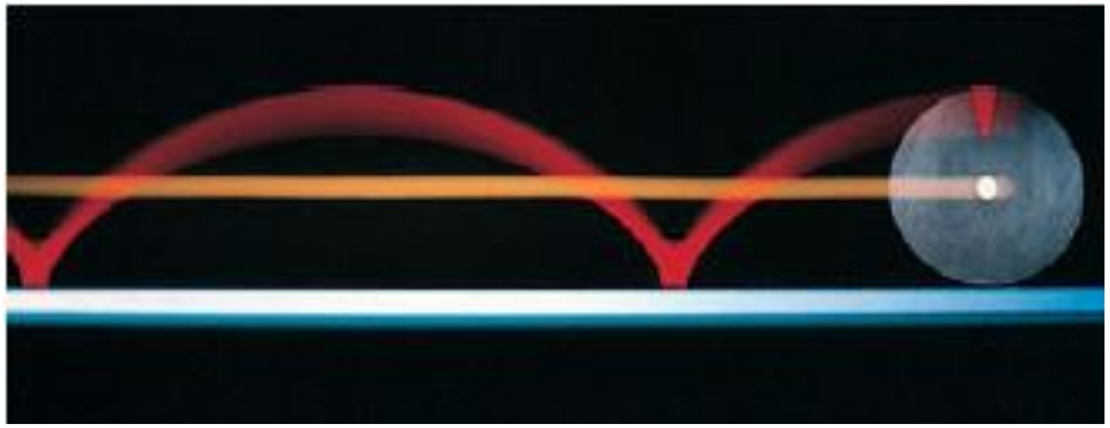
Chapter 11: Rolling, Torque, and Angular Momentum

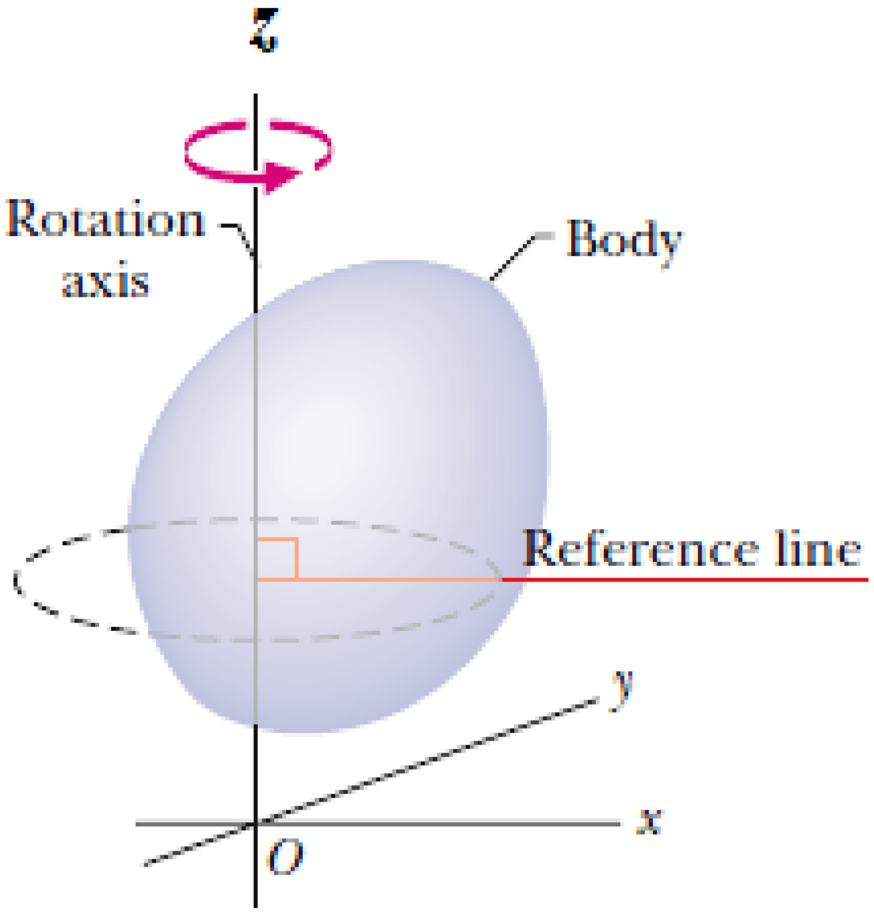
- ✓ **Rolling**
- ✓ **Torque**
- ✓ **Angular Momentum**
- ✓ **Newton’s Second Law in Angular Form**
- ✓ **Conservation of Angular Momentum**

Session 23:

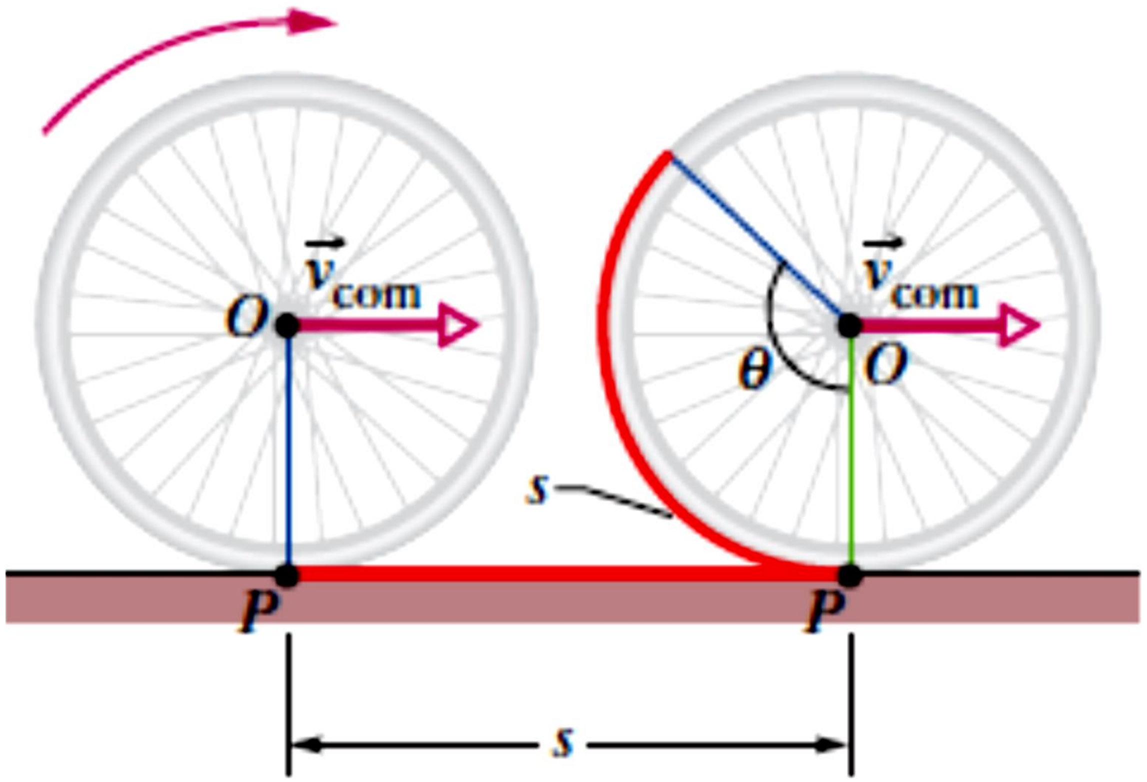
- ✓ Rolling
- ✓ Examples

Rolling





In pure rolling motion, an object rolls without slipping:



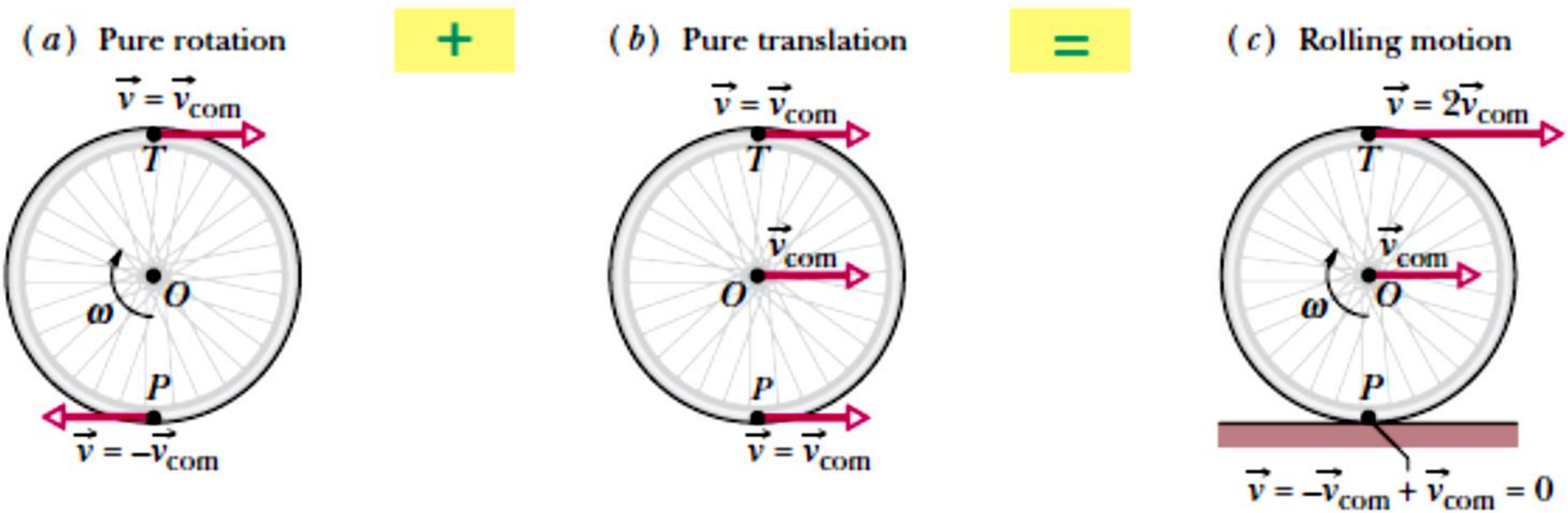
$$s = R\theta$$

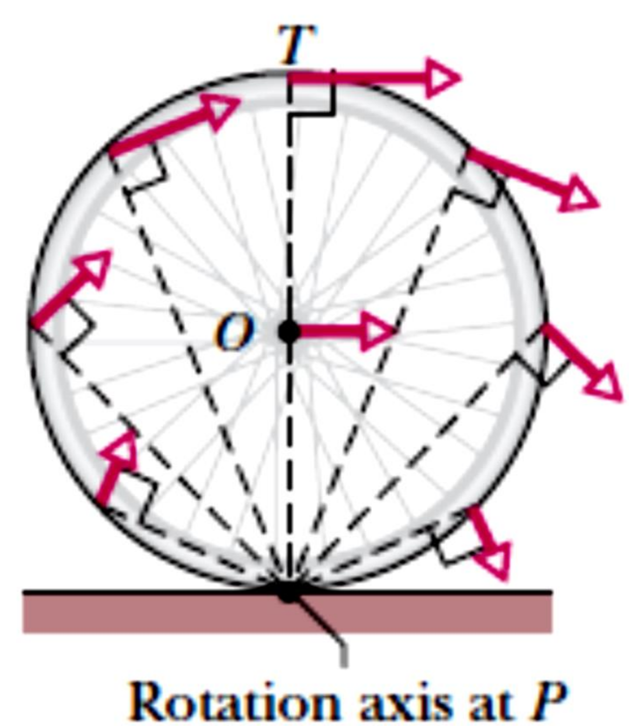
$$v_{com} = \frac{ds}{dt} = R\frac{d\theta}{dt} = R\omega$$

$$a_{com} = \frac{dv_{com}}{dt} = R\frac{d\omega}{dt} = R\alpha$$

Rolling

Rolling motion can be modeled as a **combination** of **pure translational motion** and **pure rotational motion.**



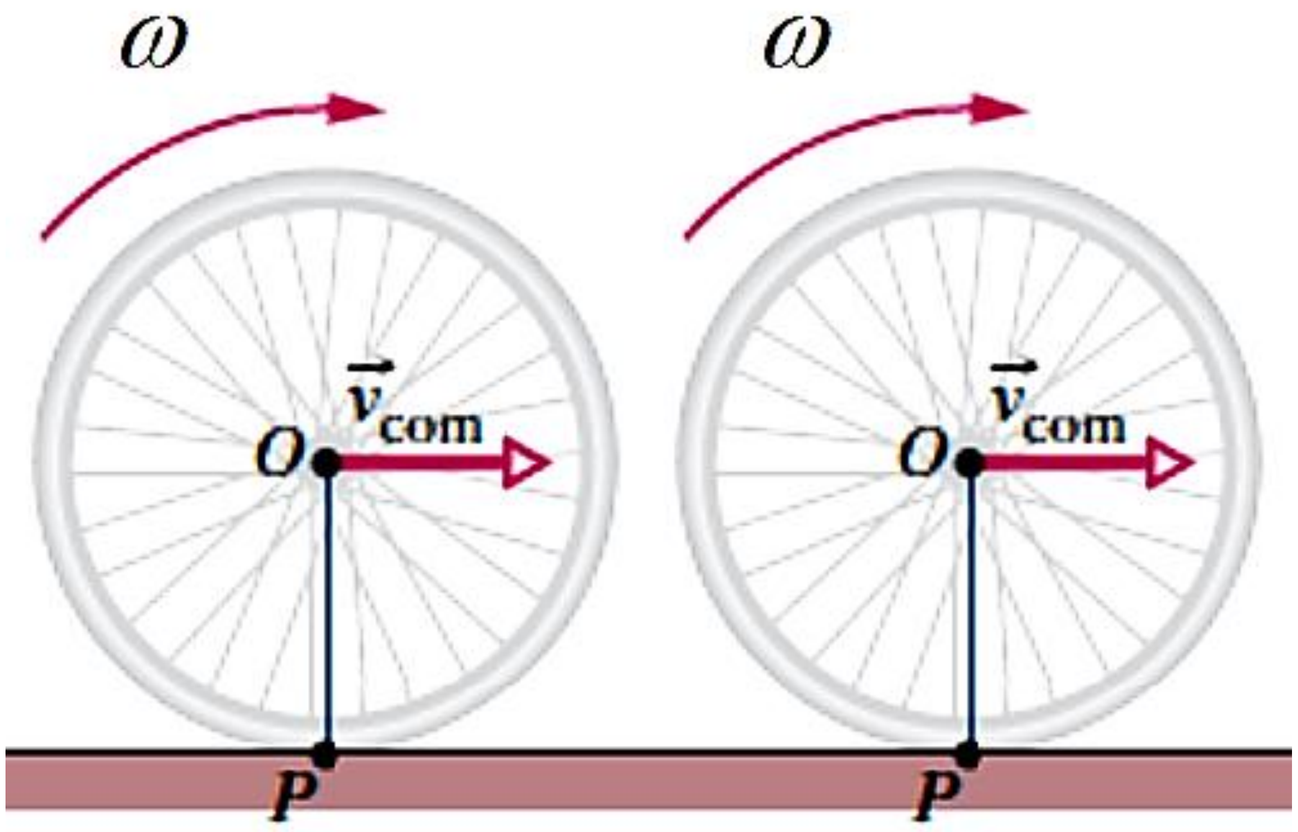


Rolling as a pure rotational motion.

$$v_{top} = 2v_{com} = 2R\omega$$

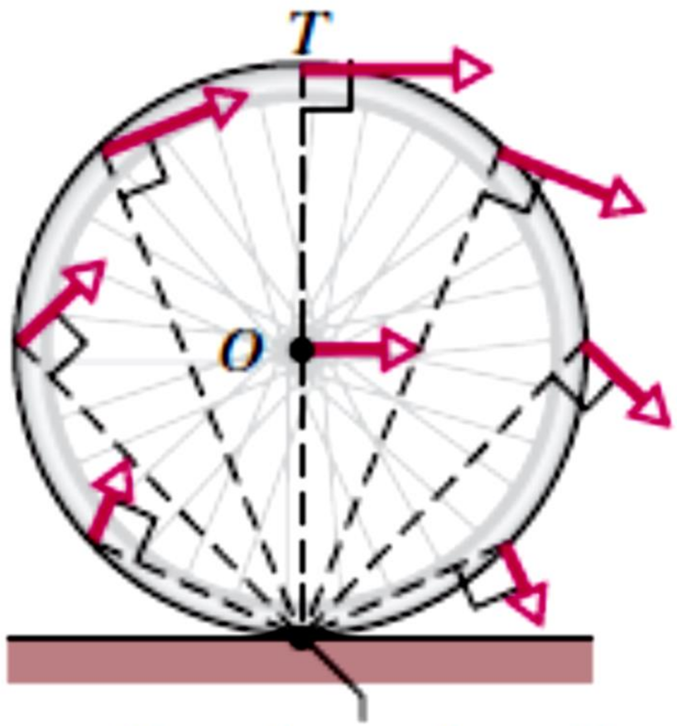
The Kinetic Energy of Rolling

$$k = k_{rot} + k_{tran}$$



$$k = \frac{1}{2} I_{com}\omega^2 + \frac{1}{2} M v_{com}^2$$

$$k = \frac{1}{2} I_{com}\omega^2 + \frac{1}{2} M(R\omega)^2 = \frac{1}{2}(I_{com} + MR^2)\omega^2$$

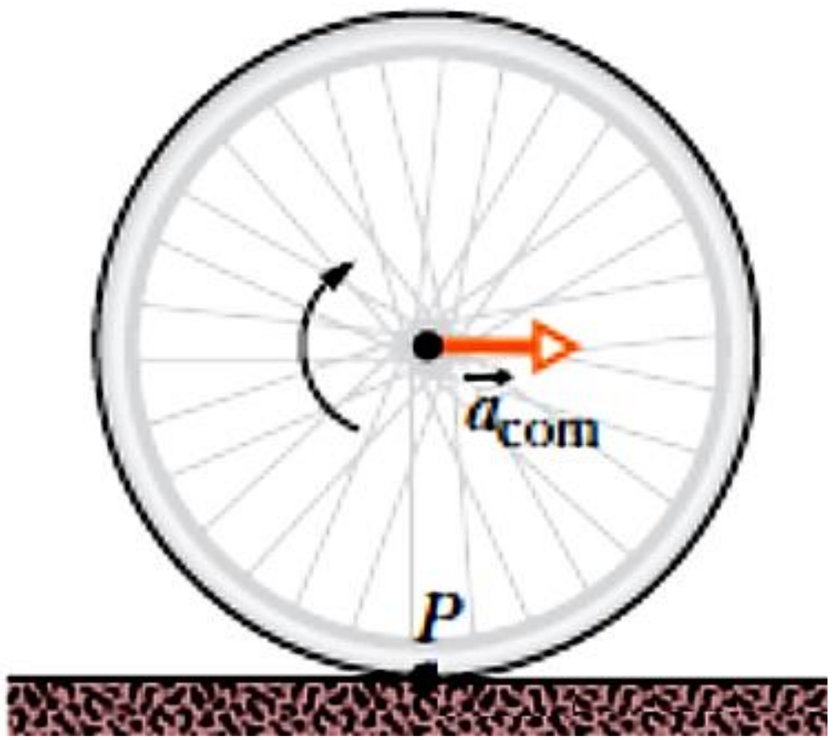


$$k = \frac{1}{2} I_P \omega^2$$

The Forces of Rolling

$$\vec{\mathbf{F}}_{net} = M\vec{\mathbf{a}}_{com}$$

$$\vec{\tau}_{net} = I_{com}\,\vec{\alpha}$$



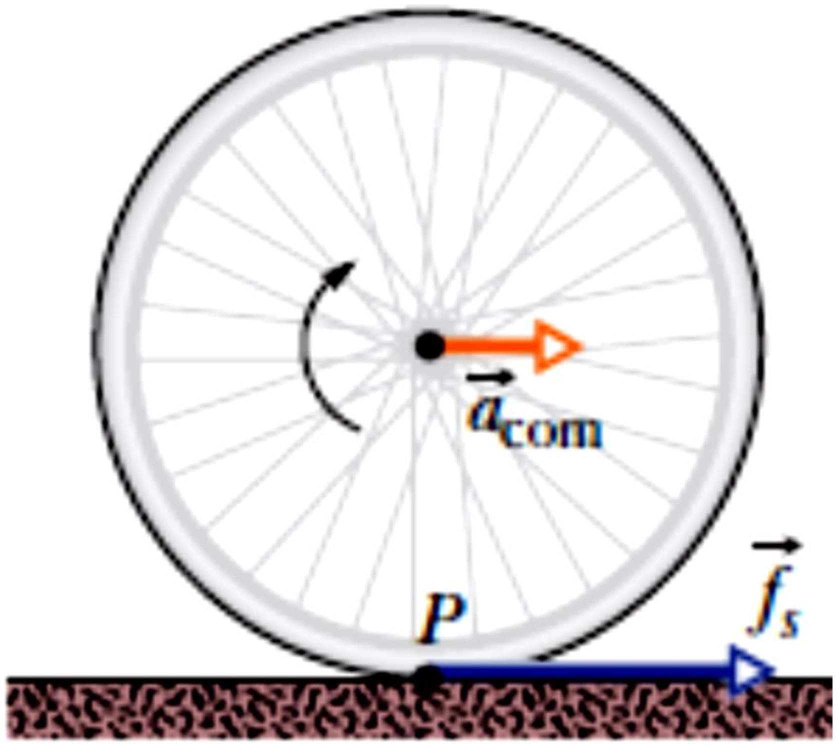
Friction and Rolling:

- If the **wheel does not slide**, the force acts at P is a **static frictional force (f_s),** the motion is **smooth rolling**

$$a_{com} = R\alpha$$

- If the **wheel does slide**, the force that acts at P is a **kinetic frictional force (f_k)**, motion is **not smooth rolling**.

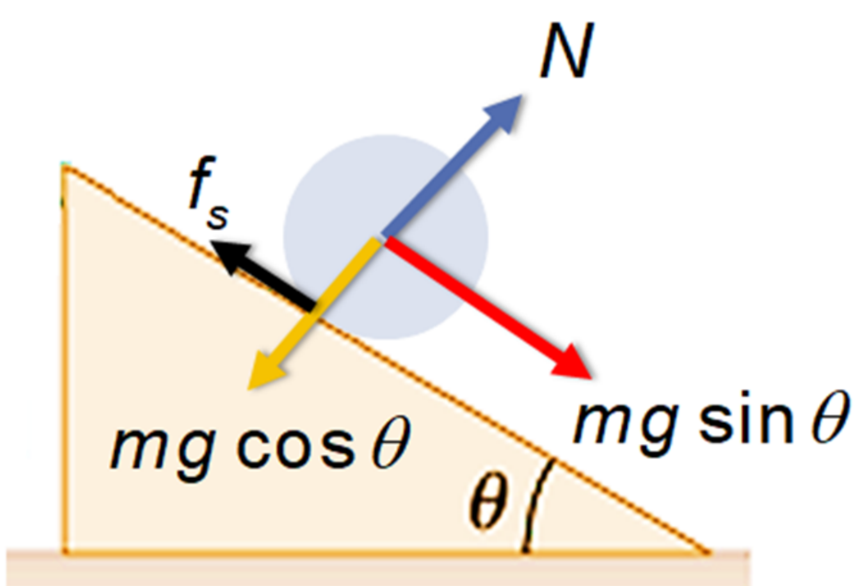
$$a_{com} \neq R\alpha$$

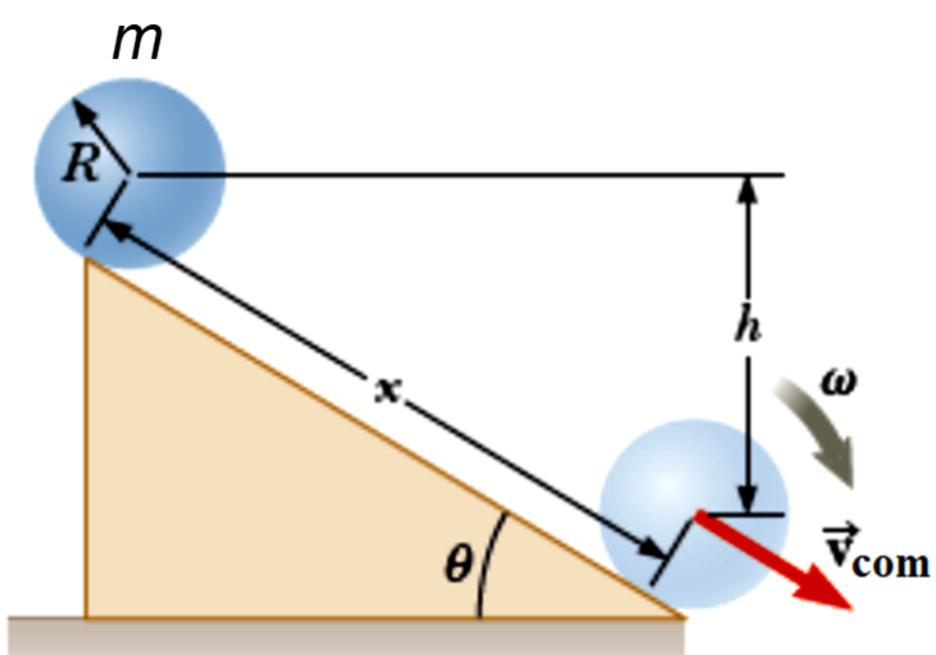


f_s acts on the wheel at *P*, opposing its tendency to slide.

Ex 1: A **solid sphere** released from **rest** and **roll without slipping** as shown in figure. Calculate the magnitude of the translational acceleration of the center of mass and the translational speed of the center of mass at the bottom of the incline.

$$I_{com} = \frac{2}{5} mR^2$$





$$\mathbf{F}_{net} = m\mathbf{a}_{com} \Rightarrow mg\sin\theta - f_s = ma_{com}$$

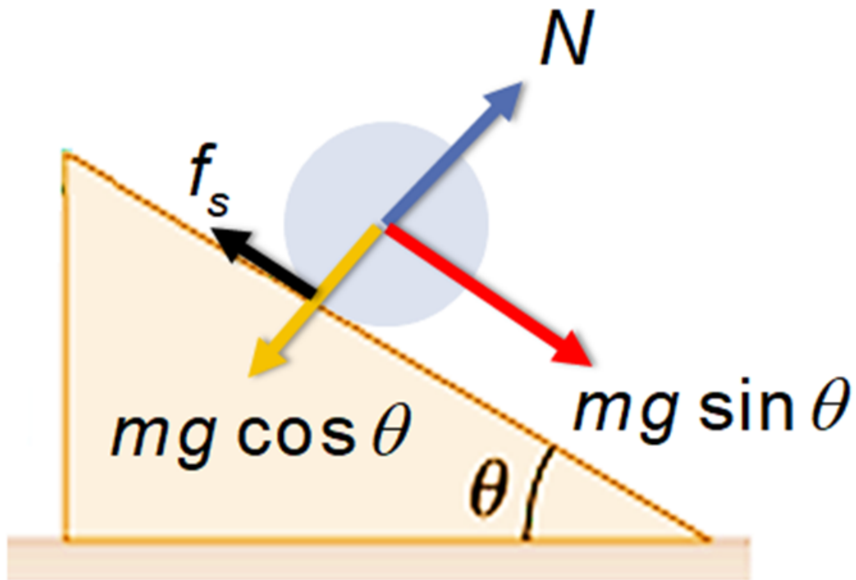
$$\tau_{net} = I_{com}\alpha \Rightarrow f_s R = I_{com}\alpha = I_{com}\frac{a_{com}}{R} \Rightarrow f_s = I_{com}\frac{a_{com}}{R^2}$$

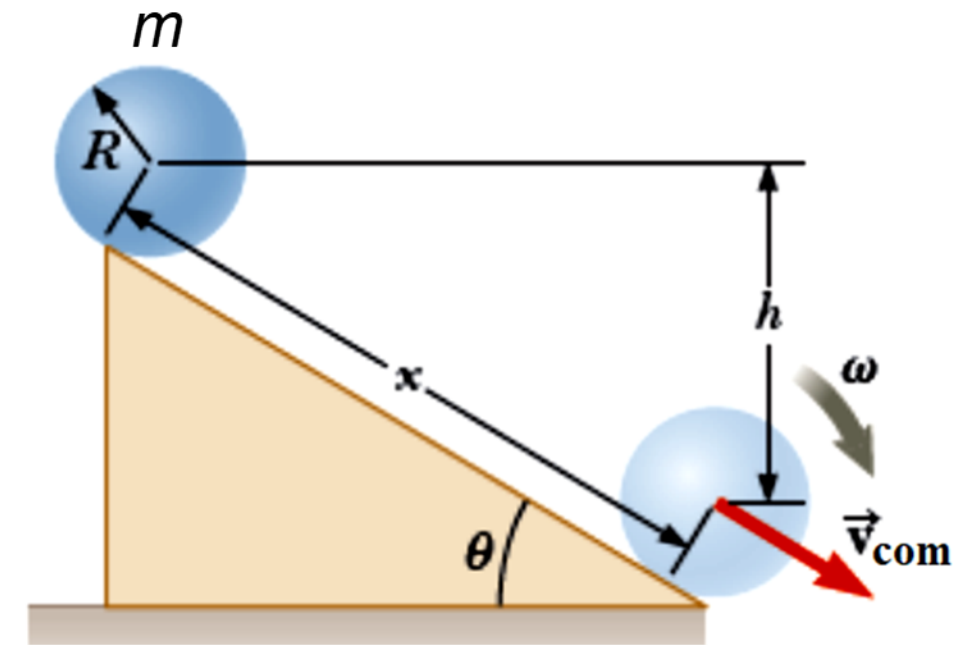
$$mg\sin\theta - I_{com}\frac{a_{com}}{R^2} = ma_{com} \Rightarrow a_{com} = \frac{g\sin\theta}{1+\frac{I_{com}}{mR^2}} = \frac{5}{7}g\sin\theta$$

$$v^2 - v_0{}^2 = 2ax \Rightarrow v_{com}^2 = 2a_{com}x = \frac{10}{7}g(x\sin\theta) \Rightarrow v_{com} = \sqrt{\frac{10}{7}gh}$$

Ex 1- Energy method

$$I_{com} = \frac{2}{5} mR^2$$





Despite the presence of friction, no loss of mechanical energy occurs because the contact point is at rest relative to the surface at any instant.

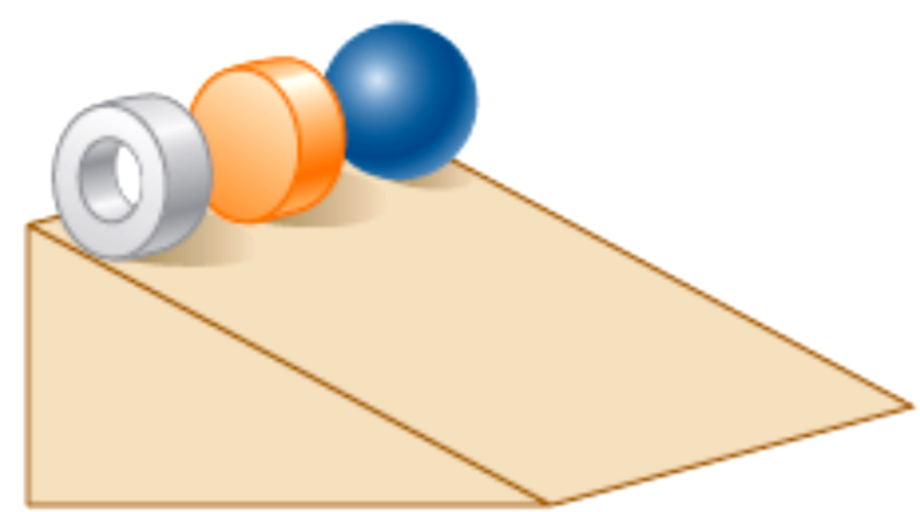
$$\Delta k + \Delta U = 0 \Rightarrow \left[\left(\frac{1}{2} I_{com}\omega^2 + \frac{1}{2} m v_{com}^2 \right) - 0 \right] + [0 - mgh] = 0$$

$$v_{com} = R\omega$$

$$\frac{1}{2} m \left(\frac{I_{com}}{mR^2} + 1 \right) v_{com}^2 = mgh \Rightarrow$$

$$v_{com} = \sqrt{\frac{2gh}{\left(1 + \frac{I_{com}}{mR^2}\right)}} = \sqrt{\frac{10}{7} gh}$$

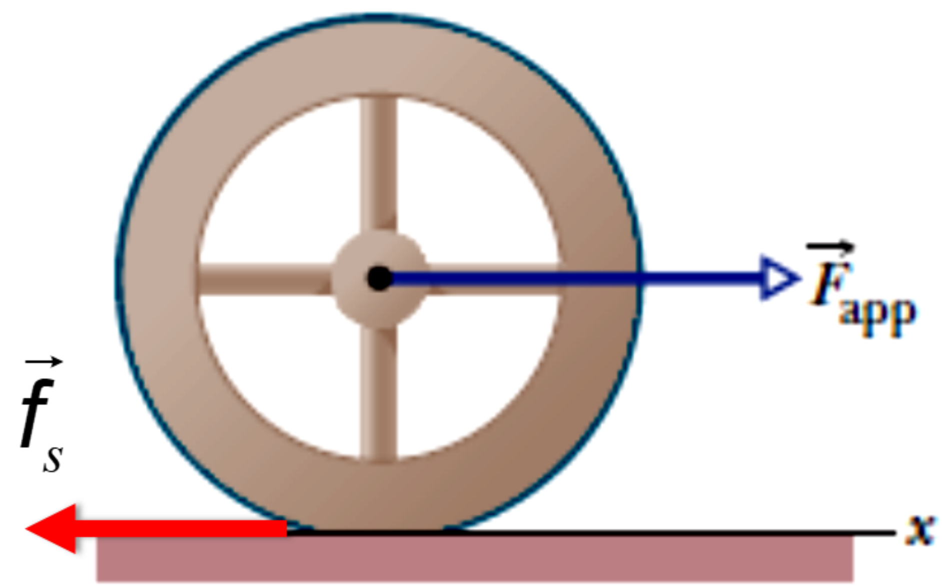
$$a_{com} = \frac{v_{com}^2}{2x} = \frac{g \sin\theta}{1 + \frac{I_{com}}{mR^2}} = \frac{5}{7} g \sin\theta$$



$$V_{Sphere} > V_{Disc} > V_{Loop}$$

Ex 2: (Problem 11. 11 Halliday)

A constant horizontal force of magnitude **10 N** is applied to a wheel of mass **10 kg** and radius **0.30 m**. The wheel **rolls smoothly** on the horizontal surface, and the **acceleration of its center of mass** has magnitude **0.60 m/s²**. (a) In unit-vector notation, what is the frictional force on the wheel? (b)What is the rotational inertia of the wheel about the rotation axis through its center of mass?



$$\mathbf{F}_{net} = m\mathbf{a}_{com} \Rightarrow F_{app} - f_s = ma_{com}$$

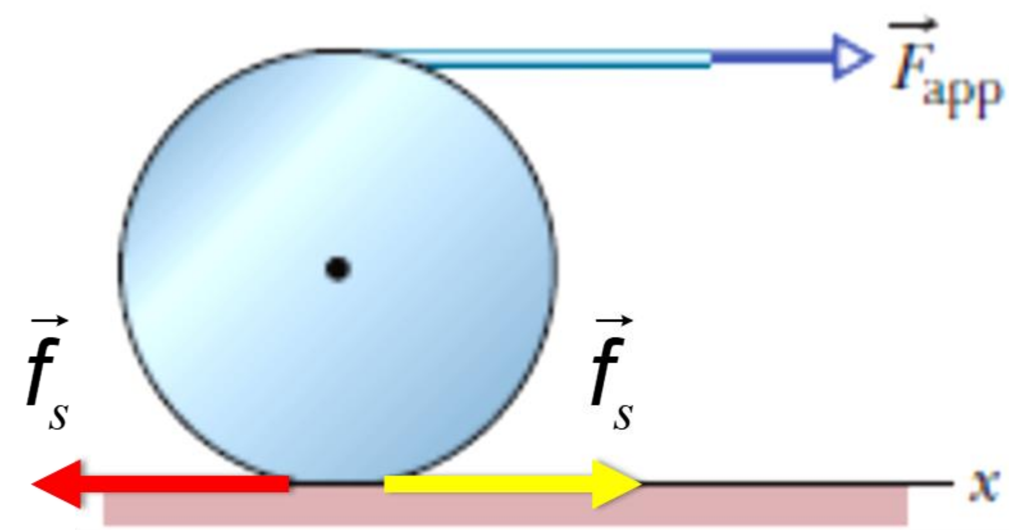
$$f_s = F_{app} - ma_{com} = 10 - 10(0.6) = 4\ N \Rightarrow \vec{f}_s = -4(\hat{\mathbf{i}})\ N$$

$$\tau_{net} = I_{com}\,\alpha \Rightarrow f_s R = I_{com}\,\alpha = I_{com}\frac{a_{com}}{R} \Rightarrow I_{com} = \frac{f_s R^2}{a_{com}}$$

$$I_{com} = 0.6\ kg.m^2$$

Ex 3: (Problem 11. 71 Halliday)

A constant horizontal force of magnitude **12 N** is applied to a **uniform solid cylinder** by fishing line wrapped around the cylinder. The mass of the cylinder is **10 kg**, its radius is **0.10 m**, and the cylinder **rolls smoothly** on the horizontal surface. (a) What is the magnitude of the acceleration of the center of mass of the cylinder? (b) What is the magnitude of the angular acceleration of the cylinder about the center of mass? (c) In unit-vector notation, what is the frictional force acting on the cylinder?



$$I_{com} = \frac{1}{2} mR^2$$

$$\mathbf{F}_{net} = m\mathbf{a}_{com} \Rightarrow F_{app} - f_s = ma_{com}$$

$$\tau_{net} = I_{com}\alpha \Rightarrow F_{app}R + f_s R = I_{com}\alpha = \left(\frac{1}{2}mR^2\right)\frac{a_{com}}{R} \Rightarrow F_{app} + f_s = \frac{1}{2}ma_{com}$$

$$2F_{app} = \frac{3}{2}ma_{com} \Rightarrow a_{com} = \frac{4F_{app}}{3m} = 1.6 \ m/s^2 \qquad \alpha = \frac{a_{com}}{R} = 16 \ rad/s^2$$

$$2f_s = -\frac{1}{2}ma_{com} \Rightarrow f_s = -\frac{1}{4}ma_{com} = -4 \ N \Rightarrow \vec{f}_s = +4(\hat{\mathbf{i}}) \ N$$