rightarrow 0 - 730 \text{ m} = v_0 \sin(-37.0^\circ)(5.00 \text{ s}) - \frac{1}{2}(9.80 \text{ m/s}^2)(5.00 \text{ s})^2$$

which yields  $v_0 = 202 \text{ m/s}$ .

(b) The horizontal distance traveled is  $x = v_0 t \cos \theta_0 = (202 \text{ m/s})(5.00 \text{ s}) \cos(-37.0^\circ) = 806 \text{ m}$ .

(c) The  $x$  component of the velocity (just before impact) is

$$v_x = v_0 \cos \theta_0 = (202 \text{ m/s}) \cos(-37.0^\circ) = 161 \text{ m/s}.$$

(d) The  $y$  component of the velocity (just before impact) is

$$v_y = v_0 \sin \theta_0 - g t = (202 \text{ m/s}) \sin(-37.0^\circ) - (9.80 \text{ m/s}^2)(5.00 \text{ s}) = -171 \text{ m/s}.$$

32. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is at ground level directly below the point where the ball was hit by the racquet.

(a) We want to know how high the ball is above the court when it is at  $x = 12.0$  m. First, Eq. 4-21 tells us the time it is over the fence:

$$t = \frac{x}{v_0 \cos \theta_0} = \frac{12.0 \text{ m}}{(23.6 \text{ m/s}) \cos 0^\circ} = 0.508 \text{ s.}$$

At this moment, the ball is at a height (above the court) of

$$y = y_0 + (v_0 \sin \theta_0) t - \frac{1}{2} g t^2 = 1.10 \text{ m}$$

which implies it does indeed clear the 0.90 m high fence.

(b) At  $t = 0.508$  s, the center of the ball is  $(1.10 \text{ m} - 0.90 \text{ m}) = 0.20$  m above the net.

(c) Repeating the computation in part (a) with  $\theta_0 = -5.0^\circ$  results in  $t = 0.510$  s and  $y = 0.040$  m, which clearly indicates that it cannot clear the net.

(d) In the situation discussed in part (c), the distance between the top of the net and the center of the ball at  $t = 0.510$  s is  $0.90 \text{ m} - 0.040 \text{ m} = 0.86$  m.

33. We first find the time it takes for the volleyball to hit the ground. Using Eq. 4-22, we have

$$y - y_0 = (v_0 \sin \theta_0) t - \frac{1}{2} g t^2 \Rightarrow 0 - 2.30 \text{ m} = (-20.0 \text{ m/s}) \sin(18.0^\circ) t - \frac{1}{2} (9.80 \text{ m/s}^2) t^2$$

which gives  $t = 0.30 \text{ s}$ . Thus, the range of the volleyball is

$$R = (v_0 \cos \theta_0) t = (20.0 \text{ m/s}) \cos 18.0^\circ (0.30 \text{ s}) = 5.71 \text{ m}$$

On the other hand, when the angle is changed to  $\theta'_0 = 8.00^\circ$ , using the same procedure as shown above, we find

$$y - y_0 = (v_0 \sin \theta'_0) t' - \frac{1}{2} g t'^2 \Rightarrow 0 - 2.30 \text{ m} = (-20.0 \text{ m/s}) \sin(8.00^\circ) t' - \frac{1}{2} (9.80 \text{ m/s}^2) t'^2$$

which yields  $t' = 0.46 \text{ s}$ , and the range is

$$R' = (v_0 \cos \theta'_0) t' = (20.0 \text{ m/s}) \cos 8.00^\circ (0.46 \text{ s}) = 9.06 \text{ m}$$

Thus, the ball travels an extra distance of

$$\Delta R = R' - R = 9.06 \text{ m} - 5.71 \text{ m} = 3.35 \text{ m}$$

34. Although we could use Eq. 4-26 to find where it lands, we choose instead to work with Eq. 4-21 and Eq. 4-22 (for the soccer ball) since these will give information about where *and when* and these are also considered more fundamental than Eq. 4-26. With  $\Delta y = 0$ , we have

$$\Delta y = (v_0 \sin \theta_0) t - \frac{1}{2} g t^2 \Rightarrow t = \frac{(19.5 \text{ m/s}) \sin 45.0^\circ}{(9.80 \text{ m/s}^2) / 2} = 2.81 \text{ s.}$$

Then Eq. 4-21 yields  $\Delta x = (v_0 \cos \theta_0) t = 38.7 \text{ m}$ . Thus, using Eq. 4-8, the player must have an average velocity of

$$\vec{v}_{\text{avg}} = \frac{\Delta \vec{r}}{\Delta t} = \frac{(38.7 \text{ m}) \hat{i} - (55 \text{ m}) \hat{i}}{2.81 \text{ s}} = (-5.8 \text{ m/s}) \hat{i}$$

which means his average speed (assuming he ran in only one direction) is 5.8 m/s.

35. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is at its initial position (where it is launched). At maximum height, we observe  $v_y = 0$  and denote  $v_x = v$  (which is also equal to  $v_{0x}$ ). In this notation, we have  $v_0 = 5v$ . Next, we observe  $v_0 \cos \theta_0 = v_{0x} = v$ , so that we arrive at an equation (where  $v \neq 0$  cancels) which can be solved for  $\theta_0$ :

$$(5v) \cos \theta_0 = v \Rightarrow \theta_0 = \cos^{-1} \left( \frac{1}{5} \right) = 78.5^\circ.$$

36. (a) Solving the quadratic equation Eq. 4-22:

$$y - y_0 = (v_0 \sin \theta_0) t - \frac{1}{2} g t^2 \Rightarrow 0 - 2.160 \text{ m} = (15.00 \text{ m/s}) \sin(45.00^\circ) t - \frac{1}{2} (9.800 \text{ m/s}^2) t^2$$

the total travel time of the shot in the air is found to be  $t = 2.352 \text{ s}$ . Therefore, the horizontal distance traveled is

$$R = (v_0 \cos \theta_0) t = (15.00 \text{ m/s}) \cos 45.00^\circ (2.352 \text{ s}) = 24.95 \text{ m}.$$

(b) Using the procedure outlined in (a) but for  $\theta_0 = 42.00^\circ$ , we have

$$y - y_0 = (v_0 \sin \theta_0) t - \frac{1}{2} g t^2 \Rightarrow 0 - 2.160 \text{ m} = (15.00 \text{ m/s}) \sin(42.00^\circ) t - \frac{1}{2} (9.800 \text{ m/s}^2) t^2$$

and the total travel time is  $t = 2.245 \text{ s}$ . This gives

$$R = (v_0 \cos \theta_0) t = (15.00 \text{ m/s}) \cos 42.00^\circ (2.245 \text{ s}) = 25.02 \text{ m}.$$

37. We designate the given velocity  $\vec{v} = (7.6 \text{ m/s})\hat{i} + (6.1 \text{ m/s})\hat{j}$  as  $\vec{v}_1$  – as opposed to the velocity when it reaches the max height  $\vec{v}_2$  or the velocity when it returns to the ground  $\vec{v}_3$  – and take  $\vec{v}_0$  as the launch velocity, as usual. The origin is at its launch point on the ground.

(a) Different approaches are available, but since it will be useful (for the rest of the problem) to first find the initial  $y$  velocity, that is how we will proceed. Using Eq. 2-16, we have

$$v_{1y}^2 = v_{0y}^2 - 2g\Delta y \Rightarrow (6.1 \text{ m/s})^2 = v_{0y}^2 - 2(9.8 \text{ m/s}^2)(9.1 \text{ m})$$

which yields  $v_{0y} = 14.7 \text{ m/s}$ . Knowing that  $v_{2y}$  must equal 0, we use Eq. 2-16 again but now with  $\Delta y = h$  for the maximum height:

$$v_{2y}^2 = v_{0y}^2 - 2gh \Rightarrow 0 = (14.7 \text{ m/s})^2 - 2(9.8 \text{ m/s}^2)h$$

which yields  $h = 11 \text{ m}$ .

(b) Recalling the derivation of Eq. 4-26, but using  $v_{0y}$  for  $v_0 \sin \theta_0$  and  $v_{0x}$  for  $v_0 \cos \theta_0$ , we have

$$0 = v_{0y}t - \frac{1}{2}gt^2, \quad R = v_{0x}t$$

which leads to  $R = 2v_{0x}v_{0y}/g$ . Noting that  $v_{0x} = v_{1x} = 7.6 \text{ m/s}$ , we plug in values and obtain

$$R = 2(7.6 \text{ m/s})(14.7 \text{ m/s})/(9.8 \text{ m/s}^2) = 23 \text{ m}.$$

(c) Since  $v_{3x} = v_{1x} = 7.6 \text{ m/s}$  and  $v_{3y} = -v_{0y} = -14.7 \text{ m/s}$ , we have

$$v_3 = \sqrt{v_{3x}^2 + v_{3y}^2} = \sqrt{(7.6 \text{ m/s})^2 + (-14.7 \text{ m/s})^2} = 17 \text{ m/s}.$$

(d) The angle (measured from horizontal) for  $\vec{v}_3$  is one of these possibilities:

$$\tan^{-1}\left(\frac{-14.7 \text{ m}}{7.6 \text{ m}}\right) = -63^\circ \text{ or } 117^\circ$$

where we settle on the first choice ( $-63^\circ$ , which is equivalent to  $297^\circ$ ) since the signs of its components imply that it is in the fourth quadrant.

38. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is at the release point (the initial position for the ball as it begins projectile motion in the sense of §4-5), and we let  $\theta_0$  be the angle of throw (shown in the figure). Since the horizontal component of the velocity of the ball is  $v_x = v_0 \cos 40.0^\circ$ , the time it takes for the ball to hit the wall is

$$t = \frac{\Delta x}{v_x} = \frac{22.0 \text{ m}}{(25.0 \text{ m/s}) \cos 40.0^\circ} = 1.15 \text{ s}.$$

(a) The vertical distance is

$$\Delta y = (v_0 \sin \theta_0)t - \frac{1}{2}gt^2 = (25.0 \text{ m/s}) \sin 40.0^\circ(1.15 \text{ s}) - \frac{1}{2}(9.80 \text{ m/s}^2)(1.15 \text{ s})^2 = 12.0 \text{ m}.$$

(b) The horizontal component of the velocity when it strikes the wall does not change from its initial value:  $v_x = v_0 \cos 40.0^\circ = 19.2 \text{ m/s}$ .

(c) The vertical component becomes (using Eq. 4-23)

$$v_y = v_0 \sin \theta_0 - gt = (25.0 \text{ m/s}) \sin 40.0^\circ - (9.80 \text{ m/s}^2)(1.15 \text{ s}) = 4.80 \text{ m/s}.$$

(d) Since  $v_y > 0$  when the ball hits the wall, it has not reached the highest point yet.

39. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is at the end of the rifle (the initial point for the bullet as it begins projectile motion in the sense of § 4-5), and we let  $\theta_0$  be the firing angle. If the target is a distance  $d$  away, then its coordinates are  $x = d$ ,  $y = 0$ . The projectile motion equations lead to  $d = v_0 \cos \theta_0$  and  $0 = v_0 t \sin \theta_0 - \frac{1}{2} g t^2$ . Eliminating  $t$  leads to  $2v_0^2 \sin \theta_0 \cos \theta_0 - gd = 0$ . Using  $\sin \theta_0 \cos \theta_0 = \frac{1}{2} \sin(2\theta_0)$ , we obtain

$$v_0^2 \sin(2\theta_0) = gd \Rightarrow \sin(2\theta_0) = \frac{gd}{v_0^2} = \frac{(9.80 \text{ m/s}^2)(45.7 \text{ m})}{(460 \text{ m/s})^2}$$

which yields  $\sin(2\theta_0) = 2.11 \times 10^{-3}$  and consequently  $\theta_0 = 0.0606^\circ$ . If the gun is aimed at a point a distance  $\ell$  above the target, then  $\tan \theta_0 = \ell/d$  so that

$$\ell = d \tan \theta_0 = (45.7 \text{ m}) \tan(0.0606^\circ) = 0.0484 \text{ m} = 4.84 \text{ cm}.$$

40. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The initial velocity is horizontal so that  $v_{0y} = 0$  and  $v_{0x} = v_0 = 161 \text{ km/h}$ . Converting to SI units, this is  $v_0 = 44.7 \text{ m/s}$ .

(a) With the origin at the initial point (where the ball leaves the pitcher's hand), the  $y$  coordinate of the ball is given by  $y = -\frac{1}{2}gt^2$ , and the  $x$  coordinate is given by  $x = v_0t$ . From the latter equation, we have a simple proportionality between horizontal distance and time, which means the time to travel half the total distance is half the total time. Specifically, if  $x = 18.3/2 \text{ m}$ , then  $t = (18.3/2 \text{ m})/(44.7 \text{ m/s}) = 0.205 \text{ s}$ .

(b) And the time to travel the next  $18.3/2 \text{ m}$  must also be  $0.205 \text{ s}$ . It can be useful to write the horizontal equation as  $\Delta x = v_0\Delta t$  in order that this result can be seen more clearly.

(c) From  $y = -\frac{1}{2}gt^2$ , we see that the ball has reached the height of  $|\frac{1}{2}(9.80 \text{ m/s}^2)(0.205 \text{ s})^2| = 0.205 \text{ m}$  at the moment the ball is halfway to the batter.

(d) The ball's height when it reaches the batter is  $-\frac{1}{2}(9.80 \text{ m/s}^2)(0.409 \text{ s})^2 = -0.820 \text{ m}$ , which, when subtracted from the previous result, implies it has fallen another  $0.615 \text{ m}$ . Since the value of  $y$  is not simply proportional to  $t$ , we do not expect equal time-intervals to correspond to equal height-changes; in a physical sense, this is due to the fact that the initial  $y$ -velocity for the first half of the motion is not the same as the "initial"  $y$ -velocity for the second half of the motion.

41. Following the hint, we have the time-reversed problem with the ball thrown from the ground, towards the right, at  $60^\circ$  measured counterclockwise from a rightward axis. We see in this time-reversed situation that it is convenient to use the familiar coordinate system with  $+x$  as *rightward* and with positive angles measured counterclockwise.

(a) The  $x$ -equation (with  $x_0 = 0$  and  $x = 25.0$  m) leads to

$$25.0 \text{ m} = (v_0 \cos 60.0^\circ)(1.50 \text{ s}),$$

so that  $v_0 = 33.3$  m/s. And with  $y_0 = 0$ , and  $y = h > 0$  at  $t = 1.50$  s, we have  $y - y_0 = v_{0y}t - \frac{1}{2}gt^2$  where  $v_{0y} = v_0 \sin 60.0^\circ$ . This leads to  $h = 32.3$  m.

(b) We have

$$\begin{aligned}v_x &= v_{0x} = (33.3 \text{ m/s})\cos 60.0^\circ = 16.7 \text{ m/s} \\v_y &= v_{0y} - gt = (33.3 \text{ m/s})\sin 60.0^\circ - (9.80 \text{ m/s}^2)(1.50 \text{ s}) = 14.2 \text{ m/s}.\end{aligned}$$

The magnitude of  $\vec{v}$  is given by

$$|\vec{v}| = \sqrt{v_x^2 + v_y^2} = \sqrt{(16.7 \text{ m/s})^2 + (14.2 \text{ m/s})^2} = 21.9 \text{ m/s}.$$

(c) The angle is

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{14.2 \text{ m/s}}{16.7 \text{ m/s}}\right) = 40.4^\circ.$$

(d) We interpret this result (“undoing” the time reversal) as an initial velocity (from the edge of the building) of magnitude 21.9 m/s with angle (down from leftward) of  $40.4^\circ$ .

42. In this projectile motion problem, we have  $v_0 = v_x = \text{constant}$ , and what is plotted is  $v = \sqrt{v_x^2 + v_y^2}$ . We infer from the plot that at  $t = 2.5$  s, the ball reaches its maximum height, where  $v_y = 0$ . Therefore, we infer from the graph that  $v_x = 19$  m/s.

(a) During  $t = 5$  s, the horizontal motion is  $x - x_0 = v_x t = 95$  m.

(b) Since  $\sqrt{(19 \text{ m/s})^2 + v_{0y}^2} = 31$  m/s (the first point on the graph), we find  $v_{0y} = 24.5$  m/s. Thus, with  $t = 2.5$  s, we can use  $y_{\text{max}} - y_0 = v_{0y} t - \frac{1}{2} g t^2$  or  $v_y^2 = 0 = v_{0y}^2 - 2g(y_{\text{max}} - y_0)$ , or  $y_{\text{max}} - y_0 = \frac{1}{2}(v_y + v_{0y}) t$  to solve. Here we will use the latter:

$$y_{\text{max}} - y_0 = \frac{1}{2}(v_y + v_{0y}) t \Rightarrow y_{\text{max}} = \frac{1}{2}(0 + 24.5 \text{ m/s})(2.5 \text{ s}) = 31 \text{ m}$$

where we have taken  $y_0 = 0$  as the ground level.

43. (a) Let  $m = \frac{d_2}{d_1} = 0.600$  be the slope of the ramp, so  $y = mx$  there. We choose our coordinate origin at the point of launch and use Eq. 4-25. Thus,

$$y = \tan(50.0^\circ)x - \frac{(9.80 \text{ m/s}^2)x^2}{2(10.0 \text{ m/s})^2(\cos 50.0^\circ)^2} = 0.600x$$

which yields  $x = 4.99 \text{ m}$ . This is less than  $d_1$  so the ball *does* land on the ramp.

(b) Using the value of  $x$  found in part (a), we obtain  $y = mx = 2.99 \text{ m}$ . Thus, the Pythagorean theorem yields a displacement magnitude of  $\sqrt{x^2 + y^2} = 5.82 \text{ m}$ .

(c) The angle is, of course, the angle of the ramp:  $\tan^{-1}(m) = 31.0^\circ$ .

44. (a) Using the fact that the person (as the projectile) reaches the maximum height over the middle wheel located at  $x = 23 \text{ m} + (23/2) \text{ m} = 34.5 \text{ m}$ , we can deduce the initial launch speed from Eq. 4-26:

$$x = \frac{R}{2} = \frac{v_0^2 \sin 2\theta_0}{2g} \Rightarrow v_0 = \sqrt{\frac{2gx}{\sin 2\theta_0}} = \sqrt{\frac{2(9.8 \text{ m/s}^2)(34.5 \text{ m})}{\sin(2 \cdot 53^\circ)}} = 26.5 \text{ m/s}.$$

Upon substituting the value to Eq. 4-25, we obtain

$$y = y_0 + x \tan \theta_0 - \frac{gx^2}{2v_0^2 \cos^2 \theta_0} = 3.0 \text{ m} + (23 \text{ m}) \tan 53^\circ - \frac{(9.8 \text{ m/s}^2)(23 \text{ m})^2}{2(26.5 \text{ m/s})^2 (\cos 53^\circ)^2} = 23.3 \text{ m}.$$

Since the height of the wheel is  $h_w = 18 \text{ m}$ , the clearance over the first wheel is  $\Delta y = y - h_w = 23.3 \text{ m} - 18 \text{ m} = 5.3 \text{ m}$ .

(b) The height of the person when he is directly above the second wheel can be found by solving Eq. 4-24. With the second wheel located at  $x = 23 \text{ m} + (23/2) \text{ m} = 34.5 \text{ m}$ , we have

$$y = y_0 + x \tan \theta_0 - \frac{gx^2}{2v_0^2 \cos^2 \theta_0} = 3.0 \text{ m} + (34.5 \text{ m}) \tan 53^\circ - \frac{(9.8 \text{ m/s}^2)(34.5 \text{ m})^2}{2(26.52 \text{ m/s})^2 (\cos 53^\circ)^2} = 25.9 \text{ m}.$$

Therefore, the clearance over the second wheel is  $\Delta y = y - h_w = 25.9 \text{ m} - 18 \text{ m} = 7.9 \text{ m}$ .

(c) The location of the center of the net is given by

$$0 = y - y_0 = x \tan \theta_0 - \frac{gx^2}{2v_0^2 \cos^2 \theta_0} \Rightarrow x = \frac{v_0^2 \sin 2\theta_0}{g} = \frac{(26.52 \text{ m/s})^2 \sin(2 \cdot 53^\circ)}{9.8 \text{ m/s}^2} = 69 \text{ m}.$$

45. Using the information given, the position of the insect is given by (with the Archer fish at the origin)

$$x = d \cos \phi = (0.900 \text{ m}) \cos 36.0^\circ = 0.728 \text{ m}$$

$$y = d \sin \phi = (0.900 \text{ m}) \sin 36.0^\circ = 0.529 \text{ m}$$

Since  $y$  corresponds to the maximum height of the parabolic trajectory (see Problem 4-30):  $y = y_{\max} = v_0^2 \sin^2 \theta_0 / 2g$ , the launch angle is found to be

$$\theta_0 = \sin^{-1} \left( \sqrt{\frac{2gy}{v_0^2}} \right) = \sin^{-1} \left( \sqrt{\frac{2(9.8 \text{ m/s}^2)(0.529 \text{ m})}{(3.56 \text{ m/s})^2}} \right) \sin^{-1} (0.9044) = 64.8^\circ$$

46. Following the hint, we have the time-reversed problem with the ball thrown from the roof, towards the left, at  $60^\circ$  measured clockwise from a leftward axis. We see in this time-reversed situation that it is convenient to take  $+x$  as *leftward* with positive angles measured clockwise. Lengths are in meters and time is in seconds.

(a) With  $y_0 = 20.0$  m, and  $y = 0$  at  $t = 4.00$  s, we have  $y - y_0 = v_{0,y}t - \frac{1}{2}gt^2$  where  $v_{0,y} = v_0 \sin 60^\circ$ . This leads to  $v_0 = 16.9$  m/s. This plugs into the  $x$ -equation  $x - x_0 = v_{0,x}t$  (with  $x_0 = 0$  and  $x = d$ ) to produce  $d = (16.9 \text{ m/s})\cos 60^\circ(4.00 \text{ s}) = 33.7$  m.

(b) We have

$$v_x = v_{0,x} = (16.9 \text{ m/s})\cos 60.0^\circ = 8.43 \text{ m/s}$$

$$v_y = v_{0,y} - gt = (16.9 \text{ m/s})\sin 60.0^\circ - (9.80 \text{ m/s}^2)(4.00 \text{ s}) = -24.6 \text{ m/s}.$$

The magnitude of  $\vec{v}$  is  $|\vec{v}| = \sqrt{v_x^2 + v_y^2} = \sqrt{(8.43 \text{ m/s})^2 + (-24.6 \text{ m/s})^2} = 26.0 \text{ m/s}$ .

(c) The angle relative to horizontal is

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{-24.6 \text{ m/s}}{8.43 \text{ m/s}}\right) = -71.1^\circ.$$

We may convert the result from rectangular components to magnitude-angle representation:

$$\vec{v} = (8.43, -24.6) \rightarrow (26.0 \angle -71.1^\circ)$$

and we now interpret our result (“undoing” the time reversal) as an initial velocity of magnitude 26.0 m/s with angle (up from rightward) of  $71.1^\circ$ .

47. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is at ground level directly below impact point between bat and ball. The *Hint* given in the problem is important, since it provides us with enough information to find  $v_0$  directly from Eq. 4-26.

(a) We want to know how high the ball is from the ground when it is at  $x = 97.5$  m, which requires knowing the initial velocity. Using the range information and  $\theta_0 = 45^\circ$ , we use Eq. 4-26 to solve for  $v_0$ :

$$v_0 = \sqrt{\frac{gR}{\sin 2\theta_0}} = \sqrt{\frac{(9.8 \text{ m/s}^2)(107 \text{ m})}{1}} = 32.4 \text{ m/s}.$$

Thus, Eq. 4-21 tells us the time it is over the fence:

$$t = \frac{x}{v_0 \cos \theta_0} = \frac{97.5 \text{ m}}{(32.4 \text{ m/s}) \cos 45^\circ} = 4.26 \text{ s}.$$

At this moment, the ball is at a height (above the ground) of

$$y = y_0 + (v_0 \sin \theta_0)t - \frac{1}{2}gt^2 = 9.88 \text{ m}$$

which implies it does indeed clear the 7.32 m high fence.

(b) At  $t = 4.26$  s, the center of the ball is  $9.88 \text{ m} - 7.32 \text{ m} = 2.56 \text{ m}$  above the fence.

48. Using the fact that  $v_y = 0$  when the player is at the maximum height  $y_{\max}$ , the amount of time it takes to reach  $y_{\max}$  can be solved by using Eq. 4-23:

$$0 = v_y = v_0 \sin \theta_0 - gt \Rightarrow t_{\max} = \frac{v_0 \sin \theta_0}{g}.$$

Substituting the above expression into Eq. 4-22, we find the maximum height to be

$$y_{\max} = (v_0 \sin \theta_0) t_{\max} - \frac{1}{2} g t_{\max}^2 = v_0 \sin \theta_0 \left( \frac{v_0 \sin \theta_0}{g} \right) - \frac{1}{2} g \left( \frac{v_0 \sin \theta_0}{g} \right)^2 = \frac{v_0^2 \sin^2 \theta_0}{2g}.$$

To find the time when the player is at  $y = y_{\max} / 2$ , we solve the quadratic equation given in Eq. 4-22:

$$y = \frac{1}{2} y_{\max} = \frac{v_0^2 \sin^2 \theta_0}{4g} = (v_0 \sin \theta_0) t - \frac{1}{2} g t^2 \Rightarrow t_{\pm} = \frac{(2 \pm \sqrt{2}) v_0 \sin \theta_0}{2g}.$$

With  $t = t_-$  (for ascending), the amount of time the player spends at a height  $y \geq y_{\max} / 2$  is

$$\Delta t = t_{\max} - t_- = \frac{v_0 \sin \theta_0}{g} - \frac{(2 - \sqrt{2}) v_0 \sin \theta_0}{2g} = \frac{v_0 \sin \theta_0}{\sqrt{2}g} = \frac{t_{\max}}{\sqrt{2}} \Rightarrow \frac{\Delta t}{t_{\max}} = \frac{1}{\sqrt{2}} = 0.707.$$

Therefore, the player spends about 70.7% of the time in the upper half of the jump. Note that the ratio  $\Delta t / t_{\max}$  is independent of  $v_0$  and  $\theta_0$ , even though  $\Delta t$  and  $t_{\max}$  depend on these quantities.

49. (a) The skier jumps up at an angle of  $\theta_0 = 9.0^\circ$  up from the horizontal and thus returns to the launch level with his velocity vector  $9.0^\circ$  below the horizontal. With the snow surface making an angle of  $\alpha = 11.3^\circ$  (downward) with the horizontal, the angle between the slope and the velocity vector is  $\phi = \alpha - \theta_0 = 11.3^\circ - 9.0^\circ = 2.3^\circ$ .

(b) Suppose the skier lands at a distance  $d$  down the slope. Using Eq. 4-25 with  $x = d \cos \alpha$  and  $y = -d \sin \alpha$  (the edge of the track being the origin), we have

$$-d \sin \alpha = d \cos \alpha \tan \theta_0 - \frac{g(d \cos \alpha)^2}{2v_0^2 \cos^2 \theta_0}.$$

Solving for  $d$ , we obtain

$$\begin{aligned} d &= \frac{2v_0^2 \cos^2 \theta_0}{g \cos^2 \alpha} (\cos \alpha \tan \theta_0 + \sin \alpha) = \frac{2v_0^2 \cos \theta_0}{g \cos^2 \alpha} (\cos \alpha \sin \theta_0 + \cos \theta_0 \sin \alpha) \\ &= \frac{2v_0^2 \cos \theta_0}{g \cos^2 \alpha} \sin(\theta_0 + \alpha). \end{aligned}$$

Substituting the values given, we find

$$d = \frac{2(10 \text{ m/s})^2 \cos(9.0^\circ)}{(9.8 \text{ m/s}^2) \cos^2(11.3^\circ)} \sin(9.0^\circ + 11.3^\circ) = 7.27 \text{ m}.$$

which gives

$$y = -d \sin \alpha = -(7.27 \text{ m}) \sin(11.3^\circ) = -1.42 \text{ m}.$$

Therefore, at landing the skier is approximately 1.4 m below the launch level.

(c) The time it takes for the skier to land is

$$t = \frac{x}{v_x} = \frac{d \cos \alpha}{v_0 \cos \theta_0} = \frac{(7.27 \text{ m}) \cos(11.3^\circ)}{(10 \text{ m/s}) \cos(9.0^\circ)} = 0.72 \text{ s}.$$

Using Eq. 4-23, the  $x$ - and  $y$ -components of the velocity at landing are

$$\begin{aligned} v_x &= v_0 \cos \theta_0 = (10 \text{ m/s}) \cos(9.0^\circ) = 9.9 \text{ m/s} \\ v_y &= v_0 \sin \theta_0 - gt = (10 \text{ m/s}) \sin(9.0^\circ) - (9.8 \text{ m/s}^2)(0.72 \text{ s}) = -5.5 \text{ m/s} \end{aligned}$$

Thus, the direction of travel at landing is

$$\theta = \tan^{-1} \left( \frac{v_y}{v_x} \right) = \tan^{-1} \left( \frac{-5.5 \text{ m/s}}{9.9 \text{ m/s}} \right) = -29.1^\circ.$$

or  $29.1^\circ$  below the horizontal. The result implies that the angle between the skier's path and the slope is  $\phi = 29.1^\circ - 11.3^\circ = 17.8^\circ$ , or approximately  $18^\circ$  to two significant figures.

50. From Eq. 4-21, we find  $t = x/v_{0x}$ . Then Eq. 4-23 leads to

$$v_y = v_{0y} - gt = v_{0y} - \frac{gx}{v_{0x}}.$$

Since the slope of the graph is  $-0.500$ , we conclude  $\frac{g}{v_{0x}} = \frac{1}{2} \Rightarrow v_{0x} = 19.6$  m/s. And from the “y intercept” of the graph, we find  $v_{0y} = 5.00$  m/s. Consequently,  $\theta_0 = \tan^{-1}(v_{0y}/v_{0x}) = 14.3^\circ$ .

51. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is at the point where the ball is kicked. We use  $x$  and  $y$  to denote the coordinates of ball at the goalpost, and try to find the kicking angle(s)  $\theta_0$  so that  $y = 3.44$  m when  $x = 50$  m. Writing the kinematic equations for projectile motion:

$$x = v_0 \cos \theta_0, \quad y = v_0 t \sin \theta_0 - \frac{1}{2} g t^2,$$

we see the first equation gives  $t = x/v_0 \cos \theta_0$ , and when this is substituted into the second the result is

$$y = x \tan \theta_0 - \frac{g x^2}{2 v_0^2 \cos^2 \theta_0}.$$

One may solve this by trial and error: systematically trying values of  $\theta_0$  until you find the two that satisfy the equation. A little manipulation, however, will give an algebraic solution: Using the trigonometric identity  $1 / \cos^2 \theta_0 = 1 + \tan^2 \theta_0$ , we obtain

$$\frac{1}{2} \frac{g x^2}{v_0^2} \tan^2 \theta_0 - x \tan \theta_0 + y + \frac{1}{2} \frac{g x^2}{v_0^2} = 0$$

which is a second-order equation for  $\tan \theta_0$ . To simplify writing the solution, we denote

$$c = \frac{1}{2} g x^2 / v_0^2 = \frac{1}{2} (9.80 \text{ m/s}^2) (50 \text{ m})^2 / (25 \text{ m/s})^2 = 19.6 \text{ m}.$$

Then the second-order equation becomes  $c \tan^2 \theta_0 - x \tan \theta_0 + y + c = 0$ . Using the quadratic formula, we obtain its solution(s).

$$\tan \theta_0 = \frac{x \pm \sqrt{x^2 - 4(y+c)c}}{2c} = \frac{50 \text{ m} \pm \sqrt{(50 \text{ m})^2 - 4(3.44 \text{ m} + 19.6 \text{ m})(19.6 \text{ m})}}{2(19.6 \text{ m})}.$$

The two solutions are given by  $\tan \theta_0 = 1.95$  and  $\tan \theta_0 = 0.605$ . The corresponding (first-quadrant) angles are  $\theta_0 = 63^\circ$  and  $\theta_0 = 31^\circ$ . Thus,

(a) The smallest elevation angle is  $\theta_0 = 31^\circ$ , and

(b) The greatest elevation angle is  $\theta_0 = 63^\circ$ .

If kicked at any angle between these two, the ball will travel above the cross bar on the goalposts.

52. For  $\Delta y = 0$ , Eq. 4-22 leads to  $t = 2v_0 \sin \theta_0 / g$ , which immediately implies  $t_{\max} = 2v_0 / g$  (which occurs for the “straight up” case:  $\theta_0 = 90^\circ$ ). Thus,

$$\frac{1}{2} t_{\max} = v_0 / g \Rightarrow \frac{1}{2} = \sin \theta_0.$$

Therefore, the half-maximum-time flight is at angle  $\theta_0 = 30.0^\circ$ . Since the least speed occurs at the top of the trajectory, which is where the velocity is simply the  $x$ -component of the initial velocity ( $v_0 \cos \theta_0 = v_0 \cos 30^\circ$  for the half-maximum-time flight), then we need to refer to the graph in order to find  $v_0$  – in order that we may complete the solution. In the graph, we note that the range is 240 m when  $\theta_0 = 45.0^\circ$ . Eq. 4-26 then leads to  $v_0 = 48.5$  m/s. The answer is thus  $(48.5 \text{ m/s}) \cos 30.0^\circ = 42.0$  m/s.

53. We denote  $h$  as the height of a step and  $w$  as the width. To hit step  $n$ , the ball must fall a distance  $nh$  and travel horizontally a distance between  $(n - 1)w$  and  $nw$ . We take the origin of a coordinate system to be at the point where the ball leaves the top of the stairway, and we choose the  $y$  axis to be positive in the upward direction. The coordinates of the ball at time  $t$  are given by  $x = v_{0x}t$  and  $y = -\frac{1}{2}gt^2$  (since  $v_{0y} = 0$ ). We equate  $y$  to  $-nh$  and solve for the time to reach the level of step  $n$ :

$$t = \sqrt{\frac{2nh}{g}}.$$

The  $x$  coordinate then is

$$x = v_{0x}\sqrt{\frac{2nh}{g}} = (1.52 \text{ m/s})\sqrt{\frac{2n(0.203 \text{ m})}{9.8 \text{ m/s}^2}} = (0.309 \text{ m})\sqrt{n}.$$

The method is to try values of  $n$  until we find one for which  $x/w$  is less than  $n$  but greater than  $n - 1$ . For  $n = 1$ ,  $x = 0.309 \text{ m}$  and  $x/w = 1.52$ , which is greater than  $n$ . For  $n = 2$ ,  $x = 0.437 \text{ m}$  and  $x/w = 2.15$ , which is also greater than  $n$ . For  $n = 3$ ,  $x = 0.535 \text{ m}$  and  $x/w = 2.64$ . Now, this is less than  $n$  and greater than  $n - 1$ , so the ball hits the third step.

54. We apply Eq. 4-21, Eq. 4-22 and Eq. 4-23.

(a) From  $\Delta x = v_{0x}t$ , we find  $v_{0x} = 40 \text{ m} / 2 \text{ s} = 20 \text{ m/s}$ .

(b) From  $\Delta y = v_{0y}t - \frac{1}{2}gt^2$ , we find  $v_{0y} = \left(53 \text{ m} + \frac{1}{2}(9.8 \text{ m/s}^2)(2 \text{ s})^2\right) / 2 = 36 \text{ m/s}$ .

(c) From  $v_y = v_{0y} - gt'$  with  $v_y = 0$  as the condition for maximum height, we obtain  $t' = (36 \text{ m/s}) / (9.8 \text{ m/s}^2) = 3.7 \text{ s}$ . During that time the  $x$ -motion is constant, so  $x' - x_0 = (20 \text{ m/s})(3.7 \text{ s}) = 74 \text{ m}$ .

55. Let  $y_0 = h_0 = 1.00$  m at  $x_0 = 0$  when the ball is hit. Let  $y_1 = h$  (the height of the wall) and  $x_1$  describe the point where it first rises above the wall one second after being hit; similarly,  $y_2 = h$  and  $x_2$  describe the point where it passes back down behind the wall four seconds later. And  $y_f = 1.00$  m at  $x_f = R$  is where it is caught. Lengths are in meters and time is in seconds.

(a) Keeping in mind that  $v_x$  is constant, we have  $x_2 - x_1 = 50.0$  m  $= v_{1x}$  (4.00 s), which leads to  $v_{1x} = 12.5$  m/s. Thus, applied to the full six seconds of motion:

$$x_f - x_0 = R = v_x(6.00 \text{ s}) = 75.0 \text{ m}.$$

(b) We apply  $y - y_0 = v_{0y}t - \frac{1}{2}gt^2$  to the motion above the wall,

$$y_2 - y_1 = 0 = v_{1y}(4.00 \text{ s}) - \frac{1}{2}g(4.00 \text{ s})^2$$

and obtain  $v_{1y} = 19.6$  m/s. One second earlier, using  $v_{1y} = v_{0y} - g(1.00 \text{ s})$ , we find  $v_{0y} = 29.4$  m/s. Therefore, the velocity of the ball just after being hit is

$$\vec{v} = v_{0x}\hat{i} + v_{0y}\hat{j} = (12.5 \text{ m/s})\hat{i} + (29.4 \text{ m/s})\hat{j}$$

Its magnitude is  $|\vec{v}| = \sqrt{(12.5 \text{ m/s})^2 + (29.4 \text{ m/s})^2} = 31.9 \text{ m/s}$ .

(c) The angle is

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{29.4 \text{ m/s}}{12.5 \text{ m/s}}\right) = 67.0^\circ.$$

We interpret this result as a velocity of magnitude 31.9 m/s, with angle (up from rightward) of  $67.0^\circ$ .

(d) During the first 1.00 s of motion,  $y = y_0 + v_{0y}t - \frac{1}{2}gt^2$  yields

$$h = 1.0 \text{ m} + (29.4 \text{ m/s})(1.00 \text{ s}) - \frac{1}{2}(9.8 \text{ m/s}^2)(1.00 \text{ s})^2 = 25.5 \text{ m}.$$

56. (a) During constant-speed circular motion, the velocity vector is perpendicular to the acceleration vector at every instant. Thus,  $\vec{v} \cdot \vec{a} = 0$ .

(b) The acceleration in this vector, at every instant, points towards the center of the circle, whereas the position vector points from the center of the circle to the object in motion. Thus, the angle between  $\vec{r}$  and  $\vec{a}$  is  $180^\circ$  at every instant, so  $\vec{r} \times \vec{a} = 0$ .

57. (a) Since the wheel completes 5 turns each minute, its period is one-fifth of a minute, or 12 s.

(b) The magnitude of the centripetal acceleration is given by  $a = v^2/R$ , where  $R$  is the radius of the wheel, and  $v$  is the speed of the passenger. Since the passenger goes a distance  $2\pi R$  for each revolution, his speed is

$$v = \frac{2\pi(15 \text{ m})}{12 \text{ s}} = 7.85 \text{ m/s}$$

and his centripetal acceleration is  $a = \frac{(7.85 \text{ m/s})^2}{15 \text{ m}} = 4.1 \text{ m/s}^2$ .

(c) When the passenger is at the highest point, his centripetal acceleration is downward, toward the center of the orbit.

(d) At the lowest point, the centripetal acceleration is  $a = 4.1 \text{ m/s}^2$ , same as part (b).

(e) The direction is up, toward the center of the orbit.

58. The magnitude of the acceleration is

$$a = \frac{v^2}{r} = \frac{(10 \text{ m/s})^2}{25 \text{ m}} = 4.0 \text{ m/s}^2.$$

59. We apply Eq. 4-35 to solve for speed  $v$  and Eq. 4-34 to find centripetal acceleration  $a$ .

(a)  $v = 2\pi r/T = 2\pi(20 \text{ km})/1.0 \text{ s} = 126 \text{ km/s} = 1.3 \times 10^5 \text{ m/s}$ .

(b) The magnitude of the acceleration is

$$a = \frac{v^2}{r} = \frac{(126 \text{ km/s})^2}{20 \text{ km}} = 7.9 \times 10^5 \text{ m/s}^2.$$

(c) Clearly, both  $v$  and  $a$  will increase if  $T$  is reduced.

60. We apply Eq. 4-35 to solve for speed  $v$  and Eq. 4-34 to find acceleration  $a$ .

(a) Since the radius of Earth is  $6.37 \times 10^6$  m, the radius of the satellite orbit is

$$r = (6.37 \times 10^6 + 640 \times 10^3) \text{ m} = 7.01 \times 10^6 \text{ m}.$$

Therefore, the speed of the satellite is

$$v = \frac{2\pi r}{T} = \frac{2\pi(7.01 \times 10^6 \text{ m})}{(98.0 \text{ min})(60 \text{ s/min})} = 7.49 \times 10^3 \text{ m/s}.$$

(b) The magnitude of the acceleration is

$$a = \frac{v^2}{r} = \frac{(7.49 \times 10^3 \text{ m/s})^2}{7.01 \times 10^6 \text{ m}} = 8.00 \text{ m/s}^2.$$

61. The magnitude of centripetal acceleration ( $a = v^2/r$ ) and its direction (towards the center of the circle) form the basis of this problem.

(a) If a passenger at this location experiences  $\vec{a} = 1.83 \text{ m/s}^2$  east, then the center of the circle is *east* of this location. The distance is  $r = v^2/a = (3.66 \text{ m/s})^2/(1.83 \text{ m/s}^2) = 7.32 \text{ m}$ .

(b) Thus, relative to the center, the passenger at that moment is located 7.32 m toward the west.

(c) If the direction of  $\vec{a}$  experienced by the passenger is now *south*—indicating that the center of the merry-go-round is south of him, then relative to the center, the passenger at that moment is located 7.32 m toward the north.

62. (a) The circumference is  $c = 2\pi r = 2\pi(0.15 \text{ m}) = 0.94 \text{ m}$ .

(b) With  $T = (60 \text{ s})/1200 = 0.050 \text{ s}$ , the speed is  $v = c/T = (0.94 \text{ m})/(0.050 \text{ s}) = 19 \text{ m/s}$ . This is equivalent to using Eq. 4-35.

(c) The magnitude of the acceleration is  $a = v^2/r = (19 \text{ m/s})^2/(0.15 \text{ m}) = 2.4 \times 10^3 \text{ m/s}^2$ .

(d) The period of revolution is  $(1200 \text{ rev/min})^{-1} = 8.3 \times 10^{-4} \text{ min}$  which becomes, in SI units,  $T = 0.050 \text{ s} = 50 \text{ ms}$ .

63. Since the period of a uniform circular motion is  $T = 2\pi r / v$ , where  $r$  is the radius and  $v$  is the speed, the centripetal acceleration can be written as

$$a = \frac{v^2}{r} = \frac{1}{r} \left( \frac{2\pi r}{T} \right)^2 = \frac{4\pi^2 r}{T^2}.$$

Based on this expression, we compare the (magnitudes) of the wallet and purse accelerations, and find their ratio is the ratio of  $r$  values. Therefore,  $a_{\text{wallet}} = 1.50 a_{\text{purse}}$ . Thus, the wallet acceleration vector is

$$a = 1.50[(2.00 \text{ m/s}^2)\hat{i} + (4.00 \text{ m/s}^2)\hat{j}] = (3.00 \text{ m/s}^2)\hat{i} + (6.00 \text{ m/s}^2)\hat{j}.$$

64. The fact that the velocity is in the  $+y$  direction, and the acceleration is in the  $+x$  direction at  $t_1 = 4.00$  s implies that the motion is clockwise. The position corresponds to the “9:00 position.” On the other hand, the position at  $t_2 = 10.0$  s is in the “6:00 position” since the velocity points in the  $-x$  direction and the acceleration is in the  $+y$  direction. The time interval  $\Delta t = 10.0 \text{ s} - 4.00 \text{ s} = 6.00 \text{ s}$  is equal to  $3/4$  of a period:

$$6.00 \text{ s} = \frac{3}{4}T \Rightarrow T = 8.00 \text{ s}.$$

Eq. 4-35 then yields

$$r = \frac{vT}{2\pi} = \frac{(3.00 \text{ m/s})(8.00 \text{ s})}{2\pi} = 3.82 \text{ m}.$$

(a) The  $x$  coordinate of the center of the circular path is  $x = 5.00 \text{ m} + 3.82 \text{ m} = 8.82 \text{ m}$ .

(b) The  $y$  coordinate of the center of the circular path is  $y = 6.00 \text{ m}$ .

In other words, the center of the circle is at  $(x,y) = (8.82 \text{ m}, 6.00 \text{ m})$ .

65. We first note that  $\vec{a}_1$  (the acceleration at  $t_1 = 2.00$  s) is perpendicular to  $\vec{a}_2$  (the acceleration at  $t_2 = 5.00$  s), by taking their scalar (dot) product.:

$$\vec{a}_1 \cdot \vec{a}_2 = [(6.00 \text{ m/s}^2)\hat{i} + (4.00 \text{ m/s}^2)\hat{j}] \cdot [(4.00 \text{ m/s}^2)\hat{i} + (-6.00 \text{ m/s}^2)\hat{j}] = 0.$$

Since the acceleration vectors are in the (negative) radial directions, then the two positions (at  $t_1$  and  $t_2$ ) are a quarter-circle apart (or three-quarters of a circle, depending on whether one measures clockwise or counterclockwise). A quick sketch leads to the conclusion that if the particle is moving counterclockwise (as the problem states) then it travels three-quarters of a circumference in moving from the position at time  $t_1$  to the position at time  $t_2$ . Letting  $T$  stand for the period, then  $t_2 - t_1 = 3.00 \text{ s} = 3T/4$ . This gives  $T = 4.00 \text{ s}$ . The magnitude of the acceleration is

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(6.00 \text{ m/s}^2)^2 + (4.00 \text{ m/s}^2)^2} = 7.21 \text{ m/s}^2.$$

Using Eq. 4-34 and 4-35, we have  $a = 4\pi^2 r / T^2$ , which yields

$$r = \frac{aT^2}{4\pi^2} = \frac{(7.21 \text{ m/s}^2)(4.00 \text{ s})^2}{4\pi^2} = 2.92 \text{ m}.$$

66. When traveling in circular motion with constant speed, the instantaneous acceleration vector necessarily points towards the center. Thus, the center is “straight up” from the cited point.

(a) Since the center is “straight up” from (4.00 m, 4.00 m), the  $x$  coordinate of the center is 4.00 m.

(b) To find out “how far up” we need to know the radius. Using Eq. 4-34 we find

$$r = \frac{v^2}{a} = \frac{(5.00 \text{ m/s})^2}{12.5 \text{ m/s}^2} = 2.00 \text{ m}.$$

Thus, the  $y$  coordinate of the center is  $2.00 \text{ m} + 4.00 \text{ m} = 6.00 \text{ m}$ . Thus, the center may be written as  $(x, y) = (4.00 \text{ m}, 6.00 \text{ m})$ .

67. To calculate the centripetal acceleration of the stone, we need to know its speed during its circular motion (this is also its initial speed when it flies off). We use the kinematic equations of projectile motion (discussed in §4-6) to find that speed. Taking the +y direction to be upward and placing the origin at the point where the stone leaves its circular orbit, then the coordinates of the stone during its motion as a projectile are given by  $x = v_0 t$  and  $y = -\frac{1}{2} g t^2$  (since  $v_{0y} = 0$ ). It hits the ground at  $x = 10$  m and  $y = -2.0$  m. Formally solving the second equation for the time, we obtain  $t = \sqrt{-2y/g}$ , which we substitute into the first equation:

$$v_0 = x \sqrt{-\frac{g}{2y}} = (10 \text{ m}) \sqrt{-\frac{9.8 \text{ m/s}^2}{2(-2.0 \text{ m})}} = 15.7 \text{ m/s}.$$

Therefore, the magnitude of the centripetal acceleration is

$$a = \frac{v^2}{r} = \frac{(15.7 \text{ m/s})^2}{1.5 \text{ m}} = 160 \text{ m/s}^2.$$

68. We note that after three seconds have elapsed ( $t_2 - t_1 = 3.00$  s) the velocity (for this object in circular motion of period  $T$ ) is reversed; we infer that it takes three seconds to reach the opposite side of the circle. Thus,  $T = 2(3.00 \text{ s}) = 6.00$  s.

(a) Using Eq. 4-35,  $r = vT/2\pi$ , where  $v = \sqrt{(3.00 \text{ m/s})^2 + (4.00 \text{ m/s})^2} = 5.00$  m/s, we obtain  $r = 4.77$  m. The magnitude of the object's centripetal acceleration is therefore  $a = v^2/r = 5.24$  m/s<sup>2</sup>.

(b) The average acceleration is given by Eq. 4-15:

$$\vec{a}_{\text{avg}} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \frac{(-3.00\hat{i} - 4.00\hat{j}) \text{ m/s} - (3.00\hat{i} + 4.00\hat{j}) \text{ m/s}}{5.00 \text{ s} - 2.00 \text{ s}} = (-2.00 \text{ m/s}^2)\hat{i} + (-2.67 \text{ m/s}^2)\hat{j}$$

which implies  $|\vec{a}_{\text{avg}}| = \sqrt{(-2.00 \text{ m/s}^2)^2 + (-2.67 \text{ m/s}^2)^2} = 3.33$  m/s<sup>2</sup>.

69. We use Eq. 4-15 first using velocities relative to the truck (subscript t) and then using velocities relative to the ground (subscript g). We work with SI units, so  $20 \text{ km/h} \rightarrow 5.6 \text{ m/s}$ ,  $30 \text{ km/h} \rightarrow 8.3 \text{ m/s}$ , and  $45 \text{ km/h} \rightarrow 12.5 \text{ m/s}$ . We choose east as the  $+\hat{i}$  direction.

(a) The velocity of the cheetah (subscript c) at the end of the 2.0 s interval is (from Eq. 4-44)

$$\vec{v}_{ct} = \vec{v}_{cg} - \vec{v}_{tg} = (12.5 \text{ m/s})\hat{i} - (-5.6 \text{ m/s})\hat{i} = (18.1 \text{ m/s})\hat{i}$$

relative to the truck. Since the velocity of the cheetah relative to the truck at the beginning of the 2.0 s interval is  $(-8.3 \text{ m/s})\hat{i}$ , the (average) acceleration vector relative to the cameraman (in the truck) is

$$\vec{a}_{avg} = \frac{(18.1 \text{ m/s})\hat{i} - (-8.3 \text{ m/s})\hat{i}}{2.0 \text{ s}} = (13 \text{ m/s}^2)\hat{i},$$

or  $|\vec{a}_{avg}| = 13 \text{ m/s}^2$ .

(b) The direction of  $\vec{a}_{avg}$  is  $+\hat{i}$ , or eastward.

(c) The velocity of the cheetah at the start of the 2.0 s interval is (from Eq. 4-44)

$$\vec{v}_{0cg} = \vec{v}_{0ct} + \vec{v}_{0tg} = (-8.3 \text{ m/s})\hat{i} + (-5.6 \text{ m/s})\hat{i} = (-13.9 \text{ m/s})\hat{i}$$

relative to the ground. The (average) acceleration vector relative to the crew member (on the ground) is

$$\vec{a}_{avg} = \frac{(12.5 \text{ m/s})\hat{i} - (-13.9 \text{ m/s})\hat{i}}{2.0 \text{ s}} = (13 \text{ m/s}^2)\hat{i}, \quad |\vec{a}_{avg}| = 13 \text{ m/s}^2$$

identical to the result of part (a).

70. We use Eq. 4-44, noting that the upstream corresponds to the  $+\hat{i}$  direction.

(a) The subscript b is for the boat, w is for the water, and g is for the ground.

$$\vec{v}_{bg} = \vec{v}_{bw} + \vec{v}_{wg} = (14 \text{ km/h}) \hat{i} + (-9 \text{ km/h}) \hat{i} = (5 \text{ km/h}) \hat{i}.$$

Thus, the magnitude is  $|\vec{v}_{bg}| = 5 \text{ km/h}$ .

(b) The direction of  $\vec{v}_{bg}$  is  $+x$ , or upstream.

(c) We use the subscript c for the child, and obtain

$$\vec{v}_{cg} = \vec{v}_{cb} + \vec{v}_{bg} = (-6 \text{ km/h}) \hat{i} + (5 \text{ km/h}) \hat{i} = (-1 \text{ km/h}) \hat{i}.$$

The magnitude is  $|\vec{v}_{cg}| = 1 \text{ km/h}$ .

(d) The direction of  $\vec{v}_{cg}$  is  $-x$ , or downstream.

71. While moving in the same direction as the sidewalk's motion (covering a distance  $d$  relative to the ground in time  $t_1 = 2.50$  s), Eq. 4-44 leads to

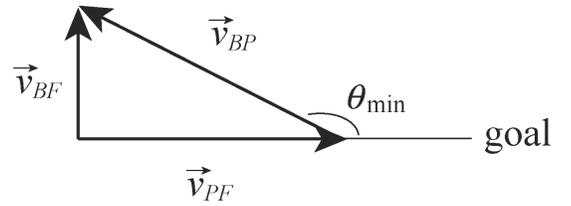
$$v_{\text{sidewalk}} + v_{\text{man running}} = \frac{d}{t_1} .$$

While he runs back (taking time  $t_2 = 10.0$  s) we have

$$v_{\text{sidewalk}} - v_{\text{man running}} = -\frac{d}{t_2} .$$

Dividing these equations and solving for the desired ratio, we get  $\frac{12.5}{7.5} = \frac{5}{3} = 1.67$ .

72. We denote the velocity of the player with  $\vec{v}_{PF}$  and the relative velocity between the player and the ball be  $\vec{v}_{BP}$ . Then the velocity  $\vec{v}_{BF}$  of the ball relative to the field is given by  $\vec{v}_{BF} = \vec{v}_{PF} + \vec{v}_{BP}$ . The smallest angle  $\theta_{\min}$  corresponds to the case when  $\vec{v}_{BF} \perp \vec{v}_{PF}$ . Hence,



$$\theta_{\min} = 180^\circ - \cos^{-1} \left( \frac{|\vec{v}_{PF}|}{|\vec{v}_{BP}|} \right) = 180^\circ - \cos^{-1} \left( \frac{4.0 \text{ m/s}}{6.0 \text{ m/s}} \right) = 130^\circ.$$

73. The velocity vectors (relative to the shore) for ships  $A$  and  $B$  are given by

$$\begin{aligned}\vec{v}_A &= -(v_A \cos 45^\circ) \hat{i} + (v_A \sin 45^\circ) \hat{j} \\ \vec{v}_B &= -(v_B \sin 40^\circ) \hat{i} - (v_B \cos 40^\circ) \hat{j},\end{aligned}$$

with  $v_A = 24$  knots and  $v_B = 28$  knots. We take east as  $+\hat{i}$  and north as  $\hat{j}$ .

(a) Their relative velocity is

$$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B = (v_B \sin 40^\circ - v_A \cos 45^\circ) \hat{i} + (v_B \cos 40^\circ + v_A \sin 45^\circ) \hat{j}$$

the magnitude of which is  $|\vec{v}_{AB}| = \sqrt{(1.03 \text{ knots})^2 + (38.4 \text{ knots})^2} \approx 38 \text{ knots}$ .

(b) The angle  $\theta$  which  $\vec{v}_{AB}$  makes with north is given by

$$\theta = \tan^{-1} \left( \frac{v_{AB,x}}{v_{AB,y}} \right) = \tan^{-1} \left( \frac{1.03 \text{ knots}}{38.4 \text{ knots}} \right) = 1.5^\circ$$

which is to say that  $\vec{v}_{AB}$  points  $1.5^\circ$  east of north.

(c) Since they started at the same time, their relative velocity describes at what rate the distance between them is increasing. Because the rate is steady, we have

$$t = \frac{|\Delta r_{AB}|}{|\vec{v}_{AB}|} = \frac{160}{38.4} = 4.2 \text{ h.}$$

(d) The velocity  $\vec{v}_{AB}$  does not change with time in this problem, and  $\vec{r}_{AB}$  is in the same direction as  $\vec{v}_{AB}$  since they started at the same time. Reversing the points of view, we have  $\vec{v}_{AB} = -\vec{v}_{BA}$  so that  $\vec{r}_{AB} = -\vec{r}_{BA}$  (i.e., they are  $180^\circ$  opposite to each other). Hence, we conclude that  $B$  stays at a bearing of  $1.5^\circ$  west of south relative to  $A$  during the journey (neglecting the curvature of Earth).

74. The destination is  $\vec{D} = 800 \text{ km } \hat{j}$  where we orient axes so that  $+y$  points north and  $+x$  points east. This takes two hours, so the (constant) velocity of the plane (relative to the ground) is  $\vec{v}_{pg} = (400 \text{ km/h}) \hat{j}$ . This must be the vector sum of the plane's velocity with respect to the air which has  $(x,y)$  components  $(500\cos 70^\circ, 500\sin 70^\circ)$  and the velocity of the air (*wind*) relative to the ground  $\vec{v}_{ag}$ . Thus,

$$(400 \text{ km/h}) \hat{j} = (500 \text{ km/h}) \cos 70^\circ \hat{i} + (500 \text{ km/h}) \sin 70^\circ \hat{j} + \vec{v}_{ag}$$

which yields

$$\vec{v}_{ag} = (-171 \text{ km/h}) \hat{i} - (70.0 \text{ km/h}) \hat{j}.$$

(a) The magnitude of  $\vec{v}_{ag}$  is  $|\vec{v}_{ag}| = \sqrt{(-171 \text{ km/h})^2 + (-70.0 \text{ km/h})^2} = 185 \text{ km/h}$ .

(b) The direction of  $\vec{v}_{ag}$  is

$$\theta = \tan^{-1} \left( \frac{-70.0 \text{ km/h}}{-171 \text{ km/h}} \right) = 22.3^\circ \text{ (south of west).}$$

75. Relative to the car the velocity of the snowflakes has a vertical component of 8.0 m/s and a horizontal component of 50 km/h = 13.9 m/s. The angle  $\theta$  from the vertical is found from

$$\tan \theta = \frac{v_h}{v_v} = \frac{13.9 \text{ m/s}}{8.0 \text{ m/s}} = 1.74$$

which yields  $\theta = 60^\circ$ .

76. Velocities are taken to be constant; thus, the velocity of the plane relative to the ground is  $\vec{v}_{PG} = (55 \text{ km}) / (1/4 \text{ hour}) \hat{j} = (220 \text{ km/h}) \hat{j}$ . In addition,

$$\vec{v}_{AG} = (42 \text{ km/h})(\cos 20^\circ \hat{i} - \sin 20^\circ \hat{j}) = (39 \text{ km/h}) \hat{i} - (14 \text{ km/h}) \hat{j}.$$

Using  $\vec{v}_{PG} = \vec{v}_{PA} + \vec{v}_{AG}$ , we have

$$\vec{v}_{PA} = \vec{v}_{PG} - \vec{v}_{AG} = -(39 \text{ km/h}) \hat{i} + (234 \text{ km/h}) \hat{j}.$$

which implies  $|\vec{v}_{PA}| = 237 \text{ km/h}$ , or  $240 \text{ km/h}$  (to two significant figures.)

77. Since the raindrops fall vertically relative to the train, the horizontal component of the velocity of a raindrop is  $v_h = 30 \text{ m/s}$ , the same as the speed of the train. If  $v_v$  is the vertical component of the velocity and  $\theta$  is the angle between the direction of motion and the vertical, then  $\tan \theta = v_h/v_v$ . Thus  $v_v = v_h/\tan \theta = (30 \text{ m/s})/\tan 70^\circ = 10.9 \text{ m/s}$ . The speed of a raindrop is

$$v = \sqrt{v_h^2 + v_v^2} = \sqrt{(30 \text{ m/s})^2 + (10.9 \text{ m/s})^2} = 32 \text{ m/s}.$$

78. This is a classic problem involving two-dimensional relative motion. We align our coordinates so that *east* corresponds to  $+x$  and *north* corresponds to  $+y$ . We write the vector addition equation as  $\vec{v}_{BG} = \vec{v}_{BW} + \vec{v}_{WG}$ . We have  $\vec{v}_{WG} = (2.0 \angle 0^\circ)$  in the magnitude-angle notation (with the unit m/s understood), or  $\vec{v}_{WG} = 2.0\hat{i}$  in unit-vector notation. We also have  $\vec{v}_{BW} = (8.0 \angle 120^\circ)$  where we have been careful to phrase the angle in the ‘standard’ way (measured counterclockwise from the  $+x$  axis), or  $\vec{v}_{BW} = (-4.0\hat{i} + 6.9\hat{j})$  m/s.

(a) We can solve the vector addition equation for  $\vec{v}_{BG}$ :

$$\vec{v}_{BG} = \vec{v}_{BW} + \vec{v}_{WG} = (2.0 \text{ m/s})\hat{i} + (-4.0\hat{i} + 6.9\hat{j}) \text{ m/s} = (-2.0 \text{ m/s})\hat{i} + (6.9 \text{ m/s})\hat{j}.$$

Thus, we find  $|\vec{v}_{BG}| = 7.2$  m/s.

(b) The direction of  $\vec{v}_{BG}$  is  $\theta = \tan^{-1}[(6.9 \text{ m/s})/(-2.0 \text{ m/s})] = 106^\circ$  (measured counterclockwise from the  $+x$  axis), or  $16^\circ$  west of north.

(c) The velocity is constant, and we apply  $y - y_0 = v_y t$  in a reference frame. Thus, in the *ground* reference frame, we have  $(200 \text{ m}) = (7.2 \text{ m/s}) \sin(106^\circ)t \rightarrow t = 29$  s. Note: if a student obtains “28 s”, then the student has probably neglected to take the  $y$  component properly (a common mistake).

79. We denote the police and the motorist with subscripts  $p$  and  $m$ , respectively. The coordinate system is indicated in Fig. 4-49.

(a) The velocity of the motorist with respect to the police car is

$$\vec{v}_{m p} = \vec{v}_m - \vec{v}_p = (-60 \text{ km/h})\hat{j} - (-80 \text{ km/h})\hat{i} = (80 \text{ km/h})\hat{i} - (60 \text{ km/h})\hat{j}.$$

(b)  $\vec{v}_{m p}$  does happen to be along the line of sight. Referring to Fig. 4-49, we find the vector pointing from one car to another is  $\vec{r} = (800 \text{ m})\hat{i} - (600 \text{ m})\hat{j}$  (from  $M$  to  $P$ ). Since the ratio of components in  $\vec{r}$  is the same as in  $\vec{v}_{m p}$ , they must point the same direction.

(c) No, they remain unchanged.

80. We make use of Eq. 4-44 and Eq. 4-45.

The velocity of Jeep  $P$  relative to  $A$  at the instant is

$$\vec{v}_{PA} = (40.0 \text{ m/s})(\cos 60^\circ \hat{i} + \sin 60^\circ \hat{j}) = (20.0 \text{ m/s})\hat{i} + (34.6 \text{ m/s})\hat{j}.$$

Similarly, the velocity of Jeep  $B$  relative to  $A$  at the instant is

$$\vec{v}_{BA} = (20.0 \text{ m/s})(\cos 30^\circ \hat{i} + \sin 30^\circ \hat{j}) = (17.3 \text{ m/s})\hat{i} + (10.0 \text{ m/s})\hat{j}.$$

Thus, the velocity of  $P$  relative to  $B$  is

$$\vec{v}_{PB} = \vec{v}_{PA} - \vec{v}_{BA} = (20.0\hat{i} + 34.6\hat{j}) \text{ m/s} - (17.3\hat{i} + 10.0\hat{j}) \text{ m/s} = (2.68 \text{ m/s})\hat{i} + (24.6 \text{ m/s})\hat{j}.$$

(a) The magnitude of  $\vec{v}_{PB}$  is  $|\vec{v}_{PB}| = \sqrt{(2.68 \text{ m/s})^2 + (24.6 \text{ m/s})^2} = 24.8 \text{ m/s}$ .

(b) The direction of  $\vec{v}_{PB}$  is  $\theta = \tan^{-1}[(24.6 \text{ m/s})/(2.68 \text{ m/s})] = 83.8^\circ$  north of east (or  $6.2^\circ$  east of north).

(c) The acceleration of  $P$  is

$$\vec{a}_{PA} = (0.400 \text{ m/s}^2)(\cos 60.0^\circ \hat{i} + \sin 60.0^\circ \hat{j}) = (0.200 \text{ m/s}^2)\hat{i} + (0.346 \text{ m/s}^2)\hat{j},$$

and  $\vec{a}_{PA} = \vec{a}_{PB}$ . Thus, we have  $|\vec{a}_{PB}| = 0.400 \text{ m/s}^2$ .

(d) The direction is  $60.0^\circ$  north of east (or  $30.0^\circ$  east of north).

81. Here, the subscript  $W$  refers to the water. Our coordinates are chosen with  $+x$  being *east* and  $+y$  being *north*. In these terms, the angle specifying *east* would be  $0^\circ$  and the angle specifying *south* would be  $-90^\circ$  or  $270^\circ$ . Where the length unit is not displayed, km is to be understood.

(a) We have  $\vec{v}_{AW} = \vec{v}_{AB} + \vec{v}_{BW}$ , so that

$$\vec{v}_{AB} = (22 \angle -90^\circ) - (40 \angle 37^\circ) = (56 \angle -125^\circ)$$

in the magnitude-angle notation (conveniently done with a vector-capable calculator in polar mode). Converting to rectangular components, we obtain

$$\vec{v}_{AB} = (-32 \text{ km/h}) \hat{i} - (46 \text{ km/h}) \hat{j} .$$

Of course, this could have been done in unit-vector notation from the outset.

(b) Since the velocity-components are constant, integrating them to obtain the position is straightforward ( $\vec{r} - \vec{r}_0 = \int \vec{v} dt$ )

$$\vec{r} = (2.5 - 32t) \hat{i} + (4.0 - 46t) \hat{j}$$

with lengths in kilometers and time in hours.

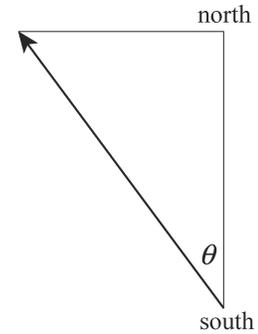
(c) The magnitude of this  $\vec{r}$  is  $r = \sqrt{(2.5 - 32t)^2 + (4.0 - 46t)^2}$ . We minimize this by taking a derivative and requiring it to equal zero — which leaves us with an equation for  $t$

$$\frac{dr}{dt} = \frac{1}{2} \frac{6286t - 528}{\sqrt{(2.5 - 32t)^2 + (4.0 - 46t)^2}} = 0$$

which yields  $t = 0.084$  h.

(d) Plugging this value of  $t$  back into the expression for the distance between the ships ( $r$ ), we obtain  $r = 0.2$  km. Of course, the calculator offers more digits ( $r = 0.225\dots$ ), but they are not significant; in fact, the uncertainties implicit in the given data, here, should make the ship captains worry.

82. We construct a right triangle starting from the clearing on the south bank, drawing a line (200 m long) due north (*upward* in our sketch) across the river, and then a line due west (upstream, leftward in our sketch) along the north bank for a distance  $(82 \text{ m}) + (1.1 \text{ m/s})t$ , where the  $t$ -dependent contribution is the distance that the river will carry the boat downstream during time  $t$ .



The hypotenuse of this right triangle (the arrow in our sketch) also depends on  $t$  and on the boat's speed (relative to the water), and we set it equal to the Pythagorean “sum” of the triangle's sides:

$$(4.0)t = \sqrt{200^2 + (82 + 1.1t)^2}$$

which leads to a quadratic equation for  $t$

$$46724 + 180.4t - 14.8t^2 = 0.$$

We solve this and find a positive value:  $t = 62.6 \text{ s}$ .

The angle between the northward (200 m) leg of the triangle and the hypotenuse (which is measured “west of north”) is then given by

$$\theta = \tan^{-1} \left( \frac{82 + 1.1t}{200} \right) = \tan^{-1} \left( \frac{151}{200} \right) = 37^\circ.$$

83. Using displacement = velocity  $\times$  time (for each constant-velocity part of the trip), along with the fact that 1 hour = 60 minutes, we have the following vector addition exercise (using notation appropriate to many vector capable calculators):

$$(1667 \text{ m } \angle 0^\circ) + (1333 \text{ m } \angle -90^\circ) + (333 \text{ m } \angle 180^\circ) + (833 \text{ m } \angle -90^\circ) + (667 \text{ m } \angle 180^\circ) + (417 \text{ m } \angle -90^\circ) = (2668 \text{ m } \angle -76^\circ).$$

(a) Thus, the magnitude of the net displacement is 2.7 km.

(b) Its direction is  $76^\circ$  clockwise (relative to the initial direction of motion).

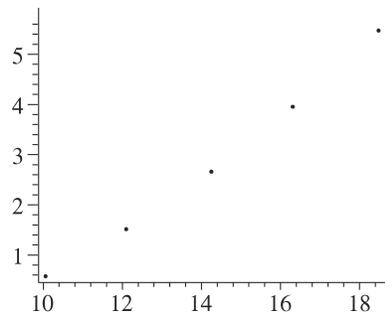
84. We compute the coordinate pairs  $(x, y)$  from  $x = (v_0 \cos \theta)t$  and  $y = v_0 \sin \theta t - \frac{1}{2}gt^2$  for  $t = 20$  s and the speeds and angles given in the problem.

(a) We obtain

$$\begin{aligned}(x_A, y_A) &= (10.1 \text{ km}, 0.56 \text{ km}) & (x_B, y_B) &= (12.1 \text{ km}, 1.51 \text{ km}) \\(x_C, y_C) &= (14.3 \text{ km}, 2.68 \text{ km}) & (x_D, y_D) &= (16.4 \text{ km}, 3.99 \text{ km})\end{aligned}$$

and  $(x_E, y_E) = (18.5 \text{ km}, 5.53 \text{ km})$  which we plot in the next part.

(b) The vertical ( $y$ ) and horizontal ( $x$ ) axes are in kilometers. The graph does not start at the origin. The curve to “fit” the data is not shown, but is easily imagined (forming the “curtain of death”).



85. Let  $v_0 = 2\pi(0.200 \text{ m})/(0.00500 \text{ s}) \approx 251 \text{ m/s}$  (using Eq. 4-35) be the speed it had in circular motion and  $\theta_0 = (1 \text{ hr})(360^\circ/12 \text{ hr [for full rotation]}) = 30.0^\circ$ . Then Eq. 4-25 leads to

$$y = (2.50 \text{ m}) \tan 30.0^\circ - \frac{(9.8 \text{ m/s}^2)(2.50 \text{ m})^2}{2(251 \text{ m/s})^2 (\cos 30.0^\circ)^2} \approx 1.44 \text{ m}$$

which means its height above the floor is  $1.44 \text{ m} + 1.20 \text{ m} = 2.64 \text{ m}$ .

86. For circular motion, we must have  $\vec{v}$  with direction perpendicular to  $\vec{r}$  and (since the speed is constant) magnitude  $v = 2\pi r / T$  where  $r = \sqrt{(2.00 \text{ m})^2 + (-3.00 \text{ m})^2}$  and  $T = 7.00 \text{ s}$ . The  $\vec{r}$  (given in the problem statement) specifies a point in the fourth quadrant, and since the motion is clockwise then the velocity must have both components negative. Our result, satisfying these three conditions, (using unit-vector notation which makes it easy to double-check that  $\vec{r} \cdot \vec{v} = 0$ ) for  $\vec{v} = (-2.69 \text{ m/s})\hat{i} + (-1.80 \text{ m/s})\hat{j}$ .

87. Using Eq. 2-16, we obtain  $v^2 = v_0^2 - 2gh$ , or  $h = (v_0^2 - v^2) / 2g$ .

(a) Since  $v = 0$  at the maximum height of an upward motion, with  $v_0 = 7.00$  m/s, we have  $h = (7.00 \text{ m/s})^2 / 2(9.80 \text{ m/s}^2) = 2.50$  m.

(b) The relative speed is  $v_r = v_0 - v_c = 7.00 \text{ m/s} - 3.00 \text{ m/s} = 4.00$  m/s with respect to the floor. Using the above equation we obtain  $h = (4.00 \text{ m/s})^2 / 2(9.80 \text{ m/s}^2) = 0.82$  m.

(c) The acceleration, or the rate of change of speed of the ball with respect to the ground is  $9.80 \text{ m/s}^2$  (downward).

(d) Since the elevator cab moves at constant velocity, the rate of change of speed of the ball with respect to the cab floor is also  $9.80 \text{ m/s}^2$  (downward).

88. Relative to the sled, the launch velocity is  $\vec{v}_{o,rel} = v_{ox} \hat{i} + v_{oy} \hat{j}$ . Since the sled's motion is in the negative direction with speed  $v_s$  (note that we are treating  $v_s$  as a positive number, so the sled's velocity is actually  $-v_s \hat{i}$ ), then the launch velocity relative to the ground is  $\vec{v}_o = (v_{ox} - v_s) \hat{i} + v_{oy} \hat{j}$ . The horizontal and vertical displacement (relative to the ground) are therefore

$$x_{\text{land}} - x_{\text{launch}} = \Delta x_{\text{bg}} = (v_{ox} - v_s) t_{\text{flight}}$$

$$y_{\text{land}} - y_{\text{launch}} = 0 = v_{oy} t_{\text{flight}} + \frac{1}{2}(-g)(t_{\text{flight}})^2.$$

Combining these equations leads to

$$\Delta x_{\text{bg}} = \frac{2 v_{ox} v_{oy}}{g} - \left( \frac{2 v_{oy}}{g} \right) v_s.$$

The first term corresponds to the “y intercept” on the graph, and the second term (in parentheses) corresponds to the magnitude of the “slope.” From Figure 4-54, we have

$$\Delta x_{\text{bg}} = 40 - 4v_s.$$

This implies  $v_{oy} = (4.0 \text{ s})(9.8 \text{ m/s}^2)/2 = 19.6 \text{ m/s}$ , and that furnishes enough information to determine  $v_{ox}$ .

(a)  $v_{ox} = 40g/2v_{oy} = (40 \text{ m})(9.8 \text{ m/s}^2)/(39.2 \text{ m/s}) = 10 \text{ m/s}$ .

(b) As noted above,  $v_{oy} = 19.6 \text{ m/s}$ .

(c) Relative to the sled, the displacement  $\Delta x_{\text{bs}}$  does not depend on the sled's speed, so  $\Delta x_{\text{bs}} = v_{ox} t_{\text{flight}} = 40 \text{ m}$ .

(d) As in (c), relative to the sled, the displacement  $\Delta x_{\text{bs}}$  does not depend on the sled's speed, and  $\Delta x_{\text{bs}} = v_{ox} t_{\text{flight}} = 40 \text{ m}$ .

89. We establish coordinates with  $\hat{i}$  pointing to the far side of the river (perpendicular to the current) and  $\hat{j}$  pointing in the direction of the current. We are told that the magnitude (presumed constant) of the velocity of the boat relative to the water is  $|\vec{v}_{bw}| = 6.4 \text{ km/h}$ . Its angle, relative to the  $x$  axis is  $\theta$ . With km and h as the understood units, the velocity of the water (relative to the ground) is  $\vec{v}_{wg} = (3.2 \text{ km/h})\hat{j}$ .

(a) To reach a point “directly opposite” means that the velocity of her boat relative to ground must be  $\vec{v}_{bg} = v_{bg}\hat{i}$  where  $v_{bg} > 0$  is unknown. Thus, all  $\hat{j}$  components must cancel in the vector sum  $\vec{v}_{bw} + \vec{v}_{wg} = \vec{v}_{bg}$ , which means the  $\vec{v}_{bw} \sin \theta = (-3.2 \text{ km/h})\hat{j}$ , so

$$\theta = \sin^{-1} [(-3.2 \text{ km/h})/(6.4 \text{ km/h})] = -30^\circ.$$

(b) Using the result from part (a), we find  $v_{bg} = v_{bw} \cos \theta = 5.5 \text{ km/h}$ . Thus, traveling a distance of  $\ell = 6.4 \text{ km}$  requires a time of  $(6.4 \text{ km})/(5.5 \text{ km/h}) = 1.15 \text{ h}$  or 69 min.

(c) If her motion is completely along the  $y$  axis (as the problem implies) then with  $v_{wg} = 3.2 \text{ km/h}$  (the water speed) we have

$$t_{\text{total}} = \frac{D}{v_{bw} + v_{wg}} + \frac{D}{v_{bw} - v_{wg}} = 1.33 \text{ h}$$

where  $D = 3.2 \text{ km}$ . This is equivalent to 80 min.

(d) Since

$$\frac{D}{v_{bw} + v_{wg}} + \frac{D}{v_{bw} - v_{wg}} = \frac{D}{v_{bw} - v_{wg}} + \frac{D}{v_{bw} + v_{wg}}$$

the answer is the same as in the previous part, i.e.,  $t_{\text{total}} = 80 \text{ min}$ .

(e) The shortest-time path should have  $\theta = 0^\circ$ . This can also be shown by noting that the case of general  $\theta$  leads to

$$\vec{v}_{bg} = \vec{v}_{bw} + \vec{v}_{wg} = v_{bw} \cos \theta \hat{i} + (v_{bw} \sin \theta + v_{wg}) \hat{j}$$

where the  $x$  component of  $\vec{v}_{bg}$  must equal  $l/t$ . Thus,

$$t = \frac{l}{v_{bw} \cos \theta}$$

which can be minimized using  $dt/d\theta = 0$ .

(f) The above expression leads to  $t = (6.4 \text{ km})/(6.4 \text{ km/h}) = 1.0 \text{ h}$ , or 60 min.

90. We use a coordinate system with +x eastward and +y upward.

(a) We note that  $123^\circ$  is the angle between the initial position and later position vectors, so that the angle from +x to the later position vector is  $40^\circ + 123^\circ = 163^\circ$ . In unit-vector notation, the position vectors are

$$\vec{r}_1 = (360 \text{ m})\cos(40^\circ)\hat{i} + (360 \text{ m})\sin(40^\circ)\hat{j} = (276 \text{ m})\hat{i} + (231 \text{ m})\hat{j}$$

$$\vec{r}_2 = (790 \text{ m})\cos(163^\circ)\hat{i} + (790 \text{ m})\sin(163^\circ)\hat{j} = (-755 \text{ m})\hat{i} + (231 \text{ m})\hat{j}$$

respectively. Consequently, we plug into Eq. 4-3

$$\Delta\vec{r} = [(-755 \text{ m}) - (276 \text{ m})]\hat{i} + (231 \text{ m} - 231 \text{ m})\hat{j} = -(1031 \text{ m})\hat{i}.$$

The magnitude of the displacement  $\Delta\vec{r}$  is  $|\Delta\vec{r}| = 1031 \text{ m}$ .

(b) The direction of  $\Delta\vec{r}$  is  $-\hat{i}$ , or westward.

91. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable.

(a) With the origin at the firing point, the  $y$  coordinate of the bullet is given by  $y = -\frac{1}{2}gt^2$ . If  $t$  is the time of flight and  $y = -0.019$  m indicates where the bullet hits the target, then

$$t = \sqrt{\frac{2(0.019 \text{ m})}{9.8 \text{ m/s}^2}} = 6.2 \times 10^{-2} \text{ s.}$$

(b) The muzzle velocity is the initial (horizontal) velocity of the bullet. Since  $x = 30$  m is the horizontal position of the target, we have  $x = v_0t$ . Thus,

$$v_0 = \frac{x}{t} = \frac{30 \text{ m}}{6.3 \times 10^{-2} \text{ s}} = 4.8 \times 10^2 \text{ m/s.}$$

92. Eq. 4-34 describes an inverse proportionality between  $r$  and  $a$ , so that a large acceleration results from a small radius. Thus, an upper limit for  $a$  corresponds to a lower limit for  $r$ .

(a) The minimum turning radius of the train is given by

$$r_{\min} = \frac{v^2}{a_{\max}} = \frac{(216 \text{ km/h})^2}{(0.050)(9.8 \text{ m/s}^2)} = 7.3 \times 10^3 \text{ m.}$$

(b) The speed of the train must be reduced to no more than

$$v = \sqrt{a_{\max} r} = \sqrt{0.050(9.8 \text{ m/s}^2)(1.00 \times 10^3 \text{ m})} = 22 \text{ m/s}$$

which is roughly 80 km/h.

93. (a) With  $r = 0.15$  m and  $a = 3.0 \times 10^{14}$  m/s<sup>2</sup>, Eq. 4-34 gives

$$v = \sqrt{ra} = 6.7 \times 10^6 \text{ m/s.}$$

(b) The period is given by Eq. 4-35:

$$T = \frac{2\pi r}{v} = 1.4 \times 10^{-7} \text{ s.}$$

94. We use Eq. 4-2 and Eq. 4-3.

(a) With the initial position vector as  $\vec{r}_1$  and the later vector as  $\vec{r}_2$ , Eq. 4-3 yields

$$\Delta r = [(-2.0 \text{ m}) - 5.0 \text{ m}]\hat{i} + [(6.0 \text{ m}) - (-6.0 \text{ m})]\hat{j} + (2.0 \text{ m} - 2.0 \text{ m})\hat{k} = (-7.0 \text{ m})\hat{i} + (12 \text{ m})\hat{j}$$

for the displacement vector in unit-vector notation.

(b) Since there is no  $z$  component (that is, the coefficient of  $\hat{k}$  is zero), the displacement vector is in the  $xy$  plane.

95. We write our magnitude-angle results in the form  $(R \angle \theta)$  with SI units for the magnitude understood (m for distances, m/s for speeds,  $\text{m/s}^2$  for accelerations). All angles  $\theta$  are measured counterclockwise from  $+x$ , but we will occasionally refer to angles  $\phi$  which are measured counterclockwise from the vertical line between the circle-center and the coordinate origin and the line drawn from the circle-center to the particle location (see  $r$  in the figure). We note that the speed of the particle is  $v = 2\pi r/T$  where  $r = 3.00$  m and  $T = 20.0$  s; thus,  $v = 0.942$  m/s. The particle is moving counterclockwise in Fig. 4-56.

(a) At  $t = 5.0$  s, the particle has traveled a fraction of

$$\frac{t}{T} = \frac{5.00 \text{ s}}{20.0 \text{ s}} = \frac{1}{4}$$

of a full revolution around the circle (starting at the origin). Thus, relative to the circle-center, the particle is at

$$\phi = \frac{1}{4}(360^\circ) = 90^\circ$$

measured from vertical (as explained above). Referring to Fig. 4-56, we see that this position (which is the “3 o’clock” position on the circle) corresponds to  $x = 3.0$  m and  $y = 3.0$  m relative to the coordinate origin. In our magnitude-angle notation, this is expressed as  $(R \angle \theta) = (4.2 \angle 45^\circ)$ . Although this position is easy to analyze without resorting to trigonometric relations, it is useful (for the computations below) to note that these values of  $x$  and  $y$  relative to coordinate origin can be gotten from the angle  $\phi$  from the relations

$$x = r \sin \phi, \quad y = r - r \cos \phi.$$

Of course,  $R = \sqrt{x^2 + y^2}$  and  $\theta$  comes from choosing the appropriate possibility from  $\tan^{-1}(y/x)$  (or by using particular functions of vector-capable calculators).

(b) At  $t = 7.5$  s, the particle has traveled a fraction of  $7.5/20 = 3/8$  of a revolution around the circle (starting at the origin). Relative to the circle-center, the particle is therefore at  $\phi = 3/8(360^\circ) = 135^\circ$  measured from vertical in the manner discussed above. Referring to Fig. 4-56, we compute that this position corresponds to

$$\begin{aligned} x &= (3.00 \text{ m})\sin 135^\circ = 2.1 \text{ m} \\ y &= (3.0 \text{ m}) - (3.0 \text{ m})\cos 135^\circ = 5.1 \text{ m} \end{aligned}$$

relative to the coordinate origin. In our magnitude-angle notation, this is expressed as  $(R \angle \theta) = (5.5 \angle 68^\circ)$ .

(c) At  $t = 10.0$  s, the particle has traveled a fraction of  $10/20 = 1/2$  of a revolution around the circle. Relative to the circle-center, the particle is at  $\phi = 180^\circ$  measured from vertical (see explanation, above). Referring to Fig. 4-56, we see that this position corresponds to  $x = 0$  and  $y = 6.0$  m relative to the coordinate origin. In our magnitude-angle notation, this is expressed as  $(R \angle \theta) = (6.0 \angle 90^\circ)$ .

(d) We subtract the position vector in part (a) from the position vector in part (c):

$$(6.0 \angle 90^\circ) - (4.2 \angle 45^\circ) = (4.2 \angle 135^\circ)$$

using magnitude-angle notation (convenient when using vector-capable calculators). If we wish instead to use unit-vector notation, we write

$$\Delta \vec{R} = (0 - 3.0 \text{ m}) \hat{i} + (6.0 \text{ m} - 3.0 \text{ m}) \hat{j} = (-3.0 \text{ m}) \hat{i} + (3.0 \text{ m}) \hat{j}$$

which leads to  $|\Delta \vec{R}| = 4.2$  m and  $\theta = 135^\circ$ .

(e) From Eq. 4-8, we have  $\vec{v}_{\text{avg}} = \Delta \vec{R} / \Delta t$ . With  $\Delta t = 5.0$  s, we have

$$\vec{v}_{\text{avg}} = (-0.60 \text{ m/s}) \hat{i} + (0.60 \text{ m/s}) \hat{j}$$

in unit-vector notation or  $(0.85 \angle 135^\circ)$  in magnitude-angle notation.

(f) The speed has already been noted ( $v = 0.94$  m/s), but its direction is best seen by referring again to Fig. 4-56. The velocity vector is tangent to the circle at its “3 o’clock position” (see part (a)), which means  $\vec{v}$  is vertical. Thus, our result is  $(0.94 \angle 90^\circ)$ .

(g) Again, the speed has been noted above ( $v = 0.94$  m/s), but its direction is best seen by referring to Fig. 4-56. The velocity vector is tangent to the circle at its “12 o’clock position” (see part (c)), which means  $\vec{v}$  is horizontal. Thus, our result is  $(0.94 \angle 180^\circ)$ .

(h) The acceleration has magnitude  $a = v^2/r = 0.30$  m/s<sup>2</sup>, and at this instant (see part (a)) it is horizontal (towards the center of the circle). Thus, our result is  $(0.30 \angle 180^\circ)$ .

(i) Again,  $a = v^2/r = 0.30$  m/s<sup>2</sup>, but at this instant (see part (c)) it is vertical (towards the center of the circle). Thus, our result is  $(0.30 \angle 270^\circ)$ .

96. Noting that  $\vec{v}_2 = 0$ , then, using Eq. 4-15, the average acceleration is

$$\vec{a}_{\text{avg}} = \frac{\Delta \vec{v}}{\Delta t} = \frac{0 - (6.30\hat{i} - 8.42\hat{j}) \text{ m/s}}{3 \text{ s}} = (-2.1\hat{i} + 2.8\hat{j}) \text{ m/s}^2$$

97. (a) The magnitude of the displacement vector  $\Delta\vec{r}$  is given by

$$|\Delta\vec{r}| = \sqrt{(21.5 \text{ km})^2 + (9.7 \text{ km})^2 + (2.88 \text{ km})^2} = 23.8 \text{ km}.$$

Thus,

$$|\vec{v}_{\text{avg}}| = \frac{|\Delta\vec{r}|}{\Delta t} = \frac{23.8 \text{ km}}{3.50 \text{ h}} = 6.79 \text{ km/h}.$$

(b) The angle  $\theta$  in question is given by

$$\theta = \tan^{-1} \left( \frac{2.88 \text{ km}}{\sqrt{(21.5 \text{ km})^2 + (9.7 \text{ km})^2}} \right) = 6.96^\circ.$$

98. The initial velocity has magnitude  $v_0$  and because it is horizontal, it is equal to  $v_x$  the horizontal component of velocity at impact. Thus, the speed at impact is

$$\sqrt{v_0^2 + v_y^2} = 3v_0$$

where  $v_y = \sqrt{2gh}$  and we have used Eq. 2-16 with  $\Delta x$  replaced with  $h = 20$  m. Squaring both sides of the first equality and substituting from the second, we find

$$v_0^2 + 2gh = (3v_0)^2$$

which leads to  $gh = 4v_0^2$  and therefore to  $v_0 = \sqrt{(9.8 \text{ m/s}^2)(20 \text{ m})} / 2 = 7.0 \text{ m/s}$ .

99. We choose horizontal  $x$  and vertical  $y$  axes such that both components of  $\vec{v}_0$  are positive. Positive angles are counterclockwise from  $+x$  and negative angles are clockwise from it. In unit-vector notation, the velocity at each instant during the projectile motion is

$$\vec{v} = v_0 \cos \theta_0 \hat{i} + (v_0 \sin \theta_0 - gt) \hat{j}.$$

(a) With  $v_0 = 30$  m/s and  $\theta_0 = 60^\circ$ , we obtain  $\vec{v} = (15\hat{i} + 6.4\hat{j})$  m/s, for  $t = 2.0$  s. The magnitude of  $\vec{v}$  is  $|\vec{v}| = \sqrt{(15 \text{ m/s})^2 + (6.4 \text{ m/s})^2} = 16$  m/s.

(b) The direction of  $\vec{v}$  is

$$\theta = \tan^{-1}[(6.4 \text{ m/s})/(15 \text{ m/s})] = 23^\circ,$$

measured counterclockwise from  $+x$ .

(c) Since the angle is positive, it is above the horizontal.

(d) With  $t = 5.0$  s, we find  $\vec{v} = (15\hat{i} - 23\hat{j})$  m/s, which yields

$$|\vec{v}| = \sqrt{(15 \text{ m/s})^2 + (-23 \text{ m/s})^2} = 27 \text{ m/s}.$$

(e) The direction of  $\vec{v}$  is  $\theta = \tan^{-1}[(-23 \text{ m/s})/(15 \text{ m/s})] = -57^\circ$ , or  $57^\circ$  measured *clockwise* from  $+x$ .

(f) Since the angle is negative, it is below the horizontal.

100. The velocity of Larry is  $v_1$  and that of Curly is  $v_2$ . Also, we denote the length of the corridor by  $L$ . Now, Larry's time of passage is  $t_1 = 150$  s (which must equal  $L/v_1$ ), and Curly's time of passage is  $t_2 = 70$  s (which must equal  $L/v_2$ ). The time Moe takes is therefore

$$t = \frac{L}{v_1 + v_2} = \frac{1}{v_1/L + v_2/L} = \frac{1}{\frac{1}{150\text{s}} + \frac{1}{70\text{s}}} = 48\text{s}.$$

101. We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The coordinate origin is at the initial position for the football as it begins projectile motion in the sense of §4-5), and we let  $\theta_0$  be the angle of its initial velocity measured from the  $+x$  axis.

(a)  $x = 46$  m and  $y = -1.5$  m are the coordinates for the landing point; it lands at time  $t = 4.5$  s. Since  $x = v_{0x}t$ ,

$$v_{0x} = \frac{x}{t} = \frac{46 \text{ m}}{4.5 \text{ s}} = 10.2 \text{ m/s}.$$

Since  $y = v_{0y}t - \frac{1}{2}gt^2$ ,

$$v_{0y} = \frac{y + \frac{1}{2}gt^2}{t} = \frac{(-1.5 \text{ m}) + \frac{1}{2}(9.8 \text{ m/s}^2)(4.5 \text{ s})^2}{4.5 \text{ s}} = 21.7 \text{ m/s}.$$

The magnitude of the initial velocity is

$$v_0 = \sqrt{v_{0x}^2 + v_{0y}^2} = \sqrt{(10.2 \text{ m/s})^2 + (21.7 \text{ m/s})^2} = 24 \text{ m/s}.$$

(b) The initial angle satisfies  $\tan \theta_0 = v_{0y}/v_{0x}$ . Thus,  $\theta_0 = \tan^{-1} [(21.7 \text{ m/s})/(10.2 \text{ m/s})] = 65^\circ$ .

102. We assume the ball's initial velocity is perpendicular to the plane of the net. We choose coordinates so that  $(x_0, y_0) = (0, 3.0)$  m, and  $v_x > 0$  (note that  $v_{0y} = 0$ ).

(a) To (barely) clear the net, we have

$$y - y_0 = v_{0y}t - \frac{1}{2}gt^2 \Rightarrow 2.24 \text{ m} - 3.0 \text{ m} = 0 - \frac{1}{2}(9.8 \text{ m/s}^2)t^2$$

which gives  $t = 0.39$  s for the time it is passing over the net. This is plugged into the  $x$ -equation to yield the (minimum) initial velocity  $v_x = (8.0 \text{ m})/(0.39 \text{ s}) = 20.3 \text{ m/s}$ .

(b) We require  $y = 0$  and find  $t$  from  $y - y_0 = v_{0y}t - \frac{1}{2}gt^2$ . This value ( $t = \sqrt{2(3.0 \text{ m})/(9.8 \text{ m/s}^2)} = 0.78 \text{ s}$ ) is plugged into the  $x$ -equation to yield the (maximum) initial velocity  $v_x = (17.0 \text{ m})/(0.78 \text{ s}) = 21.7 \text{ m/s}$ .

103. (a) With  $\Delta x = 8.0 \text{ m}$ ,  $t = \Delta t_1$ ,  $a = a_x$ , and  $v_{0x} = 0$ , Eq. 2-15 gives

$$8.0 \text{ m} = \frac{1}{2} a_x (\Delta t_1)^2,$$

and the corresponding expression for motion along the  $y$  axis leads to

$$\Delta y = 12 \text{ m} = \frac{1}{2} a_y (\Delta t_1)^2.$$

Dividing the second expression by the first leads to  $a_y / a_x = 3/2 = 1.5$ .

(b) Letting  $t = 2\Delta t_1$ , then Eq. 2-15 leads to  $\Delta x = (8.0 \text{ m})(2)^2 = 32 \text{ m}$ , which implies that its  $x$  coordinate is now  $(4.0 + 32) \text{ m} = 36 \text{ m}$ . Similarly,  $\Delta y = (12 \text{ m})(2)^2 = 48 \text{ m}$ , which means its  $y$  coordinate has become  $(6.0 + 48) \text{ m} = 54 \text{ m}$ .

104. We apply Eq. 4-34 to solve for speed  $v$  and Eq. 4-35 to find the period  $T$ .

(a) We obtain

$$v = \sqrt{ra} = \sqrt{(5.0 \text{ m})(7.0)(9.8 \text{ m/s}^2)} = 19 \text{ m/s}.$$

(b) The time to go around once (the period) is  $T = 2\pi r/v = 1.7 \text{ s}$ . Therefore, in one minute ( $t = 60 \text{ s}$ ), the astronaut executes

$$\frac{t}{T} = \frac{60 \text{ s}}{1.7 \text{ s}} = 35$$

revolutions. Thus, 35 rev/min is needed to produce a centripetal acceleration of  $7g$  when the radius is  $5.0 \text{ m}$ .

(c) As noted above,  $T = 1.7 \text{ s}$ .

105. The radius of Earth may be found in Appendix C.

(a) The speed of an object at Earth's equator is  $v = 2\pi R/T$ , where  $R$  is the radius of Earth ( $6.37 \times 10^6$  m) and  $T$  is the length of a day ( $8.64 \times 10^4$  s):

$$v = 2\pi(6.37 \times 10^6 \text{ m})/(8.64 \times 10^4 \text{ s}) = 463 \text{ m/s}.$$

The magnitude of the acceleration is given by

$$a = \frac{v^2}{R} = \frac{(463 \text{ m/s})^2}{6.37 \times 10^6 \text{ m}} = 0.034 \text{ m/s}^2.$$

(b) If  $T$  is the period, then  $v = 2\pi R/T$  is the speed and the magnitude of the acceleration is

$$a = \frac{v^2}{R} = \frac{(2\pi R/T)^2}{R} = \frac{4\pi^2 R}{T^2}.$$

Thus,

$$T = 2\pi\sqrt{\frac{R}{a}} = 2\pi\sqrt{\frac{6.37 \times 10^6 \text{ m}}{9.8 \text{ m/s}^2}} = 5.1 \times 10^3 \text{ s} = 84 \text{ min}.$$

106. When the escalator is stalled the speed of the person is  $v_p = \ell/t$ , where  $\ell$  is the length of the escalator and  $t$  is the time the person takes to walk up it. This is  $v_p = (15 \text{ m})/(90 \text{ s}) = 0.167 \text{ m/s}$ . The escalator moves at  $v_e = (15 \text{ m})/(60 \text{ s}) = 0.250 \text{ m/s}$ . The speed of the person walking up the moving escalator is

$$v = v_p + v_e = 0.167 \text{ m/s} + 0.250 \text{ m/s} = 0.417 \text{ m/s}$$

and the time taken to move the length of the escalator is

$$t = \ell / v = (15 \text{ m}) / (0.417 \text{ m/s}) = 36 \text{ s}.$$

If the various times given are independent of the escalator length, then the answer does not depend on that length either. In terms of  $\ell$  (in meters) the speed (in meters per second) of the person walking on the stalled escalator is  $\ell/90$ , the speed of the moving escalator is  $\ell/60$ , and the speed of the person walking on the moving escalator is  $v = (\ell/90) + (\ell/60) = 0.0278\ell$ . The time taken is  $t = \ell/v = \ell/0.0278\ell = 36 \text{ s}$  and is independent of  $\ell$ .

107. (a) Eq. 2-15 can be applied to the vertical ( $y$  axis) motion related to reaching the maximum height (when  $t = 3.0$  s and  $v_y = 0$ ):

$$y_{\max} - y_0 = v_y t - \frac{1}{2} g t^2 .$$

With ground level chosen so  $y_0 = 0$ , this equation gives the result  $y_{\max} = \frac{1}{2} g (3.0 \text{ s})^2 = 44 \text{ m}$ .

(b) After the moment it reached maximum height, it is falling; at  $t = 2.5$  s, it will have fallen an amount given by Eq. 2-18:

$$y_{\text{fence}} - y_{\max} = (0)(2.5 \text{ s}) - \frac{1}{2} g (2.5 \text{ s})^2$$

which leads to  $y_{\text{fence}} = 13 \text{ m}$ .

(c) Either the *range* formula, Eq. 4-26, can be used or one can note that after passing the fence, it will strike the ground in 0.5 s (so that the total "fall-time" equals the "rise-time"). Since the horizontal component of velocity in a projectile-motion problem is constant (neglecting air friction), we find the original  $x$ -component from  $97.5 \text{ m} = v_{0x}(5.5 \text{ s})$  and then apply it to that final 0.5 s. Thus, we find  $v_{0x} = 17.7 \text{ m/s}$  and that after the fence

$$\Delta x = (17.7 \text{ m/s})(0.5 \text{ s}) = 8.9 \text{ m}.$$

108. With  $g_B = 9.8128 \text{ m/s}^2$  and  $g_M = 9.7999 \text{ m/s}^2$ , we apply Eq. 4-26:

$$R_M - R_B = \frac{v_0^2 \sin 2\theta_0}{g_M} - \frac{v_0^2 \sin 2\theta_0}{g_B} = \frac{v_0^2 \sin 2\theta_0}{g_B} \left( \frac{g_B}{g_M} - 1 \right)$$

which becomes

$$R_M - R_B = R_B \left( \frac{9.8128 \text{ m/s}^2}{9.7999 \text{ m/s}^2} - 1 \right)$$

and yields (upon substituting  $R_B = 8.09 \text{ m}$ )  $R_M - R_B = 0.01 \text{ m} = 1 \text{ cm}$ .

109. We make use of Eq. 4-25.

(a) By rearranging Eq. 4-25, we obtain the initial speed:

$$v_0 = \frac{x}{\cos \theta_0} \sqrt{\frac{g}{2(x \tan \theta_0 - y)}}$$

which yields  $v_0 = 255.5 \approx 2.6 \times 10^2$  m/s for  $x = 9400$  m,  $y = -3300$  m, and  $\theta_0 = 35^\circ$ .

(b) From Eq. 4-21, we obtain the time of flight:

$$t = \frac{x}{v_0 \cos \theta_0} = \frac{9400 \text{ m}}{(255.5 \text{ m/s}) \cos 35^\circ} = 45 \text{ s.}$$

(c) We expect the air to provide resistance but no appreciable lift to the rock, so we would need a greater launching speed to reach the same target.

110. When moving in the same direction as the jet stream (of speed  $v_s$ ), the time is

$$t_1 = \frac{d}{v_{ja} + v_s},$$

where  $d = 4000$  km is the distance and  $v_{ja}$  is the speed of the jet relative to the air (1000 km/h). When moving against the jet stream, the time is

$$t_2 = \frac{d}{v_{ja} - v_s},$$

where  $t_2 - t_1 = \frac{70}{60}$  h. Combining these equations and using the quadratic formula to solve gives  $v_s = 143$  km/h.

111. Since the  $x$  and  $y$  components of the acceleration are constants, we can use Table 2-1 for the motion along both axes. This can be handled individually (for  $\Delta x$  and  $\Delta y$ ) or together with the unit-vector notation (for  $\Delta r$ ). Where units are not shown, SI units are to be understood.

(a) Since  $\vec{r}_0 = 0$ , the position vector of the particle is (adapting Eq. 2-15)

$$\vec{r} = \vec{v}_0 t + \frac{1}{2} \vec{a} t^2 = (8.0 \hat{j}) t + \frac{1}{2} (4.0 \hat{i} + 2.0 \hat{j}) t^2 = (2.0 t^2) \hat{i} + (8.0 t + 1.0 t^2) \hat{j}.$$

Therefore, we find when  $x = 29$  m, by solving  $2.0 t^2 = 29$ , which leads to  $t = 3.8$  s. The  $y$  coordinate at that time is  $y = (8.0 \text{ m/s})(3.8 \text{ s}) + (1.0 \text{ m/s}^2)(3.8 \text{ s})^2 = 45$  m.

(b) Adapting Eq. 2-11, the velocity of the particle is given by

$$\vec{v} = \vec{v}_0 + \vec{a} t.$$

Thus, at  $t = 3.8$  s, the velocity is

$$\vec{v} = (8.0 \text{ m/s}) \hat{j} + ((4.0 \text{ m/s}^2) \hat{i} + (2.0 \text{ m/s}^2) \hat{j})(3.8 \text{ s}) = (15.2 \text{ m/s}) \hat{i} + (15.6 \text{ m/s}) \hat{j}$$

which has a magnitude of

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(15.2 \text{ m/s})^2 + (15.6 \text{ m/s})^2} = 22 \text{ m/s}.$$

112. We make use of Eq. 4-34 and Eq. 4-35.

(a) The track radius is given by

$$r = \frac{v^2}{a} = \frac{(9.2 \text{ m/s})^2}{3.8 \text{ m/s}^2} = 22 \text{ m}.$$

(b) The period of the circular motion is  $T = 2\pi(22 \text{ m})/(9.2 \text{ m/s}) = 15 \text{ s}$ .

113. Since this problem involves constant downward acceleration of magnitude  $a$ , similar to the projectile motion situation, we use the equations of §4-6 as long as we substitute  $a$  for  $g$ . We adopt the positive direction choices used in the textbook so that equations such as Eq. 4-22 are directly applicable. The initial velocity is horizontal so that  $v_{0y} = 0$  and

$$v_{0x} = v_0 = 1.00 \times 10^9 \text{ cm/s.}$$

(a) If  $\ell$  is the length of a plate and  $t$  is the time an electron is between the plates, then  $\ell = v_0 t$ , where  $v_0$  is the initial speed. Thus

$$t = \frac{\ell}{v_0} = \frac{2.00 \text{ cm}}{1.00 \times 10^9 \text{ cm/s}} = 2.00 \times 10^{-9} \text{ s.}$$

(b) The vertical displacement of the electron is

$$y = -\frac{1}{2} a t^2 = -\frac{1}{2} (1.00 \times 10^{17} \text{ cm/s}^2) (2.00 \times 10^{-9} \text{ s})^2 = -0.20 \text{ cm} = -2.00 \text{ mm,}$$

or  $|y| = 2.00 \text{ mm}$ .

(c) The  $x$  component of velocity does not change:  $v_x = v_0 = 1.00 \times 10^9 \text{ cm/s} = 1.00 \times 10^7 \text{ m/s}$ .

(d) The  $y$  component of the velocity is

$$v_y = a_y t = (1.00 \times 10^{17} \text{ cm/s}^2) (2.00 \times 10^{-9} \text{ s}) = 2.00 \times 10^8 \text{ cm/s} = 2.00 \times 10^6 \text{ m/s.}$$

114. We neglect air resistance, which justifies setting  $a = -g = -9.8 \text{ m/s}^2$  (taking *down* as the  $-y$  direction) for the duration of the motion of the shot ball. We are allowed to use Table 2-1 (with  $\Delta y$  replacing  $\Delta x$ ) because the ball has constant acceleration motion. We use primed variables (except  $t$ ) with the constant-velocity elevator (so  $v' = 10 \text{ m/s}$ ), and unprimed variables with the ball (with initial velocity  $v_0 = v' + 20 = 30 \text{ m/s}$ , relative to the ground). SI units are used throughout.

(a) Taking the time to be zero at the instant the ball is shot, we compute its maximum height  $y$  (relative to the ground) with  $v^2 = v_0^2 - 2g(y - y_0)$ , where the highest point is characterized by  $v = 0$ . Thus,

$$y = y_0 + \frac{v_0^2}{2g} = 76 \text{ m}$$

where  $y_0 = y'_0 + 2 = 30 \text{ m}$  (where  $y'_0 = 28 \text{ m}$  is given in the problem) and  $v_0 = 30 \text{ m/s}$  relative to the ground as noted above.

(b) There are a variety of approaches to this question. One is to continue working in the frame of reference adopted in part (a) (which treats the ground as motionless and “fixes” the coordinate origin to it); in this case, one describes the elevator motion with  $y' = y'_0 + v't$  and the ball motion with Eq. 2-15, and solves them for the case where they reach the same point at the same time. Another is to work in the frame of reference of the elevator (the boy in the elevator might be oblivious to the fact the elevator is moving since it isn't accelerating), which is what we show here in detail:

$$\Delta y_e = v_{0e}t - \frac{1}{2}gt^2 \quad \Rightarrow \quad t = \frac{v_{0e} + \sqrt{v_{0e}^2 - 2g\Delta y_e}}{g}$$

where  $v_{0e} = 20 \text{ m/s}$  is the initial velocity of the ball relative to the elevator and  $\Delta y_e = -2.0 \text{ m}$  is the ball's displacement relative to the floor of the elevator. The positive root is chosen to yield a positive value for  $t$ ; the result is  $t = 4.2 \text{ s}$ .

115. (a) With  $v = c/10 = 3 \times 10^7$  m/s and  $a = 20g = 196$  m/s<sup>2</sup>, Eq. 4-34 gives

$$r = v^2 / a = 4.6 \times 10^{12} \text{ m.}$$

(b) The period is given by Eq. 4-35:  $T = 2\pi r / v = 9.6 \times 10^5$  s. Thus, the time to make a quarter-turn is  $T/4 = 2.4 \times 10^5$  s or about 2.8 days.

116. Using the same coordinate system assumed in Eq. 4-25, we rearrange that equation to solve for the initial speed:

$$v_0 = \frac{x}{\cos \theta_0} \sqrt{\frac{g}{2(x \tan \theta_0 - y)}}$$

which yields  $v_0 = 23$  ft/s for  $g = 32$  ft/s<sup>2</sup>,  $x = 13$  ft,  $y = 3$  ft and  $\theta_0 = 55^\circ$ .

117. The (box)car has velocity  $\vec{v}_{cg} = v_1 \hat{i}$  relative to the ground, and the bullet has velocity

$$\vec{v}_{0bg} = v_2 \cos\theta \hat{i} + v_2 \sin\theta \hat{j}$$

relative to the ground before entering the car (we are neglecting the effects of gravity on the bullet). While in the car, its velocity relative to the outside ground is  $\vec{v}_{bg} = 0.8v_2 \cos\theta \hat{i} + 0.8v_2 \sin\theta \hat{j}$  (due to the 20% reduction mentioned in the problem). The problem indicates that the velocity of the bullet in the car *relative to the car* is (with  $v_3$  unspecified)  $\vec{v}_{bc} = v_3 \hat{j}$ . Now, Eq. 4-44 provides the condition

$$\begin{aligned} \vec{v}_{bg} &= \vec{v}_{bc} + \vec{v}_{cg} \\ 0.8v_2 \cos\theta \hat{i} + 0.8v_2 \sin\theta \hat{j} &= v_3 \hat{j} + v_1 \hat{i} \end{aligned}$$

so that equating  $x$  components allows us to find  $\theta$ . If one wished to find  $v_3$  one could also equate the  $y$  components, and from this, if the car width were given, one could find the time spent by the bullet in the car, but this information is not asked for (which is why the width is irrelevant). Therefore, examining the  $x$  components in SI units leads to

$$\theta = \cos^{-1}\left(\frac{v_1}{0.8v_2}\right) = \cos^{-1}\left(\frac{85 \text{ km/h} \left(\frac{1000 \text{ m/km}}{3600 \text{ s/h}}\right)}{0.8 (650 \text{ m/s})}\right)$$

which yields  $87^\circ$  for the direction of  $\vec{v}_{bg}$  (measured from  $\hat{i}$ , which is the direction of motion of the car). The problem asks, “from what direction was it fired?” — which means the answer is not  $87^\circ$  but rather its supplement  $93^\circ$  (measured from the direction of motion). Stating this more carefully, in the coordinate system we have adopted in our solution, the bullet velocity vector is in the first quadrant, at  $87^\circ$  measured counterclockwise from the  $+x$  direction (the direction of train motion), which means that the direction from which the bullet came (where the sniper is) is in the third quadrant, at  $-93^\circ$  (that is,  $93^\circ$  measured clockwise from  $+x$ ).

118. Since  $v_y^2 = v_{0y}^2 - 2g\Delta y$ , and  $v_y=0$  at the target, we obtain

$$v_{0y} = \sqrt{2(9.80 \text{ m/s}^2)(5.00 \text{ m})} = 9.90 \text{ m/s}$$

(a) Since  $v_0 \sin \theta_0 = v_{0y}$ , with  $v_0 = 12.0 \text{ m/s}$ , we find  $\theta_0 = 55.6^\circ$ .

(b) Now,  $v_y = v_{0y} - gt$  gives  $t = (9.90 \text{ m/s})/(9.80 \text{ m/s}^2) = 1.01 \text{ s}$ . Thus,  $\Delta x = (v_0 \cos \theta_0)t = 6.85 \text{ m}$ .

(c) The velocity at the target has only the  $v_x$  component, which is equal to  $v_{0x} = v_0 \cos \theta_0 = 6.78 \text{ m/s}$ .

119. From the figure, the three displacements can be written as

$$\vec{d}_1 = d_1(\cos\theta_1\hat{i} + \sin\theta_1\hat{j}) = (5.00 \text{ m})(\cos 30^\circ\hat{i} + \sin 30^\circ\hat{j}) = (4.33 \text{ m})\hat{i} + (2.50 \text{ m})\hat{j}$$

$$\begin{aligned}\vec{d}_2 &= d_2[\cos(180^\circ + \theta_1 - \theta_2)\hat{i} + \sin(180^\circ + \theta_1 - \theta_2)\hat{j}] = (8.00 \text{ m})(\cos 160^\circ\hat{i} + \sin 160^\circ\hat{j}) \\ &= (-7.52 \text{ m})\hat{i} + (2.74 \text{ m})\hat{j}\end{aligned}$$

$$\begin{aligned}\vec{d}_3 &= d_3[\cos(360^\circ - \theta_3 - \theta_2 + \theta_1)\hat{i} + \sin(360^\circ - \theta_3 - \theta_2 + \theta_1)\hat{j}] = (12.0 \text{ m})(\cos 260^\circ\hat{i} + \sin 260^\circ\hat{j}) \\ &= (-2.08 \text{ m})\hat{i} - (11.8 \text{ m})\hat{j}\end{aligned}$$

where the angles are measured from the  $+x$  axis. The net displacement is

$$\vec{d} = \vec{d}_1 + \vec{d}_2 + \vec{d}_3 = (-5.27 \text{ m})\hat{i} - (6.58 \text{ m})\hat{j}.$$

(a) The magnitude of the net displacement is

$$|\vec{d}| = \sqrt{(-5.27 \text{ m})^2 + (-6.58 \text{ m})^2} = 8.43 \text{ m}.$$

(b) The direction of  $\vec{d}$  is

$$\theta = \tan^{-1}\left(\frac{d_y}{d_x}\right) = \tan^{-1}\left(\frac{-6.58 \text{ m}}{-5.27 \text{ m}}\right) = 51.3^\circ \text{ or } 231^\circ.$$

We choose  $231^\circ$  (measured counterclockwise from  $+x$ ) since the desired angle is in the third quadrant. An equivalent answer is  $-129^\circ$  (measured clockwise from  $+x$ ).

120. With  $v_0 = 30.0$  m/s and  $R = 20.0$  m, Eq. 4-26 gives

$$\sin 2\theta_0 = \frac{gR}{v_0^2} = 0.218.$$

Because  $\sin \phi = \sin (180^\circ - \phi)$ , there are two roots of the above equation:

$$2\theta_0 = \sin^{-1}(0.218) = 12.58^\circ \text{ and } 167.4^\circ.$$

which correspond to the two possible launch angles that will hit the target (in the absence of air friction and related effects).

(a) The smallest angle is  $\theta_0 = 6.29^\circ$ .

(b) The greatest angle is and  $\theta_0 = 83.7^\circ$ .

An alternative approach to this problem in terms of Eq. 4-25 (with  $y = 0$  and  $1/\cos^2 = 1 + \tan^2$ ) is possible — and leads to a quadratic equation for  $\tan\theta_0$  with the roots providing these two possible  $\theta_0$  values.

121. On the one hand, we could perform the vector addition of the displacements with a vector-capable calculator in polar mode  $((75 \angle 37^\circ) + (65 \angle -90^\circ) = (63 \angle -18^\circ))$ , but in keeping with Eq. 3-5 and Eq. 3-6 we will show the details in unit-vector notation. We use a 'standard' coordinate system with  $+x$  East and  $+y$  North. Lengths are in kilometers and times are in hours.

(a) We perform the vector addition of individual displacements to find the net displacement of the camel.

$$\begin{aligned}\Delta\vec{r}_1 &= (75 \text{ km})\cos(37^\circ)\hat{i} + (75 \text{ km})\sin(37^\circ)\hat{j} \\ \Delta\vec{r}_2 &= (-65 \text{ km})\hat{j} \\ \Delta\vec{r} &= \Delta\vec{r}_1 + \Delta\vec{r}_2 = (60 \text{ km})\hat{i} - (20 \text{ km})\hat{j} .\end{aligned}$$

If it is desired to express this in magnitude-angle notation, then this is equivalent to a vector of length  $|\Delta\vec{r}| = \sqrt{(60 \text{ km})^2 + (-20 \text{ km})^2} = 63 \text{ km}$  .

(b) The direction of  $\Delta\vec{r}$  is  $\theta = \tan^{-1}[(-20 \text{ km})/(60 \text{ km})] = -18^\circ$  , or  $18^\circ$  south of east.

(c) We use the result from part (a) in Eq. 4-8 along with the fact that  $\Delta t = 90 \text{ h}$ . In unit vector notation, we obtain

$$\vec{v}_{\text{avg}} = \frac{(60\hat{i} - 20\hat{j}) \text{ km}}{90 \text{ h}} = (0.67\hat{i} - 0.22\hat{j}) \text{ km/h}.$$

This leads to  $|\vec{v}_{\text{avg}}| = 0.70 \text{ km/h}$ .

(d) The direction of  $\vec{v}_{\text{avg}}$  is  $\theta = \tan^{-1}[(-0.22 \text{ km/h})/(0.67 \text{ km/h})] = -18^\circ$  , or  $18^\circ$  south of east.

(e) The average speed is distinguished from the magnitude of average velocity in that it depends on the total distance as opposed to the net displacement. Since the camel travels 140 km, we obtain  $(140 \text{ km})/(90 \text{ h}) = 1.56 \text{ km/h} \approx 1.6 \text{ km/h}$  .

(f) The net displacement is required to be the 90 km East from  $A$  to  $B$ . The displacement from the resting place to  $B$  is denoted  $\Delta\vec{r}_3$ . Thus, we must have

$$\Delta\vec{r}_1 + \Delta\vec{r}_2 + \Delta\vec{r}_3 = (90 \text{ km})\hat{i}$$

which produces  $\Delta\vec{r}_3 = (30 \text{ km})\hat{i} + (20 \text{ km})\hat{j}$  in unit-vector notation, or  $(36 \angle 33^\circ)$  in magnitude-angle notation. Therefore, using Eq. 4-8 we obtain

$$|\vec{v}_{\text{avg}}| = \frac{36 \text{ km}}{(120 - 90) \text{ h}} = 1.2 \text{ km/h}.$$

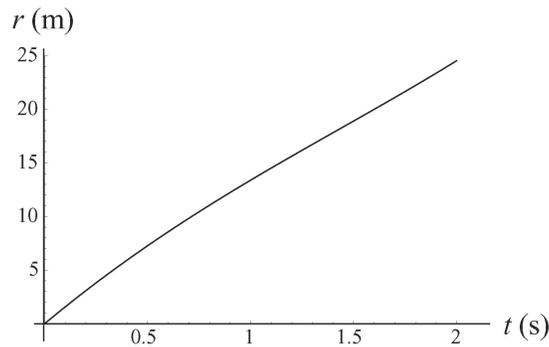
(g) The direction of  $\vec{v}_{\text{avg}}$  is the same as  $\vec{r}_3$  (that is,  $33^\circ$  north of east).

122. We make use of Eq. 4-21 and Eq.4-22.

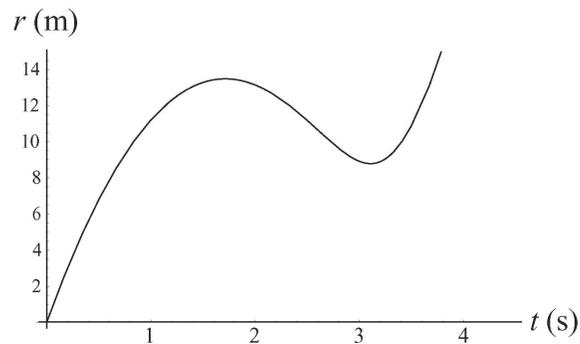
(a) With  $v_0 = 16$  m/s, we square Eq. 4-21 and Eq. 4-22 and add them, then (using Pythagoras' theorem) take the square root to obtain  $r$ :

$$\begin{aligned} r &= \sqrt{(x-x_0)^2 + (y-y_0)^2} = \sqrt{(v_0 \cos \theta_0 t)^2 + (v_0 \sin \theta_0 t - gt^2/2)^2} \\ &= t \sqrt{v_0^2 - v_0 g \sin \theta_0 t + g^2 t^2 / 4} \end{aligned}$$

Below we plot  $r$  as a function of time for  $\theta_0 = 40.0^\circ$ :



(b) For this next graph for  $r$  versus  $t$  we set  $\theta_0 = 80.0^\circ$ .



(c) Differentiating  $r$  with respect to  $t$ , we obtain

$$\frac{dr}{dt} = \frac{v_0^2 - 3v_0 g t \sin \theta_0 / 2 + g^2 t^2 / 2}{\sqrt{v_0^2 - v_0 g \sin \theta_0 t + g^2 t^2 / 4}}$$

Setting  $dr/dt = 0$ , with  $v_0 = 16.0$  m/s and  $\theta_0 = 40.0^\circ$ , we have  $256 - 151t + 48t^2 = 0$ . The equation has no real solution. This means that the maximum is reached at the end of the flight, with

$$t_{total} = 2v_0 \sin \theta_0 / g = 2(16.0 \text{ m/s}) \sin(40.0^\circ) / (9.80 \text{ m/s}^2) = 2.10 \text{ s.}$$

(d) The value of  $r$  is given by

$$r = (2.10)\sqrt{(16.0)^2 - (16.0)(9.80)\sin 40.0^\circ(2.10) + (9.80)^2(2.10)^2 / 4} = 25.7 \text{ m.}$$

(e) The horizontal distance is  $r_x = v_0 \cos \theta_0 t = (16.0 \text{ m/s}) \cos 40.0^\circ(2.10 \text{ s}) = 25.7 \text{ m.}$

(f) The vertical distance is  $r_y = 0.$

(g) For the  $\theta_0 = 80^\circ$  launch, the condition for maximum  $r$  is  $256 - 232t + 48t^2 = 0$ , or  $t = 1.71 \text{ s}$  (the other solution,  $t = 3.13 \text{ s}$ , corresponds to a minimum.)

(h) The distance traveled is

$$r = (1.71)\sqrt{(16.0)^2 - (16.0)(9.80)\sin 80.0^\circ(1.71) + (9.80)^2(1.71)^2 / 4} = 13.5 \text{ m.}$$

(i) The horizontal distance is

$$r_x = v_0 \cos \theta_0 t = (16.0 \text{ m/s}) \cos 80.0^\circ(1.71 \text{ s}) = 4.75 \text{ m.}$$

(j) The vertical distance is

$$r_y = v_0 \sin \theta_0 t - \frac{gt^2}{2} = (16.0 \text{ m/s}) \sin 80^\circ(1.71 \text{ s}) - \frac{(9.80 \text{ m/s}^2)(1.71 \text{ s})^2}{2} = 12.6 \text{ m.}$$

123. Using the same coordinate system assumed in Eq. 4-25, we find  $x$  for the elevated cannon from

$$y = x \tan \theta_0 - \frac{gx^2}{2(v_0 \cos \theta_0)^2} \quad \text{where } y = -30 \text{ m.}$$

Using the quadratic formula (choosing the positive root), we find

$$x = v_0 \cos \theta_0 \left( \frac{v_0 \sin \theta_0 + \sqrt{(v_0 \sin \theta_0)^2 - 2gy}}{g} \right)$$

which yields  $x = 715 \text{ m}$  for  $v_0 = 82 \text{ m/s}$  and  $\theta_0 = 45^\circ$ . This is 29 m longer than the 686 m found in that Sample Problem. Since the “9” in 29 m is not reliable, due to the low level of precision in the given data, we write the answer as  $3 \times 10^1 \text{ m}$ .

124. (a) Using the same coordinate system assumed in Eq. 4-25, we find

$$y = x \tan \theta_0 - \frac{gx^2}{2(v_0 \cos \theta_0)^2} = -\frac{gx^2}{2v_0^2} \quad \text{if } \theta_0 = 0.$$

Thus, with  $v_0 = 3.0 \times 10^6$  m/s and  $x = 1.0$  m, we obtain  $y = -5.4 \times 10^{-13}$  m which is not practical to measure (and suggests why gravitational processes play such a small role in the fields of atomic and subatomic physics).

(b) It is clear from the above expression that  $|y|$  decreases as  $v_0$  is increased.

125. At maximum height, the  $y$ -component of a projectile's velocity vanishes, so the given 10 m/s is the (constant)  $x$ -component of velocity.

(a) Using  $v_{0y}$  to denote the  $y$ -velocity 1.0 s before reaching the maximum height, then (with  $v_y = 0$ ) the equation  $v_y = v_{0y} - gt$  leads to  $v_{0y} = 9.8$  m/s. The magnitude of the velocity vector (or *speed*) at that moment is therefore

$$\sqrt{v_x^2 + v_{0y}^2} = \sqrt{(10 \text{ m/s})^2 + (9.8 \text{ m/s})^2} = 14 \text{ m/s}.$$

(b) It is clear from the symmetry of the problem that the speed is the same 1.0 s after reaching the top, as it was 1.0 s before (14 m/s again). This may be verified by using  $v_y = v_{0y} - gt$  again but now "starting the clock" at the highest point so that  $v_{0y} = 0$  (and  $t = 1.0$  s). This leads to  $v_y = -9.8$  m/s and  $\sqrt{(10 \text{ m/s})^2 + (-9.8 \text{ m/s})^2} = 14$  m/s.

(c) The  $x_0$  value may be obtained from  $x = 0 = x_0 + (10 \text{ m/s})(1.0\text{s})$ , which yields  $x_0 = -10$  m.

(d) With  $v_{0y} = 9.8$  m/s denoting the  $y$ -component of velocity one second before the top of the trajectory, then we have  $y = 0 = y_0 + v_{0y}t - \frac{1}{2}gt^2$  where  $t = 1.0$  s. This yields  $y_0 = -4.9$  m.

(e) By using  $x - x_0 = (10 \text{ m/s})(1.0 \text{ s})$  where  $x_0 = 0$ , we obtain  $x = 10$  m.

(f) Let  $t = 0$  at the top with  $y_0 = v_{0y} = 0$ . From  $y - y_0 = v_{0y}t - \frac{1}{2}gt^2$ , we have, for  $t = 1.0$  s,

$$y = -(9.8 \text{ m/s}^2)(1.0 \text{ s})^2 / 2 = -4.9 \text{ m}.$$

126. With no acceleration in the  $x$  direction yet a constant acceleration of  $1.4 \text{ m/s}^2$  in the  $y$  direction, the position (in meters) as a function of time (in seconds) must be

$$\vec{r} = (6.0t)\hat{i} + \left(\frac{1}{2}(1.4)t^2\right)\hat{j}$$

and  $\vec{v}$  is its derivative with respect to  $t$ .

(a) At  $t = 3.0 \text{ s}$ , therefore,  $\vec{v} = (6.0\hat{i} + 4.2\hat{j}) \text{ m/s}$ .

(b) At  $t = 3.0 \text{ s}$ , the position is  $\vec{r} = (18\hat{i} + 6.3\hat{j}) \text{ m}$ .

127. We note that

$$\vec{v}_{PG} = \vec{v}_{PA} + \vec{v}_{AG}$$

describes a right triangle, with one leg being  $\vec{v}_{PG}$  (east), another leg being  $\vec{v}_{AG}$  (magnitude = 20, direction = south), and the hypotenuse being  $\vec{v}_{PA}$  (magnitude = 70). Lengths are in kilometers and time is in hours. Using the Pythagorean theorem, we have

$$|\vec{v}_{PA}| = \sqrt{|\vec{v}_{PG}|^2 + |\vec{v}_{AG}|^2} \Rightarrow 70 \text{ km/h} = \sqrt{|\vec{v}_{PG}|^2 + (20 \text{ km/h})^2}$$

which is easily solved for the ground speed:  $|\vec{v}_{PG}| = 67 \text{ km/h}$ .

128. The figure offers many interesting points to analyze, and others are easily inferred (such as the point of maximum height). The focus here, to begin with, will be the final point shown (1.25 s after the ball is released) which is when the ball returns to its original height. In English units,  $g = 32 \text{ ft/s}^2$ .

(a) Using  $x - x_0 = v_x t$  we obtain  $v_x = (40 \text{ ft})/(1.25 \text{ s}) = 32 \text{ ft/s}$ . And  $y - y_0 = 0 = v_{0y} t - \frac{1}{2} g t^2$  yields  $v_{0y} = \frac{1}{2} (32 \text{ ft/s}^2)(1.25 \text{ s}) = 20 \text{ ft/s}$ . Thus, the initial speed is

$$v_0 = |\vec{v}_0| = \sqrt{(32 \text{ ft/s})^2 + (20 \text{ ft/s})^2} = 38 \text{ ft/s}.$$

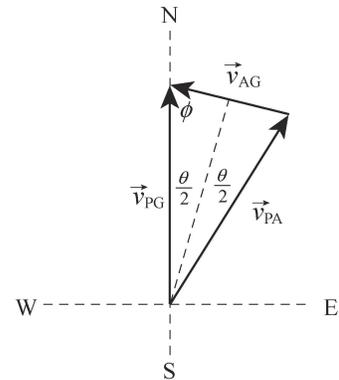
(b) Since  $v_y = 0$  at the maximum height and the horizontal velocity stays constant, then the speed at the top is the same as  $v_x = 32 \text{ ft/s}$ .

(c) We can infer from the figure (or compute from  $v_y = 0 = v_{0y} - g t$ ) that the time to reach the top is 0.625 s. With this, we can use  $y - y_0 = v_{0y} t - \frac{1}{2} g t^2$  to obtain 9.3 ft (where  $y_0 = 3 \text{ ft}$  has been used). An alternative approach is to use  $v_y^2 = v_{0y}^2 - 2g(y - y_0)$ .

129. We denote  $\vec{v}_{PG}$  as the velocity of the plane relative to the ground,  $\vec{v}_{AG}$  as the velocity of the air relative to the ground, and  $\vec{v}_{PA}$  as the velocity of the plane relative to the air.

(a) The vector diagram is shown on the right:  $\vec{v}_{PG} = \vec{v}_{PA} + \vec{v}_{AG}$ . Since the magnitudes  $v_{PG}$  and  $v_{PA}$  are equal the triangle is isosceles, with two sides of equal length.

Consider either of the right triangles formed when the bisector of  $\theta$  is drawn (the dashed line). It bisects  $\vec{v}_{AG}$ , so



$$\sin(\theta/2) = \frac{v_{AG}}{2v_{PG}} = \frac{70.0 \text{ mi/h}}{2(135 \text{ mi/h})}$$

which leads to  $\theta = 30.1^\circ$ . Now  $\vec{v}_{AG}$  makes the same angle with the E-W line as the dashed line does with the N-S line. The wind is blowing in the direction  $15.0^\circ$  north of west. Thus, it is blowing *from*  $75.0^\circ$  east of south.

(b) The plane is headed along  $\vec{v}_{PA}$ , in the direction  $30.0^\circ$  east of north. There is another solution, with the plane headed  $30.0^\circ$  west of north and the wind blowing  $15^\circ$  north of east (that is, from  $75^\circ$  west of south).

130. Taking derivatives of  $\vec{r} = 2t\hat{i} + 2\sin(\pi t/4)\hat{j}$  (with lengths in meters, time in seconds and angles in radians) provides expressions for velocity and acceleration:

$$\vec{v} = \frac{d\vec{r}}{dt} = 2\hat{i} + \frac{\pi}{2}\cos\left(\frac{\pi t}{4}\right)\hat{j}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = -\frac{\pi^2}{8}\sin\left(\frac{\pi t}{4}\right)\hat{j}.$$

Thus, we obtain:

time $t$			0.0	1.0	2.0	3.0	4.0
(a)	$\vec{r}$ position	$x$	0.0	2.0	4.0	6.0	8.0
		$y$	0.0	1.4	2.0	1.4	0.0
(b)	$\vec{v}$ velocity	$v_x$		2.0	2.0	2.0	
		$v_y$		1.1	0.0	-1.1	
(c)	$\vec{a}$ acceleration	$a_x$		0.0	0.0	0.0	
		$a_y$		-0.87	-1.2	-0.87	

And the path of the particle in the  $xy$  plane is shown in the following graph. The arrows indicating the velocities are not shown here, but they would appear as tangent-lines, as expected.

131. We make use of Eq. 4-24 and Eq. 4-25.

(a) With  $x = 180$  m,  $\theta_0 = 30^\circ$ , and  $v_0 = 43$  m/s, we obtain

$$y = \tan(30^\circ)(180 \text{ m}) - \frac{(9.8 \text{ m/s}^2)(180 \text{ m})^2}{2(43 \text{ m/s})^2(\cos 30^\circ)^2} = -11 \text{ m}$$

or  $|y| = 11$  m. This implies the rise is roughly eleven meters above the fairway.

(b) The horizontal component (in the absence of air friction) is unchanged, but the vertical component increases (see Eq. 4-24). The Pythagorean theorem then gives the magnitude of final velocity (right before striking the ground): 45 m/s.

132. We let  $g_p$  denote the magnitude of the gravitational acceleration on the planet. A number of the points on the graph (including some “inferred” points — such as the max height point at  $x = 12.5$  m and  $t = 1.25$  s) can be analyzed profitably; for future reference, we label (with subscripts) the first  $((x_0, y_0) = (0, 2)$  at  $t_0 = 0$ ) and last (“final”) points  $((x_f, y_f) = (25, 2)$  at  $t_f = 2.5$ ), with lengths in meters and time in seconds.

(a) The  $x$ -component of the initial velocity is found from  $x_f - x_0 = v_{0x} t_f$ . Therefore,  $v_{0x} = 25/2.5 = 10$  m/s. And we try to obtain the  $y$ -component from  $y_f - y_0 = 0 = v_{0y} t_f - \frac{1}{2} g_p t_f^2$ . This gives us  $v_{0y} = 1.25 g_p$ , and we see we need another equation (by analyzing another point, say, the next-to-last one)  $y - y_0 = v_{0y} t - \frac{1}{2} g_p t^2$  with  $y = 6$  and  $t = 2$ ; this produces our second equation  $v_{0y} = 2 + g_p$ . Simultaneous solution of these two equations produces results for  $v_{0y}$  and  $g_p$  (relevant to part (b)). Thus, our complete answer for the initial velocity is  $\vec{v} = (10 \text{ m/s})\hat{i} + (10 \text{ m/s})\hat{j}$ .

(b) As a by-product of the part (a) computations, we have  $g_p = 8.0 \text{ m/s}^2$ .

(c) Solving for  $t_g$  (the time to reach the ground) in  $y_g = 0 = y_0 + v_{0y} t_g - \frac{1}{2} g_p t_g^2$  leads to a positive answer:  $t_g = 2.7$  s.

(d) With  $g = 9.8 \text{ m/s}^2$ , the method employed in part (c) would produce the quadratic equation  $-4.9 t_g^2 + 10 t_g + 2 = 0$  and then the positive result  $t_g = 2.2$  s.