Impact Mechanics

Applications of 1D Theory

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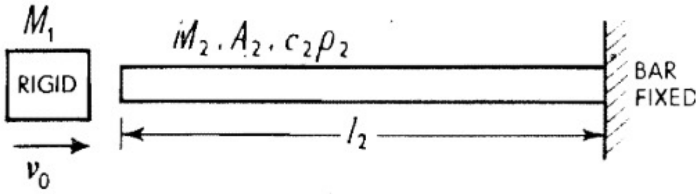
Content

- Impact of rigid mass and striker
- Pile driver
- Hopkinson pressure bar
- Split-Hopkinson Pressure Bar
- Design of Pressure Bars for a SHPB Apparatus
- Spalling or scabbing in bars and flat plates
- Impulsive surface loading
- Helical springs
- Buckling of bars and tubes due to impact

Impact of Rigid Mass on Bars

Rigid Mass



- Equation of motion for M_1: $M_1 \frac{dv_1}{dt} = -A_2 \sigma_2$
- Since we have $\sigma_2 = v_1 \sqrt{E_2 \rho_2}$

$$\frac{M_1}{A_2} \cdot \frac{dv_1}{v_1} = -\sqrt{E_2 \rho_2} dt$$

- Integrating:

$$\frac{M_1}{A_2} \int_{v_0}^{v_1} \frac{dv_1}{v_1} = -\sqrt{E_2 \rho_2} \int_0^t dt$$

$$v_1 = v_0 \exp\left(-\frac{A_2}{M_1} \cdot \sqrt{E_2 \rho_2} \cdot t\right)$$

Rigid Mass

- Alternatively,

$$\sigma_2 = \sigma_0 \cdot \exp\left(-\frac{A_2}{M_1} \cdot \sqrt{E_2\rho_2} \cdot t\right)$$
$$= \sigma_0 \cdot \exp\left(-\frac{M_2}{M_1} \cdot \frac{t}{l_2/c_2}\right)$$

- Where

$$\sigma_0 = v_0\sqrt{E_2\rho_2}$$

- Also, after time $t = l_2/c_2$ the stress at the striker σ_2 will fall to

$$= \sigma_0 \exp\left(-M_2/M_1\right)$$

Rigid Mass

- At the end of time $2l_2/c_2$, i.e. when the head of the reflected wave has arrived back at the striker and is just reflected to provide a compressive stress of $2\sigma_0$, the total compressive stress on the striker will be $[2\sigma_0 + \sigma_0 \exp(-2M_2/M_1)]$.
- Now the striker will be subject to an increased resistance, for $t > 2l_2/c_2$, due to the continuous arrival of the reflected wave from the fixed end of the bar, i.e. σ_2, and hence it will suffer an abrupt change in speed; this in turn will alter the initiating stress.
- Thus the differential equation and expression only apply for $0 < t < 2l_2/c_2$

Rigid Mass

- The new equation for $2l_2/c_2 < t < 4l_2/c_2$, or $0 < t' < 2l_2/c_2$ is

$$M_1 \cdot \frac{dv_1'}{dt'} = -A_2[2\sigma(t') + \sigma'(t')]$$

$$\frac{M_1}{\rho_2 c_2} \cdot \frac{d\sigma'}{dt'} = -A_2[2\sigma(t') + \sigma'(t')]$$

  - using $\sigma' = \rho_2 c_2 v_1'$ and where $\sigma(t') = \sigma_2(t)$.

- Simplifying,

$$-\frac{M_1}{A_2\rho_2 c_2} \cdot \frac{d\sigma'}{dt'} = 2\sigma_0 \exp\left(-\frac{A_2}{M_1} \cdot \sqrt{E_2\rho_2} \cdot t'\right) + \sigma'$$

$$\exp\left(\frac{A_2}{M_1} \cdot \sqrt{E_2\rho_2} \cdot t'\right) \cdot \frac{d\sigma'}{dt'} + \frac{A_2 \cdot \sqrt{E_2\rho_2}}{M_1} \cdot \exp\left(\frac{A_2}{M_1} \cdot \sqrt{E_2\rho_2} \cdot t'\right)\sigma' = -\frac{2A_2\rho_2 c_2 \sigma_0}{M_1}$$

Rigid Mass

- Thus

$$\frac{d}{dt'}\left[\exp\left(\frac{A_2}{M_1} \cdot \sqrt{E_2\rho_2} \cdot t'\right)\sigma'\right] = -\frac{2A_2\rho_2 c_2}{M_1}\sigma_0$$

- Integrating,

$$\sigma' \cdot \exp\left(\frac{A_2}{M_1} \cdot \sqrt{E_2\rho_2} \cdot t'\right) = -\frac{2A_2\rho_2 c_2}{M_1}\sigma_0 t' + c$$

- Now, when $t' = 0$ or $t = 2l_2/c_2$, $\sigma' = \sigma_0 \exp\left(-\frac{2M_2}{M_1}\right) = c$

$$\sigma' = \exp\left(-\frac{A_2\sqrt{E_2\rho_2}}{M_1} \cdot t'\right)\left[\sigma_0 \exp\left(-\frac{2M_2}{M_1}\right) - \frac{2A_2\rho_2 c_2}{M_1}\sigma_0 t'\right]$$

Rigid Mass

- The total compressive stress ${}_S\sigma_T$, acting on the striker during the second traversal of the bar by the stress wave, is

$$ {}_S\sigma_T = 2\sigma(t) + \sigma'(t') $$

- Hence,

$$ {}_S\sigma_T = 2\sigma_0 \exp\left(-\frac{A_2}{M_1}\sqrt{E_2\rho_2}\cdot t'\right) + \exp\left(-\frac{A_2}{M_1}\cdot\sqrt{E_2\rho_2}\cdot t'\right)\cdot\sigma_0\exp\left(-\frac{2M_2}{M_1}\right) - \exp\left(-\frac{A_2}{M_1}\cdot\sqrt{E_2\rho_2}t'\right)\cdot 2\sigma_0\cdot t'\cdot\frac{A_2\rho_2 c_2}{M_1} $$

Rigid Mass

$$ {}_s\sigma_T = \sigma_0 \exp\left(-\frac{A_2}{M_1}\sqrt{E_2\rho_2}t'\right)\left[2 + \exp\left(-\frac{2M_2}{M_1}\right) - \frac{2t'}{l_2/c_2}\cdot\frac{M_2}{M_1}\right] $$

- If the stress falls to zero during the second wave cycle, after the greatest permissible time, i.e. $t' = 2l_2/c_2$, then we have

$$ 2 + e^{-2M_2/M_1} = 4M_2/M_1 $$

- Solving, $$ M_2/M_1 \cong 0.58 $$
  - Of course, for all values of $M_2/M_1 > 0.58$, the stress falls to zero for $\frac{l_2}{c_2} < t' < \frac{2l_2}{c_2}$.
  - That the stress falls to zero does not imply that contact then ceases.

Case $M_2/M_1 = 1$

- Putting ${}_S\sigma_T = 0$ in (2.8):

$$2 + e^{-2} = \frac{2t'}{l_2/c_2}$$

- Hence,

$$\frac{t'}{l_2/c_2} = 1 + \frac{1}{2}e^{-2} = 1.068$$

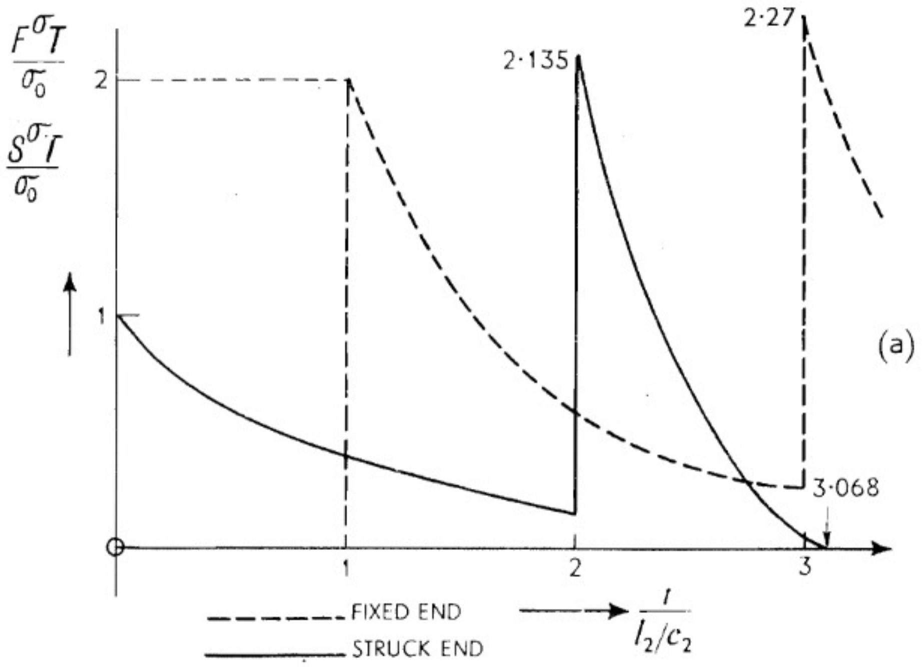
- Or,

$$\frac{t}{2l_2/c_2} = \frac{t'+2l_2/c_2}{2l_2/c_2} = 1.534$$

TABLE 2.1 VALUES OF ${}_S\sigma_T/\sigma_0$ FOR $M_2/M_1 = 1$

$t/(l_2/c_2)$	0	$\frac{1}{2}$	1	$1\frac{1}{2}$	$2-$	$2+$	$2\frac{1}{4}$	$2\frac{1}{2}$	3	3·068
$0 < t < 2l_2/c_2$	1	0·60	0·37	0·22	0·135	—	—	—	—	—
$\begin{cases} 0 < t' < 2l_2/c_2 \\ 2l_2/c_2 < t < 4l_2/c_2 \end{cases}$	—	—	—	—	—	2·135	1·27	0·685	0·050	0

Case $M_2/M_1 = 1$



Stress at the Fixed End

- For $M_2/M_1 = 1$, stress at the fixed end of the rod is:
  - Zero until $t = l_2/c_2$, when it suddenly becomes $2\sigma_0$
  - It falls to $2\sigma_0 e^{-M_2/M_1} = 2\sigma_0 e^{-1} = 0.74\sigma_0$ at $t = 2l_2/c_2$
  - At $t = 3l_2/c_2$ the stress at the fixed end due to reflection of the second wave cycle stress front, original1y initiated at the striker at $t' = 0$, is

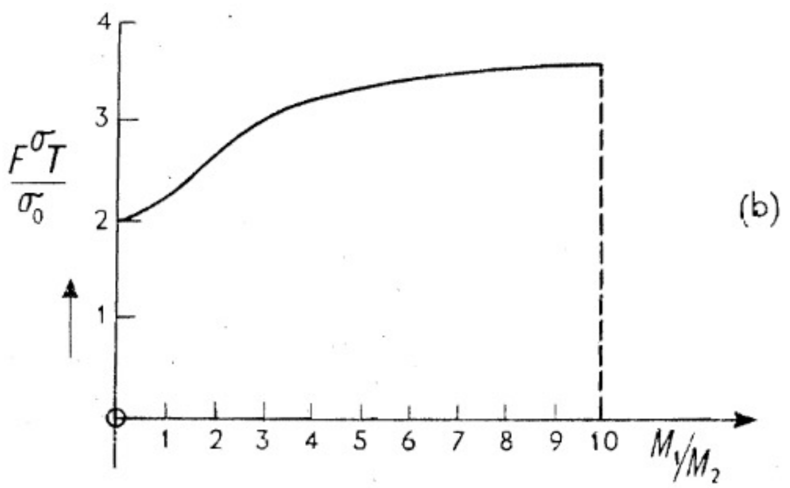
$$_F\sigma_T = 2\left(\sigma_0 + \sigma_0 e^{-2}\right) = 2 \cdot 27\sigma_0$$

- For small M_2/M_1 ratios, (M_2/M_1<0.58), there will be more than two complete passages up/down the rod.
  - The smaller is M_2/M_1 , the slower the rate of decay of stress intensity.

Stress at the Fixed End

- The maximum stress at the fixed end of a rod, impinged upon by a striker at the other end, has been shown to be well summarized by,

$$\frac{_F\sigma_T}{\sigma_0} = 1 + \sqrt{\frac{M_1}{M_2} + \frac{2}{3}}$$



Rigid Striker Impinging on One End of a Free Rod

Rigid Striker on Free Rod

- When a rigid striker impinges on one end of a long free rod with speed v_0, the stress at the striker-rod interface is

$$\sigma = \sigma_0 \exp\left(-\frac{A_2\rho_2 c_2}{M_1} \cdot t\right)$$

  - This equation applies until $t = l_2/c_2$
  - At $t = 2l_2/c_2$, the stress at the striker-rod interface is a $\sigma_0 \exp(-2M_2/M_1)$, and the particle speed at the struck end of the rod is $\mathrm{v}_0 \exp(-2M_2/M_1)$.

Rigid Striker on Free Rod

- Th us the total stress and particle speed near the interface after including the arriving reflected wave is,
  - A tensile stress $\sigma_0[1 - \exp(-2M_2/M_1)]$,
  - A speed of $v_0[1 + \exp(-2M_2/M_1)]$
- Thus separation of the striker and the rod occurs at $t = 2l_2/c_2$.
  - At this instant also, the free end of the struck rod has a speed of $2v_0 \exp(-2M_2/M_1)$.

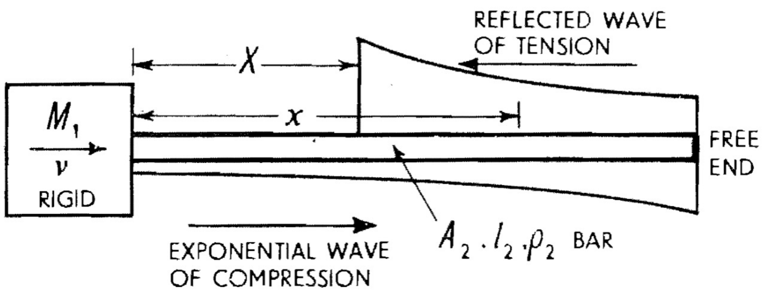
Energy Distribution

- The speed of a particle, v, in a section at distance x from the striker during $l_2/c_2 < t < 2l_2/c_2$ is

$$v = v_0\left[\exp\left(-\frac{M_2}{M_1}\cdot\frac{x-X}{l_2}\right) + \exp\left(-\frac{M_2}{M_1}\left(2-\frac{x+X}{l_2}\right)\right)\right]$$

- The normal stress is

$$\sigma_T = \sigma_0\left[\exp\left(-\frac{M_2}{M_1}\cdot\frac{x-X}{l_2}\right) - \exp\left(-\frac{M_2}{M_1}\left(2-\frac{x+X}{l_2}\right)\right)\right]$$



Energy Distribution

- Thus, the kinetic energy of the bar will be,

$$E_B = \int_x^{l_2} \frac{1}{2}(A_2 \cdot \rho_2 \cdot dx)v^2$$
$$= \int_x^{l_2} \frac{1}{2} \cdot \frac{M_2 v_0^2}{l_2}\left[\exp\left(-\frac{M_2}{M_1} \cdot \frac{x-X}{l_2}\right) + \exp\left(-\frac{M_2}{M_1}\left(2 - \frac{x+X}{l_2}\right)\right)\right] \cdot dx$$
$$= \frac{1}{4}M_1 v_0^2\left[1 + 4\frac{M_2}{M_1}\exp\left\{-2\frac{M_2}{M_1}\left(1-\frac{X}{l_2}\right)\right\} \cdot \left(1-\frac{X}{l_2}\right) - \exp\left\{-4\frac{M_2}{M_1}\left(1-\frac{X}{l_2}\right)\right\}\right]$$

- And the strain energy,

$$U = \int_x^{l_2} \frac{A_2 \sigma_T^2}{2E_2} dx$$
$$= \frac{1}{4}M_1 v_0^2\left[1 - 4\frac{M_2}{M_1}\exp\left\{-2\frac{M_2}{M_1}\left(1-\frac{X}{l_2}\right)\right\} \cdot \left(1-\frac{X}{l_2}\right) - \exp\left\{-\frac{M_2}{M_2}\left(1-\frac{X}{l_2}\right)\right\}\right]$$

Energy Distribution

- Hence, the total energy is,

$$U + E_B = \frac{1}{2}M_1 v_0^2\left[1 - \exp\left\{-4\frac{M_2}{M_1}\left(1-\frac{X}{l_2}\right)\right\}\right]$$

- When X/L_2=0, i.e. when the head of the stress wave has just returned to the striker

$$U + E_B = \frac{1}{2}M_1 v_0^2\left[1 - \exp\left(-4\frac{M_2}{M_1}\right)\right]$$

- The kinetic energy of the striker:

$$E_s = 1/2 M_1 v_0^2 \exp(-4M_2/M_1)$$

- So, we have:

$$U + E_B + E_S = 1/2 M_1 v_0^2$$

Pile Driver

- The equation of motion of the hammer:

$$M_1 \frac{dv}{dt} = M_1 g - A_2 \rho_0 c v$$

$$\frac{dv}{dt} = g - \frac{M_2}{M_1} \cdot \frac{c}{l} \cdot v$$

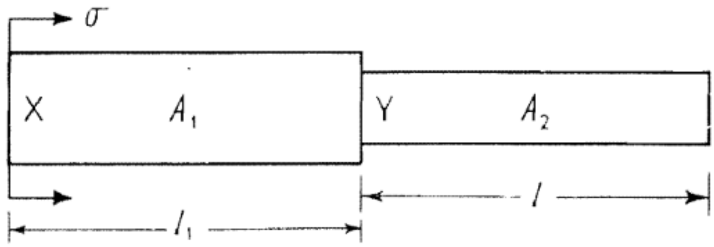
  - M_2 is the pile mass.

- Integrating,

$$v = \frac{M_1 g l}{M_2 c}\left[1 - \left(1 - \frac{M_2}{M_1 g} \cdot \frac{c}{l} \cdot v_0\right) \exp\left(-\frac{M_2}{M_1} \frac{c}{l} \cdot t\right)\right]$$

Momentum Traps

Momentum Traps



- The incident compressive stress is transmitted by the attached bar as a compressive wave, and later reflected from its free end as a tensile wave.
  - It then reaches the plane common to X and Y and immediately contact ceases.
  - Y moves off possessing a certain kinetic energy and having trapped some of the momentum in the incident pulse.

Momentum Traps

- From previous equations, we have,

$$\sigma_T = \frac{2A_1}{A_2+A_1}\cdot\sigma \quad \text{and} \quad \sigma_R = \frac{A_2-A_1}{A_2+A_1}\cdot\sigma$$

- The total reflected impulse is $-A_1.2l.\rho(\sigma_R/\rho c)$ and the transmitted impulse is $A_2.2l.\rho(\sigma_T/\rho c)$.
- The initial impulse delivered is $A_1.2l.\rho(\sigma/\rho c)$:

$$-A_1\cdot 2l\cdot\rho(\sigma_R/\rho c) + A_2\cdot 2l\cdot\rho(\sigma_T/\rho c)$$

$$= \frac{2l\rho}{\rho c}(-A_1\sigma_R + A_2\sigma_T)$$

$$= \frac{2l}{c}\left\{-A_1\cdot\frac{A_2-A_1}{A_2+A_1} + A_2\cdot\frac{2A_1}{A_2+A_1}\right\}\sigma$$

$$= \frac{2l}{c}\cdot A_1\cdot\sigma$$

Momentum Traps

- Hence the fraction of the original impulse, f, trapped in the end bar is

$$f = \frac{A_2 l\rho \cdot 2\sigma_T/\rho c}{A_1 2l\rho \cdot \sigma/\rho c} = \frac{A_2\sigma_T}{A_1\sigma} = \frac{A_2}{A_1} \cdot \frac{2A_1}{A_2 + A_1}$$
$$= \frac{2}{(1 + A_1/A_2)}$$

  - In particular, if $A_1 = A_2,\ f = 1$.

- The addition of a bar lightly attached, is often used to trap transmitted momentum for either of two reasons:
  - (i) In order to mitigate the deleterious effects of impact, and
  - (ii) for use in experimental work for measuring pulse length and applied stress.

Hopkinson Pressure Bar

Hopkinson Pressure Bar

- In a paper delivered in November 1913, Bertram Hopkinson described a simple technique whereby "it is possible to measure both the duration of a blow and the maximum pressure developed by it".
- If the (long cylindrical) rod be divided at a point a few inches from the far end, the opposed surfaces of the cut being in firm contact and carefully faced, the wave of pressure travels practically unchanged through the joint.
- At the joint the pressure continues to act until the head of the reflected tension wave arrives there.

Hopkinson Pressure Bar

- If the tail of the pressure wave has then passed the joint, the end-piece flies off, having trapped within it the whole of the momentum of the blow, and the rest of the rod is left completely at rest.
- The length of end-piece which is just sufficient completely to stop the rod is half the length of the pressure wave, and the duration of the blow is twice the time taken by the pressure wave to travel the length of the end-piece.
- The momentum trapped in quite short end-pieces will be equal to the maximum pressure multiplied by twice the time taken by the wave in traversing the end-piece.

Hopkinson Pressure Bar

- Thus by experimenting with different lengths of end-pieces and determining the momentum with which each flies off the rod as the result of the blow, it is possible to measure both the duration of the blow and maximum pressure developed by it.
- A steel rod is hung up as a ballistic pendulum and the piece is held on to the end by magnetic attraction.
- A bullet is fired at the other end and the end-piece is caught in a ballistic pendulum and its momentum measured.

Hopkinson Pressure Bar

- Experimentally, the time-piece is first made short and its length is increased in steps up to and beyond the point where its momentum reaches a constant value.
- This form of Hopkinson pressure bar is limited
  - (i) because longitudinal elastic waves are distorted to some extent when propagated,
  - (ii) because the stress intensity must not become too large, since the time-piece itself may then scab (or fracture) and
  - (iii) because the decay may be too rapid to provide very useful pressure-time results.

Split-Hopkinson Pressure Bar

Historical Background

- The design or performance assessment of a component or structure requires accurate knowledge of the elastic and inelastic deformational and strength properties of the materials involved.
- During World War II, strength properties associated with shock waves were developed using light-gas gun or explosively driven flyer-plate impact experiments, producing high hydrostatic pressures and strain rates in excess of $10^4/s$.
  - The time duration or the material strain rates due to many explosive, ballistic impact, crashes and other accident scenarios of interest for both military and civilian applications range from $10^2 - 10^4/s$.

Historical Background

- The Hopkinson's pressure bar techniques based on impact stress wave measurement were developed to obtain the material properties at these strain rates.
  - This was further extended by RM Davies in 1948 and Herbert Kolsky in 1949, improved on Hopkinson's device, adding displacement gages and oscillographic recording techniques to obtain complete impact stress pulse amplitude and wave forms in similar elastic bars.
  - Kolsky used a two-bar system, sandwiching a short compression specimen between them.
  - In the early 1960s, Ulric Lindholm of Southwest Research Institute, Texas, modified the Kolsky technique primarily by altering the bar lengths and placement of the strain gages.

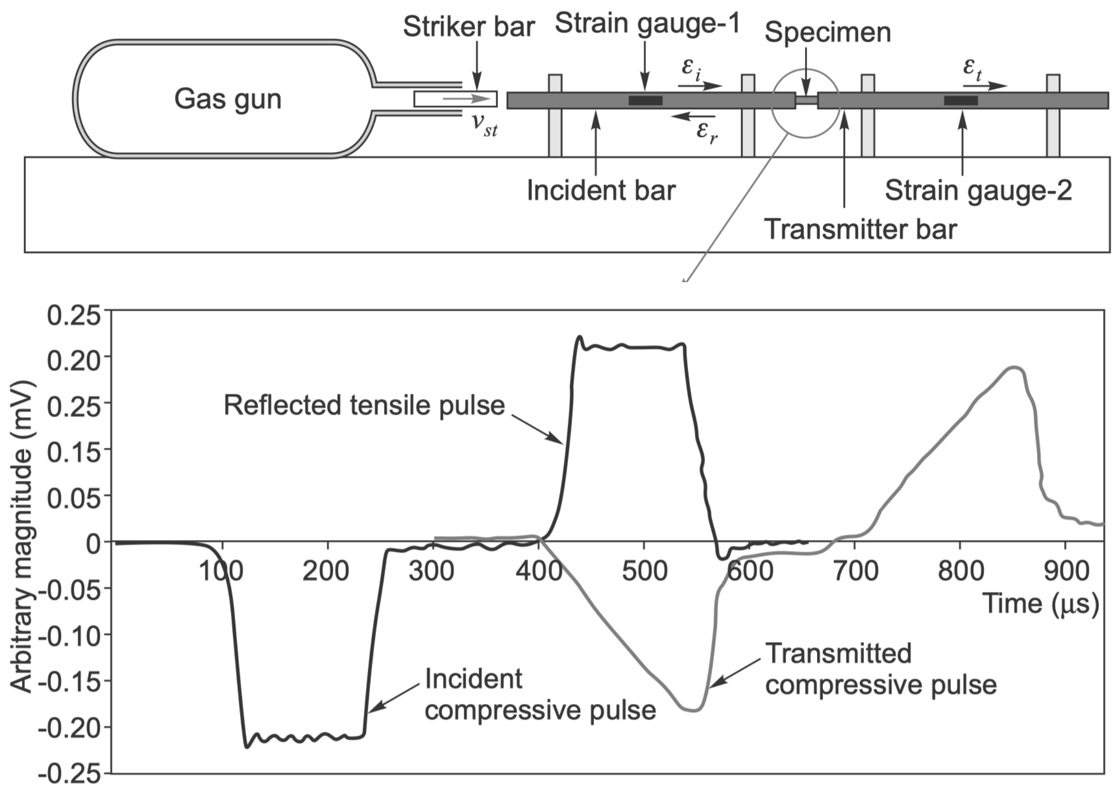
Historical Background

- Nemat-Nasser et al. (1991) developed novel techniques to the SHPB that provided the possibility of conducting compression tests followed by tension to analyse the Bauschinger effect under high strain rates.
- The techniques reported in Nemat-Nasser et al. (1991) additionally permitted the dynamic recovery experiments in which the specimen is subjected to a preassigned stress cycle and then recovered without additional loading for post-test microstructure analysis.

Principle of SHPB Test

- All SHPB apparatus share common design elements such as:
  - Air cannon/compressed gas gun that fires a projectile;
  - Sensing device to determine the projectiles velocity;
  - Two long symmetric pressure bars;
  - Bearings and alignment tooling to allow the pressure bars to move freely;
  - Strain gauges mounted on both pressure bars;
  - Test specimen; and
  - Instrumentation to record stress, strain, and strain rate information.

Principle of SHPB Test



(c) Typical strain history in incident and transmitter pressure bars

Theory Behind SHPB

- The displacements u_1 and u_2 at the left and right ends of the specimen are given by

$$u_1 = \int_0^t c_0 \varepsilon_1 dt \quad \text{and} \quad u_2 = \int_0^t c_0 \varepsilon_2 dt$$

  - where ε_1 and ε_2 are respectively the longitudinal strains at the left and right ends of the specimen and c_0 is the velocity of elastic stress wave in the bar.

- The displacements u_1 and u_2 can be written in terms of the incident, reflected and transmitted pulses as

$$u_1 = c_0 \int_0^t (\varepsilon_i - \varepsilon_r) dt \quad \text{and} \quad u_2 = c_0 \int_0^t \varepsilon_t dt$$

- The strain in the specimen ε_s is given by,

$$\varepsilon_s = \frac{u_1 - u_2}{L_s}$$

Theory Behind SHPB

- Substituting,

$$\varepsilon_s = \frac{c_0}{L_s} \int_0^t (\varepsilon_i - \varepsilon_r - \varepsilon_t) dt$$

- The forces P_1 and P_2 acting at the ends of the specimen are given by

$$P_1 = EA(\varepsilon_i + \varepsilon_r), \quad P_2 = EA\varepsilon_t$$

  - where A is the cross sectional area of the incident and transmitter bars.

- The specimen is in equilibrium under the action of the above forces, i.e. $P_1 = P_2 = P$. So,

$$\varepsilon_t = \varepsilon_i + \varepsilon_r$$

Theory Behind SHPB

- Substituting,

$$\varepsilon_s = \frac{c_0}{L_s}\int_0^t (\varepsilon_i - \varepsilon_r - \varepsilon_i - \varepsilon_r)dt$$

- Finally, the stress, strain and the strain rate acting on the specimen can be obtained respectively from the following equations.

$$\sigma_s = \frac{P}{A_s} = E\varepsilon_t\frac{A}{A_s}$$

$$\varepsilon_s = \frac{-2c_0}{L_s}\int_0^t \varepsilon_r dt$$

$$\dot{\varepsilon}_s = \frac{-2c_0}{L_s}\varepsilon_r$$

- where A_s is the cross sectional area of the specimen.

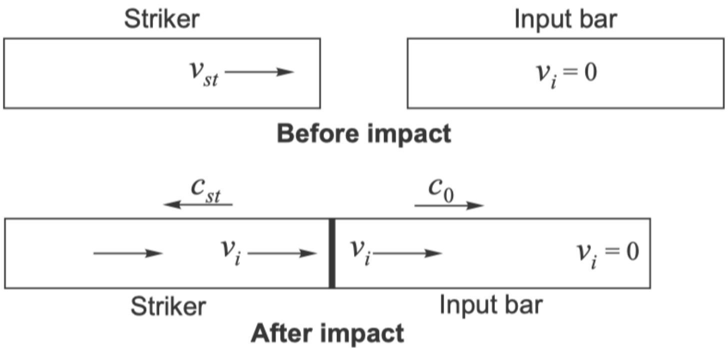
Design of Pressure Bars for a SHPB Apparatus

Design of Pressure Bars

- The most important requirements to be considered in designing a SHPB are the stress, strain and strain rate within the specimen.
- These requirements directly dictate some of the essential design variables such as:
  - The length of the impact stress pulse;
  - The level of stress in the bars;
  - The cross sectional area of bars and the specimen; and
  - The impact velocity of the striker.

Design of Pressure Bars

- This design is based on the assumption that the striker, input and output bars are loaded within the material's elastic limit.
- So it is important to ensure during the design that the stresses are well below the yield limit of the material.
- The striker travels with a velocity v_{st} and the input bar stays at rest just before the impact.



Design of Pressure Bars

- The force and velocity on the striker (P_{st} and v_{st}) and the input bar (P_i and v_i) at their common interface is equal during the course of impact.
- The velocity v_i in the input bar is $0 < v_i < v_{st}$after the impact. Therefore,

$$P_{st} = P_i \quad \rightarrow \quad A_{st}\sigma_{st} = A_i\sigma_i$$

- The stresses generated are dependent on the velocity at the common interface of striker and the input bar,

$$\sigma_{st} = \rho_{st}c_{st}(v_{st} - v_i), \quad \sigma_i = \rho_i c_0 v_i$$

- Substituting,

$$v_i = \frac{\beta}{1+\beta}v_{st}, \qquad \beta = \frac{A_{st}\rho_{st}c_{st}}{A_i\rho_i c_0}$$

Design of Pressure Bars

- So, we have,

$$\sigma_{st} = \frac{\rho_{st}c_{st}v_{st}}{1+\beta}, \qquad \sigma_i = \frac{\beta}{1+\beta}\rho_i c_0 v_{st}$$

- Just after the impact, the striker and the input bar remain in contact until the pulse generated in the striker reflects from its end as a tensile pulse and travels towards the contact interface.
- The time t_p taken by the pulse to return to the contact interface is given by

$$t_p = \frac{2L_{st}}{c_{st}}$$

  - where L_{st} is the length of the striker bar.

Design of Pressure Bars

- The pulse length generated in the input bar is then obtained as,

$$L_p = c_0 t_p = 2L_{st} \frac{c_0}{c_{st}}$$

- This pulse length in the input bar is the important parameter that determines the strain level in the specimen.
- The stress pulse generated in the input bar due to the impact of striker reaches the specimen which is then partially reflected and partially transmitted.
- The intensity of the transmitted pulse σ_t is $\sigma_t = \frac{A_s}{A}\sigma_s$
  - and σ_t should ensure to load the specimen with a specific stress level σ_s .

Design of Pressure Bars

- The reflected pulse σ_r is dependent on the strain rate in the specimen,

$$\sigma_r = \frac{EL_s}{-2c_0}\dot{\varepsilon}_s$$

- Both σ_t and σ_r are design variables in the design of a split Hopkinson pressure bar.
- The incident stress pulse σ_i that travels through the input bar must satisfy these design variables.
- The strain rate in the specimen in turn is a design requirement.

Design of Pressure Bars

- The striker velocity v_{st}, another design variable can be derived as:

$$v_{st} = \frac{1+\beta}{\beta \rho_i c_0}\left(\frac{A_s}{A}\sigma_s + \frac{EL_s}{2c_0}\dot{\varepsilon}_s\right)$$

  - Note that at the incident/sample interface we have $\sigma_i + \sigma_r = \sigma_t$.

- The above equation helps to arrive at an important design variable v_{st} so that a specified stress and strain rate in the specimen can be achieved.

- The final requirement is the maximum strain that the specimen is subjected to and is proportional to the reflected stress pulse and the time duration of the incident pulse.

Design of Pressure Bars

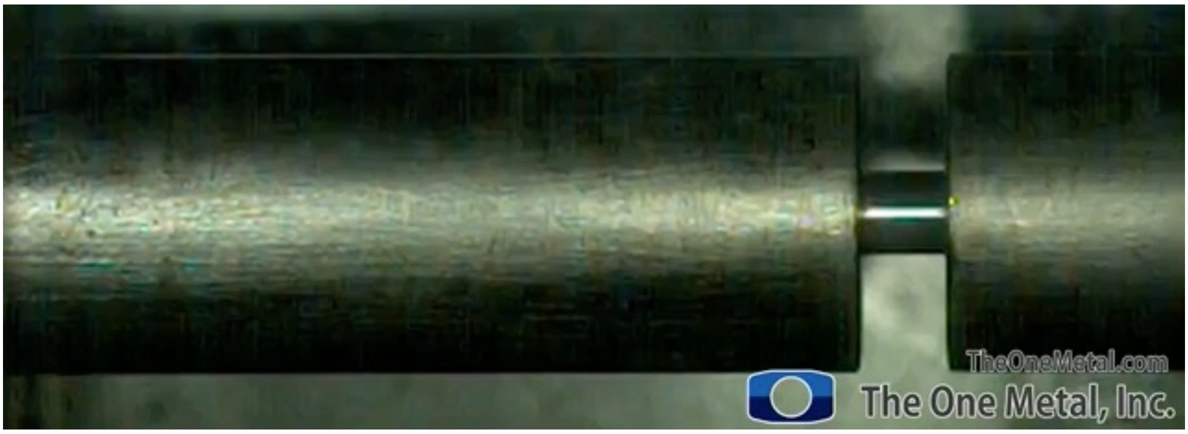
- Assuming that the reflected pulse is constant, the duration of the pulse t_p is,

$$t_p = \frac{L_s \varepsilon_s}{-2c_0 \varepsilon_r} \quad \rightarrow \quad \varepsilon_s = -\frac{2c_0 t_p \varepsilon_r}{L_s}$$

- The length of the pulse L_p, another design variable, can now be expressed in terms of the required strain in specimen as,

$$L_p = \frac{L_s \varepsilon_s}{-2\varepsilon_r}$$

  - The above equation helps to finalize the length of the input and output bars because they must be greater than the length of the greatest pulse that can be transmitted by them as per traditional SHPB technique.



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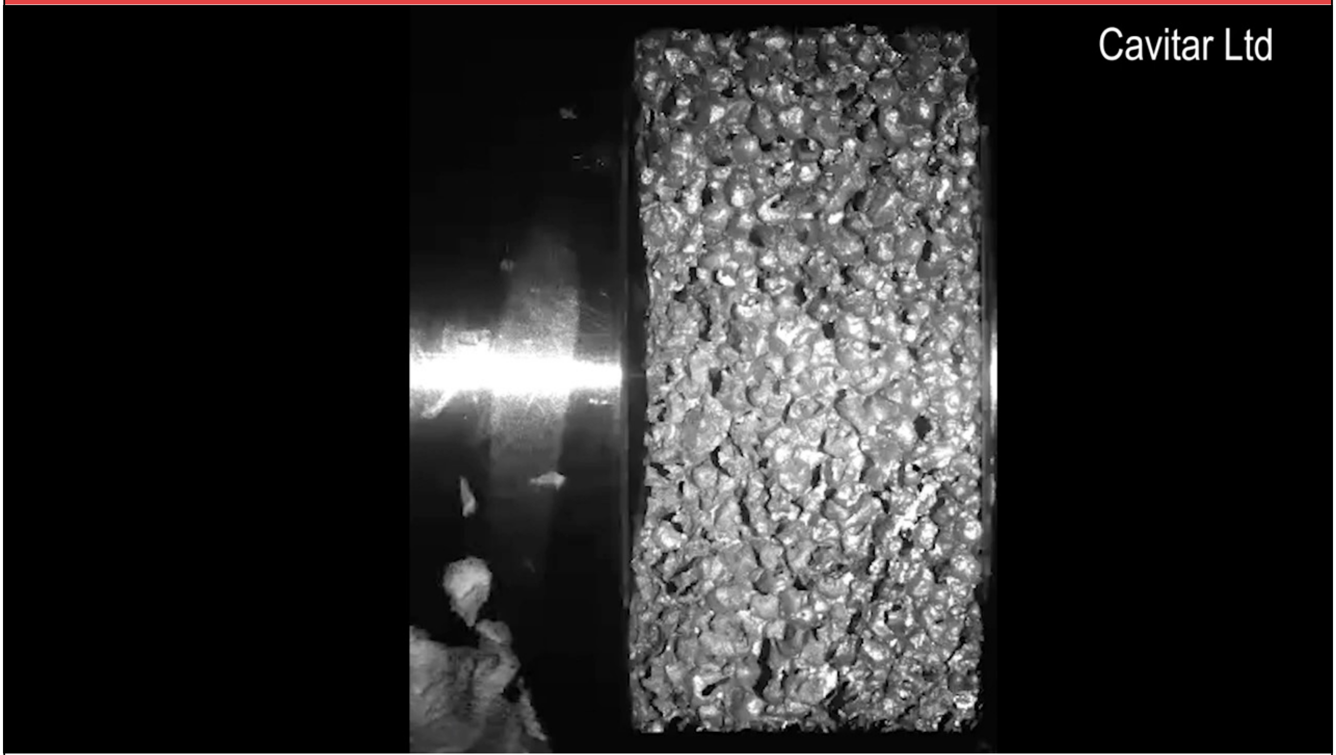
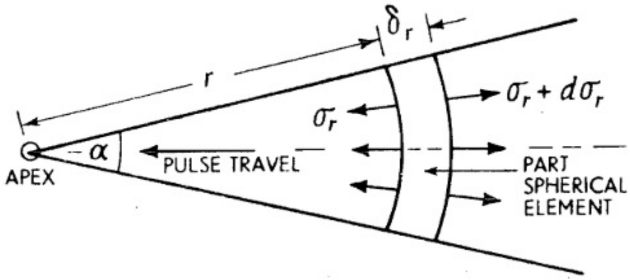


Propagation of Compressive Stress Waves in a Conical Bar

Conical Bar



- Equations of motion:

$$\rho\alpha r^2 dr \cdot \frac{\partial^2 u}{\partial t^2} = d\left(\alpha r^2 \sigma_r\right)$$

$$\sigma_r = E\partial u/\partial r$$

$$\rho r \frac{\partial^2 u}{\partial t^2} = 2\sigma_r + r\frac{\partial \sigma_r}{\partial r}$$

$$\rho \frac{\partial^2}{\partial t^2}(ur) = E\left(2\frac{\partial u}{\partial r} + r\frac{\partial^2 u}{\partial r^2}\right)$$

$$\frac{\partial^2 (ur)}{\partial t^2} = \frac{E}{\rho}\frac{\partial^2}{\partial r^2}(ur) = c^2 \frac{\partial^2}{\partial r^2}(ur)$$

Conical Bar

- Which is the equation for the spherical wave with the solution,

$$ur = f(r - ct) + F(r + ct)$$

- To describe a pulse moving in the direction of r decreasing, i.e. toward the apex of the cone, we must choose from

$$u = \frac{1}{r}F(r + ct)$$

- Thus

$$\sigma_r = E\frac{\partial u}{\partial r} = \frac{E}{r}\cdot F'(r + ct) - \frac{E}{r^2}\cdot F(r + ct)$$

- The particle speed:

$$v = \frac{\partial u}{\partial t} = \frac{c}{r}\cdot F'(r + ct)$$

Conical Bar

- For $F(r + ct)$ we choose $C\,\{\exp[-(r + ct)/\lambda] - 1\}$; λ is a characteristic length of the pulse which determines its 'sharpness'.

- This choice is satisfactory because it ensures that,
  - (i) at the pulse head where $u = 0, r = -ct$, which is entirely consistent with
  - (ii) measuring t negatively and taking $t = 0$ to be the instant at which the head of the pulse reaches the apex of the cone.
  - (iii) when $r < |ct|$, the undisplaced part of the cone is referred to.

Conical Bar

- Thus

$$u = \frac{C}{r}\left(e^{-(r+ct)/\lambda} - 1\right)$$

- Using the above,

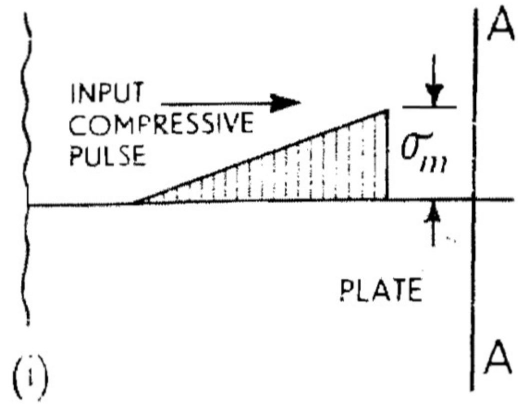
$$\sigma_r = E\frac{\partial u}{\partial r} = -\frac{EC}{r^2}\left[\exp\left(-\frac{(r+ct)}{\lambda}\right) - 1\right] - \frac{EC}{\lambda r}\cdot\exp\left(-\frac{(r+ct)}{\lambda}\right)$$

  - The pressure at the head of the wave is $-EC/\lambda r$, which is obviously the greater the nearer the pulse head to the cone apex.
  - In regions well behind the head of the pulse, i.e. when $r \gg |ct|$, $\exp\{-(r + ct)/\lambda\} \to 0$ and $\sigma_r \to EC/r^2$, i.e. the stress is tensile.

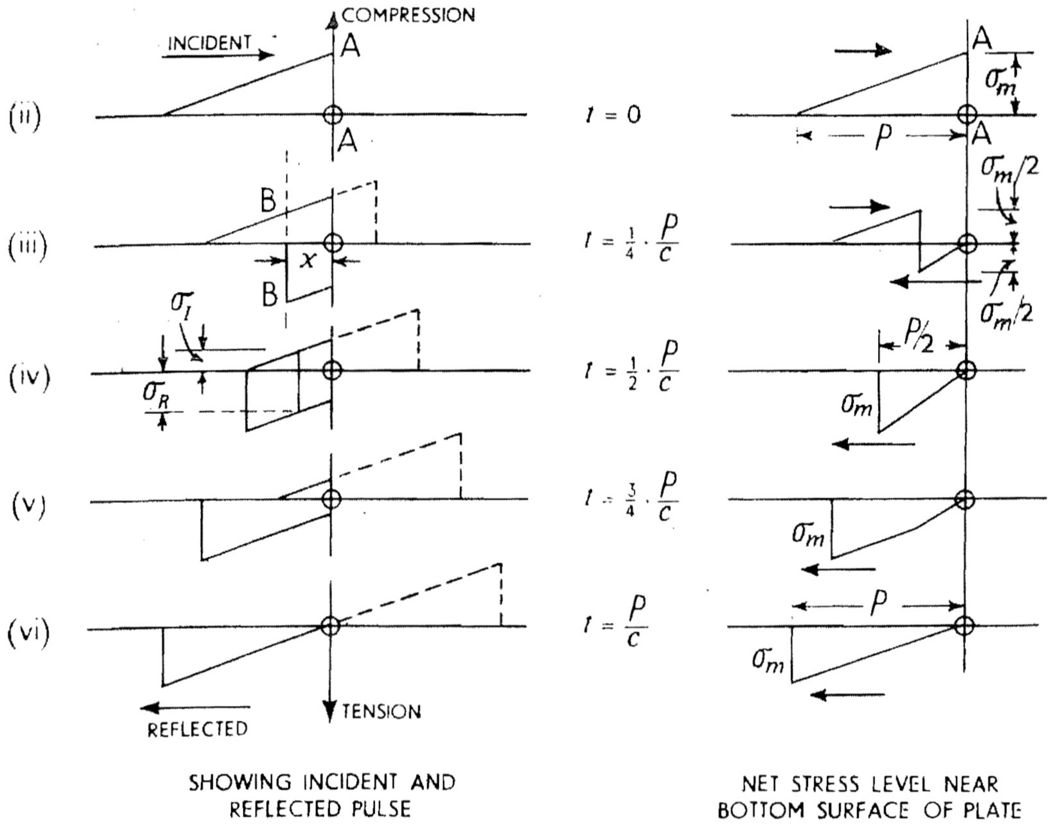
Spalling or Scabbing in Bars and Flat Plates

Spalling

- Spalling or scabbing is a form of fracture in a plate of material which occurs near a free surface remote from the area to which the causative impulsive load is applied.



Spalling



Spalling

- The greatest tensile stress first arises at $t = P/(2c)$; this greatest tensile stress is σ_0 and first occurs at one half the pulse length from the rear face.
- All the particles in this zone of one half of the pulse length (between the first plane of maximum tensile stress and the rear face) have the same speed, v_N

$$v_N = v_I + v_R$$
$$= \frac{\sigma_I}{\rho c} + \frac{(\sigma_m - \sigma_I)}{\rho c} = \frac{\sigma_m}{\rho c}$$

Spalling

- Then at a distance x into the plate from the rear face, tensile stress σ' in a typical plane, is

$$\sigma' = \sigma_m\left(1 - \frac{P-2x}{P}\right) = \frac{2x}{P} \cdot \sigma_m$$

- This equation is valid for $x \leq P/2$ and as we have already seen is greatest when $x = P/2$. The particle speed associated with σ' and applying for all parallel planes between it and the bottom face is v'

$$\rho c v' = \frac{2x}{P}\sigma_m$$

- Again, v' is greatest when $x = P/2$ and then $v' = \frac{\sigma_m}{\rho c}$

Spalling

- If a material undergoes fracture when the tensile stress reaches some critical value σ_F, then fracture will occur in a plate in a layer where σ_F is first reached, i.e. where $\sigma' = \sigma_F$.
- If fracture occurs, then the material below the fracture surface will have trapped in it an amount of downward momentum equal to the spall mass times v'.

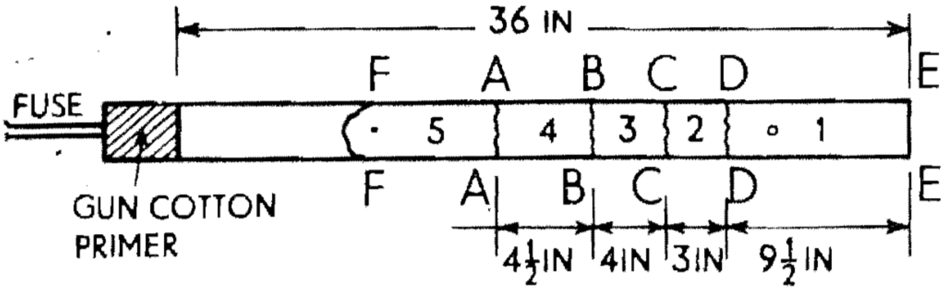
	σ_F, lbf/in^2	Differential particle speed, ft/sec
Copper	410 000	264
Brass	310 000	216
4130 Steel	440 000	235
1020 Steel	160 000	84

Multiple Scabbing

- Once a fracture has occurred, the remainder of the incident compressive impulse in the mass of the plate will be approaching a fresh, new surface and correspondingly will be reflected as a tension wave.
- Thus further fractures may occur.
- If no scab is thrown off, the total effect may be a series of more or less parallel cracks.

Multiple Scabbing

- Landon and Quinney described an experiment in which a concrete bar, resting a horizontal timber baulk, three feet long and three inches diameter, was subject to the detonation one end of an 8 oz primer of gun cotton.

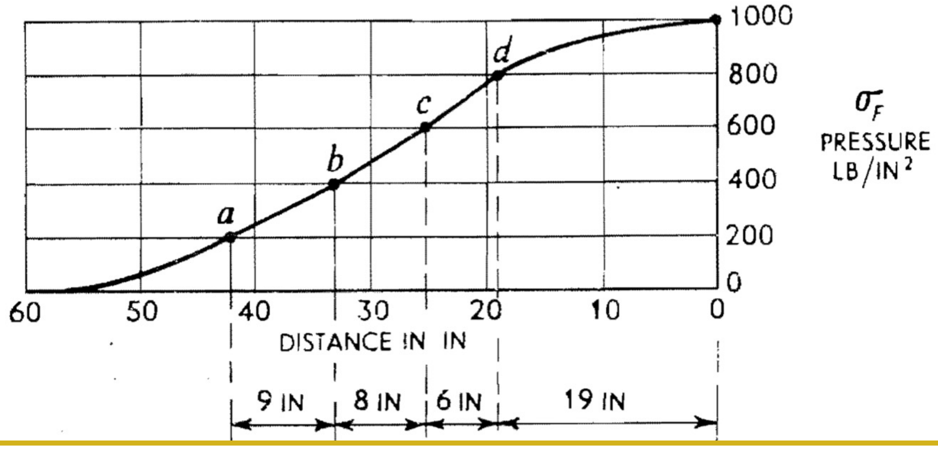


CONCRETE BAR

- Piece 5 was scarcely moved and the portion of the bar at the primer end was pulverized by the detonation.

Multiple Scabbing

- By assuming the wave front generated by the gun cotton to be vertical and that fracture of a piece occurred when the difference between the propagated pressure wave and the reflected tension wave was equal to the tensile strength of the concrete, it was found possible to deduce an approximate curve for the pressure propagated along the bar:

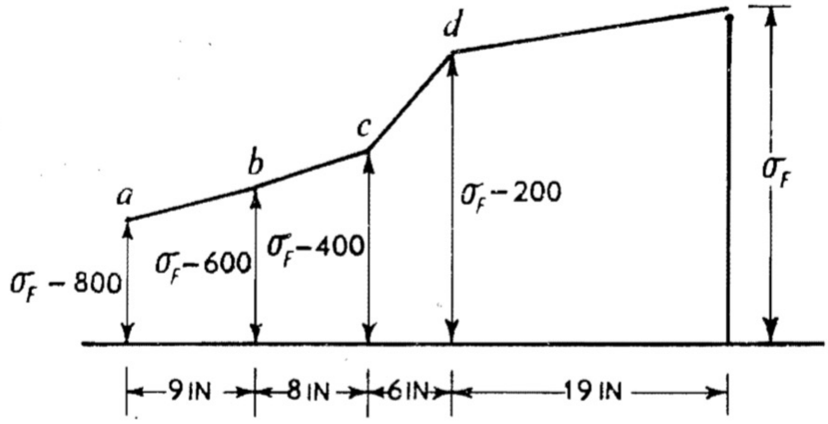


Multiple Scabbing

- Static test on the concrete established its tensile strength as $200\ lb/in^2$ and its compressive strength as $800\ lb/in^2$.
- When fracture occurs in plane DD, it will be because the head of the initially compressive stress wave of intensity σ_F, after reflection from free end EE as a tensile wave, exerts a net tensile stress in plane DD of $200\ lb/in^2$.
- The compressive stress in the incident pulse in plane DD at the instant fracture occurs is thus $(\sigma_F - 200)$ and this occurs at distance $(2{\times}9½)\ in$ behind the head of the incident pulse.
- Associated with a distance further back in the pulse of $(2 \times 3)\ in$ there will be a further decrease in pulse stress of $200\ lb/in^2$ because a second fracture, at CC, occurs…

Multiple Scabbing

- Due to the given information about the location of the further fractures, the diagram may be constructed.



- Since no further fracture occurs after AA, the absolute stress at AA should be $\leq 200\ psi$.

$$\sigma_F - 800 = 200 \quad \rightarrow \quad \sigma_F = 1000$$

Multiple Scabbing

- Piece 5 will have a forward speed $v_5 = \sigma_5/\rho c$ where σ_5 is $200\ lb/in^2$.
- For concrete, $E = 10^6\ lb/in^2$, $\rho = 0.03\ lb/in^3$, then $v = 2\ ft/s$.
- Piece 4 will have a forward speed $v_4 = \sigma_4/\rho c = (200 + 400)/\rho c = 6\ \text{ft}/s$.
- Pieces 3, 2 and 1 have speeds of approximately 10, 14 and 18 ft/sec respectively.
- It should be remembered that statically-loaded concrete structures which are very rapidly unloaded may fracture due to the unloading pulse and its reflection.

Bar Fracture

- If a bar under impact loading fractures, effectively, a new free end to the bar is created, the fraction of the length detached may easily be estimated.
- The critical net tensile stress S is first reached, say, at a section X_0 from the striker, and will be given by

$$\sigma_T = S = \sigma_0 \left[1 - \exp\left(-\frac{M_2}{M_1} \cdot 2\left(1 - \frac{X_0}{l_2}\right)\right)\right]$$

$$\frac{X_0}{l_2} = 1 - \frac{M_1}{M_2} \ln\left(\frac{1}{\sqrt{1 - S/\sigma_0}}\right)$$

- So, the length of the fractured rod is,

$$l_2 \cdot \frac{M_1}{M_2} \ln \frac{1}{\sqrt{1 - S/\sigma_0}}$$

Bar Fracture

- Writing the fracture stress $S = \rho_2 c_2 v_c$ so that we associate with it a critical particle speed v_c , then the initial striker impact speed to cause fracture (or necking) at a section distant X_O from the striker is given by

$$v_0 = v_c / \left[1 - \exp\left\{-2\left(1 - \frac{X_0}{l_2}\right)\frac{M_2}{M_1}\right\}\right]$$

- Evidently the least speed to cause fracture, which in the limit is at the striker-rod interface, is $v_c/(1 - e^{-2M_2/M_1})$
- If for hard aluminum $S = 20000 lb/in$, $v_c = 32\ ft/s$,and if $M_2/M_1 = 1$ and $X_0/l_2 = ½$, then v_0 would be 50 ft/s .

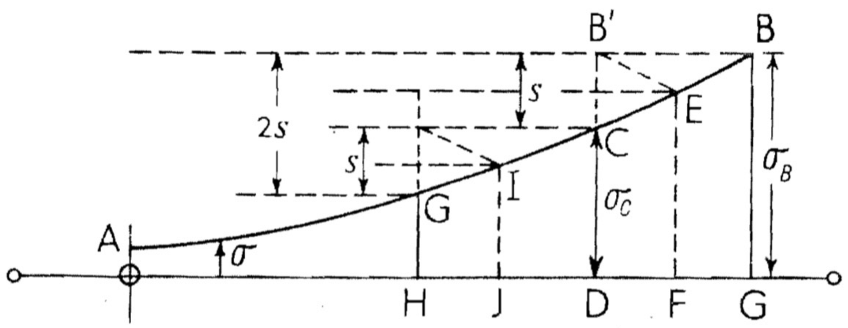
Bar fracture

- After the far portion of the bar has separated from that contiguous with the striker, it will carry with it a total energy given by (2.16).
- Also a new tensile stress will be generated in the remaining portion of the bar, from the new free end.
- The stress intensity at the head of the wave is

$$\sigma_0 \cdot e^{-2(M_2/M_1)\cdot(1-X_0/l_2)}$$

  - This new wave of tension will now be superimposed on the tail of the striker-initiated compressive wave and thus there will be a tendency to promote another fracture in the portion of the rod between the striker and the first plane of fracture.

Bar Fracture



- The striker-initiated stress wave is illustrated as AB, the greatest value of which is reduced from σ_B to σ_C, and $(\sigma_B - \sigma_C) = S$.
- Taking EF to represent the end of the bar and B'E (dotted) to represent the reflected tensile wave-identical with the incident portion BE-the portion which leaves the original bar on fracture is of length DF where DF is one half of DG.

Bar Fracture

- Similarly a second plane of fracture might occur at a distance from the new free end of DJ or HJ.
- The new or second length of fractured rod would be

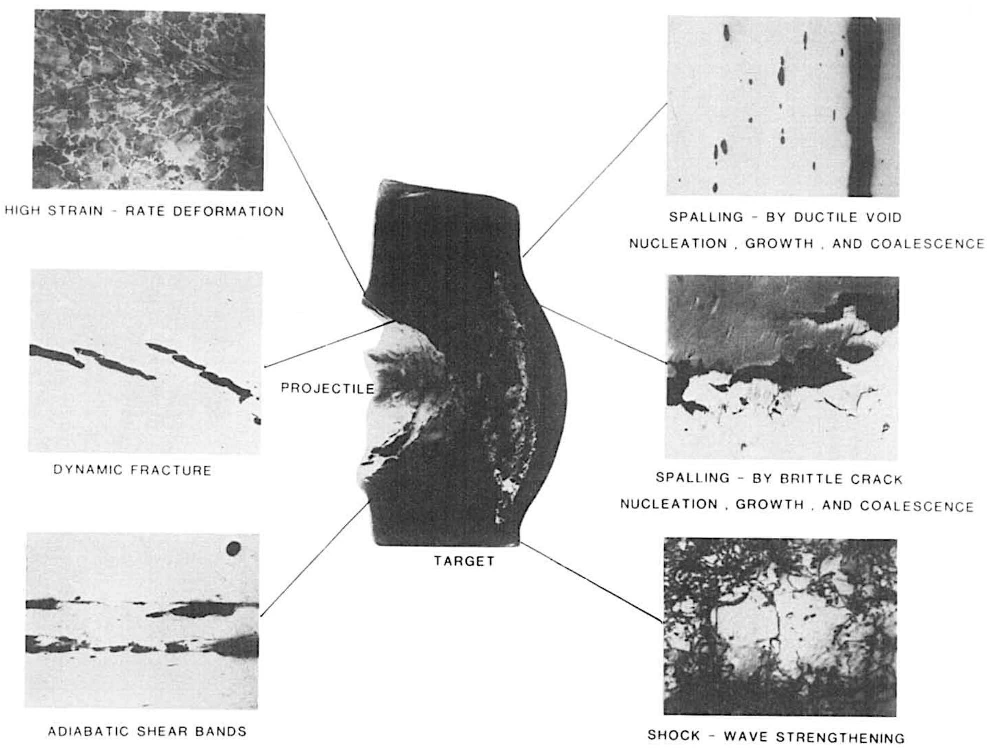
$$l_2 \cdot \frac{M_1}{M_2} \ln \frac{1}{\sqrt{1 - 2S/\sigma_0}}$$

- Generally the *n*th fracture length is

$$l_2 \cdot \frac{M_1}{M_2} \ln \frac{1}{\sqrt{1 - nS/\sigma_0}}$$

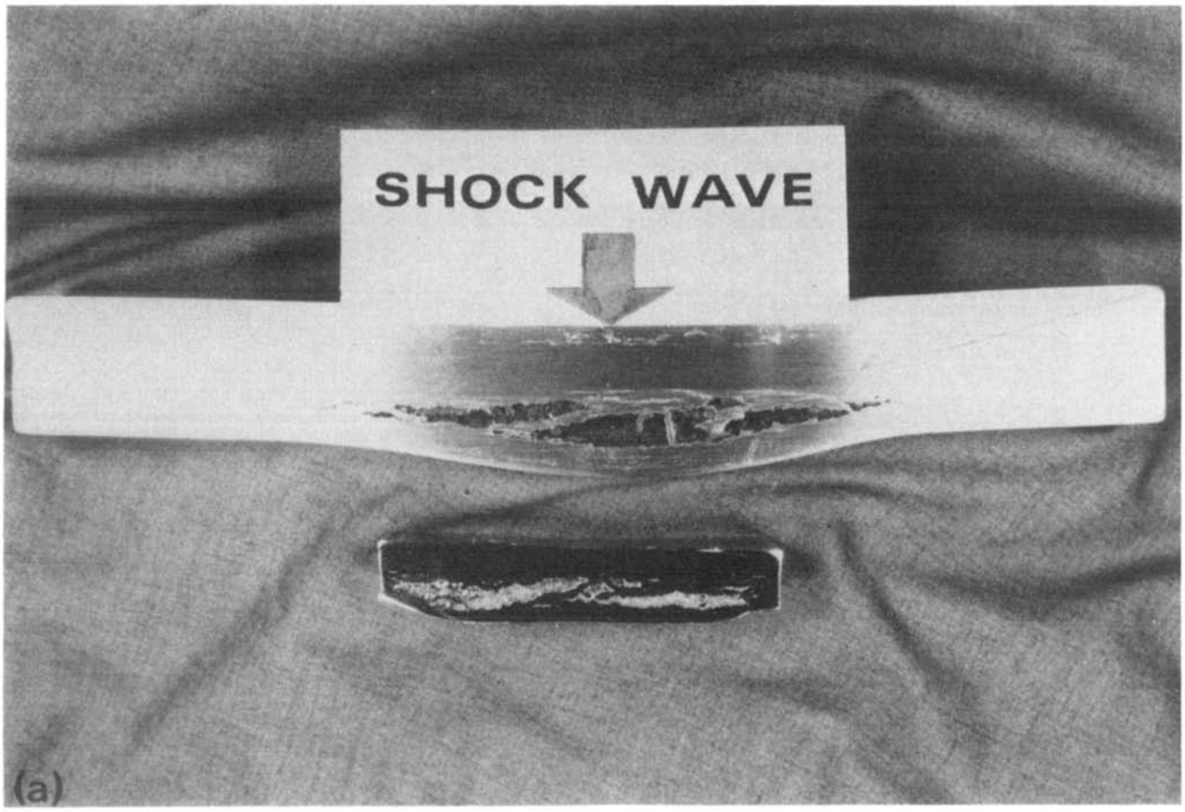
- This is only applicable as long as $S < \sigma_0/n$.

Dynamic Deformation



M.A. Meyers, C.T. Aimone, Dynamic Fracture (Spalling) of Metals, Progress in Materials Science Vol. 28, pp. 1 to 96, 1983

Complete spalling of AISI 1008 steel (large plate) and nickel plate.



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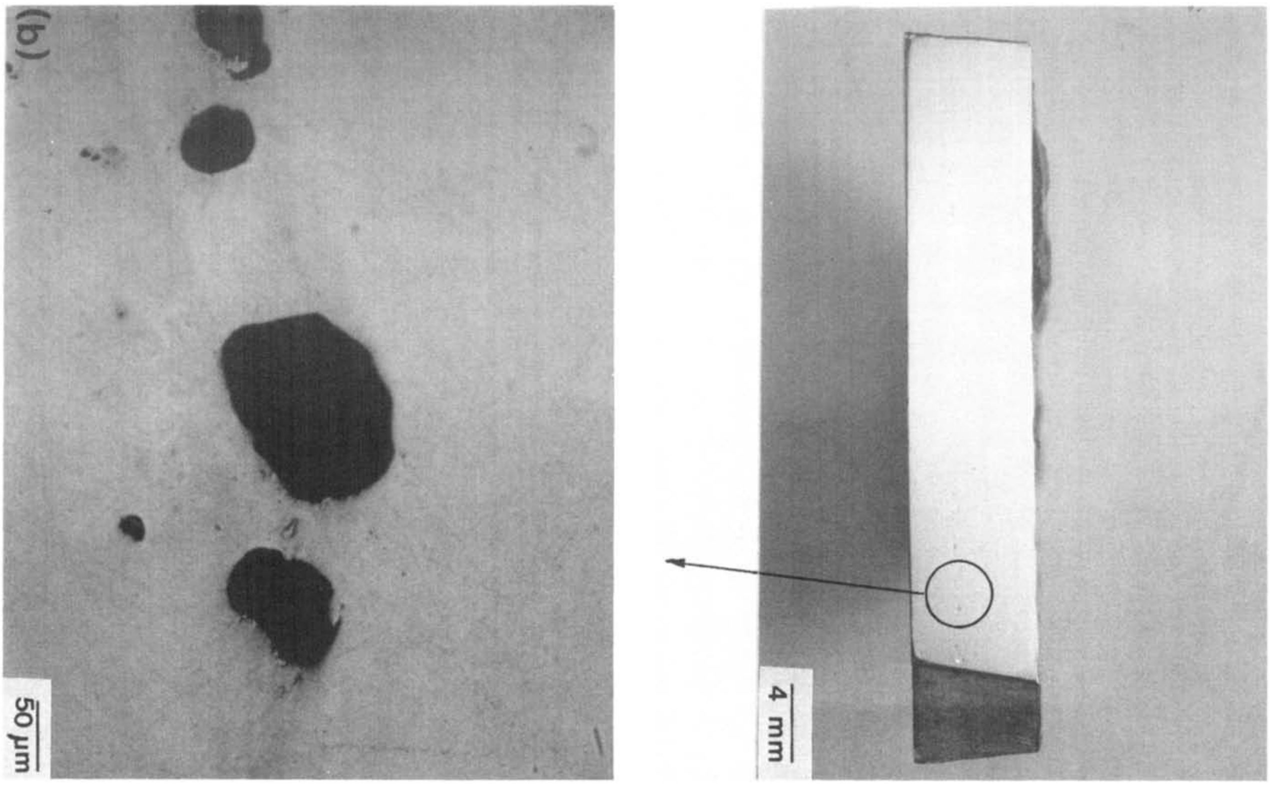
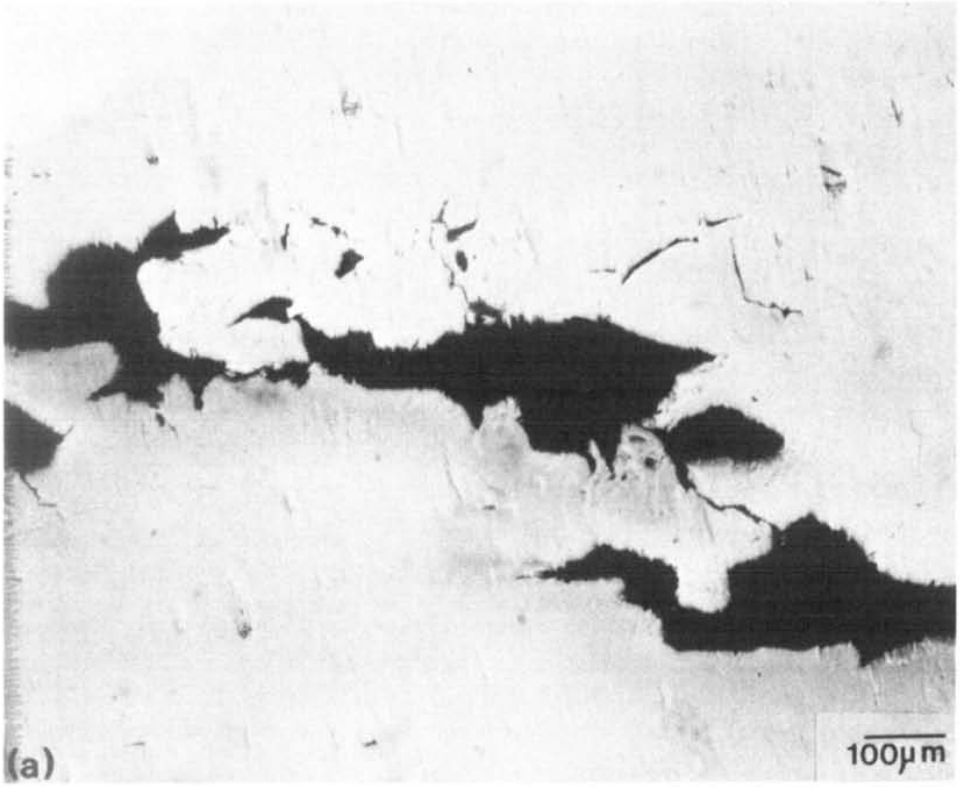
Incipient spall damage in copper



M.A. Meyers, C.T. Aimone, Dynamic Fracture (Spalling) of Metals, Progress in Materials Science Vol. 28, pp. 1 to 96, 1983

Spalling by brittle fracture in low-carbon steel.



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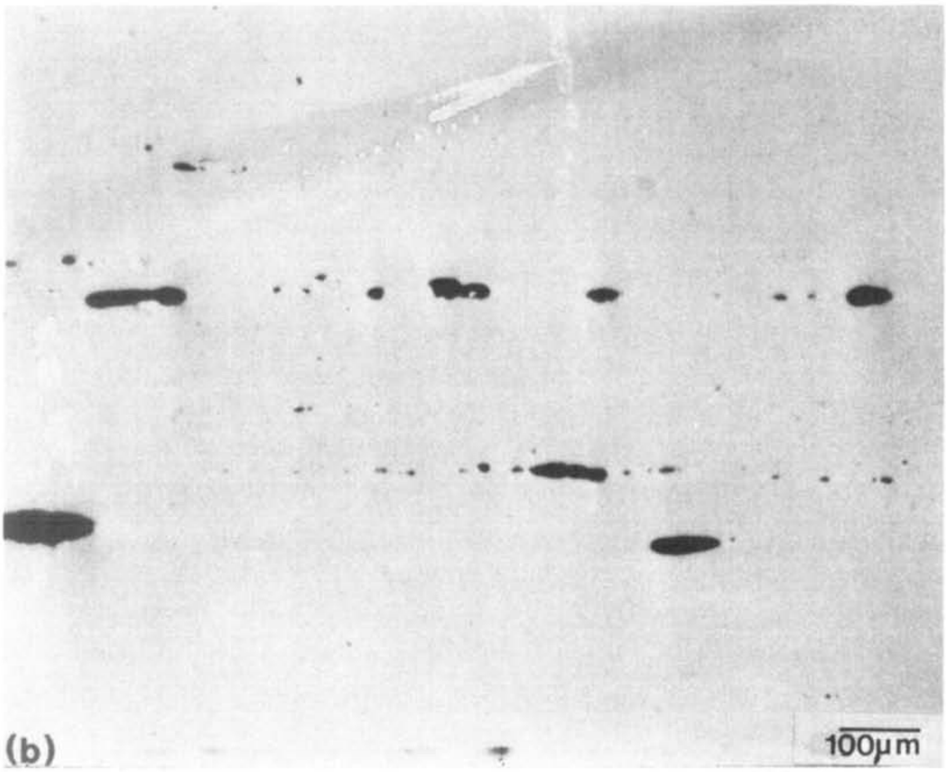
Spalling by ductile void formation in nickel.



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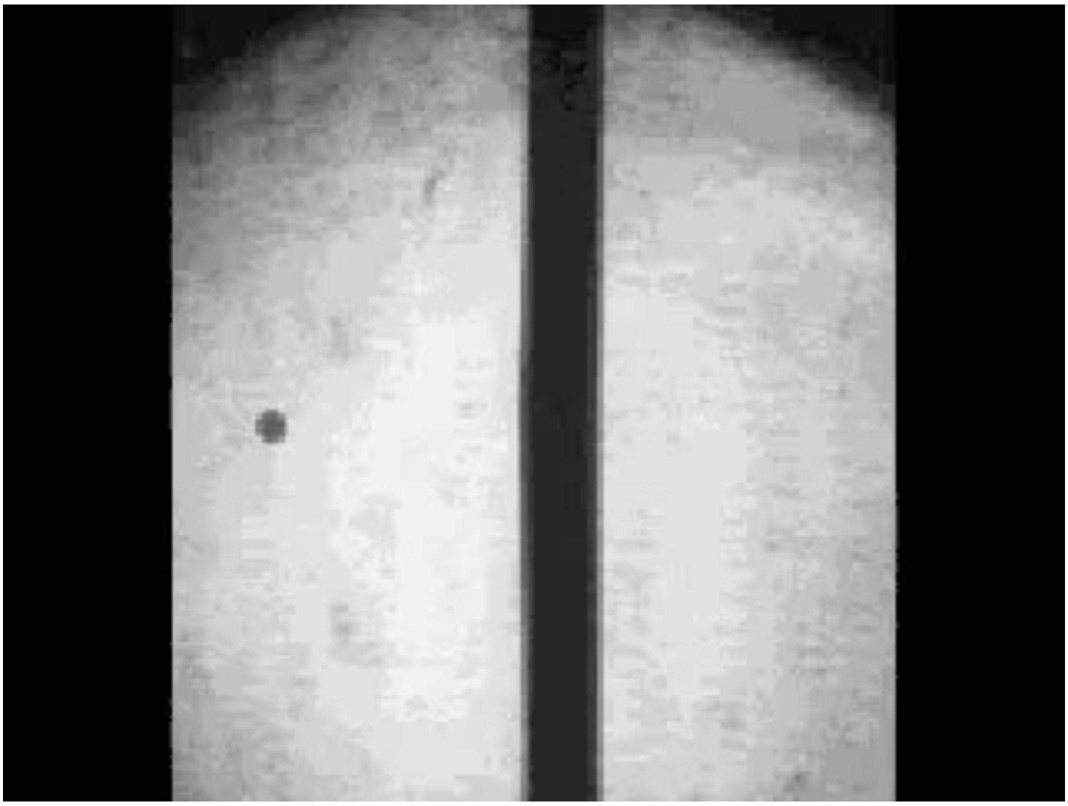


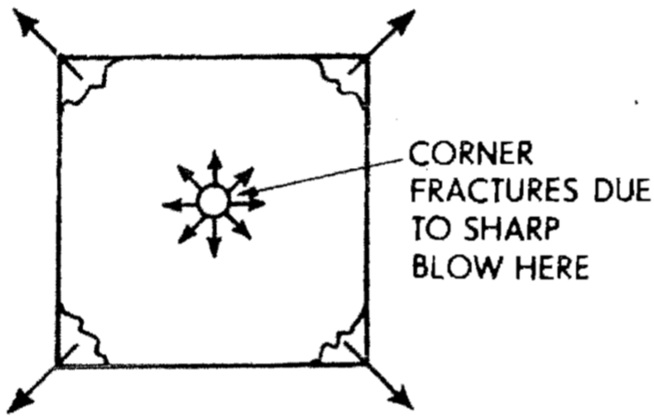
Some Fracture Patterns Due to Impulsive Surface Loading

Conical Specimens

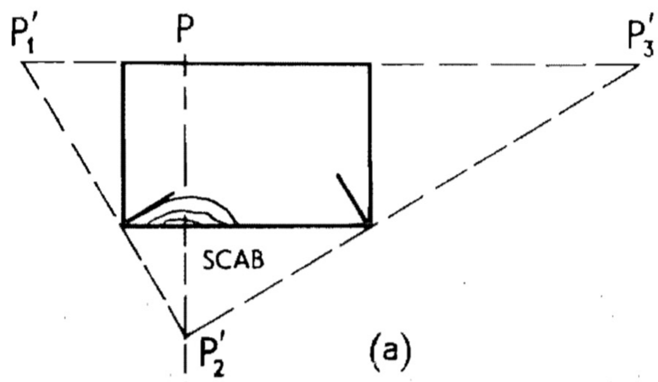
- For a cone in which the longitudinal pulse length is large in comparison with the cross-section diameter, a compressive pulse develops a tension tail.
- The length of the compression region becomes progressively shorter as the apex is approached and simultaneously the maximum stress amplitude in the tail increases so that fracture at the tip of the cone may follow.

Disks or Plates

- As the radiating compression wave moves into each of the four comers between a pair of adjacent sides, it is amplified.



Rectangular Slabs



- The explosive pulse being provided by a small live charge of lead azide, P.
- In this situation the reflected tensile pulse is obtained from the sides of the block.
- The bottom face will be responsible for a scab while the side faces will give rise to surface fractures parallel to the sides at a distance less than one half the pulse length.

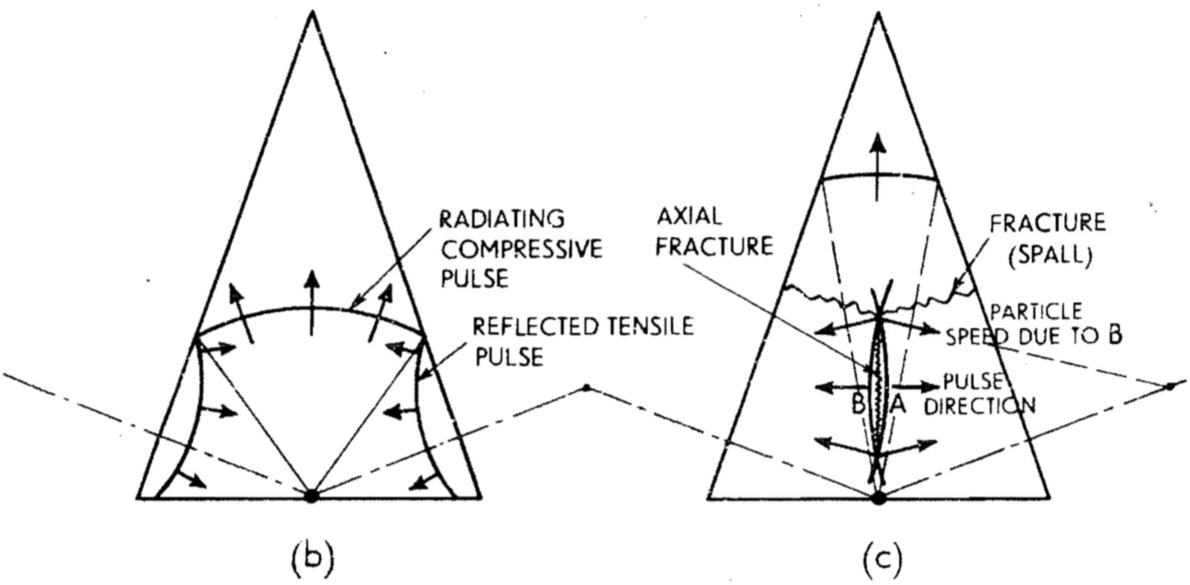
Wedge-Shaped Plates

- When a charge is detonated along the whole base of a flat wedge-shaped plate, multiple spalling near the apex is conspicuous.
- It seems that the first spall to occur is that which is furthest from the apex.
- The tensile tail of the pulse which grows in intensity the nearer the pulse is to the apex of the plate is presumably responsible for the first fracture and subsequent fractures develop from the bottom upwards in the spall tip.
- This order of development of the spalls is opposite to that which occurs in a uniform bar.

Wedge-Shaped Plates

- If a point or concentrated charge is detonated at the center of the base of a flat wedge-shaped plate, the consistent fracture patterns which are developed are different from those just described.
- The compressive pulse radiated from the center of the base is reflected from the sloping sides of the wedge as a predominantly tensile pulse at the same time as the pulse proceeds towards the wedge apex.
- The reflected waves intersect on the axis of the plate and there tend to tear it apart.

Wedge-Shaped Plates

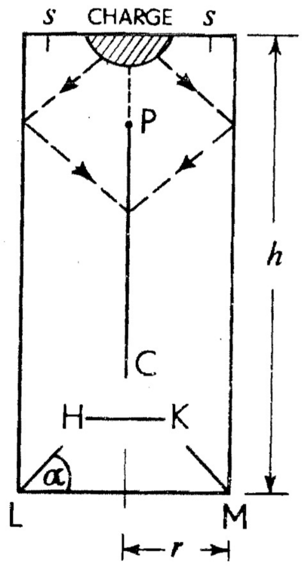


Cylindrical Blocks

- According to Kolsky, a charge detonated at the center of one end of a cylindrical block of *Perspex* gives rise to fracture patterns which are very similar to those already described for conical specimens.
  - (a) Compression damage beneath the charge, and
  - (b) a circular crack on the top surface, SS, just inside the outside curved surface; this is due to the reflection of the compressive pulse as a tensile pulse.
  - (c) Line fracture extending axially downwards from the charge; this is due to tensile wave reflection, interference and reinforcement from the curved sides of the block. The length of PC is proportional to the weight of charge, and

Cylindrical Blocks

- (d) with short cylinders, a flat region of fracture, HK, occurs; this is similar to scab formation.
- (e) A conical corner fracture, LM, arises in a manner similar to that described elsewhere



Thick-Walled Cylinders

- The cylinder fractures through the thickest part of the section.

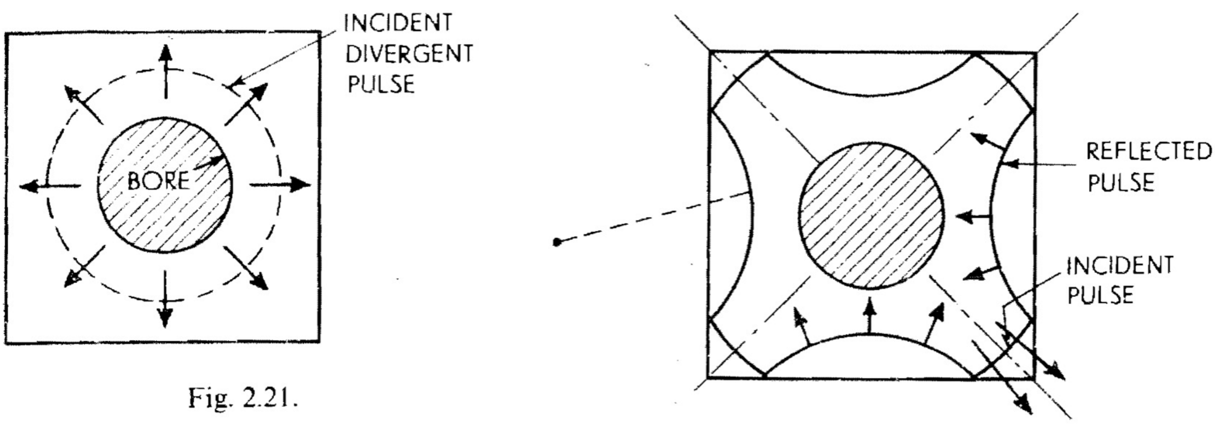
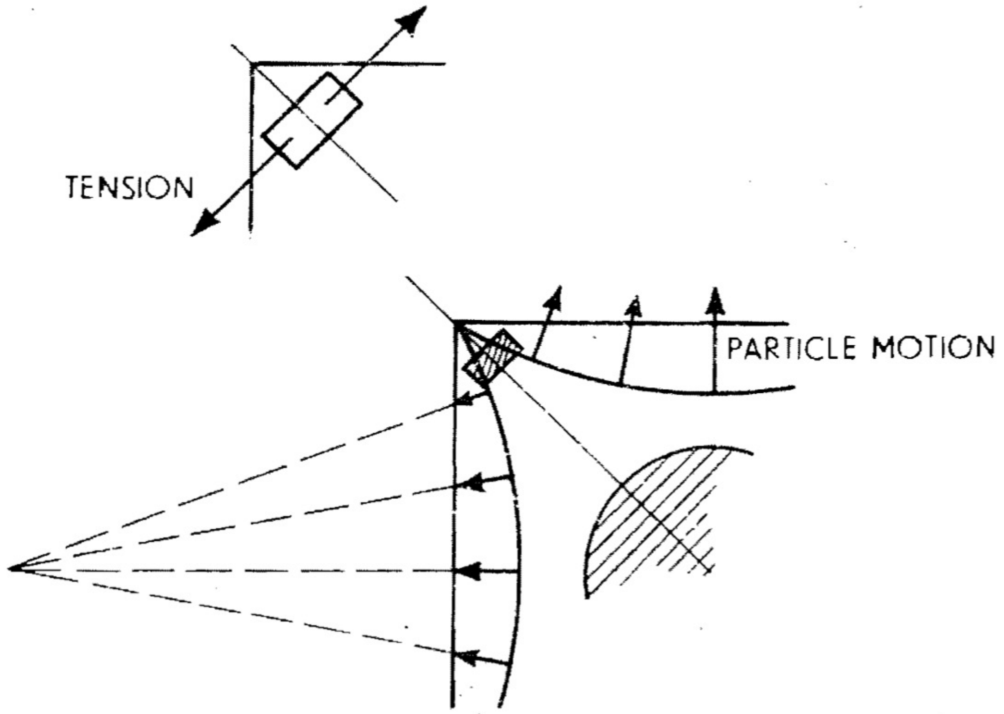


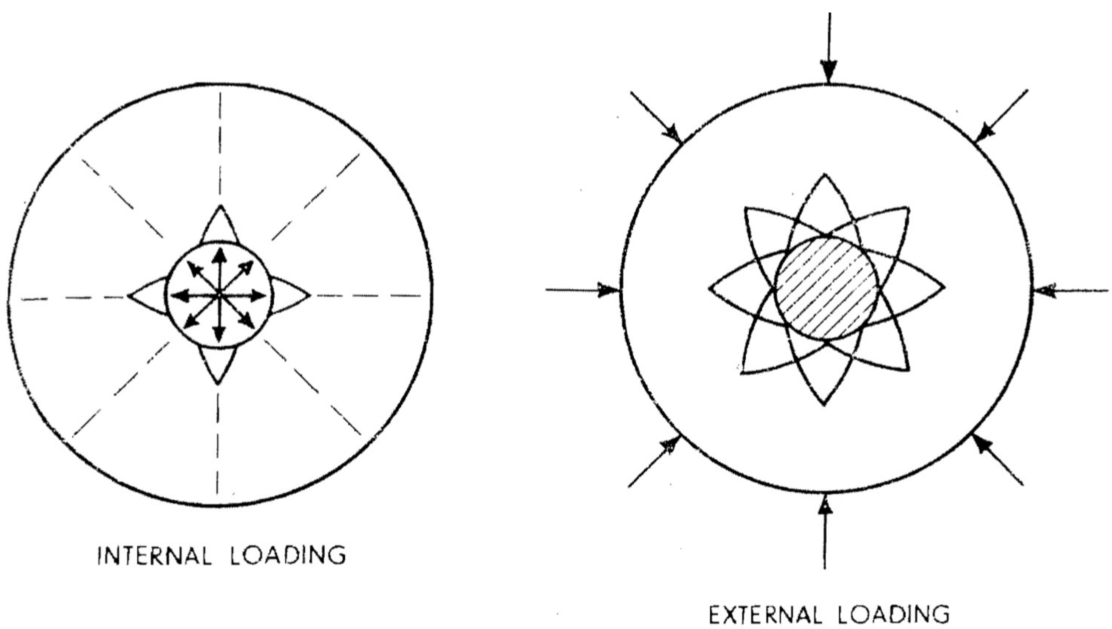
Fig. 2.21.

Thick-Walled Cylinders

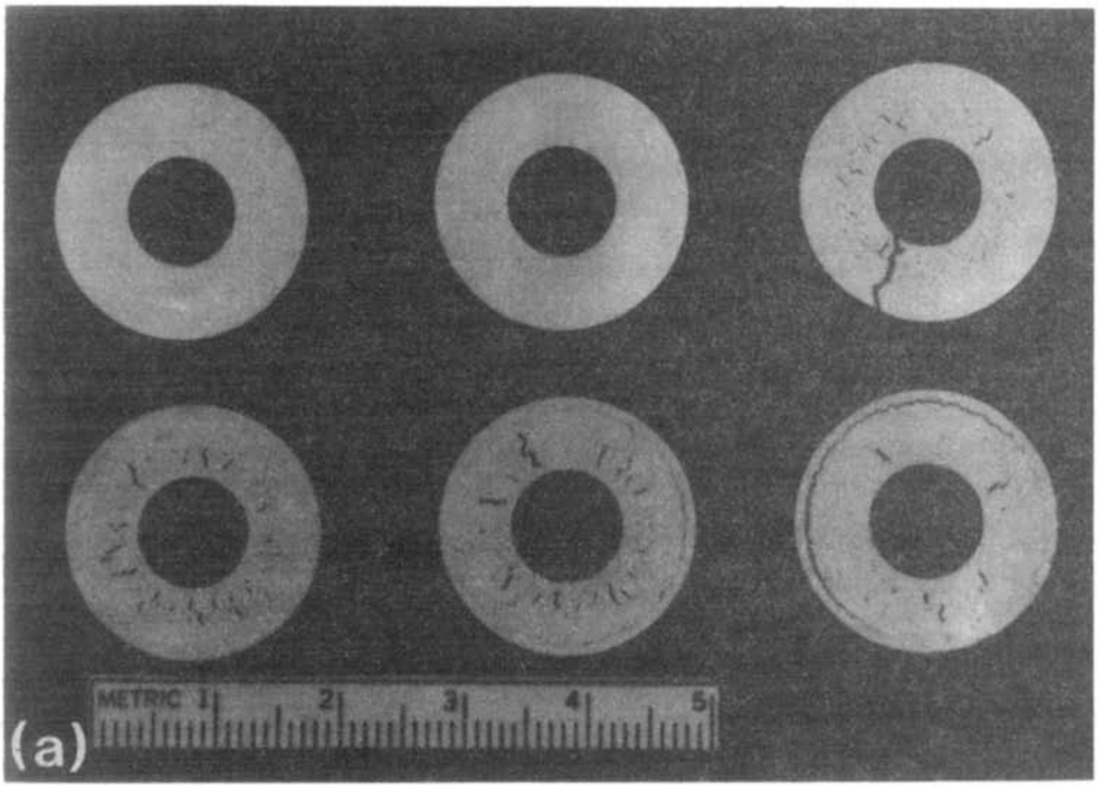


Circular Cylinders

- Shear fractures at the bore due to pulse intensification as it converges on the internal surface and is reflected.

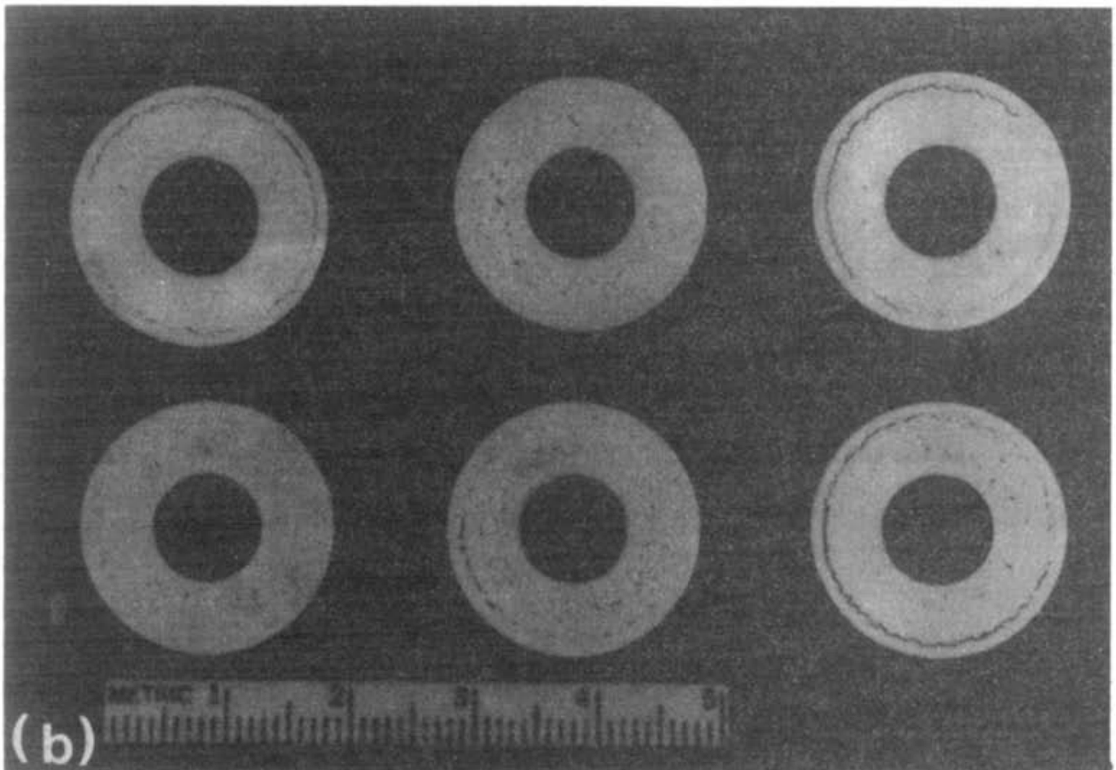


Effect of aging treatment on spalling produced in Al2024



M.A. Meyers, C.T. Aimone, Dynamic Fracture (Spalling) of Metals, Progress in Materials Science Vol. 28, pp. 1 to 96, 1983

Effect of aging treatment on spalling produced in Al6061



M.A. Meyers, C.T. Aimone, Dynamic Fracture (Spalling) of Metals, Progress in Materials Science Vol. 28, pp. 1 to 96, 1983

Thin Wires

- *B. Hopkinson* in a paper of 1905, remarks that J. Hopkinson published an account of "An investigation into the effect of a blow by a falling weight on the lower and free end of a wire, the upper end of which is fixed", in 1872.
- It was stated that when the tension wave is reflected at the fixed end, the stress and strain there are suddenly doubled.
- Rough experiments were carried out by J. Hopkinson which were said to confirm his expectations.
- The wire was most likely to break at the upper end.

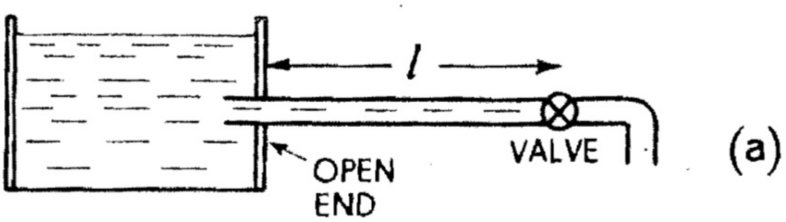
Thin Wires

- B. Hopkinson writes, "The general result that I have obtained is that iron and copper wires may be stressed much beyond the static elastic limit and even beyond their static breaking loads without the proportionality of stresses and strains being substantially departed from, provided that the time during which the stress exceeds the elastic limit is of the order of $0.001\ \mathrm{sec}$ or less."
- This is an early reference to the effects of time on the strength properties d metals and is an example of how caution should be exercised in respect of theoretical prediction.

Water Hammer

Water Hammer

- For a rigid pipe, in which a liquid flows with uniform speed v, instantaneous arrest at a point by valve closure, gives rise to a pressure in excess of that of the atmosphere of $p = \rho c v$ where $c = \sqrt{K/\rho}$



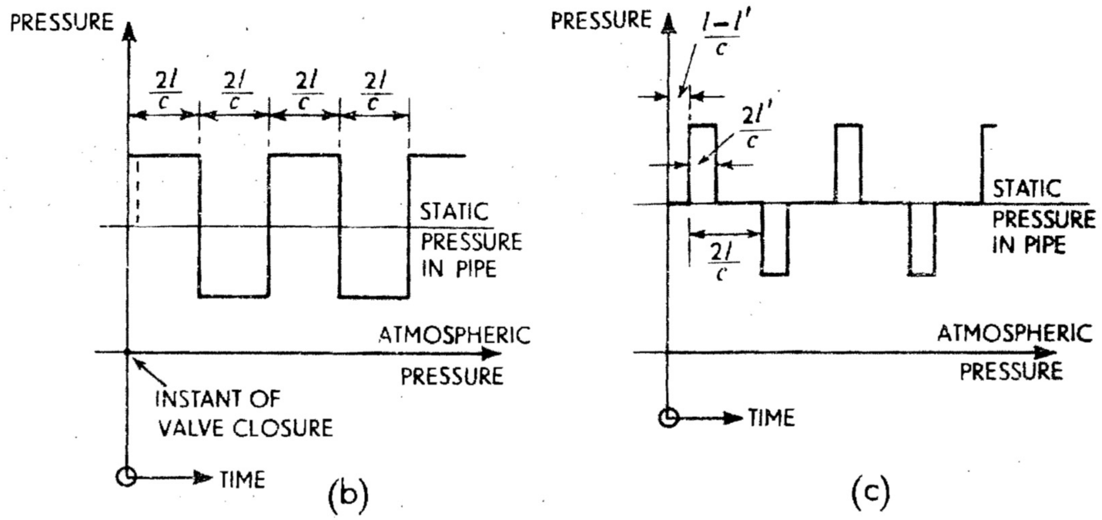
- A pressure wave in a liquid travels from the valve when closed, with speed c towards the open end of the pipe of length l.

Water Hammer

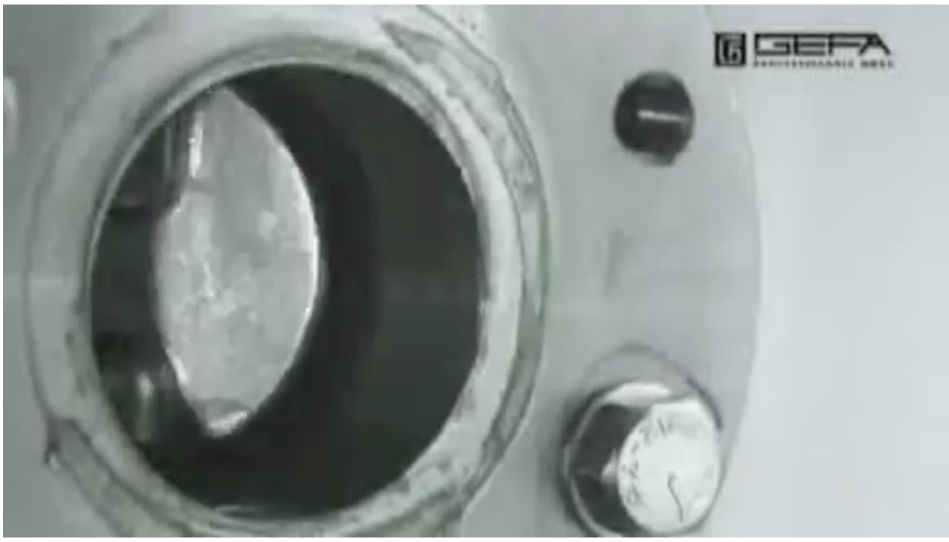
- The wave reflects from the open end of the pipe and at time $t = 2l/c$.
- The water column cannot leave the valve unless the pressure there drops to zero and its motion is consequently checked.
- If the water is completely checked, a wave at $t = 2l/c$ moves towards the open end of the pipe so that at $t = 3l/c$ the pressure is p (or ρcv) below atmospheric.
- Reflection at $t = 3l/c$ from the open end ensures that by $t = 4l/c$, the pressure is restored to just atmospheric and the column is again moving as a whole, with speed v, towards the valve.

Water Hammer

- (Idealized) Pressure-time graph:



- Factors:
  - Dissolved air, pipe friction, pipe elasticity, valve stretch



High Speed Liquid Impact with a Solid Surface

Liquid Impact

- When a liquid drop moves at high speed and impinges on a solid surface, very high pressures are developed for about one microsecond after impact.
- If a square-ended, liquid circular cylinder having a speed v impinges normally on a rigid anvil,

$$p = \rho v c_0$$

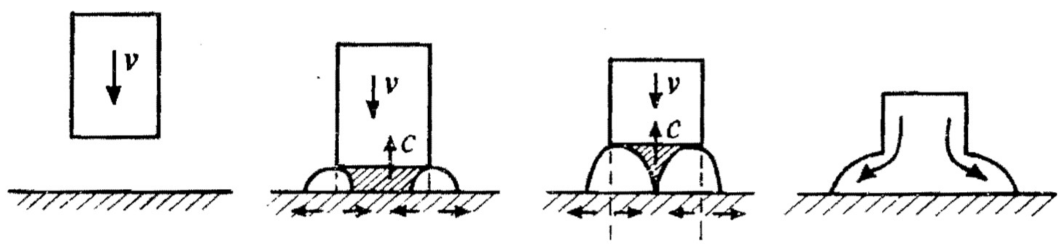
  - c_0 is the elastic wave speed in water.
- If impact takes place compressibly against a solid plane surface whose acoustic impedance is $(\rho c)_s$,

$$p = v \cdot \frac{(\rho c_0) \cdot (\rho c)_s}{\rho c_0 + (\rho c)_s}$$

  - Refer to slide 70 of chapter 1.

Liquid Impact

- The pressure, p, applies over the whole area of contact at the instant of impact.
- Its duration depends upon the time taken for unloading waves to move inwards to the axis of the cylinder from the curved surface of the cylinder.
- After this, the pressure on the anvil will fall to the steady state flow, incompressible pressure of $\rho v^2/2$ (*Bernoulli equation*).



Liquid Impact

- For a jet of diameter $3\ mm$, the release wave takes a time $t = r/c_0$, i.e. about $1\ \mu\text{sec}$.
- A jet having a speed of $520\ m/s$ would cause a peak pressure of about $770\ MPa$!
  - Note that c_0 depends on the pressure. At this pressure levels, c_0 doubles, and so do the obtained pressure!
  - Compare to yield strength of typical mild steel 250 MPa.
- Thus the subject of liquid droplet impact takes on great importance such as steam turbines, where water droplets impinge on blading, or in the front regions of aircrafts where, observation window may be hit by water droplets as it passes through a rain cloud.

Liquid Impact

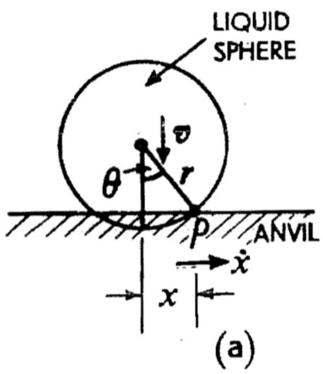
- In the very early stages of the impact of a spherical drop with a rigid plane surface, outward flow is prevented.
- Point P at the junction of the curved surface of the sphere and the anvil is moving radially outwards with speed $\dot{x}$

$$\dot{x} = \frac{d}{dt}(r\sin\theta) = r\cdot\cos\theta\cdot\dot{\theta}$$

$$v = \frac{d}{dt}(r\cos\theta) = -r\sin\theta\cdot\dot{\theta}$$

- So,

$$\dot{x} = -v\cdot\cot\theta$$



Liquid Impact

- The compressible flow pressure can only begin to decay if $\dot{x} < c_0$.
- The limiting case, as defined by a radius of contact x_0 is

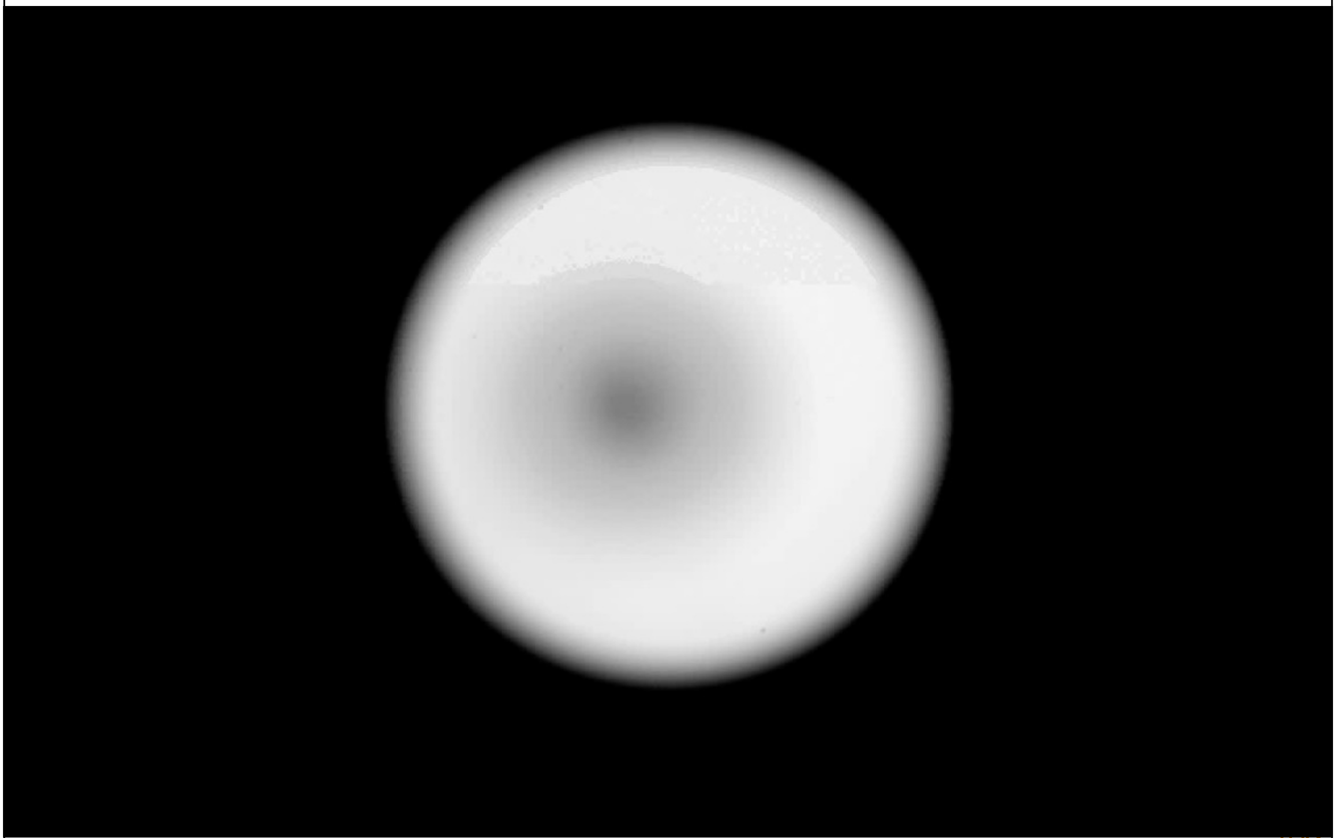
$$\dot{x} = c_0 = v \cot \theta \simeq v \cdot \frac{r}{x_0}$$

$$x_0 = v \cdot \frac{r}{c_0}$$

  - For a sphere of radius 2 mm and with $v = 607\, m/s$, $x_0 \cong 0.8\, mm$

Liquid Impact

- In summary, when a jet or drop collides with a solid surface the sequence of events is usually assumed to be as follows:
  - First there is the formation of a small central area of first contact, under uniform pressure p.
  - This initial area of contact grows as impact continues with little or no reduction in pressure until an outward flow begins.
  - As outward flow continues the compression is progressively relieved across the interface until the maximum pressure acting on the surface is only the stagnation pressure for incompressible flow $\rho v^2/2$.



Simple Elastic Wave Propagation in Helical Springs

Extensional Waves

- If Δ denotes the axial displacement of an element of the spring at initial distance x from the end of the spring to which the impact load is applied at time $t = 0$, the other end of the spring being fixed, and if F is the actual tensile force in the spring at x, then

$$F = \frac{GJ}{LR^2} l_0 \frac{\partial \Delta}{\partial x}$$

- Axial equation of motion:

$$dF = \frac{W}{g} \cdot \frac{dx}{l_0} \cdot \frac{\partial^2 \Delta}{\partial t^2}$$

- Combining,

$$\frac{dF}{dx} = \frac{W}{g} \cdot \frac{1}{l_0} \cdot \frac{\partial^2 \Delta}{\partial t^2} = \frac{GJ}{LR^2} \cdot l_0 \frac{\partial^2 \Delta}{\partial x^2}$$

Extensional Waves

$$\frac{\partial^2 \Delta}{\partial t^2} = \frac{JGl_0^2}{LR^2 \cdot W/g} \cdot \frac{\partial^2 \Delta}{\partial x^2}$$

- Thus, as would have been expected, we have derived the usual simple one-dimensional wave equation and hence the axial speed of an extensional wave is,

$$c_e = \frac{l_0}{R} \left(\frac{JG}{L \cdot W/g} \right)^{1/2}$$

- The surge time, t_s, which is the time taken for the wave to traverse the length of the spring is,

$$t_s = R \left(\frac{LW/g}{JG} \right)^{1/2}$$

Extensional Waves

- Alternatively, putting $W = wL$, where L is the helical length of the spring, w its weight per unit length, ρ the density and A the cross-sectional area,

$$t_s = RL\left(\frac{A\rho}{JG}\right)^{1/2}$$

- The wave concerned actually travels along the helix with, say, speed V_H so that,

$$V_H = \frac{L}{t_s} = \frac{1}{R}\left(\frac{JG}{A\rho}\right)^{1/2}$$

- Recalling the torsional wave speed in a straight rod,

$$c_T = \left(\frac{GJ}{I}\right)^{1/2} = \frac{1}{k_T}\left(\frac{GJ}{A\rho}\right)^{1/2}$$

- where k_T is the polar radius of gyration of the cross-section of the spring wire, $V_H/c_T = k_T/R$

Extensional Waves

- In the particular case of a circular section wire of radius a,

$$\frac{V_H}{c_T} = \frac{a}{R\sqrt{2}}$$

- and for a square section wire of side $2b$, for which $k_T^2 = 2b^2/3$,

$$\frac{V_H}{c_T} = \frac{b}{R} \cdot \left(\frac{2}{3}\right)^{1/2}$$

Rotational Waves

- The sudden application of a twisting moment about the axis of a close-coiled spring causes a rotational wave to be propagated along it.
- The torque-rotation equation for an element of initial length dx, is,

$$T = EI \cdot \frac{l_0}{L} \cdot \frac{\partial \psi}{\partial x}$$

- The rotational equation of motion of the element is,

$$dT = \frac{W}{g} \cdot R^2 \cdot \frac{dx}{l_0} \cdot \frac{\partial^2 \psi}{\partial t^2}$$

- Eliminating T from these equations,

$$\frac{\partial^2 \psi}{\partial t^2} = \frac{EI l_0^2}{LR^2 W/g} \cdot \frac{\partial^2 \psi}{\partial x^2}$$

Rotational Waves

- Thus the axial speed of a rotational wave is,

$$c_R = \frac{l_0}{R} \cdot \left(\frac{EI}{LW/g} \right)^{1/2}$$

- and the surge time t_s' is,

$$t_s' = \frac{l_0}{c_R} = R \cdot \left(\frac{LW/g}{EI} \right)^{1/2}$$

- Alternatively,

$$t_s' = RL \left(\frac{A\rho}{EI} \right)^{1/2}$$

Rotational Waves

- The speed V_H' of a rotational wave along the helix itself is

$$V_H' = \frac{L}{t_s'} = \frac{1}{R} \cdot \left(\frac{EI}{A\rho}\right)^{1/2}$$

- Now $I = Ak^2$ and $w/g = A\rho$ where k is the radius of gyration of the section. Thus,

$$V_H' = \frac{k}{R} \cdot \left(\frac{E}{\rho}\right)^{1/2}$$

  - In the particular case of a circular section wire of radius a, $k = a/2$, so that

$$V_H' = \frac{a}{2R} \cdot \left(\frac{E}{\rho}\right)^{1/2} \quad \text{or} \quad \frac{V_H'}{c_0} = \frac{a}{2R}$$

  - and for a square section wire of side $2b$,

$$k = b/\sqrt{3} \text{ and } \frac{V_H'}{c_0} = \frac{b}{R} \cdot \frac{1}{\sqrt{3}}$$

The Buckling of Rods and Tubes Due to Impact

Buckling

- The stress to which a rod is subjected at impact with speed v is,

$$\sigma = \rho c v$$

- Recalling the Euler buckling stress formula for a pin-ended strut of length λ,

$$\sigma = \frac{\pi^2 E}{(\lambda/k)^2}$$

- Eliminating σ,

$$\frac{\lambda}{k} = \pi\sqrt{\frac{c}{v}}$$

  - k is the radius of gyration of the section

Buckling

- Comparison of some experimental results: