Related Rate Problems - Solutions

1. **Air is being pumped into a spherical balloon so that its volume increases at a rate of 100 cm^3/s. How fast is the radius of the balloon increasing when the diameter is 50 cm?**

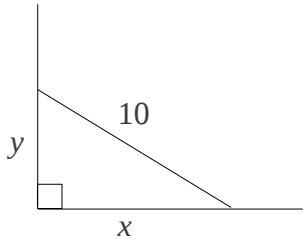
$$V=\frac{4}{3}\pi r^3$$

$$\frac{dV}{dt}=4\pi r^2\frac{dr}{dt}$$

$$100=4\pi 25^2\frac{dr}{dt}$$

$$\frac{dr}{dt}=\frac{1}{25\pi}\ \text{cm/s.}$$

2. **A ladder 10ft long rests against a vertical wall. If the bottom of the ladder slides away from the wall at a rate of 1 ft/s. how fast is the top of the ladder sliding down the wall when the bottom of the ladder is 6 ft from the wall?**



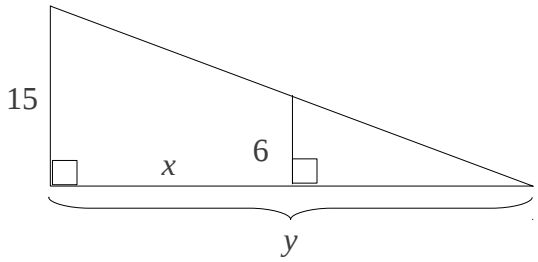
$$x^2+y^2=100$$

$$2x\frac{dx}{dt}+2y\frac{dy}{dt}=0$$

$$(2)(6)(1)+(2)(8)\frac{dy}{dt}=0$$

$$\frac{dy}{dt}=-\frac{3}{4}\ \text{ft/s.}$$

3. A street light is mounted at the top of a 15-ft-tall pole. A man 6 ft tall walks away from the pole with a speed of 5 ft/s along a straight path. How fast is the tip of his shadow moving when he is 40 ft from the pole?



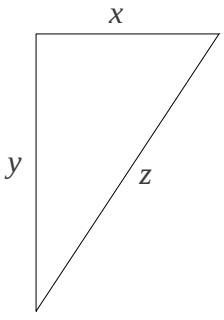
By similar triangles, $\frac{15}{y}=\frac{6}{y-x}$. That is, $9y=15x$.

$$9\frac{dy}{dt}=15\frac{dx}{dt}$$

$$9\frac{dy}{dt}=(15)(5)$$

$$\frac{dy}{dt}=\frac{25}{3}\ \text{ft/s.}$$

4. Car A is traveling west at 50 mi/h and car B is traveling north at 60 mi/h. Both are headed for the intersection of the two roads. At what rate are the cars approaching each other when car A is 0.3 mi and car B is 0.4 mi from the intersection?



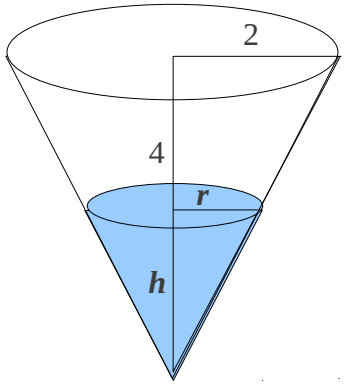
$$x^2+y^2=z^2$$

$$2x\frac{dx}{dt}+2y\frac{dy}{dt}=2z\frac{dz}{dt}$$

$$(2)(0.3)(-50)+(2)(0.4)(-60)=(2)(0.5)\frac{dz}{dt}$$

$$\frac{dz}{dt}=-78\ \text{mi/h.}$$

5. **A water tank has the shape of an inverted circular cone with base radius 2m and height 4m. If water is being pumped into the tank at a rate of 2 m^3/min, find the rate at which the water level is rising when the water is 3 m deep.**



$$V=\frac{1}{3}\pi r^2 h$$

by similar triangles $\frac{r}{h}=\frac{2}{4}$ and hence $r=\frac{h}{2}$

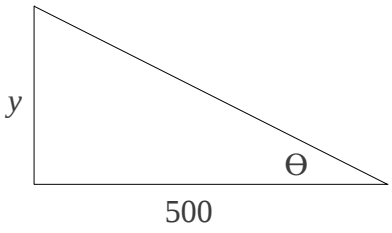
$$V=\frac{1}{12}\pi h^3$$

$$\frac{dV}{dt}=\frac{1}{12}\pi 3h^2\frac{dh}{dt}$$

$$2=\frac{1}{12}\pi 3(3)^2\frac{dh}{dt}$$

$$\frac{dh}{dt}=\frac{8}{9\pi}\ \text{m/min.}$$

6. **A hot air balloon rising straight up from a level field is being tracked from a spectator 500 ft from the liftoff point. At the moment the angle of elevation is $\frac{\pi}{4}$, the angle is increasing at the rate of 0.14 rad/min. How fast is the balloon rising at that moment?**



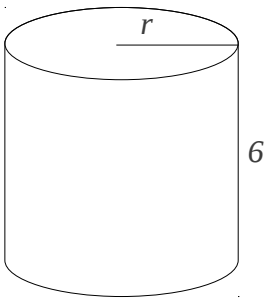
$$\tan\theta=\frac{y}{500}$$

$$\sec^2\theta\frac{d\theta}{dt}=\frac{1}{500}\frac{dy}{dt}$$

$$\sec^2\left(\frac{\pi}{4}\right)(0.14)=\frac{1}{500}\frac{dy}{dt}$$

$$\frac{dy}{dt}=140\ \text{ft/min.}$$

7. A mechanic is reboring a 6-in-deep cylinder to fit a new piston. The machine they are using increases the cylinder's radius one-three thousandth of an inch every minute. How rapidly is the cylinder volume increasing when the bore diameter is 3.8 inches?



$$V = \pi r^2 6$$

$$\frac{dV}{dt} = 12\pi r \frac{dr}{dt}$$

$$\frac{dV}{dt} = 12\pi (1.9) \frac{1}{3000}$$

$$\frac{dr}{dt} = \frac{19\pi}{2500} \quad \text{in}^3/\text{min}$$