

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos \frac{n\pi}{L} x + b_n \sin \frac{n\pi}{L} x)$$

$P = 2L$

توجه کنید:

$$a_n = \frac{1}{L} \int_{-L}^L f(x) \cos \frac{n\pi}{L} x dx$$

$$b_n = \frac{1}{L} \int_{-L}^L f(x) \sin \frac{n\pi}{L} x dx$$

$$\frac{a_0}{2} = \frac{1}{2L} \int_{-L}^L f(x) dx = \frac{\text{سطح فضاها زیر نمودار درین بازه}}{\text{طول بازه}}$$

تمرین 1: سری فوریه  $f(x) = x + |x|$ ,  $-1 < x < 1$  را بسازید.

حل:  $f(x) = \begin{cases} 0 & -1 < x < 0 \\ 2x & 0 \leq x < 1 \end{cases}$

$$\frac{a_0}{2} = \frac{1/2}{2} = \frac{1}{4} \quad P = 2L = 2$$

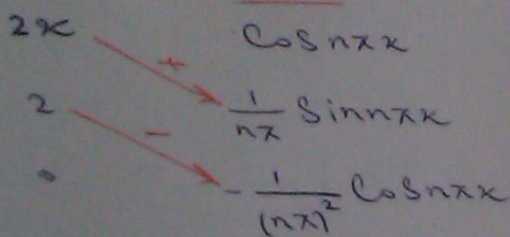
$$a_n = \frac{1}{1} \int_0^1 2x \cos n\pi x dx = \frac{2x}{n\pi} \sin n\pi x + \frac{1}{(n\pi)^2} \cos n\pi x \Big|_0^1 = \frac{(-1)^n}{(n\pi)^2} - \frac{1}{(n\pi)^2}$$

$$a_n = \begin{cases} 0 & n: \text{زوج} \\ \frac{-2}{(n\pi)^2} & n: \text{فرد} \end{cases}$$

مشتق

انتگرال

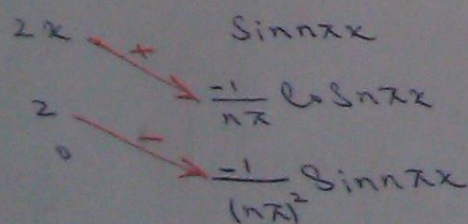
توضیح برای معادله ی انتگرال \*



$$b_n = \int_0^1 2x \sin n\pi x dx = -\frac{2x}{n\pi} \cos(n\pi)x + \frac{2}{(n\pi)^2} \sin(n\pi)x \Big|_0^1 = \frac{2(-1)^{n+1}}{n\pi}$$

مشتق

انتگرال



$$f(x) = \frac{1}{4} + \sum_{n=1}^{\infty} \left( \frac{-2}{(2n-1)\pi^2} \cos(2n-1)\pi x + \frac{2(-1)^{n+1}}{n\pi} \sin(n\pi x) \right)$$

در نهایت:

را با ...  

$$f(x) = \begin{cases} \cos 3x & 0 < x < \pi \\ f(-x) & -\pi < x < 0 \end{cases}$$

...  
 $P(x) = f(x)$  فرد با دوره تناوب  $P = 2\pi$  و  $a_n = 0$  و  $b_n$  را با ...

$$f(x) = \begin{cases} \cos 3x & 0 < x < \pi \\ -\cos 3x & -\pi < x < 0 \end{cases} \quad b_n = \frac{2}{\pi} \int_0^{\pi} \cos 3x \sin nx \, dx =$$

$$\frac{1}{\pi} \int_0^{\pi} [\sin(n+3)x + \sin(n-3)x] \, dx = \frac{1}{\pi} \left[ \frac{-\cos(n+3)x}{n+3} - \frac{\cos(n-3)x}{n-3} \right]_0^{\pi}$$

$$= \frac{1}{\pi} \left( \frac{-6 \sin n \sin 3 - 2n \cos n \cos 3}{n^2 - 9} + \frac{2n}{n-3} \right)$$

$$f(x) = \frac{1}{\pi} \sum_{n=1}^{\infty} \left( \frac{-6 \sin n \sin 3 - 2n \cos n \cos 3}{n^2 - 9} + \frac{2n}{n-3} \right) \sin nx$$

...  

$$f(x) = \begin{cases} x^2 & 0 < x < 1 \\ -x^2 & -1 < x < 0 \end{cases}$$

...  
 $P(x) = f(x)$  فرد است با دوره تناوب  $P = 2$  و  $a_n = 0$  و  $b_n$  را با ...

$$b_n = 2 \int_0^1 x^2 \sin n\pi x \, dx = \left[ \frac{-x^2}{n\pi} \cos n\pi x + \frac{2x}{(n\pi)^2} \sin n\pi x + \frac{2}{(n\pi)^3} \cos n\pi x \right]_0^1$$

توضیح:

$x^2$	$\times$	$\sin n\pi x$	$\rightarrow$	$\frac{1}{n\pi} \cos n\pi x$
$2x$	$-$	$\frac{1}{n\pi} \cos n\pi x$	$\rightarrow$	$-\frac{1}{(n\pi)^2} \sin n\pi x$
$2$	$+$	$-\frac{1}{(n\pi)^2} \sin n\pi x$	$\rightarrow$	$\frac{1}{(n\pi)^3} \cos n\pi x$

$$\Rightarrow b_n = \frac{2(-1)^{n+1}}{n\pi} + \frac{4(-1)^n}{(n\pi)^3} + \frac{4}{(n\pi)^3}$$

و چون  $n$  زوج  $b_n = \frac{8 - 2(n\pi)^2}{(n\pi)^3}$

و چون  $n$  فرد  $b_n = \frac{2}{n\pi}$

$$f(x) = \sum_{n=1}^{\infty} \left[ \frac{2}{(2n-1)\pi} \sin(2n-1)\pi x + \frac{8 - 2(2n\pi)^2}{(2n\pi)^3} \sin(2n\pi x) \right]$$

$$\frac{a_0}{2} = \frac{1 + \frac{1}{2}}{2} = \frac{3}{4}$$

...  

$$f(x) = \begin{cases} x & 0 < x < 1 \\ 1 & -1 < x < 0 \end{cases}$$

...  

$$a_n = \int_{-1}^1 f(x) \cos n\pi x \, dx = \int_{-1}^0 1 \cos n\pi x \, dx + \int_0^1 x \cos n\pi x \, dx =$$

$$\left( \frac{1}{n\pi} \sin n\pi x \right) \Big|_{-1}^0 + \left( \frac{x}{n\pi} \sin n\pi x + \frac{1}{(n\pi)^2} \cos n\pi x \right) \Big|_0^1 = \frac{(-1)^n}{(n\pi)^2} + \frac{1}{(n\pi)^2}$$

$$\Rightarrow a_n = \begin{cases} \frac{2}{(n\pi)^2} & \text{زوج } n \\ 0 & \text{فرد } n \end{cases}$$

$$b_n = \int_{-1}^1 f(x) \sin n\pi x dx = \int_{-1}^0 \sin n\pi x dx + \int_0^1 x \sin n\pi x dx =$$

$$\frac{-1}{n\pi} + \frac{(-1)^n}{n\pi} + \frac{(-1)^{n+1}}{n\pi} \Rightarrow b_n = \frac{-1}{n\pi}$$

$$f(x) = \frac{3}{4} + \sum_{n=1}^{\infty} \left( \frac{1}{2(n\pi)^2} \cos n\pi x - \frac{1}{n\pi} \sin n\pi x \right)$$

سوال (۵) سری فوریه  $f(x) = e^{2x}$ ,  $-\pi < x < \pi$

$$p = 2\pi, \quad \frac{a_0}{2} = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{2x} dx = \frac{1}{2\pi} \left( \frac{1}{2} e^{2x} \right) \Big|_{-\pi}^{\pi} = \frac{1}{4\pi} (e^{2\pi} - e^{-2\pi})$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} e^{2x} \cos n\pi x dx$$

$$\text{فرض: } I = \frac{1}{\pi} \int_{-\pi}^{\pi} e^{2x} \cos n\pi x dx \Rightarrow$$

$$I = \frac{e^{2x}}{n^2} (n \sin n\pi x + 2 \cos n\pi x) - \frac{4}{n^2} I \Rightarrow$$

$$I \left( 1 + \frac{4}{n^2} \right) = \frac{e^{2x}}{n^2} (n \sin n\pi x + 2 \cos n\pi x) \Rightarrow I = \frac{e^{2x}}{4+n^2} (n \sin n\pi x + 2 \cos n\pi x)$$

$$\Rightarrow a_n = \frac{1}{\pi} I \Big|_{-\pi}^{\pi} = \frac{2e^{2\pi}}{4+n^2} (2(-1)^n) - \frac{2e^{-2\pi}}{4+n^2} (-1)^n = \frac{2(-1)^n}{4+n^2} (e^{2\pi} - e^{-2\pi})$$

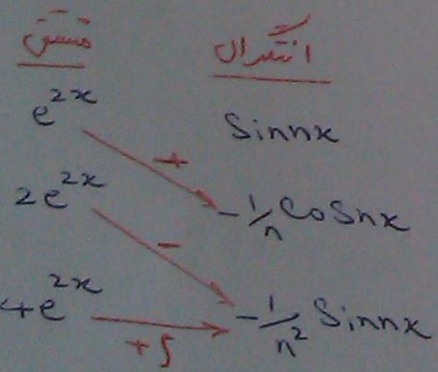
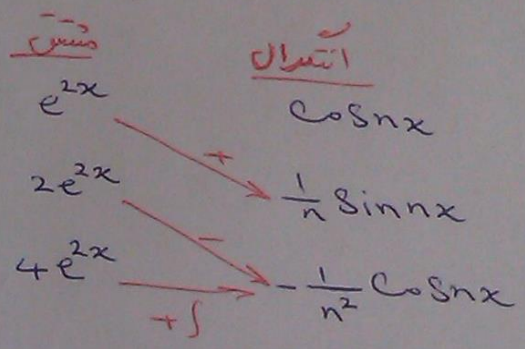
$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} e^{2x} \sin n\pi x dx$$

$$\text{فرض: } I = \int e^{2x} \sin n\pi x dx \Rightarrow$$

$$I = \frac{e^{2x}}{n^2+4} (2 \sin n\pi x - n \cos n\pi x)$$

$$b_n = \frac{1}{\pi} I \Big|_{-\pi}^{\pi} = \frac{n(-1)^{n+1}}{n^2+4} (e^{2\pi} - e^{-2\pi})$$

$$f(x) = (e^{2\pi} - e^{-2\pi}) \left[ \frac{1}{4\pi} + \sum_{n=1}^{\infty} \frac{1}{n^2+4} (2(-1)^n \cos n\pi x + n(-1)^{n+1} \sin n\pi x) \right]$$



سوال 14 سے نمونہ طور پر  $f(x) = \sin lx, -\pi < x < \pi$  اور  $a_n = 0, p = 2\pi n$  کے ساتھ  $P(x)$  تابعی فرد ہے۔

$$\tilde{f}(x) = \frac{e^x - e^{-x}}{2}$$

سوال 14 سے نمونہ طور پر  $f(x) = \sin lx, -\pi < x < \pi$  اور  $a_n = 0, p = 2\pi n$  کے ساتھ  $P(x)$  تابعی فرد ہے۔

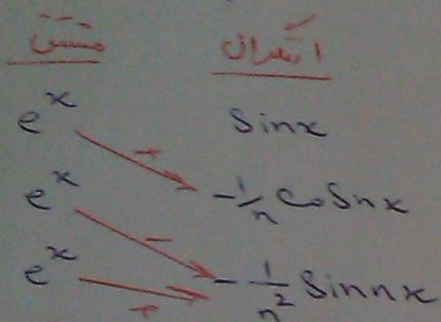
$$b_n = \frac{2}{\pi} \int_0^{\pi} \left( \frac{e^x - e^{-x}}{2} \right) \sin nx \, dx = \frac{1}{\pi} \left[ \int_0^{\pi} (e^x) \sin nx \, dx - \int_0^{\pi} e^{-x} \sin nx \, dx \right]$$

فرض:  $I = \int_0^{\pi} e^x \sin nx \, dx$

$$I = \frac{-e^x}{1+n^2} \cos nx + \frac{e^x}{n^2} \sin nx - \frac{1}{n^2} I \Rightarrow$$

$$I = \frac{e^x}{1+n^2} (\sin nx - n \cos nx) \Rightarrow$$

$$\int_0^{\pi} e^x \sin nx \, dx = \frac{n}{1+n^2} (e^{\pi} (-1)^{n+1} + 1)$$



دوسرے طرف سے  $\int_0^{\pi} e^{-x} \sin nx \, dx = \frac{n}{n^2-1} (1 + e^{-\pi} (-1)^{n+1})$

$$b_n = \frac{1}{\pi} \left\{ \frac{n}{1+n^2} e^{\pi} (-1)^{n+1} - \frac{n}{n^2-1} e^{-\pi} (-1)^{n+1} - \frac{2n}{n^2-1} \right\}$$

$$f(x) = \frac{1}{\pi} \sum_{n=1}^{\infty} \left[ \frac{n}{1+n^2} e^{\pi} (-1)^{n+1} - \frac{n}{n^2-1} e^{-\pi} (-1)^{n+1} - \frac{2n}{n^2-1} \right] \sin nx$$

سوال 17 سے نمونہ طور پر  $f(x) = \sin lx, -\pi < x < \pi$

$\sin$  تابعی فرد ہے اس لیے  $f(x)$  تابعی فرد ہر دورہ میں  $p = 2\pi$  ہے۔

$a_n = 0$

$$b_n = \frac{2}{\pi} \int_0^{\pi} \sin lx \sin nx \, dx = \frac{1}{\pi} \int_0^{\pi} [\cos(l-n)x - \cos(l+n)x] \, dx$$

$$= \frac{1}{\pi} \left[ \frac{1}{l-n} \sin(l-n)x - \frac{1}{l+n} \sin(l+n)x \right]_0^{\pi} =$$

$$\frac{1}{\pi} \left[ \frac{1}{l-n} \sin(l-n)\pi - \frac{1}{l+n} \sin(l+n)\pi \right] \Rightarrow$$

$$f(x) = \frac{1}{\pi} \sum_{\substack{n=1 \\ n \neq l}}^{\infty} \left[ \frac{1}{l-n} \sin(l-n)\pi - \frac{1}{l+n} \sin(l+n)\pi \right]$$