Chapter 3: Gauss’s Law

- ✓ **Electric Flux**
- ✓ **Gauss’s Law**
- ✓ **Applying Gauss’s Law**
- ✓ **Examples**

Session 6:

- ✓ **Electric Flux**
- ✓ **Gauss's Law**
- ✓ **Examples**

Introduction

❖ **Gauss' Law is convenient for calculating the electric field of highly symmetric charge distributions.**

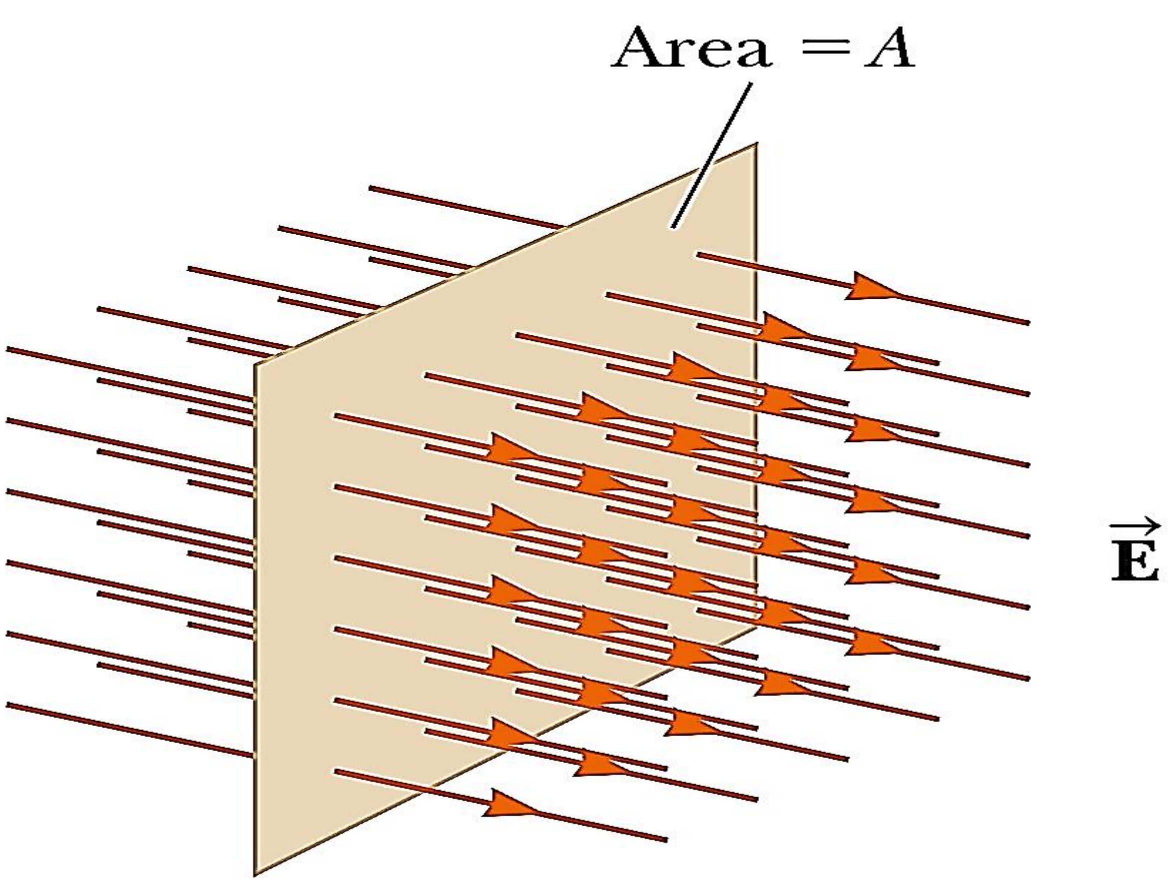
❖ **Gauss' Law is important in understanding and verifying the properties of conductors in electrostatic equilibrium.**

Electric Flux

- ***Electric flux*** is the product of the magnitude of the electric field and the surface area, *A*, perpendicular to the field.

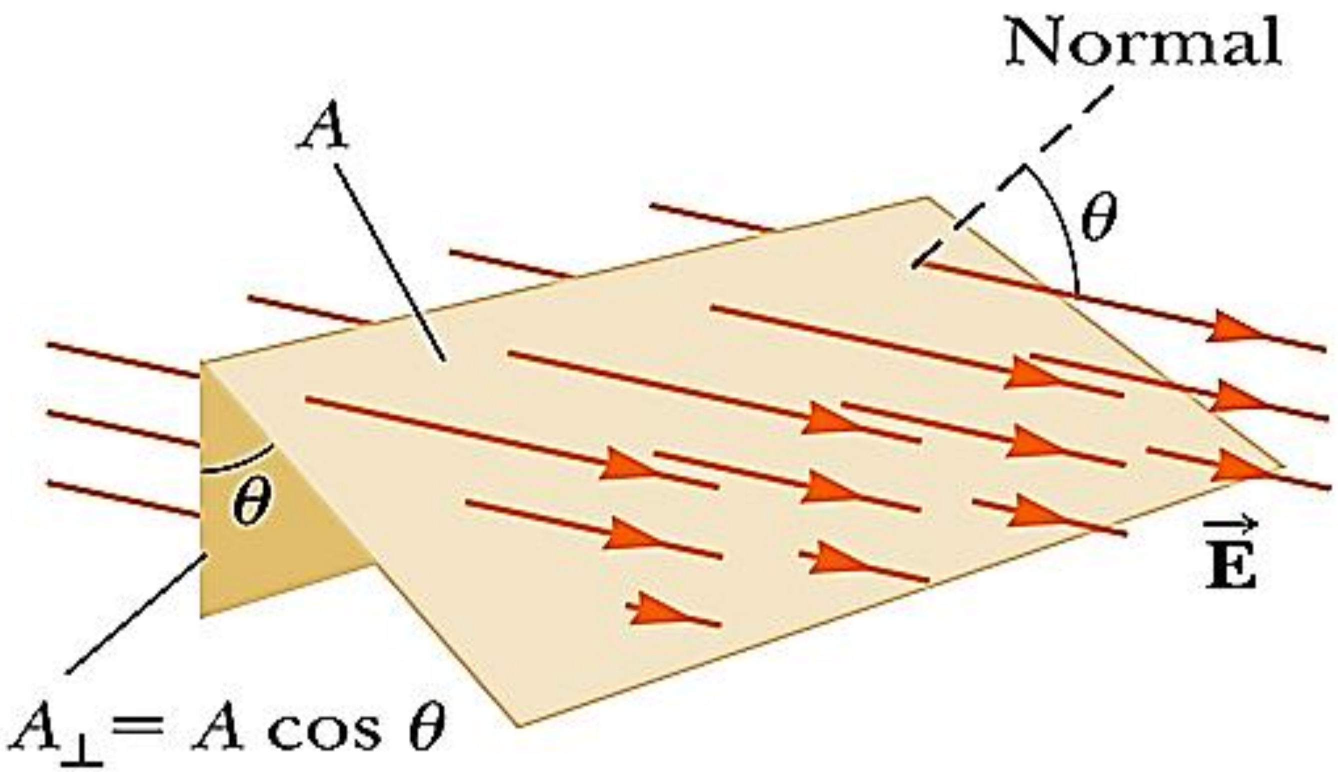
$$\Phi_E = E\,A$$

Units: $N \cdot m^2 / C$



Electric Flux, General Area

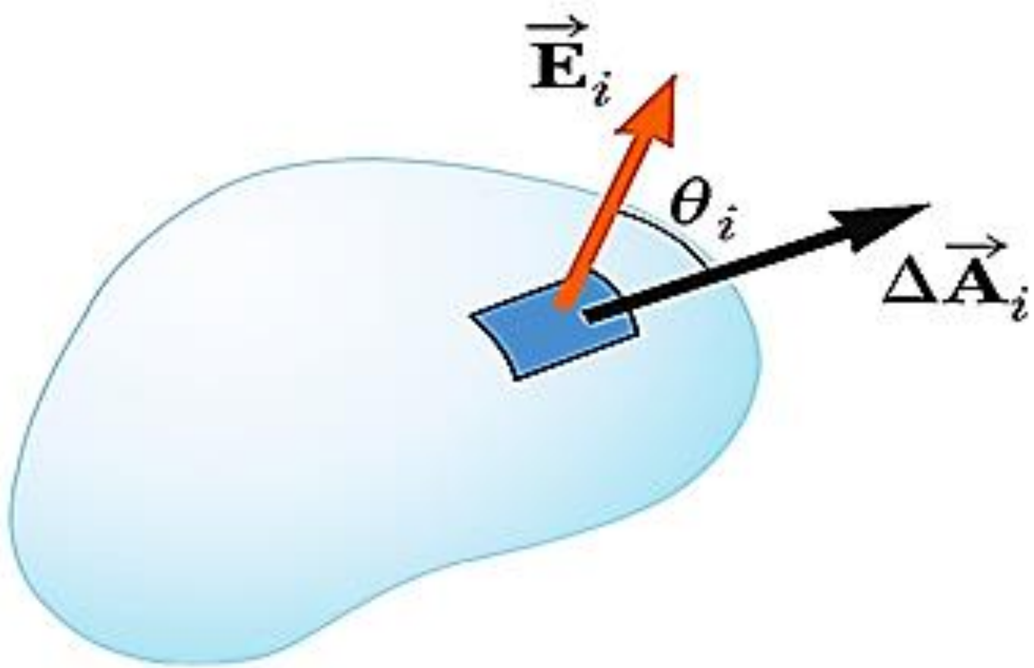
$$\Phi_E = E\,A\cos\theta = E.A$$



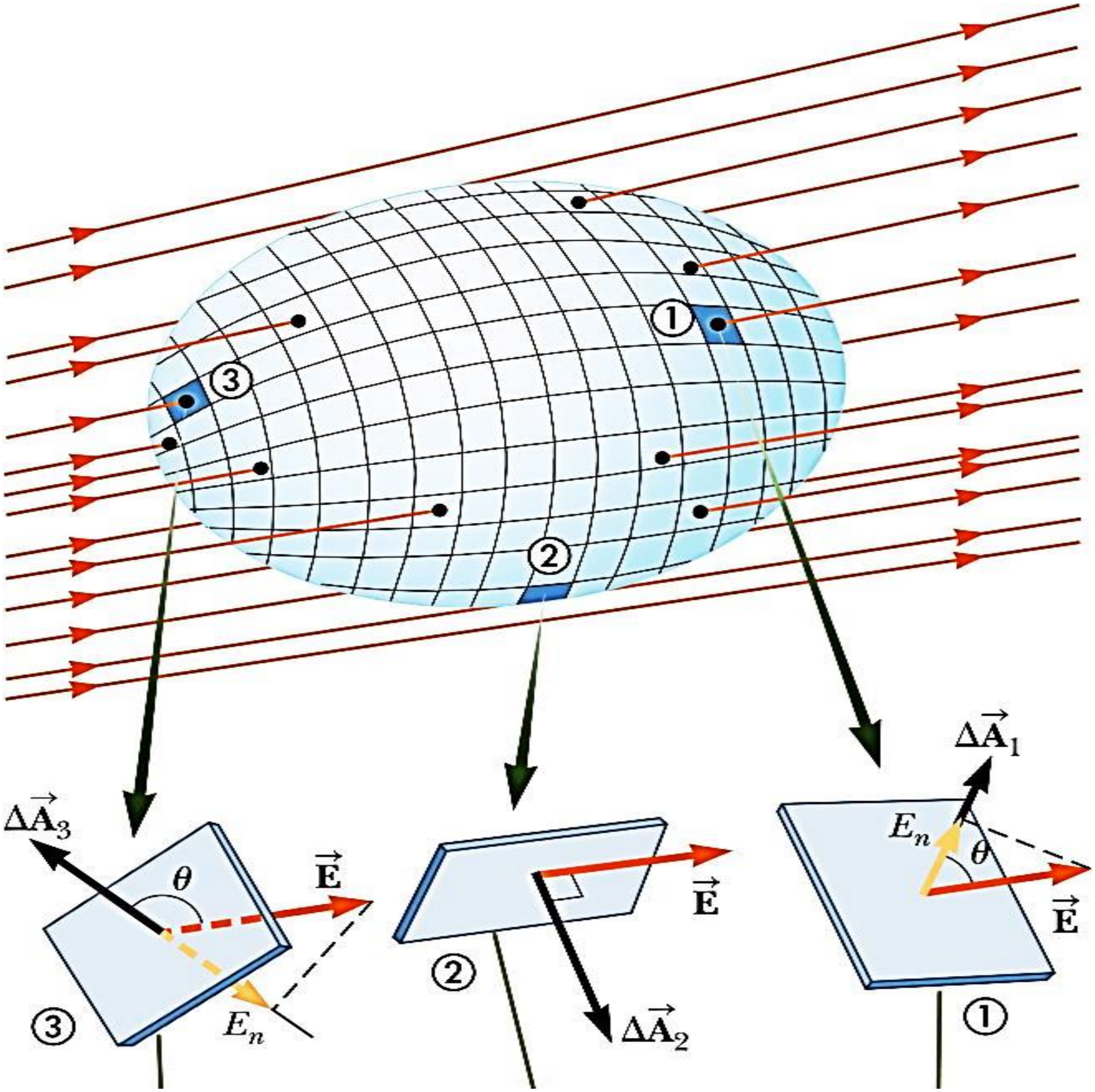
$$\Delta\Phi_E = E_i \Delta A_i \cos\theta_i = \vec{\mathbf{E}}_i \cdot \Delta\vec{\mathbf{A}}_i$$

$$\Phi_E = \lim_{\Delta A_i \to 0} \sum E_i \cdot \Delta A_i$$

$$\Phi_E = \int_{\text{surface}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}$$



Electric Flux, Closed Surface

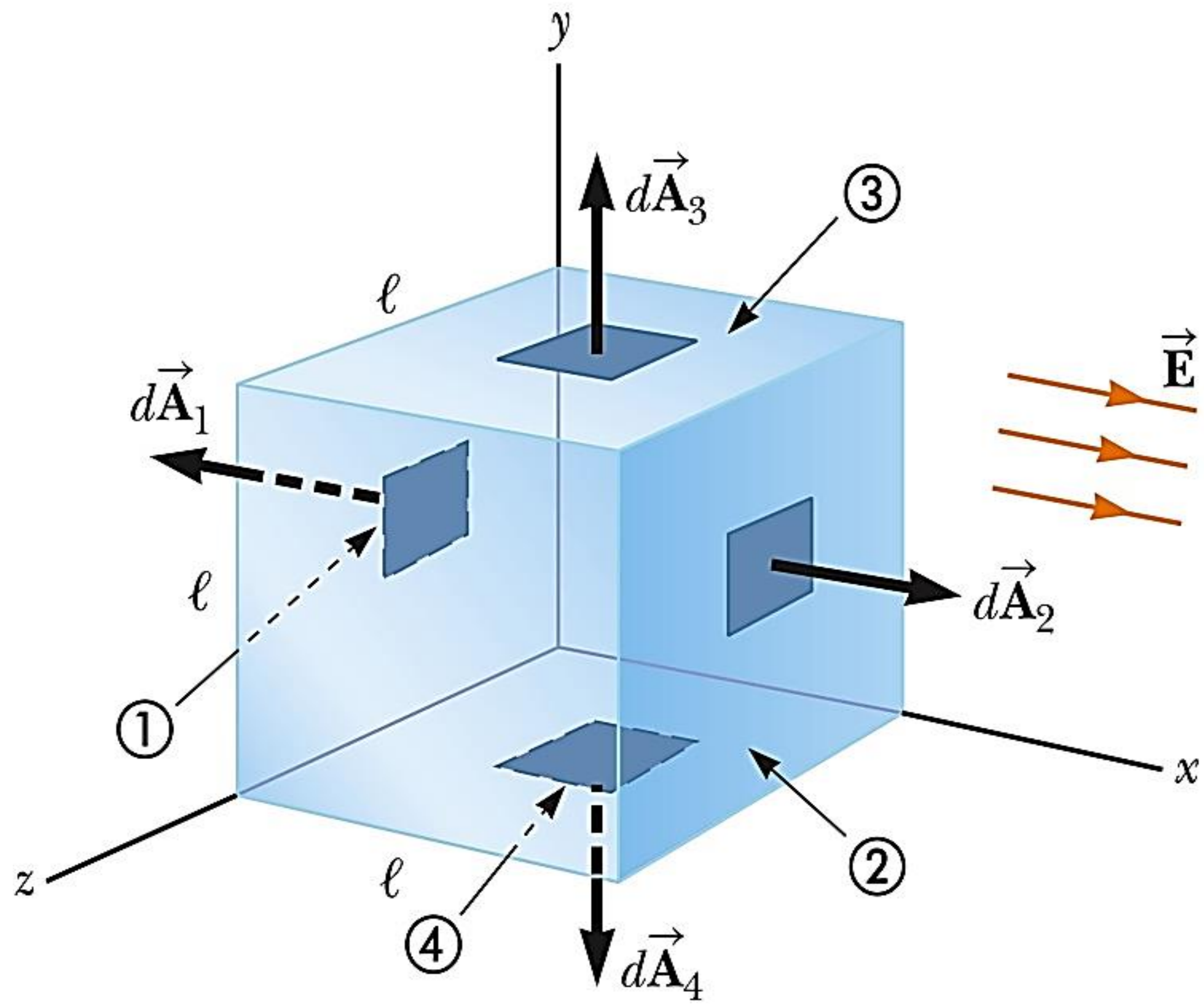


$\Phi_E < 0$ $\quad\quad$ $\Phi_E = 0$ $\quad\quad$ $\Phi_E > 0$

Electric Flux, Closed Surface

$$\Phi_E = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \oint E_n dA$$

- For face 1, $\boldsymbol{\Phi_E = -E\,l^2}$
- For face 2, $\boldsymbol{\Phi_E = +E\,l^2}$
- For the other sides, $\boldsymbol{\Phi_E = 0}$
- Therefore, $\boldsymbol{\Phi_{E\ total} = 0}$



The ***net*** **flux** through the surface is proportional to the net **number of lines leaving the surface**.

Gauss's Law

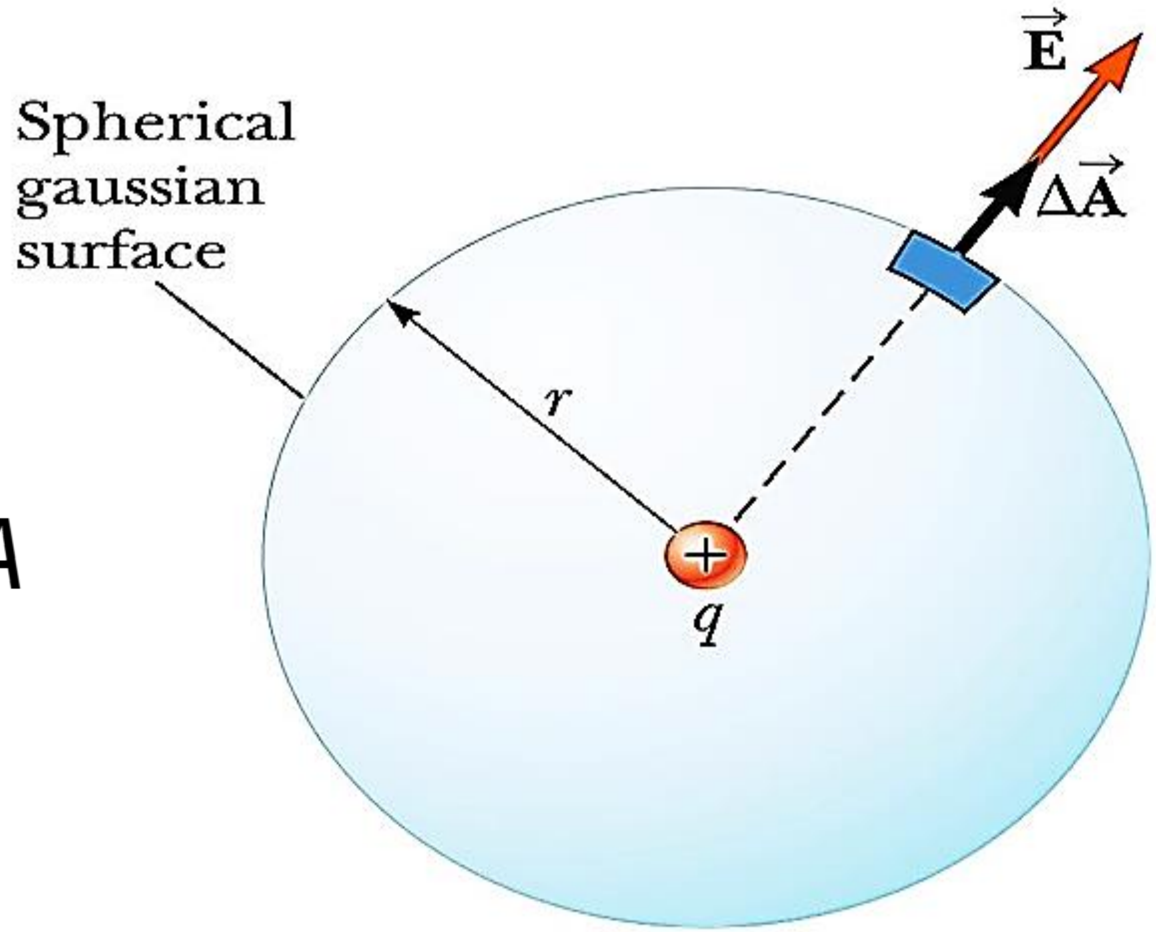
Gauss's law is an expression of the general **relationship between the net electric flux through a closed surface and the charge enclosed by the surface**.

The closed surface is often called a ***Gaussian surface.***

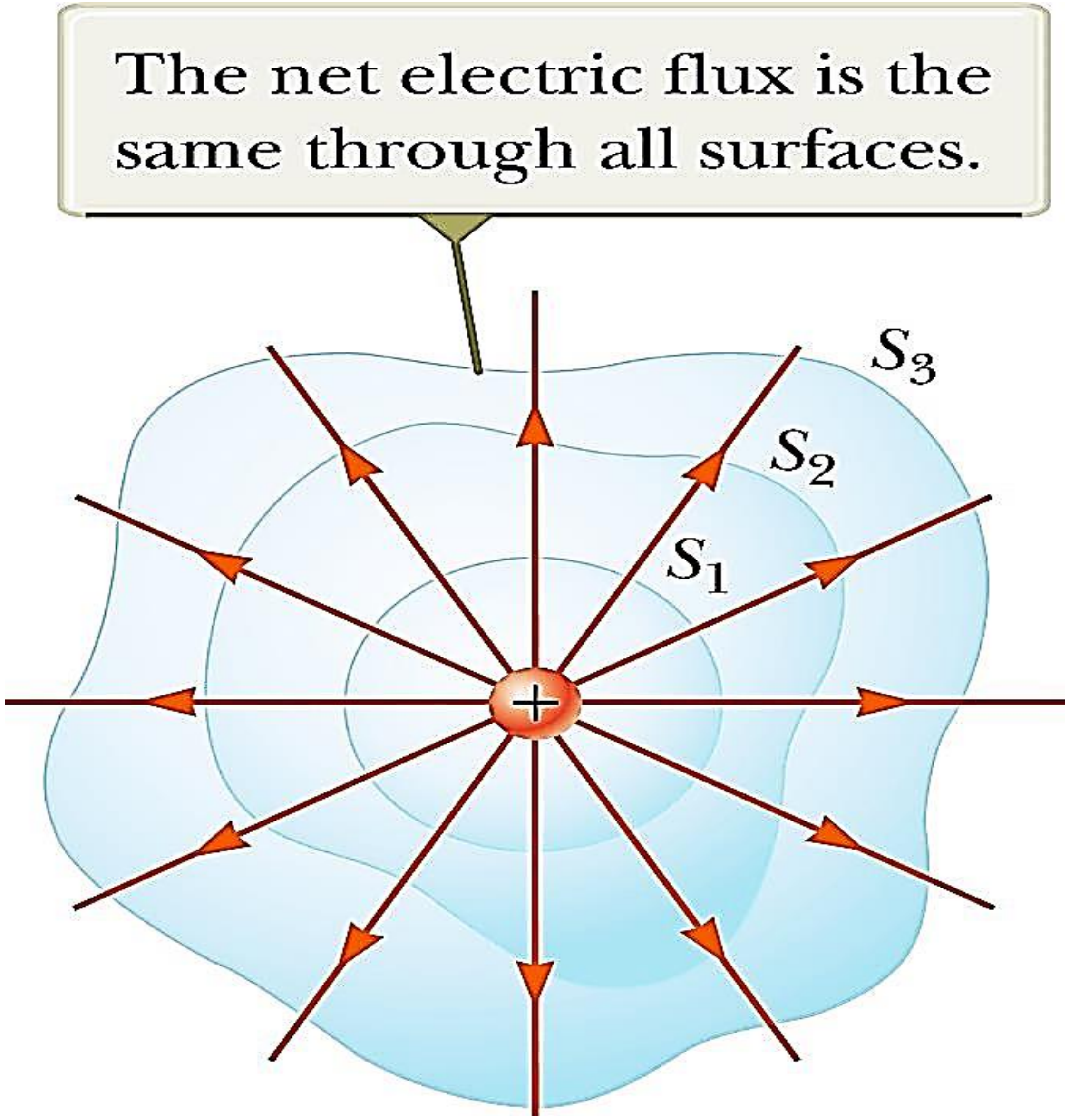
$$\mathbf{E} = \frac{kq}{r^2}$$

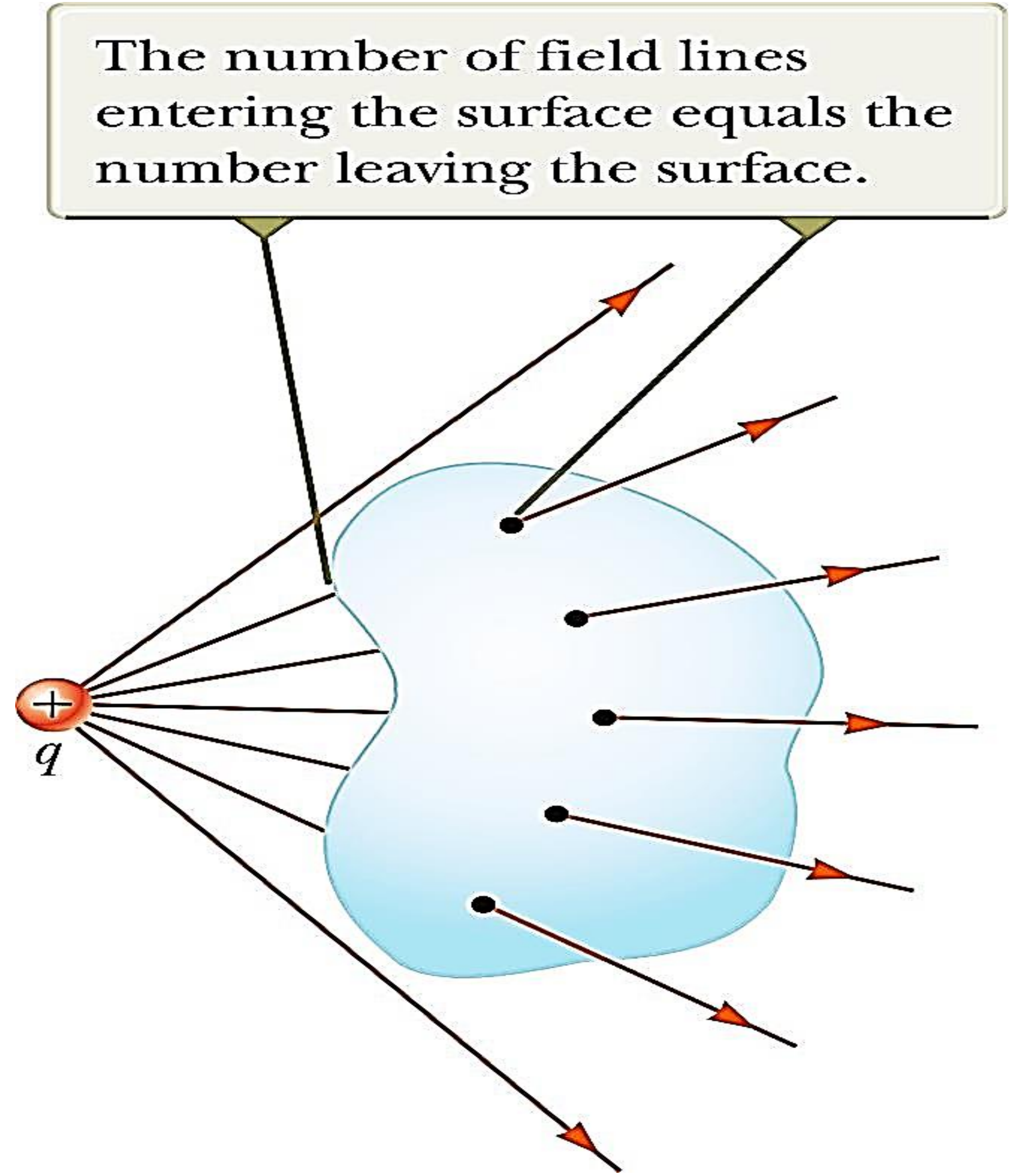
$$\Phi_E = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \oint E\,dA = E\oint dA$$

$$\Phi_E = \frac{kq}{r^2}(4\pi r^2) = 4\pi kq = \frac{q}{\varepsilon_o}$$



Gauss's Law





$$\Phi_E = \frac{q}{\varepsilon_o}$$

$$\Phi_E = 0$$

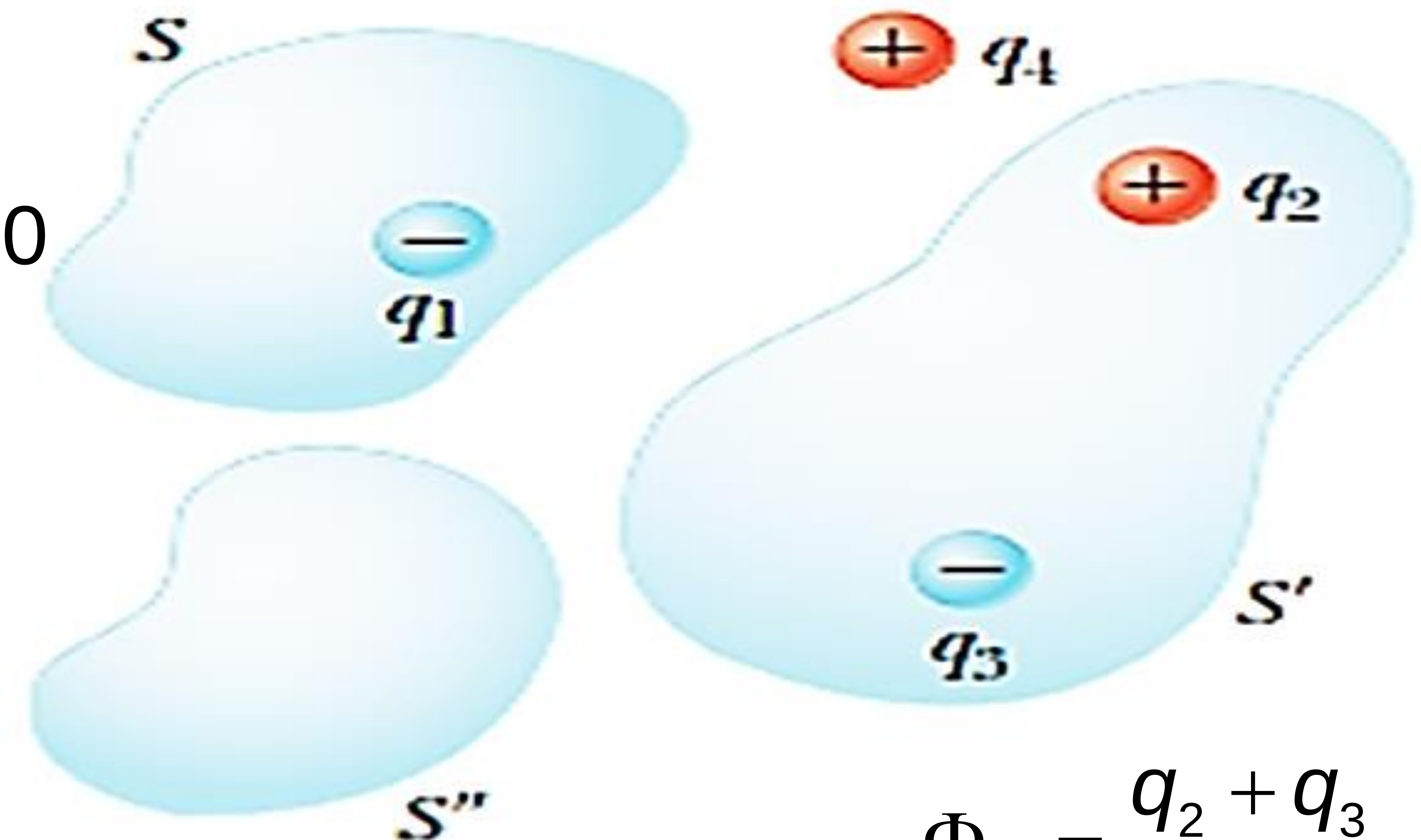
Gauss's Law

❖ The **mathematical form of Gauss's law** states:

$$\Phi_E = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{\text{in}}}{\varepsilon_o}$$

Carl Friedrich Gauss
(German mathematician)

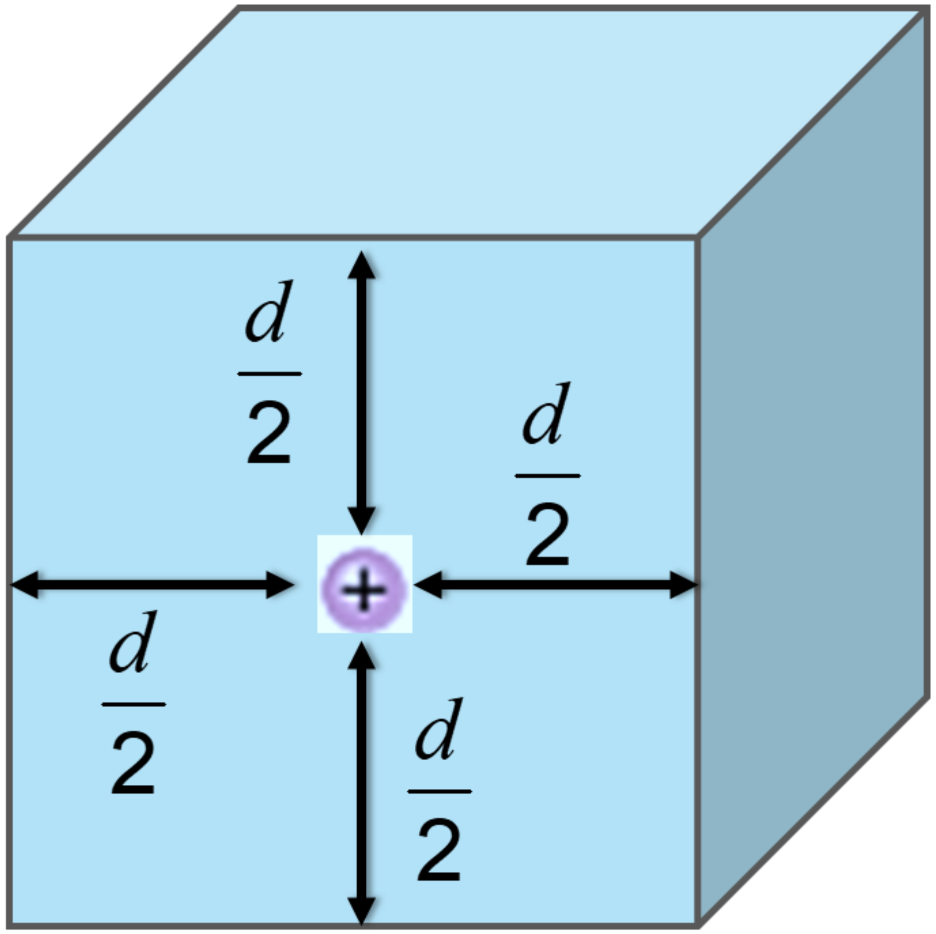
$$\Phi_E = \frac{q_1}{\varepsilon_o} < 0$$

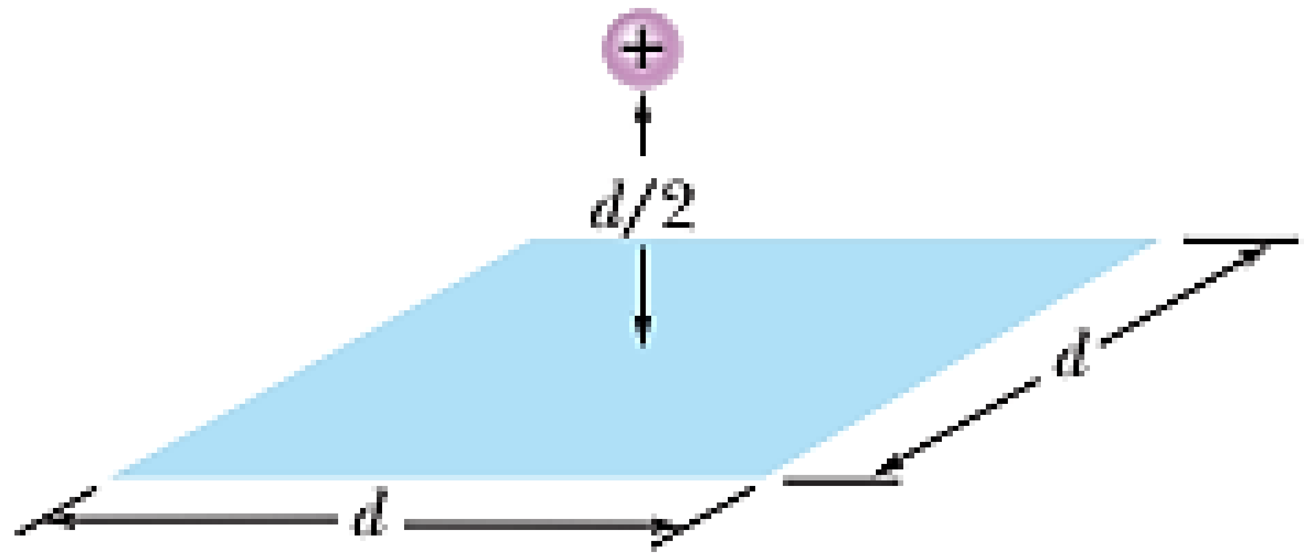


$$\Phi_E = 0$$

$$\Phi_E = \frac{q_2 + q_3}{\varepsilon_o}$$

Ex 1 (Prob 23. 5). A proton is a distance **d/2** directly above the center of a square of side **d**. What is the magnitude of the electric flux through the square? (Hint: Think of the square as one face of a cube with edge d.)





$$\Phi_E = \oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q}{\varepsilon_o} \quad \Rightarrow \quad \Phi_E{}' = \frac{\Phi_E}{6} = \frac{q}{6\,\varepsilon_o}$$

$$\Phi_E{}' = \frac{q}{6\,\varepsilon_o} = \frac{1.6 \times 10^{-19}}{6(8.85 \times 10^{-12})} \approx 3 \times 10^{-9} \quad \left(\frac{\mathrm{N.m^2}}{\mathrm{C}}\right)$$