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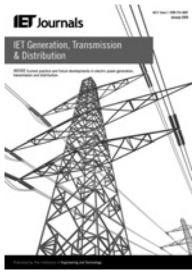

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Max–min fair Financial Transmission Rights payment-based AC optimal power flow locational marginal price decomposition

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Abstract: The decomposition of locational marginal price obtained from AC optimal power flow is disputable because of its dependency on the choice of energy reference. The dispute arises because the difference of congestion component, on which the Financial Transmission Rights (FTR) payments are made, is not constant and depends on the energy reference. Prior art aimed at obtaining reference-independent decomposition. The authors look at this dispute as a fairness issue and formulate the decomposition as a fairness problem. The fairness issue is addressed by invoking max–min fairness criteria which ensures that the FTR payment (FP) received (made) is not at the expense of other ill-positioned FTR holders. Max–min fairness algorithm is used to arrive at an energy reference choice that aims at maximising the FP of each FTR. This in turn ensures that revenue received (payment made) is the maximum (minimum) possible under the set of constraints, without adversely affecting the FP of other FTR holders. Results on PJM 5 bus and IEEE 30 bus systems indicate that a fair solution is indeed possible.

Nomenclature

$\boldsymbol{\alpha}$	slack weight vector
$\boldsymbol{\delta}$	vector of load angles except at angle reference bus
$\boldsymbol{\lambda}$	LMP vector
$\boldsymbol{P}$	vector of net nodal power injections
$\boldsymbol{V}$	vector of voltage magnitudes
$\boldsymbol{\lambda}^{c}$	congestion component of LMP
$\boldsymbol{\lambda}^{e}$	energy component of LMP
$\boldsymbol{\lambda}^{l}$	loss component of LMP
$\boldsymbol{LF}$	loss factors vector
η_{ftr}	number of FTRs
λ_i^{c}	ith entry in $\boldsymbol{\lambda}^{c}$
λ_i^{l}	ith entry in $\boldsymbol{\lambda}^{l}$
$\lambda_{\text{si},i}^{c}$	congestion component of sink node of ith FTR
$\lambda_{\text{so},i}^{c}$	congestion component of source node of ith FTR
μ	Lagrangian multiplier of line power flow constraint
FP_i	FTR payment of ith FTR holder
$\text{LF}_{\text{si},\,i}$	loss sensitivity of sink node of ith FTR
$\text{LF}_{\text{so},\,i}$	loss sensitivity of source node of ith FTR
M	number of transmission lines
n	number of buses
P_i^{ftr}	MW amount of ith FTR
P_{loss}	total active power loss
T_{l-i}	power flow sensitivity of line l with respect to injection at bus i

1 Introduction

The decomposition of locational marginal price (LMP) using the AC optimal power flow (ACOPF) model has gained widespread attention with respect to energy reference-independent decomposition. The LMP can be obtained using either DCOPF or ACOPF. Power markets that use a centralised dispatch philosophy have moved from a completely lossless model (DCOPF) to marginal loss DCOPF models [1–4] for market settlement. Reference-independent decomposition (constant difference of congestion component) has already been achieved with the marginal loss DCOPF models [1, 3, 4]. Although the report prepared by the Federal Energy Regulatory Commission (FERC), providing an overview of current practices and future plans of the electricity markets in the US [5], does not reveal the use of ACOPF in real-life markets, there appears to be immense interest in utilising the full ACOPF model for practical applications. This can also be corroborated from the fact that addressing the issues pertaining to the use of ACOPF modelling to be properly formulated and solved has been one of the major themes in the technical conferences conducted by FERC for the years 2012 [6] and 2013 [7].

A characteristic feature of the ACOPF model is that it does not require any additional loss modelling effort. The equality constraint for power at all buses accounts for power loss. The ACOPF model can handle voltage and reactive power-related constraints and can fully reflect actual operating conditions in terms of system modelling. The solution obtained from ACOPF (cost, dispatch, state variables and LMP) does not depend on slack (angle reference) bus.

However, another characteristic feature of ACOPF is the non-unique decomposition of LMP into its components. The LMP may be notionally seen as comprising of three

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components: energy, loss and congestion. These components are necessary to hedge congestion and/or loss-related costs incurred because of the volatile nature of LMP. The individual components as such do not have any physical relevance and their only purpose is for congestion and loss hedging, if at all they exist. Financial Transmission Rights (FTR) [8] are risk-hedging instruments designed mainly with the aim of minimising the congestion price risk for forward contracts [9]. Ability to hedge loss price risk [10–12], while proposed with loss hedging rights (LHR), has not gained much acceptance with regard to practical implementation. The proposal in [10] is to hedge both losses and congestion (total LMP) using unbalanced FTRs, overcoming the necessity to decompose the LMP. A detailed analysis on FTR and LHR is given in [9] and [12] respectively.

FTR holders are entitled to a stream of revenues or charges accumulated from settlement because of the spatial variation of LMP to hedge congestion costs [13]. An FTR is defined between a source and sink, and for a MW amount that has a certain validity period. The necessity to decompose LMP into its components arises from the fact that in any OPF model (DC or AC) that incorporates losses, FTRs do not hedge against LMPs anymore, only for difference in congestion component [14]. Also, there is no mechanism to issue revenue adequate FTRs when considering network losses [15]. Further, volatility of LMP is primarily because of network congestion and not transmission losses. For example, outage of a critical line could cause a large price (LMP) difference between the areas that the line connects. However, system losses will not change drastically enough to cause a spike in the LMP.

Numerous interpretations of spot price decomposition have been reported in [16–19]. Interpretation of slack bus and system lambda in spot price decomposition is summarised in [16]. Importance of reference bus selection for decomposition of spot price is highlighted in [17], where it is stated that 'bus with lowest marginal cost and having available capacity is a logical choice'. A comparison of spot price decomposition for various OPF-based pricing models is provided in [18]. In [19], a detailed description of each nodal price is provided by breaking it down into a variety of parts corresponding to various concerned factors and its application to reducing electricity price volatility is proposed in [20].

In case of ACOPF, the difference in loss or congestion components between any two buses is not constant because the marginal effect of loss is not the same for different choices of reference bus. Hence, a policy is specified in [21] that fixes the loss component of marginal buses to zero, which determines the loss component of non-marginal buses, thereby making the difference of congestion component a constant value. The work reported in [21] is a special case of the general formulation of LMP evaluation reported in [22]. Recent work reported in [23] aims at obtaining a market-oriented decomposition by minimising the difference between the so-called ideal hedge and actual hedge.

In this paper, we too aim to propose a market-oriented decomposition by looking into the problem from a fairness perspective. The authors in [21] raise concerns about market fairness when utilising LMP components based on arbitrary decomposition. Arbitrary choice of energy reference may lead to certain FTR holders receiving higher payments compared to others, creating winners and losers. Hence, an efficient mechanism to fairly share revenues is proposed. For a given set of FTRs already issued and for which settlement is yet to take place, we exploit the flexibility associated with LMP decomposition so that payments to be received (made) by each FTR holder is maximised (minimised) to an extent permitted by a fairness criteria called 'max–min fairness'.

The max–min algorithm is used for the benefit allocation problem aimed at achieving fairness [24]. The dual of max–min, min–max fairness, has already been used in resolving fairness issue in transmission pricing [25, 26]. Since the congestion component of LMP exhibits strong dependence on the choice of energy reference, FTR payments (FPs) received (made) will vary for different choices of reference. The max–min fair algorithm tries to maximise (minimise) payment received (made) by each FTR holder without lowering payment of other FTR holders whose payment is lesser, thus ensuring a fair decomposition.

This paper is organised as follows. An overview of the FTR mechanism and motivation for the proposed work is given in Section 2. The concept of max–min fairness and formulation of the proposed work is described in Section 3. Results are given in Section 4 and Section 5 concludes the paper.

2 Overview of FTR mechanism

FTRs are financial instruments awarded to bidders in the FTR auctions that entitle the holder to a stream of revenues (or charges) based on the hourly day-ahead congestion price differences across the path [13]. FTR is defined between a source–sink pair of nodes for which the LMP is calculated. The node can be a bus or a zone. FTRs can also be bought between a node and a zone. FTR is issued for a specific time period (its validity) and for a MW amount. FTRs can be categorised into two types: options and obligations. An obligation FTR can be a benefit or a liability depending on the direction of congestion. An option FTR is never a liability because it becomes invalid if congestion occurs in the reverse direction. FTRs are generally acquired in an auction held annually and/or monthly. They can also be traded bilaterally in a secondary market. The FP is given by the FTR MW amount times the difference in congestion price between sink and source nodes [13], as shown in

$$\mathrm{FP}_i = (\lambda^{\mathrm{c}}_{\mathrm{si},i} - \lambda^{\mathrm{c}}_{\mathrm{so},i}) \times P^{\mathrm{ftr}}_i \quad (1)$$

Note that the difference in price corresponds to the congestion component of LMP. An FTR provides a perfect hedge to the congestion charge only in the case when LMP is calculated using the lossless DCOPF approach. In this case, the LMP differential equals the difference in congestion component of LMP. With the incorporation of network loss in the dispatch, FTRs do not provide hedge against LMPs anymore, only for difference in congestion component [14]. The current practice of issuing FTRs is to hedge congestion costs only, which has necessitated decomposition of LMP into components.

FTRs are to be issued only up to an extent to which it can be paid back using the congestion collection because the ISO should be able to collect enough revenue from settling the market in order to make FTR payments. This is ensured using a simultaneous feasibility test (SFT) [27]. The SFT is valid only for a linear and lossless DC model because there is no well-defined mechanism to ensure revenue adequacy when the dispatch considers network losses [15]. In fact, in [28], it is shown that revenue adequacy cannot be proven for the general non-linear AC power flow model.

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2.1 Motivation

The decomposition of LMP from ACOPF, by virtue, is dependent on the reference bus. This is because any marginal increase in load within the system is served by a generator or a set of generators at different locations and the effect of their response varies because of the non-linearity of the system model. The analysis in [22] reveals the limitation of any decomposition technique into its components because of the underlying structural interdependencies among them.

The unique decomposition claimed by authors in [19] does not apply for an application which requires LMP to be decomposed into energy, loss and congestion components. This essentially depends on the bus whose nodal power balance is eliminated to accommodate the expression for summated power balance for which Lagrangian multiplier is to be calculated. The so-called 'value of real power loss' corresponding to that Lagrangian multiplier is dependent on the bus whose power balance equation is eliminated. In fact, the price of 1 MWh of physical losses in the LMP-based market is undefined and it can only be suggested that price of MWh of physical losses equals energy price at market reference [14].

Prior art on LMP decomposition is focused on obtaining a reference-independent solution, which results in the difference in congestion component of LMP being a constant value, irrespective of the choice of energy reference. In [22], a general formulation of LMP decomposition is presented that gives better insight into the various decomposition techniques that exist. The authors opine that although there can be no unique decomposition, the 'policy' adopted for decomposition of LMP at marginal nodes (nodes with generation not at their limits) determines the decomposition at all other non-marginal nodes. The reference-independent decomposition proposed in [21] is, in fact, a solution when loss component of marginal nodes is zero (the so-called 'policy'). In [23], instead of obtaining a reference-independent decomposition, the authors propose a market-oriented optimal solution that hedges the congestion cost to the maximum extent possible. The problem is tackled by reducing the difference between, what the authors define as, ideal hedge (difference in LMP) and actual hedge (difference in congestion components).

In the work proposed in this paper, we look at the decomposition of LMP from a fairness point of view. Since there exists an inter-relationship between congestion components of all buses, already reported in [23], when FTR payment for *a* FTR holder is calculated choosing any arbitrary reference, it can impact the payments of other FTR holders in either a positive or negative way. Hence, the nature of the problem is such that choosing a reference with the aim of maximising payment of a FTR holder need not result in maximum payment for other FTR holders simultaneously.

Therefore, we propose to invoke the issue of fairness in decomposing the LMP into its respective components while making FTR payments. The proposed technique is also a market-oriented decomposition from the point of view of a fair decomposition. This is achieved using the max–min fairness criteria, generally used for benefit allocation. Further insights into the max–min fairness problem and how LMP decomposition is obtained by applying max–min fairness criteria is dealt with in the next section.

This paper does not intend to propose a hedging mechanism to guard against the total LMP (congestion + losses). The issue of revenue adequacy comes to the fore when considering any proposal to hedge both congestion and losses, whether in an AC or DC model. Also, there is no certainty as to what hedging mechanism will be employed if ACOPF were to be implemented for LMP evaluation. Currently, hedging is available only against congestion price risk and there is no tool to hedge loss price risk. Therefore, it is plausible to assume that the current scheme of hedging 'only congestion' will continue until such a time when a practically acceptable and implementable scheme of hedging the LMP (congestion and losses) is proposed. Hence, the proposed methodology is implementable when FTRs are balanced, but LMP is evaluated using ACOPF, which is a hybrid of hedging congestion component of LMP, which is the current practice, and the use of ACOPF for LMP calculation. Considering the fact that only congestion hedging instrument is available to market participants, the methodology proposed in this paper attempts to make fair payments to FTR holders, thereby choosing a fair reference for LMP decomposition.

3 LMP decomposition as a fairness problem

The decomposition of LMP into its components can be seen as a fairness problem as there can be infinitely many solutions for the choice of energy reference when a distributed slack bus formulation is employed. For a choice of reference, the FTR payments are fixed because there can be only one set of congestion components. However, this may not be entirely fair because FTR holders may receive varying payments, that is, some could receive higher payments at the expense of others. This issue arises because of the non-unique nature of ACOPF LMP decomposition and the underlying structural interdependencies.

Posing LMP decomposition as a fairness problem involves exploring the multiplicity of the solution space for the choice of energy reference. The use of a distributed slack formulation provides flexibility in choosing a reference or a set of references for which the FTR payments meet max–min fairness criteria.

3.1 Max–min fairness

The concept of fairness in cost/benefit sharing can be posed as an optimisation problem. In [25], a comprehensive assessment of fairness as an optimisation problem is discussed. The problem of transmission fixed cost allocation is addressed using min–max fairness criteria. The authors in [25] state the use of max–min fairness for benefit allocation and min–max fairness for cost allocation which is logical because any market player would like to reduce his/her cost and maximise benefit.

It is worthwhile to note that the notion of fairness is a subjective term and its interpretation may vary depending on application. However, we have used a globally accepted fairness criterion used extensively in communication networks for bandwidth sharing [24, 29]. A solution is said to be max–min fair if it is not possible to increase the benefit of an entity without simultaneously decreasing the benefit of another entity whose benefit is already equal or lower.

The concept of max–min fairness is better understood using the benefit sharing example depicted in Fig. 1. For the sake of simplicity, the inter-relationship of benefits among players is not depicted in the figure. In the process of maximising the

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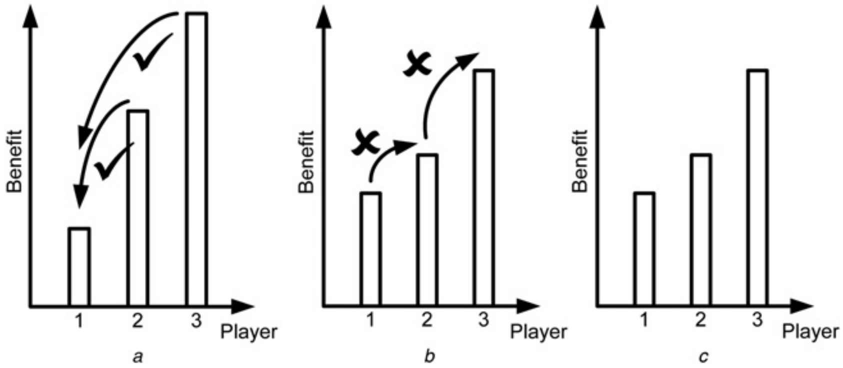

Fig. 1 *Max–min fair benefit sharing*

a Maximising benefit of player 1
b Maximising benefit of players 2 and 3
c Max-min fair benefit allocation

benefit of player 1, the max–min fairness criteria allows for the decrease of benefits of players 2 and 3 only up to the extent such that they are not lower than the benefit received by player 1. In Fig. 1*a*, the benefit of players 2 and 3 is decreased to increase the benefit of player 1. Hence, the max–min optimality is not yet reached in Fig. 1*a*.

When the benefit of player 2 is to be maximised, depicted in Fig. 1*b*, solution space is such that it is not possible to increase the benefit of player 2 without decreasing the benefit of player 1 and the same holds true when maximising benefit of player 3. Since the max–min constraints are such that it does not allow decreasing the benefit of a player whose benefit is already lower, max–min optimality is reached in Fig. 1*c*.

3.2 Distributed slack LMP formulation

The LMP is a by-product of the solution of an optimisation problem whose objective is cost minimisation (or social welfare maximisation if loads are elastic). The expressions for the calculation of LMP using a distributed slack approach are provided in [30], and may be expressed as the sum of three components as follows

$$\boldsymbol{\lambda} = \lambda^{\mathrm{e}} + \boldsymbol{\lambda}^{\mathrm{l}} + \boldsymbol{\lambda}^{\mathrm{c}} \tag{2}$$

where

$$\lambda^{\mathrm{e}} = \sum_{i=1}^{n} \alpha_i \times \lambda_i \tag{3}$$

$$\boldsymbol{\lambda}_i^{\mathrm{l}} = -\lambda^{\mathrm{e}} \times \frac{\partial P_{\mathrm{loss}}}{\partial P_i} \tag{4}$$

$$\boldsymbol{\lambda}_i^{\mathrm{c}} = -\sum_{l=1}^{M} \mu_l \times T_{l-i} \tag{5}$$

In (3), the distributed slack node is a fictitious node at which the energy component is calculated. The loss component is the product of the loss factor (LF) and the negated value of energy component at the fictitious node. The loss factors are calculated using the first-order sensitivities obtained from the inverse of the Jacobian matrix, as given in

$$\boldsymbol{LF} = \frac{\partial P_{\mathrm{loss}}}{\partial \boldsymbol{P}} = \begin{bmatrix} \dfrac{\partial P_{\mathrm{loss}}}{\partial \boldsymbol{\delta}} & \dfrac{\partial P_{\mathrm{loss}}}{\partial \boldsymbol{V}} \end{bmatrix} \times \begin{bmatrix} \dfrac{\partial \boldsymbol{\delta}}{\partial \boldsymbol{P}} \\ \dfrac{\partial \boldsymbol{V}}{\partial \boldsymbol{P}} \end{bmatrix} \tag{6}$$

Although all three components can be expressed in terms of the slack distribution variable (α_i), only the energy and loss components are explicitly written in terms of α_i. The congestion component is expressed as the difference of the total LMP and the sum of energy and loss components, as given in (7). For a given solution of ACOPF, for which λ is already available, the components can be computed using the state variables by assuming a suitable value of $\boldsymbol{\alpha}$. This also includes the case when the energy reference is same as the angle reference bus.

$$\boldsymbol{\lambda}^{\mathrm{c}} = \boldsymbol{\lambda} - \boldsymbol{\lambda}^{\mathrm{e}} - \boldsymbol{\lambda}^{\mathrm{l}} \tag{7}$$

From (1), (3), (4) and (7), FP_i can be expressed using (8). It is amply clear that the choice of $\boldsymbol{\alpha}$ affects the values of all three components. A certain choice of $\boldsymbol{\alpha}$ may benefit an FTR or set of FTRs at the expense of another/other FTR/s. Hence, there is a need and scope for developing a rationale to establish $\boldsymbol{\alpha}$. In the proposed methodology, instead of pre-specifying it, $\boldsymbol{\alpha}$ is made a decision variable.

3.3 Max–min algorithm

Since congestion component difference exhibits dependency on energy reference, by making $\boldsymbol{\alpha}$ a decision variable, the proposed algorithm arrives at that choice of $\boldsymbol{\alpha}$ for which FTR payments satisfy max–min fairness constraints. The formulation of the max–min fair FTR payment problem for one instance, a subsidiary problem (SP), is as follows:

$$\mathrm{FP}_i = \left[\lambda_{\mathrm{si},i} - \left(-\sum_{j=1}^{n} \alpha_j \lambda_j \times \mathrm{LF}_{\mathrm{si},i}\right) - \lambda_{\mathrm{so},i} + \left(-\sum_{j=1}^{n} \alpha_j \lambda_j \times \mathrm{LF}_{\mathrm{so},i}\right)\right] \times P_i^{\mathrm{ftr}} \tag{8}$$

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Problem **SP(*i*)**

$$\text{maximise} \qquad \mathrm{FP}_i(\boldsymbol{\alpha}) \tag{9}$$

such that

$$\mathrm{FP}_i(\boldsymbol{\alpha}) \le \mathrm{FP}_k(\boldsymbol{\alpha}) \quad \forall\, k \ne i \text{ and } \mathrm{FP}_i(\bar{\boldsymbol{\alpha}}) < \mathrm{FP}_k(\bar{\boldsymbol{\alpha}}) \tag{10}$$

$$\mathrm{FP}_k(\boldsymbol{\alpha}) \ge \mathrm{FP}_k(\bar{\boldsymbol{\alpha}}) \quad \forall\, k \ne i \text{ and } \mathrm{FP}_i(\bar{\boldsymbol{\alpha}}) > \mathrm{FP}_k(\bar{\boldsymbol{\alpha}}) \tag{11}$$

$$\mathrm{FP}_i(\boldsymbol{\alpha}) \ge \mathrm{FP}_i(\bar{\boldsymbol{\alpha}}) \tag{12}$$

$$\sum \alpha_j = 1, \quad j = 1, 2, \ldots, n \tag{13}$$

$$0 \le \alpha_j \le 1, \quad \forall\, j = 1, 2, \ldots, n \tag{14}$$

$$\sum \mathrm{FP}_m(\boldsymbol{\alpha}) \le \mathrm{MS}, \quad m = 1, 2, \ldots, \eta_{\mathrm{ftr}} \tag{15}$$

$$\mathrm{FP}_m(\boldsymbol{\alpha}) \ge 0 \quad \forall\, \lambda_{\mathrm{si},m} - \lambda_{\mathrm{so},m} > 0, \quad m = 1, 2, \ldots, \eta_{\mathrm{ftr}} \tag{16}$$

$$\mathrm{FP}_m(\boldsymbol{\alpha}) \le 0 \quad \forall\, \lambda_{\mathrm{si},m} - \lambda_{\mathrm{so},m} < 0, \quad m = 1, 2, \ldots, \eta_{\mathrm{ftr}} \tag{17}$$

where $\mathrm{FP}_i(\boldsymbol{\alpha})$ is as given in (8). Here, i indicates the FTR holder whose payment is currently being maximised and k is the index of remaining $\eta_{\mathrm{ftr}} - 1$ FTR holders. It may be noted here that (2)–(5) will always hold true. In fact, it is this inter-relationship among components and the total LMP, which is bound by the inter-relationship, that plays a key role in ensuring max–min fair solution.

To begin with, any random reference is chosen for LMP decomposition such that it results in feasible FTR payments [feasibility in terms of satisfaction of all constraints (13)–(17), and are denoted using $\bar{\boldsymbol{\alpha}}$]. This forms the initialisation of max–min algorithm with any feasible solution. These payments are sorted in ascending order and this begins the iterative process where a series of optimisation problems begin to be solved. One such optimisation problem is **SP(*i*)**, represented using (9)–(17). **SP(*i*)** is a non-linear problem with linear and non-linear constraints.

In the process of maximising FTR payment of ith holder ($\mathrm{FP}_i(\boldsymbol{\alpha})$), using (9), a set of constraints invoking the max–min fairness criteria is enforced using (10)–(12). They ensure that it is not at the cost of some other FTR payment which is already lower. Note that among (10)–(12), the constraint that applies to $\mathrm{FP}_k(\boldsymbol{\alpha})$ depends on its FTR payment in the previous SP, in this case the initial feasible solution.

For FTR holders whose payments ($\mathrm{FP}_k(\bar{\boldsymbol{\alpha}})$) were greater than the FTR holder whose payment is currently being maximised (ith FTR holder's payment $\mathrm{FP}_i(\bar{\boldsymbol{\alpha}})$), equation (10) ensures that their new payment $\mathrm{FP}_k(\boldsymbol{\alpha})$ cannot be reduced below $\mathrm{FP}_i(\boldsymbol{\alpha})$. Equation (11) ensures that for FTR payments ($\mathrm{FP}_k(\bar{\boldsymbol{\alpha}})$) lesser than $\mathrm{FP}_i(\bar{\boldsymbol{\alpha}})$, their new FP must be greater than or equal to the FP from the previous SP, in this case the initial feasible solution. For the FTR payment that is being maximised ($\mathrm{FP}_i(\boldsymbol{\alpha})$), one constraint requires that it should not be less than the payment it received from the previous SP ($\mathrm{FP}_i(\bar{\boldsymbol{\alpha}})$), enforced using (12).

In addition to max–min fairness constraints, (13) ensures summation of all participation factors to unity and non-negativity of α_i is enforced using (14). To ensure financial consistency, the sum of all FTR payments to be made is restricted to the merchandising surplus (MS) using (15). To maintain consistency in flow of revenue, another set of constraints are imposed in each optimisation problem that ensures the direction of payments. Since the payment for holding an FTR is dependent on the difference in congestion component, the direction of payments must be taken care of in the constraint set. This ensures that FTR holders paying congestion charge to the ISO will receive payments for holding the FTR and vice versa. Hence, in addition to the max–min constraints, (16) and (17) are added to ensure this consistency. This constraint set can be restricted to only those FTRs which have bilateral contracts.

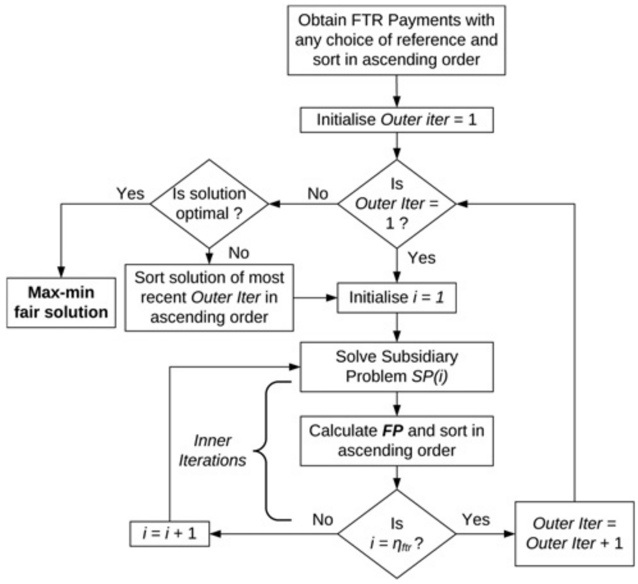

Fig. 2 *Flowchart of max–min fair LMP decomposition*

After obtaining solution of one SP, which constitutes one inner iteration, the FTR payments are updated using the new value of $\boldsymbol{\alpha}$ obtained (which then become $\mathrm{FP}(\bar{\boldsymbol{\alpha}})$ for the next SP). These payments are sorted in ascending order once again and the next SP is solved. The FTR holder whose payment is to be maximised depends on the counter and the ranking of FTR payments after every sort operation. The order in which they are maximised is from the lowest to the highest. In all, η_{ftr} SPs are solved and this constitutes one outer iteration. Conditions for enforcing max–min fairness constraints of current SP depend on solution of previous SP.

Fig. 2 gives an overview of the flowchart for arriving at a max–min fair solution. Optimality is said to be reached when there is no movement between the solutions of two successive outer iterations. The reader is referred to the appendix for proof of convergence of the proposed algorithm.

4 Results

The results for the proposed method are obtained on the PJM 5 bus and IEEE 30 bus systems. To minimise the effect of reactive power, a power factor of 0.95 is assumed and voltage limits at all buses are between 0.94 and 1.06 pu. ACOPF LMPs are obtained using Matpower [31]. It is assumed that the results of the FTR auction are already available and they are all FTR obligations. Necessary discussion regarding results from each system is given in subsequent subsections. For both systems, results are given for four different cases, whose description is given in Table 1. A comprehensive analysis of the results obtained for the IEEE 30 bus system is performed in order to bring out the effectiveness of the proposed method.

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Table 1 Description of various cases

Case	Pre-defined $\boldsymbol{\alpha}$ A	B	C	D
Reference	$\alpha_1 = 1$	$\alpha_i = (1/n)$	$\alpha_i = \frac{Pd_i}{\sum Pd_i}$	max–min fairness

4.1 PJM 5 bus system

The data for PJM 5 bus system are obtained from [2]. The generators are assumed to have reactive power limits from +150 and −150 MVar. The FTRs for this snapshot and the payments for four different cases are given in Table 2, and the LMP and its components are given in Table 3. The slack distribution at the max–min fair solution is $[0\ 0\ 0\ 1\ 0]^T$.

From Table 2, it is quite evident that the FTR payments vary widely for different reference choices. In case D, the max–min solution, FTRs # 3 and 5 benefit as the payment to FTR #3 has increased to \$416.47 and the payment by FTR #5 to ISO has decreased to \$301.40, ensuring minimum payment, compared to the remaining three cases (A–C). It could be said that FTRs # 1, 2 and 4 were actually receiving higher payments at the expense of FTRs # 3 and 5. The max–min fair solution is unique for a set of FTRs, and this was observed for the three different starting points (either A, B or C). A more detailed analysis regarding various aspects of max–min solution is given for the IEEE 30 bus system.

4.2 IEEE 30 bus system

The branch data for the IEEE 30 bus system are obtained from [32]. All bus shunts, taps and line susceptance are disabled. The generator limits and cost curves are obtained from [33]. The power flow limits on lines are as given in [34]. ACOPF is run on a system load as given in Table 6. The set of FTRs, given in Table 5, for which payments are to be made, are obtained after an auction, are simultaneously feasible when using the DC approximation and load certain lines up to their limits. The following observations are worth noting.

4.2.1 Minimum and maximum FTR payments:

In addition to the four cases studied earlier for the PJM 5 bus system, Table 4 summarises three additional cases against which the results of the proposed method are compared. Cases E and F obtain the minimum and maximum possible payments for each FTR holder. The respective objective functions are solved in isolation for each FTR. The constraints column indicates that no max–min-related constraints are used in obtaining the respective solution and neither any direction of payment constraints nor any constraint relating FTR payment with merchandising surplus is used. Case G involves calculating FTR payments when the sum of payments of all FTR holders is maximised. Note that for this objective, constraints (13)–(17) are enforced.

The last four columns of Table 5 provide a comparison of the max–min solution, the corresponding minimum and maximum values of FTR payment and the payments when summated maximisation of FTR payments is chosen as the objective (cases D–G). The minimum and maximum span the range of possible payments within the solution space. Any choice of reference must result in a payment that lies between their corresponding minimum and maximum values. Whether the FTR holder receives a positive payment or it has to pay the ISO entirely depends on the resulting decomposition. For most of the FTRs, the

Table 4 Additional cases with their objectives and constraints

Case	Objective function	Constraints
E	minimise FP_i	(13), (14)
F	maximise FP_i	(13), (14)
G	maximise $\sum_{i=1}^{n_{\text{ftr}}} FP_i$	(13)–(17)

Table 2 FTR payments for PJM 5 bus system

FTR No.	Source	Sink	MW	FTR payment, \$ A	B	C	D
1	5	3	340	5680.34	5624.68	5571.91	5515.75
2	5	4	210	5207.19	5189.87	5173.44	5155.96
3	3	4	50	404.46	408.52	412.37	416.47
4	1	4	70	1334.75	1330.73	1326.92	1322.87
5	3	2	100	−304.42	−303.40	−302.43	−301.40

Table 3 LMP decomposition comparison for PJM 5 bus system

Bus	LMP, \$/MW	LMP components, \$/MW A λ^e	A λ^l	A λ^c	B λ^e	B λ^l	B λ^l	C λ^e	C λ^l	C λ^c	D λ^e	D λ^l	D λ^c
1	15.79	15.79	0.00	0.00	22.39	−0.21	−6.39	28.72	−0.48	−12.45	35.00	−0.31	−18.90
2	24.04	15.79	0.32	7.93	22.39	0.24	1.42	28.72	0.09	−4.77	35.00	0.39	−11.34
3	27.11	15.79	0.34	10.98	22.39	0.27	4.45	28.72	0.13	−1.74	35.00	0.44	−8.33
4	35.00	15.79	0.14	19.07	22.39	−0.01	12.62	28.72	−0.22	6.51	35.00	0.00	0.00
5	10.00	15.79	−0.06	−5.73	22.39	−0.29	−12.09	28.72	−0.59	−18.13	35.00	−0.45	−24.55

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Table 5 FTR data and payments in various cases for IEEE 30 bus system

FTR				LMP, \$/MW		FTR payment, \$						
No.	Source	Sink	MW	λ_{so}	λ_{si}	A	B	C	D	E	F	G
1	2	25	16.29	11.00	20.26	135.14	127.48	126.59	128.37	112.21	136.29	136.29
2	2	27	6.73	11.00	22.00	67.85	64.87	64.52	65.22	58.92	68.30	68.30
3	8	29	22.29	11.50	23.34	241.04	229.90	228.59	231.19	207.68	242.72	242.72
4	23	12	10.08	16.00	16.01	1.72	2.49	2.58	2.40	1.60	4.03	1.60
5	1	5	60.00	11.50	13.27	63.57	42.88	40.47	45.28	1.65	66.68	66.68
6	2	23	46.15	11.00	16.00	194.56	176.94	174.89	178.99	141.83	197.21	197.21
7	3	10	90.00	13.71	25.72	1055.74	1043.52	1042.10	1044.94	1019.18	1057.58	1057.58
8	14	13	40.00	16.00	16.00	16.83	25.03	25.99	24.08	15.60	41.36	15.60
9	14	20	19.07	16.00	31.20	279.01	273.71	273.09	274.33	263.14	279.81	279.81
10	18	12	4.46	17.00	16.01	−1.32	0.17	0.35	0.00	−1.55	3.16	−1.55
11	1	3	95.00	11.50	13.71	144.97	113.33	109.64	117.01	50.29	149.72	149.72
12	2	14	29.52	11.00	16.00	116.63	101.55	99.79	103.30	71.49	118.90	118.90
13	13	18	4.46	16.00	17.00	1.39	−0.11	−0.28	0.07	−3.08	1.61	1.61
14	13	21	3.11	16.00	30.39	44.07	43.76	43.72	43.79	43.15	44.11	44.11
15	22	12	34.00	16.00	16.01	4.62	6.66	6.90	6.42	4.32	10.72	4.32
16	22	16	6.00	16.00	20.16	24.56	24.37	24.35	24.39	23.99	24.59	24.59
17	22	19	10.00	16.00	32.04	152.20	148.21	147.75	148.68	140.27	152.80	152.80
18	14	12	1.18	16.00	16.01	0.51	0.76	0.79	0.73	0.48	1.24	0.48
19	23	7	30.00	16.00	14.44	−48.87	−49.87	−49.98	−49.75	−51.86	−48.72	−48.72
20	23	10	20.00	16.00	25.72	191.47	190.05	189.89	190.22	187.23	191.68	191.68
21	18	23	4.49	17.00	16.00	−2.10	−0.93	−0.80	−1.07	−2.27	1.39	−2.27
22	27	8	5.00	22.00	11.50	−51.15	−50.50	−50.42	−50.58	−51.25	−49.19	−51.25
23	27	10	15.00	22.00	25.72	55.51	55.38	55.36	55.39	55.12	55.53	55.53
24	12	18	6.74	16.01	17.00	2.00	−0.26	−0.53	0.00	−4.78	2.34	2.34
25	11	1	25.85	11.50	11.50	23.42	34.82	36.15	33.50	21.70	57.55	21.70
26	12	1	11.06	16.01	11.50	−42.62	−39.07	−38.65	−39.48	−43.15	−31.98	−43.15
27	3	12	11.50	13.71	16.01	26.76	26.89	26.91	26.88	26.74	27.16	26.74

payments are either entirely positive or entirely negative. For a few FTR holders though, four FTRs in particular, the payments can either be positive or negative. For example, the minimum and maximum FTR payments for FTR #1 lie between \$112.21 and \$136.29, signifying that any choice of reference will only result in positive payments for this FTR

Table 6 LMP decomposition for different starting points

Bus	Load, MW	λ, \$/MW	Bus 1 as reference				Bus 5 as reference				Bus 15 as reference			
			α	LMP components, \$/MW			α	LMP components, \$/MW			α	LMP components, \$/MW		
				λ^e	λ^l	λ^c		λ^e	λ^l	λ^c		λ^e	λ^l	λ^c
1	0	11.50	0.72	17.00	−0.55	−4.95	0	17.70	−1.25	−4.95	0	18.02	−1.57	−4.95
2	0	11.00	0	17.00	−0.51	−5.50	0	17.70	−1.21	−5.50	0	18.02	−1.52	−5.50
3	80.75	13.71	0	17.00	0.42	−3.72	0	17.70	−0.28	−3.72	0	18.02	−0.59	−3.72
4	0	14.21	0	17.00	0.52	−3.31	0	17.70	−0.18	−3.31	0	18.02	−0.49	−3.31
5	42.5	13.27	0	17.00	0.46	−4.20	0.75	17.70	−0.24	−4.20	0	18.02	−0.55	−4.20
6	0	15.01	0	17.00	0.62	−2.62	0	17.70	−0.08	−2.62	0	18.02	−0.39	−2.62
7	25.5	14.44	0	17.00	0.71	−3.28	0	17.70	0.01	−3.28	0	18.02	−0.30	−3.28
8	12.75	11.50	0	17.00	0.41	−5.92	0	17.70	−0.29	−5.92	0	18.02	−0.60	−5.92
9	0	11.50	0	17.00	0.75	−6.25	0	17.70	0.05	−6.25	0	18.02	−0.27	−6.25
10	95.2	25.72	0	17.00	0.82	7.89	0	17.70	0.12	7.89	0	18.02	−0.19	7.89
11	0	11.50	0	17.00	0.74	−6.25	0	17.70	0.04	−6.25	0	18.02	−0.27	−6.25
12	25.5	16.01	0	17.00	0.39	−1.38	0	17.70	−0.31	−1.38	0	18.02	−0.62	−1.38
13	10.2	16.00	0	17.00	0.39	−1.40	0	17.70	−0.31	−1.40	0	18.02	−0.62	−1.40
14	25.5	16.00	0	17.00	0.99	−2.00	0	17.70	0.29	−2.00	0	18.02	−0.02	−2.00
15	25.5	15.52	0	17.00	0.93	−2.42	0	17.70	0.23	−2.42	0.84	18.02	−0.08	−2.42
16	4.25	20.16	0	17.00	0.66	2.50	0	17.70	−0.04	2.50	0	18.02	−0.35	2.50
17	0	24.03	0	17.00	0.78	6.25	0	17.70	0.08	6.25	0	18.02	−0.23	6.25
18	17	17.00	0	17.00	1.38	−1.38	0	17.70	0.68	−1.38	0	18.02	0.37	−1.38
19	8.5	32.04	0.10	17.00	1.74	13.30	0.09	17.70	1.04	13.30	0.06	18.02	0.73	13.30
20	29.75	31.20	0.09	17.00	1.81	12.39	0.08	17.70	1.11	12.39	0.05	18.02	0.80	12.39
21	17	30.39	0.09	17.00	0.68	12.70	0.08	17.70	−0.02	12.70	0.05	18.02	−0.33	12.70
22	0	16.00	0	17.00	0.57	−1.57	0	17.70	−0.13	−1.57	0	18.02	−0.45	−1.57
23	42.5	16.00	0	17.00	0.61	−1.62	0	17.70	−0.08	−1.62	0	18.02	−0.40	−1.62
24	4.25	17.12	0	17.00	0.73	−0.61	0	17.70	0.03	−0.61	0	18.02	−0.28	−0.61
25	4.25	20.26	0	17.00	0.87	2.38	0	17.70	0.18	2.38	0	18.02	−0.14	2.38
26	0	20.26	0	17.00	0.87	2.38	0	17.70	0.18	2.38	0	18.02	−0.14	2.38
27	13.6	22.00	0	17.00	0.80	4.20	0	17.70	0.10	4.20	0	18.02	−0.21	4.20
28	0	24.52	0	17.00	0.87	6.64	0	17.70	0.17	6.64	0	18.02	−0.14	6.64
29	17	23.34	0	17.00	1.88	4.45	0	17.70	1.18	4.45	0	18.02	0.87	4.45
30	0	22.74	0	17.00	1.40	4.34	0	17.70	0.70	4.34	0	18.02	0.39	4.34

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holder. Similarly, for FTR #19, any reference choice will result in payments lying between $ − 51.86 and $ − 48.72, which is entirely negative. However, for FTR #24, the minimum is $ − 4.78 and maximum is $2.34.

It is interesting to note that maximising sum of FTR payments (case G) does not lead to the maximum payments to all FTR holders. In fact, for FTRs # 4, 8, 10, 15, 18, 21, 22 and 25–27, the payments are at their minimum. FTR payments for individual maximisation objective and maximising sum of payments objective will lead to the same solution if and only if the references at which the individual maximisation occurs are same for all FTRs. The difference in max–min solution (case D) and summated maximisation (case G) can be attributed to the difference in references at which individual maximisation occurs as they are different for different FTR holders. In such situations, there will always be a set of winners and losers when using summated maximisation as an objective to make FTR payments.

For example, of all the 27 FTRs under consideration, 10 of them receive their minimum payment in case G. This is, however, avoided when invoking the max–min fairness criteria because these 10 FTRs receive payments higher than their respective minimum values and the remaining 17 FTRs receive a payment that is less than their maximum value. This characteristic feature is observed in case D, which is the max–min solution.

4.2.2 Variation in FTR payments with reference: In Table 5, FTR payments for cases A–D along with the LMPs at the source and sink locations are provided. For different choices of reference, it may be observed that the payments vary widely for all FTR holders.

The inter-relationship between congestion components of buses has an effect on the eventual LMP decomposition, which in-turn affects the payments received (made) by FTR holders. For FTR #13 between buses 13 and 18, the payment made under case A is positive, which implies that the FTR holder receives a payment and this is consistent with the LMP difference. However, when the reference is evenly distributed between buses or made proportional to load (cases B and C), the FTR holder ends up paying the ISO. A similar observation can be made for FTR #24. A contrasting set of payments result for FTR #10, wherein, payment is negative for case A and positive for cases B and C. This wide variation is due to the multiplicity of the solution space in LMP decomposition. Therefore any arbitrary choice of reference can change the direction of payment for an FTR holder. The results obtained in case D are, however, unique and satisfy the max–min fairness constraints, also taking into account the direction of payment when arriving at max–min fair solution.

Of the three cases of pre-fixed reference under consideration (cases A–C), FTRs # 4, 8, 15, 18, 25 and 27 receive their minimum payments when choosing bus 1 as reference (case A) for decomposition. Their maximum payments occur when reference is made proportional to load (case C). These FTR holders may argue that they were better off with the decomposition in case C rather than case A. This situation is, however, avoided when choosing a max–min fair reference because the payment received by an FTR holder is not at the expense of some other FTR holder whose payment is already lesser, which can be deemed more acceptable. Hence, the reference at which solution is max–min can be termed as a trade-off reference at which all FTR holders are ensured of the maximum payment possible without adversely affecting payment of other FTR holders.

4.2.3 Importance of direction of payments constraint: The importance of the direction of payment constraints can be seen when considering FTRs # 10, 13 and 24. The LMP difference between sink and source nodes for FTR #10 is negative and for FTRs # 13 and 24, it is positive. However, the max–min fair FTR payments ensure that FTRs # 10 and 24 neither receive nor pay and for FTR #13, the max–min fair solution reveals a credit of $0.07. This is due to the fact that any attempt to increase payment of FTR #10 will imply that it receives a payment for holding the FTR, which is against the direction of payment constraint and the LMP difference. Similarly, any attempt to increase the payment of FTR #13 beyond $0.07 would have to be at the expense of FTRs # 10 and 24 and some other FTRs whose payment is lesser than the payment of FTR #13, which is against the max–min principle.

4.2.4 Uniqueness of solution: The max–min solution is a characteristic feature of given set of FTRs and the snapshot. In other words, for a given set of FTRs and snapshot, the max–min solution is unique, that is, indifferent to starting point. In order to ascertain that the max–min fair solution is indeed unique, the max–min algorithm was initialised using different starting points. These starting points are nothing but the resulting payments after LMP decomposition using a particular choice of reference. In Table 6, a comparison of the three components of LMP at max–min fair reference for three different starting points is given. The max–min fair $\boldsymbol{\alpha}$ after convergence is also provided. The results reveal that the congestion component for all the three starting points is the same. However, the energy and loss components are different. This can be attributed to the fact that there can exist more than one reference choice for the same set of congestion components. Hence, the resulting FTR payment at max–min fair references for all three starting points remains the same.

5 Conclusion

The decomposition of LMP obtained from ACOPF is seen as a fairness problem and a set of constraints using the widely accepted max–min fairness criteria is enforced so as to obtain a fair LMP decomposition. Through the results, it has been shown that a max–min fair set of payments is indeed possible and that the solution is indifferent to the starting point, in other words ‘unique’. A comparison of the proposed method with the minimum and maximum FTR payments possible is done so as to highlight the advantage of using the max–min fair reference. Constraints on direction of payment ensure consistency in payments when involving bilateral contracts.

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8 Appendix

Proof of convergence of max–min algorithm

Proof: Let $\mathrm{FP}_1(\bar{\boldsymbol{\alpha}})$ represent lowest of FTR payments after arranging them in ascending order before solving **SP(1)** of any particular iteration. Similarly, let $\mathrm{FP}_2(\bar{\boldsymbol{\alpha}})$ be next lowest and so on until $\mathrm{FP}_{\eta_{\mathrm{ftr}}}(\bar{\boldsymbol{\alpha}})$ being the highest.
Suppose, in the first inner iteration, **SP(1)** shows movement in $\mathrm{FP}_1(\bar{\boldsymbol{\alpha}})$. Constraint of type (12) ensures that this movement has to be in the upward direction. While passing through remaining instances, that is, while maximising FTR payment of the other holders, in the outer iteration, constraints of type (10) and (11) ensure that

$$\mathrm{FP}_1(\boldsymbol{\alpha}) > \mathrm{FP}_1(\bar{\boldsymbol{\alpha}}) \tag{18}$$

However, if there is no movement while solving **SP(1)**, (12) ensures that it is at least as much as $\mathrm{FP}_1(\bar{\boldsymbol{\alpha}})$ using

$$\mathrm{FP}_1(\boldsymbol{\alpha}) > \mathrm{FP}_1(\bar{\boldsymbol{\alpha}}) \tag{19}$$

In other words, the lowest payment will remain at same value or can only increase. It will never decrease. Now, if **SP(1)** does not show movement in $\mathrm{FP}_1(\bar{\boldsymbol{\alpha}})$, then depending on whether **SP(2)** shows movement in $\mathrm{FP}_2(\bar{\boldsymbol{\alpha}})$, at least the following will be ensured

$$\mathrm{FP}_2(\boldsymbol{\alpha}) \geq \mathrm{FP}_2(\bar{\boldsymbol{\alpha}}) \tag{20}$$

While doing so, because of the constraints of type (11) ($\mathrm{FP}_1(\bar{\boldsymbol{\alpha}})$ is less than $\mathrm{FP}_2(\bar{\boldsymbol{\alpha}})$, $i=2$ and $k=1$), (19) will be obtained automatically. In general, if in a particular outer iteration, if solving first instance to second last instance do not show any movement, then while solving last instance, it is ensured that

$$\mathrm{FP}_t(\boldsymbol{\alpha}) \geq \mathrm{FP}_t(\bar{\boldsymbol{\alpha}}), \quad t = 1, 2, \ldots, \eta_{\mathrm{ftr}} - 1 \tag{21}$$

In effect, the FTR payment currently being maximised cannot be reduced below its value from the earlier iteration. Thus, FTR payments will either remain stationary, or move in only one direction, that is, upward. Max–min fair solution is obtained when $t = \eta_{\mathrm{ftr}} - 1$. The combination of arranging the payments in ascending order after every iteration, constraint modelling using (10)–(12), solving inner iteration at most η_{ftr} times and movement in only one direction will ensure convergence.

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