

Date of publication xxxx 00, 0000, date of current version xxxx 00, 0000.

Digital Object Identifier 10.1109/ACCESS.2017.Doi Number

A Non-singular Fast Terminal Sliding Mode Control Based on Third-Order Sliding Mode Observer for a Class of Second-Order Uncertain Nonlinear Systems and Its Application to Robot Manipulators

Van-Cuong Nguyen¹, Anh-Tuan Vo², and Hee-Jun Kang³

¹Graduate School of Electrical Engineering, University of Ulsan, Ulsan 44610, South Korea

²Graduate School of Electrical Engineering, University of Ulsan, Ulsan 44610, South Korea

³School of Electrical Engineering, University of Ulsan, Ulsan 44610, South Korea

Corresponding author: Hee-Jun Kang (e-mail: hjkang@ulsan.ac.kr)

This research was supported by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education (2019R1D1A3A03103528).

ABSTRACT This paper proposes a controller-observer strategy for a class of second-order uncertain nonlinear systems with only available position measurement. The third-order sliding mode observer is first introduced to estimate both velocities and the lumped uncertain terms of system with high accuracy, less chattering, and finite time convergency of estimation errors. Then, the proposed controller-observer strategy is designed based on non-singular fast terminal sliding mode sliding control and proposed observer. Thanks to this combination, the proposed strategy has some superior properties such as high tracking accuracy, chattering phenomenon reduction, robustness against the effects of the lumped uncertain terms, velocity measurement elimination, finite time convergence, and faster reaching sliding motion. Especially, two period times, before and after the convergence of the velocity estimation takes place, are considered. The finite time stability of proposed controller-observer method is proved by using the Lyapunov stability theory. Final, the proposed strategy is applied to robot manipulator system and its effectiveness is verified by simulation results, in which a PUMA560 robot manipulator is employed.

INDEX TERMS Uncertain Nonlinear Systems, Non-singular Fast Terminal Sliding Mode Control, Third-Order Sliding Mode Observer, Controller-Observer Strategy, Uncertainty Compensation, Robot Manipulators.

I. INTRODUCTION

In the past decades, controlling uncertain nonlinear systems have been a topic that attracts attention from many researchers theoretically [1]–[3]. This topic is also crucial in practical because almost real-world systems have nonlinear dynamic characteristics. Generally, the dynamic model of the system is not clearly known because of the unknown uncertainties and/or external disturbances - in this paper, for more convenience and avoiding duplication, we will treat it as the lumped uncertainties. They affect directly to the control signal thus reduce the accuracy of the system. This problem has been a big challenge in control theory. To deal with the lumped uncertainties, numerous control strategies

have been proposed by researchers, such as PID control [4], [5], adaptive control [6]–[8], fuzzy logic control [9], [10], neural network control [11], [12], and sliding mode control (SMC) [13]–[17], etc. Among them, sliding mode control (SMC) has been widely used in controlling uncertain system by many researchers because of its attractive properties such as fast dynamic response, robustness against the lumped uncertainties and a quite simple design procedure. It is suitable for various types of real systems such as DC-DC converters, motors, helicopters, magnetic levitation, aircraft, and robot manipulators. Besides the great benefits, the utilization of a linear sliding function in conventional SMC causes the finite-time convergence of system state error

cannot be guaranteed. To overcome this limitation, the terminal SMC (TSMC) has been proposed, in which nonlinear sliding functions are utilized instead of the linear sliding function in design procedure [18]–[20]. By carefully designing parameters, TSMC provides higher accuracy and finite-time convergence; unfortunately, the conventional TSMC generates two main limitations: 1) slower convergence time comparing to SMC; 2) singularity problem. Various great researches have been focused to overcome these drawbacks. Each problem has been solved by using fast TSMC (FTSMC) [21]–[23] and nonsingular TSMC (NTSMC) [24]–[26], separately. To handle both problems simultaneously, nonsingular fast TSMC (NFTSMC) has been developed [15], [27]–[30]. Thanks to the superior properties of the NFTSMC such as finite-time convergence, singularity elimination, high tracking error performance, and robustness against the lumped uncertainties, this controller has been extensively utilized to a variety of systems. However, both conventional SMC and NFTSMC still utilize a switching element in reaching phase with a big fixed sliding gain against the effects of the lumped uncertainties leads to the chattering phenomenon. It harms the system and thus reduces the practical applicability of both control methods. On the other hand, the design procedure requires real velocity information which is not usually available in a practical system because of saving cost and reducing the size of the device.

In order to reduce or eliminate the chattering phenomenon, the basic idea is to reduce the sliding gain in the switching element. Accordingly, the lumped uncertainties must be completely or partially estimated and applied to the control signals to compensate for its effects. Consequently, the switching element is now used to handle the effects of the uncertainty's estimation error instead of the lumped uncertainties; therefore, the sliding gain will be selected smaller than the original method to guarantee the sliding mode can be reached. As a result, the chattering phenomenon will be reduced depending on the precision of the estimation method. In the literature, various model-based techniques have been developed to estimate the lumped uncertainties such as time delay estimation (TDE) [29], [31], neural network (NN) observer [32], [33], second-order sliding mode (SOSM) observer [34]–[37], third-order sliding mode (TOSM) observer [17], [34], [38]–[40]. Among them, the TDE technique can only provide the ability to estimate unknown inputs; therefore, an additional observer is needed to estimate the system velocities [29]. It leads the system more complex and increases the computational time. Thanks to the learning capability and excellent approximation, the NN observer can supply an arbitrary accuracy of estimation information. Especially, it can not only have the capability to approximate the lumped uncertainties but also the system velocities. Therefore, only one observer is employed in the system. However, the drawback of using learning techniques is that the transient performance in the existence of external

disturbance can be reduced because of the requirement of the online learning procedure. Moreover, the complex training process of neural weights requires a large computation of the system thus degrades the implementation ability in a practical system. Compared to others, the SOSM observer stands out due to its capability to approximate both system velocities and the lumped uncertainties with the finite-time convergence of estimation error. Although providing high precision and less chattering in the estimation of velocities, the equivalent output injection of SOSM observer which is used to obtain the estimation of the lumped uncertainties is a discontinuous term that causes an undesired chattering phenomenon. Therefore, a lowpass filter is needed to reconstruct the lumped uncertainties from the equivalent output injection. However, it causes the estimation delay and error thus reduces the estimation accuracy of the SOSM observer. For that reason, the TOSM observer which has the ability to provide a continuous equivalent output injection, has been investigated. Consequently, the required filtration in the SOSM observer is eliminated. Compared with the SOSM observer, the TOSM observer provides the estimation of lumped uncertainties with less chattering and higher estimation accuracy. Moreover, the TOSM observer maintains almost all the advantages of the SOSM observer. Thanks to the superior benefits, the TOSM observer has been widely applied to control uncertain systems by many researchers [41]–[43]. In [41], the TOSM observer has been employed to estimate system velocities; however, the author did not consider the ability to approximate the lumped uncertainties of the observer. In contrast, an SMC combined with the TOSM observer is presented in [42]. Unfortunately, only the estimation of lumped uncertainties is considered to eliminate its effects. A combination of the two algorithms above, both obtained velocities and lumped uncertainties from the TOSM observer is applied to design a conventional SMC, in [43]. However, the actual velocity signal of the system is replaced by the estimated velocity which is after the convergence takes place instead of the original one. This makes the controller design simpler but leads to some components in the control signal not being clearly considered. Consequently, the system is sometimes unstable due to the incorrect selection of control parameters, especially, in the period before the convergence occurs.

In this paper, the TOSM observer is first designed to estimate not only system velocities but also the lumped uncertainties for the class of second-order uncertain nonlinear systems without any filtration. Based on the obtained information, an NFTSMC is proposed for position tracking trajectory without the requirement of system velocities. With this control strategy, we can obtain a control law that provides high accuracy, non-singularity, robustness against the lumped uncertainties, low chattering, and finite-time convergence without the need of velocity measurement. In summary, the major contributions of this paper are as follow: 1) proposes a NFTSM control law based on the

obtained information from the TOSM observer; 2) proves the global stability of the system when combining controller and observer by using the Lyapunov stability theory; 3) reduces the chattering phenomenon in control output signal by compensating the lumped uncertainties; 4) eliminates the requirement of velocity measurement in the system; 5) obtains higher performance of the NFTSM controller by using higher accuracy compensation method.

This paper is constructed into seven sections. After the introduction, Section II declares the problem statement. In Section III, the design of the TOSM observer is presented, followed by the design method of the NFTSMC for the class of second-order uncertain nonlinear systems is proposed in Section IV. The application of the controller-observer strategy for robotic manipulators is presented in Section V. In Section VI, numerical simulations on a PUMA560 robot manipulator are shown to prove the effectiveness of the proposed method. Finally, some conclusions are provided in Section VII.

II. PROBLEM STATEMENT

Consider the following second-order nonlinear control systems with dynamic uncertainties and/or external disturbances as

$$\begin{aligned}\dot{x}_1 &= x_2 \\ \dot{x}_2 &= f(x, t) + g(x, t)u(t) + \Delta(x, u, t)\end{aligned}\quad (1)$$

where $x_1 \in \mathbb{R}^n$ and $x_2 \in \mathbb{R}^n$, $x = [x_1^T \ x_2^T]^T$ denote the system state vectors, $f(x, t) \in \mathbb{R}^n$ and $g(x, t) \in \mathbb{R}^{n \times n}$ are given nonlinear functions, $g(x, t)$ is invertible, $\Delta(x, u, t) \in \mathbb{R}^n$ presents lumped uncertainties which includes the dynamic uncertainties and/or external disturbances, and $u(x, t) \in \mathbb{R}^n$ denotes the control input.

The main purpose of this paper is to design a controller-observer strategy which can eliminate the effects of the lumped uncertainties without the requirement of velocity measurement. This control method is designed based on the following assumptions:

Assumption 1: The system states are bounded at all time.

Assumption 2: The lumped uncertainties $\Delta(x, u, t)$ of the system (1) are bounded as

$$\|\Delta(x, u, t)\| \leq \Lambda \quad (2)$$

where Λ is a positive constant.

Assumption 3: The first-time derivative lumped uncertainties $\dot{\Delta}(x, u, t)$ exist and are bounded as

$$\left\| \frac{d}{dt} \Delta(x, u, t) \right\| \leq \bar{\Lambda} \quad (3)$$

where $\bar{\Lambda}$ is a positive constant.

III. STATE OBSERVER DESIGN AND UNCERTAINTY IDENTIFICATION

First, the TOSM observer is introduced to estimate both system velocities and the lumped uncertainties. Then, the estimated information will be applied to design the control signal.

A. STATE OBSERVER DESIGN

Based on system (1), the TOSM observer is designed as [17]

$$\begin{aligned}\dot{\hat{x}}_1 &= \gamma_1 |x_1 - \hat{x}_1|^{2/3} \text{sign}(x_1 - \hat{x}_1) + \hat{x}_2 \\ \dot{\hat{x}}_2 &= f(\hat{x}, t) + g(\hat{x}, t)u(t) + \gamma_2 |x_1 - \hat{x}_1|^{1/3} \text{sign}(x_1 - \hat{x}_1) - \hat{z} \\ \dot{\hat{z}} &= -\gamma_3 \text{sign}(x_1 - \hat{x}_1)\end{aligned}\quad (4)$$

where \hat{x} denotes the estimation of x and γ_i denote the observer gains.

Subtracting (1) to (4), we can get the estimation errors as

$$\begin{aligned}\dot{\tilde{x}}_1 &= -\gamma_1 |\tilde{x}_1|^{2/3} \text{sign}(\tilde{x}_1) + \tilde{x}_2 \\ \dot{\tilde{x}}_2 &= -\gamma_2 |\tilde{x}_1|^{1/3} \text{sign}(\tilde{x}_1) + \Delta(x, u, t) - d(x, \tilde{x}, u, t) + \hat{z} \\ \dot{\tilde{z}} &= -\gamma_3 \text{sign}(\tilde{x}_1)\end{aligned}\quad (5)$$

where $\tilde{x} = x - \hat{x}$ denote the system state's estimation errors and the estimation errors of the lumped uncertainties are described as

$d(x, \tilde{x}, u, t) = \{f(\hat{x}, t) + g(\hat{x}, t)u(t)\} - \{f(x, t) + g(x, t)u(t)\}$. Based on the Assumption 1, the estimation errors, $d(x, \tilde{x}, u, t)$, are bounded as $\|d(x, \tilde{x}, u, t)\| \leq \Xi$.

Denoting the estimation of the lumped uncertainties, $\hat{\Delta}(x, \hat{x}, u, t)$, as $\hat{\Delta}(x, \hat{x}, u, t) = \Delta(x, u, t) - d(x, \tilde{x}, u, t)$, the estimation errors (5) can be rewritten as follow

$$\begin{aligned}\dot{\tilde{x}}_1 &= -\gamma_1 |\tilde{x}_1|^{2/3} \text{sign}(\tilde{x}_1) + \tilde{x}_2 \\ \dot{\tilde{x}}_2 &= -\gamma_2 |\tilde{x}_1|^{1/3} \text{sign}(\tilde{x}_1) + \hat{\Delta}(x, \hat{x}, u, t) + \hat{z} \\ \dot{\tilde{z}} &= -\gamma_3 \text{sign}(\tilde{x}_1)\end{aligned}\quad (6)$$

Now, let define $\hat{z}_0 = \hat{\Delta}(x, \hat{x}, u, t) + \hat{z}$, the system (6) becomes

$$\begin{aligned}\dot{\tilde{x}}_1 &= -\gamma_1 |\tilde{x}_1|^{2/3} \text{sign}(\tilde{x}_1) + \tilde{x}_2 \\ \dot{\tilde{x}}_2 &= -\gamma_2 |\tilde{x}_1|^{1/3} \text{sign}(\tilde{x}_1) + \hat{z}_0 \\ \dot{\tilde{z}}_0 &= -\gamma_3 \text{sign}(\tilde{x}_1) + \dot{\hat{\Delta}}(x, \hat{x}, u, t)\end{aligned}\quad (7)$$

The estimation errors (7) is in the standard form of second-order robust exact differentiator and its finite-time stability has successfully proved in literature [44]. Therefore, by selecting suitable observer gains, the estimation errors, \tilde{x}_1 , \tilde{x}_2 , and \hat{z}_0 will converge to zero in finite time.

Remark 1: The observer gains of (4) could be selected based on [44] as $\gamma_1 = \alpha_1 L^{1/3}$, $\gamma_2 = \alpha_2 L^{2/3}$, and $\gamma_3 = \alpha_3 L$ where $\alpha_1 = 2$, $\alpha_2 = 2.12$, and $\alpha_3 = 1.1$ with $L = \Lambda + \Xi$.

B. UNCERTAINTY IDENTIFICATION

After the convergence process, the differentiators will converge to zero. In other words, the estimated states will achieve the real states ($\hat{x}_1 = x_1$, $\hat{x}_2 = x_2$) after finite time. Thus, the uncertainty estimation errors will be equal to zero, $d(x, \tilde{x}, u, t) = 0$. The third equation of system (7) turn into

$$\dot{\hat{z}}_0 = -\gamma_3 \text{sign}(\tilde{x}_1) + \hat{\Delta}(x, \hat{x}, u, t) \equiv 0 \quad (8)$$

The lumped uncertainties can be reconstructed as

$$\hat{\Delta}(x, \hat{x}, u, t) = \int \gamma_3 \text{sign}(\tilde{x}_1) \quad (9)$$

Since the estimation of lumped uncertainties in (9) include integral element; therefore, it can be reconstructed immediately from the output injection term without requirement of lowpass filter. Thanks to this preminent feature, the TOSM observer can provide higher estimation accuracy than that of SOSM observer, which is designed in Appendix A. Moreover, the finite time convergence of both system velocities and the lumped uncertainties is guaranteed. The obtained lumped uncertainties can also be utilized for fault detection and applied to the fault tolerant control. The estimated velocities are used in the control design procedure instead of the measured velocities in the next Section.

Remark 2: It is worth noting that when applying the estimation states from the observer to the closed-loop controlling system, the exact estimation of system states can only achieve after some transient time. In another word, there exists additional errors in the period before the convergence occurs. These errors will affect in choosing the parameters of the controller. If it is not carefully considered, it will lead to a wrong selection of control parameters. Consequently, the system will be unstable in some cases.

IV. DESIGN OF OBSERVER-BASED NFTSMC ALGORITHM

In this part, a NFTSMC algorithm is proposed for the class of second-order uncertain nonlinear systems (1) to handle the effect of the lumped uncertainties with low chattering and minimum tracking errors. Especially, only position measurements are required. The design method is expressed in the two following steps.

A. DESIGN OF SLIDING FUNCTION

The tracking errors and velocity errors are defined as

$$\begin{aligned} e &= x_1 - x_d \\ \dot{e} &= \dot{x}_2 - \dot{x}_d \end{aligned} \quad (10)$$

where x_d, \dot{x}_d denote the desired trajectories and velocities, respectively.

A terminal sliding function is chosen as the following expression [12]

$$s = \dot{e} + \int_0^t \left(\kappa_2 |\dot{e}|^{\alpha_2} \text{sign}(\dot{e}) + \kappa_1 |e|^{\alpha_1} \text{sign}(e) \right) dt \quad (11)$$

where constants κ_1, κ_2 denote sliding gains which can be chosen such that the polynomial $\kappa_2 p + \kappa_1$ is Hurwitz and α_1, α_2 can be selected as

$$\begin{aligned} \alpha_1 &= (1 - \varepsilon, 1), \quad \varepsilon \in (0, 1) \\ \alpha_2 &= \frac{2\alpha_1}{1 + \alpha_1} \end{aligned} \quad (12)$$

Generally, for saving the cost and reducing the weight of devices, the tachometers in the devices will be cut off by manufacturers. Therefore, in this article, we assume that only the position measurements are available in the system (1). Consequently, the variables \dot{e} in sliding functions, s , in (11), are not available. To achieve applicable sliding functions, we define the tracking errors and estimation of velocity errors as

$$e = x_1 - x_d \quad (13)$$

$$\hat{e} = \hat{x}_2 - \dot{x}_d \quad (14)$$

With the above defining, the estimation of sliding function (11) can be obtained as

$$\hat{s} = \hat{e} + \int_0^t \left(\kappa_2 |\hat{e}|^{\alpha_2} \text{sign}(\hat{e}) + \kappa_1 |e|^{\alpha_1} \text{sign}(e) \right) dt \quad (15)$$

B. DESIGN OF CONTROLLER

In order to obtain the control signal for the uncertain nonlinear system (1), an NFTSMC based on TOSM observer as described in Fig.1 is proposed. The control law is proposed as below

$$u = -g(x, t)^{-1} (u_{eq} + u_{sw}) \quad (16)$$

In (16), the equivalent control law, u_{eq} , holds the trajectory of the error state on the sliding surface, is designed as

$$\begin{aligned} u_{eq} &= f(x, t) + \gamma_2 |\tilde{x}_1|^{1/3} \text{sign}(\tilde{x}_1) \\ &+ \int \lambda_3 \text{sign}(\tilde{x}_1) + \kappa_2 |\hat{e}|^{\alpha_2} \text{sign}(\hat{e}) + \kappa_1 |e|^{\alpha_1} \text{sign}(e) - \ddot{x}_d \end{aligned} \quad (17)$$

The switching control law, u_{sw} , is constructed to compensate for the estimation errors as follows

$$u_{sw} = (\Xi + \mu) \text{sign}(\hat{s}) \quad (18)$$

where μ is a small positive constant.

The control design method for the system is described in Theorem below.

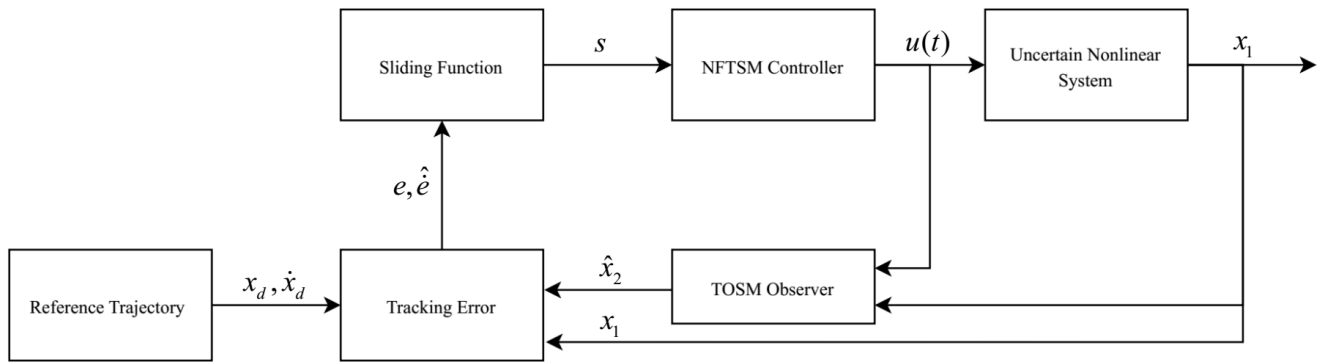


Figure 1. Block diagram of the proposed controller-observer strategy.

Theorem 1: Consider the class of second-order uncertain nonlinear systems given by (1), if the NFTSM control input is designed as (16-18), then the origin of the sliding function (15) is globally finite-time stable equilibrium point and the sliding function (15) will converge to zero in finite time defined by $T_r = \frac{\|\hat{s}(0)\|}{\mu}$.

Proof:

Taking the first-time derivative of the estimated sliding function (15) yields

$$\dot{\hat{s}} = \frac{d}{dt} \hat{e} + \kappa_2 |\hat{e}|^{\alpha_2} \text{sign}(\hat{e}) + \kappa_1 |e|^{\alpha_1} \text{sign}(e) \quad (19)$$

We can obtain the first-time derivative of tracking velocity errors (14) as follows

$$\frac{d}{dt} \hat{e} = \dot{\hat{x}}_2 - \ddot{x}_d \quad (20)$$

Substituting the second equation of observer (4) into (20), we can get

$$\begin{aligned} \frac{d}{dt} \hat{e} &= -\ddot{x}_d + f(\hat{x}, t) + g(\hat{x}, t)u(t) \\ &\quad + \gamma_2 |\tilde{x}_1|^{1/3} \text{sign}(\tilde{x}_1) + \int \gamma_3 \text{sign}(\tilde{x}_1) \\ &= -\ddot{x}_d + f(x, t) + g(x, t)u(t) + d(x, \tilde{x}, u, t) \\ &\quad + \gamma_2 |\tilde{x}_1|^{1/3} \text{sign}(\tilde{x}_1) + \int \gamma_3 \text{sign}(\tilde{x}_1) \end{aligned} \quad (21)$$

Substituting (21) into (20), we can obtain

$$\begin{aligned} \dot{\hat{s}} &= -\ddot{x}_d + f(x, t) + g(x, t)u(t) + d(x, \tilde{x}, u, t) \\ &\quad + \gamma_2 |\tilde{x}_1|^{1/3} \text{sign}(\tilde{x}_1) + \int \gamma_3 \text{sign}(\tilde{x}_1) \\ &\quad + \kappa_2 |\hat{e}|^{\alpha_2} \text{sign}(\hat{e}) + \kappa_1 |e|^{\alpha_1} \text{sign}(e) \end{aligned} \quad (22)$$

Employing the control input from (16) to (18) into (22) yields

$$\dot{\hat{s}} = -(\Xi + \mu) \text{sign}(\hat{s}) + d(x, \tilde{x}, u, t) \quad (23)$$

Define the Lyapunov function as following

$$V = \frac{1}{2} \hat{s}^T \hat{s} \quad (24)$$

Taking the first-time derivative of Lyapunov function (24) and substituting the result from (23) yields

$$\begin{aligned} \dot{V} &= \hat{s}^T \dot{\hat{s}} \\ &= \hat{s}^T (-(\Xi + \mu) \text{sign}(\hat{s}) + d(x, \tilde{x}, u, t)) \\ &= -(\Xi + \mu) \sum_{i=1}^n |\hat{s}_i| + d(x, \tilde{x}, u, t)^T \hat{s} \leq -\mu \sum_{i=1}^n |\hat{s}_i| \\ &\leq -\mu \|\hat{s}\| = -\sqrt{2} \mu V^{1/2} < 0, \forall \hat{s} \neq 0 \end{aligned} \quad (25)$$

As a result, according to [45], we can conclude that the origins $s_i = 0, i = 1, 2, \dots, n$ of sliding function (15) are globally finite-time stable equilibrium points and the sliding function will converge to zero in finite time $T_r = \frac{\|\hat{s}(0)\|}{\mu}$.

Theorem 1 is successful proved.

The proposed controller-observer method provides high position tracking accuracy, non-singularity, robustness against the lumped uncertainties, low chattering, and finite-time convergence without the need of velocity measurement. Its effectiveness will be illustrated by the simulation results.

Remark 3: We can see that the estimation of the lumped uncertainties term, $\int \gamma_3 \text{sign}(\tilde{x}_1)$, which is obtained from the TOSM observer (4), contains in the equivalent control signal (17). Accordingly, in the switching control law, only a small value of sliding gain, Ξ , is selected to compensate the effects of the lumped uncertainties' estimation errors, $d(x, \tilde{x}, u, t)$. By using this way, the chattering is significantly reduced in control input torque.

Remark 4: It is worth noting that there exists an additional component, $\gamma_2 |\tilde{x}_1|^{1/3} \text{sign}(\tilde{x}_1)$, in the equivalent control signal (17) compared with the control signal (60), in which the converged estimation velocities is used in controller design procedure, see Appendix D. After the convergence time, this term will be equal to zero; however, the presence of this component ensures the proper functioning of the system when a steady state has not been established.

V. APPLICATION TO ROBOT MANIPULATORS

The controller-observer method is designed for the class of second-order uncertain nonlinear systems; therefore, it can be applied to many systems which have the same characteristic such as motors, helicopters, aircraft, and robot manipulators. In this part, the proposed controller-observer strategy is employed for tracking trajectory a serial n-link robotic manipulator with the dynamic equation is given in Lagrange form as

$$M(\theta)\ddot{\theta} + C(\theta, \dot{\theta}) + G(\theta) + F(\dot{\theta}) = \tau(t) + \tau_d(t) \quad (26)$$

where $\theta, \dot{\theta}, \ddot{\theta} \in \mathbb{R}^n$ denote position, velocity, and acceleration of robot joints, respectively. $\tau(t) \in \mathbb{R}^n$ represents the control input torque, $M(\theta) \in \mathbb{R}^{n \times n}$ represents the inertia matrix, $C(\theta, \dot{\theta}) \in \mathbb{R}^n$ represents the Coriolis and centripetal forces, $G(\theta) \in \mathbb{R}^n$ represents the gravitational force term. $F(\dot{\theta}) \in \mathbb{R}^n$ denotes the friction vector, $\tau_d(t) \in \mathbb{R}^n$ denotes the disturbance vector.

Generally, because of the different between the mathematical and practical model, there exist uncertain component of the model of the robot manipulators as

$$M(\theta) = M_0(\theta) + \Delta M(\theta) \quad (27)$$

$$C(\theta, \dot{\theta}) = C_0(\theta, \dot{\theta}) + \Delta C(\theta, \dot{\theta}) \quad (28)$$

$$G(\theta) = G_0(\theta) + \Delta G(\theta) \quad (29)$$

where $M_0(\theta)$, $C_0(\theta, \dot{\theta})$, and $G_0(\theta)$ denote the nominal terms; and $\Delta M(\theta)$, $\Delta C(\theta, \dot{\theta})$, and $\Delta G(\theta)$ denote the uncertain terms. Therefore, the robot dynamic equation (26) becomes

$$M_0(\theta)\ddot{\theta} + C_0(\theta, \dot{\theta}) + G_0(\theta) = \tau(t) + \Upsilon(\theta, \dot{\theta}, t) \quad (30)$$

where

$$\Upsilon(\theta, \dot{\theta}, t) = -\Delta M(\theta) - \Delta C(\theta, \dot{\theta}) - \Delta G(\theta) - F(\dot{\theta}) + \tau_d(t).$$

The robot dynamic equation (30) can be converted to the below form

$$\ddot{\theta} = M_0^{-1}(\theta) \left[\tau(t) - C_0(\theta, \dot{\theta}) - G_0(\theta) + \Upsilon(\theta, \dot{\theta}, t) \right] \quad (31)$$

For simply in designing, the robot dynamic (31) can be rewritten in state space form as

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= f^*(x) + g^*(x)u(t) + \Delta^*(x, t) \end{aligned} \quad (32)$$

where $x_1 = \theta$, $x_2 = \dot{\theta}$, $x = [x_1^T \ x_2^T]^T$, $u(t) = \tau(t)$, $f^*(x) = M_0^{-1}(\theta) [-C_0(\theta, \dot{\theta}) - G_0(\theta)]$, $g^*(x) = M_0^{-1}(\theta)$, and $\Delta^*(x, t) = M_0^{-1}(\theta) \Upsilon(\theta, \dot{\theta}, t)$.

It can be shown that the robot dynamic system (28) is in the same form as (1). Thus, the proposed controller-observer algorithm, which are designed in Section III and Section IV, can be applied directly.

VI. NUMERICAL SIMULATIONS

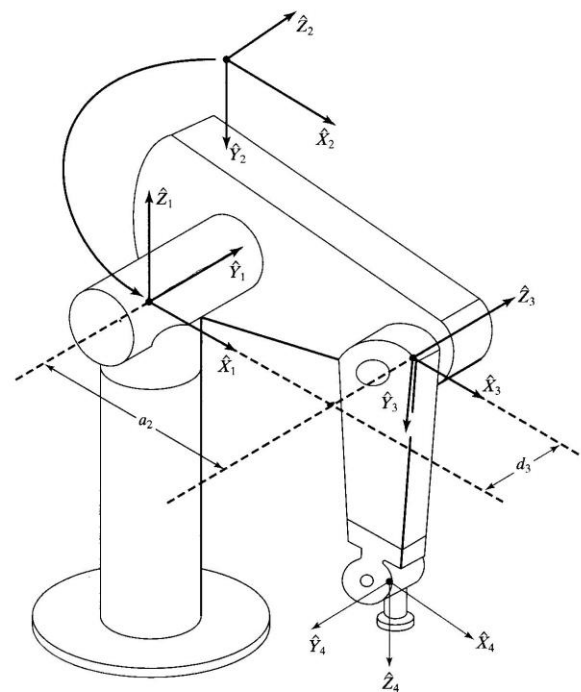


FIGURE 2. PUMA560 robot manipulator.

In this section, a PUMA560 robot manipulator (the last three joints are blocked) as shown in Fig. 2 is used for computer simulation to demonstrate the significance and applicability of the proposed controller-observer method. The specific dynamic model with required parameter values of PUMA560 robot are provided in [46]. In this paper, the simulation analysis is performed by using the MATLAB/Simulink program and the sampling time is 10-3s.

In this work, it is assumed that the desired trajectories to be tracked are

$$\theta_d = \begin{bmatrix} \theta_{d_1} \\ \theta_{d_2} \\ \theta_{d_3} \end{bmatrix} = \begin{bmatrix} 2 \cos(\pi t/6) - 1 \\ 3 \sin(\pi t/7 + \pi/2) - 1 \\ 1.5 \sin(\pi t/5 + \pi/2) - 1 \end{bmatrix} \quad (33)$$

The initial states are chosen as $\theta_1(0) = \theta_2(0) = \theta_3(0) = -0.5$ and $\dot{\theta}_1(0) = \dot{\theta}_2(0) = \dot{\theta}_3(0) = 0$.

The dynamic uncertainties, friction, and external disturbances are assumed as

$$\Delta = \begin{bmatrix} \Delta_1 \\ \Delta_2 \\ \Delta_3 \end{bmatrix} = \begin{bmatrix} -1.1\dot{q}_1 + 1.2 \sin(3q_1 + \pi/2) - \cos(t) \\ 1.65\dot{q}_2 - 2.14 \cos(2q_2) + 0.5 \sin(t) \\ -0.5\dot{q}_3 + 1.3 \sin(2.5q_3 - \pi/2) + 0.7 \sin(0.5t) \end{bmatrix} \quad (34)$$

$$F(\dot{\theta}) = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix} = \begin{bmatrix} 1.9 \cos(2\dot{q}_1) \\ 2.03 \sin(\dot{q}_2 + \pi/2) - 1 \\ 1.76 \cos(0.9\dot{q}_3) \end{bmatrix} \quad (35)$$

$$\tau_d = \varsigma(t - T_f) \begin{bmatrix} \tau_{d_1} \\ \tau_{d_2} \\ \tau_{d_3} \end{bmatrix} = \varsigma(t - T_f) \begin{bmatrix} -12.5 \sin(\pi t/4 + \pi/3) \\ 13.7 \cos(\pi t/5 + \pi/2) \\ 7.5 \sin(\pi t/3) \end{bmatrix} \quad (36)$$

where T_f denotes the time of occurrence and

$\varsigma(t - T_f) = \text{diag}\{\varsigma_1(t - T_f), \varsigma_2(t - T_f), \dots, \varsigma_n(t - T_f)\}$ represents the time profile of the of the external disturbances.

With $\varsigma_i(t - T_f) = \begin{cases} 0 & \text{if } t \leq T_f \\ 1 - e^{-\sigma_i(t - T_f)} & \text{if } t \geq T_f \end{cases}$ and $\sigma_i > 0$

denote the evolution rate.

In this simulation, the external disturbances occur at $T_f = 20s$. The parameters of the controllers using in this simulation are chosen as $L = 9$, $\alpha_1 = 1/2$, $\alpha_2 = 2/3$, $\Xi = 1$, $\kappa_1 = \text{diag}(15, 15, 15)$, $\kappa_2 = \text{diag}(10, 10, 10)$.

The simulation consists three parts. First, the estimation results of the TOSM observer is compared with that of the SOSM observer - which is designed in Appendix A. Second, the proposed NFTSM controller-observer method is compared with NFTSM controllers with and without compensating the estimation of lumped uncertainties - which are designed in Appendix B, C, and D. Finally, we will compare the proposed controller-observer strategy with the NFTSM controller with compensation of obtained lumped uncertainties from TOSM observer, which is designed in Appendix D, in term of changing value of the switching gain.

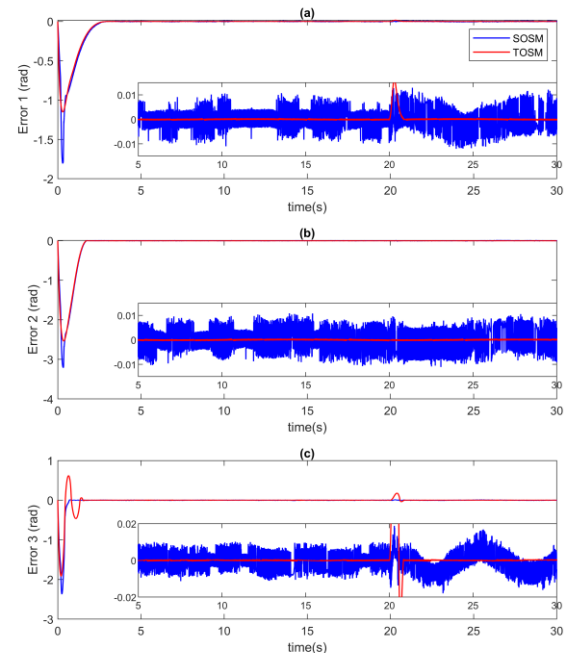


Figure 3. Velocity estimation errors of the TOSM observer compared with the SOSM observer.

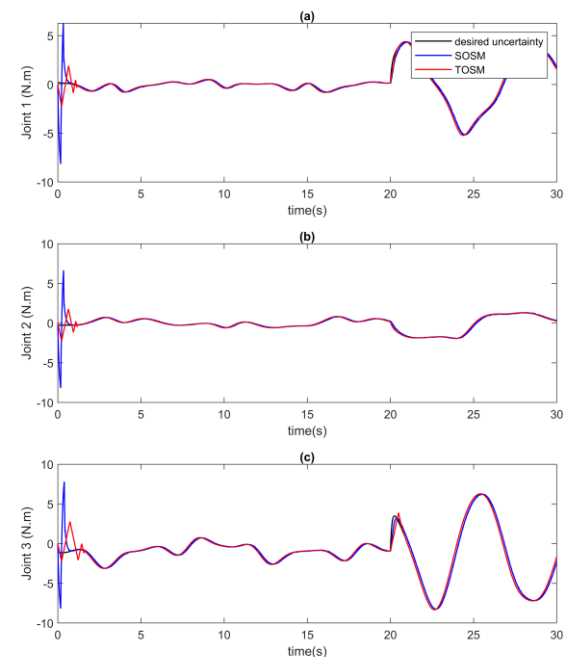


Figure 4. Uncertainty estimation of the TOSM observer compared with the SOSM observer.

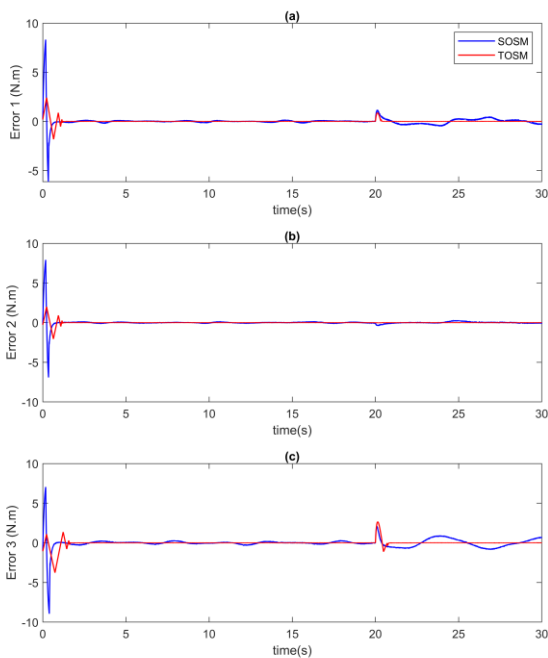


Figure 5. Uncertainties estimation error of the TOSM observer compared with the SOSM observer.

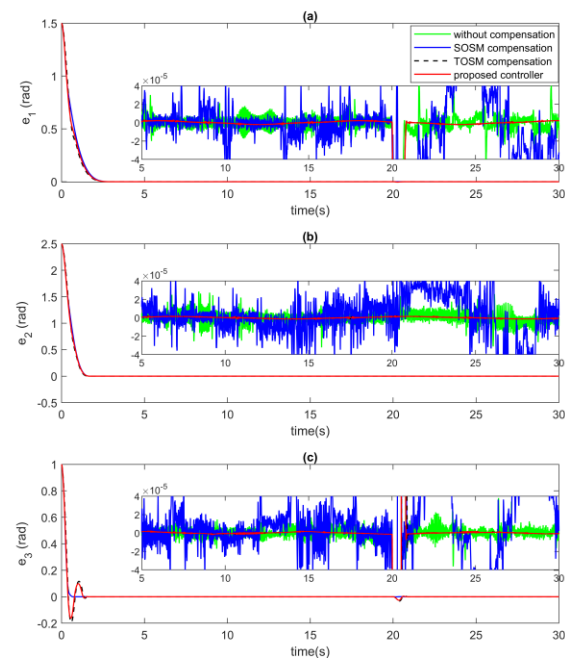


Figure 7. Tracking error of the proposed controller-observer method compared with the NFTSM control with and without uncertainty compensation.

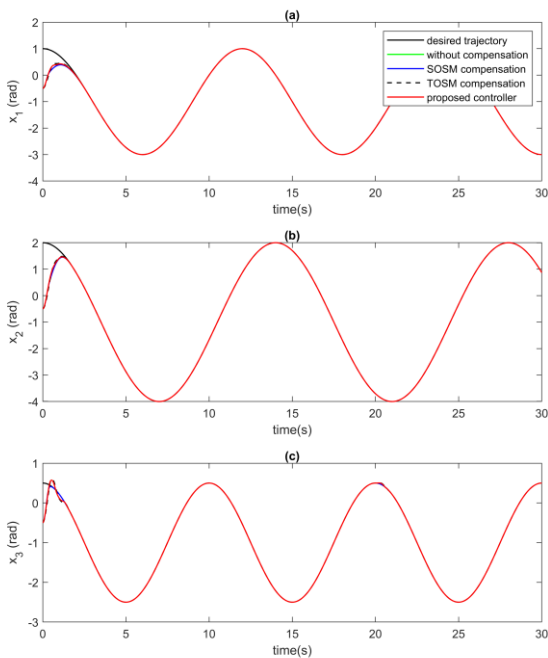


Figure 6. Tracking position of the proposed controller-observer method compared with the NFTSM control with and without uncertainty compensation.

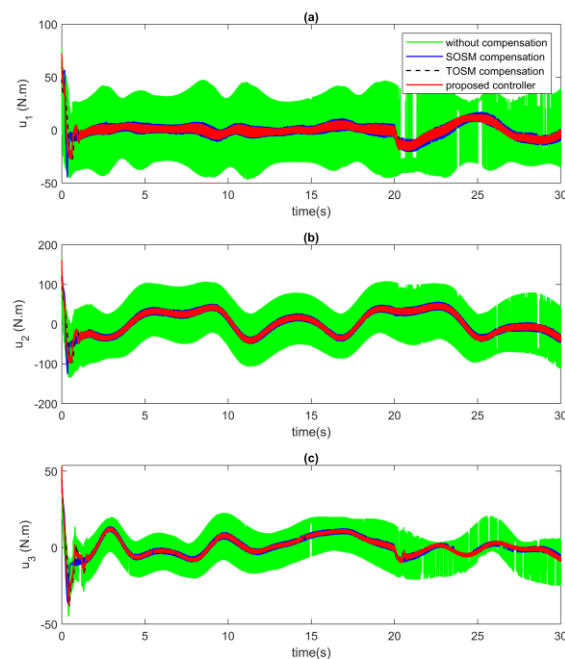


Figure 8. Control input of proposed controller-observer method compared with the NFTSM control with and without uncertainty compensation.

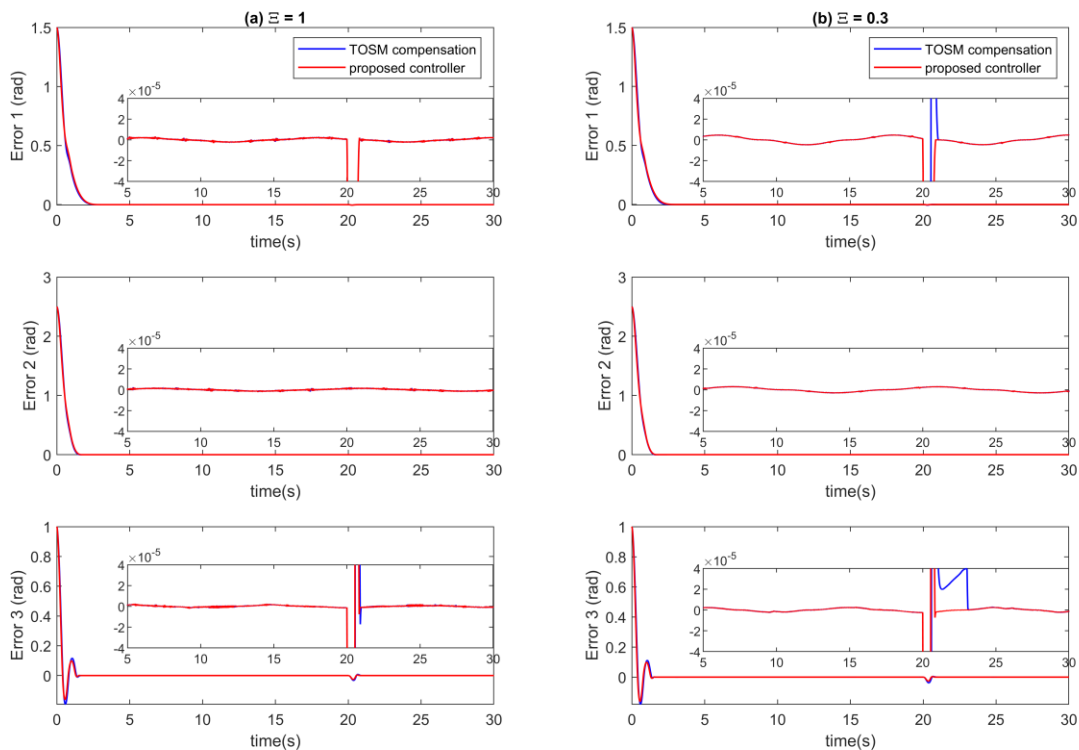


Figure 9. Position tracking error of the controller-observer control method compared with NFTSMC-TOSMO: a) $\Xi = 1$, b) $\Xi = 0.3$.

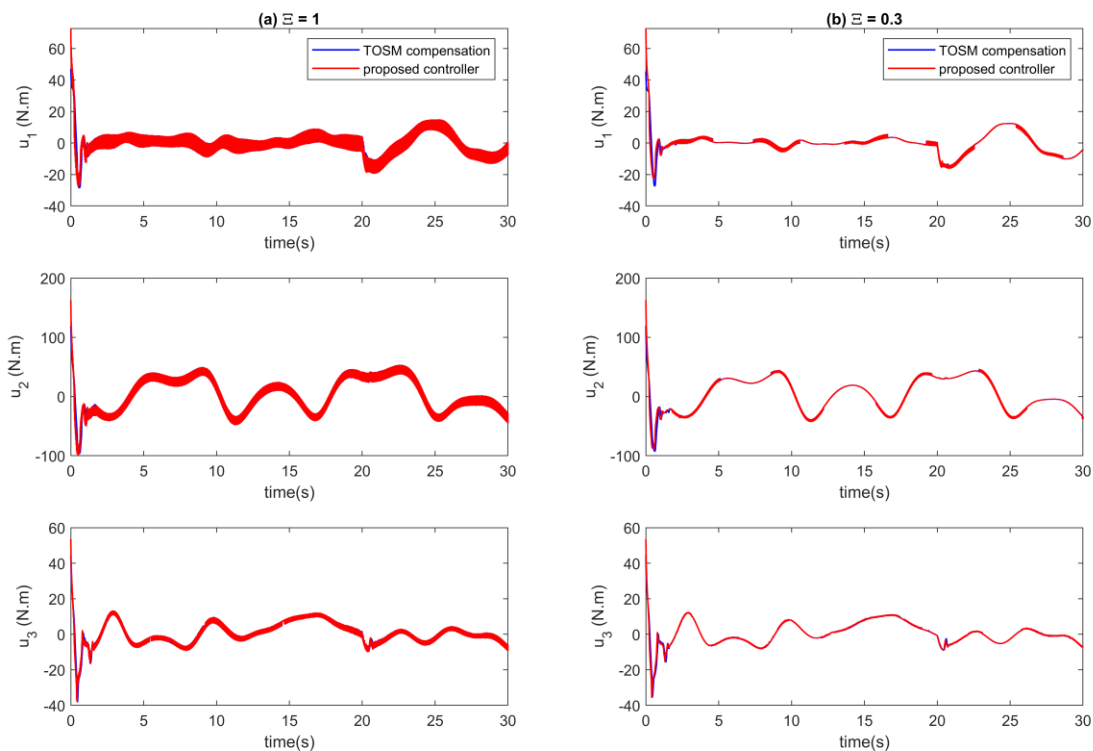


Figure 10. Control input of the proposed controller-observer method compared with NFTSMC-TOSMO: a) $\Xi = 1$, b) $\Xi = 0.3$.

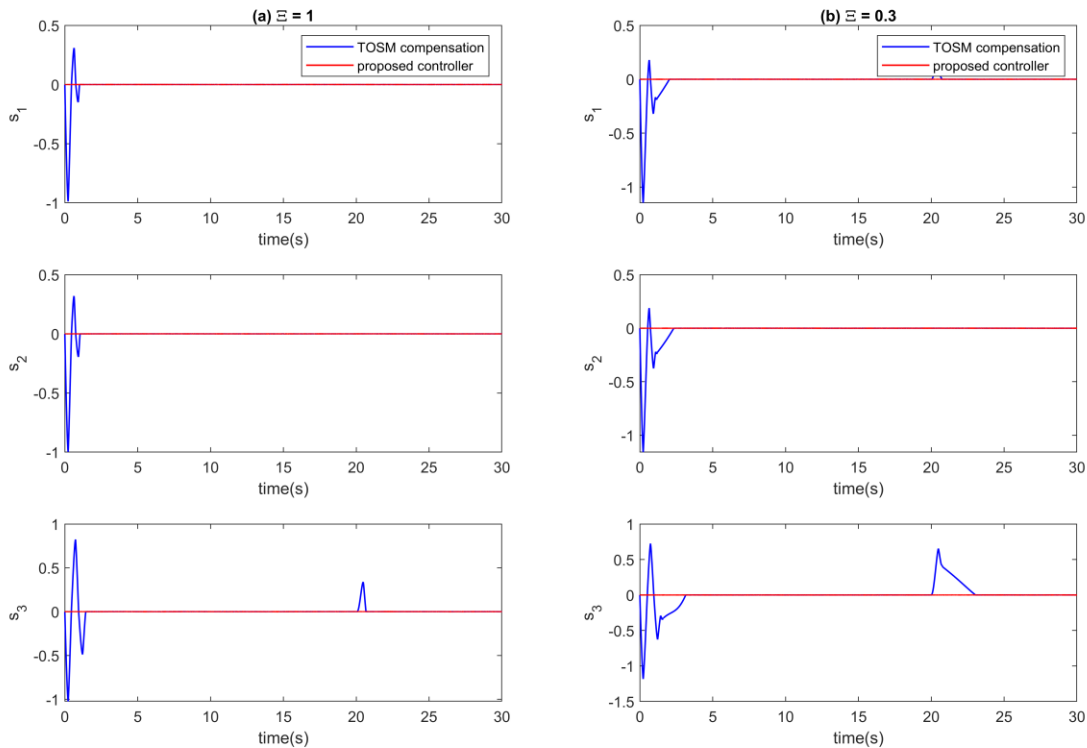


Figure 11. Sliding function of the proposed controller-observer method compared with NFTSMC-TOSMO: a) $\Xi = 1$, b) $\Xi = 0.3$.

For the first part of the simulation, the comparison results between the TOSM observer and the SOSM observer are shown in Fig. 3, Fig. 4, and Fig. 5. Fig. 3 shows the obtained estimation error of velocity, as we can see that the TOSM observer can estimate the system velocity with higher precision whereas the SOSM observer provides a larger chattering in the estimation signal. In term of the estimation of lumped uncertainties, the SOSM observer requires a lowpass filter to reconstruct the estimation information that is the cause of time delay reducing the precision of this observer. On the contrary, the TOSM observer can construct the lumped uncertainties directly without the need of using lowpass filter. The simulation results of the estimation of lumped uncertainties and estimation errors in Fig. 4 and Fig. 5 indicate that the TOSM observer can obtain higher estimation accuracy than that of the SOSM observer. However, as a trade-off, the convergence time is little slower. It is worth noting that the more accurate the estimation information, the higher the control performance.

For the second part of the simulation, the comparison results among the proposed controller-observer strategy in Eq. 16-18, the NFTSM controller without the uncertainties compensation in Eq. 43-45, the NFTSM controller with SOSM observer compensation (NFTSMC-SOSMO) in Eq. 50-53, and the NFTSM controller with TOSM observer

compensation (NFTSMC-TOSMO) in Eq. 58-61 are presented in Fig. 6, Fig. 7 and Fig. 8. The tracking position and the tracking error among three joints are shown in Fig. 6 and Fig. 7, respectively. Fig. 8 presents the control input of controllers among three joints. As shown in the figures, the proposed controller-observer strategy can provide higher tracking precision and less chattering in control input compared with others, except for the NFTSMC-TOSMO.

In order to show the superior properties of the proposed controller-observer strategy compared with the NFTSMC-TOSMO, we go into the third part of the simulation in which the switching gain, Ξ , of the switching control law, u_{sw} , is changed. The comparison of position tracking error between the proposed control method and the NFTSMC-TOSMO is shown in Fig. 9. The results show that when the switching gain, $\Xi = 1$, the proposed controller-observer method and the NFTSMC-TOSMO can provide almost the same position tracking performance. However, when reducing the switching gain Ξ to 0.3, the NFTSMC-TOSMO provides a slower convergence time. This happened because the switching gain, Ξ , in the proposed controller-observer strategy is only used against the effect of the uncertainties' estimation error whereas in the NFTSMC-TOSMO, this gain is used to handle the effect of both the uncertainties' estimation error and its overshooting. It is worth mentioning that the smaller the selected switching gain, Ξ , the lower the chattering in the

control input signal, which is presented in Fig. 10. The comparison of sliding function is illustrated in Fig. 11. As we can see that the proposed controller-observer method can provide fast convergence of sliding function for both cases. On the contrary, the convergence of sliding function of the NFTSMC-TOSMO increases when the value of switching gain, Ξ , is reduced. It means that the sliding mode will reach slower.

VII. CONCLUSIONS

This paper has proposed an effective controller-observer method for the class of second-order uncertain nonlinear systems. The ability to approximate system velocities of the TOSM observer eliminates the requirement of tachometer in the system thus the device's cost and size can be reduced. Moreover, the obtained lumped uncertainties with high accuracy increases the controller performance when applying the estimated information to compensate the uncertainties' effects. The proposed NFTSMC method provides high tracking accuracy, fast response time, low chattering phenomenon, robustness against the lumped uncertainties, faster reaching to the sliding motion and finite-time convergence of the system states. The finite-time stability of both observer and controller have been demonstrated in theory. The proposed controller-observer algorithm has been successfully applied to robot manipulator and its effectiveness has been verified by simulation results.

APPENDIX

A. DESIGN OF SOSM OBSERVER

Based on system (1), the SOSM observer is designed in [37] as

$$\begin{aligned}\dot{\hat{x}}_1 &= \hat{x}_2 + k_1 |x_1 - \hat{x}_1|^{1/2} \text{sign}(x_1 - \hat{x}_1) \\ \dot{\hat{x}}_2 &= f(\hat{x}, t) + g(\hat{x}, t)u(t) + k_2 \text{sign}(x_1 - \hat{x}_1)\end{aligned}\quad (37)$$

where \hat{x} denotes the estimation of x and k_i denote the observer gains.

By subtracting (1) to (33) we can obtain the estimation error as

$$\begin{aligned}\dot{\tilde{x}}_1 &= -k_1 |\tilde{x}_1|^{1/2} \text{sign}(\tilde{x}_1) + \tilde{x}_2 \\ \dot{\tilde{x}}_2 &= -k_2 \text{sign}(\tilde{x}_1) + \Delta(x, u, t) - d(x, \tilde{x}, u, t)\end{aligned}\quad (38)$$

where $\tilde{x} = x - \hat{x}$ and $d(x, \tilde{x}, u, t) = \{f(\hat{x}, t) + g(\hat{x}, t)u(t)\} - \{f(x, t) + g(x, t)u(t)\}$.

After the convergence process, the differentiators will converge to zero, thus the estimated states will reach the real states ($\hat{x}_1 = x_1$, $\hat{x}_2 = x_2$) and the lumped uncertainties' estimation will be equal to zero, $d(x, \tilde{x}, u, t) = 0$. The lumped uncertainties can be reconstructed as

$$\hat{\Delta}(x, u, t) = k_2 \text{sign}(\tilde{x}_1) \quad (39)$$

As we can see, the equivalent output injection of SOSM observer is the result of the discontinuous terms $k_2 \text{sign}(\tilde{x}_1)$, which cause the chattering phenomenon in estimation signal. For that reason, a lowpass filter is required to reconstruct the estimation of the lumped uncertainties.

The observer gains in (38) could be selected based on [44] as $k_1 = \alpha_1 L^{2/3}$, and $k_2 = \alpha_2 L$ where $\alpha_1 = 2.12$, and $\alpha_2 = 1.1$.

B. DESIGN OF NFTSMC WITHOUT UNCERTAINTIES COMPENSATION

The tracking errors and velocity errors as (10), the terminal sliding function is selected as (11). The control law is designed as follows:

$$u = -g(x, t)^{-1} (u_{eq} + u_{sw}) \quad (40)$$

$$u_{eq} = f(x, t) + \kappa_2 |\dot{e}|^{\alpha_2} \text{sign}(\dot{e}) + \kappa_1 |e|^{\alpha_1} \text{sign}(e) - \ddot{x}_d \quad (41)$$

$$u_{sw} = (\Lambda + \mu) \text{sign}(s) \quad (42)$$

After substituting the estimation of system velocities from the TOSM observer (4), the control law becomes

$$u = -g(\hat{x}, t)^{-1} (u_{eq} + u_{sw}) \quad (43)$$

$$u_{eq} = f(\hat{x}, t) + \kappa_2 |\hat{\dot{e}}|^{\alpha_2} \text{sign}(\hat{\dot{e}}) + \kappa_1 |e|^{\alpha_1} \text{sign}(e) - \ddot{x}_d \quad (44)$$

$$u_{sw} = (\Lambda + \mu) \text{sign}(\hat{s}) \quad (45)$$

The switching control law here is used to compensate for the lumped uncertainties.

C. DESIGN OF NFTSMC WITH SOSM OBSERVER COMPENSATION

The tracking errors and velocity errors as (10), the terminal sliding function is selected as (11). The control law is designed as follows:

$$u = -g(x, t)^{-1} (u_{eq} + u_{sw} + u_c) \quad (46)$$

$$u_{eq} = f(x, t) + \kappa_2 |\dot{e}|^{\alpha_2} \text{sign}(\dot{e}) + \kappa_1 |e|^{\alpha_1} \text{sign}(e) - \ddot{x}_d \quad (47)$$

$$u_{sw} = (\Xi + \mu) \text{sign}(s) \quad (48)$$

$$u_c = k_2 \text{sign}(\tilde{x}_1) \quad (49)$$

Substituting the estimation of system velocities from the SOSM observer (37), the control law becomes

$$u = -g(\hat{x}, t)^{-1} (u_{eq} + u_{sw} + u_c) \quad (50)$$

$$u_{eq} = f(\hat{x}, t) + \kappa_2 |\hat{\dot{e}}|^{\alpha_2} \text{sign}(\hat{\dot{e}}) + \kappa_1 |e|^{\alpha_1} \text{sign}(e) - \ddot{x}_d \quad (51)$$

$$u_{sw} = (\Xi + \mu) \text{sign}(\hat{s}) \quad (52)$$

$$u_c = k_2 \text{sign}(\tilde{x}_1) \quad (53)$$

The switching control law here is used to compensate for the uncertainty's estimation error of the SOSM observer and the compensation element, u_c , is obtained from (39).

D. DESIGN OF NFTSMC WITH TOSM OBSERVER COMPENSATION

The tracking errors and velocity errors as (10), the terminal sliding function is selected as (11). The control law is designed as follows:

$$u = -g(x, t)^{-1} (u_{eq} + u_{sw} + u_c) \quad (54)$$

$$u_{eq} = f(x, t) + \kappa_2 |\dot{e}|^{\alpha_2} \text{sign}(\dot{e}) + \kappa_1 |e|^{\alpha_1} \text{sign}(e) - \ddot{x}_d \quad (55)$$

$$u_{sw} = (\Xi + \mu) \text{sign}(s) \quad (56)$$

$$u_c = \int \gamma_3 \text{sign}(\tilde{x}_1) \quad (57)$$

Substituting the estimation of system velocities from the TOSM observer (4), the control law becomes

$$u = -g(\hat{x}, t)^{-1} (u_{eq} + u_{sw} + u_c) \quad (58)$$

$$u_{eq} = f(\hat{x}, t) + \kappa_2 |\hat{e}|^{\alpha_2} \text{sign}(\hat{e}) + \kappa_1 |e|^{\alpha_1} \text{sign}(e) - \ddot{x}_d \quad (59)$$

$$u_{sw} = (\Xi + \mu) \text{sign}(\hat{s}) \quad (60)$$

$$u_c = \int \gamma_3 \text{sign}(\tilde{x}_1) \quad (61)$$

The switching control law here is used to compensate for the uncertainty's estimation error of the TOSM observer and the compensation element, u_c , is obtained from (9).

ACKNOWLEDGMENT

This research was supported by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education (2019R1D1A3A03103528).

REFERENCES

- [1] Y. Wang, L. Xie, and C. E. De Souza, "Robust control of a class of uncertain nonlinear systems," *Syst. Control Lett.*, vol. 19, no. 2, pp. 139–149, 1992.
- [2] B. Xian, D. M. Dawson, M. S. de Queiroz, and J. Chen, "A continuous asymptotic tracking control strategy for uncertain nonlinear systems," *IEEE Trans. Automat. Contr.*, vol. 49, no. 7, pp. 1206–1211, 2004.
- [3] M. C. M. Teixeira and S. H. Zak, "Stabilizing controller design for uncertain nonlinear systems using fuzzy models," *IEEE Trans. Fuzzy Syst.*, vol. 7, no. 2, pp. 133–142, 1999.
- [4] Y. Song, X. Huang, and C. Wen, "Robust adaptive fault-tolerant PID control of MIMO nonlinear systems with unknown control direction," *IEEE Trans. Ind. Electron.*, vol. 64, no. 6, pp. 4876–4884, 2017.

- [5] J. Zhang and L. Guo, "Theory and design of PID controller for nonlinear uncertain systems," *IEEE Control Syst. Lett.*, vol. 3, no. 3, pp. 643–648, 2019.
- [6] J. Zhou, C. Wen, and Y. Zhang, "Adaptive backstepping control of a class of uncertain nonlinear systems with unknown backlash-like hysteresis," *IEEE Trans. Automat. Contr.*, vol. 49, no. 10, pp. 1751–1759, 2004.
- [7] C. Wen, J. Zhou, Z. Liu, and H. Su, "Robust adaptive control of uncertain nonlinear systems in the presence of input saturation and external disturbance," *IEEE Trans. Automat. Contr.*, vol. 56, no. 7, pp. 1672–1678, 2011.
- [8] W. Deng, J. Yao, and D. Ma, "Adaptive control of input delayed uncertain nonlinear systems with time-varying output constraints," *IEEE Access*, vol. 5, pp. 15271–15282, 2017.
- [9] H.-K. Lam, F. H. F. Leung, and P. K.-S. Tam, "Stable and robust fuzzy control for uncertain nonlinear systems," *IEEE Trans. Syst. Man, Cybern. A Syst. Humans*, vol. 30, no. 6, pp. 825–840, 2000.
- [10] M. Van, H.-J. Kang, and Y.-S. Suh, "A novel fuzzy second-order sliding mode observer-controller for a TS fuzzy system with an application for robot control," *Int. J. Precis. Eng. Manuf.*, vol. 14, no. 10, pp. 1703–1711, 2013.
- [11] D. Wang, "Neural network-based adaptive dynamic surface control of uncertain nonlinear pure-feedback systems," *Int. J. Robust Nonlinear Control*, vol. 21, no. 5, pp. 527–541, 2011.
- [12] A. T. Vo and H.-J. Kang, "Adaptive neural integral full-order terminal sliding mode control for an uncertain nonlinear system," *IEEE Access*, vol. 7, pp. 42238–42246, 2019.
- [13] M. Chen and W.-H. Chen, "Sliding mode control for a class of uncertain nonlinear system based on disturbance observer," *Int. J. Adapt. Control Signal Process.*, vol. 24, no. 1, pp. 51–64, 2010.
- [14] X.-T. Tran and H.-J. Kang, "Adaptive hybrid high-order terminal sliding mode control of MIMO uncertain nonlinear systems and its application to robot manipulators," *Int. J. Precis. Eng. Manuf.*, vol. 16, no. 2, pp. 255–266, 2015.
- [15] M. Van, "An enhanced robust fault tolerant control based on an adaptive fuzzy PID-nonsingular fast terminal sliding mode control for uncertain nonlinear systems," *IEEE/ASME Trans. Mechatronics*, vol. 23, no. 3, pp. 1362–1371, 2018.
- [16] A. T. Vo and H.-J. Kang, "An adaptive terminal sliding mode control for robot manipulators with non-singular terminal sliding surface variables," *IEEE Access*, vol. 7, pp. 8701–8712, 2018.
- [17] V.-C. Nguyen, A.-T. Vo, and H.-J. Kang, "Continuous PID Sliding Mode Control Based on Neural Third Order Sliding Mode Observer for Robotic Manipulators," in *International Conference on Intelligent Computing*, 2019, pp. 167–178.
- [18] M. Zhihong, A. P. Paplinski, and H. R. Wu, "A robust MIMO terminal sliding mode control scheme for rigid robotic manipulators," *IEEE Trans. Automat. Contr.*, vol. 39, no. 12, pp. 2464–2469, 1994.
- [19] H. Wang *et al.*, "Design and implementation of adaptive terminal sliding-mode control on a steer-by-wire equipped road vehicle," *IEEE Trans. Ind. Electron.*, vol. 63, no. 9, pp. 5774–5785, 2016.
- [20] Y. Wu, X. Yu, and Z. Man, "Terminal sliding mode control design for uncertain dynamic systems," *Syst. Control Lett.*, vol. 34, no. 5, pp. 281–287, 1998.
- [21] S. Mobayen, "Fast terminal sliding mode controller design for nonlinear second-order systems with time-varying uncertainties," *Complexity*, vol. 21, no. 2, pp. 239–244, 2015.
- [22] C. U. Solis, J. B. Clempner, and A. S. Poznyak, "Fast terminal sliding-mode control with an integral filter applied to a Van Der Pol oscillator," *IEEE Trans. Ind. Electron.*, vol. 64, no. 7, pp. 5622–5628, 2017.
- [23] T. Madani, B. Daachi, and K. Djouani, "Modular-controller-design-based fast terminal sliding mode for articulated exoskeleton systems," *IEEE Trans. Control Syst. Technol.*, vol. 25, no. 3, pp. 1133–1140, 2016.
- [24] C.-K. Lin, "Nonsingular terminal sliding mode control of robot manipulators using fuzzy wavelet networks," *IEEE Trans. Fuzzy Syst.*, vol. 14, no. 6, pp. 849–859, 2006.
- [25] M. Jin, J. Lee, and K. K. Ahn, "Continuous nonsingular terminal sliding-mode control of shape memory alloy actuators using time

- delay estimation," *IEEE/ASME Trans. Mechatronics*, vol. 20, no. 2, pp. 899–909, 2014.
- [26] S. Eshghi and R. Varatharajoo, "Nonsingular terminal sliding mode control technique for attitude tracking problem of a small satellite with combined energy and attitude control system (CEACS)," *Aerosp. Sci. Technol.*, vol. 76, pp. 14–26, 2018.
- [27] L. Yang and J. Yang, "Nonsingular fast terminal sliding-mode control for nonlinear dynamical systems," *Int. J. Robust Nonlinear Control*, vol. 21, no. 16, pp. 1865–1879, 2011.
- [28] V. Anh Tuan and H.-J. Kang, "A new finite time control solution for robotic manipulators based on nonsingular fast terminal sliding variables and the adaptive super-twisting scheme," *J. Comput. Nonlinear Dyn.*, vol. 14, no. 3, 2019.
- [29] M. Van, S. S. Ge, and H. Ren, "Finite time fault tolerant control for robot manipulators using time delay estimation and continuous nonsingular fast terminal sliding mode control," *IEEE Trans. Cybern.*, vol. 47, no. 7, pp. 1681–1693, 2016.
- [30] M. Van, M. Mavrouniotis, and S. S. Ge, "An adaptive backstepping nonsingular fast terminal sliding mode control for robust fault tolerant control of robot manipulators," *IEEE Trans. Syst. Man, Cybern. Syst.*, vol. 49, no. 7, pp. 1448–1458, 2018.
- [31] M. Jin, S. H. Kang, and P. H. Chang, "Robust compliant motion control of robot with nonlinear friction using time-delay estimation," *IEEE Trans. Ind. Electron.*, vol. 55, no. 1, pp. 258–269, 2008.
- [32] Q. Zhou, P. Shi, S. Xu, and H. Li, "Observer-based adaptive neural network control for nonlinear stochastic systems with time delay," *IEEE Trans. Neural Networks Learn. Syst.*, vol. 24, no. 1, pp. 71–80, 2012.
- [33] F. Abdollahi, H. A. Talebi, and R. V. Patel, "A stable neural network-based observer with application to flexible-joint manipulators," *IEEE Trans. Neural Networks*, vol. 17, no. 1, pp. 118–129, 2006.
- [34] A. Chalanga, S. Kamal, L. M. Fridman, B. Bandyopadhyay, and J. A. Moreno, "Implementation of super-twisting control: Super-twisting and higher order sliding-mode observer-based approaches," *IEEE Trans. Ind. Electron.*, vol. 63, no. 6, pp. 3677–3685, 2016.
- [35] A. T. Vo, H.-J. Kang, and V.-C. Nguyen, "An output feedback tracking control based on neural sliding mode and high order sliding mode observer," in *2017 10th International Conference on Human System Interactions (HSI)*, 2017, pp. 161–165.
- [36] J. A. Moreno and M. Osorio, "A Lyapunov approach to second-order sliding mode controllers and observers," in *2008 47th IEEE conference on decision and control*, 2008, pp. 2856–2861.
- [37] J. Davila, L. Fridman, and A. Levant, "Second-order sliding-mode observer for mechanical systems," *IEEE Trans. Automat. Contr.*, vol. 50, no. 11, pp. 1785–1789, 2005.
- [38] F. A. Ortiz-Ricardez, T. Sánchez, and J. A. Moreno, "Smooth Lyapunov function and gain design for a second order differentiator," in *2015 54th IEEE Conference on Decision and Control (CDC)*, 2015, pp. 5402–5407.
- [39] M. Van, H.-J. Kang, Y.-S. Suh, and K.-S. Shin, "Output feedback tracking control of uncertain robot manipulators via higher-order sliding-mode observer and fuzzy compensator," *J. Mech. Sci. Technol.*, vol. 27, no. 8, pp. 2487–2496, 2013.
- [40] H. Imine, A. Benallegue, T. Madani, and S. Srairi, "Rollover risk prediction of heavy vehicle using high-order sliding-mode observer: Experimental results," *IEEE Trans. Veh. Technol.*, vol. 63, no. 6, pp. 2533–2543, 2013.
- [41] Q. Xu, "Continuous integral terminal third-order sliding mode motion control for piezoelectric nanopositioning system," *IEEE/ASME Trans. Mechatronics*, vol. 22, no. 4, pp. 1828–1838, 2017.
- [42] X.-T. Tran, H. Oh, I.-R. Kim, and S. Kim, "Attitude stabilization of flapping micro-air vehicles via an observer-based sliding mode control method," *Aerosp. Sci. Technol.*, vol. 76, pp. 386–393, 2018.
- [43] M. Van, P. Franciosa, and D. Ceglarek, "Fault diagnosis and fault-tolerant control of uncertain robot manipulators using high-order sliding mode," *Math. Probl. Eng.*, vol. 2016, 2016.
- [44] A. Levant, "Higher-order sliding modes, differentiation and output-feedback control," *Int. J. Control*, vol. 76, no. 9–10, pp. 924–941, 2003.
- [45] S. P. Bhat and D. S. Bernstein, "Finite-time stability of continuous autonomous systems," *SIAM J. Control Optim.*, vol. 38, no. 3, pp.

751–766, 2000.

- [46] B. Armstrong, O. Khatib, and J. Burdick, "The explicit dynamic model and inertial parameters of the PUMA 560 arm," in *Proceedings. 1986 IEEE international conference on robotics and automation*, 1986, vol. 3, pp. 510–518.



Van-Cuong Nguyen received his B.S. degree in Electronic and Telecommunication Engineering from Danang University of Technology, Danang city, Vietnam, in 2014. He is currently working toward a Ph.D. degree in the School of Electrical Engineering, University of Ulsan, Ulsan, Korea. His research interests include robot fault diagnosis, fault tolerant control, sliding mode control.



Anh Tuan Vo received his B.S. degree in Electrical Engineering from Danang University of Technology, Danang city, Vietnam, in 2008. He is currently working toward a Ph.D. degree in the School of Electrical Engineering, University of Ulsan, Ulsan, Korea. His research interests include intelligent control, sliding mode control, robot fault diagnosis, fault tolerant control.



Hee-Jun Kang received his B.S. degree in Mechanical Engineering from Seoul National University, Korea in 1985. He received his M.S and Ph.D. degrees in Mechanical Engineering from The University of Texas at Austin, USA in 1988 and 1991, respectively. Since March in 1992, he has been a Professor of Electrical Engineering in University of Ulsan. His current research interests are sensor based robotic application, robot calibration, haptics, robot fault diagnosis and mechanism analysis.