



# Combined heat and power economic dispatch using exchange market algorithm



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## ABSTRACT

Combined heat and power economic dispatch (CHPED) is one of the critical issues in power systems, playing key role in economic performance of the system. CHPED is a challenging optimization problem of non-linear and non-convex type. Thus, evolutionary and heuristic algorithms are employed as effective tools in solving this problem. This paper applies newly proposed exchange market algorithm (EMA) on CHPED problem. EMA is a powerful and robust algorithm. With two powerful absorbing operators pulling solutions toward optimality and two smart searching operators, EMA is able to extract optimum point in optimization problem. In order to examine the proposed algorithm's capabilities and find optimum solution for CHPED problem, several test systems considering valve-point effect, system power loss and system constraints are optimized. The obtained results prove high capability of EMA in extracting optimum points. The results also show that this algorithm can be utilized as an efficient and reliable tool in solving CHPED problem.

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## Introduction

In conventional thermal generating units, all of the produced heat energy is not converted to the electric power and a considerable fraction of the power is lost as heat loss. Combined heat and power (CHP) as a cogeneration system can lead to the simultaneous production of heat and electric power from one fuel source. Thus, supplying simultaneous heat and power required for customers is possible [1,2]. In CHP system, output energy of a generating unit can be utilized as input energy for the other system. The use of CHP system is, therefore, can increase fuel efficiency up to 90% [3], decrease production cost by 10–40% [4] and environmental pollution by 13–18% [5]. In order to effectively utilizing of cogeneration units, economic dispatch problem is solved for optimal combination of output heat and power of generating units to satisfy the heat and power demand in system. That is, the economic dispatch problem with cogeneration units called the CHP economic dispatch (CHPED) problem is solved [6].

The aim of solving CHPED problem is to determine optimal heat and power of generating units with the minimized cost of total system and satisfied constraints of problem. In addition, the heat and power demand should be met. The presence of heat-power

feasibility constraints of cogeneration units may result in more complicated ED problem in comparison to conventional economic dispatch problems [7,8]. In recent two decades, much research has been reported in literature for solving CHPED problem using mathematical methods and optimization algorithms. In [9], a two-level strategy was proposed to solve CHPED problem. The lower level determines the outputs of units under given Lagrangian multipliers, and the upper level updates the multipliers by a Newton-based iterative process. The procedure is repeated until the heat and power demands are met. In [10], CHPED problem was divided into subproblems: heat dispatch and power dispatch. These two subproblems were correlated in heat-power feasible operation region for CHP units. Afterwards, Lagrangian relaxation algorithm was utilized to solve this problem. In [11], Makkonen and Lahdelma proposed a mixed integer programming model to solve CHP problem. In order to accelerate optimization process, the problem is divided into two hourly subproblems and a customized branch-and-bound algorithm was applied to solve these subproblems. All mentioned techniques could successfully solve CHPED problem assuming a convex fuel cost. However, generating units have non-convex fuel cost in practice leading to inability of the aforementioned techniques in solving non-convex CHPED problem. Heuristic algorithms can optimize various problems by generating random numbers without considering complexity and constraints of the problem. Thus, various intelligent techniques,

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including improved ant colony search algorithm [12], evolutionary programming [13] genetic algorithm [14], harmony search algorithm [15] and multi objective particle swarm optimization [16] have been proposed to successfully solve CHPED problem with convex and non-convex fuel cost function.

Heuristic algorithms have an operator for generating random number and another operator for absorbing random numbers toward optimum numbers. In other words, heuristic algorithms find optimum points in optimization problems by generating random numbers. Due to their randomized structure, evolutionary algorithms may encounter with problems and constraints such as trapping in local minima and, in turn, premature convergence, inability to extract optimum-neighborhood points and convergence to non-matched solutions in each program run [17].

Exchange market algorithm as a heuristic algorithm was first proposed by N. Ghorbani and E. Babaei in 2014. Inspired by human intelligence and the process of trading shares in stock market, EMA is proposed mainly to solve optimization problems. EMA's structure is same as the other optimization algorithms in terms of generating random numbers. However, this has two simultaneous intelligent operators generating random numbers and two efficient operators absorbing random numbers towards optimal numbers. This leads to the best-generated numbers. Thus, some of drawbacks and issues in other optimization algorithms mentioned above are highly obviated [18].

EMA is a population-based algorithm inspired by stock market in which a number of stocks are selected by shareholders. Then, they make decisions on the selected stocks based on their own policies. In the proposed algorithm, two market states are available per program run: (1) balanced market, where the algorithm absorbs individuals toward elite person, (2) oscillated market, where the algorithm produces random numbers. In this algorithm, the fitness of individuals is evaluated after each market state. Then, they are ranked based on their conditions and placed in different groups.

Considering high capability of EMA in finding optimum point, this algorithm can be applied on various CHPED problems including power-only units, CHP units, and heat-only units with valve-point effect, system power loss and operational constraints. The results obtained by this technique are compared with those of obtained by intelligent methods. These results show the superiority of the proposed algorithm over the other intelligent techniques.

The rest of this paper is organized as follows. Section "Problem formulation": gives the formulation of the CHPED problem; Section "Exchange market algorithm": explains the EMA; Section "Exchange market algorithm implementation pattern in solving CHPED problem": shows implementation pattern of EMA in solving CHPED problem; Section "Numerical studies": shows implementation of the proposed algorithm to the test systems and obtained results; and Section "Conclusion" gives our conclusions.

### Problem formulation

Authors in [12–15] formulated CHPED problem constraints in details. In general, the aim of solving CHPED problem is to determine the generating unit power and heat production such that the system's production cost is minimized while the power and heat demands and other constraints are met appropriately.

#### Objective function

The objective function of CHPED problem is given by:

$$\min \sum_{i=1}^{N_p} C_i(P_i^p) + \sum_{j=1}^{N_c} C_j(P_j^c, H_j^c) + \sum_{k=1}^{N_h} C_k(H_k^h) \quad (\$/h) \quad (1)$$

where  $C_i$ ,  $C_j$  and  $C_k$  are production cost of the power-only, GHP and heat-only units, respectively.  $N_p$ ,  $N_c$ ,  $N_h$  are the number of above mentioned units, respectively.  $i$ ,  $j$  and  $k$  are the indices used for power-only, CHP and heat-only units, respectively. In Eq. (1),  $H$  and  $P$  indicate the heat and power output of unit, respectively. The production cost of different unit types are defined as:

$$C_i(P_i^p) = \alpha_i(P_i^p)^2 + \beta_i P_i^p + \gamma_i \quad (\$/h) \quad (2)$$

$$C_j(P_j^c, H_j^c) = a_j(P_j^c)^2 + b_j P_j^c + c_j + d_j(H_j^c)^2 + e_j H_j^c + f_j H_j^c P_j^c \quad (\$/h) \quad (3)$$

$$C_k(H_k^h) = a_k(H_k^h)^2 + b_k H_k^h + c_k \quad (\$/h) \quad (4)$$

where  $C_i(P_i^p)$ ,  $C_j(P_j^c, H_j^c)$  and  $C_k(H_k^h)$  are cost function of the power-only, CHP and heat-only units, respectively.  $\alpha_i$ ,  $\beta_i$  and  $\gamma_i$  stand for cost coefficients of  $i$ th conventional thermal unit.  $a_j$ ,  $b_j$ ,  $c_j$ ,  $d_j$ ,  $e_j$  and  $f_j$  are cost coefficients of  $j$ th CHP unit. In Eq. (3),  $a_k$ ,  $b_k$  and  $c_k$  show the cost coefficients of  $k$ th heat-only unit.  $P_i^p$  and  $P_j^c$  are the power outputs of power and CHP units.  $H_j^c$  and  $H_k^h$  are the heat production by cogeneration and heat-only units.

In a practical generation unit, steam-valve admission effects lead to the ripple in the production cost. In order to model this effect more accurately, a sinusoidal term is added to the quadratic cost function. In this case, Eq. (5) is used to show the valve-point effects in cost function of power units instead of Eq. (2).

$$C_i(P_i^p) = \alpha_i(P_i^p)^2 + \beta_i P_i^p + \gamma_i + |\lambda_i \sin(\rho_i(P_i^p - P_i^{\min}))| \quad (\$/h) \quad (5)$$

where  $\lambda_i$  and  $\rho_i$  are the cost coefficients of power unit  $i$  for reflecting valve-point effects [19].

#### Equality and inequality constraints

In order to balance the supply and demand, the power equality constraint should be met. Total generated power of the power-only and CHP units should be equal to total system demand which can be evaluated by Eq. (6). If there are power losses in the system, they should be added to the system demand power.

$$\sum_{i=1}^{N_p} P_i^p + \sum_{j=1}^{N_c} P_j^c = P_d \quad (6)$$

$$\sum_{i=1}^{N_p} P_i^p + \sum_{j=1}^{N_c} P_j^c = P_d + P_{loss} \quad (7)$$

$$P_{loss} = \sum_{i=1}^{N_p} \sum_{m=1}^{N_p} P_i^p B_{im} P_m^p + \sum_{i=1}^{N_p} \sum_{j=1}^{N_c} P_i^p B_{ij} P_j^c + \sum_{j=1}^{N_c} \sum_{n=1}^{N_c} P_j^c B_{jn} P_n^c \quad (8)$$

where  $P_d$  is the system demand. Parameter  $P_{loss}$  is the power losses of transmission line and a function of units output power evaluated by Eq. (8). Total generated heat of cogeneration and heat units should be equal to total system demand heat in order to balance the heat demand:

$$\sum_{j=1}^{N_c} H_j^c + \sum_{k=1}^{N_h} H_k^h = H_d \quad (9)$$

where  $H_d$  is the system heat demand.

The outputs of electricity units and heat units are restricted by their own upper and lower boundaries. The power and heat outputs of cogeneration units should be placed in feasible operation region. Fig. 1 illustrates the heat-power feasible operation region of a CHP unit. The inequality constraints of each generating unit in the CHPED problem are given by:

$$P_i^{\min} \leq P_i^p \leq P_i^{\max} \quad i = 1, 2, \dots, N_p \quad (10)$$

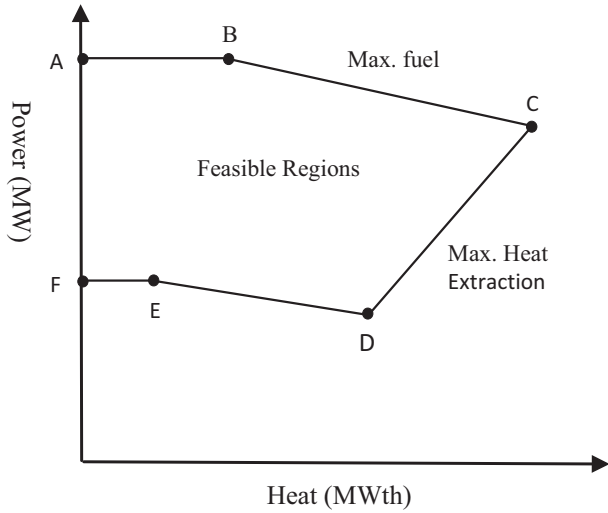


Fig. 1. Feasible operating region of a cogeneration unit.

$$P_j^{c \min}(H_j^c) \leq P_j^c \leq P_j^{c \max}(H_j^c) \quad j = 1, 2, \dots, N_c \quad (11)$$

$$H_j^{c \min}(P_j^c) \leq H_j^c \leq H_j^{c \max}(P_j^c) \quad j = 1, 2, \dots, N_c \quad (12)$$

$$H_k^{h \min} \leq H_k^h \leq H_k^{h \max} \quad k = 1, 2, \dots, N_h \quad (13)$$

where  $P_i^{p \min}$  and  $P_i^{p \max}$  are the minimum and maximum power generation boundaries of the power-only units.  $P_j^{c \min}(H_j^c)$  and  $P_j^{c \max}(H_j^c)$  are the minimum and maximum power generation boundaries of the CHP units.  $H_j^{c \min}(P_j^c)$  and  $H_j^{c \max}(P_j^c)$  in (11) indicate the minimum and maximum heat generation boundaries of the cogeneration units. In (13)  $H_k^{h \min}$  and  $H_k^{h \max}$  are the minimum and maximum heat generation boundaries of the heat units.

### Exchange market algorithm

The aim of EMA, inspired by the method of selling and purchasing of shares by elite stockholders, is to solve the optimization problems. Investigating the performance of elite stockholders results in launching of EMA. With decreasing in the possessions of these stockholders, there is a trend to take greater risks [26].

The performance of elite stockholders varies in the market with oscillation and balanced markets. In each iteration of the algorithm, it is assumed that two different market situations are available. Successful stockholders have different performances when they have high or low success. The behavior and performance of elite stockholders have been assessed when their level of possessions is low, mean and high, and the results have been employed using EMA. In this algorithm, the successful individuals are taking the necessary measures to introduce themselves as the most successful stockholders in the market; hence, they compete with each other. Considering the above-mentioned points, in EMA, there are two market situations. The stockholders' fitness is investigated after each iteration and individual stockholders will be ranked based on the value of their possessions. After each market condition, the individuals with high, medium, and low ranks will be named as group 1 (G1), group 2 (G2), and group 3 (G3), respectively. The members of G1, will not trade in all iterations. Members of G2 and G3 tend to sell and purchase shares through special separate equations. In the balanced market, the algorithm is responsible for absorbing individuals toward elite stockholders and in an oscillated market; the algorithm is responsible for searching

process. The algorithm in the balanced market and oscillated market has two absorbing operators and two searching operators which cause the most appropriate creation and organization of random number in EMA.

#### The exchange market in balanced condition

In this section, the market is balanced and there exist no oscillations. The stockholders are trying to search for the optimum points as follows: without taking non-market risks, using experiences of elite stockholders, and close consideration of the existing situations. In this section, each individual is ranked based on the number of each type of shares s/he holds and the fitness function.

#### Shareholders with high ranks

This group's members lead the stock market and preserve their ranking, they do not change their shares and do not undergo the trade risk. The individuals of the group are the elite stockholders, or the best solutions for the problems which are necessary to stay intact and unchanged.

#### Shareholders with mean ranks

This group of shareholders comprises of 20–50 percent of the stock market. The members of this group use the successful experiences of elite stockholders. They tend to take the least possible risk in changing their shares. They cleverly and consciously utilize the differences of the values of the G1's shares. In this section, a comparison is done between the shares of the two shareholders. As mentioned earlier, the members of this group change the number of their shares based on the Eq. (14) to achieve further profits.

$$pop_j^{group(2)} = r \times pop_{1,i}^{group(1)} + (1 - r) \times pop_{2,i}^{group(1)} \quad (14)$$

$$i = 1, 2, 3, \dots, n_i \text{ and } j = 1, 2, 3, \dots, n_j$$

where,  $n_i$  is the  $n$ th individual of the first group,  $n_j$  is the  $n$ th individual of the second group and  $r$  is a random number in interval  $[0, 1]$ .  $pop_{1,i}^{group(1)}$  and  $pop_{2,i}^{group(1)}$  are the members of the first group and  $pop_j^{group(2)}$  is the  $j$ th individual of the second group.

#### Shareholders with low ranks

This group of individuals are the end-placed ranking shareholders. The behavioral characteristics of this group are as follows: their risk is high compared to the G2; they make use of small changes and differences of G1's shares; unlike second group individuals, they utilize the differences of share values of the first group as well as their share values' differences compared to the first group individuals and change their shares. In order to earn more profits, the members of this group would change the number of their shares based on the Eq. (16):

$$S_k = 2 \times r_1 \times (pop_{i,1}^{group(1)} - pop_k^{group(3)}) + 2 \times r_2 \times (pop_{i,2}^{group(1)} - pop_k^{group(3)}) \quad (15)$$

$$pop_k^{group(3),new} = pop_k^{group(3)} + 0.8 \times S_k \quad k = 1, 2, 3, \dots, n_k \quad (16)$$

where  $r_1$  and  $r_2$  are random numbers in interval  $[0, 1]$  and  $n_k$  is the  $n$ th member of the third group.  $pop_k^{group(3)}$  is the  $k$ th member and  $s_k$  is the share variations of the  $k$ th member of the third group.

#### The exchange market in oscillated condition

In this section, having assessed the shareholders and ranked them based on their fitness values, the shareholders would start trading their shares [1]. With regard to their fitness, shareholders are categorized into 3 separate groups:

**Shareholders with high ranks**

This part of the population includes the elite stockholders or the individuals who are the best solutions to the problem. This group leads the stock market and preserves their rank, they do not modify their shares and do not take any trading risks. This group consists of 10–30 percent of the population.

**Shareholders with mean ranks**

In this section, the sum of the shares held by individuals tends to be constant and only some of each type of shares increase and some decrease such that the sum remains constant. At first, the number of shares held by each individual increases based on the following equation:

$$\Delta n_{t1} = n_{t1} - \delta + (2 \times r \times \mu \times \eta_1) \tag{17}$$

$$\mu = \left( \frac{t_{pop}}{n_{pop}} \right) \tag{18}$$

$$n_{t1} = \sum_{y=1}^n |s_{ty}| \quad y = 1, 2, 3, \dots, n \tag{19}$$

$$\eta_1 = n_{t1} \times g_1 \tag{20}$$

$$g_1^k = g_{1,max} - \frac{g_{1,max} - g_{1,min}}{iter_{max}} \times k \tag{21}$$

where  $\Delta n_{t1}$  is the amount of shares should be added randomly to some shares,  $n_{t1}$  is total shares of  $t$ th member before applying the share changes.  $s_{ty}$  is the shares of the  $t$ th member,  $\delta$  is the information of exchange market.  $r$  is a random number in interval  $[0, 1]$ .  $\eta_1$  is risk level related to each member of the second group,  $t_{pop}$  is the number of the  $t$ th member in exchange market.  $n_{pop}$  is the number of the last member in exchange market,  $\mu$  is a constant coefficient for each member and  $g_1$  is the common market risk amount that decreases with the increase in iteration number.  $iter_{max}$  is the last iteration number and  $k$  is the number of program iteration.  $g_{1,max}$  and  $g_{1,min}$  indicate the maximum and minimum values of risk in market, respectively.

In the second part of this section, it is required that each individual sells some of his/her shares randomly being equal to the number  $s$ /he has purchased in a way that the sum of each individual's shares remain constant. In this section, it is essential that each individual reduces the number of her/his shares in  $\Delta n_{t2}$  amount. In this state, the  $\Delta n_{t2}$  of each individual equals by:

$$\Delta n_{t2} = n_{t2} - \delta \tag{22}$$

where  $\Delta n_{t2}$  is the amount of shares are to be decreased randomly from some shares and  $n_{t2}$  is the sum share amount of  $t$ th member after applying the share variations.

**Shareholders with low ranks**

The risk percentage of individuals in this group is variable. With reduction of their fitness, this risk increases. In this section, unlike G2, the sum of the individual's number of shares would change after each trade. In other words, in each section, the individual purchases or sells a number of shares. The shareholders of this group change some of their shares based on the following equation:

$$\Delta n_{t3} = (4 \times r_s \times \mu \times \eta_2) \tag{23}$$

$$r_s = (0.5 - rand) \tag{24}$$

$$\eta_2 = n_{t1} \times g_2 \tag{25}$$

$$g_2^k = g_{2,max} - \frac{g_{2,max} - g_{2,min}}{iter_{max}} \times k \tag{26}$$

where  $\Delta n_{t3}$  is the share amount are to be randomly added to the shares of each member,  $r_s$  is a random number in  $[-0.5, 0.5]$  and  $\eta_2$  is the risk coefficient related to each member of the third group.  $g_2$  is the variable risk of the market in the third group and  $\mu$  is the risk increase coefficient which forces lower ranked shareholders from fitness function viewpoint to perform more risk in comparison with successful competitors to increase their finance.  $g_2$  is the variable risk coefficient of the market and determines what percentage of shares should be changed by shareholders.

**Exchange market algorithm implementation pattern in solving CHPED problem**

The CHPED problem optimization is accomplished using the exchange market algorithm by taking the following steps:

- 1) Selecting initial values and allocating share to the initial shareholders.
- 2) Calculating shareholders fitness by Eq. (1), ranking them, and classifying of shareholders in three separate groups. (Beginning balanced mode).
- 3) Applying variations on the shares of the second group members in normal market mode (balanced market) by Eq. (14).
- 4) Applying variations on the shares of the third group members in normal market mode by Eq. (16).
- 5) Recalculating shareholders fitness by Eq. (1), ranking and classifying shareholders in three separate groups. (Beginning oscillation mode).
- 6) Trading the shares of the second group members using Eq. (17) in oscillated market mode.
- 7) Trading the shares of the third group members using Eq. (23) in oscillated market mode.
- 8) Jumping to step 2 until the program ending criterion is satisfied.

In this step, the market oscillation condition is finished and the program starts to operate in order to evaluate the shareholders from step 2 if end up conditions are not satisfied. If end up conditions are satisfied, that is the number of program iteration, the program operation is ended up.

Flowchart of the EMA's implementation for solving the CHPED problem is shown in Fig. 2.

**Numerical studies**

In order to evaluate the effectiveness of EMA and extract optimum point in CHPED problem, this algorithm is applied successfully on 5 different systems considering valve-point effect and implemented network losses. For each of test system, 50 independent experiments are done so as to compare problem solving quality and convergence characteristics. EMA based methodology is developed by Matlab 7.8 in 2.5 GHz, i5, personal computer. For all case studies initial population size is 100 and adjustable parameters of the algorithm are coefficients ' $g_1$ ' and ' $g_2$ ', and their optimal values are included in Table 1.

*Test System-I*

The study system is composed of one power-only unit, two CHP units and one heat-only unit. All information related to power-only unit (unit-1) and heat-only unit (unit-4) and data of CHP units and feasible regions are presented in [2]. Power and heat demands are 200 MW and 115 MWth, respectively. The obtained results from solving above problem using EMA are given in Table 2. In addition, these results are compared with those of particle swarm optimiza-

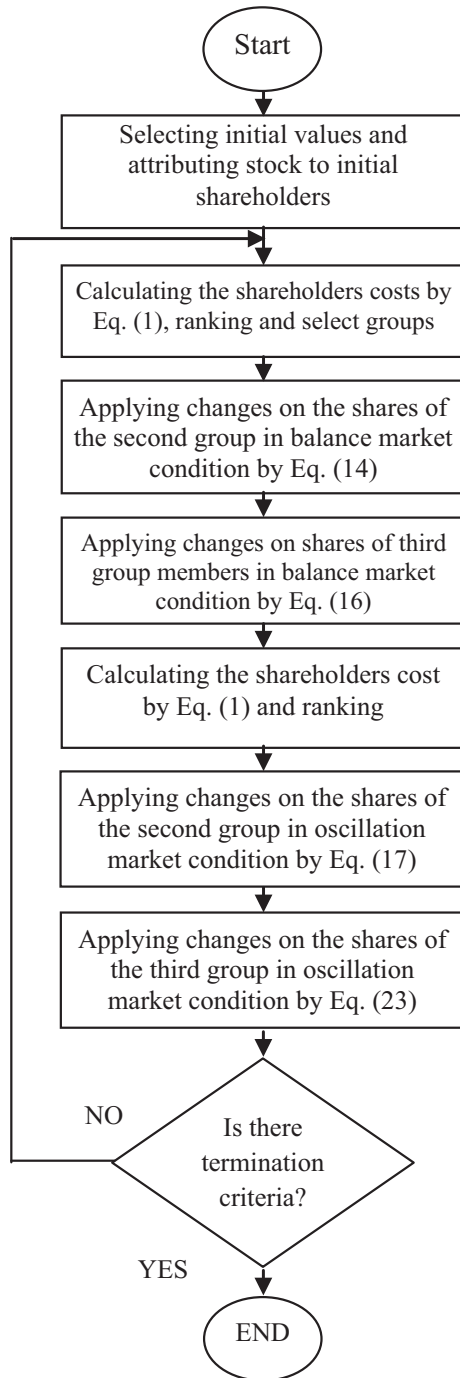


Fig. 2. Program implementation flowchart of exchange market algorithm.

Table 2

Comparison of simulation results for case I.

Output	PSO	TVAC -PSO	EDHS <sup>a</sup>	IACS	EMA
$P_1$	0.05	0.00	0	0.08	0
$P_2$	159.43	160.00	200	150.93	160.00
$P_3$	40.57	40.00	0	49	40.00
$H_2$	39.97	40.00	0	48.84	40.00
$H_3$	75.03	75.00	115	65.79	75.00
$H_4$	0	0.00	0	0.37	0.00
TP	200.05	200.00	200	200.01	200.00
TH	115	115.00	115	115	115.00
TC	9265.1	9257.07	8606.07	9452.2	9257.07
CT	1.42	1.33	NA	23.4	0.9846

P: Power (MW); H: Heat (MWth); TP: Total Power (MW); TH: Total Heat (MWth); TC: Total Cost (\$); CT: CPU Time (s).

<sup>a</sup> Not feasible.

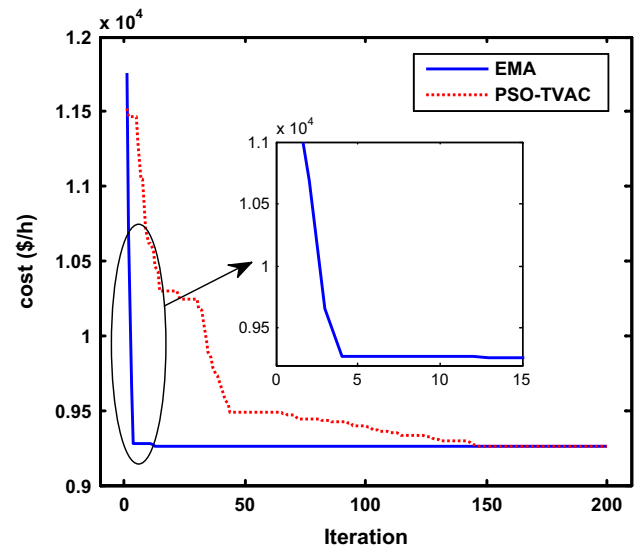


Fig. 3. Convergence characteristics of EMA for test case I.

As seen from Table 2, both EMA and PSO-TVAC could reach cost of 9257.0701 \$ much better than PSO and IACS. The results obtained by EDHS are not in feasible region [2]. The iteration number of program is 200. Fig. 3 shows convergence trend of EMA in comparison with PSO-TVAC. As can be seen from Fig. 3, due to high capability of EMA in producing random numbers, this algorithm could find optimum point neighborhood in few initial iterations. The mean obtained cost for the study system, after 50 times runs, is \$ 9257.0730.

#### Test System-II

The tests were accomplished on a system comprised of five generating units, including one power-only unit, three CHP units and one heat-only unit. Cost function of power-only unit (unit-1) and heat-only unit (unit-5) and data of CHP units and feasible regions are given in [4].

This problem is optimized in terms of three different load profiles (LP). Power and heat demands in LP1 are 300 MW and 150 MWth, respectively. While they are in LP2 are 250 MW and 175 MWth, respectively. Finally, the power and heat demand stands at 160 MW and 220 MWth, respectively. The obtained results from solving above problem using EMA compared with those of genetic algorithm (GA) [4], harmony search (HS) [15], PSO [2] and PSO-TVAC [2] are given in Table 3. As seen from Table 3,

Table 1

Adjustable parameters of EMA for numerical experimentations.

Risk value	$g_1$ [max, min]	$g_2$ [max, min]
Case I	[0.005, 0.0005]	[0.01, 0.001]
Case II	[0.02, 0.002]	[0.01, 0.001]
Case III	[0.05, 0.04]	[0.04, 0.03]
Case IV	[0.02, 0.002]	[0.01, 0.001]
Case V	[0.02, 0.002]	[0.01, 0.001]

tion (PSO) [16], particle swarm optimization with time varying acceleration coefficients (PSO-TVAC) [2], economic dispatch harmony search (EDHS) [20], improved ant colony search (IACS) algorithm [21].

**Table 3**  
Comparison of simulation results for case II.

Load	Method	$P_1$	$P_2$	$P_3$	$P_4$	$H_2$	$H_3$	$H_4$	$H_5$	TP	TH	TC
LP1	GA	135.00	70.81	10.84	83.28	80.54	39.81	0.00	29.64	299.93	149.99	13779.50
	HS	134.74	48.20	16.23	100.85	81.09	23.92	6.29	38.70	300.02	150.00	13723.20
	PSO	135.0000	40.7309	19.2728	105.0000	64.4003	26.4119	0.0000	59.1955	300.00	150.00	13692.5212
	PSO-TVAC	135.0000	41.4019	18.5981	105.0000	73.3562	37.4295	0.0000	39.2143	300.00	150.00	13672.8892
	EMA	135.0000	40.7163	19.2837	105.0000	73.7022	36.7183	0.0000	39.5829	300.00	150.00	13672.7407
LP2	GA	119.2200	45.1200	15.8200	69.8900	78.9400	22.6300	18.4000	54.9900	250.05	174.96	12327.3700
	HS	134.6700	52.9900	10.1100	52.2300	85.6900	39.7300	4.1800	45.4000	250.00	175.00	12284.4500
	PSO	135.0000	40.3446	10.0506	64.6060	70.9318	39.9918	4.0773	60.0000	250.00	175.00	12132.8579
	PSO-TVAC	135.0000	40.0118	10.0391	64.9491	74.8263	39.8443	16.1867	44.1428	250.00	175.00	12117.3895
	EMA	135.0000	40.0000	10.0002	64.9997	74.9980	40.0001	14.0624	45.9394	250.00	175.00	12117.0785
LP3	GA	37.9800	76.3900	10.4100	35.0300	106.000	38.3700	15.8400	59.9700	159.81	220.18	11837.4000
	HS	41.4100	66.6100	10.5900	41.3900	97.7300	40.2300	22.8300	59.2100	160.00	220.00	11810.8800
	PSO	35.5972	57.3554	10.0070	57.0587	89.9767	40.0025	30.0232	60.0000	160.02	220.00	11781.3690
	PSO-TVAC	42.1433	64.6271	10.0001	43.2295	96.2593	40.0001	23.7407	60.0000	160.00	220.00	11758.0625
	EMA	42.1433	64.6378	10.0000	43.2188	96.2653	40.0000	23.7338	60.0000	160.00	220.00	11757.9124

optimization of above problem with three different load profiles using EMA leads to better results compared to the other methods. Convergence characteristics of EMA and PSO-TVAC in terms of LP1 are illustrated in Fig. 4.

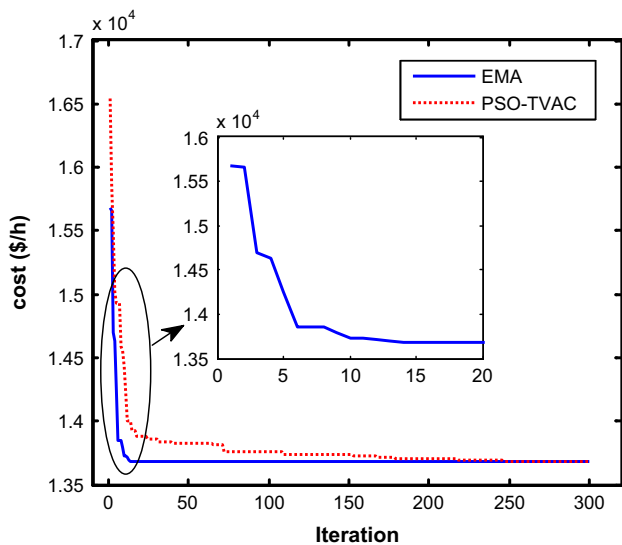


Fig. 4. Convergence characteristics of EMA for test case II for load profile 1.

Test System-III

Now, tests were performed on a non-convex system with seven generating units considering valve-point effects and system loss. This system comprised of four power-only units, two CHP units and one heat-only unit. Power and heat demands are 600 MW and 150 MWth, respectively. Related data of generating units and coefficients to B-matrix of network losses are given by [10].

The obtained results from solving non-convex optimization problem using EMA compared with those of evolutionary programming (EP) [22], differential evolution (DE) [22], real-coded genetic algorithm (RCGA) [23], bee colony optimization (BCO) [23], PSO [23] and PSO-TVAC [2] are included in Table 4. As seen in Table 4,

**Table 5**  
Determination of  $g_1$  and  $g_2$  for EMA in case III.

Case	$g_{1,max}$	$g_{2,max}$	Minimum cost (\$)	Average cost (\$)
1	0.2	0.2	10120.1426	10151.2640
2	0.2	0.1	10114.1844	10159.1043
3	0.15	0.1	10116.3531	10148.8594
4	0.1	0.05	10111.8954	10132.2528
5	0.07	0.04	10111.1194	11161.6128
<b>6</b>	<b>0.05</b>	<b>0.04</b>	<b>10111.0732</b>	<b>10111.0932</b>
7	0.04	0.03	10111.0901	10111.6932
8	0.02	0.02	10117.4698	10199.4110
9	0.005	0.005	10123.0011	10284.1364
10	0.002	0.005	10217.7419	10529.3515

**Table 4**  
Comparison of simulation results for case III.

Output	EP	DE	RCGA	BCO	PSO	PSO-TVAC	EMA
$P_1$	61.361	44.2118	74.6834	43.9457	18.4626	47.3383	52.6847
$P_2$	95.1205	98.5383	97.9578	98.5888	124.2602	98.5398	98.5398
$P_3$	99.9427	112.6913	167.2308	112.932	112.7794	112.6735	112.6734
$P_4$	208.7319	209.7741	124.9079	209.771	209.8158	209.8158	209.8158
$P_5$	98.8	98.8217	98.8008	98.8	98.8140	92.3718	93.8341
$P_6$	44	44	44.0001	44	44.0107	40.0000	40.0000
$H_5$	18.0713	12.5379	58.0965	12.0974	57.9236	37.8467	29.2420
$H_6$	77.5548	78.3481	32.4116	78.0236	32.7603	74.9999	75.0000
$H_7$	54.3739	59.1139	59.4919	59.879	59.3161	37.1532	45.7579
$P_{loss}$	7.9561	8.0372	7.5808	8.0384	8.1427	0.7329 <sup>a</sup>	7.5479
TP	607.9561	608.0372	607.5808	608.038	608.1427	600.7392 <sup>a</sup>	607.5479
TH	150	149.9999	150	150	150	150.0000	150.0000
TC	10,390	10,317	10,667	10,317	10,613	10100.3164 <sup>a</sup>	10111.0732
CT	5.2750	5.26	6.4723	5.1563	5.3844	3.25	2.0654

CT: CPU Time (s); (\$).

<sup>a</sup> Invalid.

**Table 6**  
Comparison of simulation results for case IV.

Output	CPSO	PSO-TVAC	TLBO	OTLBO	GSO	IGSO	GWO	EMA
$P_1$	680	538.5587	628.3240	538.5656	627.7455	628.1520	538.5840	628.3171
$P_2$	0.0000	224.4608	227.3588	299.2123	76.2285	299.4778	299.3426	299.1859
$P_3$	0.0000	224.4608	225.9347	299.1220	299.5794	154.5535	299.3423	299.1624
$P_4$	180.0000	109.8666	110.3721	109.992	159.4386	60.8460	109.9653	109.8665
$P_5$	180.0000	109.8666	110.2461	109.9545	61.2378	103.8538	109.9653	109.8605
$P_6$	180.0000	109.8666	160.1761	110.4042	60.0000	110.0552	109.9653	109.8650
$P_7$	180.0000	109.8666	108.3552	109.8045	157.1503	159.0773	109.9653	60.0000
$P_8$	180.0000	109.8666	110.5379	109.6862	107.2654	109.8258	109.9653	109.8664
$P_9$	180.0000	109.8666	110.5672	109.8992	110.1816	159.9920	109.9653	109.8564
$P_{10}$	50.5304	77.5210	75.7562	77.3992	113.9894	41.103	77.6223	40.0000
$P_{11}$	50.5304	77.5210	41.8698	77.8364	79.7755	77.7055	77.6223	77.0195
$P_{12}$	55.0000	120.0000	92.4789	55.2225	91.1668	94.9768	55.0000	55.0000
$P_{13}$	55.0000	120.0000	57.5140	55.0861	115.6511	55.7143	55.0000	55.0000
$P_{14}$	117.4854	88.3514	82.5628	81.7524	84.3133	83.9536	83.4650	81.0000
$P_{15}$	45.9281	40.5611	41.4891	41.7615	40.0000	40.0000	40.0000	40.0000
$P_{16}$	117.4854	88.3514	84.7710	82.2730	81.1796	85.7133	82.7732	81.0000
$P_{17}$	45.9281	40.5611	40.5874	40.5599	40.0000	40.0000	40.0000	40.0000
$P_{18}$	10.0013	10.0245	10.0010	10.0002	10.0000	10.0000	10.0000	10.0000
$P_{19}$	42.1109	40.4288	31.0978	31.4679	35.0970	35.0000	31.4568	35.0000
$H_{14}$	125.2754	108.9256	105.6717	105.2219	106.6588	106.4569	106.0991	104.8002
$H_{15}$	80.1174	75.4844	76.2843	76.5205	74.9980	74.9980	75.0000	75.0000
$H_{16}$	125.2754	108.9256	106.9125	105.5142	104.9002	107.4073	105.7890	104.8002
$H_{17}$	80.1175	75.484	75.5061	75.4833	74.9980	74.9980	75.0000	75.0000
$H_{18}$	40.0005	40.0104	39.9986	39.9999	40.0000	40.0000	40.0000	40.0000
$H_{19}$	23.2322	22.4676	18.2205	18.3944	19.7385	20.0000	18.3782	20.0000
$H_{20}$	415.9815	458.7020	468.2278	468.9043	469.3368	466.2575	469.7337	470.3996
$H_{21}$	60.0000	60.0000	59.9867	59.9994	60.0000	60.0000	60.0000	60.0000
$H_{22}$	60.0000	60.0000	59.9814	59.9999	60.0000	60.0000	60.0000	60.0000
$H_{23}$	120.0000	120.0000	119.9854	119.9854	119.6511	120.0000	120.0000	120.0000
$H_{24}$	120.0000	120.0000	119.6030	119.9768	119.7176	119.8823	120.0000	120.0000
Min. cost (\$)	59736.2635	58122.7460	58006.99	57856.26	58225.7450	58049.0197	57846.84	57825.4792
Ave. cost (\$)	59853.4780	58198.3106	58014.3685	57883.2105	58295.9243	58156.5192	-	57832.7361
Max. cost (\$)	60076.6903	58359.552	58038.5273	57913.7731	58318.8792	58219.1413	-	57841.1469
T/I <sup>a</sup>	0.0266	0.0261	0.0189	0.0194	0.1184	0.1184	0.0773	0.01167

<sup>a</sup> T/I: Time to Iteration (s).

**Table 7**  
Comparison of simulation results for case V.

Output	CPSO	PSO-TCAV	TLBO	OTLBO	EMA	Output	CPSO	PSO-TVAC	TLBO	OTLBO	EMA
$P_1$	359.0392	538.5587	538.5693	628.3199	628.3166	$P_{31}$	10.0002	10.0031	10.5480	10.0832	10.0000
$P_2$	74.5831	75.1340	225.3021	225.3313	299.1692	$P_{32}$	56.7153	35.0000	52.7180	39.3110	35.0000
$P_3$	74.5831	75.1340	229.9473	223.9653	224.3017	$P_{33}$	109.1877	95.4799	82.1522	82.0236	81.0000
$P_4$	139.3803	140.6146	159.1352	159.8516	109.8618	$P_{34}$	65.6006	54.9235	52.0606	40.1105	40.0000
$P_5$	139.3803	140.6146	160.0561	109.9150	109.8665	$P_{35}$	109.1877	95.4799	82.7394	81.3039	81.0000
$P_6$	139.3803	140.6146	109.7821	159.7795	109.8415	$P_{36}$	65.6006	54.9235	45.7398	45.6700	40.0000
$P_7$	139.3803	140.6146	159.6609	109.8946	109.8663	$P_{37}$	10.6158	23.4981	10.0075	13.8709	10.0000
$P_8$	139.3803	140.6146	159.6492	109.9321	109.8583	$P_{38}$	60.5994	54.0882	30.0332	30.3881	35.0000
$P_9$	139.3803	140.6146	109.9660	159.9569	109.8665	$H_{27}$	111.4458	108.1177	105.0678	107.5951	104.8002
$P_{10}$	74.7998	112.1998	40.3726	40.8970	40.0000	$H_{28}$	125.6898	88.9006	78.9162	125.4997	75.0000
$P_{11}$	74.7998	112.1998	77.5821	41.3115	40.0000	$H_{29}$	111.4458	108.1177	104.8270	105.1942	104.8002
$P_{12}$	74.7998	74.7999	92.2489	55.1748	55.0000	$H_{30}$	125.6898	88.9006	119.6006	82.6853	75.0000
$P_{13}$	74.7998	74.7999	55.1755	92.4003	55.0000	$H_{31}$	40.0001	40.0013	40.2345	40.0346	40.0000
$P_{14}$	679.8810	269.2794	448.6854	448.8359	628.3185	$H_{32}$	29.8706	20.0000	28.0508	21.9568	20.0000
$P_{15}$	148.6585	299.1993	149.4238	225.7871	298.6422	$H_{33}$	120.6188	112.9260	105.4339	105.3622	104.8002
$P_{16}$	148.6585	299.1993	224.7173	75.4600	299.0560	$H_{34}$	97.0997	87.8827	85.4086	75.0938	75.0000
$P_{17}$	139.0809	140.3973	109.9355	160.1192	109.8685	$H_{35}$	120.6188	112.9260	105.7694	104.9667	104.8002
$P_{18}$	139.0809	140.3973	159.9052	110.3532	109.8667	$H_{36}$	97.0997	87.8827	79.9447	79.8936	75.0000
$P_{19}$	139.0809	140.3973	159.7255	159.8190	159.7331	$H_{37}$	40.2639	45.7849	40.0001	41.6554	40.0000
$P_{20}$	139.0809	140.3973	159.7820	159.7765	109.8386	$H_{38}$	31.6361	28.6765	17.7401	17.9018	20.0000
$P_{21}$	139.0809	140.3973	60.0777	159.7370	109.8667	$H_{39}$	357.9456	433.9113	394.6160	445.0937	470.3802
$P_{22}$	139.0809	140.3973	110.0689	160.1751	109.8613	$H_{40}$	59.9916	60.0000	59.9300	59.9967	60.0000
$P_{23}$	74.7998	74.7998	77.6818	40.1140	40.0000	$H_{41}$	59.9916	60.0000	59.9578	59.9974	60.0000
$P_{24}$	74.7998	74.7998	40.2707	40.3042	40.0000	$H_{42}$	120.0000	120.0000	118.5797	119.8834	120.0000
$P_{25}$	112.1993	112.1997	92.4108	92.4149	55.0000	$H_{43}$	120.0000	120.0000	118.3425	119.5231	120.0000
$P_{26}$	112.1993	112.1997	55.0956	92.5012	55.0000	$H_{44}$	370.6214	415.9741	480.6566	428.7605	470.4190
$P_{27}$	92.8423	86.9119	81.4882	85.9857	81.0000	$H_{45}$	59.9999	60.0000	59.9346	59.9957	60.0000
$P_{28}$	98.7199	56.1027	44.5478	98.5005	40.0000	$H_{46}$	59.9999	60.0000	59.9810	59.9638	60.0000
$P_{29}$	92.8423	86.9119	81.0560	81.7197	81.0000	$H_{47}$	119.9856	119.9989	117.8207	119.5025	120.0000
$P_{30}$	98.7199	56.1027	91.6819	48.9055	40.0000	$H_{48}$	119.9856	119.9989	119.1898	119.4440	120.0000
Method	CPSO		PSO-TVAC			TLBO		OTLBO		EMA	
Cost (\$)		119708.8818		117824.8956		116739.3640		116579.2390		115611.8447	

the obtained loss by PSO-TVAC algorithm is 0.7329 MW, less than ten times the other techniques. However, based on the examination these results are invalid. The least cost is obtained by EMA (10111.0732 \$) less than PSO, BCO, RCGA, DE and EP by 502 \$, 206 \$, 556 \$, 206 \$ and 279 \$. As seen from Table 4, the mean run time of program by EMA is 2.0654 s that is less than EP, DE, RCGA, BCO, PSO and PSO-TVAC Techniques.

How selecting optimal values for EMA's adjustable parameters is explained in [18]. In order to show the effect of EMA's adjustable parameters in converging to optimal point, the results of solving CHPED problem in 7-unit system in terms of various values for  $g_{1,max}$  and  $g_{2,max}$  after fifty program implementations are given in Table 5.

#### Test System-IV

In this section, tests were done on a large system with non-convex fuel cost. This system consists of thirteen power-only units, six CHP units, and five heat-only units. Power and heat demands are 2350 MW and 1250 MWth, respectively. Related data of generating units are given in [2]. The obtained results from solving 24 units test system using EMA compared with those of PSO-TVAC [2], CPSO [2], teaching learning based optimization (TLBO) [24], oppositional TLBO (OTLBO) [24], group search optimization (GSO) [25], Improved GSO (IGSO) [25] and gray:gray wolf optimization (GWO) [26] are included in Table 6. Data of thirteen power-only units has many local optimized points. Thus, finding an optimum point of this test system is a difficult benchmark for evolutionary algorithms. However, EMA could successfully extract this point by cost of 57825.4792 \$ that is less than CPSO, PSO-TVAC, TLBO, OTLBO, GSO, IGSO and GWO by 1910.7843 \$, 297.2668 \$, 181.5108 \$, 30.7808 \$, 400.2658 \$, 223.5405 \$ and 21.3608 \$ respectively, that indicating its great superiority over the other well-behaved algorithms. As seen from Table 6, the time to iteration of proposed EMA in solving 24 units test system is 0.01167 s that is lower than compared optimization algorithms.

#### Test System-V

In this section, tests were done on a large system with non-convex fuel cost as proposed in [2]. This system consists of twenty-six power-only units, twelve CHP units, and ten heat-only units. Power and heat demands are 4700 MW and 2500 MWth,

respectively. Related data of study system is given in [2]. In this case study units 1 to 26 are power-only units, units 27–38 are CHP units and units 39–48 are heat-only units.

The obtained results from solving above problem using EMA compared with those of PSO-TVAC [2], CPSO [2], TLBO [24] and OTLBO [24] are included in Table 7. As seen from this table, EMA could successfully extract optimal point by cost of 115611.8447 \$ that is less than CPSO, PSO-TVAC, TLBO, OTLBO by 4097.0371 \$, 2213.0509 \$, 1127.5193 \$ and 967.3943 \$, respectively. The obtained results shows EMA's great superiority over the other well-behaved algorithms. Convergence trend of EMA compared with CPSO and PSO-TVAC is illustrated in Fig. 5.

#### Conclusion

This paper introduces the exchange market algorithm to solve the CHPED problem. The exchange market algorithm has two search operators (members of G2 and G3 in oscillation mode) results in simultaneously exploration in two limited and wide search domains. Searching in limited domain leads to exploration of points adjacent to the optimum point and searching in wide domain results in exploiting unknown points as well as two absorbent operators for individuals to be absorbed to the elite person (members of G2 and G3 in non-oscillation mode), which leads to create and organize the random numbers in the most appropriate manner.

This algorithm is applied on 5 different CHPED problems with fuel convex (non-convex) cost in order to examine EMA's capability and extracting optimum point of CHPED problem. The obtained results of EMA in solving various systems are compared to intelligent techniques, including EDHS, HS, PSO-TVAC, PSO, RCGA, BCO, DE, EP, IACS, TLBO, OTLBO, GSO, IGSO and GWO. The obtained results by EMA in various tests revealed its superiority over other techniques. Test systems in Sections "Exchange market algorithm implementation pattern in solving CHPED problem" and "Numerical studies" tests are difficult benchmark because of non-convex fuel cost. However, EMA could obtain the best cost compared other intelligent techniques where the EMA could obtain cost of 57825.4792 \$ for test system IV that is less than CPSO, PSO-TVAC, TLBO, OTLBO, GSO, IGSO and GWO by 1910.7843 \$, 297.2668 \$, 181.5108 \$, 30.7808 \$, 400.2658 \$, 223.5405 \$ and 21.3608 \$ respectively and where the EMA could obtain cost of 115611.8447 \$ for test system V that is less than CPSO, PSO-TVAC, TLBO, OTLBO by 4097.0371 \$, 2213.0509 \$, 1127.5193 \$ and 967.3943 \$, respectively.

The results prove the robustness and effectiveness of the proposed algorithm in solving CHPED problem over the other compared optimization algorithms. Considering the results of this paper, EMA could be efficiently employed on various power system problems.

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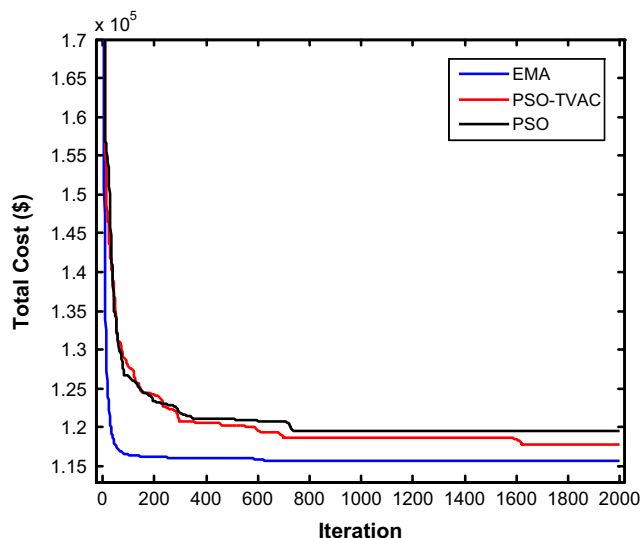


Fig. 5. Convergence characteristics of EMA for test case V.



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