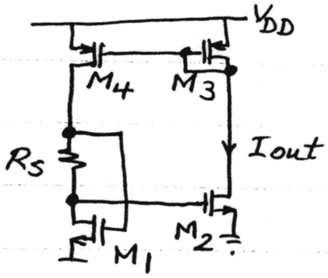
Chapter 11

11.1 Assuming all transistors are in saturation, we have

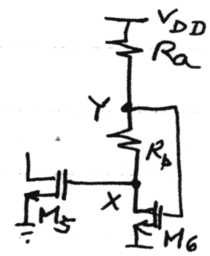


$$I_{out} R_S + \sqrt{\frac{2 I_{out}}{\mu_n C_{ox} (\frac{W}{L})_2}} + V_{TH2} = \sqrt{\frac{2 I_{out}}{\mu_n C_{ox} (\frac{W}{L})_2}} + V_{TH1},$$

where we have assumed $(\frac{W}{L})_4 = (\frac{W}{L})_3$ and $\lambda = 0$.

Thus, $$I_{out} = \frac{1}{\mu_n C_{ox} R_S^2} \left(\sqrt{(\tfrac{L}{W})_1} - \sqrt{(\tfrac{L}{W})_2} \right)^2$$

11.2 When the circuit turns on, initially both M_5 and M_6 are off and V_X and V_Y rise together, i.e., $V_X = V_Y$. When V_Y reaches V_{TH6}, V_X is also near V_{TH5}. Thus, M_6 and M_5 turn on almost simultaneously.



The surge in the drain current of M_5 turns the rest of the circuit on. As V_Y increases further, V_X begins to drop if M_6 is turned on sufficiently because the voltage gain of M_6 and R_b exceeds unity. For high values of V_Y, V_X can be lower than V_{TH5}.

Since $(V_{DD} - I_{D6} \cdot R_a - V_{TH})^2 \cdot \mu_n C_{ox} (\frac{W}{L})_6 = I_{D6}$, we solve the quadratic equation:

$$R_a^2 I_{D6}^2 - I_{D6}\left(2 R_a (V_{DD}-V_{TH}) + \frac{1}{\mu_n C_{ox} (\frac{W}{L})_6}\right) + (V_{DD} - V_{TH})^2 = 0$$

$$\Rightarrow I_{D6} = \frac{2 R_a (V_{DD} - V_{TH}) + \frac{1}{\mu_n C_{ox} (\frac{W}{L})_6} + \sqrt{\left[2 R_a (V_{DD} - V_{TH}) + \frac{1}{\mu_n C_{ox} (\frac{W}{L})_6}\right]^2 - 4 R_a^2 (V_{DD} - V_{TH})^2}}{2 R_a^2}$$

This value is substituted in the other condition:

$$V_{DD} - I_{D6}(R_a + R_b) \leq V_{TH5}$$

to give the condition for turning off M_5.

11.3 (a) Since the output voltage is near 2.5V whereas $V_X \approx 2V_{BE}$,

we write $\dfrac{I_{D1}}{I_{D2}} \approx \dfrac{1+\lambda(V_{DD}-2V_{BE})}{1+\lambda(V_{DD}-2.5V)}$

$$\approx 1+\lambda(2.5V-2V_{BE})$$

$\Rightarrow V_{BE2}-V_{BE4} = V_T \ln n + V_T \ln \dfrac{I_{D1}}{I_D}$ $\quad \ln(1+\varepsilon)\approx\varepsilon$

$$= V_T \ln n + V_T\lambda(2.5V-2V_{BE})$$

The error $V_T\lambda(2.5V-2V_{BE})$ directly appears in V_{out}.

This error is also divided by R_1 and multiplied by R_2, giving another error component at the output. So the overall error is equal to $(1+\frac{R_2}{R_1})V_T\lambda(2.5V-2V_{BE})$.

(b) $\dfrac{I_{D3}}{I_{D4}} \approx \dfrac{1+\lambda(V_{DD}-V_{BE1})}{1+\lambda(V_{DD}-V_{BE1}+V_T\ln n)}$

$$\approx 1-\lambda V_T \ln n$$

The output error is then equal to $V_T \ln(1-\lambda V_T \ln n)$

$$\approx -V_T^2\lambda \ln n.$$

(c) $V_{TH1}=V_{TH}$, $V_{TH2}=V_{TH}+\Delta V_{TH}$

For small V_{TH}, we have $I_{D2}=I_{D1}+g_m\Delta V_{TH}$, where g_m is the mean transconductance of M_1 and M_2. Thus,

$\dfrac{I_{D1}}{I_{D2}} = 1-\dfrac{g_m\Delta V_{TH}}{I_{D2}} = 1-\dfrac{2\Delta V_{TH}}{|V_{GS}-V_{TH}|_2}$. Using the method of part (a), we have: output error $= (1+\frac{R_2}{R_1})(-V_T)\dfrac{2\Delta V_{TH}}{|V_{GS}-V_{TH}|_2}$.

(d) $\dfrac{I_{D3}}{I_{D4}} = 1-\dfrac{2\Delta V_{TH}}{|V_{GS}-V_{TH}|_4} \Rightarrow$ output error $= -V_T\cdot\dfrac{2\Delta V_{TH}}{|V_{GS}-V_{TH}|_4}$

11.4 $\quad -V_{XY}\cdot A_1 = V_{DD} - |V_{GS2}| \qquad |V_{GS2}| = \sqrt{\dfrac{2(V_T \ln n)/R_1}{\mu_n C_{ox}\left(\frac{W}{L}\right)_2}} + |V_{TH2}|$

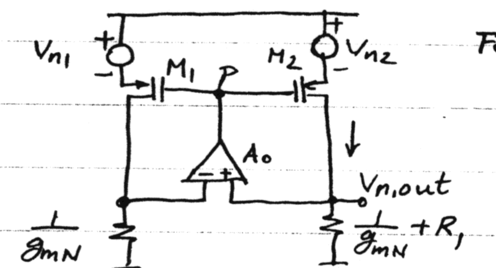
$$A_1 \ge \left[V_{DD} - \sqrt{\dfrac{2(V_T \ln n)/R_1}{\mu_n C_{ox}\left(\frac{W}{L}\right)_2}} - |V_{TH2}|\right]\Big/(-V_e)$$

11.5 The collector current of Q_4 is less than its emitter current. Thus, the current thru R_1 and R_2 is given by $\dfrac{(V_T \ln n)}{R_1}\times\dfrac{\beta+1}{\beta}$, and hence the output has an error equal to $\dfrac{1}{\beta}\,\dfrac{V_T \ln n}{R_1}\,R_2$.

Another source of error is the flow of base currents of Q_2 and Q_4 from M_3 and M_4, respectively. That is, $|V_{BE1}|$ and $|V_{BE3}|$ are slightly less than the predicted value.

$$\text{error} = V_T \ln\frac{\beta}{\beta+1}.$$

11.6



For the noise due to M_1:

$$\frac{V_{n,out}}{R_1+g_{mN}^{-1}}\cdot\frac{1}{g_{mp}} = V_P, \quad \left(+\frac{V_P}{A_o}+V_{n,out}\right)g_{mN} = |I_{D1}|$$

$\Rightarrow$

$$\underbrace{\left(\frac{V_{n,out}}{R_1+g_{mN}^{-1}}\cdot\frac{1}{g_{mp}}\cdot\frac{1}{A_o}+V_{n,out}\right)g_{mN}}_{I_{D1}}\left(\frac{1}{g_{mp}}\right) = \frac{V_{n,out}}{R_1+g_{mN}^{-1}}\cdot\frac{1}{g_{mp}}+V_{n1}$$

$$\Rightarrow V_{n,out} = V_{n1}\,\frac{1}{\left(\frac{1}{R_1+g_{mN}^{-1}}\cdot\frac{1}{g_{mp}}\cdot\frac{1}{A_o}+1\right)\frac{g_{mN}}{g_{mp}} - \frac{1}{(R_1+g_{mN}^{-1})}\cdot\frac{1}{g_{mp}}},$$

where $\overline{V_{n1}^2} = 4kT\left(\dfrac{2}{3g_{mp}}\right) + \dfrac{K_{F,P}}{WLC_{ox}}\cdot\dfrac{1}{f}$

For the noise due to M_2:

$$\begin{cases} \dfrac{V_{n,out}}{R_1+g_{mN}^{-1}} \cdot \dfrac{1}{g_{mP}} + V_{n2} = V_P \\[2ex] \left(\dfrac{V_P}{A_0} + V_{n,out}\right) g_{mN} = |I_{D1}| \end{cases}$$

$$\Rightarrow \left[\left(\frac{V_{n,out}}{R_1+g_{mN}^{-1}} \cdot \frac{1}{g_{mP}} + V_{n2}\right)\frac{1}{A_0} + V_{n,out}\right] g_{mN}\left(\frac{1}{g_{mP}}\right) = \frac{V_{n,out}}{R_1+g_{mN}^{-1}} \cdot \frac{1}{g_{mP}} + V_{n2}$$

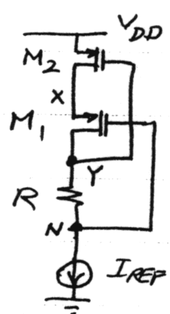
$$\Rightarrow V_{n,out}\left\{\left[\frac{1}{(R_1+g_{mN}^{-1})g_{mP}A_0} + 1\right] \times \frac{g_{mN}}{g_{mP}} - \frac{1}{R_1+g_{mN}^{-1}} \cdot \frac{1}{g_{mP}}\right\} = V_{n2}\left[1 - \frac{g_{mN}}{A_0\, g_{mP}}\right]$$

$$\Rightarrow V_{n,out} = V_{n2}\,\frac{1 - \dfrac{g_{mN}}{g_{mP}}}{\dfrac{g_{mN}/g_{mP}}{(R_1+g_{mN}^{-1})g_{mP}A_0} + \dfrac{g_{mN}}{g_{mP}} - \dfrac{1}{R_1+g_{mN}^{-1}} \cdot \dfrac{1}{g_{mP}}},$$

where $\overline{V_{n2}^2} = 4kT\left(\dfrac{2}{3g_{mP}}\right) + \dfrac{K_{F,P}}{WLC_{ox}} \cdot \dfrac{1}{f}$

The overall noise is obtained by adding the noise powers.

11.7 For M_1 to be in saturation, $R I_{REF} \le |V_{TH1}|$.



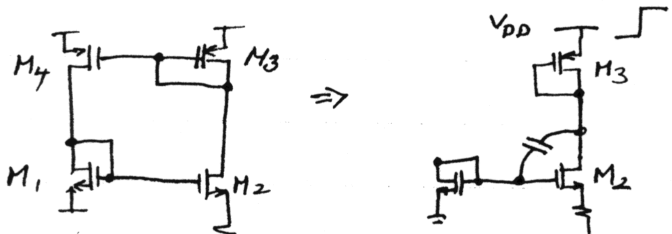
For M_2 to be in saturation,

$$V_N + |V_{GS1}| \le \underbrace{V_{DD} - |V_{GS2}|}_{V_Y = V_N + RI_{REF}} + |V_{TH2}|$$

$$\Rightarrow |V_{GS1}| \le R I_{REF} + |V_{TH2}|$$

$$\Rightarrow |V_{GS1}| - |V_{TH2}| \le R I_{REF} \le |V_{TH1}|$$

11.8



When V_{DD} rises, M_3 turns on because the gate-drain overlap capacitance of M_2 must charge. The current flowing thru this capacitance may increase the gate voltage of M_2 sufficiently, turning this transistor on as well. When M_3 turns on, M_4 also turns on.

11.9 $\dfrac{\partial V_{BE}}{\partial T} = \dfrac{V_{BE} - E_g/q}{T} - (4+m)\dfrac{k}{q}$

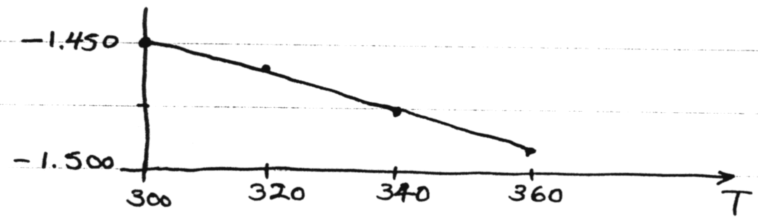
As T increased, V_{BE} drops. Thus, the TC becomes more negative.
We can sketch the behavior by a piecewise linear approximation.

$T = 300°K$, $V_{BE} \approx 750\ mV \Rightarrow TC = -1.45\ mV/°K$

$T = 320°K$, $V_{BE} \approx 750 - 20(1.45) = 721\ mV \Rightarrow TC = -1.46\ mV/°K$

$T = 340°K$, $V_{BE} \approx 721 - 20(1.46) = 692\ mV \Rightarrow TC = -1.476\ mV/°K$

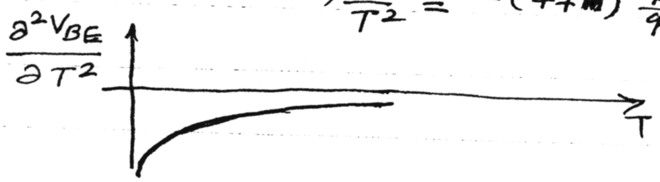
$T = 360°K$, $V_{BE} \approx 692 - 20(1.476) = 662\ mV \Rightarrow TC = -1.489\ mV/°K$



11.10 $\dfrac{\partial^2 V_{BE}}{\partial T^2} = \dfrac{(\partial V_{BE}/\partial T)T - (V_{BE} - \frac{E_g}{q})}{T^2} = \dfrac{1}{T}\dfrac{\partial V_{BE}}{\partial T} - \dfrac{1}{T^2}(V_{BE} - \dfrac{E_g}{q})$

$= \dfrac{V_{BE} - (4+m)V_T - E_g/q}{T^2} - \dfrac{1}{T^2}V_{BE} + \dfrac{1}{T^2}\dfrac{E_g}{q}$

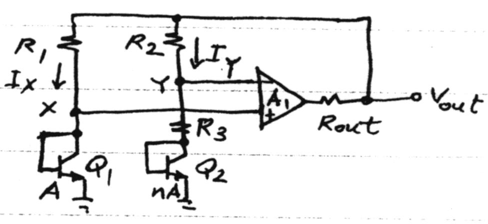
$= -(4+m)\dfrac{V_T}{T^2} = -(4+m)\dfrac{k}{q}\cdot\dfrac{1}{T}$



11.11 $V_Y - V_X = R_3 I_Y - V_T \ln n$

$\Rightarrow -A_1(R_3 I_Y - V_T \ln n) - R_{out}(2 I_Y) = V_{out}$

Assumed $I_X \approx I_Y$ here.



we also note that : $I_Y = \dfrac{V_{out} - V_{BE2}}{R_2 + R_3}$.

Thus, $V_{out} = \dfrac{(R_2+R_3)(V_T \ln n)A_1 + (A_1 R_3 + 2R_{out})V_{BE2}}{R_2 + 2R_{out} + A_1 R_3 + R_3}$

Dividing the numerator and denominator by $A_1 R_3$ and assuming $\frac{R_3+R_2+2R_{out}}{A_1 R_3} \ll 1$, we have:

$$V_{out} \cong \left[\left(1+\frac{R_2}{R_3}\right)V_T \ln n + V_{BE} + \left(\frac{2R_{out}}{A_1 R_3} V_{BE}\right)\right]\left(1-\frac{R_3+R_2+2R_{out}}{A_1 R_3}\right)$$

The error is then equal to:

$$\frac{2R_{out}}{A_1 R_3} V_{BE} - \frac{R_3+R_2+2R_{out}}{A_1 R_3}\left[\left(1+\frac{R_2}{R_3}\right)V_T \ln n + V_{BE}\right]$$

11.12 $R_3 = 1\,k\Omega$ $I_{R3} = 50\,\mu A$ $R_1 = R_2$

$$V_{out} = V_{BE2} + (V_T \ln n)\left(1+\frac{R_2}{R_3}\right) = 1.25\,V, \qquad V_{BE2} \approx 750\,mV$$

$$I_{R3} = \frac{V_{out}-V_{BE2}}{R_2+R_3} = \frac{(V_T \ln n)\left(1+\frac{R_2}{R_3}\right)}{R_2+R_3} = 50\,\mu A$$

$$(\ln n)\left(1+\frac{R_2}{R_3}\right) \cong 17.2 \quad \Rightarrow R_2 = 7.944\,k\Omega, \quad \Rightarrow n \approx 6.84.$$

Some iteration is usually necessary to arrive at an integer n. (Of course, the current thru R_3 will be slightly different from 50 μA.)

11.13 $I_{C1} = I_{C2} = 100\,\mu A$ $I_{C3} = I_{C4} = 50\,\mu A$ $R_1 = 1\,k\Omega$.

V_{DD} must be equal to 3 V.

Since $V_{out} \cong 2.5\,V$, M_2 and hence M_1 must be sized such that they remain in saturation.

$$\begin{cases} V_{BE3} + V_{BE4} + \left(1+\frac{R_2}{R_1}\right)(2V_T)\ln(\overset{I_{C2}/I_{C3}}{m}\, n) \cong 2.5\,V \\ V_{out} - (V_{BE4}+V_{BE3}) = (50\,\mu A)(R_1+R_2) \end{cases}$$

The two unknowns here are R_2 and n. Since Q_3 and Q_4 are biased at a relatively low current, we assume

$V_{BE3} = V_{BE} \approx 700\,mV \Rightarrow \left(1+\frac{R_2}{R_1}\right)(2V_T)\ln(mn) \approx 1.8\,V$

From the second equation, $R_1 + R_2 \approx 36\,k\Omega$, $\Rightarrow R_2 = 35\,k\Omega$.

Dividing the numerator and denominator by $A_1 R_3$ and assuming $\frac{R_3+R_2+2R_{out}}{A_1 R_3} \ll 1$, we have:

$$V_{out} \cong \left[\left(1+\frac{R_2}{R_3}\right)V_T \ln n + V_{BE} + \left(\frac{2R_{out}}{A_1 R_3} V_{BE}\right)\right]\left(1 - \frac{R_3+R_2+2R_{out}}{A_1 R_3}\right)$$

The error is then equal to:

$$\frac{2R_{out}}{A_1 R_3} V_{BE} - \frac{R_3+R_2+2R_{out}}{A_1 R_3}\left[\left(1+\frac{R_2}{R_3}\right)V_T \ln n + V_{BE}\right]$$

11.12 $R_3 = 1\,k\Omega$ $I_{R3} = 50\,\mu A$ $R_1 = R_2$

$V_{out} = V_{BE2} + (V_T \ln n)\left(1+\frac{R_2}{R_3}\right) = 1.25\,V$, $V_{BE2} \approx 750\,mV$

$$I_{R3} = \frac{V_{out} - V_{BE2}}{R_2 + R_3} = \frac{(V_T \ln n)\left(1+\frac{R_2}{R_3}\right)}{R_2 + R_3} = 50\,\mu A$$

$$(\ln n)\left(1+\frac{R_2}{R_3}\right) \cong 17.2 \quad \Rightarrow R_2 = 7.944\,k\Omega,\ \Rightarrow n \approx 6.84.$$

Some iteration is usually necessary to arrive at an integer n. (Of course, the current thru R_3 will be slightly different from $50\,\mu A$.)

11.13 $I_{C1} = I_{C2} = 100\,\mu A$ $I_{C3} = I_{C4} = 50\,\mu A$ $R_1 = 1\,k\Omega$.

V_{DD} must be equal to 3 V.

Since $V_{out} \cong 2.5\,V$, M_2 and hence M_1 must be sized such that they remain in saturation.

$$\begin{cases} V_{BE3} + V_{BE4} + \left(1+\frac{R_2}{R_1}\right)(2V_T)\ln(\overset{I_{C2}/I_{C3}}{m}\,n) \cong 2.5\,V \\ V_{out} - (V_{BE4} + V_{BE3}) = (50\,\mu A)(R_1 + R_2) \end{cases}$$

The two unknowns here are R_2 and n. Since Q_3 and Q_4 are biased at a relatively low current, we assume $V_{BE3} = V_{BE} \approx 700\,mV \Rightarrow \left(1+\frac{R_2}{R_1}\right)(2V_T)\ln(mn) \approx 1.8\,V$.

From the second equation, $R_1 + R_2 \approx 36\,k\Omega$, $\Rightarrow R_2 = 35\,k\Omega$.

From the first equation, $n \cong 1.31$.

Since $|V_{DS2}| \approx 0.5\ V$ with a 3-V supply, with $|I_{DS}| = 50\ \mu A$, we have $(W/L)_2 \geq 10.4$. with $I_{D1} = 2\ I_{D2}$, $(W/L)_1 = 2(W/L)_2$. Similarly, $(W/L)_3 = 2(W/L)_2$ and $(W/L)_4 = (W/L)_2$.

11.14 When we set (11.34) to zero, we obtain a relationship that is valid at only one temperature. Thus, (11.35) is only valid at one temperature and so is (11.36). In other words, the V_{BE} in (11.35) is at a single temperature T_0 whereas the V_{BE} in (11.33) is at a general temperature T. When we say $V_{REF} \to \frac{E_g}{q}$ if $T \to 0$, we really mean if V_{REF} is extrapolated, it reaches E_g/q.

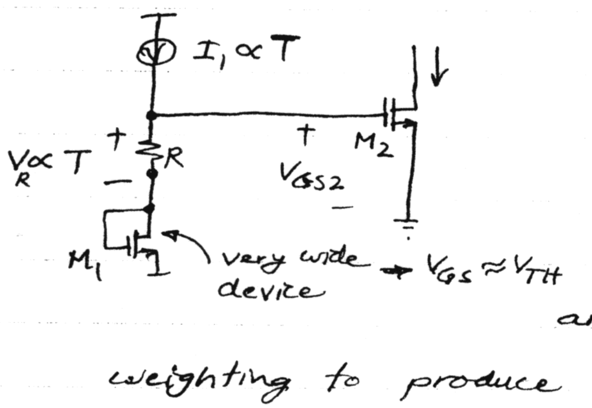
11.15 $\frac{\partial}{\partial T}(g_m R_D) = 0 \qquad g_m = \sqrt{2 \mu_n C_{ox} \frac{W}{L} I_D} \qquad \mu_n \propto T^{-3/2}$

Thus, $I_D \propto T^{3/2} \approx \alpha T + \beta T^2$

$$\left.\begin{aligned} &\Rightarrow T^{1/2} \approx \alpha + \beta T \\ \text{derivative} \to\ &\tfrac{3}{2} T^{1/2} \approx \alpha + 2\beta T \end{aligned}\right\} \text{using } T = 300^\circ K,\text{ we have}$$

$\alpha \approx 8.66$ and $\beta = 0.0289$.

$\Rightarrow I_D \propto \underbrace{8.66\,T}_{\text{PTAT}} + \underbrace{0.0289\,T^2}_{\text{PTAT}^2}$ (Note that the coefficient of T^2 is quite small.)



$V_{GS2} \cong V_R + V_{TH1} \Rightarrow$

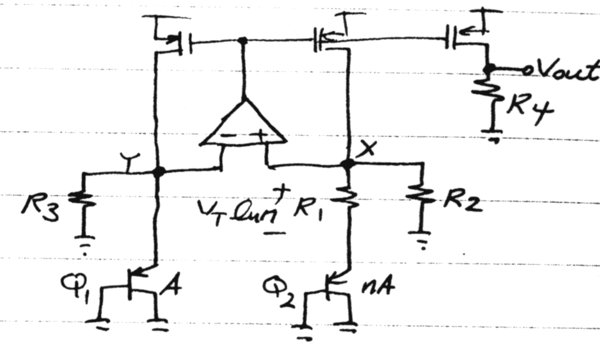
$I_{D2} = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L}\right)_2 (V_R)^2$

$\propto T^2$

This current and a PTAT current are simply added with proper weighting to produce $8.66T + 0.0289T^2$.

11.16 $g_m \propto T^{-3/4}$. Thus, $\frac{\partial R}{\partial T} = T^{3/4}$.

11.17 The current thru R_1 is PTAT and $V_X = V_Y = V_{BE1}/R_3$.



The current thru each PMOS device is $\frac{V_T \ln n}{R_1} + \frac{V_{BE1}}{R_3}$ and hence

$$V_{out} = R_4\left(\frac{V_T \ln n}{R_1} + \frac{V_{BE1}}{R_3}\right)$$
$$= \frac{R_4}{R_3} V_{BE1} + \frac{R_4}{R_1} V_T \ln n.$$

Since V_{BE1} is multiplied by R_4/R_3, the output voltage can be arbitrarily scaled.

11.18 $V_X = V_Y - V_{OS} \Rightarrow V_{REF} = V_{BE1}\frac{R_4}{R_3} + \frac{R_4}{R_1} V_T \ln n$

$$- \frac{R_4}{R_3}\left(1 + \frac{R_2}{R_1}\right) V_{OS}.$$

11.19 (a) When S_1 is on and S_2 is off, $V_{out} \approx V_T \ln \frac{I_1}{I_{S1}}$.

(b) when S_1 turns off and S_2 turns on,

$V_X = V_T \ln \frac{I_1 + I_2}{I_{S1}}$. This change is amplified by $\frac{C_2}{C_1}+1$ and added to the original voltage across C_2:

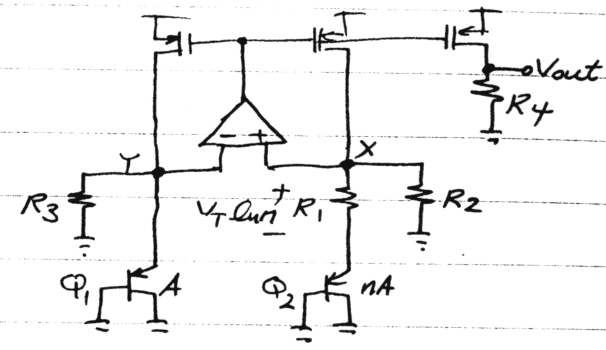
$$V_{out} = \left(1 + \frac{C_2}{C_1}\right)\left(V_T \ln \frac{I_1+I_2}{I_{S1}} - V_T \ln \frac{I_1}{I_{S1}}\right) + \underbrace{V_T \ln \frac{I_1}{I_{S1}}}_{V_{BE}}$$

$$= \left(1+\frac{C_2}{C_1}\right) V_T \ln \Big(\overbrace{1 + \frac{I_2}{I_1}}^{m}\Big) + V_{BE}$$

$$= \left(1+\frac{C_2}{C_1}\right) V_T \ln m + V_{BE}$$

11.16 $g_m \propto T^{-3/4}$. Thus, $\frac{\partial R}{\partial T} = T^{3/4}$.

11.17 The current thru R_1 is PTAT and $V_X = V_Y = V_{BE1}/R_3$.



The current thru each PMOS device is $\frac{V_T \ln n}{R_1} + \frac{V_{BE1}}{R_3}$ and hence

$$V_{out} = R_4\left(\frac{V_T \ln n}{R_1} + \frac{V_{BE1}}{R_3}\right)$$
$$= \frac{R_4}{R_3} V_{BE1} + \frac{R_4}{R_1} V_T \ln n.$$

Since V_{BE1} is multiplied by R_4/R_3, the output voltage can be arbitrarily scaled.

11.18 $V_X = V_Y - V_{OS} \Rightarrow V_{REF} = V_{BE1}\frac{R_4}{R_3} + \frac{R_4}{R_1} V_T \ln n - \frac{R_4}{R_3}\left(1 + \frac{R_2}{R_1}\right) V_{OS}$.

11.19 (a) When S_1 is on and S_2 is off, $V_{out} \approx V_T \ln \frac{I_1}{I_{S1}}$.

(b) when S_1 turns off and S_2 turns on, $V_X = V_T \ln \frac{I_1 + I_2}{I_{S1}}$. This change is amplified by $\frac{C_2}{C_1}+1$ and added to the orginal voltage across C_2:

$$V_{out} = \left(1 + \frac{C_2}{C_1}\right)\left(V_T \ln \frac{I_1 + I_2}{I_{S1}} - V_T \ln \frac{I_1}{I_{S1}}\right) + \underbrace{V_T \ln \frac{I_1}{I_{S1}}}_{V_{BE}}$$

$$= \left(1 + \frac{C_2}{C_1}\right) V_T \ln \overbrace{\left(1 + \frac{I_2}{I_1}\right)}^{m} + V_{BE}$$

$$= \left(1 + \frac{C_2}{C_1}\right) V_T \ln m + V_{BE}$$

(c) for zero TC: $(1+\frac{C_2}{C_1})\ln(1+\frac{I_2}{I_1}) \approx 17.2$.

11.20 $V_{out} = (1+\frac{C_2}{C_1})V_T \ln(1+\frac{I_2}{I_1}) + V_{BE}$

If $\frac{I_2}{I_1} = N+\varepsilon \Rightarrow V_{out} = (1+\frac{C_2}{C_1})V_T\ln(1+N+\varepsilon)+V_{BE}$

$$= (1+\frac{C_2}{C_1})V_T\left[\ln(1+N) + \ln(1+\frac{\varepsilon}{1+N})\right] + V_{BE}$$

$$\cong (1+\frac{C_2}{C_1})V_T\left[\ln m + \frac{\varepsilon}{1+N}\right] + V_{BE}$$

The error is thus equal to $(1+\frac{C_2}{C_1})V_T\frac{\varepsilon}{1+N}$.

11.21 $R_1 = 1\,k\Omega$, $R_2 = 2\,k\Omega$

(a) $V_{out} = \frac{V_T \ln n}{R_1}\cdot R_2 + V_{BE3} \Rightarrow \ln n \approx \frac{17.2}{2} = 8.6$

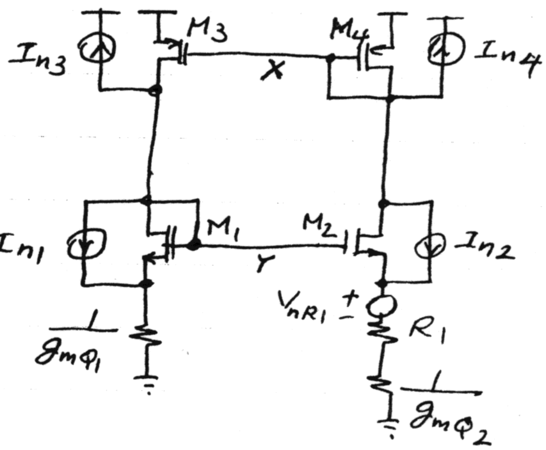
$\Rightarrow n = 5432$ (!)

Alternatively, we can scale $(W/L)_5$ up by a factor α such that: $V_{out} = \frac{V_T \ln n}{R_1}\alpha\cdot R_2 + V_{BE3}$.

For example, for $\alpha = 4$, $n = 8.58$.

(b) V_Y is given by:

$$\underbrace{(\underbrace{-g_{m3}V_X - I_{n3}}_{I_{D3}} - I_{n1})\frac{1}{g_{m1}}}_{\text{current thru } M_1} + \underbrace{(-g_{m3}V_X - I_{n3})}_{\text{current thru } \frac{1}{g_{mQ}}}\frac{1}{g_{mQ1}}$$



Also:

$$(-g_{m4}V_X - I_{n4})(R_1 + \frac{1}{g_{mQ2}}) + V_{nR1} + (\underbrace{-g_{m4}V_X - I_{n4} - I_{n2}}_{I_{D2}})\frac{1}{g_{m2}} = V_Y$$

Equating these, we have

$$V_X = \frac{1}{g_{m4}R_1}\left[I_{n3}(\frac{1}{g_{m1}}+\frac{1}{g_{mQ1}}) + \frac{I_{n1}}{g_{m1}} + I_{n4}(R_1+\frac{1}{g_{mQ2}}+\frac{1}{g_{m2}}) + \frac{I_{n2}}{g_{m2}} + V_{nR1}\right]$$

This noise is amplified by $g_{m5}(R_2+\frac{1}{g_{mQ3}})$ when it appears at the output.

$$\overline{V_{n,out,tot}^2} = \frac{g_{m5}^2 \left(R_2 + \frac{1}{g_{mQ3}}\right)^2}{(g_{m4} R_1)^2} \left[2\, I_{n3}^2 \left(\frac{1}{g_{m1}} + \frac{1}{g_{mQ1}}\right)^2 + \frac{2\, I_{n1}^2}{g_{m1}^2} + I_{n4}^2 R_1^2 + V_{nR_1}^2 \right]$$

$$+ I_{n5}^2 \left(R_2 + \frac{1}{g_{mQ3}}\right)^2 + V_{nR_2}^2$$

11.22 $f_{CK} = 50\,MHz$ power budget $= 1\,mW$. $g_{m1} = \frac{1}{500\Omega}$

$g_{m1} = \frac{2}{R_S}\left(1 - \frac{1}{\sqrt{K}}\right)$ $R_S = \frac{1}{f_{CK} C_S}$ $I_{D1} = I_{D2} = \frac{0.5\,mW}{3V}$

$= 167\,\mu A$

$$I_{out} = \frac{2}{\mu_n C_{ox} (W/L)_N} \cdot \frac{1}{R^2}\left(1 - \frac{1}{\sqrt{K}}\right)^2 = \frac{2}{\mu_n C_{ox} (W/L)_N} \left(\frac{g_{m1}}{2}\right)^2$$

$\Rightarrow \left(\frac{W}{L}\right)_N = 89.4$

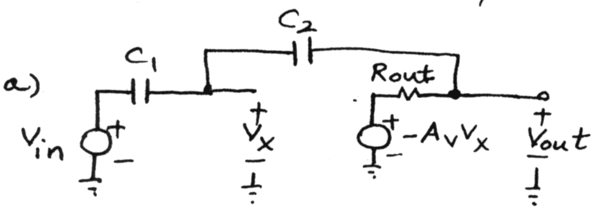
We assume $K = 4$. $\Rightarrow$ $\frac{1}{500\Omega} = \frac{2}{R_S}\left(1 - \frac{1}{2}\right) = \frac{1}{R_S}$

$\Rightarrow R_S = 500\,\Omega \Rightarrow C_S = 40\,pF$.

$\left(\frac{W}{L}\right)_2 = 4 \times 89.4$ For M_3 and M_4, there is some freedom so long as the transistors remain saturated. For example $\left(\frac{W}{L}\right)_3 = \left(\frac{W}{L}\right)_4 = 50$

Chapter 12

12.1 (a)



$$V_X = (V_{out} - V_{in})\frac{C_2}{C_1+C_2} + V_{in}$$

$$\frac{V_{out} - (-A_V V_X)}{R_{out}} = -\frac{V_{out} - V_X}{\frac{1}{C_2 s}} \Rightarrow \quad = \frac{C_2}{C_1+C_2} V_{out} - \frac{C_1}{C_1+C_2} V_{in}$$

$$\Rightarrow V_{out}\left[1 + \frac{A_V C_2}{C_1+C_2} + \frac{C_2}{C_1+C_2} R_{out} C_2 s\right] = V_{in}\left[A_V \frac{C_1}{C_1+C_2} + \frac{C_1}{C_1+C_2} R_{out} C_2 s\right]$$

$$\Rightarrow \frac{V_{out}}{V_{in}} = A_V \frac{C_1}{C_2} \frac{1 + R_{out} C_2 s}{1 + \frac{C_1}{C_2} + A_V + R_{out} C_2 s}$$

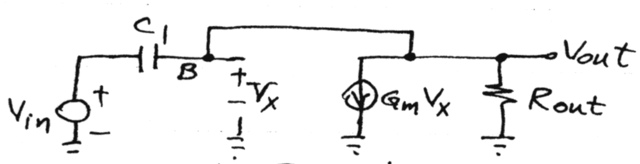
(b) $$A_V(s) = -\frac{R_F \| \frac{1}{C_2 s}}{\frac{1}{C_1 s}} = -\frac{R_F C_1 s}{R_F C_2 s + 1}$$

Nominal Gain $= 4$ $\qquad \dfrac{R_F C_1 \omega}{\sqrt{1 + R_F^2 C_2^2 \omega^2}} = 3.96 \qquad \omega = 2\pi(1\,MHz)$

$$(3.96^2 R_F^2 C_2^2 - R_F^2 C_1^2)\omega^2 + 3.96^2 = 0 \Rightarrow R_F^2 = \frac{3.96^2}{\omega^2(C_1^2 - 3.96^2 C_2^2)}$$

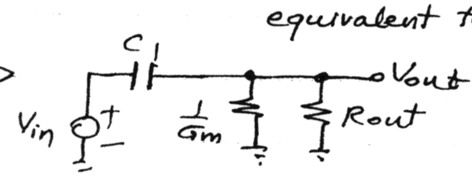
$\Rightarrow R_F = 2.23\ M\Omega$

12.2 (a)



equivalent to a resistor of $\frac{1}{G_m}$

$\Rightarrow$

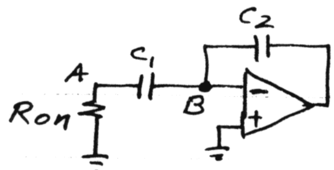


$$\frac{V_{out}}{V_{in}} = \frac{(\frac{1}{G_m} \| R_{out}) C_1 s}{(\frac{1}{G_m} \| R_{out}) C_1 s + 1} \approx \frac{(1/G_m) C_1 s}{\frac{C_1}{G_m} s + 1}$$

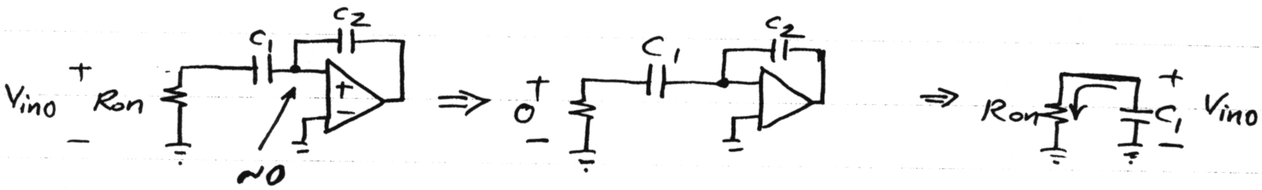
(b) $\omega = 2\pi\ (100\ MHz)$, $C_1 = 1\,pF$, $\frac{1}{G_m} = 100\ \Omega \Rightarrow \frac{C_1}{G_m}\omega = 0.0628$

$\Rightarrow \dfrac{V_{out}}{V_{in}} \approx 0.0628$, with a phase shift of nearly $90°$.

12.3



Since node B is at virtual ground, $\tau \approx R_{on} C_1$.



$\Rightarrow$ Total energy is that stored on C_1 $= \frac{1}{2} C_1 V_{ino}^2$

12.4 (a) $\left(\frac{W}{L}\right)_1 = \frac{20}{0.5}$, $C_H = 1\ pF$ $\quad I_{D,sat} = 20.8\ mA$

$\Rightarrow t_1 = 146\ ps$

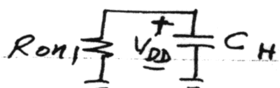
$$+1\ mV = \frac{2(2.3V)\exp\left[-(2.3V)\frac{\mu_n C_{ox} W/L}{C_H}(t-t_1)\right]}{1+\exp\left[-(2.3V)\frac{\mu_n C_{ox} W/L}{C_H}(t-t_1)\right]}$$

$$\Rightarrow \exp\left[-(2.3V)\frac{\mu_n C_{ox} W/L}{C_H}(t-t_1)\right] \approx \frac{+1\ mV}{2(2.3V)}$$

$\Rightarrow t - t_1 = 465\ ps \Rightarrow$ total time $= 611\ ps$

(b) $R_{on_1} = 55\Omega \Rightarrow \tau = 55\ ps$

$V_{out} = V_{DD} \exp\frac{-t}{\tau} \Rightarrow t = 440\ ps$

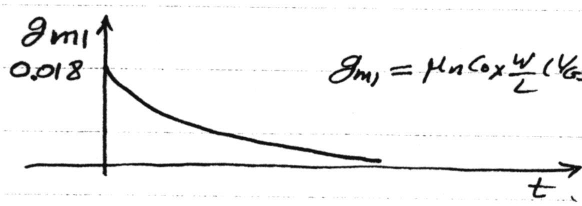


It underestimates the required time.

12.5 (a)
$$2.1 = 2.3 - \frac{1}{\frac{1}{2}\frac{\mu_n C_{ox}}{C_H}\frac{W}{L}t + \frac{1}{2.3}} \qquad (\gamma = 0)$$

$$\Rightarrow \frac{1}{2}\frac{\mu_n C_{ox}}{C_H}\frac{W}{L}t + \frac{1}{2.3} = 5 \quad \Rightarrow t \approx 1.16\ ns$$

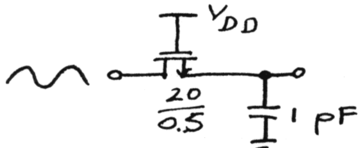
(b)



$g_{m1} = \mu_n C_{ox}\frac{W}{L}(V_{GS} - V_{TH})$

$\Rightarrow g_{m1}(t=0) = 0.018\ \mho$

12.6



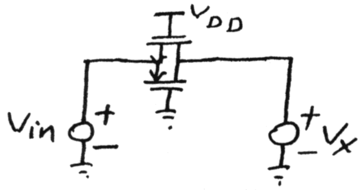
(a) $R_{on1} = 55\,\Omega$

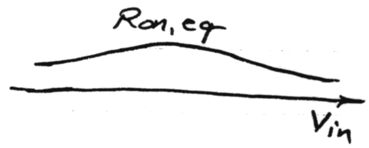
$|\theta| = \tan^{-1}(RC\omega)$

$= 1.98^\circ$

(b) $R_{on1} = 97.6\,\Omega \Rightarrow |\theta| \approx 3.96^\circ$

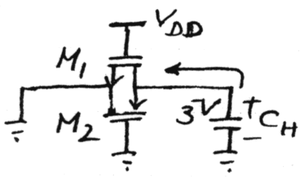
12.7





V_X is a voltage-dependent voltage source that follows V_{in} with a, say, 20-mV difference. We can then monitor the current drawn by either source, invert it, and normalize it to 20 mV in a dc sweep that varies V_{in} across the range of interest.

12.8



$R_{on1} = 55\,\Omega \quad R_{on2} = 64.3\,\Omega,\ C_H = 1\,pF$

$R_{on1} \| R_{on2} = 29.6\,\Omega \Rightarrow \tau = 29.6\,ps$

$\Rightarrow +1\,mV = +3V \exp\frac{-t}{\tau}$

$\Rightarrow t \approx 237\,ps.$

12.9 $V_{GS} - V_{TH} = 2.3V \Rightarrow V_{error} = \dfrac{WL_{eff}C_{ox}(V_{GS} - V_{TH})}{C_H}$

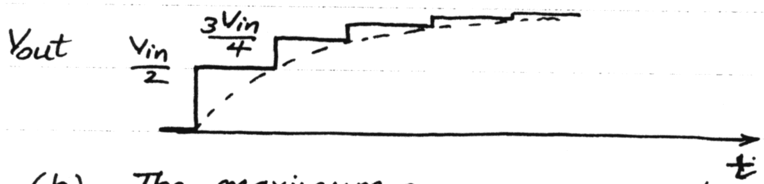
$= 60\,mV$

For clock feedthrough: $C_{ov} = (0.4 \times 10^{-11}\ F/m) \times 20\,\mu m = 0.08\,fF$

$V_{error} \cong \dfrac{C_{ov}}{C_H} V_{CK} = 0.24\,mV$

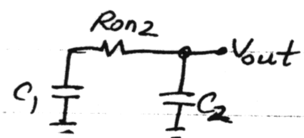
The overlap capacitance in Table 2.1 should actually be 0.4e-9 for NMOS. Thus, the error due to clock feedthrough will be about 24 mV, somewhat less than that due to worst-case charge injection.

12.10 (a) C_1 ~~together~~ with M_1 and M_2 can be viewed as a resistor. Thus, C_2 charges to $2V$ with an envelope given by $1-\exp\frac{-t}{\tau}$, where $\tau = \frac{1}{f_{CK} C_1} \cdot C_2$.



(b) The maximum error occurs when $V_{GS}-V_{TH}$ is maximum. If all of M_1 channel charge is injected onto C_1, then after V_{C_1} has reached V_{in} and M_1 turns off, V_{C_1} incurs an error equal to $(V_{GS}-V_{in} - V_{TH})WLC_{ox}/C_1$. When M_2 turns on, it absorbs some charge into its channel and when it turns off, it injects the charge back onto C_1 and C_2. Thus, only the charge due to M_1 need be considered. This error is divided equally between C_1 and C_2, yielding an overall output error of $\frac{WLC_{ox}}{2C_1}(V_{GS}-V_{in}-V_{TH})$.

(c) When M_1 turns off, a voltage equal to $\sqrt{\frac{kT}{C_1}}$ is stored across C_1. When M_2 is on, this voltage is distributed between C_1 and C_2. Moreover, M_2 itself produces thermal noise:

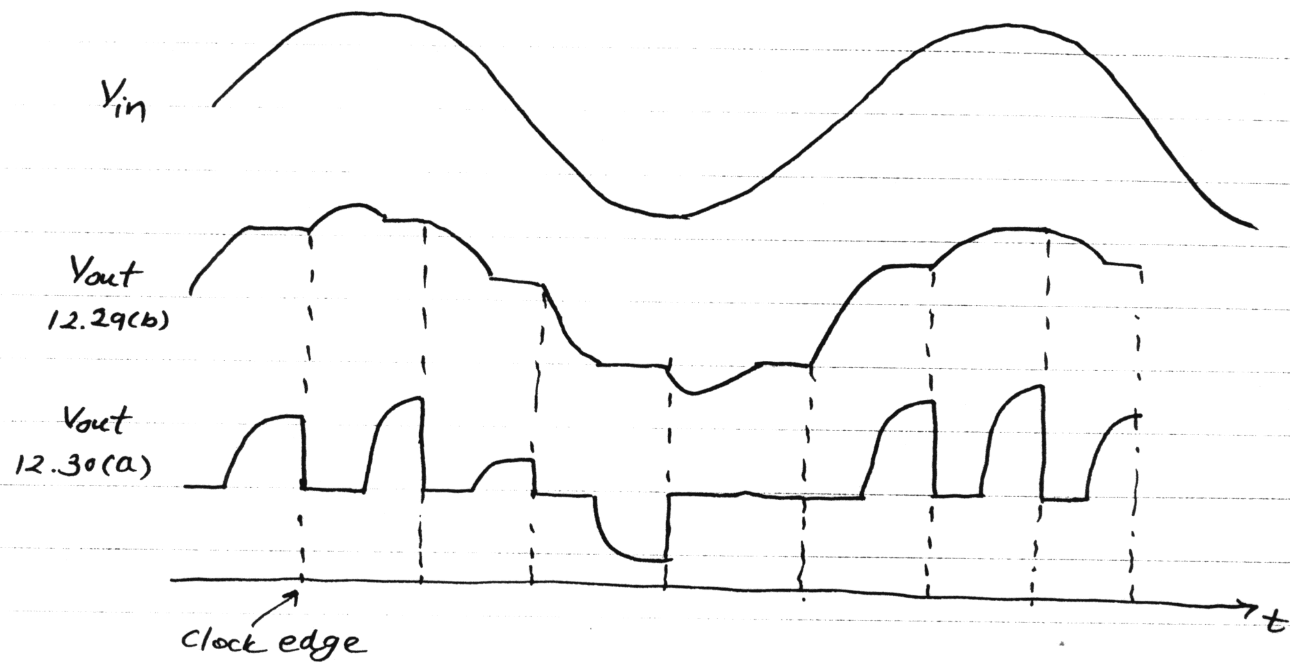


$$\Rightarrow \quad V_{n,out} = \sqrt{\frac{kT}{2C_1}}$$

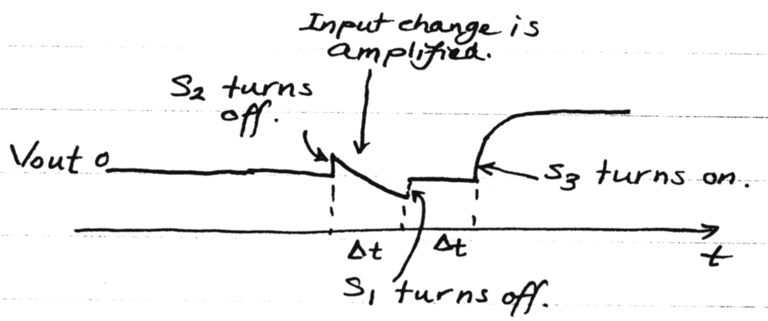
$$\Rightarrow \quad V_{n,out,tot}^2 = \frac{1}{4}\frac{kT}{C_1} + \frac{kT}{2C_1} = \frac{3kT}{4C_1}$$

$$\Rightarrow V_{n,out,tot} = \sqrt{\frac{3kT}{4C_1}}$$

12.5



12.12



12.13 Gain error $\approx (C_2 + C_1 + C_{in})/(C_2 A_{v_1}) = 0.01$

$$\Rightarrow 1 + \frac{C_1}{C_2} + \frac{C_{in}}{C_2} = 10 \Rightarrow 9C_2 = C_1 + C_{in} = 2.2\,\text{pF}$$

$$\Rightarrow C_2 = \frac{2.2\,\text{pF}}{9} \Rightarrow$$

$$\frac{C_1}{C_2} = 8.2 \rightarrow 8.0$$

12.14 $G_m = 100^{-1}\,℧$ $\quad \tau_{amp} = \dfrac{C_L C_{eq} + C_L C_2 + C_{eq} C_2}{G_m C_2}$ $\quad C_{eq} = C_1 + C_{in}$

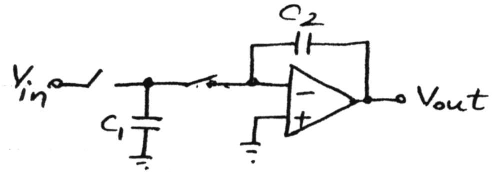
$$= \frac{C_{eq}}{G_m} \quad \text{because } C_L = 0$$

$$= 2\,\text{ns} \Rightarrow C_{eq} = 20\,\text{pF}$$

Since $C_{in} = 0.2\,\text{pF}$, $C_1 = 19.8\,\text{pF}$. Also, $9C_2 = 20\,\text{pF} \Rightarrow$

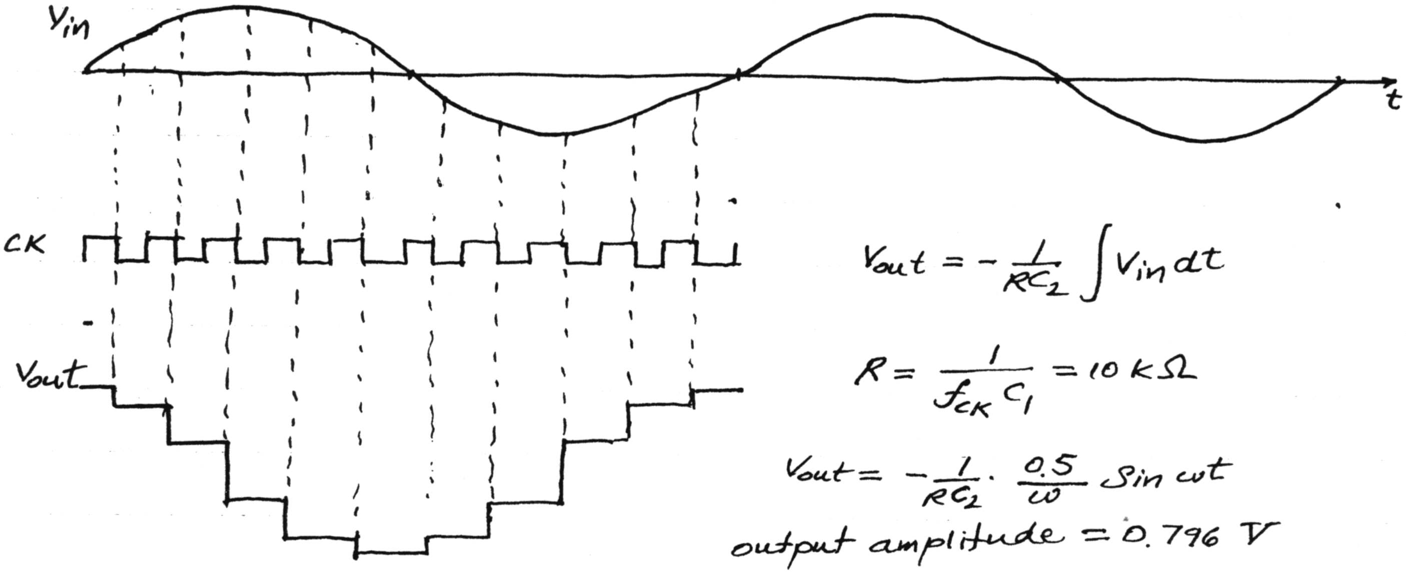
$\frac{C_1}{C_2} = 44$

12.15



$C_1 = C_2 = 1\ pF \quad f_{CK} = 100\ MHz$

$V_{in} = 0.5 \cos\left[2\pi(10\ MHz)t\right]$



$$V_{out} = -\frac{1}{RC_2}\int V_{in}\,dt$$

$$R = \frac{1}{f_{CK}\,C_1} = 10\,k\Omega$$

$$V_{out} = -\frac{1}{RC_2}\cdot\frac{0.5}{\omega}\sin\omega t$$

output amplitude $= 0.796\ V$

12.16 (a) The minimum level $= 1.5V - V_{TH1,2} \approx 0.8V$.

The maximum level places M_3 or M_4 at the edge of the triode region. $|V_{GS} - V_{TH}|_{3,4} = 0.421\ V \Rightarrow$ max. level $= 2.58V$.

$\Rightarrow$ Max. Swing $= 1.78\ V$.

(b) $A_{v,open} \approx g_{m1,2}(r_{o1} \| r_{o2}) = 27.3$

Gain Error $= \dfrac{C_1 + C_3}{C_3 A_v} = 18.3\%$

(c) $\tau_{amp} \approx \dfrac{C_1}{G_m} = 0.488\ ns$

12.17 (a) same.

(b) The gate-source cap is equal to $\frac{2}{3} W L_{eff} C_{ox} + W C_{ov}$

$\approx 44\ fF$

(The overlap cap in Table 2.1 must actually be 0.4e-9, in which case $C_{in} \cong 64\ fF$.)

The gate-drain overlap capacitance (of $M_{1,2}$) changes the gain equation because it appears in parallel with the feedback capacitor:

$$\frac{V_{out}}{V_{in}} \approx -\frac{C_1}{C_1 + W C_{ov}}\left(1 - \frac{C_3 + W C_{ov} + C_1 + C_{in}}{C_3 + W C_{ov}} \cdot \frac{1}{A_v}\right)$$

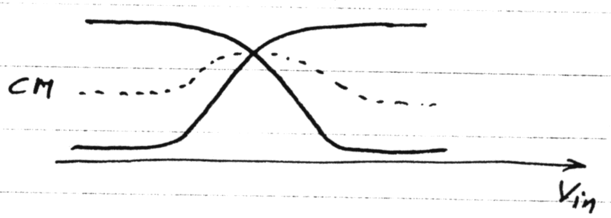
$$\approx -\frac{C_1}{C_3}\left(1 - \frac{W C_{ov}}{C_3}\right)\left(1 - \frac{C_3 + W C_{ov} + C_1 + C_{in}}{C_3 + W C_{ov}} \cdot \frac{1}{A_v}\right)$$

Thus, the gain error rises to $\frac{W C_{ov}}{C_3} + \frac{C_3 + C_1 + W C_{ov} + C_{in}}{C_3 + W C_{ov}} \cdot \frac{1}{A_v}$.

Assuming $C_{ov} = 0.4e{-}9$, we obtain a gain error of 22.2%.

(c) Neglecting the drain junction caps at the output, we have $\tau_{amp} \approx \frac{(C_1 + C_{in})}{G_m} \cong 0.503$ ns

12.18 Plotting the CM level, we see that it changes with the differential output.



This usually means that the CM feedback network, in particular the devices sensing the CM level, are quite nonlinear.

12.19 Since $I_{D5} = 1$ mA, $(V_{GS} - V_{TH})_5 = 319$ mV $\Rightarrow$ Minimum input level $= V_{GS1,2} + 319$ mV $\cong 1.245$ V (neglecting body effect.)

Since $I_{D6} = 50\,\mu A$, $(V_{GS} - V_{TH})_6 = 71.3$ mV $\Rightarrow$

$V_{out,CM} = 71.3\,mV + V_{TH6} + V_{GS5} \cong 1.79$ (neglecting body effect.)

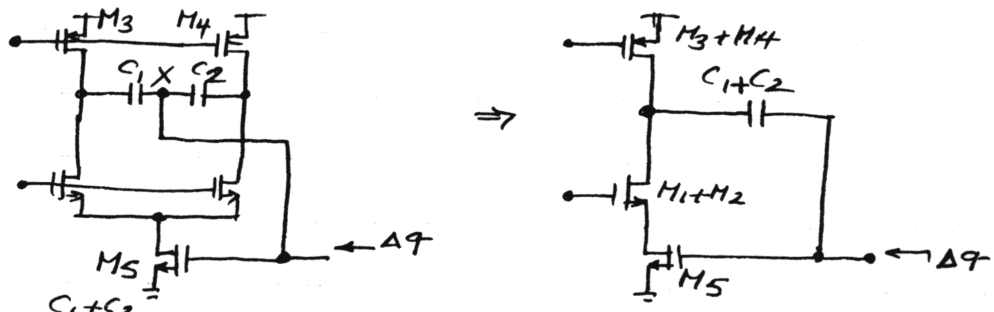
$\Rightarrow V_{in,max} = 1.79 + V_{TH1,2} \approx 2.49$ V

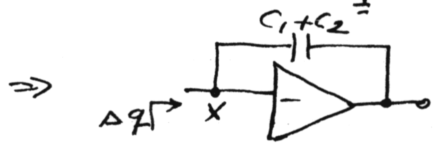
· 12.20 $V_{in,min}$ remains the same.

$$V_{in,max} = V_{GS6} + V_{GS5} + V_{TH1,2}$$

$$= 0.859 + 1.019 + 0.7 = 2.578\text{ V} \quad \text{(neglecting body effect)}$$

12.21



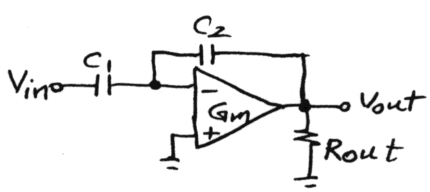


The charge resides on C_1+C_2 ⇒ output CM level changes by $\sim \frac{\Delta q}{C_1+C_2}$.

The voltage at node X changes by $\frac{\Delta q}{C_1+C_2}\cdot\frac{1}{A_V}$,

where $A_V = g_{m5}\left[r_{O3+4} \,||\, (g_{m1+2}\, r_{O1+2}\, r_{O5})\right]$.

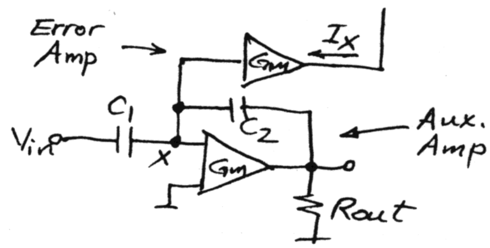
12.22 For a simple stage :

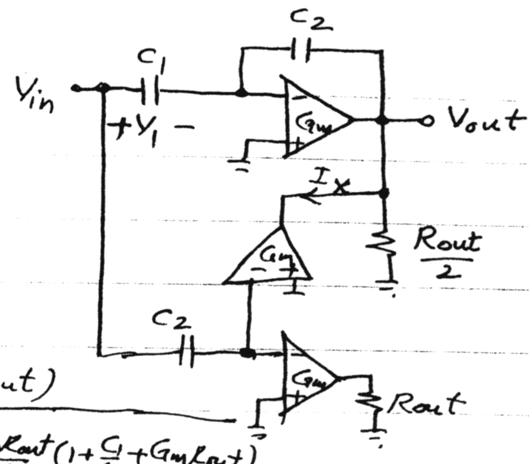


$$\frac{V_{out}}{V_{in}} = -\frac{C_1}{C_2}\,\frac{1}{1+\left(1+\frac{C_1}{C_2}\right)\frac{1}{G_m R_{out}}}$$

Thus, the voltage at node X and hence the current drawn by the error amplifier can be easily calculated.

$$I_X = \frac{C_1}{C_2}\,\frac{G_m}{1+\frac{C_1}{C_2}+G_m R_{out}}\cdot V_{in}$$





$$\left[-G_m(V_{in}-V_1)+\frac{G_m \frac{C_1}{C_2} V_{in}}{1+\frac{C_1}{C_2}+G_m R_{out}}\right]\frac{R_{out}}{2}=V_{out}$$

$$V_1=\frac{V_{in}-V_{out}}{1+\frac{C_1}{C_2}}$$

$$\Rightarrow \frac{V_{out}}{V_{in}}=-\frac{C_1}{C_2}\,\frac{\frac{G_m R_{out}}{2}\left(2+2\frac{C_1}{C_2}+G_m R_{out}\right)}{\left(1+\frac{C_1}{C_2}\right)\left(1+\frac{C_1}{C_2}+G_m R_{out}\right)+\frac{G_m R_{out}}{2}\left(1+\frac{C_1}{C_2}+G_m R_{out}\right)}$$

$$=-\frac{C_1}{C_2}\,\frac{1+2\frac{1+\frac{C_1}{C_2}}{G_m R_{out}}}{1+3\frac{1+C_1/C_2}{G_m R_{out}}+2\left(\frac{1+C_1/C_2}{G_m R_{out}}\right)^2}$$

$$=-\frac{C_1}{C_2}\,\frac{1+2X}{(1+X)(1+2X)} \qquad X=\frac{1+C_1/C_2}{G_m R_{out}}$$

$$=-\frac{C_1}{C_2}\,\frac{1}{1+\frac{1+C_1/C_2}{G_m R_{out}}}$$

Interestingly, the gain error is the same. But if the Gm stage in the error amplifier has a very high output impedance, then the load resistor of the main amplifier is Rout rather than Rout/2 and

$$\frac{V_{out}}{V_{in}}=-\frac{C_1}{C_2}\,\frac{1+2\frac{1+C_1/C_2}{G_m R_{out}}}{1+2\frac{1+C_1/C_2}{G_m R_{out}}+\left(\frac{1+C_1/C_2}{G_m R_{out}}\right)^2}$$

$$=-\frac{C_1}{C_2}\,\frac{1}{1+\frac{(1+C_1/C_2)^2/(G_m R_{out})^2}{\underbrace{1+2\frac{1+C_1/C_2}{G_m R_{out}}}_{\approx 1}}}$$

$$\cong-\frac{C_1}{C_2}\,\frac{1}{1+\left(\frac{1+C_1/C_2}{G_m R_{out}}\right)^2},$$ as if the open-loop gain of the amplifier is squared.

Chapter 13

13.1 $\quad y(t) = \alpha_1 x(t) + \alpha_2 x^2(t) \quad x = [0 \;\; x_{max}]$

(a) Straight line passing through the end points:

$$y_1 = \frac{\alpha_1 x_{max} + \alpha_2 x_{max}^2}{x_{max}} \cdot x = (\alpha_1 + \alpha_2 x_{max})\, x$$

$$y(t) - y_1 = -\alpha_2 x_{max} \cdot x + \alpha_2 x^2$$

$\Rightarrow$ Error is maximum at $x = \frac{x_{max}}{2}$ and equal to $-\frac{\alpha_2 x_{max}^2}{4}$. This value is usually normalized to the maximum output level.

(b)

$$y(t) = \alpha_1 \frac{x_{max} \cos \omega t}{2} + \alpha_1 \frac{x_{max}}{2} + \frac{\alpha_2}{4} x_{max}^2 \underbrace{\cos^2 \omega t}_{\frac{1+\cos 2\omega t}{2}} + \frac{\alpha_2}{4} 2 x_{max}^2 \cos \omega t + \frac{\alpha_2}{4} x_{max}^2$$

$\Rightarrow$ Fundamental: $\left(\frac{\alpha_1 x_{max}}{2} + \frac{\alpha_2}{2} x_{max}^2\right) \cos \omega t$

Second Harmonic: $\frac{\alpha_2}{8} x_{max}^2 \cos 2\omega t$

$$\Rightarrow \quad THD = \frac{\alpha_2^2 x_{max}^4 / 64}{\left(\frac{\alpha_1 x_{max}}{2} + \frac{\alpha_2}{2} x_{max}^2\right)^2}$$

$$= \frac{\alpha_2^2 x_{max}^2}{16(\alpha_1 + \alpha_2 x_{max})^2}$$

13.2 $\quad$ For Fig. 13.6 (a) : $\frac{A_{HD2}}{A_F} = \frac{V_m}{4(V_{GS} - V_{TH})}$ $\qquad V_{GS} - V_{TH} = 356\,mV$

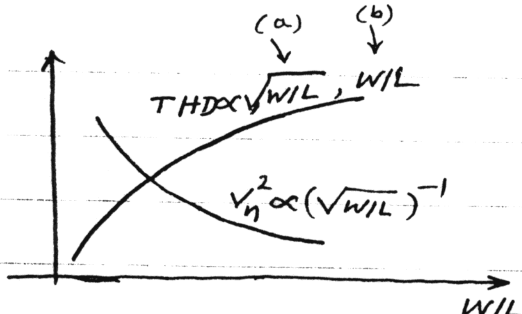
$$\Rightarrow \quad \frac{A_{H2}}{A_F} = 7\% \quad (-23\ dB)$$

For Fig. 13.6 (b) : $\frac{A_{HD3}}{A_F} \approx \frac{V_m^2}{32(V_{GS} - V_{TH})^2}$

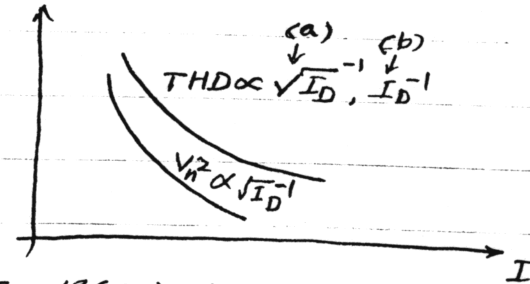
$$= 0.25\% \quad (-52\ dB)$$

- If we double W/L, $V_{GS} - V_{TH}$ is divided by $\sqrt{2}$.
  $\Rightarrow$ For (a), distortion goes up by $\sqrt{2}$ (3 dB) and for (b), by 2 (6 dB).
- If we double I, $V_{GS} - V_{TH}$ is multiplied by $\sqrt{2}$.
  $\Rightarrow$ Distortion goes down by $\sqrt{2}$ and 2 for (a) & (b), respectively.

13.3



Figs. 13.6 (a), (b)



Figs. 13.6(a), (b)

(Note that here THD is the ratio of voltages. If we take the ratio of powers, the relations must be squared.)

Increasing I and hence power dissipation decreases both the noise and the nonlinearity, whereas increasing W/L degrades the linearity while reducing the noise.

13.4 (1) As (W/L) is increased to increase the voltage gain, the linearity degrades (with a constant I).

(2) As I is increased to linearize the circuit, the load resistance must be decreased to maintain the same voltage headroom $\Rightarrow$ gain $\downarrow$.

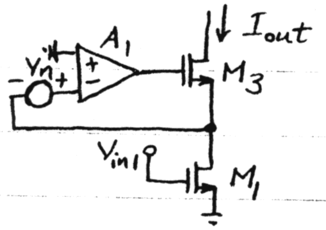
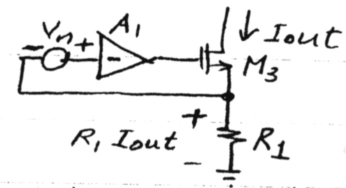
13.5 $$\frac{b}{a} = \frac{\alpha_2}{2} V_m \frac{1}{\alpha_1} \frac{1}{(1+\beta\alpha_1)^2}$$

$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS0} + V_m \cos\omega t - V_{TH})^2 \qquad V_{GS0}-V_{TH} = \text{overdrive with } I_D = 1mA$$

$$= \frac{1}{2}\mu_n C_{ox}\frac{W}{L}\left[V_m^2\cos^2\omega t + 2V_m\cos\omega t \cdot (V_{GS0}-V_{TH}) + (V_{GS0}-V_{TH})^2\right]$$

$$\Rightarrow \begin{cases} \left|\frac{\alpha_2}{\alpha_1}\right| = \frac{1}{2\cdot(V_{GS0}-V_{TH})} = 1.57\ V^{-1} \\ \alpha_1 = \mu_n C_{ox}\frac{W}{L}(V_{GS0}-V_{TH})R_D = 6286\ \mu A/V \times 2k\Omega = 12.57 \end{cases}$$

$$\Rightarrow \frac{b}{a} = 6.36\times 10^{-4}$$

13.6 
$\Rightarrow$ 

$$R_1^{-1} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 \left[2(V_{GS}-V_{TH}) - 2V_{DS}\right]$$

$$\left[(R_1 I_{out} + V_n)(-A_1) - R_1 I_{out}\right] g_{m3} = I_{out}$$

$$\Rightarrow \quad I_{out} = \frac{-g_{m3} A_1}{1 + g_{m3} R_1 A_1 + g_{m3} R_1} V_n \approx \frac{-A_1}{R_1(A_1+1)} V_n$$

$$\approx \frac{-1}{R_1} V_n$$

$$\Rightarrow |V_{n,in}| = \frac{\frac{1}{R_1} V_n}{g_{m1}}, \quad g_{m1} = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (2V_{DS})$$

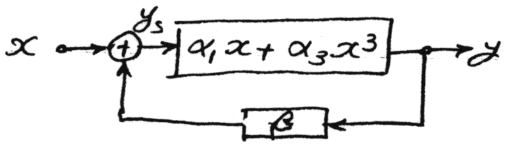
13.7 while increasing W/L raises the open-loop gain, it also makes the circuit more nonlinear (if I remains constant.) Since $\frac{W}{L}$ is multiplied by a factor of 4 $\Rightarrow \left|\frac{\alpha_2}{\alpha_1}\right| \uparrow$ by 2X, and $\alpha_1 \uparrow$ by 2X $\Rightarrow$

$$\frac{b}{a} = 4.32 \times 10^{-4}$$

13.8 $\beta\alpha_1 = 5.03 \Rightarrow$

$$\frac{b}{a} \approx \frac{\alpha_2}{\alpha_1} \cdot \frac{V_m}{2} \cdot \frac{1}{\beta^2 \alpha_1^2}$$

$$= 3.1 \times 10^{-4}$$

13.9 

Assume $y = a\cos\omega t + b\cos(3\omega t)$ and $x = V_m \cos\omega t$.

$$y_s = V_m \cos\omega t - \beta(a\cos\omega t + b\cos 3\omega t)$$

$$\Rightarrow y(t) = \alpha_1(V_m - \beta a)\cos\omega t - \alpha_1 \beta b \cos 3\omega t + \alpha_3 (V_m - \beta a)^3 \cos^3\omega t$$
$$- \alpha_3 \beta^3 b^3 \cos^3 3\omega t - 3\alpha_3 (V_m - \beta a)^2 \cos^2\omega t \cdot \beta b \cos 3\omega t$$
$$+ 3\alpha_3 (V_m - \beta a)\cos\omega t \cdot \beta^2 b^2 \cos^2 3\omega t.$$

Neglecting higher order terms: $a \approx \frac{\alpha_1}{1+\beta\alpha_1} V_m$, $V_m - \beta a \approx \frac{a}{\alpha_1}$

$$b \approx -\alpha_1 \beta b + \frac{\alpha_3}{4}(V_m - \beta a)^3$$

$$\Rightarrow \quad \frac{b}{a} \approx \frac{1}{4}\,\frac{\alpha_3}{\alpha_1}\,\frac{V_m^2}{(1+\beta\alpha_1)^3}$$

13.10 $\quad I_D = I_o \exp\dfrac{V_{GS}}{\zeta V_T} \qquad V_{GS} = V_{GSO} + V_m \cos\omega t$

$$\Rightarrow I_D = \left(I_o \exp\frac{V_{GSO}}{\zeta V_T}\right)\left[1 + \frac{V_m \cos\omega t}{\zeta V_T} + \frac{1}{2}\left(\frac{V_m \cos\omega t}{\zeta V_T}\right)^2 + \cdots\right]$$

If $V_m \ll \zeta V_T$, only second harmonic is significant: $\frac{1}{4}\left(\frac{V_m}{\zeta V_T}\right)^2 \cos 2\omega t$.

For the differential pair, $I_{D1} + I_{D2} = I_{SS}$, and

$V_{in} - V_{GS1} + V_{GS2} = 0 \Rightarrow \quad V_{in} = \zeta V_T \ln\dfrac{I_{D1}}{I_o} - \zeta V_T \ln\dfrac{I_{D2}}{I_o}$

$$= \zeta V_T \ln\frac{I_{D1}}{I_{D2}}$$

It follows that: $\quad I_{D1} = \dfrac{I_{SS}\exp[V_{in}/(\zeta V_T)]}{1+\exp[V_{in}/(\zeta V_T)]}$

$$I_{D2} = \frac{I_{SS}}{1+\exp[V_{in}/(\zeta V_T)]}$$

Thus, $\quad I_{D1} - I_{D2} = I_{SS}\tanh\dfrac{V_{in}}{2\zeta V_T} \qquad \tanh\varepsilon \approx \varepsilon - \dfrac{\varepsilon^3}{3}$

$$\approx I_{SS}\left[\frac{V_{in}}{2\zeta V_T} - \left(\frac{V_{in}}{2\zeta V_T}\right)^3\right]$$

If $V_{in} = V_{ino} + V_m\cos\omega t$, the third harmonic is given by $\quad -I_{SS}\dfrac{1}{(2\zeta V_T)^3} V_m^3 \dfrac{1}{4}\cos 3\omega t$.

13.11 $\quad I_D = \dfrac{1}{2}\,\dfrac{\mu_o C_{ox}}{1+\theta(V_{GS}-V_{TH})}\,\dfrac{W}{L}(V_{GS}-V_{TH})^2$

$$\approx \frac{1}{2}\mu_o C_{ox}\frac{W}{L}\left[1-\theta(V_{GS}-V_{TH})\right](V_{GS}-V_{TH})^2$$

If $V_{GS} = V_{GSO} + V_m\cos\omega t$, then the third harmonic is given by $\quad \frac{1}{2}\mu_o C_{ox}\frac{W}{L}(-\theta)\frac{V_m^3}{4}\cos 3\omega t$.

13.12 (a) $\Delta V_{TH} = \dfrac{0.1\, t_{ox}}{\sqrt{WL}}$ $t_{ox} = 90\,\text{Å}$

$\Rightarrow W = 6.5\ \mu m$

(b) $THD = \dfrac{V_m^2}{32(V_{GS}-V_{TH})^2}$ $I_D = 1\,mA$ $\frac{W}{L} = \frac{6.5}{0.5}$

$\Rightarrow V_{GS}-V_{TH} = 1.07\ V$

$\Rightarrow V_{m,max} = 0.61\ V$

13.13 (a) $W = 6.5\ \mu m \times \left(\dfrac{5\ mV}{2\ mV}\right)^2 = 41\ \mu m.$

(b) $V_{GS}-V_{TH} = 1.07\,V \times \sqrt{\dfrac{6.5}{41}} = 0.426\ V$

$\Rightarrow V_{m,max} = 0.61 \times \sqrt{\dfrac{6.5}{41}} = 0.243\ V$

We see a trade-off between input offset and nonlinearity (if the channel length remains constant.)

13.14 $\left|\dfrac{\Delta I_D}{I_D}\right| = \dfrac{2\,\Delta V_{TH}}{V_{GS}-V_{TH}} = 0.02 \Rightarrow \Delta V_{TH} = 5\ mV = \dfrac{0.1 \times 90\text{Å}}{\sqrt{WL}}$

$I_D = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)(V_{GS}-V_{TH})^2 \Rightarrow \frac{W}{L} = 29.9$

$\Rightarrow \begin{cases} L = 0.033\ \mu m \\ W = 0.984 \end{cases}$ But if $L_{min} \approx 0.5\ \mu m \Rightarrow \dfrac{W}{L} = \dfrac{15\ \mu m}{0.5\ \mu m}.$

13.15 $I_D R_S + \sqrt{\dfrac{2 I_D}{\mu_n C_{ox} W/L}} + V_{TH} = V_b$

Take the total differential of both sides and substitute $g_m = \dfrac{2 I_D}{V_{GS}-V_{TH}}$. Then, the result is obtained.

13.16 $y_1(t) = \alpha_1 A\cos\omega t + \alpha_2 A^2\cos^2\omega t + \alpha_3 A^3\cos^3\omega t$

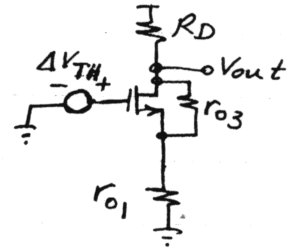
$y_2(t) = \alpha_1 A\cos(\omega t+\theta) + \alpha_2 A^2\cos^2(\omega t+\theta) + \alpha_3 A^3\cos^3(\omega t+\theta)$

The second harmonic arises from $\alpha_2 A^2\left[\cos^2\omega t - \cos^2(\omega t+\theta)\right]$

$= \alpha_2 A^2 \dfrac{\cos(2\omega t) - \cos(2\omega t + 2\theta)}{2}$

$= \dfrac{\alpha_2 A^2}{4} \sin\theta \, \sin(2\omega t)$

13.17 We calculate the output offset first. Viewing offset as noise, we have the following circuit:



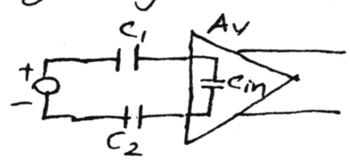
$\Rightarrow V_{out} = \Delta V_{TH} \dfrac{-g_{m3} r_{O3} R_D}{R_D + r_{O1} + r_{O3} + g_{m3} r_{O3} r_{O1}}$

This must be divided by the voltage gain, which for moderate R_D is given by $g_{m1}R_D \Rightarrow |V_{OS,in}| \approx \dfrac{\frac{g_{m3}}{g_{m1}} r_{O3}}{R_D + g_{m3} r_{O3} r_{O1}}$.

If $R_D \to \infty$, $|V_{out}| \to \Delta V_{TH} \cdot g_{m3} r_{O3}$. The voltage gain is obtained from Eq. (3.119) as $\sim g_{m1} r_{O1} g_{m3} r_{O3} \Rightarrow$

$|V_{OS,in}| \approx \dfrac{\Delta V_{TH}}{g_{m1} r_{O1}}$. This is why we usually neglect the offset contributed by cascode devices.

13.18 With a finite input capacitance, the gain of the circuit is no longer A_v.



$A_{v,tot} = \dfrac{C_1}{C_1 + 2C_{in}} \cdot A_v$

$\Rightarrow V_{OS,in} = \dfrac{V_{OS}}{\frac{C_1}{C_1+2C_{in}} \cdot A_v}$.

13.19 If W is doubled, the gain increases by approximately a factor of $\sqrt{2}$. Also, the input devices exhibit a smaller mismatch. For example, the threshold voltage mismatch decreases by a factor of $\sqrt{2}$. Thus, if the input devices dominate the offset, the overall input offset drops by a factor of 2.

13.20 To minimize the input offset, we <u>maximize</u> the overdrive of M_3 and M_4. But this limits the high level of the output swings.

Chapter 14

14.1

$$I_{D,scaled} = \frac{1}{2}\mu_n C_{ox} \frac{W/\alpha}{L/\alpha}\left(\frac{V_{GS}}{\alpha} - \frac{V_{TH}}{\alpha}\right)^2$$
$$= \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{TH})^2 \frac{1}{\alpha^2}$$

$$C_{ch,scaled} = \frac{W}{\alpha}\cdot\frac{L}{\alpha} C_{ox}$$
$$= \frac{1}{\alpha^2} WLC_{ox}$$

If the junction capacitances of S/D are neglected, then,

$$T_{d,scaled} = \frac{C/\alpha^2}{I/\alpha^2}\cdot\frac{V_{DD}}{\alpha}$$
$$= \left(\frac{C}{I}V_{DD}\right)\frac{1}{\alpha}.$$

Same as ideal scaling. But,

$$g_{m,scaled} = \mu C_{ox}\frac{W/\alpha}{L/\alpha}\;\frac{V_{GS}-V_{TH}}{\alpha}$$
$$= \mu C_{ox}\frac{W}{L}(V_{GS}-V_{TH})\cdot\frac{1}{\alpha}$$

14.2

$$W_{d,scaled} \approx \sqrt{\frac{2\varepsilon_{si}}{q}\Big(\underbrace{\frac{1}{N_A}}_{S/D}+\underbrace{\frac{1}{\alpha N_D}}_{Sub.}\Big)\frac{V_R}{\alpha}} \qquad N_A \gg \alpha N_D$$

$$\approx \frac{1}{\sqrt{\alpha}}\sqrt{\frac{2\varepsilon_{si}}{q}\frac{1}{N_A}V_R}$$

The depletion region capacitance per unit area therefore increases by $\sqrt{\alpha}$ rather than α. The series resistance increases.

DIBL arises primarily from the depletion region in the substrate rather than in the drain. Thus, DIBL remains relatively constant.

14.3 (a) Since $V_{n,rms} = \sqrt{\frac{kT}{C}}$, the capacitors must increase by a factor of 4.

(b) G_m should increase by a factor of 4.

(c) For square-law devices, W/L and bias current must increase by a factor of 4. $\Rightarrow$ Power increases by a factor of 2.

(d) $SR = I/C \Rightarrow$ Bias current must increase by a factor of 4.

14.4

$$I_D = \mu C_d \frac{W}{L} V_T^2 \left(\exp \frac{V_{GS}-V_{TH}}{\xi V_T}\right)\left(1-\exp \frac{-V_{DS}}{V_T}\right)$$

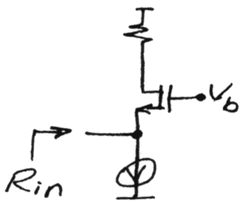
$C_d = \sqrt{\varepsilon_{si} q N_{sub}/(4\Phi_B)}$ $\quad N_{sub} \to \alpha N_{sub}$, $\Phi_B \sim$ constant.

$V_{GS}-V_{TH} \to \dfrac{V_{GS}-V_{TH}}{\alpha}$ $\qquad \xi = 1+\dfrac{C_d}{C_{ox}} \to 1+\dfrac{\sqrt{\alpha}}{\alpha}\dfrac{C_d}{C_{ox}}$

$V_{DS} \to \dfrac{V_{DS}}{\alpha}$

$S_{scaled} = 2.3 V_T \left(1+\dfrac{\sqrt{\alpha} C_d}{\alpha C_{ox}}\right) \Rightarrow$ $S\downarrow$, i.e., subthreshold behavior improves.

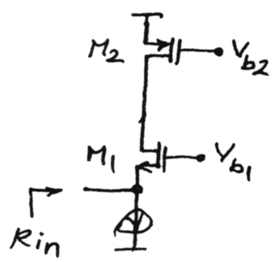
14.5



$$R_{in} = \frac{1}{\sqrt{\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{TH})}} = 50\,\Omega$$

$$R_{in,scaled} = \frac{1}{\sqrt{\alpha \frac{\alpha}{\alpha}\frac{1}{\alpha}}} \times 50\,\Omega = 50\,\Omega$$

14.6



$$R_{in} = \frac{r_{o2}+r_{o1}}{1+(g_{m1}+g_{mb1})r_{o1}} \approx \frac{r_{o2}+r_{o1}}{(g_{m1}+g_{mb1})r_{o1}}$$

$$R_{in,scaled} \cong \frac{r_{o2}+r_{o1}}{(g_{m1}+g_{mb1\,scaled})r_{o1}}$$

$$g_{mb1,scaled} = \frac{\gamma_{scaled}}{2\sqrt{2\Phi_F+\frac{V_{sub}}{\alpha}}}\, g_{m1} \qquad \gamma_{scaled} = \frac{\sqrt{2q\varepsilon\,\alpha N_{sub}}}{\alpha C_{ox}}$$

If $\dfrac{V_{sub}}{\alpha} \gg 2\Phi_F \Rightarrow g_{mb1,scaled} = g_{mb1}$.

14.7

$$\left.\frac{g_m}{I_D}\right|_{sat.} = \frac{\sqrt{2\mu_n C_{ox}\frac{W}{L} I_D}}{I_D} = \sqrt{\frac{2\mu_n C_{ox}\frac{W}{L}}{I_D}}$$

$$\left.\frac{g_m}{I_D}\right|_{sub.} \cong \frac{\frac{I_D}{\xi V_T}}{I_D} = \frac{1}{\xi V_T}$$

The two are equal at $I_D = \dfrac{2\mu_n C_{ox}\frac{W}{L}}{(\xi V_T)^2}$.

14.4

$$I_D = \mu C_d \frac{W}{L} V_T^2 \left(\exp \frac{V_{GS}-V_{TH}}{\xi V_T}\right)\left(1-\exp \frac{-V_{DS}}{V_T}\right)$$

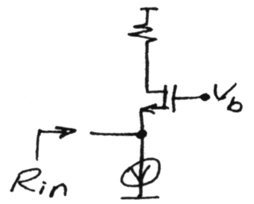
$C_d = \sqrt{\varepsilon_{si} q N_{sub}/(4\Phi_B)}$ $\quad N_{sub} \to \alpha N_{sub}$, $\Phi_B \sim$ constant.

$V_{GS}-V_{TH} \to \dfrac{V_{GS}-V_{TH}}{\alpha}$ $\qquad \xi = 1+\dfrac{C_d}{C_{ox}} \to 1+\dfrac{\sqrt{\alpha}}{\alpha}\dfrac{C_d}{C_{ox}}$

$V_{DS} \to \dfrac{V_{DS}}{\alpha}$

$S_{scaled} = 2.3 V_T \left(1+\dfrac{\sqrt{\alpha}\, C_d}{\alpha\, C_{ox}}\right) \Rightarrow S\downarrow$, i.e., subthreshold behavior improves.

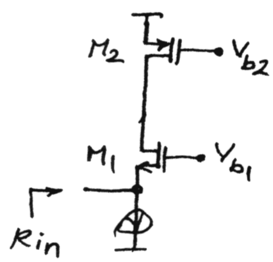
14.5



$$R_{in} = \frac{1}{\sqrt{\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{TH})}} = 50\,\Omega$$

$$R_{in,scaled} = \frac{1}{\sqrt{\alpha\frac{\alpha}{\alpha}\frac{1}{\alpha}}} \times 50\,\Omega = 50\,\Omega$$

14.6



$$R_{in} = \frac{r_{o2}+r_{o1}}{1+(g_{m1}+g_{mb1})r_{o1}} \approx \frac{r_{o2}+r_{o1}}{(g_{m1}+g_{mb1})r_{o1}}$$

$$R_{in,scaled} \cong \frac{r_{o2}+r_{o1}}{(g_{m1}+g_{mb1\,scaled})r_{o1}}$$

$$g_{mb1,scaled} = \frac{\gamma_{scaled}}{2\sqrt{2\Phi_F+\frac{V_{sub}}{\alpha}}}\, g_{m1} \qquad \gamma_{scaled} = \frac{\sqrt{2q\varepsilon\,\alpha N_{sub}}}{\alpha C_{ox}}$$

If $\dfrac{V_{sub}}{\alpha} \gg 2\Phi_F \Rightarrow g_{mb1,scaled} = g_{mb1}$.

14.7

$$\left.\frac{g_m}{I_D}\right|_{sat.} = \frac{\sqrt{2\mu_n C_{ox}\frac{W}{L} I_D}}{I_D} = \sqrt{\frac{2\mu_n C_{ox}\frac{W}{L}}{I_D}}$$

$$\left.\frac{g_m}{I_D}\right|_{sub.} \cong \frac{\frac{I_D}{\xi V_T}}{I_D} = \frac{1}{\xi V_T}$$

The two are equal at $I_D = \dfrac{2\mu_n C_{ox}\frac{W}{L}}{(\xi V_T)^2}$.

14.8 Since $I = v \cdot Q$, if Q drops to zero, $v \to \infty$. But the velocity is limited to v_{sat}. Thus, at the pinch-off point, the charge density is not zero. Carriers reach their saturated velocity and shoot through the depletion region surrounding the drain.

14.9
$$I_D = \frac{1}{2}\frac{\mu_0 C_{ox}}{1+\theta(V_{GS}-V_{TH})}\frac{W}{L}(V_{GS}-V_{TH})^2$$

$$\frac{\partial I_D}{\partial V_{GS}} = \frac{1}{2}\mu_0 C_{ox}\frac{W}{L}\left[\frac{-\theta(V_{GS}-V_{TH})^2}{[1+\theta(V_{GS}-V_{TH})]^2} + \frac{2(V_{GS}-V_{TH})}{1+\theta(V_{GS}-V_{TH})}\right]$$

$$= \mu_0 C_{ox}\frac{W}{L}\frac{V_{GS}-V_{TH}}{1+\theta(V_{GS}-V_{TH})}\left[1-\frac{\frac{\theta}{2}(V_{GS}-V_{TH})}{1+\theta(V_{GS}-V_{TH})}\right]$$

$$= \mu_0 C_{ox}\frac{W}{L}\frac{V_{GS}-V_{TH}}{1+\theta(V_{GS}-V_{TH})}\frac{1+\frac{\theta}{2}(V_{GS}-V_{TH})}{1+\theta(V_{GS}-V_{TH})}$$

$$= \frac{2I_D}{V_{GS}-V_{TH}}\cdot\frac{1+\frac{\theta}{2}(V_{GS}-V_{TH})}{1+\theta(V_{GS}-V_{TH})}$$

For small overdrives, $g_m \to \frac{2I_D}{V_{GS}-V_{TH}}$. For large overdrives, $g_m \to \frac{I_D}{V_{GS}-V_{TH}}$.

14.10 Using the results of Prob. 14.9 and replacing θ with $\frac{\mu_0}{2v_{sat}L}+\theta$, we have:

$$g_m = \frac{I_D}{V_{GS}-V_{TH}}\,\frac{2+\left(\frac{\mu_0}{2v_{sat}L}+\theta\right)(V_{GS}-V_{TH})}{1+\left(\frac{\mu_0}{2v_{sat}L}+\theta\right)(V_{GS}-V_{TH})}$$

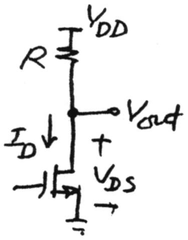
$$= \frac{I_D}{V_{GS}-V_{TH}}\left[1+\frac{1}{1+\left(\frac{\mu_0}{2v_{sat}L}+\theta\right)(V_{GS}-V_{TH})}\right]$$

14.11 $$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{TH})^2\left(1+\frac{\lambda}{1+kV_{DS}}V_{DS}\right)$$

$$r_o^{-1} = \frac{\partial I_D}{\partial V_{DS}} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{TH})^2\,\frac{\lambda V_{DS}}{(1+kV_{DS})^2}$$

$$\approx I_D\,\frac{\lambda V_{DS}}{(1+kV_{DS})^2}$$

$$\Rightarrow r_o = \frac{1}{\lambda\,\frac{I_D V_{DS}}{(1+kV_{DS})^2}}$$



$$I_D \approx \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{TH})^2\left[1+\lambda V_{DS}(1-kV_{DS})\right] \quad \text{if } kV_{DS}\ll 1$$
$$= \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{TH})^2(1+\lambda V_{DS}-\lambda k V_{DS}^2)$$

We note that the voltage across R_D $= V_{DD} - I_D R_D$
$= V_{DD} - \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{TH})^2(1+\lambda V_{DS}-\lambda k V_{DS}^2)R_D$.
Thus, even if V_{GS} changed by a very small value, the nonlinear dependence on V_{DS} results in nonlinearity in the voltage across R_D.

14.12 (a) $g_m = v_{sat} W C_{ox} \Rightarrow |A_v| \approx v_{sat} W C_{ox}$

(b) $|A_v| \approx \frac{g_{m1}}{g_{m3}} = \frac{v_{sat} W_1 C_{ox}}{v_{sat} W_3 C_{ox}} = \frac{W_1}{W_3}$.

14.13 $$g_{mb} = \frac{\partial I_D}{\partial V_{BS}} = \mu C_{ox}\frac{W}{L}\left(-\frac{2}{3}\gamma\right)\left(\frac{-\frac{3}{2}}{\sqrt{V_{DS}-V_{BS}+2\phi_F}} - \frac{-3/2}{\sqrt{-V_{BS}+2\phi_F}}\right)$$

$$= \mu C_{ox}\frac{W}{L}\,\gamma\,\frac{\sqrt{-V_{BS}+2\phi_F}-\sqrt{V_{DS}-V_{BS}+2\phi_F}}{\sqrt{V_{DS}-V_{BS}+2\phi_F}\sqrt{-V_{BS}+2\phi_F}}$$

14.14 $$\frac{\partial E_g}{\partial T} = -7.02\times10^{-4}\,\frac{2T(T+1108)-T^2}{(T+1108)^2}$$

$$= -7.02\times10^{-4}\,\frac{T^2+2216T}{(T+1108)^2}$$

For example, at $T=300\,°K$, $\frac{\partial E_g}{\partial T} = -0.267\ meV/°K$

For bandgap references, Eq. (11.10) must be modified:

$$\frac{\partial I_S}{\partial T} = b(4+m)T^{3+m}\exp\frac{-E_g}{kT} + bT^{4+m}\left(\exp\frac{E_g}{kT}\right)\left(\frac{E_g}{kT^2}-\frac{1}{kT}\frac{\partial E_g}{\partial T}\right)$$

$$\Rightarrow \frac{V_T}{I_S}\cdot\frac{\partial I_S}{\partial T} = (4+m)\frac{V_T}{T} + \left(\frac{E_g}{kT^2}-\frac{1}{kT}\frac{\partial E_g}{\partial T}\right)V_T$$

$$\Rightarrow \frac{\partial V_{BE}}{\partial T} = \frac{V_{BE}-(4+m)V_T}{T} - \left(\frac{E_g}{qT}-\frac{1}{q}\frac{\partial E_g}{\partial T}\right)$$

Thus, the TC of V_{BE} is slightly more positive.

14.15 (a) $|A_V| = \frac{g_{m1}}{g_{m2}} = \sqrt{\frac{\mu_n C_{ox}(\frac{W}{L})_1}{\mu_p C_{ox}(\frac{W}{L})_2}}$ $\Rightarrow |A_V|$ is highest for fast N, slow P, etc.

Input thermal noise voltage:

$\overline{v_n^2} = 4kT\frac{2}{3g_{m1}} + 4kT\frac{2}{3}\frac{g_{m2}}{g_{m1}^2}$ $\Rightarrow v_n$ is lowest for fast N, slow P, etc.

(b) $|A_V| = g_{m1}(r_{o1}\|r_{o2})$ $\Rightarrow |A_V|$ highest for fast N.

input noise: same as above.

14.16 (a) If V_{GS1} and V_{GS2} are constant $\Rightarrow g_m = \mu_n C_{ox}\frac{W}{L}(V_{GS}-V_{TH})$

$\Rightarrow |A_V| = \frac{\mu_n C_{ox}(\frac{W}{L})_1(V_{GS}-V_{TH})_1}{\mu_p C_{ox}(\frac{W}{L})_2(V_{GS}-V_{TH})_2}$ $\Rightarrow |A_V|$ highest for fast N, slow P, etc.

same result for thermal noise.

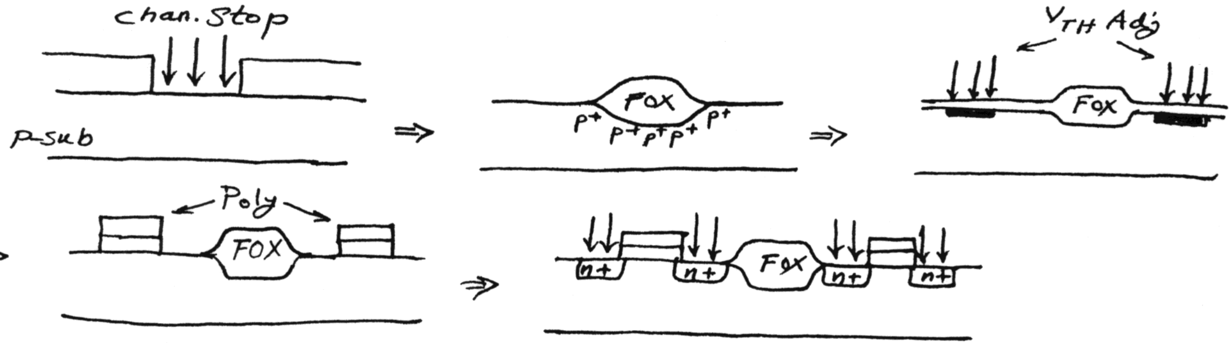
(b) $|A_V| = \mu_n C_{ox}(\frac{W}{L})_1(V_{GS}-V_{TH})_1\left[\frac{1}{\lambda_1\frac{1}{2}\mu_n C_{ox}(\frac{W}{L})_1(V_{GS}-V_{TH})_1^2}\,\Big\|\,\frac{1}{\frac{\lambda_2}{2}\mu_p C_{ox}(\frac{W}{L})_2(V_{GS}-V_{TH})_2^2}\right]$

$\Rightarrow |A|$ highest for fast N and slow P, etc.

Noise is as before.

Chapter 15

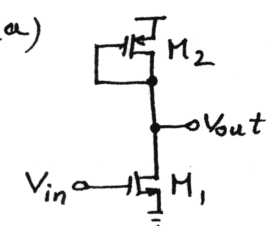
15.1 Simplifying the flow shown in Fig. 15.8, we note that n-well is not necessary.



The back-end processing is similar to that shown in Figs. 15.10 and 15.11. Thus, the process requires one fewer mask.

15.2 Since the dopants are not concentrated near the surface, their effect is less than expected. For example, if the implant aims to increase the threshold of an NFET from zero to 0.5 V, the actual value will be less than 0.5 V.

15.3 (a)



For M_1 in saturation:

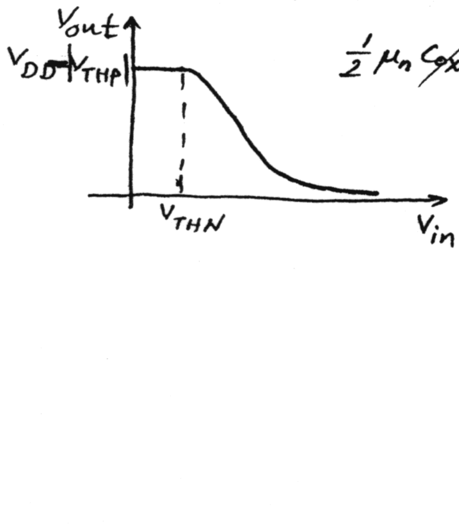
$$\frac{1}{2}\mu_n \cancel{C_{ox}}\left(\frac{W}{L}\right)_1 (V_{in}-V_{THN})^2 = \frac{1}{2}\mu_p \cancel{C_{ox}}\left(\frac{W}{L}\right)_2 (V_{out}-|V_{THP}|)^2$$

$\Rightarrow$ result independent of C_{ox}.

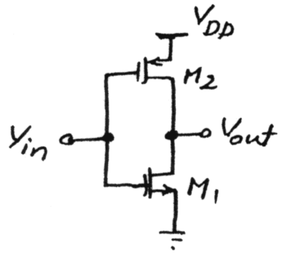
when M_1 enters the triode region:

$$\frac{1}{2}\mu_n \cancel{C_{ox}}\left(\frac{W}{L}\right)_1 \left[2(V_{in}-V_{THN})V_{out}-V_{out}^2\right] = \frac{1}{2}\mu_p \cancel{C_{ox}}\left(\frac{W}{L}\right)_2 (V_{out}-|V_{THP}|)^2$$

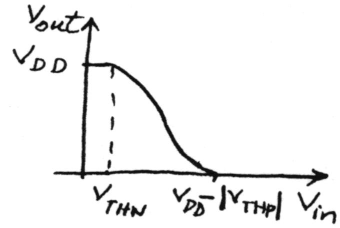
$\Rightarrow$ same.



(b)

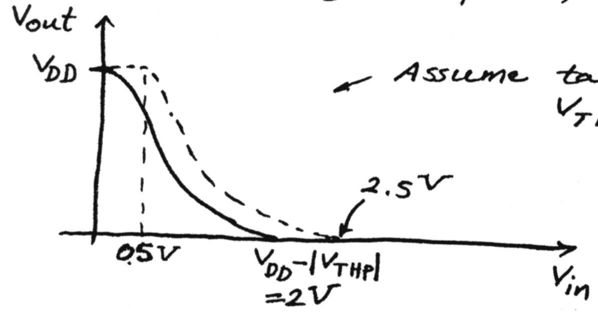


Using similar arguments, one can show that the result does not depend on C_{ox}.



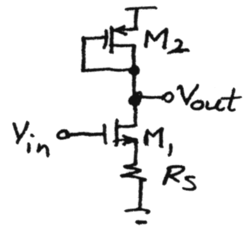
15.4 Without a threshold-adjust implant, $V_{THN} \approx 0$ and $V_{THP} \approx -1V$.

$V_{DD}=3V$



Assume target values are: $V_{THN}=+0.5\,V$, $V_{THP}=-0.5\,V$

15.5

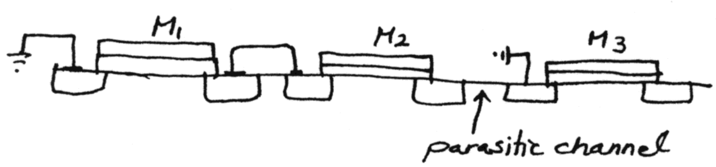


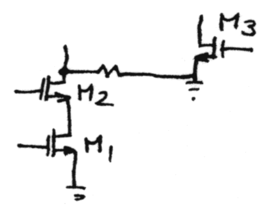
(a) Source of M_1 is spiked to the substrate, shorting R_S out.

(b) Drain of M_2 is spiked to its n-well.

15.6 (a) Channeling during S/D implant leads to deep junctions, intensifying DIBL. But the effect is not significant as far as the output impedance is concerned. (Just slightly lower.)

(b) With no channel-stop implant, it is possible that an unrelated high-voltage line passing over the field oxide between the transistors creates a channel between them.:

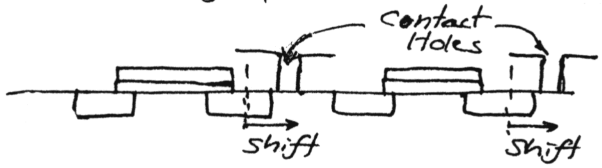




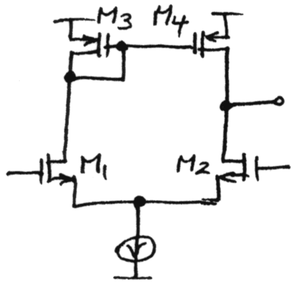
(c) Insufficient gate oxide growth typically does not degrade the output impedance.

15.7

The zero output current is probably caused by a contact misalignment.



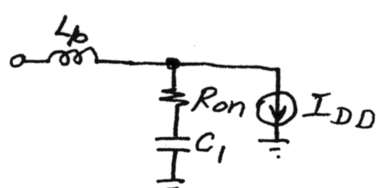
15.8



Long gate oxidation cycle is probably the reason: $A_V = g_{m1,2}(r_{O2} \| r_{O4})$, $g_{m1,2}$ is lower than expected. The output resistance remains constant or decreases as $t_{ox}\uparrow$.

15.9 (b) If the bottom plate of C_1 is heavily doped, then the oxide grows faster in C_1, leading to a smaller value for the capacitor. From Chapter 12, we note that if the input capacitance of the op amp is taken into account, then a lower value of C_1 yields a higher gain error.

15.10 (a) $R_{on} = \left[\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{GS}-V_{TH})\right]^{-1} = 11\,\Omega$



$C_1 = 100 \times 0.34 \times 3.84 + 100 \times 2 \times 0.4$

$= 210.6\,fF$

For critically-damped response: $R_{on} = 2\sqrt{\frac{L_b}{C_1}} \Rightarrow$
$L_b \leq 6.37\,pH$.

15.11 $\quad g_m r \dfrac{N(N+1)}{2} = 0.01$

$g_m = \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH}) = 1/(254\,\Omega)$

$\Rightarrow \quad r = 4.8\, m\Omega.$

15.12 $\quad t = 1\,\mu m,\ h = 3\,\mu m$ $\quad$ Parallel Plate $\propto \frac{W}{h}$, the remaining terms determine the fringe capacitance:

$\frac{W}{3} = 0.77 + 1.06\left(\frac{W}{3}\right)^{0.25} + 1.06\left(\frac{1}{3}\right)^{0.5}$

$\Rightarrow W \approx 8.25\,\mu m$

If $h = 5\,\mu m$, then:

$\frac{W}{8} = 0.77 + 1.06\left(\frac{W}{8}\right)^{0.25} + 1.06\left(\frac{1}{8}\right)^{0.5}$

$\Rightarrow W \approx 19.7\,\mu m$

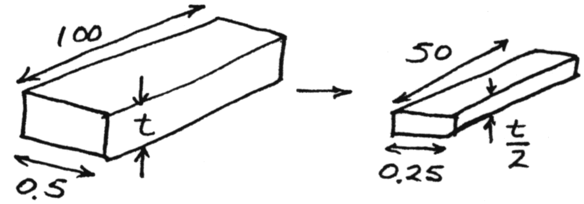
Chapter 16

16.1 $R_{\square,poly} = 30\ \Omega/\square \quad R_{\square,M1} = 80\ m\Omega/\square$

$$R_\square = \frac{\rho}{t} \Rightarrow \frac{\rho_{poly}}{\rho_{M1}} = \frac{R_{\square,poly} \times t_{poly}}{R_{\square,M_1} \times t_{M_1}} = \frac{30 \times 0.2}{0.08 \times 1.0}$$

$$= 75$$

16.2 $\frac{W}{L} = \frac{100}{0.5} \longrightarrow \frac{50}{0.25}$



The sheet resistivity increases by a factor of 2. Since the number of squares is constant, the total gate resistance also increases by a factor of 2.

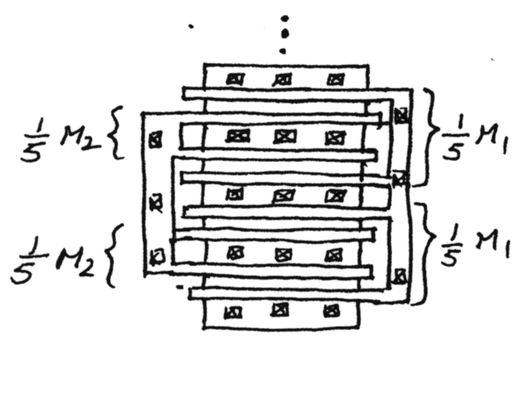
16.3 For a total gate resistance of $10\ \Omega$, suppose each device consists of N fingers each $\frac{100\ \mu m}{N}$ wide. The total gate resistance is then equal to

$$R_G = \left(\frac{200}{N}\right) \cdot \frac{1}{N} \cdot (5\ \Omega/\square)$$

$$= \frac{1000}{N^2}\ \Omega$$

$$\Rightarrow N = 10$$

From Fig. 16.13(c), a possible solution is:



16.4. A_1: a finite resistance may appear between the drains, degrading the voltage gain.

A_2: a large resistance may appear in series with the sources, introducing unwanted degeneration or, more importantly, input-referred offset.

A_3: Gate of NMOS current source on the bottom may be shorted to its source.

A_4: Part of contact hole may fall on FOX, increasing the contact resistance: source degeneration or offsets.

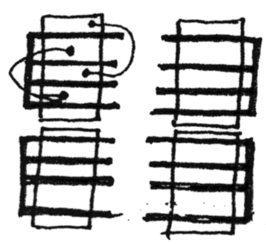
A_5: If the poly contact area is too close to the active area, the active area may be damaged during the etching of poly ⇒ offsets, even poor transistor operation.

A_6: Latch-up may occur.

A_7: Latch-up may occur.

A_8: A finite resistance may appear between the gates of the input transistors.

16.5. In principle, only two layers of interconnect are sufficient for any routing. However, for reasonable symmetry, interconnect resistance, and area, approximately four layers are needed here.



16.6 In Fig. 6.22, temp. gradients introduce threshold and mobility mismatch between M_{REF} and each of $M_1 - M_n$. Thus, the output currents suffer from additional mismatches.

In Fig. 6.23, temp. gradients have much less effect because M_{REF1} and M_{REF2} are quite close to their mirrors.

16.7 $R = 500\,\Omega \Rightarrow$ poly must be $\frac{500}{60}$ squares long and

n-well must be $\frac{500}{2000}$ square long.

Poly width $= 3\,\mu m \Rightarrow$ Poly length $= 25\,\mu m$

n-well length $= 6\,\mu m \Rightarrow$ n-well width $= 24\,\mu m$.

$\Rightarrow$ Poly Cap $= 3 \times 25 \times 100\ aF/\mu m^2 = 7.5\ fF$

n-well Cap $= 6 \times 24 \times 1000\ aF/\mu m^2 = 144\ fF$.

Thus, the poly structure is preferable.

16.8 Assuming $C_1 = C_2 = C_3 = 40\ aF/\mu m^2$ and $C_4 = 60\ aF/\mu m^2$ in Fig. 16.34(d), we have (neglecting fringe cap.):

Fig. 16.34(a): $C_1 = 40\ aF/\mu m^2$, $C_p = 9\ aF/\mu m^2$

(b): $C_1 + C_2 = 80\ aF/\mu m^2$, $C_p = 15\ aF/\mu m^2$

(c): $C_1 + C_2 + C_3 = 120\ aF/\mu m^2$, $C_p = 30\ aF/\mu m^2$

(d): $C_1 + \cdots + C_4 = 180\ aF/\mu m^2$, $C_p = 90\ aF/\mu m^2$

Thus, the lowest C_p/C occurs for (b).

16.9 Wire Propagation Delay $\approx \frac{R_{tot}\,C_{tot}}{2} = \frac{40 \times 37\ fF}{2}$

$= 0.74\ ps$

Lumped Delay $\approx 500\,\Omega \times 37\ fF$

$= 18.5\ ps$

Thus, the propagation delay thru the wire is negligible.

16.10 Wire Delay $\approx \dfrac{20\,\Omega \times 44\,aF}{2}$

$= 0.44\,ps$

Lumped Delay $\approx 22\,ps$

16.11 Metal 1 : $C_{tot} = (1000\,\mu m \times 0.35\,\mu m \times 30\,aF/\mu m^2) + 1000\,\mu m \times 80\,aF/\mu m$

$= 90.5\,fF$

Metal 2 : $C_{tot} = (1000\,\mu m \times 0.45\,\mu m \times 15\,aF/\mu m^2) + 1000\,\mu m \times 50\,aF/\mu m$

$= 56.75\,fF$

Metal 3 : $C_{tot} = (1000\,\mu m \times 0.5\,\mu m \times 9\,aF/\mu m^2) + 1000\,\mu m \times 40\,aF/\mu m$

$= 44.5\,fF$

Metal 4 : $C_{tot} = (1000\,\mu m \times 0.6\,\mu m \times 7\,aF/\mu m^2) + 1000\,\mu m \times 30\,aF/\mu m$

$= 34.2\,fF$

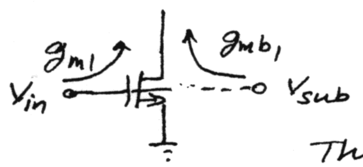
Thus, metal 4 provides the smallest delay.

16.12 The results do not change because the capacitance of metal 4 is still largest.

16.13 $(W/L)_1 = 100/0.5$, $I_{D1} = 1mA \Rightarrow g_{m1} = \sqrt{2 \times 1mA \times \dfrac{100}{0.34} \times 134\,\mu A/V^2}$

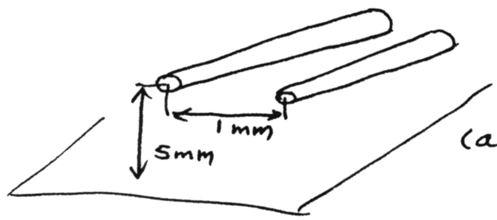
$= 8.88\,m\mho$

$$g_{mb1} = \frac{\gamma\, g_{m1}}{2\sqrt{\underbrace{V_{SB}}_{0} + |2\phi_F|}} = \frac{0.45}{2\sqrt{0.9}} \times 8.88\,m\mho = 2.11\,m\mho$$



V_{sub} generates a drain current of $g_{mb1} V_{sub}$. Thus, referred to the gate, the effect is equal to $\dfrac{g_{mb1}}{g_{m1}} = \dfrac{\gamma}{2\sqrt{|2\phi_F|}} = 4.21^{-1} \Rightarrow$ input-referred noise $= 11.9\,mV_{pp}$.

16.14.



(a) $L_m = 0.1 \ln\left[1+\left(\frac{10}{1}\right)^2\right] \times 4\text{ mm}$

$= 1.85\text{ nH}.$

(b) $V = L_m \frac{di}{dt}$

$= 1.85\text{ nH} \times 2\pi \times 10^8 \times 1\text{ mA}$

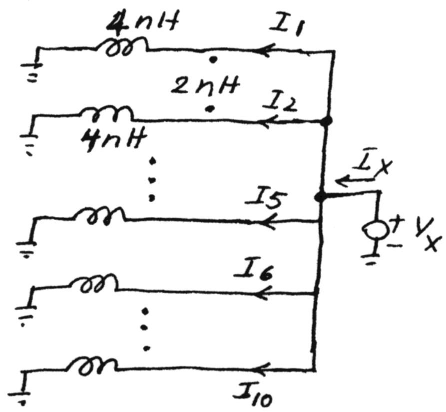
$= 1.16\text{ mV}_p.$

16.15 L_m must decrease by a factor of 4. $\Rightarrow$

$0.1 \ln\left[1+\left(\frac{2h}{d}\right)^2\right] \times 4\text{ mm} = \frac{1.85}{4}$

$\Rightarrow \frac{2h}{d} = 1.476 \Rightarrow d = 6.78\text{ mm}.$

16.16 (a)



By symmetry:

$I_1 = I_{10},\ I_2 = I_9,\ \cdots,\ I_5 = I_6$

we then construct equations for

$I_1 - I_5$

$$\begin{cases} (4\text{ nH})sI_1 + (2\text{ nH})sI_2 = V_x \\ (4\text{ nH})sI_2 + (2\text{ nH})sI_1 + (2\text{ nH})sI_3 = V_x \\ \vdots \end{cases}$$

$\Rightarrow I_1 = \frac{5}{22}\frac{V_x}{s},\ I_2 = \frac{1}{22}\frac{V_x}{s},\ I_3 = \frac{4}{22}\frac{V_x}{s},\ I_4 = \frac{2}{22}\frac{V_x}{s},\ I_5 = \frac{3}{22}\frac{V_x}{s}$

$I_x = 2(I_1 + \cdots + I_5) \Rightarrow L_{eq} = \frac{22}{30}\text{ nH}$ for each of ground and V_{DD} lines.

16.17 (a) $L_a = 0.2 \ln \frac{2h}{25\mu m}$ nH $\quad C_a = 100^2 C_0$

(b) $L_b = 0.2 \ln \frac{2h}{12.5\mu m}$ nH $\quad C_b = 50^2 C_0$

$$\frac{L_a C_a}{L_b C_b} = \frac{\ln \frac{2h}{25}}{\ln \frac{2h}{12.5}} \cdot \frac{4}{1}$$

Is the first fraction greater or less than 1/4 ?

$$\frac{\ln \frac{2h}{25}}{\ln \frac{2h}{12.5}} = \frac{1}{4} \Rightarrow h \approx 15.7\ \mu m$$

Thus, for $h > 15.7\ \mu m$ (which is quite realistic), case b is certainly preferable. For $h \ll 15.7\ \mu m$, case (a) may be preferable.

Design of Analog CMOS Integrated Circuits

Behzad Razavi

Errata in Problem Sets

Chapter 2

- In Eq. (2.44), μ_n must be in the numerator.

Chapter 3

- Call the third problem 3.2'.
- In Problem 3.2, Fig. 3.68(d), change the gate voltage of M_2 to V_{b2}.
- In Problem 3.4, Fig. 3.71(a), change the gate voltage of $M_=1$ to V_{b1}.
- In Fig. 3.72(e), V_{b1} must be changed to V_{in}.
- In Fig. 3.73(h), the output is at the source of M_2.
- In Problem 3.10(c), the question must be phrased as: Which device enters the triode region first as V_{out} falls?
- In Problem 3.13, first sentence should read: ... with $W/L = 50/0.5$ ...
- In Problem 3.16(a), do not neglect channel-length modulation in the triode region.

Chapter 4

- In Problem 4.2, assume $I_{SS} = 1$ mA and change part (a) to: Determine the voltage gain.
- In Problem 4.6, assume $\lambda = 0$.
- In Problem 4.9, assume $\lambda = \gamma = 0$.
- In Problem 4.11, assume $I_{D5} = 20$ μA.
- In Problem 4.13, change the figure number to 4.8(a).

Chapter 5

- In Problem 5.16(d), assume V_{TH} does not vary with temperature.

Chapter 6

- In Problem 6.4(b) and (d), assume $\lambda \neq 0$.

Chapter 7

- The second sentence of Problem 7.2 should read: Assume $(W/L)_1 = 50/0.5$, $I_{D1} = I_{D2} = 0.1$ mA ...
- In Problem 7.20, change I_{D1} and I_{D2} to 0.05 mA.
- In Problem 7.24, change the bias current to 0.1 mA.

Chapter 8

- In Problem 8.10, change the tolerable gain error to 5%.
- In Problem 8.15, Fig. 8.55(b), call label the top G_m block G_{m2}. The output is at the output nodes of G_{m2}.

Chapter 10

- In Problem 10.11, change I_{SS} to 0.25 mA and $(W/L)_{5,6}$ to 60/0.5.
- In Problem 10.12, add: Maximize $V_{GS14} = V_{GS15}$ while leaving at least 0.5 V across I_1. Also, in part (b), change M_2 to M_1.
- Problem 10.17 should read: ... between the gate and the drain of M_2 or M_3.
- In Fig. 10.42, change the gate voltage of $M_{3,4}$ to V_{b1}.
- In Problem 10.19(c), change A_0 in the numerator to A.

Chapter 11

- In Problem 11.13, ... such that the circuit operates with $V_{DD} = 3$ V.
- In Problems 11.17 and 11.18, the top terminal of R_2 should be connected to the top terminal of R_1.
- In Problem 11.22, assume $K = 4$.

Chapter 12

- In Problem 12.8, assume $C_H = 1$ pF.
- In Problem 12.12, assume all switches are NMOS devices.
- In Problem 12.14, assume $C_{in} = 0.2$ pF and calculate C_1 and C_2.
- In Problem 12.16, the output is sensed at the drains of M_1 and M_2.

Chapter 13

- In Problem 13.5, change the figure number to 13.6(a).