

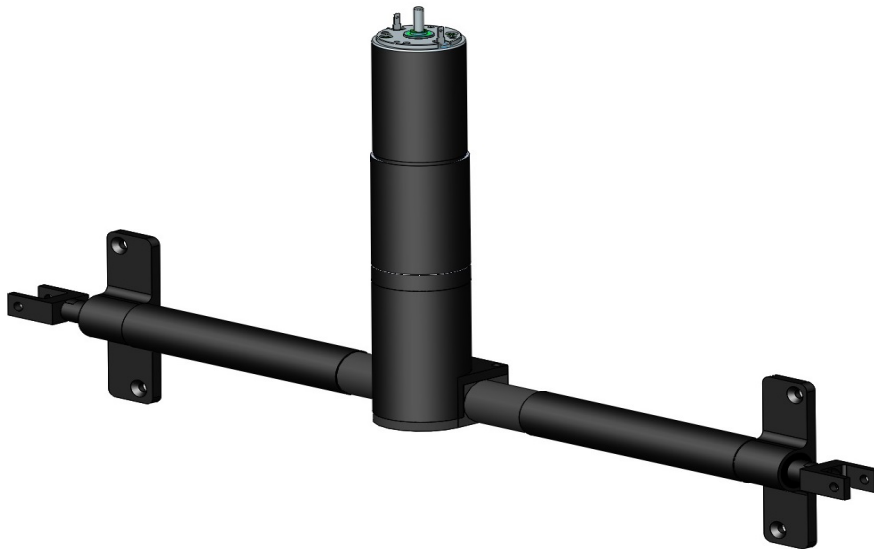


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# Mechanisms for rear wheel steering on a Formula Student car

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**Examensarbete TRITA-ITM-EX 2018:89**

**Mekanismer för bakhjulsstyrning av en Formula  
Student-bil**

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## Sammanfattning

I detta arbete undersöktes kraven som ställs på ett bakhjulsstyrningssystem för implementering på KTH:s Formula Student bil. Kraven togs fram genom att använda redan kända laster, en analys av hur snabbt föraren svänger utifrån videomaterial samt regelboken som alla formulastudentbilar är konstruerade efter. Efter att kraven tagits fram utvärderades ett antal koncept där det bästa konceptet, kuggstångsmekanism med ett roterande ställdon utvecklades vidare.

Prototypdesignen gjordes genom att först välja ett ställdon med planetväxellåda som skulle klara av kraven och därefter med hjälp av CAD skapa en integrerbar design. Det färdiga konceptet väger ca 1,7 kg och klarar alla ställda krav.

**Nyckelord:** Bakhjulsstyrning, Formula Student, fyrhjulstyrning, mekanism



BachelorThesis TRITA-ITM-EX 2018:89

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# Abstract

In this thesis the requirements for a rear wheel steering mechanism aimed to be implemented on KTH:s Formula Student car were evaluated. The requirements were obtained by using already known loads, an analysis of how the quickly the driver turns the steering wheel during track driving from video material and the Formula Student rulebook which the car is designed after. After the requirements were produced a number of concepts were produced and evaluated against each other. The concept that was given the highest score was a rack and pinion concept with a rotary actuator which was developed further.

The design was made by first selecting an actuator with a planetary gearbox that could fulfill the speed and load requirements and afterwards with CAD a design that could be integrated on the car was made. The concept weighed around 1,7 kg and was compatible with the requirements.

**Keywords:** Formula Student, Four wheel steering, mechanism, Rear wheel steering



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# Chapter 1

## Introduction

*In this chapter the Formula Student competition, the KTH team and its car are introduced. Furthermore, the general concept of four wheel steering is discussed and the purpose of the thesis, its limitations and the overall methodology are also presented.*

### 1.1 Background

#### Formula Student

Formula Student is an engineering competition for students at technical universities around the world. A team consists of a number of students from one university that together design and build a formula style car according to the Formula Student rulebook[3].

The cars are scrutinized by judges and the cars score points in different aspects of the competition. The cars undergo tests and among them there are dynamic events where they are driven on track to their limit by one of the team members. There are also static events such as the Business Plan Presentation and the Engineering Design event. More information about these can be found in the rulebook.

The implementation of a 4WS system has the potential to score points in both categories since the design and implementation can give Engineering Design points and additional points in the dynamic events as a result of better performance.

#### KTH Formula Student

KTH Formula Student was founded in 2003 and the current car, the eV14, is an all electric rear wheel drive formula car using two EMRAX 207 synchronous electric motors with planetary gearboxes and is built on a steel tube chassis with a double wishbone suspension in both the front and rear. This overall design has been fairly consistent since the eV12 but the next car, eV15, will likely be modified and incorporate new design features. One possible design feature with potential for implementation is four wheel steering and in this thesis mechanisms for implementing 4WS are explored.

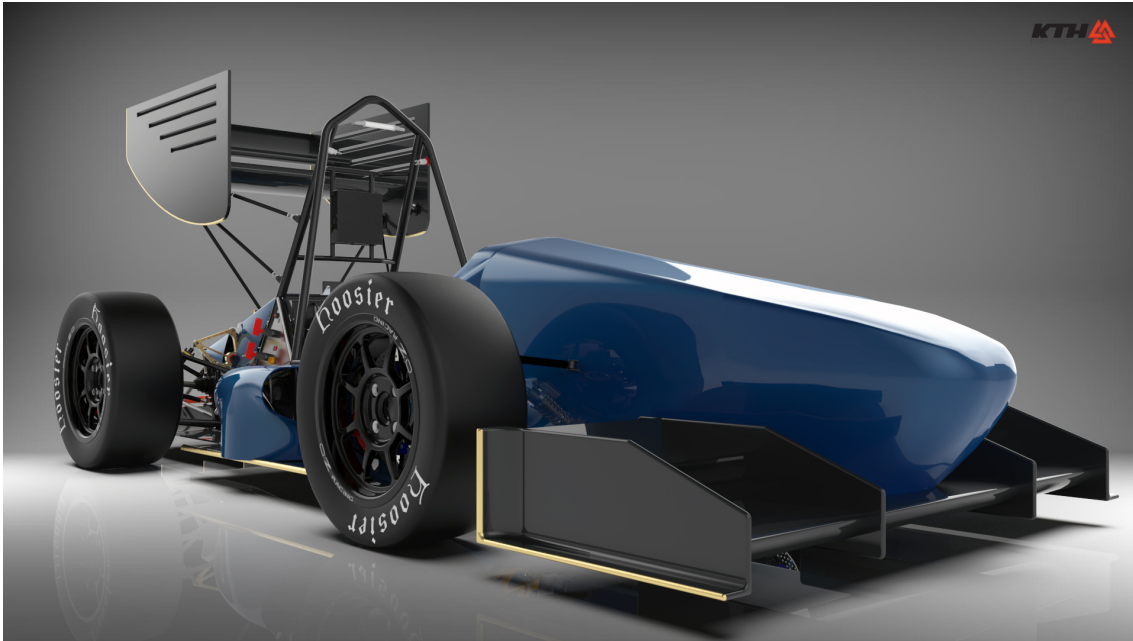


Figure 1.1: A render of the current KTH Formula Student car eV14.

The design of the eV14 is potentially well suited for the implementation of a 4WS system. The rear spindle with the wheel assembly on each side of the car is held in place by two A-arms which fixes its position while a tie rod fixes its rotation. The A-arms attach to the spindles with spherical bearings which allow the rear spindles to be rotated around the upright axle, this is done on purpose so that the rear toe angle can be adjusted by changing the length of the tie rods by adding or removing shims. The components and their positions is shown in Figure 1.2.

A side effect of this adjustability is that the rear spindle and A-arm setup can be used in a RWS system without much modification. By removing the tie rods and replacing them with a RWS mechanism mounted on the backplate the angle of the rear wheels can be adjusted continuously while driving.

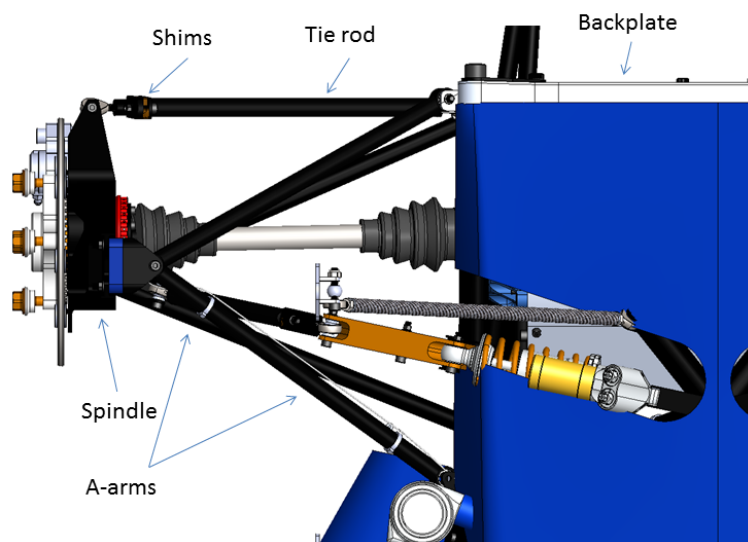


Figure 1.2: The current rear wheel setup.



## 1.2 State of the art

Four wheel steering has been used in commercial (often high performance or sports) vehicles since the late 80s with Japanese manufacturers such as Nissan, Toyota, Mazda and Mitsubishi paving the way[8]. Today the technology is often available as an optional extra in some commercial vehicles such as the Porsche 911 Targa 4S[6]. The most common implementation of a 4WS system is with computer controlled actuators but there are entirely mechanical implementations where the rear wheel steering angle is controlled directly by the steering wheel as well as entirely passive systems[8].

Four wheel steering has also seen some use in motorsport. It was implemented on the Peugeot 405 T16 which raced in the 1988 Pikes Peak Hill Climb rally where a record breaking time was set[2]. Its use is currently banned in Formula 1[1].

### Advantages of four wheel steering

There are two main reasons to implement 4WS on a car, the first one being reducing the vehicle's turning radius. This is useful to increase slow-speed maneuverability primarily during parking or slow driving around tight bends. The reduced turning radius is achieved by turning the rear wheels in the opposite direction to the front wheels which is called negative 4WS or counter-steering. Negative steering will make the car turn as if the wheel base was shorter and is not suitable for high speed driving since it reduces the car's stability[4].

The second reason to implement 4WS is to increase the high speed stability and response of the car by turning the rear wheels in the same direction as the front wheels[8]. This is called positive 4WS or same-steer which in a sense makes the car handle as if the wheelbase was longer, increasing stability. The two variations are shown in Figure 1.3 and by comparison a conventional car has its center of rotation in line with the rear wheels.

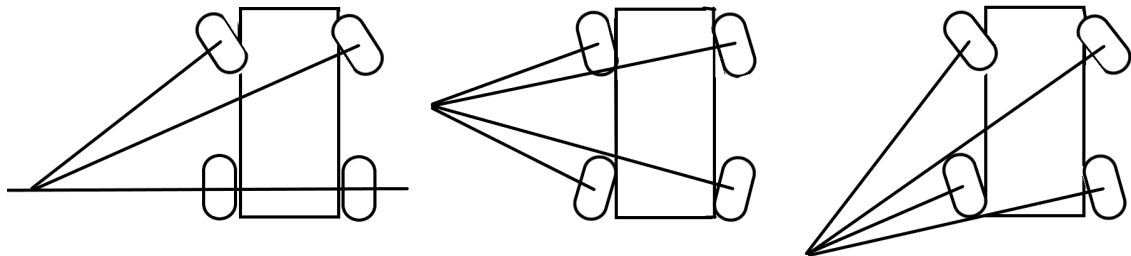


Figure 1.3: Normal steering to the left, negative 4WS in the middle and positive 4WS to the right.

### Literature review

In the Vehicle Dynamics project course SD2229/SD2230, KTH Formula Student team member Suvansh Kasliwal investigated the performance of the current torque vectoring setup and discussed the effects of 4WS on the car's handling[5]. The conclusion was that the car showed a more stable behavior with 4WS which could help alleviate oversteer issues. In addition, if 4WS can achieve the same performance gains that torque vectoring does the powertrain of the car can be changed to a single motor setup which means that the weight can be reduced by at least 12kg<sup>1</sup>

4WS systems in Formula Student are fairly uncommon and most literature that can be found publicly focus on vehicle dynamics optimization rather than the mechanisms to

<sup>1</sup>12kg is the weight of one of the EMRAX 207 motors currently used on the car. Additional hardware would need to be changed as well like removing a gearbox and changing the motor mounts.

implement it. One team that have implemented 4WS is WashU Racing at Washington University in St. Louis[12]. They used two linear actuators that replaced the rear tie rods which is detailed in a thesis but it is not known if this setup was ever properly implemented and brought to competition.

### 1.3 Purpose

Overall there appears to be a general lack of public information on actual 4WS mechanisms for Formula Student cars and their achievable performance. Therefore the purpose of this thesis is to evaluate different 4WS solutions to develop a prototype that can be implemented on the car in the future. The conclusions can then serve as a solid foundation for future analysis of the feasibility and performance benefits of implementing a 4WS system.

As an aid in achieving this and clarifying the purpose a set of research questions have been posed, listed below.

- What are the requirements for a 4WS system?
- Which concepts are feasible?
- Which concept fulfills the requirements the best?
- How should the concept be developed and integrated in the car?

### 1.4 Limitations

The scope of this thesis could potentially become very large due to the subject spanning many disciplines, from vehicle dynamics to control theory and electronics. Therefore a set of limitations have been imposed to limit the scope.

- No hardware testing will be done.
- No integration of electronics will be done apart from required sensors and actuators.
- No regulator design will be done.
- The vehicle dynamics characteristics of the car will not be optimized.
- The steering geometry will not be optimized in detail (like the Ackermann angle).

### 1.5 Methodology

The overall methodology used is listed below.

- Orientation
  - Information gathering
  - Specification of requirements
- Development of concepts
  - Evaluating concepts based on the requirements
  - Selection of the most promising candidate
- Prototype design
  - Detailed design of the selected mechanism
  - Verification of performance in relation to set requirements and goals

### 1.5.1 Establishing a set of requirements

There are several approaches to implement a 4WS system, each with their own strengths and weaknesses. Therefore before a prototype can be developed a set of requirements have to be determined that the different concepts can be evaluated against. The requirements are introduced below and will be quantified in the next chapter.

- **Formula Student rulebook**

The car is designed after the Formula Student Germany rulebook[3] and there are some rules directly concerning rear wheel steering mechanisms and others indirectly that the system needs to adhere to.

- **Responsiveness**

The system needs to be able to change the steering angle of the rear wheels fast enough to match the steering input from the driver during a normal lap on a track.

With the FWS system the driver is in direct control through a rack and pinion mechanism but depending on the selected mechanism for the rear wheel steering that may not be the case, e.g. if it is electronically controlled with actuators, there will be some delay in the control loop and the actuators themselves may not be fast enough to match the driver input. Minimizing this delay will be vital in order to achieve good handling characteristics after implementation.

- **Loads**

The system must be able to withstand the loads it is subjected to during a lap. The loads on a Formula car will be dynamic with the possibility of shock loads, like from the car hitting a bump, and the mechanism must be able to withstand the worst load case. Fatigue will be of less concern since the number of load cycles is fairly low.

- **Packaging**

The chosen mechanism must not interfere with other systems on the car and is preferably easy to access and maintain. The current car design utilizes an aluminium and carbon fibre back-plate seen in Figure 1.4 and it is preferable if the mechanism can be mounted on it directly without involving too much surrounding hardware.

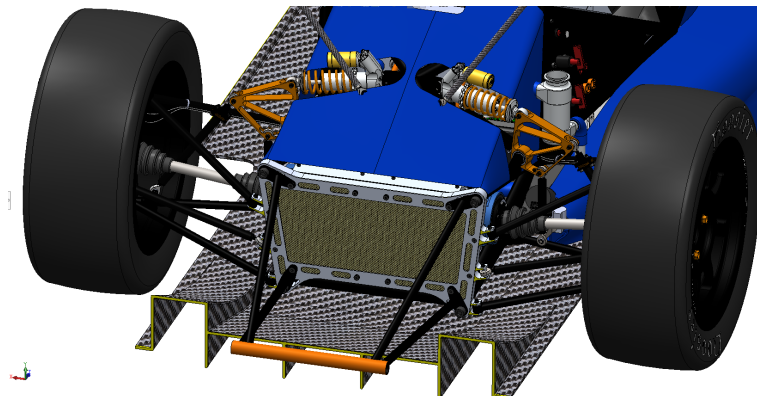


Figure 1.4: Rear view of the current car eV14 showing the composite backplate.

- **Vehicle dynamic controllability**

The different mechanisms concepts will be varyingly well suited from a vehicle dynamics standpoint. In order to give the 4WS mechanism the greatest possibility to improve the cars handling, a good idea is to keep as few variables regarding the RWS

locked as possible in order to make room for future vehicle dynamics optimizations. For example allowing the rear wheels to be controlled independent from the FWS allows for further tweaking of the car's dynamics than for them to be locked to the front steering.

The 4WS system may be controlled by more variables in addition to the steering angle such as vehicle speed and leaving the possibility to implement an arbitrary control strategy is advantageous.

- **Mass**

Mass is at a premium in a Formula car as more mass will decrease the car's acceleration and braking performance as well as decrease the handling and cornering acceleration. The positioning of the mass also plays a part in how detrimental the mass is to the cars performance. The system will preferably not add to the unsprung or rotating mass.

- **Cost**

Actuators and other components for the car can be very expensive and keeping the cost down will make the step toward hardware testing and potentially implementing 4WS on the car lower. This can in part be negated through sponsorships.

- **Safety**

The safety of the driver is always the main concern, therefore the result of potential failures in the mechanisms must be assessed.

### **1.5.2 Selection of design concept, detailed design and evaluation**

With the requirements as a foundation, different concepts were drafted and evaluated. The scores for each design determined which one of the concepts that had the most potential which was used as the basis for the detailed design. The detailed design was modeled in Solidworks and analyzed with the inbuilt FEM tool. Lastly the final design was evaluated against the design requirements.

## Chapter 2

# Conceptual Design

*In this chapter the requirements introduced previously are analyzed and quantified. A number of concepts are evaluated against each other based on the requirements and thereafter a more detailed design of the chosen concept is made.*

### 2.1 Quantifying the requirements

Before the requirements first introduced in the previous chapter can become a useful tool they first have to be made more specific, therefore in this chapter the quantifiable requirements will be developed and the unquantifiable requirements will be concretized.

#### Rulebook

In the Formula Student Germany 2018 rules there is only one paragraph specifically addressing RWS.

T 1.6.11 *"Rear wheel steering, which can be electrically actuated, is permitted if mechanical stops limit the range of angular movement of the rear wheels to a maximum of 6 degrees. This must be demonstrated with a driver in the vehicle and the team must provide the equipment for the steering angle range to be verified at technical inspection."*[3]

In addition, there are also rules that affect the system indirectly, the most important ones are listed below.

T 1.6.1 *"Steering systems using cables or belts for actuation are prohibited."*

T 9.1.1 *"Critical fasteners are defined as bolts, nuts, and other fasteners utilized in the primary structure, the steering, braking, driver's harness, suspension systems and those specifically designated as critical fasteners in the respective rule."*

T 9.2.1 *"All critical fasteners must be secured from unintentional loosening by the use of positive locking mechanisms<sup>1</sup>."*

Since all fasteners used in the RWS system are considered critical they will have to be positively affixed. The prohibition of the use of belt drives means that some linear actuators are not allowed.

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<sup>1</sup>I.e. safety wire, nylon lock nuts, tab washers etc. The full list can be found in the rules. Adhesive or devices based on pre-tensioning such as Nord-Lock washers are not considered to be positive locking mechanisms.

The Formula Student Rulebook does not have a specific rule regarding the mechanical system free play of a potential RWS system but there is however a rule limiting the free play of the "steering system" stated in the rule T 6.5.3 to seven degrees. Since the allowed steering angle for the RWS system is so small and for many possible designs there will not be a mechanical linkage between the steering wheel and the RWS mechanism this rule will be considered non-applicable.

## Responsiveness

In order to quantify the responsiveness requirement for the system an analysis of how quickly the driver turns the steering wheel during a lap was performed. The maximum input angle for the steering wheel is  $90^\circ$  to the left or right, which means that the steering has a total input span of  $180^\circ$ . Assuming that the maximum angle of the rear wheel steering will occur at  $90^\circ$  of steering wheel input, the required actuation time to full stroke can be measured. During a twitchy corner while testing the eV13 the two following captures were taken from the video[11].



Figure 2.1: Picture showing steering angle in a twitchy corner.

The camera operated at a frame rate of 29,97Hz which means that the time for each frame was 0,0334s and the time between the two captures was 5 frames or 0,167s. The steering wheel angle difference between the captures is approximately  $90^\circ$ . This means that the RWS must be able to reach its maximum angle in 0,167s or less, otherwise the RWS will not be able to keep up with driver input and unwanted behavior could arise.

Since the maximum angle allowed in the rules that the RWS can use is  $6^\circ$  the steering mechanism chosen must be able to change the steering angle from 0 to  $6^\circ$  in under 0,167s. To easier calculate the angular acceleration needed for the RWS angle, the angular acceleration was set as constant and a kinematic equation was used.

$$\theta(t) = \frac{\ddot{\theta}_0 t^2}{2} + \dot{\theta}_0 t + \theta_0 \quad (2.1)$$

At the start of a corner both the initial turning angle,  $\theta_0 = 0$  and the turning angle speed,  $\dot{\theta}_0 = 0$  leaving only the first term.

$$\ddot{\theta}_0 = \frac{2\theta(t_{end})}{t_{end}^2} \quad (2.2)$$

Entering the desired time  $t_{end} = 0,167s$  and turning angle  $\theta(t_{end}) = 6^\circ$  leaves us with the turning angle acceleration to be  $430,3^\circ s^{-2}$ . The maximum angular speed that the steering will need to achieve is calculated using Equation 2.3.

$$\dot{\theta}_{max} = \ddot{\theta}_0 t_{end} \quad (2.3)$$

and is found to be  $43,03^\circ \text{s}^{-1}$ . The theoretical step response with constant angular acceleration is shown in Figure 2.2.

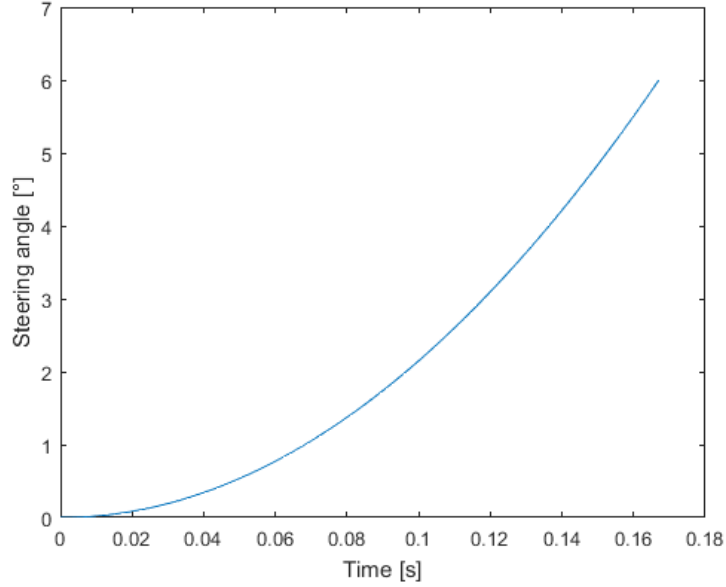


Figure 2.2: Steering angle plotted against time for a constant acceleration response.

This is the theoretical step response for the RWS assuming a constant acceleration to a  $90^\circ$  driver steering input, however in reality this would have to be decelerated before reaching  $6^\circ$ .

Angular acceleration	Maximum speed	Stroke time
$430^\circ \text{s}^{-2}$	$43^\circ \text{s}^{-1}$	0,167s

In practice the system needs to be even faster since the system needs to be slowed down before reaching the end stops. The results are however still relevant since they provide a lowest acceptable value for both the angular acceleration and the maximum stroke time.

## Loads

The given loads for the tie rods in the current setup of the car are shown in Table 2.1. These load cases have been calculated by the Vehicle Dynamics group from simulation data

Table 2.1: Loads on the tie rods given by VD simulations.

	Acceleration (1.7g)		Braking (-1.7g)		Cornering (1.82g)		Combined*	
	Left	Right	Left	Right	Left	Right	Left	Right
Dynamic Coef. *								
0	34	34	15	15	-173	583	-125	442
2.5	-81	-81	-29	-29	-203	451	-194	298

and are considered accurate enough for this thesis. The worst load case forces that will be used in this thesis are highlighted in red. The force needed by the mechanism driving the

RWS will have to both overcome these forces and the additional force needed to accelerate the rear wheels when turning. The force needed can be calculated using Equation 2.4 where  $\theta$  is the rear steering angle and  $I_{rot}$  denotes the wheel assembly's moment of inertia around the rotation axis.

$$M_o = I_{rot}\ddot{\theta} \quad (2.4)$$

With  $\ddot{\theta} = 7,5\text{rad s}^{-2}$  and from the CAD  $I_{rot} = 0,25\text{kg m}^2$   $M_o = 1,9\text{N m}$  per rear tire. With the  $D = 115\text{mm}$  lever on the rear spindle, which is shown in Figure 2.3, the additional force needed to turn each of the wheels is 16N.

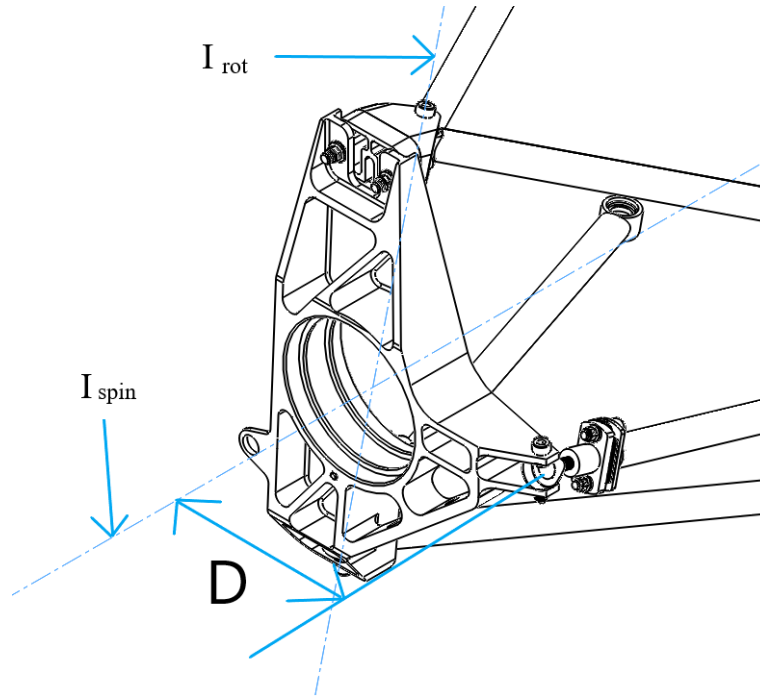


Figure 2.3: The rear spindle with marked axis of rotations and D.

This is a simplification in that the acceleration is assumed constant until the system hits its limits, which in reality it will not be. It is also assumed that the effective lever is constant as the spindle turns. By using more advanced models of the acceleration of the system more exact forces can be found, however since the forces are small in relation to the static forces when dimensioning these forces were simply doubled, 32N per wheel which gives a total force of 816N required.

An addition that does not specifically affect the RWS mechanism but is worth noting, since the wheel is spinning a gyroscopic torque will affect the spindle in the direction of travel. It can be shown by using Euler's second law that the torque will be as shown in Equation 2.5.

$$M_{gyro} = \omega\dot{\theta}I_{spin} \quad (2.5)$$

if the car is driving  $100\text{km h}^{-1}$  and with a tire diameter of  $0,5\text{m}$ ,  $\omega = 111\text{rad s}^{-1}$ . From the CAD  $I_{spin} = 0,326\text{kg m}^2$  and as shown in the speed section  $\dot{\theta} = 43^\circ \text{s}^{-1} = 0,75\text{rad s}^{-1}$  resulting in a torque  $M_{gyro} = 27\text{N m}$ . The A-arm hardpoints are 115mm from the center of rotation which means an additional 60N load on each of the A-arms perpendicular to the spindle.



## Packaging

There is no good way of quantifying how good a mechanism is packaging wise but a few positive and negative traits can be specified. A system that is mostly positioned on the backplate of the car will be easy to access and tweak and a system that have mechanical components in multiple places is less desirable for the opposite reasons. Interference with other mechanical components in a way that limits any systems functionality is not acceptable. The approximate boundaries for the mechanism around the backplate is shown in Figure 2.4.

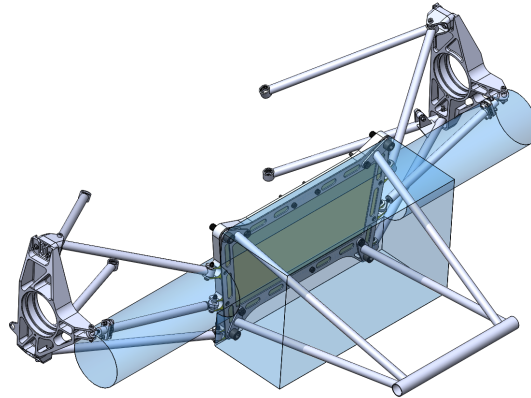


Figure 2.4: The approximate volume the mechanism can use.

## Vehicle dynamic controllability

To describe the level of vehicle dynamic controllability of the concepts three main levels of controllability have been determined.

1. **Arbitrary control** The rear wheels steering angles can be controlled independently of the FWS and also independently from each other. This gives the full range of possibilities regarding control of the RWS system.
2. **Independent from FWS** The rear wheels steering angles can be controlled independently of the FWS but are locked to each other through a mechanical linkage.
3. **Locked to FWS** The rear wheels steering angles are fully dependent on the driver input to the FWS.

## Mass

The maximum mass that the system can have without negating the benefits of integrating a FWS system is not known, however as explained in the previous chapter the mass should be as low as possible. A threshold value of 4kg was selected but systems with lower mass were premiered.

## Cost

No hard limitation on cost will be set, however the relative cost of the concepts were compared based on the use of expensive components like actuators to give the score. The cost will also be evaluated using the Formula Student standard component cost report repository.

## Safety

Three main failure modes have been identified for the RWS system, listed below in falling order of severity.

1. **Loss of RWS control, spindle free to rotate.** This is the equivalent of the rear tie rods breaking. Will most likely cause a detrimental performance loss and potentially big load spikes on the A-arms and surrounding structure if the wheels turn in unfavorable directions. Fatal error, will lead to DNF for the event.
2. **Loss of RWS control, spindle rotation locked.** Depending on the angle that the wheels are locked in it may have a small to big impact on performance as in one direction will have a degree of positive 4WS and the other negative. Not a fatal error but potentially detrimental to performance.
3. **Unstable behaviour, eg. oscillations.** If the control loop becomes unstable the steering system will start oscillating with increasing amplitude which will make the car difficult to control. By using a killswitch for the control loop that returns the wheels to their neutral position the driver can regain control at the loss of whatever benefit the 4WS was giving. Entirely mechanical systems will not suffer from this issue apart from those induced by the driver ;).

## 2.2 Design concepts

As explained in the previous chapter 4WS can be implemented relatively easily on the current car because of the rear spindles' rotation being fixed only by the tie rods and the use of a backplate. The evaluated concepts are listed and explained below.

### Concept 1: Two linear actuators replacing tie rods

In this concept shown in Figure 2.5 each of the tie rods on the car are replaced entirely with electrically controlled linear actuators. The actuators would be controlled individually and allow the control system to adjust steering angle on each of the rear wheels independently which means that it has VD controllability class 1.

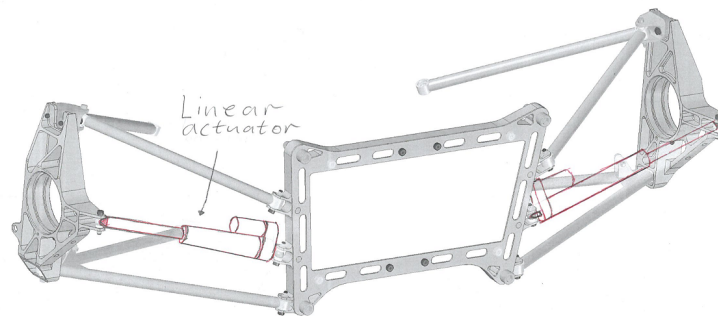


Figure 2.5: Concept 1: Linear actuators replacing tie rods.

- **Advantages:** This concept has a high possibility of optimization of the car's dynamic behavior and would for example allow the car to modify its rear toe angle on the go. The implementation is fairly straightforward since the current rear spindles can be reused and gives a high flexibility since the RWS geometry is controlled entirely through software.
- **Disadvantages:** The negative side of this concept is that it would be heavy with two actuators instead of one. By looking at the loads in table 2.1 for cornering one of the actuators will experience essentially zero static load during the turn but since the car needs to be able to turn both ways both actuators have to be dimensioned for the higher load. Having two actuators increases the complexity in the design process of the control system and the amount of wiring. Having two actuators might also increase the risk of system failure and increase maintenance. It is also dubious whether there are commercially sold actuators that combine the requirements of a short stroke with high speed and force at a reasonable price point.
- **Failure modes:** A system failure would take the shape of either a type two or type three failure as linear actuators are self-locking which would fix the position of the wheels.

## Concept 2: Rack and pinion with rotary actuator

This concept which is shown in Figure 2.6 uses a traditional rack and pinion mechanism connected to the tie rods that is actuated by an electronic motor connected to the pinion gear. This is similar to the FWS system which is a rack and pinion steering, however a gearbox will have to be used between the electric motor and the pinion. The concept has VD controllability class 2.

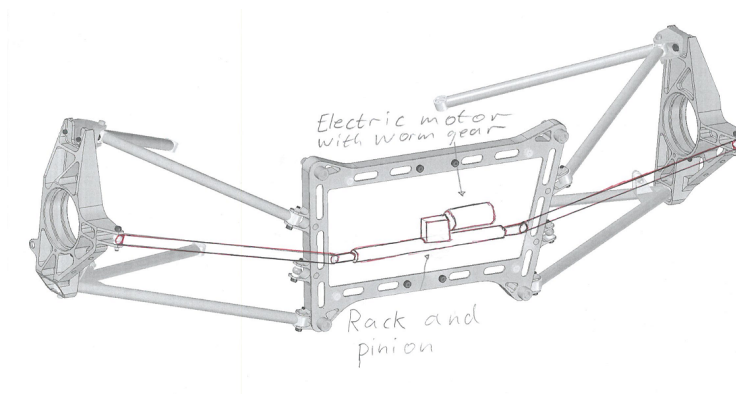


Figure 2.6: Concept 2: Rack and pinion with rotary actuator.

- **Advantages:** The system will be fairly lightweight since it only uses one actuator and it will be very similar in construction to the front steering rack which is a proven design. The regulator design will also be fairly straight forward and it will be easy to maintain. The stops required by the rules can easily be incorporated in the rack and pinion mechanism.
- **Disadvantages:** The main disadvantage is that unlike with the two linear actuator design you do not get the same amount of control over the individual tire angles for optimization.
- **Failure modes:** Either one and three or two and three depending on if the gearbox on the motor is self-locking or not.

### Concept 3: Ackermann mechanism with linear actuator

This concept which is shown in Figure 2.7 is similar to Concept 2 but instead of a rotary actuator acting on a pinion gear a linear actuator would act directly on the steering link. It also has VD controllability class 2.

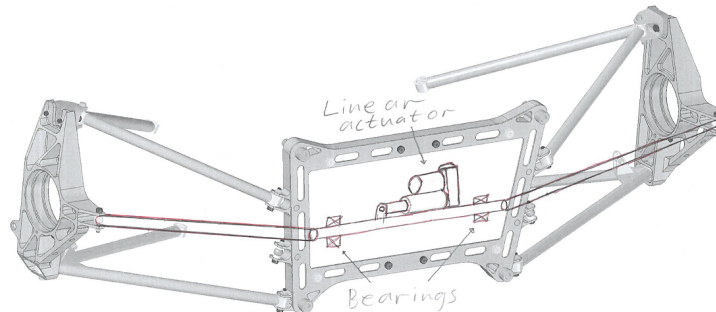


Figure 2.7: Concept 3: Ackermann with linear actuator.

- **Advantages:** The mechanism has many of the same advantages as concept 2, with the addition that it is a bit simpler since fewer components are needed.
- **Disadvantages:** Like concept 2 the ability to exactly tweak the steering angles is more limited than they are for concept 1 and in addition the mechanism is not as standard as a rack and pinion design. Speed is about the same as Concept 1 which is possibly too low.
- **Failure modes:** The concept has failure modes two and three since the actuator is self locking.

## Concept 4: Proportional hydraulic system

In this concept the front steering mechanism would be connected to a hydraulic cylinder which drives the rear steering mechanism with another cylinder at the back of the car as shown in Figure 2.8. This means that the rear steering angle is directly proportional to the front steering angle and that the ratio between the two is determined by the mechanical advantage of the system. This concept has VD controllability class 3.

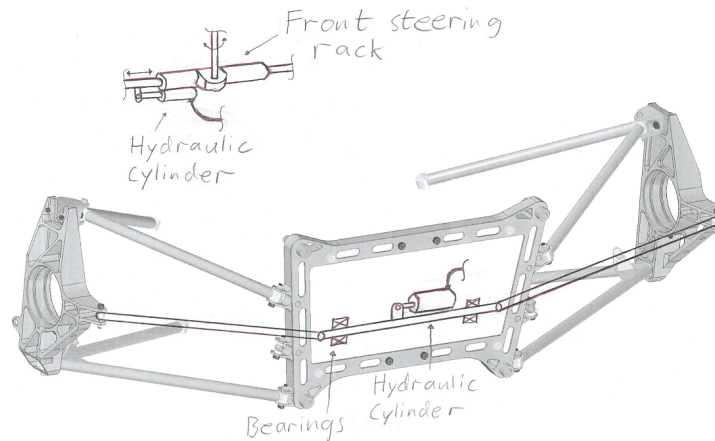


Figure 2.8: Concept 4: Proportional hydraulic system.

- **Advantages:** The main advantages of the proportional hydraulic system stems from its simplicity. Since the mechanism is directly actuated by the driver the input lag is virtually zero and there is no need to design a control loop. The system can also potentially be made very lightweight because all the actuation force comes directly from the driver without the need for actuators.
- **Disadvantages:** The main disadvantage of the system is because of the lack of a control loop it lacks a lot of flexibility for optimization from a vehicle dynamics standpoint; It does not take into account the effects of the car's speed, the ratio between the front and rear steering angle is the same at 5 and 100 km/h and there is no way of changing from positive to negative 4WS or turning the system off. Another disadvantage is that the system is difficult to adjust, if there came a need to change the steering angle ratio the cylinders or the mechanism would have to be changed out.
- **Failure modes:** If the system would fail, e.g. if a hydraulic hose were to come loose, the system would fail in failure mode one. This is however not very likely since the overall design would be fairly similar to the hydraulic brake system which rarely if ever has issues with this.

## Concept 5: Non proportional hydraulic system

This concept is a refinement of the proportional hydraulic system that uses a mechanism that makes the relation between the front and rear steering angle non proportional to improve the vehicle dynamics characteristics of the car, conceptually shown in Figure 2.9. This kind of mechanism can be made more or less complex depending on the requested relationship between driver input and rear steering angle. The VD controllability class of the system is somewhere between class 2 and class 3 depending on how complex the nonlinear mechanism is.

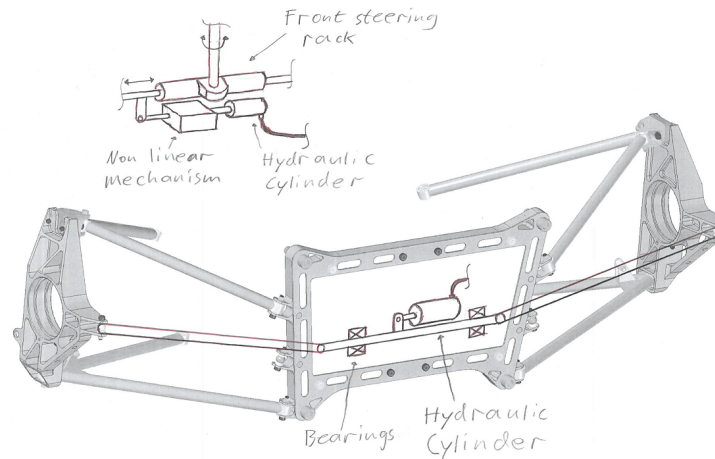


Figure 2.9: Concept 5: Non proportional hydraulic system.

- **Advantages:** The main advantages is that it potentially maintains the simplicity of implementation of the proportional hydraulic system with the added benefit of better vehicle dynamical behavior.
- **Disadvantages:** The mechanism is however still nowhere near the flexibility of an electronically actuated RWS system to tweak and optimize, especially when it comes to incorporating experience from testing, and the complexity of the non proportional mechanism will increase drastically with higher demands on vehicle dynamical behavior. It essentially becomes making a mechanical controller instead of an electrical controller like the governor found on steam engines. This also adds to the overall weight of the system.
- **Failure modes:** Like the proportional system it would fail in failure mode one.

## 2.3 Evaluation of concepts

To evaluate the concepts a Pugh-matrix analysis was done which is shown in Table 2.2. It is based of the advantages and disadvantages listed above with the *Two linear actuators* concept chosen as baseline since it is the previously most explored concept.

The rules compliance and loads requirements have not been evaluated as there is little point in developing concepts that cannot actually be used. These requirements with loads in particular will however affect the others like mass and speed.

**Concept 1:** Two linear actuators.

**Concept 2:** Rack and pinion with rotary actuator.

**Concept 3:** Ackermann with linear actuator.

**Concept 4:** Proportional hydraulic system.

**Concept 5:** Non proportional hydraulic system.

Table 2.2: Pugh-matrix comparison of concepts.

Requirements:	Concept 1	Concept 2	Concept 3	Concept 4	Concept 5
Responsiveness	0	+	=	++	++
Packaging	0	=	+	+	-
VD controllability	0	-	-	---	--
Mass	0	++	+	++	+
Cost	0	+	+	+	+
Safety	0	=	=	-	-
$\Sigma+$	0	4	3	6	4
$\Sigma-$	0	1	1	4	4
$\Sigma$	0	3	2	2	0

All concepts have received a lower VD controllability score than concept 1 since it is the only one evaluated that is VD controllability class 1. The hydraulic systems have as explained previously superior responsiveness characteristics and all systems are believed to have lower mass than concept 1 since they only use one actuator. Based on the results of the Pugh-matrix, concept 2 will be developed further.



## 2.4 Design

At this point everything needed to design the system in detail is known. The overall layout of the system as seen from the rear of the car is shown in Figure 2.10.

### 2.4.1 Motor and Gearbox selection

Required traits for the motor driving the RWS are high enough power to move the mechanism at the desired rate under the prevalent forces and a high efficiency while at the same time using a gearbox that is self locking. As the system's mechanical advantage can be adjusted with the pinion gear size the exact gearing ratio in the actuator gearbox is not a determining factor, instead the focus is making sure that the motor can handle cornering continuously i.e a high enough power rating.

To determine an approximated mechanical power that the actuator will need to put out in the worst load case a simple model of the steering actuation was made.

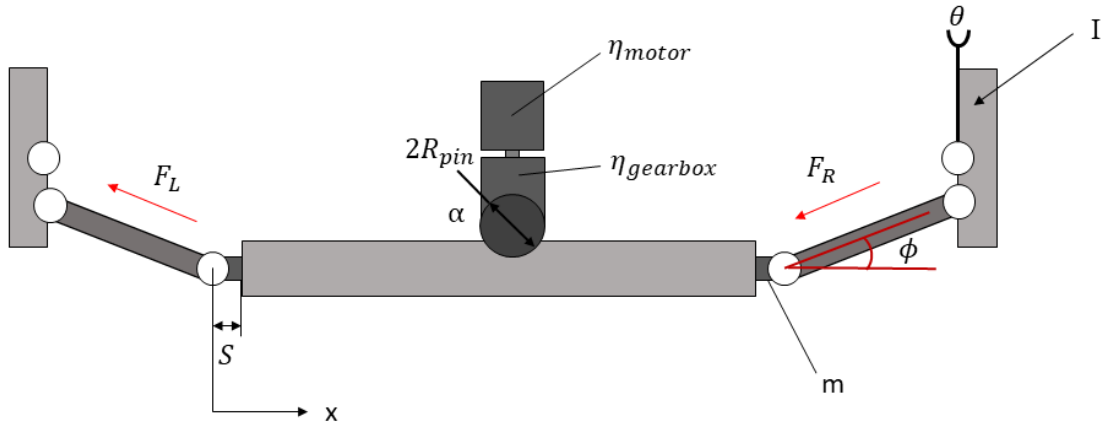


Figure 2.10: The overall layout of the system and used variables.

In this model the angle  $\phi$  is regarded as negligible and will not affect the leverage of the rack towards the spindle, however the forces  $F_L$  and  $F_R$  will be affected when acting on the rack by this angle. Because  $\theta$  is also going under small angle differences the angle  $\phi$  is regarded as constant, making the rack and the tie rods act as a solid body in this model. Since the moment of inertia is the same on both the left and right wheel the model becomes simple. The power generated from the motor,  $P_{pin}$  on the pinion is constant with respect to rotational speed, which in reality it would not be, depending on the motor specifications.

From Figure 2.10 the following equations can be derived using Newtons 2:nd law. An explanation to the variables are given in Table 2.3.

$$F_{pin} - \frac{(F_R + F_L)}{\cos \phi} = m\ddot{\theta}D + \frac{2I_{rot}\ddot{\theta}}{D} \quad (2.6)$$

The force acting on the rack from the pinion will be

$$F_{pin} = \frac{M_{pin}}{R_{pin}} \quad (2.7)$$

and the moment  $M_{pin}$  acting on the pinion will vary with the pinion rotational speed  $\dot{\alpha}$  similar to

$$M_{pin} = \frac{P_{pin}}{\dot{\alpha}}. \quad (2.8)$$

The kinematic relation between  $\dot{\alpha}$  and  $\dot{\theta}$  for small angles of  $\theta$  can be written as

$$\dot{\alpha} = \frac{\dot{\theta}D}{R_{pin}}. \quad (2.9)$$

Combining equations 2.6, 2.7, 2.8, 2.9 gives us

$$\ddot{\theta} = \frac{P_{pin}}{D^2\dot{\theta} + 2I_{rot}} - \frac{F_R + F_L}{\cos\phi(mD + \frac{2I_{rot}}{D})}. \quad (2.10)$$

And additionally, 2.11 can be used to calculate  $P_{pin}$  from motor specifications.

$$P_{pin} = P_{motor}\eta_{motor}\eta_{gearbox} \quad (2.11)$$

Table 2.3: Table of variables used during design.

Variable	Description	Expression/Value Range
$\alpha$	Rotational angle of pinion	$\frac{\theta D}{R_{pin}}$
$\theta$	Steering angle of spindle	$-6 \rightarrow 6^\circ$
$\eta_{motor}$	Efficiency coefficient of motor	$0 \rightarrow 1$
$\eta_{gearbox}$	Efficiency coefficient of gearbox	$0 \rightarrow 1$
$R_{pin}$	Pitch radius of the pinion	$0,5 \rightarrow 2\text{cm}$
$M_{pin}$	The moment acting on the pinion	$\frac{P_{pin}}{\dot{\alpha}}$
$P_{motor}$	The electrical power draw of the motor	-

Table 2.4: Table of constants used in the model and during design.

Constant	Description	Value
$\phi$	Angle between tie rod and main rod	$17^\circ$
$I_{rot}$	Moment of inertia of one wheel assembly	$0,25\text{kg m}^2$
$F_R$	Force from the right tierod	$583\text{N}$
$F_L$	Force from the left tierod	$173\text{N}$
$D$	The leverage against the spindle	$115\text{mm}$
$S$	The maximum travel of the rack	$1,2\text{cm}$
$F_{pin}$	Force of the pinion on the rack	$816\text{N}$
$m$	Mass of the main rod	$1\text{kg}$
$P_{pin}$	The mechanical power acting on the pinion	$65\text{W}$

Using the differential equation in 2.10 with starting values  $\theta = 0$  and  $\dot{\theta} \approx 0$ <sup>2</sup> and  $P_{pin} = 65\text{W}$  the response in Figure 2.11 was obtained. As a note  $P_{pin}$  is the mechanical power that the pinion will act with, after the losses in the gearbox and motor itself.

Figure 2.11 shows that around  $P_{pin} = 65\text{W}$  of mechanical power is enough to turn the wheels  $7.3^\circ$  within  $0,167\text{s}$  however the motor itself will need a higher power rating according to equation 2.11 since there are losses in both gearbox and motor.

Maxon Motor, a Swiss based company that make high performance motors for aerospace and industry among other fields, was selected because of their motors having good performance at a low weight and their ability to provide a complete solution with both motor

<sup>2</sup> $\dot{\theta}$  can not be exactly zero since the model would be undefined, instead a small positive value close to zero is used.

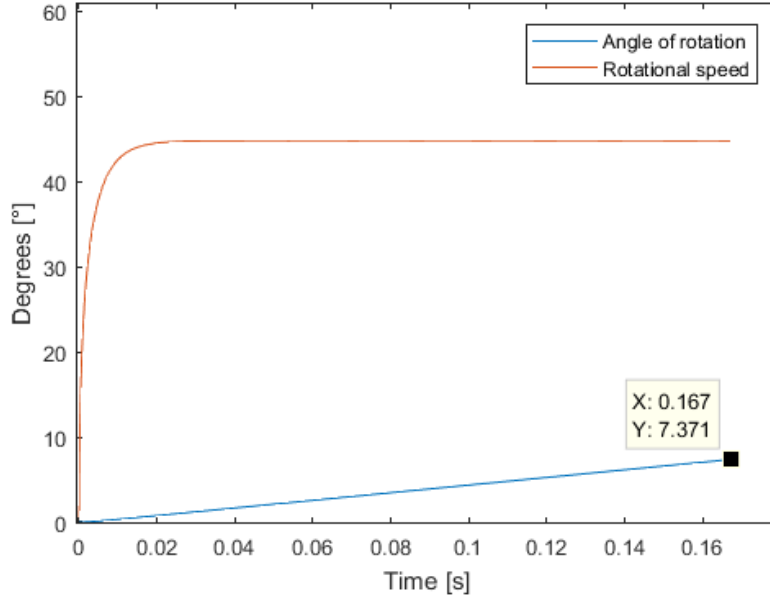


Figure 2.11: Response of the system with 65 W mechanical power.

and gearbox. After comparing motor categories and gearboxes it was found that the 24 V DC motor RE 40 150 W, part number 148867 and a planetary gearbox in the GP 42 C series was the most suitable combination<sup>3</sup>. The DC motors, while still expensive, have better performance for their price point than their brushless counterparts and the planetary gearbox allows for good packaging of the system.

It is worth noting that originally the mechanism was envisioned to use a worm drive in the concept stage for its self-locking capabilities. Planetary gearboxes can be self-locking depending on its construction and some studies have been done into this[7], in this case the gearbox is most certainly self-locking since it is in three stages and has a fairly high reduction.

Choosing the reduction in the gearbox was made in conjunction with choosing a pinion gear. The torque needed from the motor to overcome the forces from the tie rods would ideally be around the nominal torque of the motor. Equation 2.12 is the result of applying equilibrium to the pinion gear where  $M_{nom}$  is the nominal torque of the motor,  $u$  is the reduction of the gearbox,  $\eta_{gearbox}$  is the efficiency of the gearbox and  $R_d$  is the pitch radius of the pinion. The right hand side of the equation is according to Figure 2.10.

$$\frac{M_{nom}u\eta_{gearbox}}{R_d} = \frac{F_R + F_L}{\cos \phi} \quad (2.12)$$

This can be rearranged to show gearbox reduction  $u$  as a function of pinion gear size  $R_d$  which results in Equation 2.13. With  $M_{nom} = 0,177\text{N m}$ ,  $\eta_{gearbox} = 0,72$  according to the motor and gearbox datasheets and other parameters according to Table 2.1 and Table 2.3 the resulting plot is shown in Figure 2.12.

$$u(R_d) = \frac{R_d(F_R + F_L)}{M_{nom}\eta_{gearbox} \cos \phi} \quad (2.13)$$

<sup>3</sup>Datasheets on the motor and gearboxes can be found in Appendix A

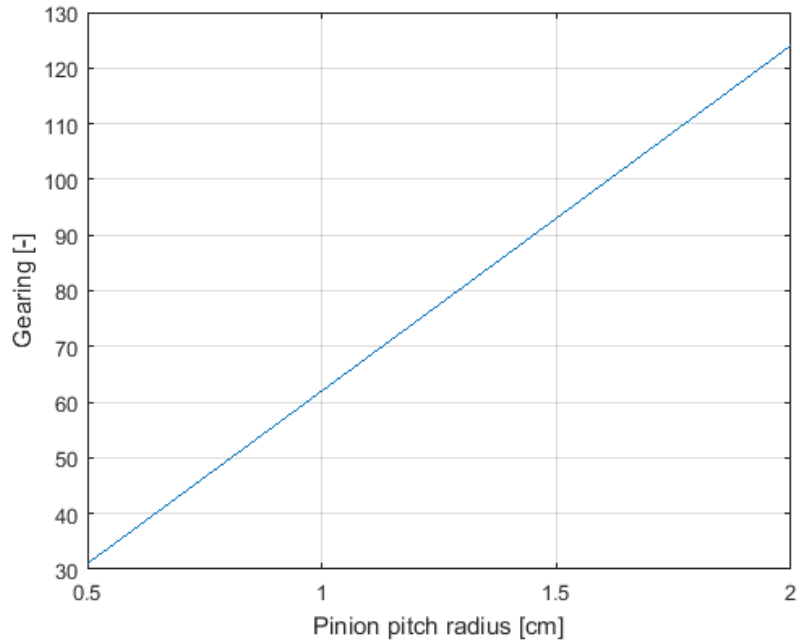


Figure 2.12: Required gearbox gearing as a function of pinion pitch radius.

Based on Figure 2.12 and the gearbox datasheets the gearbox model 203122 with a reduction of 66 and a pinion with a pitch radius of 1cm were selected. The final motor and gearbox combination is shown in Figure 2.13.

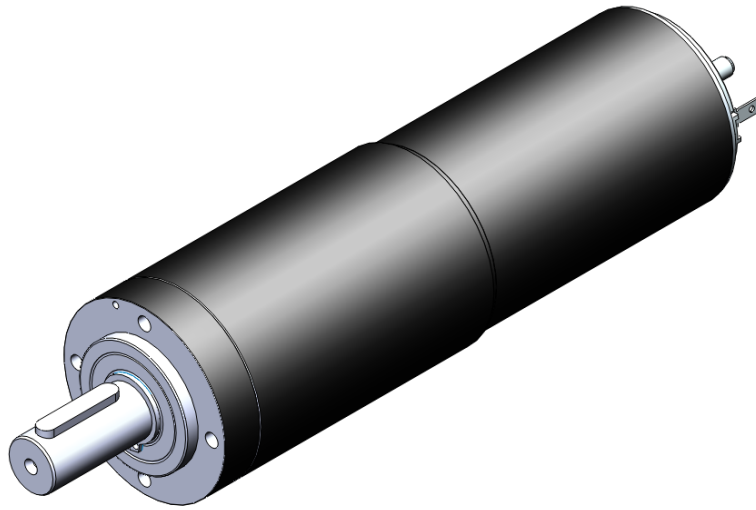


Figure 2.13: The selected motor and gearbox combination.

The motor is rated at 150W electrical power and the mechanical power on the pinion can be calculated with 2.11 to be around 100W, well above the 65W used in 2.10 This motor and gearbox combo will put out a nominal torque on the pinion which the pinion will be dimensioned against.

$$M_{pin,nom} = M_{nom}u\eta_{gearbox} \quad (2.14)$$

With  $M_{nom} = 0,177\text{N}\cdot\text{m}$  and  $\eta_{gearbox} = 0,72$  and the gearing ratio  $u = 66$ . The nominal torque on the pinion is  $M_{nom,pin} = 8,4\text{N}\cdot\text{m}$ . If the stalling torque is calculated instead the maximum moment the motor can apply is received. With the stalling torque  $= 2,42\text{N}\cdot\text{m}$  the maximum torque is  $115\text{N}\cdot\text{m}$ . The nominal speeds can be calculated with

$$\omega_{pin,nom} = \frac{\omega_{motor,nom}}{u} \quad (2.15)$$

$$\dot{\theta} = \frac{\omega_{pin,nom} R_{pin}}{D} \quad (2.16)$$

Inserting the  $\omega_{motor,nom} = 726,75 \text{rad s}^{-1}$  the  $\omega_{pin,nom} = 11,01 \text{rad s}^{-1}$  and the  $\dot{\theta} = 0,95 \text{rad s}^{-1}$  which is equal to  $\dot{\theta} = 54,4^\circ \text{s}^{-1}$ .

### 2.4.2 Design of other components

With the motor selected the other components such as housings, the rack and pinion mechanism and their mounts could be designed. This was done through an iterative process where the components were drafted and refined using analytical and numerical methods. The calculations and FEM analysis of components are found in Appendix B.

## Chapter 3

# Results

*In this chapter the final design of the RWS system is shown and certain features are highlighted. Thereafter a verification of the system against the requirements is made.*

### 3.1 Overall design

The mechanism in its entirety is shown in Figure 3.1. The brackets are screwed into the backplate of the car and the tie rods are attached to the mounts on each side.

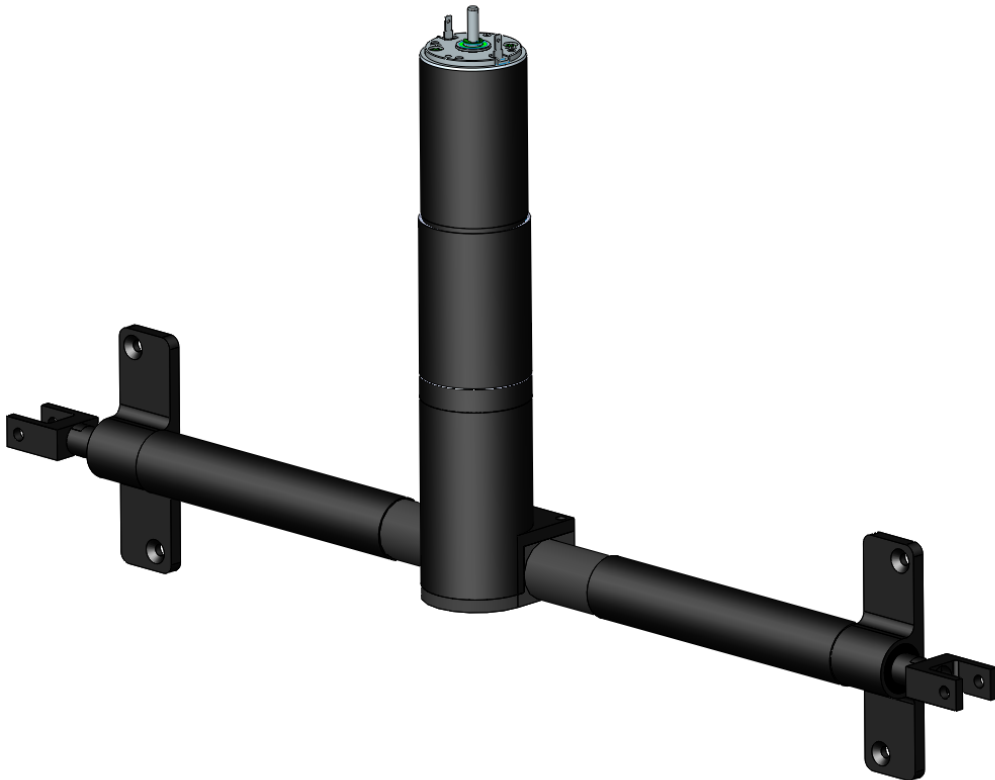


Figure 3.1: The final RWS mechanism.

In Figure 3.2 the main rod, the rack and pinion and bearings among other components are shown.

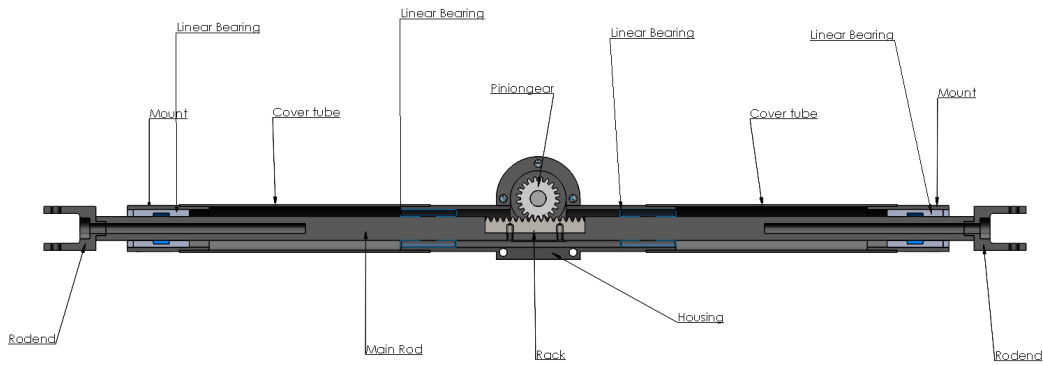


Figure 3.2: Section view of the rack and pinion mechanism

Figure 3.3 shows a closeup of the coupling between the gearbox and the rack and pinion mechanism and its bushings.

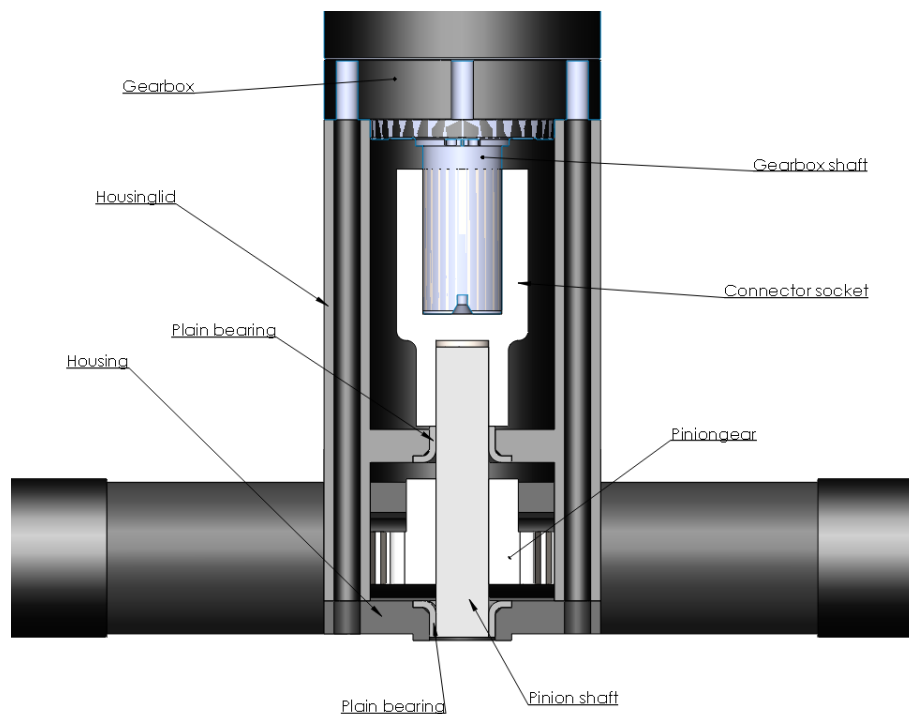


Figure 3.3: Section view of the housing.

The mechanism as installed on the backplate is shown in Figure 3.4 and Figure 3.5. The old tie rods are still shown on the figures but would in reality be shortened and connected to the RWS mechanism instead of the backplate.

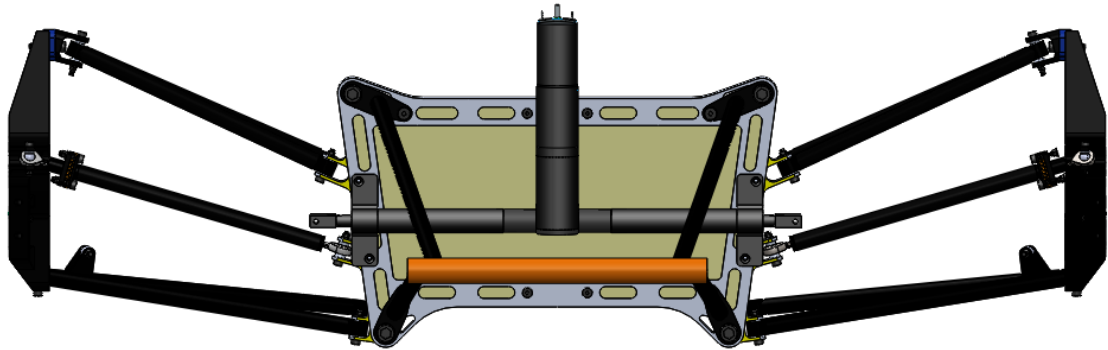


Figure 3.4: The mechanism installed on the backplate from behind.

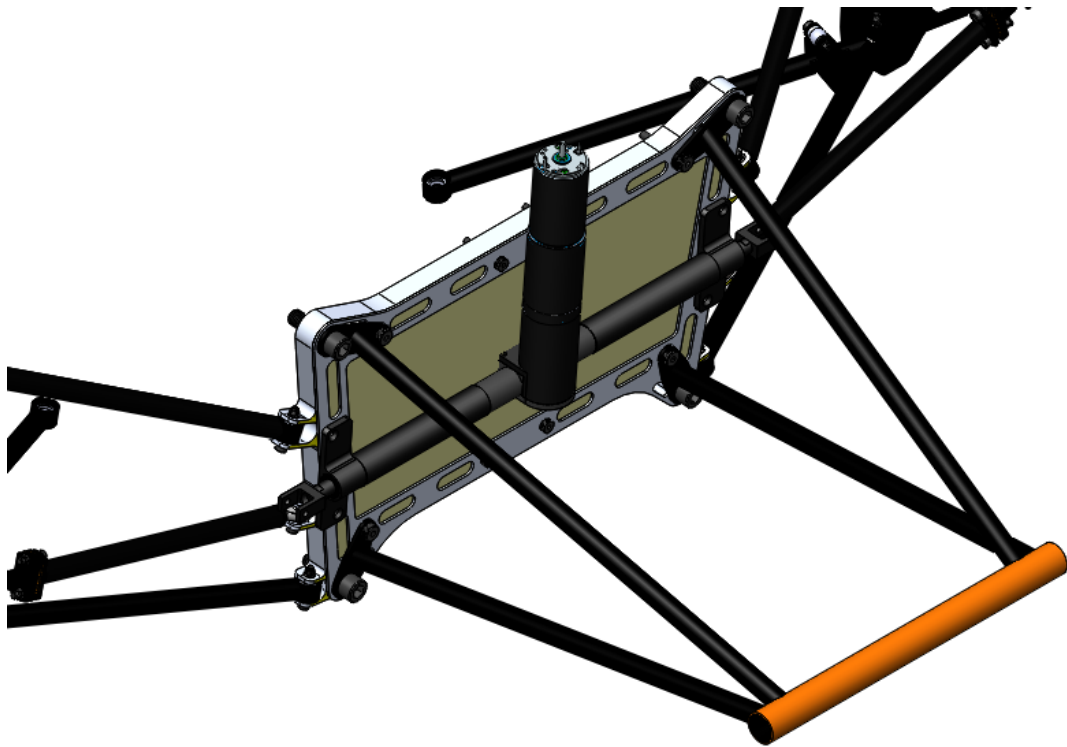


Figure 3.5: The mechanism installed on the backplate, isometric view.



## 3.2 Components

The different components are listed and described in detail below.

### 3.2.1 Pinion

The pinion shown in Figure 3.6 is dimensioned according to SS 1871[10] and the calculations can be found in Appendix B. The pinion is a standard component and its datasheet can be found in Appendix A. It is made out of SS 1672/EN 1.1191 steel that needs to be nitered and hardened to at least 36HRC and has a pitch radius of 10mm, tooth width of 8mm, a modulus of 1mm, a bore of 8mm, 20 teeth and a total width of 16mm. The rack is made of the same material and given the same heat treatment and its datasheet can be found in Appendix B.

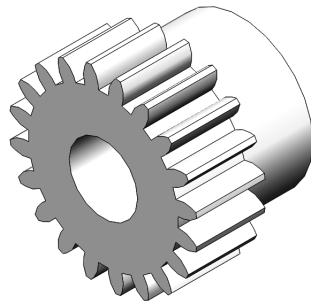


Figure 3.6: Showing the pinion.

### 3.2.2 Main rod

The diameter of the main rod was chosen to be a 12 mm cold drawn rod of EN 6063 T6 aluminium. The rack was fastened to the main rod with two M3 screws.

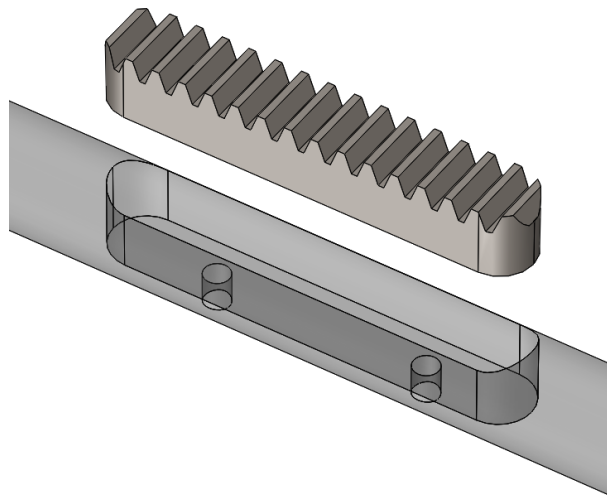


Figure 3.7: Showing the rack slot in the main rod.

The main rod is held in place by four plastic plain linear bearings of the type SKF LPBR 12 which are dimensioned in Appendix B. Plain bearings were chosen over linear

ball bearings since both the travel speed and travel distance will be very low while loads are fairly high. Another advantage is that they are maintenance free.

The choice to use four bearings stems from having one on each end of the rack and one on each side of the pinion housing. In this way, bending loads from the tie rods acting on the main rod are minimized and the bending loads from the pinion engagement are also minimized. The outer bearings are also fitted with a seal to keep particles and dust out of the mechanism as shown in Figure 3.8.

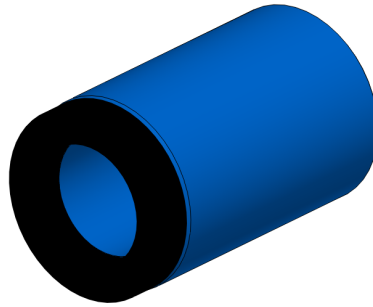


Figure 3.8: SKF LPBR bearing with seal.

### 3.2.3 Pinion shaft

As the selected gearbox is not able to withstand the radial loads from the pinion an intermediate shaft was required which is shown in Figure 3.9. The smaller shaft is held in place by two bronze flange bushings of the type PCMF 081005.5 E on both sides of the pinion which are dimensioned in Appendix B. The shaft diameter was selected based on the pinion bore to 8mm. The keyway is dimensioned in Appendix B.

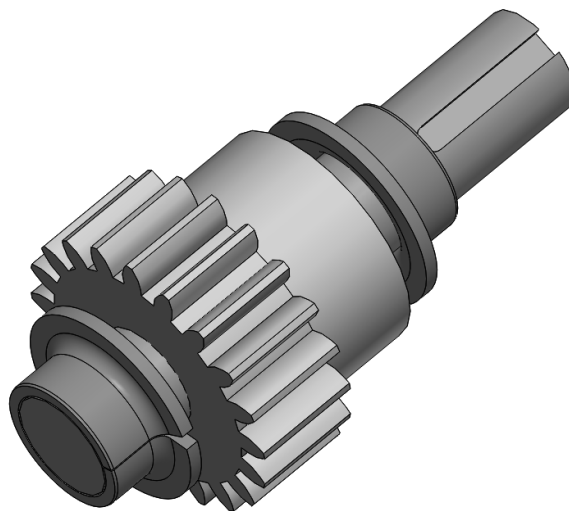


Figure 3.9: The pinion on its shaft and bushings.

Bushings are cheap, easy to install and will not require any maintenance, however the

friction is higher than equivalent roller bearings. They are dimensioned in Appendix B where their frictional moment is also calculated and found to be  $0,443Nm$ . This can also be implemented in the motor dimensioning calculations but was not done.

### 3.2.4 Housing

The housing envelops the rack and pinion and keeps dirt out and lubricant inside and consists of two parts, the upper and lower housing. The lower housing which is shown in Figure 3.10 contains the rack and holds two of the plain linear bearings and one of the plain bushings. The linear bearings are press-fitted in each tube end and on the ends aluminum tubes are pressed on.

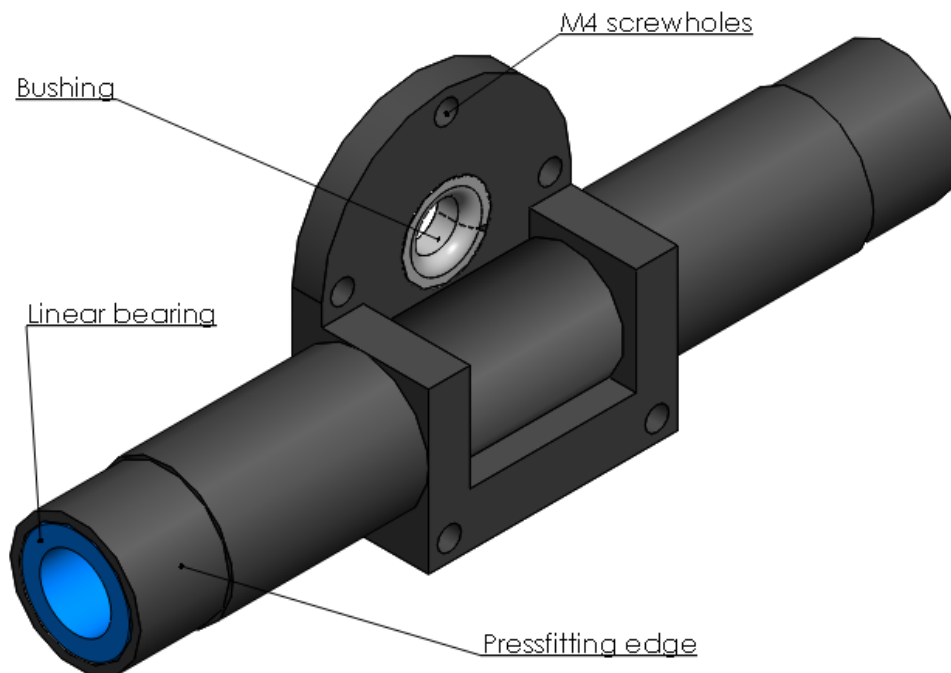


Figure 3.10: The lower housing.

The upper housing which is shown in Figure 3.11 is mounted to the gearbox and holds the other bushing and contains the coupling between the gearbox and the pinion shaft. Both housing components are machined from Alumecc 89<sup>1</sup>.

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<sup>1</sup>Alumecc 89 is a high strength aluminium alloy made by Uddeholm AB that is somewhat similar to EN 7075 and has good machining characteristics. Uddeholm is one of KTH Formula Student's sponsors which is why it is used where possible for aluminium components.

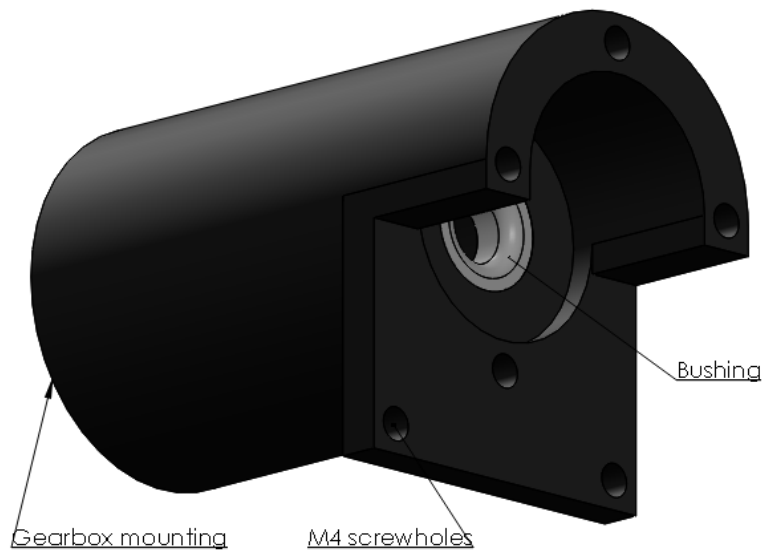


Figure 3.11: The upper housing.

### 3.2.5 Mounts

The mechanism is mounted to the backplate with four M4 screws, two on each side. The mounts themselves shown in Figure 3.12 contain the other two of the linear plain bearings and are also made from Alumecc 89.

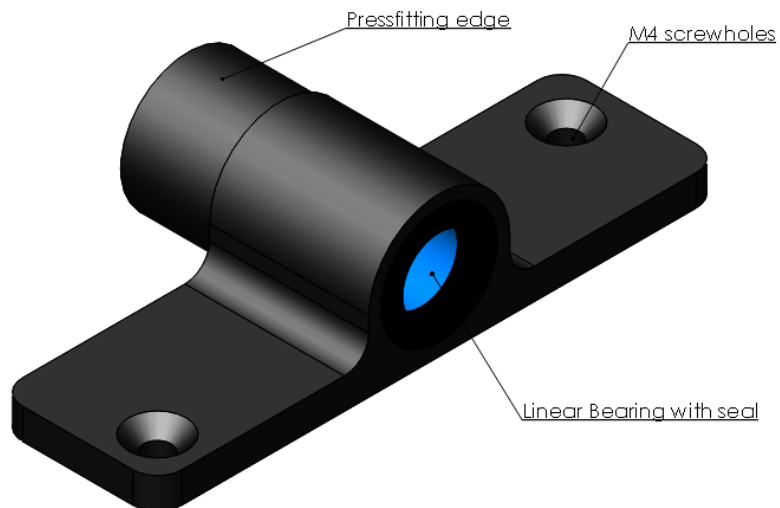


Figure 3.12: The mounts with bearings.

### 3.2.6 Assembly

The assembly procedure is as follows. First the rack is installed in the main rod with two M3 screws. The main rod is then inserted in the housing and the two linear bearings are press-fitted inside the housing. The two cover tubes are then press-fitted on the housings edges. The two sealed linear bearings are then press-fitted in the mounts and the mounts are press-fitted in the cover tubes. This is shown in Figure 3.13.

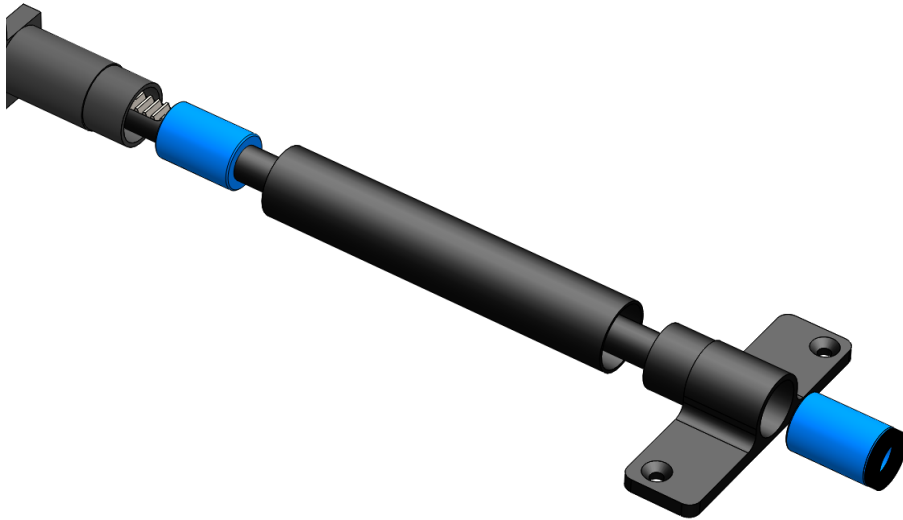


Figure 3.13: Assembly of the main rod, lower housing and mounts.

Now the rod ends can be screwed on with M4 screws. The coupling between the gearbox and pinion shaft is fitted on the shaft from the gearbox and the bushings are mounted in the upper and lower housings. The pinion is shrink fitted on the shaft and is assembled with the lower housing. The upper housing is then screwed on with five M4 screws.

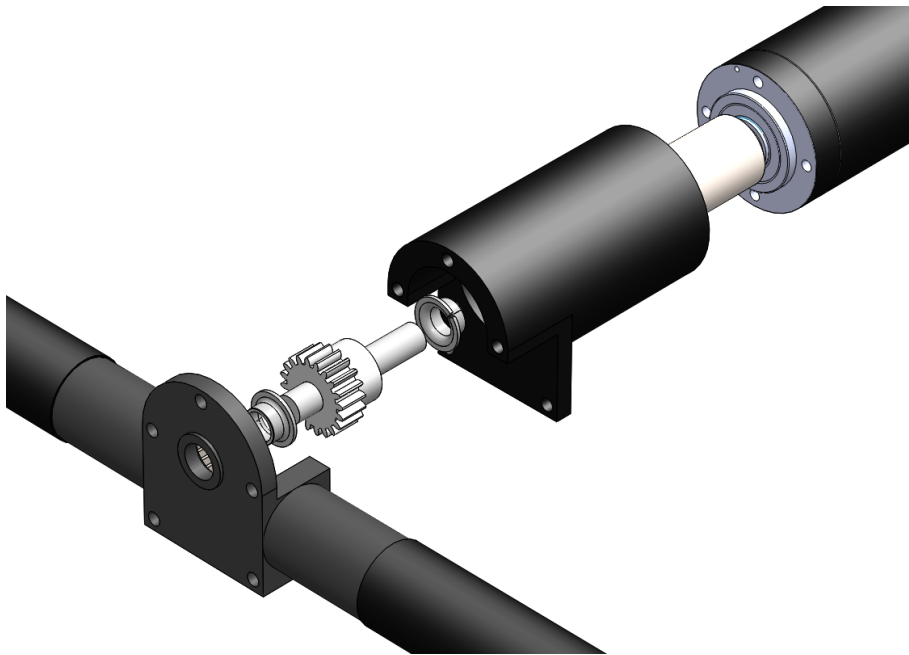


Figure 3.14: Assembling the upper and lower housings.

### 3.3 Verification

#### Rulebook

The mechanism is made to be compatible with all formula student rules. The requirement to have mechanical stops is provided by the mounts and the length of the main rod. The fasteners used can be equipped with the required positive locking mechanisms. As stated previously the limitation on steering system free play is considered to be non applicable on the RWS mechanism and has not been taken into consideration, however the play in the mechanism should be very low since the play in the planetary gearbox is only  $1^\circ$  according to the datasheet.

#### Loads

Every part in the mechanism can withstand the loads with the smallest safety factor being 1,46 for the pinion as shown in Appendix B.

#### Responsiveness

The system fulfills the responsiveness requirement of reaching full travel in 1,66s as shown in Figure 3.15. Because of the low inertia of the motor and gearbox and the self locking characteristics of the latter the mechanism will also stop very quickly.

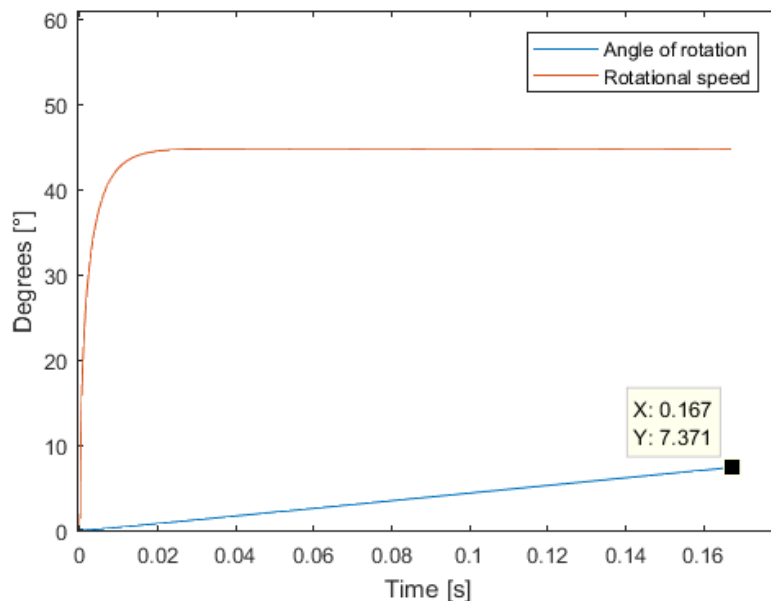


Figure 3.15: System response

#### Packaging

The system does not interfere with the function of any neighboring system. The housing can be taken apart with relative ease for inspection so that the pinion and rack can be greased, and or cleaned. The bearings are maintenance free and requires no lubrication and should keep contaminants out with their seals. The requirement to use positive locking mechanisms means that disassembly and reassembly is rather cumbersome.













## Vehicle dynamic controllability

Because of the electronic control the system has potentially good VD characteristics, however since no VD simulations have been performed it is impossible to draw any conclusions.

## Mass

The total weight of the mechanism is 1,678kg, and a breakdown is shown in Table 3.1. The motor and gearbox are the heaviest components followed by the housing. This summary does not include the weight of fasteners, cables and sensors.

Table 3.1: Breakdown of component weight.

Part	Quantity	Weight[g]	Subtotal Weight[g]	Proportion
Whole Assembly	1	1678	1678	
Motor and Gearbox	1	1040	1040	
Housing	1	211	211	
Main Rod	1	114	114	
Mount	2	41	82	
Connector Socket	1	54	54	
Cover Tubes	2	25	50	
Linear Bearing	4	10	40	
Pinion	1	27	27	
Rack	1	21	21	
Rod end	2	10	20	
Pinion Shaft	1	18	18	
Bushing	2	0,5	1	

## Cost

The cost of the components are calculated according to FSAE cost report rules where applicable.

Table 3.2: Cost breakdown of the system.

Part/Process	Quantity	Material Cost	Manufacturing	Subtotal Cost
Motor and Gearbox	1	€689,28	-	€689,28
Housing And Mounts	1	€68,34	€11,40	€79,74
Bearings	6	€1,96	-	€11,76
Fasteners	9	€0,123	€0,13	€2,28
Interference fit	9	-	€0,19	€1,71
Total Cost	-	-	-	€784,77

It should be noted that this is a very rough estimate of the real world cost of the system. Certain parts that were not found in the cost report repository for formula student were not included such as, pinion and rack and also the pinion shaft.

## Safety

With a self locking mechanism the the danger that the driver is exposed to during a system failure is minimized, however no estimate of the likeliness of system failure has been made.

## Chapter 4

# Discussion and Conclusions

*In this chapter further improvements and future work are discussed together with drawn conclusions based on the results.*

### 4.1 Discussion

#### 4.1.1 Passive 4WS

One concept of RWS that was not investigated in this thesis is passive RWS. The concept revolves around using the forces that are acting on the wheels during a corner to turn them in towards the corner, and applying control via a damper attached to the tie rod. This concept can only use the same steer principle and the vehicle dynamic controllability is not very extensive, however it is a simple solution to possibly achieving the same goals.

The reason this concept was not an option is because the spindles and probably the whole suspension would have to be redesigned for it to provide any benefit. On the current setup the forces on the tires tries to turn the wheels outwards in the corner, and the actuator designed must work against the forces, this is because the A-arm points are in front of the centre of the wheel.

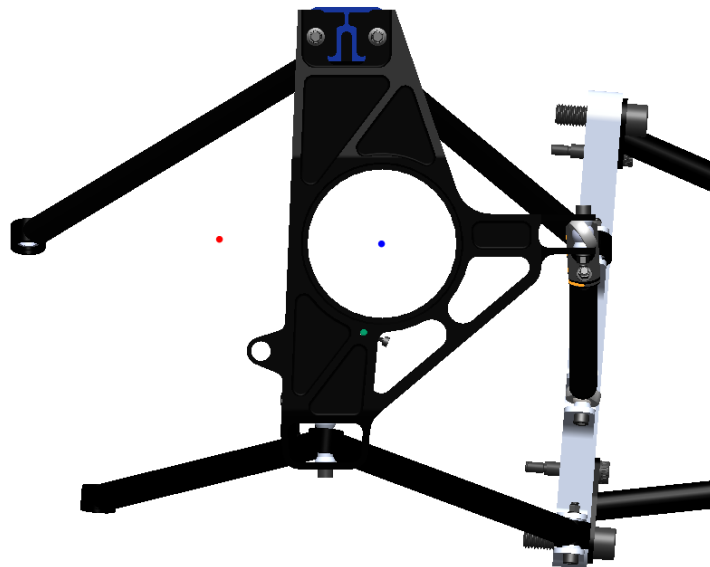


Figure 4.1: Side view of current spindles



During a corner the outer wheel is pushing in towards the corner and if the centre of the wheel is behind the A-arm mounting points the wheels are turned outwards of the corner. By redesigning the uprights so that the centre is in front of the A-arm mounting points passive RWS can be achieved. The tie rods could be disconnected and replaced with dampers with certain characteristics and a fixed travel length. In 4.1 the current centre is illustrated with a blue dot and the potential passive RWS compatible centre in red.

#### **4.1.2 Use of a weaker actuator**

Because of limited data the forces used in this thesis to dimension the actuator may be much larger than the forces that it would be exposed to in reality. The forces used are worst case loads with a very high g-force in mid corner. The actual work that the actuator will have to achieve might be lower since the forces at the beginning of the turn might be significantly smaller. The conclusion of this is that the mechanism can probably be designed with a smaller motor which would reduce the weight, however real world testing or a more comprehensive analysis of the forces during cornering would have to be made.

#### **4.1.3 Optimizing machined aluminium parts**

As can be seen in both B.4 and B.3 the maximum stresses occurring are very low in comparison to the yield strength of Alumeec89. There are also big areas that have low stresses overall. This means that there is potential for further optimization of the geometry of the different parts which can reduce the weight of the mechanism, however a more thorough FEM analysis would have to be made.

#### **4.1.4 Connector socket replaced with claw coupling**

A weak point in the overall design of the system is the manufacturability of the connector socket connecting the shaft from the gearbox and the shaft holding the pinion. As it is designed now it is very difficult to manufacture without using EDM or additive technologies. This connector socket could be replaced with a claw connector which would require modifying the shaft from the gearbox. This would not only remove the connector socket but also allow the housing to be made smaller and lighter. Another possibility is to use the same diameter on the pinion shaft as the one from the gearbox. This would allow the use of a sleeve with integrated keyway instead.

#### **4.1.5 Endpoint killswitches**

Because of the immense stalling torque from the motor it is quite possible for the motor to break the mechanism if it is given power when the rod has reached its endpoint. Therefore it is vital that in designing the control system for the motor this is taken in to consideration, and properly prevented.

## **4.2 Conclusions**

The answers to the research questions are here answered individually.

### **What are the requirements for a 4WS system?**

The main requirements for a 4WS system can be divided into 8 categories, Rulebook compliance, responsiveness, loads, packaging, vehicle dynamic controllability, mass, cost and safety. Exact values for these are found in Chapter 2.

### **Which concepts are feasible?**

Five concepts were found feasible and worth further investigation and are as follows.

**Concept 1:** Two linear actuators.

**Concept 2:** Rack and pinion with rotary actuator.

**Concept 3:** Ackermann with linear actuator.

**Concept 4:** Proportional hydraulic system.

**Concept 5:** Non proportional hydraulic system.

### **Which concept fulfills the requirements the best?**

It was found that Concept 2, Rack and pinion with rotary actuator, fulfills the requirements the best based on estimates and a Pugh matrix analysis.

### **How should the concept be developed and integrated in the car?**

The design was developed by firstly selecting a motor that would cope with the requirements and thereafter designing the rest of the system.

This method worked well.

## **4.3 Recommendations for future work**

There are two main things that need doing before the system can be integrated on the car, a further analysis of the VD behavior to determine if it is worth implementing in the first place and the second one is the regulator design and electronics integration.

In addition to this there are a few improvements on the design that can be made, firstly redesigning the coupling between the pinion shaft and gearbox and secondly optimizing the housings to reduce weight.

### **4.3.1 Comprehensive analysis of RWS vehicle dynamics**

A question that will warrant or discard the implementation of a RWS system is determining if the system will actually benefit the car. A comprehensive vehicle dynamic analysis of 4WS has to be made to determine, for example, if the car would theoretically be able to set a better time on a certain track. The analysis could also help validate the requirements produced in this thesis, or perhaps falsify them. It would also be interesting, assuming that a RWS system actually benefits the car, to know how much mass can be added to the car to negate its benefits as this sets a clear limit on how much the system can weigh, perhaps rendering the system designed in this thesis too heavy.

It is however not certain that the performance gain would appear in simulations, rather the benefits of implementing 4WS might be that the car simply becomes easier to drive. This would allow the inexperienced students driving the car to push it further to its limits without losing control. After all the track performance of the car depends heavily on the driver and enabling the driver to perform more consistently and controlled could potentially be very beneficial.

Regarding the specific formula car there is potential to replace the torque-vectoring system with a 4WS system which would remove the need for two electric motors if the analysis comes to the same conclusions as Suvansh did.

### **4.3.2 Regulator design and electronic integration**

As stated in the introduction no regulator or integration of electronics was made in this thesis. This is however a very important field to tackle before manufacturing the system, and for further research a control would be interesting to design. It is related to the previous point in that it would require a vehicle dynamic analysis. The control loop could read many states apart from steering angle for example, vehicle speed, acceleration, lateral acceleration, angular acceleration an so on.

The electronic integration would also need an analysis of how much electrical energy the system consumes during driving and a new low voltage battery would likely have to be designed.

# Bibliography

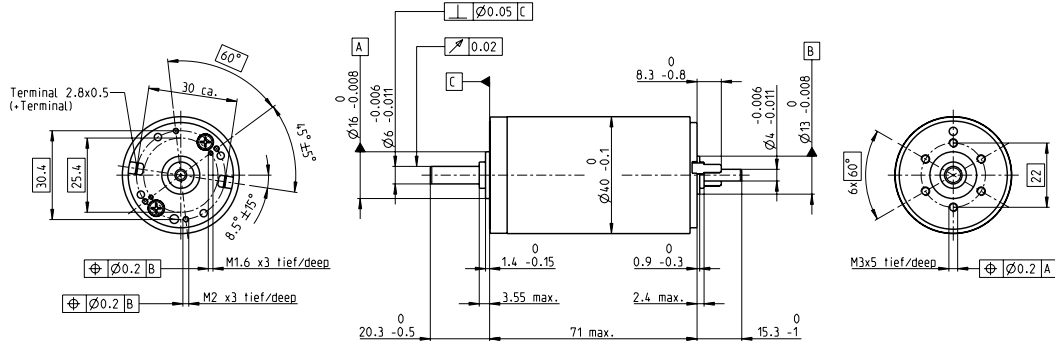
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## Appendix A

# Datasheets of standard components

All datasheets of standard components can be found in this Appendix.

## RE 40 Ø40 mm, Graphite Brushes, 150 Watt



M 1:2

- Stock program
- Standard program
- Special program (on request)

### Part Numbers

148866 **148867** 148877 218008 218009 218010 218011 218012 218013 218014

### Motor Data

Values at nominal voltage		12	24	48	48	48	48	48	48	48
1	Nominal voltage	V	12	24	48	48	48	48	48	48
2	No load speed	rpm	6920	7580	7590	6420	5560	3330	2690	2130
3	No load current	mA	241	137	68.6	53.7	43.7	21.9	16.6	12.5
4	Nominal speed	rpm	6380	6940	7000	5810	4930	2710	2060	1510
5	Nominal torque (max. continuous torque)	mNm	94.9	177	187	186	180	189	190	192
6	Nominal current (max. continuous current)	A	6	6	3.17	2.66	2.23	1.4	1.13	0.909
7	Stall torque	mNm	1720	2420	2560	2040	1620	1020	814	655
8	Stall current	A	105	80.2	42.4	28.6	19.7	7.43	4.79	3.06
9	Max. efficiency	%	88	91	92	91	91	89	89	88
Characteristics										
10	Terminal resistance	Ω	0.115	0.299	1.13	1.68	2.44	6.46	10	15.7
11	Terminal inductance	mH	0.024	0.082	0.33	0.46	0.613	1.7	2.62	4.14
12	Torque constant	mNm/A	16.4	30.2	60.3	71.3	82.2	137	170	214
13	Speed constant	rpm/V	581	317	158	134	116	69.7	56.2	44.7
14	Speed / torque gradient	rpm/mNm	4.05	3.14	2.97	3.16	3.45	3.29	3.31	3.27
15	Mechanical time constant	ms	5.89	4.67	4.28	4.2	4.19	4.16	4.15	4.15
16	Rotor inertia	gcm <sup>2</sup>	139	142	137	127	116	121	120	120

### Specifications

Thermal data		
17	Thermal resistance housing-ambient	4.7 K/W
18	Thermal resistance winding-housing	1.9 K/W
19	Thermal time constant winding	41.5 s
20	Thermal time constant motor	809 s
21	Ambient temperature	-30...+100°C
22	Max. winding temperature	+155°C

Mechanical data (ball bearings)		
23	Max. speed	12000 rpm
24	Axial play	0.05 - 0.15 mm
25	Radial play	0.025 mm
26	Max. axial load (dynamic)	5.6 N
27	Max. force for press fits (static) (static, shaft supported)	110 N
28	Max. radial load, 5 mm from flange	1200 N
		28 N

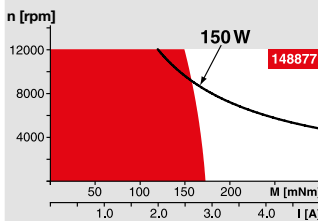
Other specifications		
29	Number of pole pairs	1
30	Number of commutator segments	13
31	Weight of motor	480 g

Values listed in the table are nominal. Explanation of the figures on page 64.

**Option**  
Preloaded ball bearings

- \* Industrial version with radial shaft seal ring (resulting in increased no load current). IP54 protection only if mounted on brush side, in compliance with maxon modular system.

### Operating Range



### Comments

- Continuous operation**  
In observation of above listed thermal resistance (lines 17 and 18) the maximum permissible winding temperature will be reached during continuous operation at 25°C ambient. = Thermal limit.
- Short term operation**  
The motor may be briefly overloaded (recurring).
- Assigned power rating**

### maxon Modular System

Overview on page 28-36

<p><b>Planetary Gearhead</b> Ø42 mm 3 - 15 Nm Page 345</p> <p><b>Planetary Gearhead</b> Ø52 mm 4 - 30 Nm Page 350</p>	<p><b>Recommended Electronics:</b> <b>Notes</b> Page 30</p> <ul style="list-style-type: none"> <li>ESCON Mod. 50/5 427</li> <li>ESCON 50/5 428</li> <li>ESCON 70/10 428</li> <li>EPOS2 24/5 435</li> <li>EPOS2 50/5 435</li> <li>EPOS2 70/10 435</li> <li>EPOS2 P 24/5 438</li> <li>EPOS4 Module/CB 50/5 442</li> <li>EPOS4 Module 50/8 443</li> <li>EPOS4 Comp. 50/8 CAN 443</li> <li>MAXPOS 50/5 447</li> </ul>	<p><b>Encoder MR</b> 256 - 1024 CPT, 3 channels Page 405</p> <p><b>Encoder HED_ 5540</b> 500 CPT, 3 channels Page 413/416</p> <p><b>Brake AB 28</b> 24 VDC 0.4 Nm Page 458</p> <p><b>Industrial Version IP54*</b> <b>Encoder HEDL 9140</b> Page 419</p> <p><b>Brake AB 28</b> Page 459</p> <p><b>End cap</b> Page 463</p>
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Figure A.1: Datasheet for the selected motor, part number 148867.



### Technical Data

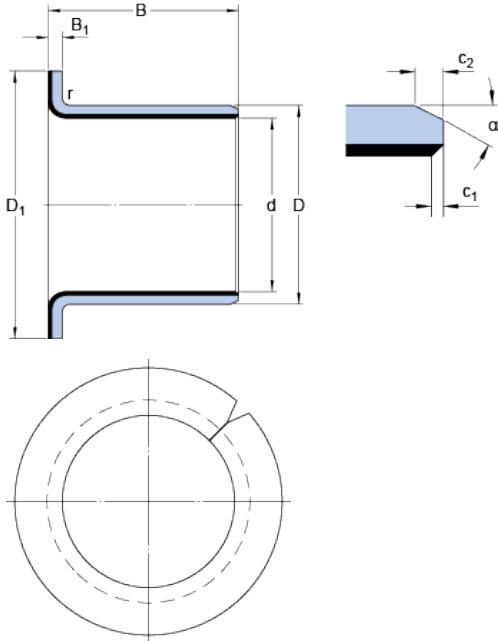
<b>Fw[mm]</b>	12
<b>Accessories</b>	w/o Shaft Seals
<b>Ordering key</b>	LPBR 12
<b>Basic Load Ratings - Dynamic at 0.1 m/s (C)[N]</b>	965
<b>Basic Load Ratings - Dynamic at 4 m/s (C)[N]</b>	24
<b>Basic Load Ratings - Static (Co)[N]</b>	3350
<b>Mass[kg]</b>	0.006

Figure A.3: Datasheet for the selected Linear Bearings.



## PCMF 081005.5 E

### Dimensions



$d$	8	mm
$D$	10	mm
$B$	5.5	mm
$D_1$	15	mm
$B_1$	1	mm
$c_1$	min. 0.1	mm
$c_1$	max. 0.7	mm
$c_2$	min. 0.2	mm
$c_2$	max. 1	mm
$\alpha$	$\pm 8$ 20	$^\circ$
$r$	max. 1	mm

### Recommended fits

Tolerance shaft	$f7$
Tolerance housing	$H7$

### Calculation data

Basic dynamic load rating - radial direction	$C$	2.24	kN
Basic static load rating - radial direction	$C_0$	6.95	kN
Basic dynamic load rating - axial direction	$C_a$	5.1	kN
Basic static load rating - axial direction	$C_{0a}$	16	kN
Specific dynamic load factor	$K$	80	$N/mm^2$
Specific static load factor	$K_0$	250	$N/mm^2$
Factor depending on material and bearing type	$K_M$	480	
Permissible sliding velocity	$v$ min.	0	m/s
Permissible sliding velocity	$v$ max.	2	m/s
Coefficient of friction	$\mu$ min.	0.03	
Coefficient of friction	$\mu$ max.	0.25	

### Mass

Mass bushing	0.0021	kg
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Figure A.4: Datasheet for the selected bushings

# CYLINDRISKA KUGGHJUL

## Modul 1

### Material

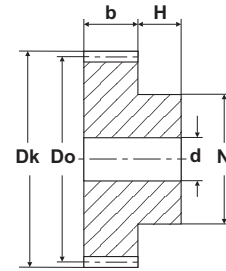
Stål SS EN 10083-1 - C45E (1.1191), kuggkvalitet 8e25



### KUGGBREDD b = 15 mm

Kugg	Artikelnr	Typ	Dk	Do	H	N	d H7
12	10012-S15	N	14	12	10	9	-
13	10013-S15	N	15	13	10	10	-
14	10014-S15	N	16	14	10	11	-
15	10015-S15	N	17	15	10	12	-
16	10016-S15	N	18	16	10	13	-
17	10017-S15	N	19	17	10	14	-
18	10018-S15	N	20	18	10	15	8
19	10019-S15	N	21	19	10	15	8
20	10020-S15	N	22	20	10	16	8
21	10021-S15	N	23	21	10	16	8
22	10022-S15	N	24	22	10	16	8
23	10023-S15	N	25	23	10	18	8
24	10024-S15	N	26	24	10	20	10
25	10025-S15	N	27	25	10	20	10
26	10026-S15	N	28	26	10	20	10
27	10027-S15	N	29	27	10	20	10
28	10028-S15	N	30	28	10	20	10
29	10029-S15	N	31	29	10	20	10
30	10030-S15	N	32	30	10	20	10
31	10031-S15	N	33	31	10	25	10
32	10032-S15	N	34	32	10	25	10
33	10033-S15	N	35	33	10	25	10
34	10034-S15	N	36	34	10	25	10
35	10035-S15	N	37	35	10	25	10
36	10036-S15	N	38	36	10	25	10
37	10037-S15	N	39	37	10	25	10
38	10038-S15	N	40	38	10	25	10
39	10039-S15	N	41	39	10	25	10
40	10040-S15	N	42	40	10	25	10
41	10041-S15	N	43	41	10	30	10
42	10042-S15	N	44	42	10	30	10
43	10043-S15	N	45	43	10	30	10
44	10044-S15	N	46	44	10	30	10
45	10045-S15	N	47	45	10	30	10
46	10046-S15	N	48	46	10	30	10
47	10047-S15	N	49	47	10	30	10
48	10048-S15	N	50	48	10	30	10
49	10049-S15	N	51	49	10	30	10
50	10050-S15	N	52	50	10	30	12
52	10052-S15	N	54	52	10	40	12
54	10054-S15	N	56	54	10	40	12
56	10056-S15	N	58	56	10	40	12
58	10058-S15	N	60	58	10	40	12
60	10060-S15	N	62	60	10	40	12
62	10062-S15	N	64	62	10	50	12
64	10064-S15	N	66	64	10	50	12
66	10066-S15	N	68	66	10	50	12
68	10068-S15	N	70	68	10	50	12
70	10070-S15	N	72	70	10	50	12
72	10072-S15	L	74	72	-	-	12
75	10075-S15	L	77	75	-	-	12
76	10076-S15	L	78	76	-	-	12
80	10080-S15	L	82	80	-	-	12
85	10085-S15	L	87	85	-	-	12
90	10090-S15	L	92	90	-	-	12
95	10095-S15	L	97	95	-	-	12
100	10100-S15	L	102	100	-	-	12
110	10110-S15	L	112	110	-	-	12
114	10114-S15	L	116	114	-	-	12
120	10120-S15	L	122	120	-	-	12
127	10127-S15	L	129	127	-	-	12

TYP N



TYP L

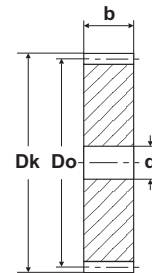


Figure A.5: Datasheet of the pinion, part number 10020-S15

## Appendix B

# Dimensioning of components

### B.1 Dimensioning of rack and pinion gears

The rack and pinion mechanism was dimensioned in accordance with SS 1871 which is based on surface pressure and bending stress.

The dimensioning load is calculated using Equation B.1 where  $M_1$  is the moment on the driving gear and  $d_1$  is its pitch diameter.  $K_I$  is a load factor that depends on the evenness of the driving and driven machine and  $K_v$  is a dynamic load factor that depends on the gear geometry and its rotational speed.

$$F_{ber} = \frac{2T_1}{d_1} \cdot K_I \cdot K_v \quad (\text{B.1})$$

Here  $T_1 = 8,4Nm$ ,  $d_1 = 20mm$  and since the driving machine is considered to run somewhat uneven and the driven mechanism is very uneven  $K_I$  is selected to be 2. Since the rotational speed is very low  $K_v = 1$ . This gives  $F_{ber} = 1680N$ .

For spur gears the surface pressure is given by Equation B.2 where  $b$  is the gear tooth width,  $Z_M$  is a material factor,  $Z_\epsilon = \sqrt{\frac{4-\epsilon}{3}}$  is the transverse contact ratio factor and  $u$  is the gear ratio.

$$\sigma_H = 1,76 \cdot Z_M \cdot Z_\epsilon \sqrt{\frac{F_{ber}(u+1)}{bd_1u}} \quad (\text{B.2})$$

For steel  $Z_M = 268 \frac{\sqrt{N}}{mm}$  and for a transverse contact ratio  $\epsilon = 1,8$ ,  $Z_\epsilon = 0,86$ . The gear tooth width is 8 mm and since it is a rack and pinion mechanism the gear ratio approaches infinity which in total gives  $\sigma_H = 1314MPa$ . Allowed surface pressure is calculated using Equation B.3 where  $\sigma_{Hlim}$  is the fatigue limit for the material,  $S_H$  is a safety factor and  $K_{HN}$  is a lifetime factor depending on the number of load cycles.

$$\sigma_{HP} = \frac{\sigma_{Hlim} \cdot K_{HN}}{S_H} \quad (\text{B.3})$$

$K_{HN} = 1,75$  corresponds to  $10^5$  load cycles and for nitrided SS 1672/EN1.1191 hardened to at least 36HRC  $\sigma_{Hlim} = 1100MPa$  according to SMS 2995[9] a safety factor of 1,46 is achieved.

For spur gears the bending stress is calculated using Equation B.4 where  $Y_\epsilon = \frac{1}{\epsilon}$  and  $Y_F$  can be approximated using Equation B.5 where  $z$  is the number of gear teeth. This gives  $\sigma_F = 343MPa$ .

$$\sigma_F = Y_F \cdot Y_\epsilon \cdot \frac{F_{ber}}{b \cdot m} \quad (\text{B.4})$$

$$Y_F = 2,2 + 3,1 \cdot e^{\frac{-z}{14}} \quad (\text{B.5})$$

Allowed bending stress is given by Equation B.6 where  $\sigma_{Flim}$  is the fatigue limit for the material,  $K_{FN}$  is a lifetime factor and  $S_F$  is a safety factor.

$$\sigma_{FP} = \frac{\sigma_{Flim} \cdot K_{FN}}{S_F} \quad (\text{B.6})$$

For steel hardened to 36HRC ( $\approx 340HB$ ) and  $10^5$  load cycles,  $K_{FN} = 1,5$ . SMS 2995 gives  $\sigma_{Flim} = 350\text{MPa}$  for nitrided SS 1672/EN1.1191 which gives a safety factor  $S_F = 1.53$ .

## B.2 Dimensioning of pinion shaft

The torque that a key can transmit is calculated using Equation B.7.

### B.2.1 Keyway

$$M_v = \frac{phld}{4} \quad (\text{B.7})$$

With a maximum allowed surface perssure  $p = 200\text{MPa}$ ,  $h = 4\text{mm}$ ,  $l = 12\text{mm}$  and  $d = 8\text{mm}$  the calculated maximal moment for the keyway was  $M_v = 19,2\text{N m}$ .

### B.2.2 Bushings

#### Loads

The bushings had a Basic Dynamic Load rating in the radial direction which was  $C = 2,24\text{kN}$  which is well above the approximate bearing load of 400N each. The bushings were not analyzed for longevity.

#### Friction

To calculate the approximate moment of friction that would arise in the two bearings holding the shaft, an average of the maximum and the minimum frictional coefficients from the specification sheet was used. The normal force on the bearings is calculated using Equation B.8 and the frictional moment is the calculated using Equation B.9.

$$F_N = \frac{F_R + F_L}{\cos \phi} \quad (\text{B.8})$$

$$M_f = \frac{dF_N \mu_{mid}}{2} \quad (\text{B.9})$$

With  $d = 8\text{mm}$  and  $\mu_{mid} = 0,14$  and  $F_N = 791\text{N}$  the moment of friction is  $M_f = 0,443\text{N m}$  which can be implemented in the motor dimensioning calculations, however this has not been done.

## B.3 Dimensioning of linear bearings

The Basic Load Rating at  $0,1\text{ms}^{-1}$ , which is similar to the velocity the rack will be traveling, was 965N. The maximal load in the radial direction for one of the linear bearings is  $F_R \sin \phi = 167,5\text{N}$ . The longevity and friction for the linear bearings was not analyzed.

## B.4 Dimensioning of other components with FEM

### B.4.1 FEM analysis

The different parts in the mechanism were analyzed with SolidWorks FEM tool. Mesh sizing and mesh control were basic and the accuracy and the convergence of the analysis was not studied thoroughly, instead the FEM analysis was used to indicate problems in the design, or to verify the reasonableness of different design choices.

### B.4.2 Analysis of housing

A FEM model of the housing was made where the ends of the tubes were fixed. The bearing forces of 800N was added and the torque of 8 Nm from the motor mount was added in.

The FEM analysis of the housing revealed a large stress concentration in the edge between the tube part of the housing. However this could be alleviated by adding a radius on the inside edge.

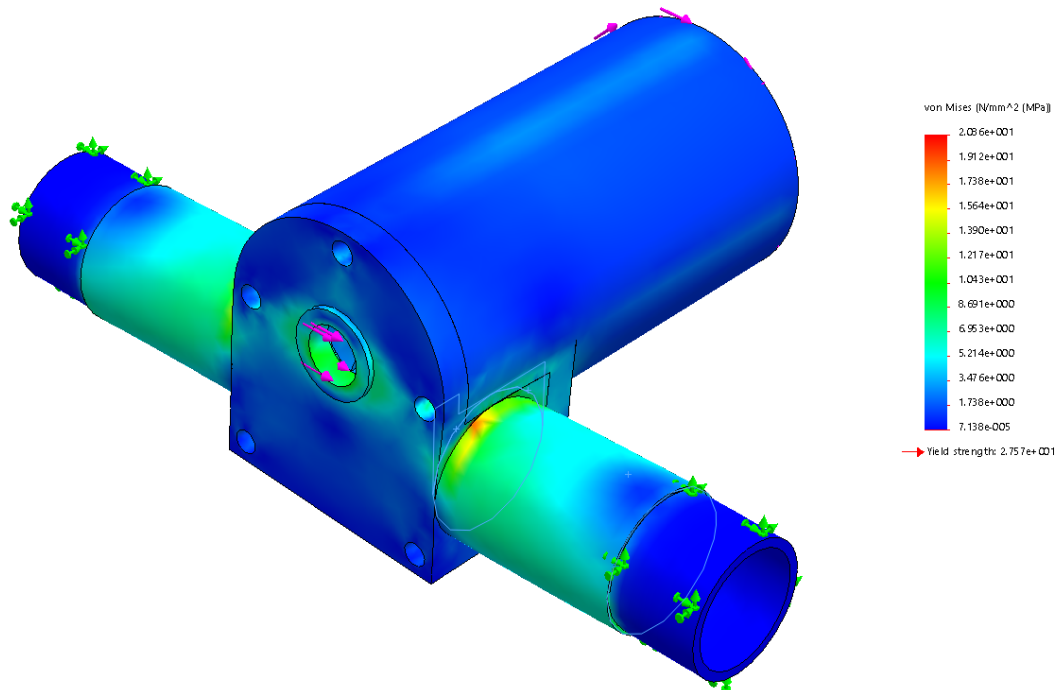


Figure B.1: FEM of the housing.

Initially in B.1 a course mesh was used and a stress concentration appeared at the edge between the round part and the rectangular part.

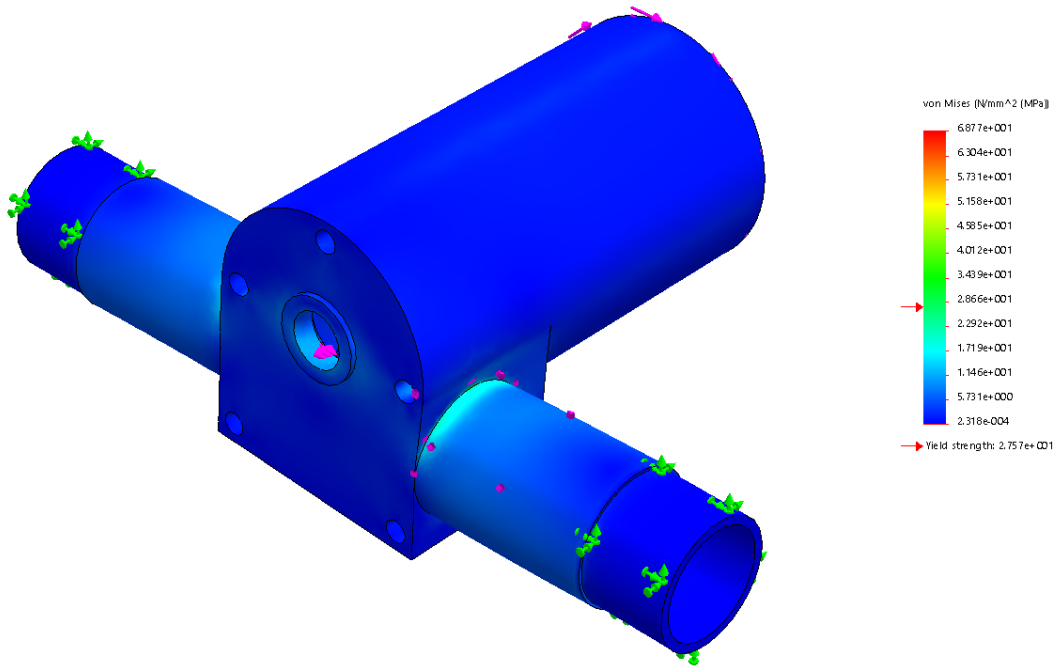


Figure B.2: FEM of the housing with a finer mesh.

Using a refinement of the mesh in the specific corner it shows in B.2 that there is a diverging stress concentration in the corner.

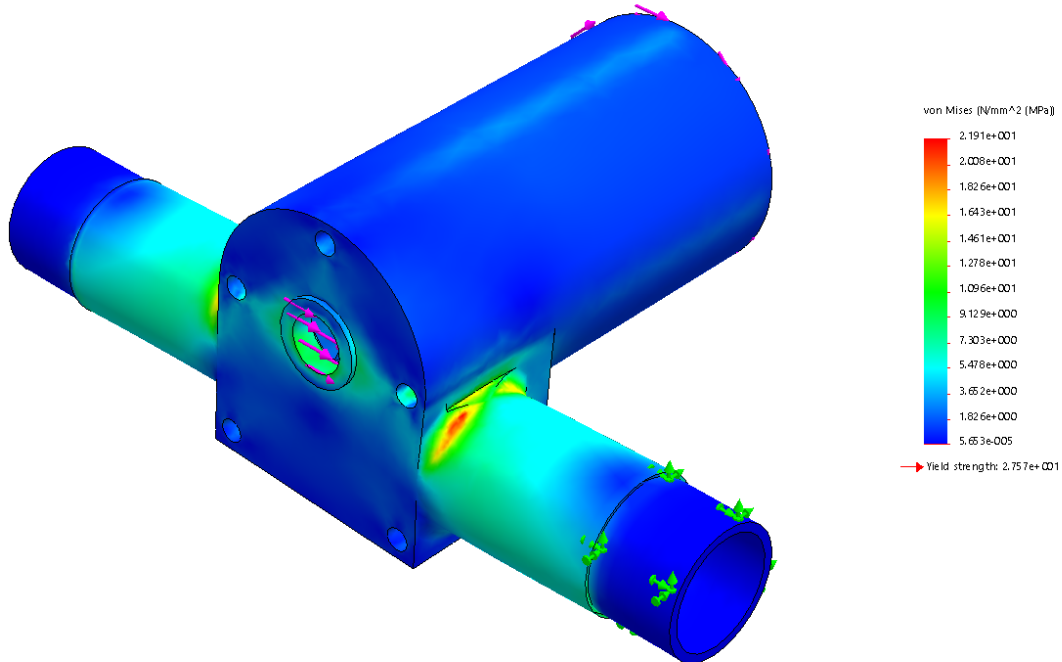


Figure B.3: FEM of the housing with an added radius in problem corner.

After adding a radius in the corner of 3mm in B.3 the problem was attenuated, however the corner remains a weak point in the design. The maximum von Mises stress was around 22MPa. The Yield strength of Alumec is 520MPa.

### B.4.3 Analysis of the main rod

A FEM model of the main rod was also made. The rod was extended to its end point, simulating a full steering angle lock out so that the bending forces from the rod ends would be maximized. The Linear bearings were fixed and both the transversal and the longitudinal component of the rod end forces were added in.

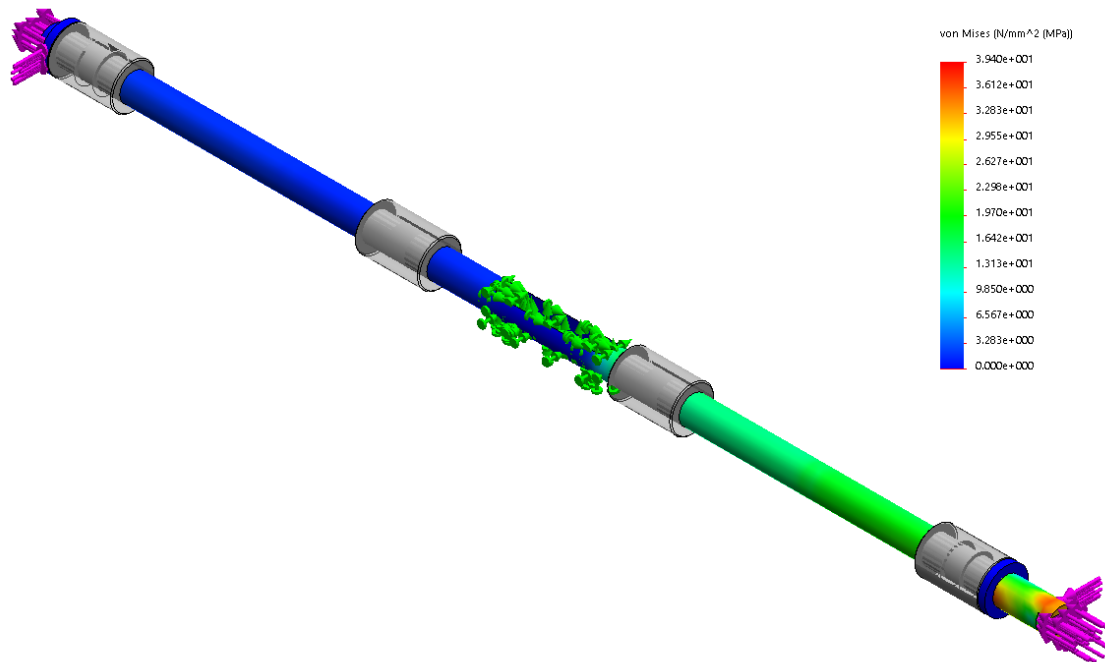


Figure B.4: Showing the Von Mises Stress of the main rod in the worst load case

The maximum stress occurring was around 40MPa with stresses stemming from the bending forces in the rod ends.

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