# INDUSTRIAL CONTROL
***(A B.Sc. Course)***

**Control Engineering Group**
**School of Engineering**
**Shiraz University**

***A Summary of The Course***

**Lectured By: Dr. Ali Akbar Safavi**

**Winter 1384**

***Please note****: This is only a summary of the material discussed during the lectures. Reading the reference books and/or attending lectures are essential for understanding the material.*

## Course Outline for ***Industrial Control***

* Introduction
* Industrial Processes:
    - Liquid Level Systems
    - Thermal Systems
    - Others
* Linearization of Non-Linear Systems
* Empirical Process Modelling
    -First Order Plus Time Delay
    - Higher Orders
    - Others
    - System Identification
* Classical Controllers and Tuning
    - On-Off Switches
    - PID Controllers
    - PID Tuning
        - Z N
        - ISE
        - Others
* Advanced Control Loops
    - Feedforward Control
    - Feedforward/Feedback
    - Cascade Control
    - Delay Compensator
    - Decoupling Control
*Various Types of Controllers
    - Electronic
    - Pneumatic
    - Hydraulic

* Industrial Control Platforms
    - On-Off Switches
    - Industrial PIDs
    - PLC
    - DCS
    - Fieldbus

**مرجع اصلی درس:** اصول و روشهاي كنترل صنعتي
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**Other References for** ***Industrial Control***

1. Process Dynamics, Modeling, and Control (1994)
   By: B.A. Ogaunnaike & W.H. Ray (Oxford University Press)
2. Modern Control Engineering, Fourth Edition (2002)
   By: K. Ogata

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**Some Internet Resources**

**The ECOSSE Control HyperCourse (UK)**
http://eweb.chemeng.ed.ac.uk/courses/control/course/map/intro.html

**Resource Center for**
**Engineering Laboratories on the Web (USA)**
http://chem.engr.utc.edu

**Virtual Control Lab (Germany)**
http://www.esr.ruhr-uni-bochum.de/VCLab/main.html

**MIT Open courses (USA):**
For general (e.e.)
http://ocw.mit.edu/OcwWeb/Electrical-Engineering-and-Computer-Science/index,html

# 1. INTRODUCTION

Control is a vital part of almost all types of industries, from chemical industries to power industries, or from food industries to aerospace industries.

Though the control problems could be different in these industries, the formulation of the problem and the methodologies applied to them are very much similar. A true understanding of the control concepts, however, is essential in order to apply the available control techniques to various applications.

Furthermore, there may always be some close relation between the subject of control and some other subjects such as modeling, optimisation, etc., so that a reasonable understanding of such subjects may also be esential for a control study.

In what follows, an industrially oriented example of a typical chemical process control problem will extensively be introduced. Through this example various industrial control concepts are briefly reviewed. Almost all aspects of this process control example can be extended to other types of control problems.

## 1.1. AN INDUSTRIAL PERSPECTIVE OF A TYPICAL CONTROL PROBLEM

The primary objective of a process industry is to combine processing units, such as reactors, distillation columns, extractors, heat exchangers, etc., integrated in a rational fashion into a process in order to transform raw materials and inut energy into finished products. Such a concept is illustrated in Figure 1.1.

For control study, we define a **plant** or **a process** to be any single processing unit, or combinations of processing unites, used for conversion of raw materials into finished products.

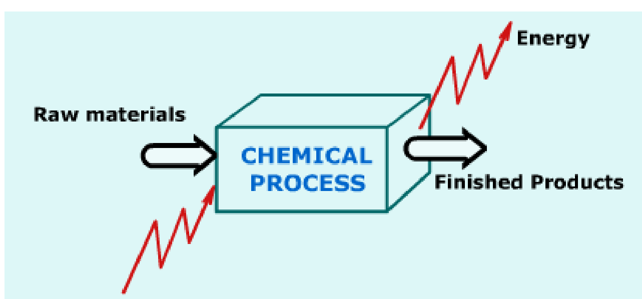



**Figure 1.1.**: A process.

A good example of these ideas is the crude oil fractionation section of a typical oil refinary as shown in Figure 1.2. Here the crude oil as the raw material is pumped from some tanks through the gas-fired preheater furnace, into the fractionator, where separation into such useful products and high boiling resedue take palce. In this example,

- The *processing units* are the storage tanks, the furnace, and the fractionator, along with their respective auxiliary equipment.
- The *raw material* is basically the crude oil; the air and fuel gas fed into the furnace *provide the energy input* realized via firing in the furnace.

- The condensation of lighter material at the top of the fractionator, effected by the cooling unit, constitutes *energy output*.
- The finished products are the naptha and the residue streams from the top and bottom, and the gas oil streams from the mid-sections.

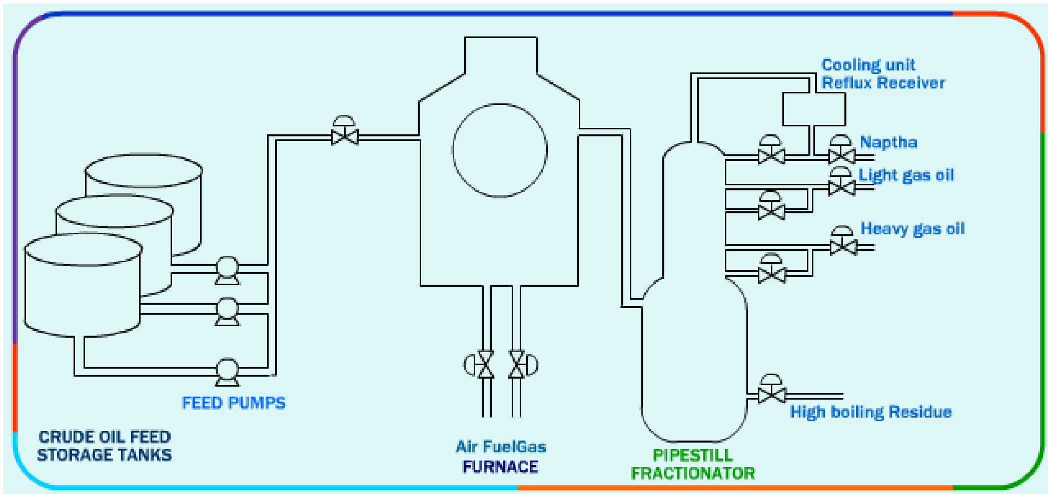



**Figure 1.2.** The upstream end of an oil refinery.

## 1.2 The Basic Principles of Operation

In operation of the processing units of a plant various objectives are usually considered. The following are examples of such objectives in a broad sense.

1. *It is desirable to operate the processing units safely*. This is to say that no unit should be operated at, or near conditions which make the operation unsafe for the equipment itself or for the people around it. In our example, some operating constraints mandated by safety are that the temperature of the furnace tubes and the pressure of the fractionation unit should not exceed their limits.
2. *Specified production rates must be maintained.* The amount of product output required of a plant in time is usually dictated by market requirements. Thus the output should be carefully maintained in the required amount.
3. *Product quality specifications must be maintained*. Product not meeting the required quality specifications must either be discarded as waste, or, where possible, reprocessed at extra cost. The need for economic utilization of resources therefore provides the motivation to satisfy product quality specifications.

For the process shown in Figure 1.2, some operating constraints mandated by *safety* would be that the furnace tubes should not exceed their metallurgical temperature limit and the fractionation unit should not exceed its pressure rating.

The issues of maintaining *production rates* and *product quality* are linked for this process. The products available from crude oil are determined by their boiling points, as shown in Figure 1.3. Thus a lighter crude oil feed could produce more naptha and light gas oil, while a heavier crude oil would produce more heavy gas oil and high boiling residue.

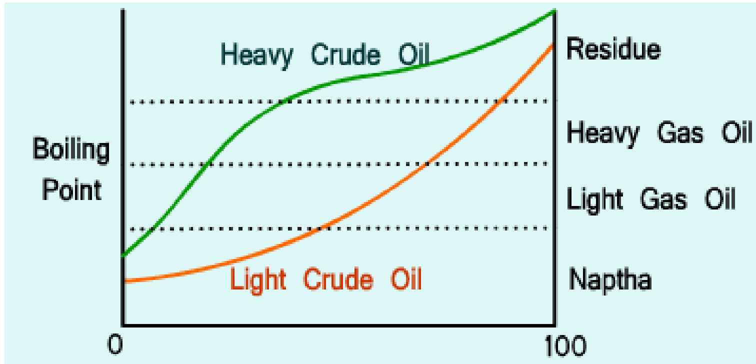



**Figure 1.3**. Crude oil boiling point curve illustrating the product distribution of a light crude oil and a heavy crude oil.

Hence, the production rate possible for each of the products depends on the particular crude oil being fractionated and the quality specifications (usually a maximum boiling point for each fraction above the bottom).

Thus by shifting the maximum boiling point upwards for a product such as naptha or gas oil, one could produce more of it, but it would have a lower quality (i.e., more high-boiling materials

Now, the processes are , by nature, dynamic, by which we mean that their variables are always changing with time. It is clear, therefore, that to achieve the above noted objectives, there is the need to monitor, and be able to induce change in, those key process variables that are related to *safety*, *production rate,* and *product quality*.

This dual task of:

1. Monitoring certain process condition indicator variables, and,
2. Inducing changes in the appropriate process variables in order to improve process conditions

is the job of the *control system*. To achieve good designs for these control systems one must embark on the study of a new field, defined as follows:

.•*Process Dynamics and Control is that aspect of engineering concerned with the analysis, design, and implementation of control systems that facilitate the achievement of specified objectives of process safety, production rates, and product quality.*

## 1.3 AN INDUSTRIAL PERSPECTIVE OF A TYPICAL PROCESS CONTROL PROBLEM

The *next* phase of our presentation of introductory concepts involves the definition of certain terms that are routinely used in connection with various components of a process, and an introduction to the concept of a process control system. This will be done in next sub-sections.

To motivate the discussion, however, let us first examine atypical industrial control problem, and present what may well be a typical attempt to solve such problems, by following a simulated discussion between a plant engineer (PE) and a control engineer (CE).

As industrial systems go, this particular example is deliberately chosen to be simple, yet possessing enough important problematic features to capture the essence of control applications in the process industry. This allows us to focus on the essentials and avoid getting bogged down with complex details that may only be distracting at this point.

### 1.3.1 The Problem

The process unit under consideration is the furnace in Figure 1.2 used to preheat the crude oil feed material to the fractionator. A more detailed schematic diagram is shown in Figure 1.4. Such units are typically found in refineries and petrochemical plants.

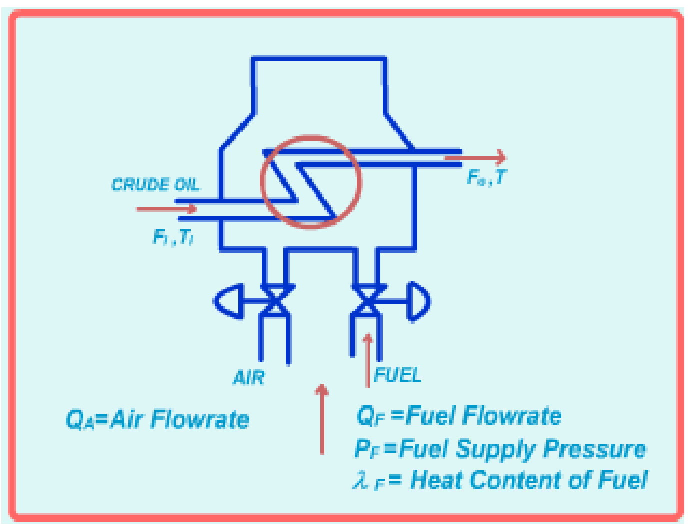



Figure 1.4. Crude oil preheater furnace.

The crude oil flowrate F and temperature Ti; at the inlet of the furnace tend to fluctuate substantially. The flowrate and temperature of the crude oil at the outlet of the furnace are, respectively, $F_0$ and $T$.

It is desired to deliver the crude oil feed to the fractionator at a *constant* temperature T*, regardless of the conditions at the furnace inlet. For plant safety reasons, and because of metallurgical limits, it is mandatory that the furnace tube temperature not exceed the value $T_m$.

The heat content of the heating fuel, as well as the fuel supply pressure, are also known to vary because of disturbances in the fuel gas coming from a different processing unit in the refinery complex.

The furnace control problem may be summarized as follows:

*Deliver crude oil feed to the fractionator at a* constant *temperature $T^*$, and flowrate $F_0$, regardless of all the factors potentially capable of causing the furnace outlet temperature $T$ to deviate from this desired value, making sure that the temperature of the tube surfaces within the furnace does not, at any time, exceed the value $T_m$.*

Observe the presence of the three objectives related to *safety, product quality, and production rate,* namely: furnace temperature limit $T_m$, the required target temperature $T^*$, for the furnace "product", and the crude oil throughput $F_0$, respectively.

### 1.3.2 Evolving Effective Solutions

The various phases in the evolution of an acceptable solution to typical industrial control problems are illustrated by the following dialogue between a *plant engineer* (PE), charged with the responsibility of smooth operation of the plant (in this case, the furnace), and the *control engineer* (CE), who is responsible for assisting in providing solutions to *control-related* process operation problems.

**Phase 1**

CE: What are your operating objectives?

PE: We would like to deliver the crude oil to the fractionation unit downstream at a
constant target temperature T* . The value of this *set-point* is usually determined

by the crude oil type, and desired refinery throughput; it therefore changes every

2-3 days. Also, we have an upper limit constraint (Tm) on how high the furnace tube temperatures can get.

CE: So, of your two process outputs, Fo, and T, the former is set externally by the

fractionator, while the latter is the one you are concerned about controlling?

PE: Yes.

CE: Your control objective is therefore to regulate the process output T as well as deal with the servo problem of set-point changes every 2-3 days?

PE: Yes.

CE: Of your input variables which ones do you really have control over?

PE: Only the air flowrate, and the fuel flowrate; and even then, we usually preset the air flowrate and change only the fuel flowrate when necessary. Our main *control variable* is the air-to-fuel ratio.

CE: The other input variables, the crude oil feed rate F, and inlet temperature Ti, are therefore disturbances'?

PE: Yes.

CE: Any other process variables of importance that I should know of?

PE: Yes, the fuel supply pressure Pf, and the fuel's heat content /lf; they vary significantly, and we don't have any control over these variations. They are also disturbances.

CE: What sort of *instrumentation* do you have for *data acquisition* and *control action implementation* ?

PE: We have thermocouples for measuring the temperatures *T* and 7"; ,- flow meters for measuring F, $Q_F$ • and a control valve on the fuel line. We have an *optical pyrometer* installed for monitoring the furnace tube temperature. An *alarm* is tripped if the temperature gets within a few degrees of the upper limit constraint.

## Phase 2

CE: Do you have a *process model* available for this furnace?

PE: No; but there's an *operator* who understands the process behavior quite well. We have tried running the process on *manual* (control) using this operator, but the results weren't acceptable. The record shown below, taken off the outlet temperature *strip-chart recorder,* is fairly representative. This is the response to a *step increase* in the inlet feedrate F. (See Figure 1.5).

CE: Do you have an idea of what might be responsible?

PE: Yes. We think it has to do with basic human limitations; his anticipation of the effect of the feed disturbance is ingenious, but imperfect, and he just couldn't react fast enough, or accurately enough, to the influence of the additional disturbance effects of variations in fuel supply pressure and heat content.

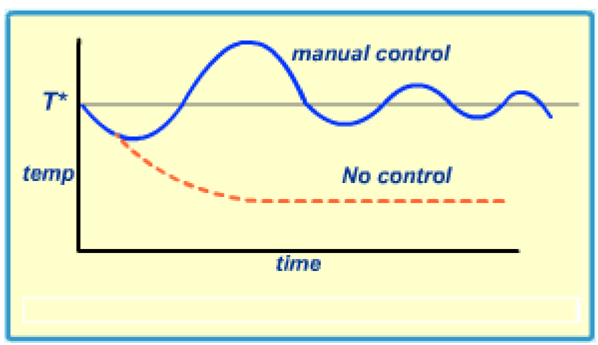



**Figure 1.5.** System performance under manual control.

CE: Let's start with a simple *feedback* system then. Let's install a temperature *controller* that uses measurements of the furnace outlet temperature T to adjust the fuel flowrate $Q_F$ accordingly [Figure 1.6(a)]. We will use a PID *controller* with these *controller parameter values* to start with (*proportional band* = 70%, *reset rate* = 2 repeats/min, *derivative time* = 0). Feel free to *retune* the controller if necessary. Let's discuss the results as soon as you are ready.

## Phase 3

PE: The performance of the feedback system [see Figure 1.6(b)], even though better than with manual control, is still not acceptable; too much low-temperature feed is sent to the fractionator during the first few hours following each throughput increase.

CE: (After a little thought) What we need is a mean by which we can change fuel flow the instant we detect a change in the feed flowrate. Try this *feedforward* control strategy by itself first (Figure 1.7); augment this *with feedback* only if you find it necessary (Figure 1.8(a)).

## *Phase* 4

PE: With the *feedforward* strategy by itself, there was the definite advantage of quickly compensating for the effect of the disturbance, at least initially. The main problem was the nonavailability of the furnace outlet temperature measurement to the controller, with the result that we had *offsets.* Since we can't afford the persistent offset, we had to activate the feedback system. As expected the addition of feedback rectified this problem (Figure 1.8(b)).

PE: We have one major problem left: the furnace outlet temperature still fluctuates, sometimes rather unacceptably, whenever we observe variations in the fuel delivery pressure. In addition, we are pretty sure that the variations in the fuel's heat content contributes to these fluctuations, but we have no easy way of *quantitatively* monitoring these heat content variations. At this point, however, they don't seem to be as significant as supply pressure variations.

CE: Let's focus on the problem caused by the variations in fuel supply pressure. It is easy to see why this should be a problem. The controller can only adjust the valve on the fuel line; and even though we expect that specific valve positions should correspond to specific fuel flowrates, this will be so only if the delivery pressure is constant. Any fluctuations in delivery pressure means that the controller will not get *the fuel flowrate* it asks for.

We must install an additional *loop* to ensure that the temperature controller gets the actual flowrate change it demands; a mere change in valve position will not ensure this.

We will install a *flow controller* in between the temperature controller and the control valve on the fuel line. The task of this *inner loop controller* will be to ensure that the fuel flowrate demanded by the temperature controller is actually delivered to the furnace regardless of supply pressure variations. The addition of this *cascade control system* should work well. (See Figure 1.9 for the final control system and its performance.)

Having overheard the successful design and installation of a control system, let us now continue with our introduction to the basic concepts and terminology of process control.

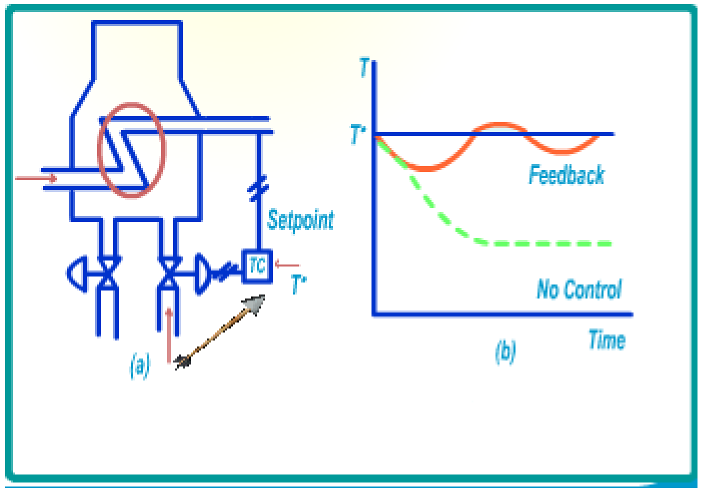



**Figure 1.6.** The feedback control system.

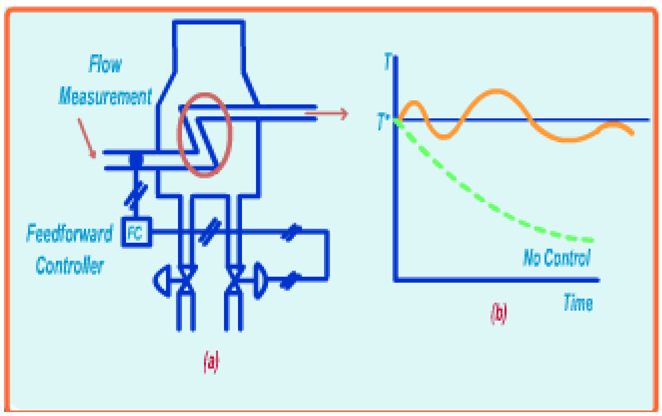



**Figure 1.7.** The feedforward control system.

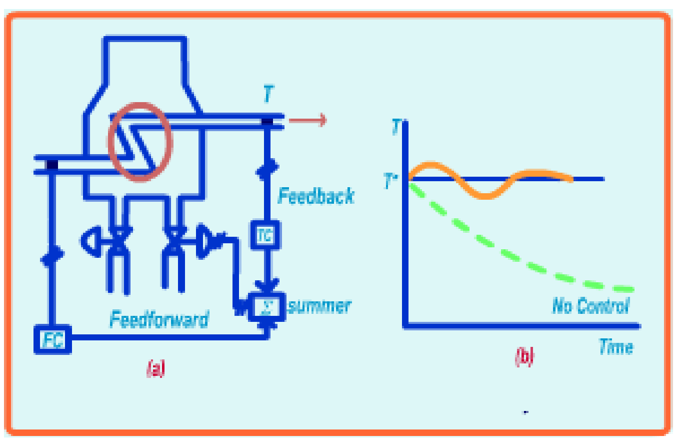



**Figure 1.8.** The feedforward/feedback control system.

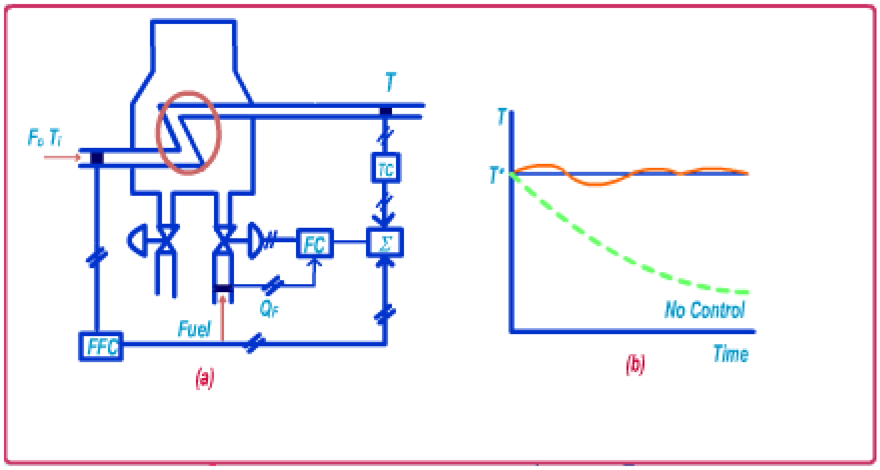



**Figure 1.9.** The final control system (feedforward/feedback-plus-cascade).

## 1.3 VARIABLES OF A PROCESS

The state of affairs within, or in the immediate environment of, a typical processing unit is usually indicated by such quantities as *temperature, flowrates in and out of containing vessels, pressure, composition, etc.* These are referred to as the *variables* of the process, or *process variables.* Recall that in our discussion of the furnace control problem we frequently referred to such variables as these.

It is customary to classify these variables according to whether they simply *provide information about process conditions,* or whether they are *capable of influencing process conditions.* On the first level, therefore, there are two categories of process variables: *input* and *output* variables.

*Input variables are those that independently stimulate the system and can thereby induce change in the internal conditions of the process.*

*Output variables are those by which one obtains information about the internal state of the process.*

*It is appropriate at this point to introduce what is called a state variable and distinguish it from an output variable. State variables are generally recognized as :*

*That minimum set of variables essential for completely describing the internal state (or condition) of a process.*

The state variables are therefore the *true* indicators of the internal state of the process system. The actual *manifestation* of these *internal states* by measurement is what yields an *output.*

Thus the *output* variable is, in actual fact, some *measurement* either of a single state variable or a combination of state variables.

On a second level, it is possible to further classify *input* variables as follows:

1. Those input variables that are at our disposal to manipulate freely as we choose are called *manipulated (or control) variables.*

2. Those over which we have no control (i.e., those whose values we are in no position to decide at will) are called *disturbance variables.*

Finally, we must note that some process variables *(output* as well as *input* variables) are directly available for measurement while some are not. Those process variables whose values are made available by direct on-line measurement are classified as *measured variables;* the others are called *unmeasured variables*, (see Figure 1.10.)

Although output variables are defined as measurements, it is possible that some outputs are not measured on-line, but require infrequent samples to be taken to the laboratory for analysis. Thus for control system design these are usually considered unmeasured output in the sense that the measurements are not available frequently enough for control purposes.

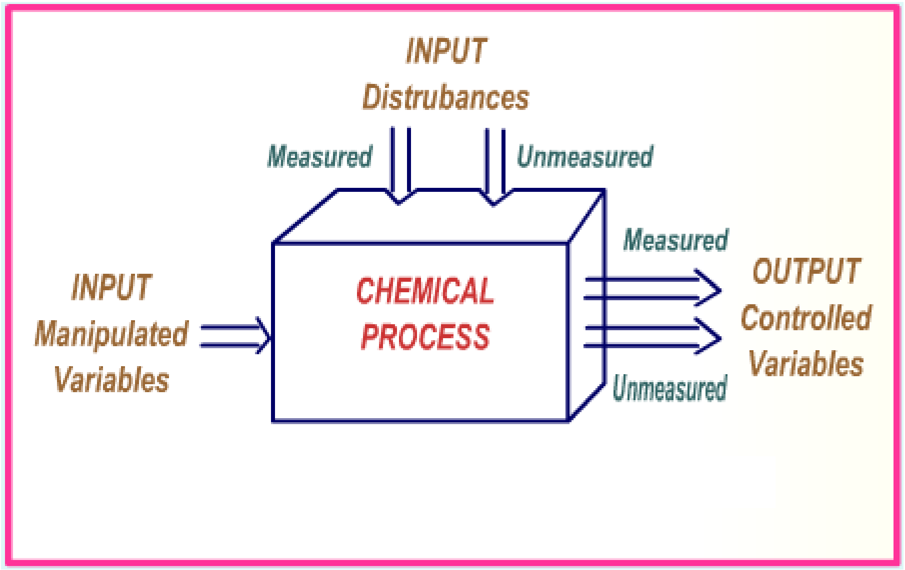



**Figure 1.10**. The variables of a process.

1.4 THE CONCEPT OF A PROCESS CONTROL SYSTEM

As earlier noted, the dynamic *(i.e.*, ever changing) nature of processes makes it imperative that we have some means of effectively monitoring, and inducing change in the process variables of interest.

In a typical process, the process control system is the entity that is charged with the responsibility for monitoring outputs, making decisions about how best to manipulate inputs so as to obtain desired ouput behaviour, and effectively implementing such decisions on the process.

It is therefore convenient to break down the responsibility of the control system into the following three major tasks:

- Monitoring process output variables by measurement
- Making rational *decisions* regarding what corrective action is needed on the basis of the information about the current and desired state of the process.
- Effectively *implementing* these decisions on the process.

When these tasks are carried out *manually* by a human operator we have a *manual control system;* on the other hand, a control system in which these tasks are carried out in an *automatic* fashion by a machine is known as an *automatic control system;* in particular, when the machine involved is a computer,, we have a *computer control system*.

With the possible exception of the manual control system, all other control systems require certain hardware elements for carrying out each of the above itemized tasks. Let us now introduce these hardware elements, reserving a more detailed discussion of the principles and practice of control system implementation to next chapters.

### 1.4.1 Control System Hardware Elements

The hardware elements required for the realization of the control system's tasks of *measurement, decision making, and corrective action implementation* typically fall into the following categories: *sensors, controllers, transmitters, and final control elements.*

*Sensors*

The first task, that of acquiring information about the status of the process output variables, is carried out by sensor (also called measuring device or primary elements). In most process control applications, the sensors are usually needed for pressure, temperature, liquid level, flow, and composition measurements. Typical examples are: *thermocouples* (for temperature measurements), *differential pressure cells* (for liquid level measurements), *gas/liquid chromatographs* (for composition measurements), *etc.*

*Controllers*

The decision maker, and hence the "heart" of the control system, is the *controller;* it is the hardware element with "built-in" capacity for performing the only task requiring some form of "intelligence."

The controller hardware may be *pneumatic* in nature (in which case it operates on air signals), or it may be *electronic* (in which case, it operates on electrical signals). Electronic controllers are more common in more modern industrial process control applications.

The pneumatic and electronic controllers are limited to fairly simple operations which we shall have to discuss more fully later. When more complex control operations are required, the *digital computer* is usually used as a controller.

*Transmitters*

How process information acquired by the sensor gets to the controller, and the controller decision gets back to the process, is the responsibility of devices known as *transmitters.* Measurement and control signals may be transmitted as *air pressure signals,* or as *electrical signals.* Pneumatic transmitters are required for the former, and electrical ones for the latter.

*Final Control Elements*

Final control elements (or actuators) have the task of actually implementing, on the process, the control command issued by the controller. Most final control elements are control valves (usually pneumatic, *i.e.*, they are air-driven), and they occur in various shapes, sizes, and have several modes of specific operation. Some other examples of final control elements include: *variable speed fans, pumps, and compressors; conveyors; and relay switches.*

*Other Hardware Elements*

In transmitting information back and forth between the process and the controller, the need to *convert* one type of signal to another type is often unavoidable. For example, it will be necessary to convert the electrical signal from an electronic controller to a pneumatic signal needed to operate a control valve. The devices used for such signal transformations are called ***transducers,*** and various types are available for various signal transformations.

Also, for computer control applications, it is necessary to have devices known as *analog-to-digital* (A/D) and *digital-to-analog* (D/A) *converters*. This is because while the rest of the control system operates on *analog* signals (electric voltage or pneumatic pressure), the computer operates *digitally,* giving out, and receiving, only binary numbers. A/D converters make the process information available in recognizable form to the computer, while the D/A converters make the computer commands accessible to the process.

### 1.4.2 Control System Configuration

Depending primarily upon the structure of the decision-making process in relation to the information-gathering and decision-implementation ends, a process control system can be configured in several different ways. Let us introduce some of the most common configurations.

***Feedback Control***

The control system illustrated in Figure 1.12 operates *by feeding* process output information *back* to the controller. Decisions based on such "fed back" information are then implemented on the process. This is known as a *feedback control structure,* and it is one of the simplest, and by far the most common, control structures employed in process control. It was introduced for the furnace example in Figure 1.6(a).

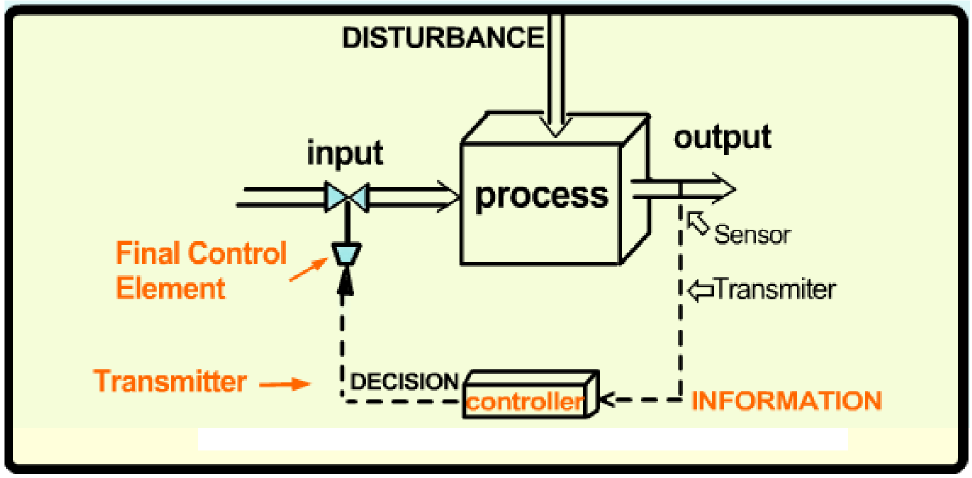



**Figure 1.12.** The feedback control configuration.

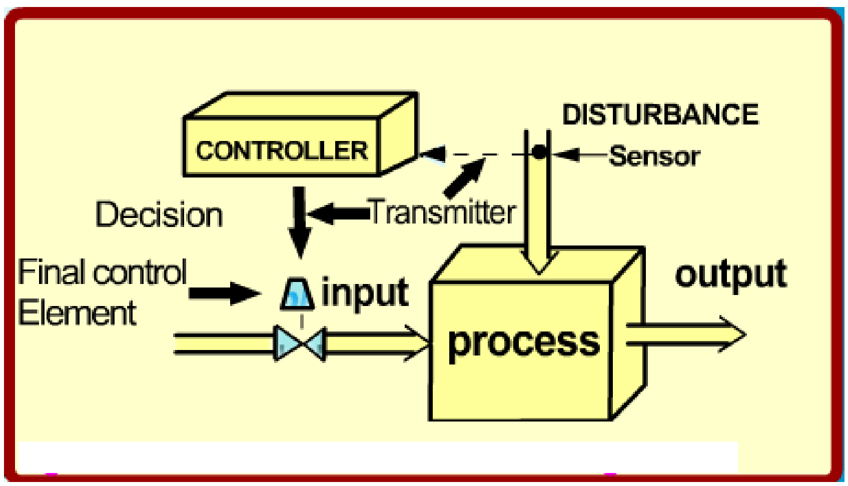



**Figure 1.13.** The feedforward control configuration.

**It is important to point out the intuitively appealing nature of this control structure. Observe that it makes use of *current* information about the output of the process to determine what action to take in regulating process behavior.. We must note, however, that with such a structure, the effect of any disturbance entering the, process must first be registered by the process as an upset in its output before corrective control action can be taken; *i.e.*, controller decisions are 'taken "after the fact."***

*Feedforward Control*

In Figure 1.13 we have a situation in which it is information about an incoming disturbance that gets directly communicated to the controller instead of actual system output information. **With this configuration, the controller decision is taken *before* the process is affected by the incoming disturbance. This is the *feedforward control* structure** (compare with Figure 1.12) since the controller decision is based on information that is being "fed forward." As we shall see later, feedforward control has proved indispensable in dealing with certain process control problems.

The main feature of the feedforward configuration is the choice of measuring the *disturbance* variable rather than the output variable that we desire to regulate. The potential advantage of this strategy has already been noted. Further reflection on this strategy will, however, also reveal a potential drawback: the controller has *no information* about the conditions existing at the process output, the actual process variable we are concerned about regulating.

**Thus the controller detects the entrance of disturbances and before the process is upset attempts to compensate for their effects somehow (typically based on an imperfect process model); however,** the controller **is unable to determine the accuracy of this compensation,** since this strategy does not call for a measurement of the process output. This is often a significant disadvantage as was noted before.

*Open-Loop Control*

When, as shown in Figure 1.14, the controller decision is *not* based upon any measurement information gathered from any part of the process, but upon some sort of internally generated strategy, we have an *open-loop control structure.* This is because the controller makes decisions *without* the advantage of information that "closes the loop" between the output and input variables of the process, as is the case with the feedback control configuration (see Figure 1.12.) This loop is "open." However, this does not necessarily constitute a handicap.

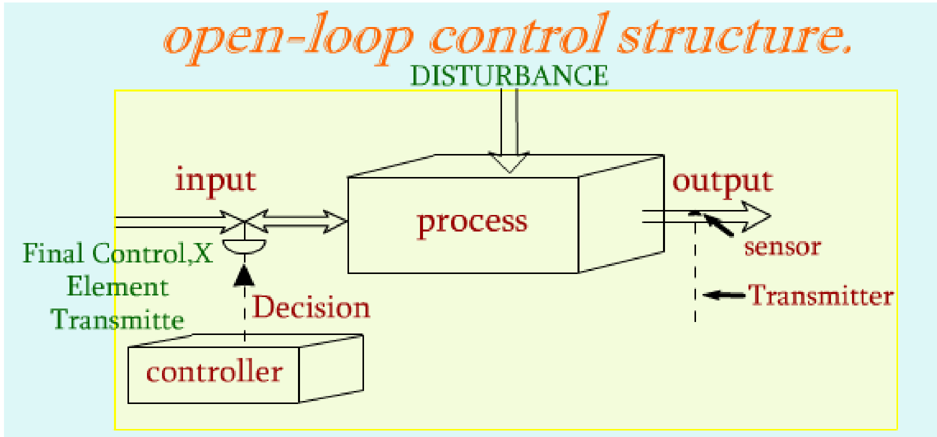



**Figure 1.14.** The open loop control configuration.

**Perhaps the most common example of an open-loop control system can be found in in the simple timing device used for some traffic lights. Regardless of the volume of traffic, the timer is set such that the period of time for which the light remains green, yellow, or red is predetermined.** We shall study these and other control system structures in greater detail later.

### 1.4.3 Some Additional Control System Terminology

Important process variables that have been selected to receive the attention of the control system typically have target values at which they are required to be maintained. **These target values are called *set-points.*** Maintaining these process variables at their prescribed set-points is, of course, the main objective of the process control system, be it manual or automatic. However, output variables deviate from their set-points:

1. Either as a result of the effect of disturbances, or
2. Because the set-point itself has changed.

We have ***regulatory control*** when the control system's task is solely that of counteracting the effect of disturbances in order to maintain the output at its set-point (as was the case in the furnace example of Section 1.2). When the objective is to cause the output to track the changing set-point, we have ***servo control*** (see Figure 1.15).

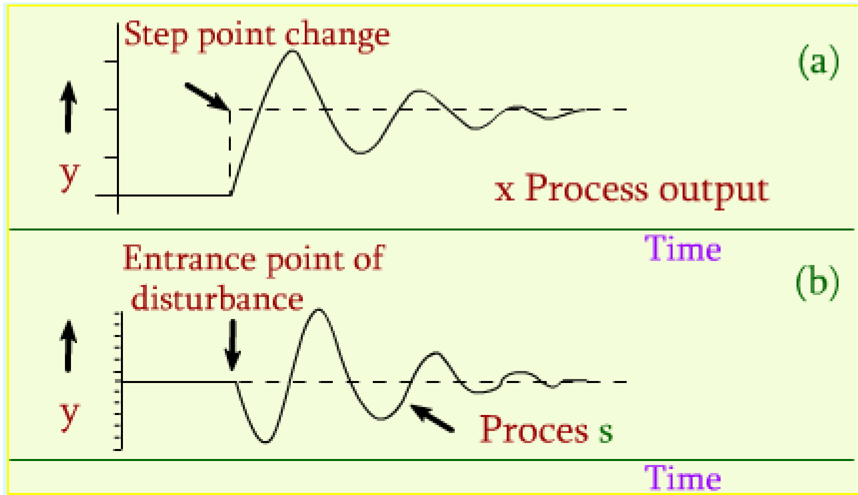



**Figure 1.15.** Possible process responses under (a) servo; (b) regulatory control.

## 1.5 OVERVIEW OF CONTROL SYSTEM DESIGN

The design of effective control systems is the main objective of the process control engineer. The following is an overview of the steps involved in successfully carrying out the task of control system design.

### 1.5.1 General Principles

*Step 1. Assess the process and define control objectives.* The issues to be resolved in this step include the following:

1. Why is there a need for control?
2. Can the problem be solved only by control, or is there another alternative (such as redesigning part of the process)?
3. What do we expect the control system to achieve?

*Step 2. Select the process variables to be used in achieving the control objectives articulated in Step 1.*

Here we must answer the following questions:

1. Which output variables are crucial and therefore must be measured in order to facilitate efficient monitoring of process conditions?
2. Which disturbances are most serious? Which ones can be measured?
3. Which input variables can be manipulated for effective regulation of the process?

*Step 3. Select control structure.*

What control configuration is chosen depends on the nature of the control problem posed by the process system. The usual alternatives are: *Feedback, Feedforward, Open Loop,* and others which we shall discuss later.

*Step 4. Design controller.*

This step can be carried out to varying degrees of sophistication, but it essentially involves the following:

*Obtain a* control law *by which, given information about the process (current and past outputs, past inputs and disturbances, and sometimes even future predictions of the system output), a* control decision is *determined which the controller implements by adjusting the appropriate manipulated variables accordingly.*

The process control engineer requires a thorough understanding of the process itself as well as a proper understanding of the principles of *Process Dynamics and Control* in order to accomplish these steps to a successful control system design.

## Some Concluding Remarks

In order not to encourage an unduly false, and simplistice, view of process control problems on the basis of the some simple illustrative examples, the following is just a sample of some typical complications one would normally expect to encounter in practice.

### *1. Nonlinearities*

The process model equations we have dealt with have been linear, and thus easy to analyze. This is not always the case.

### *2. Modeling Errors*

With the exception of the most trivial processes, it is *impossible* for a mathematical model to represent *exactly* all aspects of process behavior. This fact notwithstanding, however, the usefulness of the mathematical model should not be underestimated; we just need to keep its limitations in proper perspective. The effectiveness of any control system designed on the basis of a process model will, of course, depend on the *integrity* of such a model in representing the process.

### *3. Other Implementation Problems*

The simple illustrative examples are strictly trivial processes.

Observe that were we dealing with a thermal system, in which liquid streams at different temperatures are moved around in the pipes, or if our system were to involve mixing streams of different liquid compositions, then the situation would be different. To effect temperature, or composition, changes by moving such liquids around, we must now consider the fact that the time it takes to flow from one point to the other within a pipe can quite often be so significant as to introduce a *delay* in the system's response to the effect of control action. As we will see later, the influence of such delays can become a most serious consideration in the design *of a control system.*

Even when a control system is impeccably designed, perfect implementations may be limited by, among other things, such factors as imperfect measurements, inaccurate transmission, or control valve inertia (leading to inaccurate valve actuation), factors that by and large are unavoidable in practice.

### *4. **Complicated Process** Structure*

In many processes, the variables involved are usually more numerous, and their interrelationships more complicated. It is in fact not unusual to have to deal with an integration of severeal such processes. Nevertheless, the knowledge gained from investigating simple processes can be gainfully applied to the more complicated versions, sometimes with only minimal, quite often obvious, additional considerations, and sometimes with considerable modifications that may not be immediately obvious. Figure 1.16 shows a typical example.

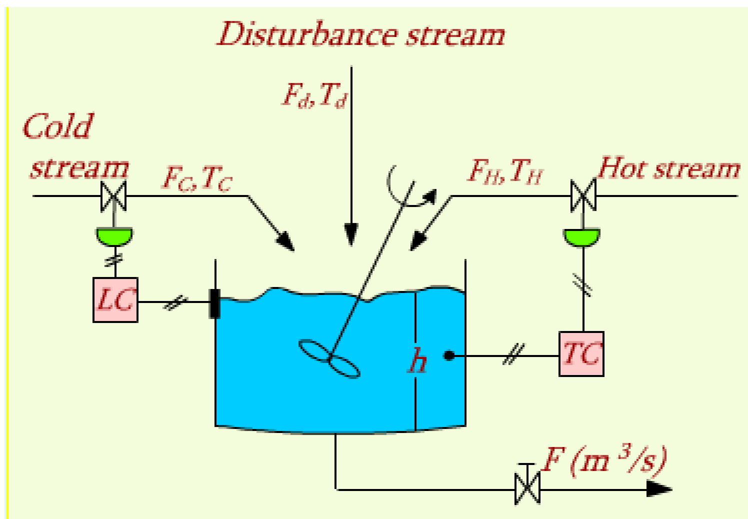



**Figure 1.16.** Dual liquid level and temperature control system.

REVIEW QUESTIONS

1. What are the three broad objectives on which the basic guiding principles of process operation are based?

2. Based on the guiding principles in operating a chemical process, can you guess why pneumatic controllers, actuators, and transmitters can be found in many plants?

3. What are the main concerns of Process Dynamics and Control as a subject matter within the process engineering discipline?

4. What is the difference between the input and the output variables of a process?

5. What is a state variable? How are they related to output variables?

6. How can you distinguish a manipulated (control) variable from a disturbance variable?

7. What are the three main responsibilities of the control system? Assign to each of these responsibilities the hardware elements required for carrying out the indicated tasks.

8. Differentiate between a manual and an automatic control system.

9. What makes an automatic control system a computer control system?

10. What are transducers used for?

11. What differentiates a feedback control system configuration from the feedforward configuration?

12. What is unique about the open-loop control system configuration?

13. Differentiate between a servo control problem and a regulatory control problem. Can you guess which will be more common in a plant in which the processes operate predominantly in the neighborhood of steady-state conditions for long periods of time?

# 2. INDUSTRIAL PROCESSES

In this chapter some typical examples of industrial systems and their related concepts and terminologies are introduced. Besides physical or mechanistic modeling of such process are also reviewed

## 2.1 LIQUID-LEVEL SYSTEMS

Fluid flow systems are very common in process industries. In general, one may divide fluid flows into two categories:

- Laminar flow (for Reynolds no. around 2000)
- Turbulent flow(for Reynolds no.  around 3000-4000)

In laminar case, fluid flow occurs in stream lines with no turbulence. Systems involving turbulent flow often have to be represented by non-linear differential equations, while the other case may be represented by linear differential equation. Industrial processes often involve flow of liquids through connecting pipes and tanks. The flow in such processes is often turbulent.

In this section we shall derive mathematical models of liquid – level systems. By introducing the concepts of resistance and capacitance for such systems, it is possible to describe the dynamic characteristics of such systems in simple forms.

### 2.1.1 Resistance and Capacitance

Consider the flow through a short pipe connecting two tanks. The resistance R for liquid flow in such a pipe or restriction is defined as the change in the level difference necessary to cause a unit change in flow rate: that is:

$$R = \frac{Change\ in\ level\ difference,\ m}{Change\ in\ flow\ rate,\ m^3/\sec}$$

Such a relationship for the laminar flow and turbulent flow are different. Both cases will be considered in the following. Consider the tank system in Figure 2.1.

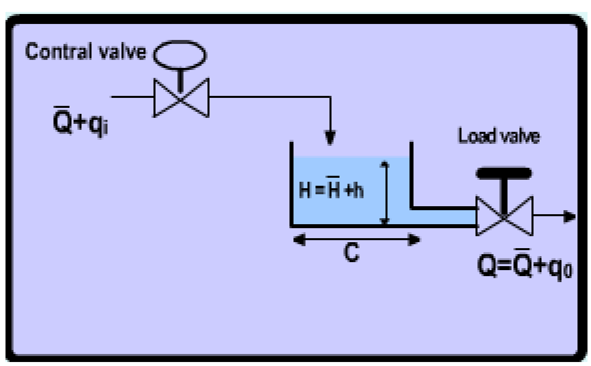



**Figure 2.1**. The tank system. Variables with a "bar" index are steady state values.

If the flow through this restriction (load valve) is laminar, we have

$$Q=KH \tag{2.1}$$

Where

Q= output flow rate, $m^3/\sec$

K= coefficient, $m^2/\sec$

H=liquid head (or height), m

Note that in this case Q is proportional to potential difference (height). For laminar flow,

$$R_L = \frac{H}{Q}$$

If the flow through the restriction is turbulent the steady state flow rate is given by

$$Q = K\sqrt{H} \qquad (2.2)$$

The resistance $R_T$ for turbulent flow is obtained from

$$R_T = \frac{dH}{dQ}$$

From Eq. (2.2) we obtain,

$$dQ = \frac{K}{2\sqrt{H}} dH$$

$$R_T = \frac{dH}{dQ} = \frac{2\sqrt{H}}{K} = \frac{2\sqrt{H}\sqrt{H}}{Q} = \frac{2H}{Q}$$

The value of the turbulent flow $R_T$ depends on the flow rate and the head. However, it may be considered constant if the changes in head and flow rate are small.

In many practical cases, the value of the coefficient K in Equation (2.2) is not known. Then the resistance may be determined by plotting the head verses flow rate curve based on experimental data and measuring the slop of the operating condition.

The capacitance C of a tank is defined to be the change in quantity of stored liquid necessary to cause a unit change in the potential (head).

$$C = \frac{Change\ in\ \ liquid\ \ stored,\ \ m^3}{Change\ \ in\ \ head,\ \ \ m}$$

It should be noted that the capacity ( $m^3$) and capacitance ( $m^2$) are different. The capacitance of the tank is equal to its cross sectional area. Thus, if A is constant, the capacitance is constant for any head.

### *2.1.2 Modeling Liquid Level Systems*

Consider the liquid level system in Figure 2.1. As described before, the system is assumed linear if the flow is laminar. Even if, flow is turbulent, the system can be linearized when changes in the variables are kept small. Based on this assumption, the differential equation of this system can be obtained as follows. The additional amount of liquid stored in the tank is equal to the inflow minus outflow during the small time interval dt. That is,

$$Cdh = (q_i - q_0)\,dt$$

From the definition of resistance, the relationship between $q_0$ and h is given by

$$q_0 = \frac{h}{R}$$

Assuming R is constant, we have

$$RC\frac{dh}{dt} + h = Rq_i$$

Note that RC is the time constant of the system.

Taking Laplace transforms of both sides of the above equation, and assuming zero initial condition, we obtain

$$(RCs+1)H(s) = RQ_i(s)$$

$$\frac{H(s)}{Q_i(s)} = \frac{R}{RCs+1}$$

If, however $q_0$ is taken as the output, and the input is the same, then

$$(RCs+1)H(s) = RQ_i(s)$$

And

$$Q_0(s).R = H(s)$$

$$\Rightarrow (RCs+1)Q_0(s).R = RQ_i(s)$$

$$\frac{Q_0(s)}{Q_i(s)} = \frac{1}{(RCs+1)}$$

### ***2.1.3.* Liquid Level Systems with Interactions**

Consider the system shown in Figure2.2.

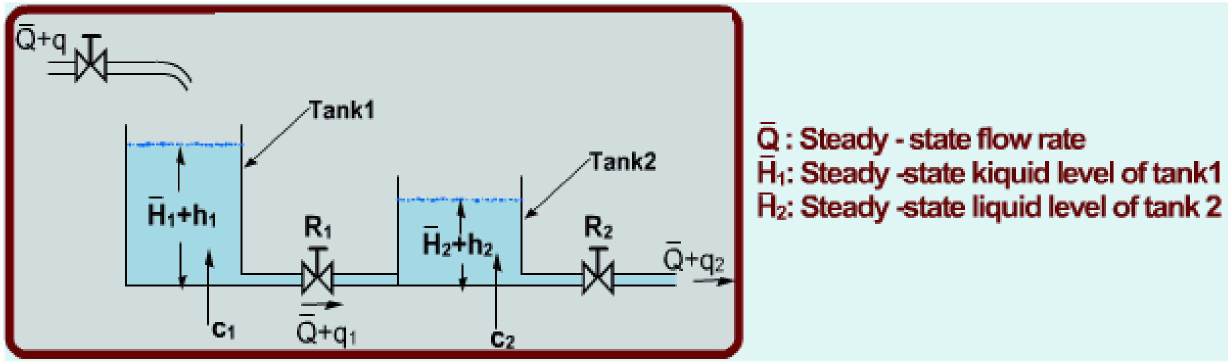



**Figure 2.2 .** Two tanks with interaction

In this system, the overall transfer function is not the product of simple transfer functions. Assuming only small variations of the variables from the steady-state values, the following equations can be obtained for the system:

$$q_1 = \frac{h_l - h_2}{R_1}$$

$$C_1 \frac{dh_l}{dt} = q - q_1$$

$$\frac{h_2}{R_2} = q_2$$

$$C_2 \frac{dh_2}{dt} = q_l - q_2$$

If we consider q as the input and $q_2$ as the output, the transfer function of the system is

$$\frac{Q_2(s)}{Q(s)} = \frac{1}{R_1C_1R_2C_2S^2 + (R_1C_1 + R_2C_2 + R_2C_1)S + 1}$$

Students should compare this transfer function with the case when there is no interaction between the two tanks and the results is just simple multiplications of two single tank systems.

### *2.1.4. . Level Control Using Pumps*

It is common in liquid level systems to use a centrifuge pump in outlet of the tank as in Figure 2.3.

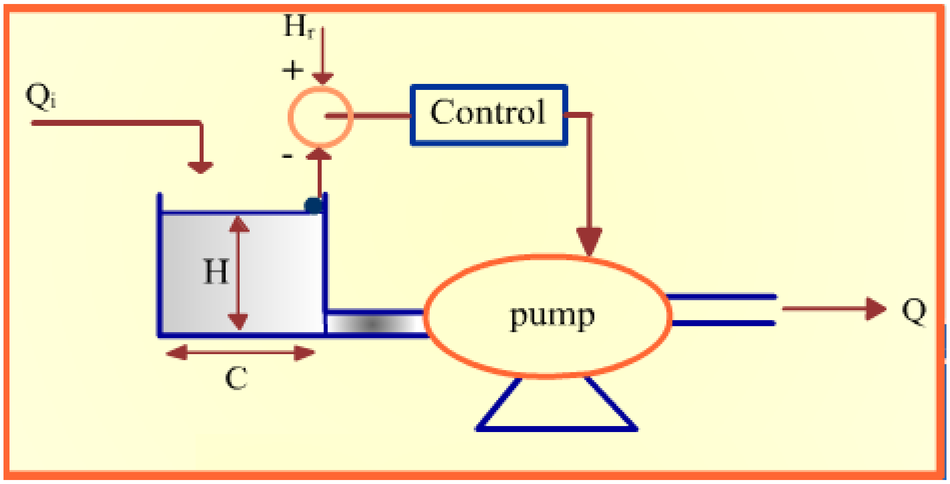



**Figure 2.3**. Level control using pumps.

The outflow of the pump depends on the difference of the heads on two sides of the pump and the speed (rpm) of the pump. Figure 2.4 shows example of such relationships.

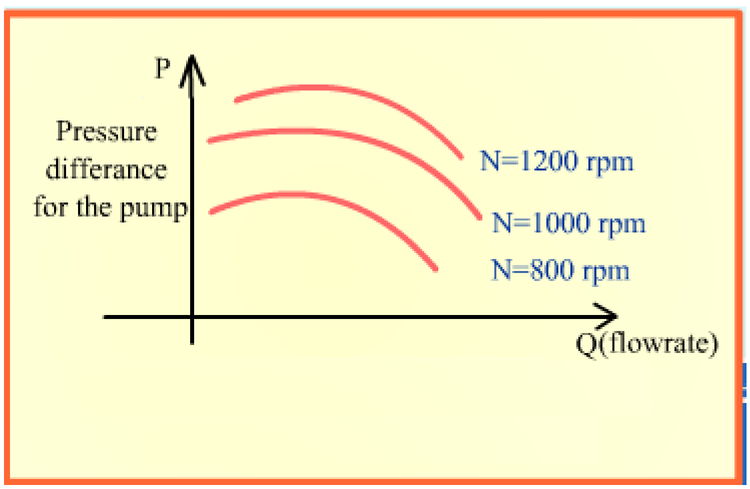



**Figure 2.4**. Pressure versus flow rate of a typical centrifuge pump.

The dynamic equation for such systems, assuming small variations around steady state is

$$A\frac{dh}{dt} = q_i - q \quad , \qquad Q = Q(P,N) \quad \begin{cases} P: Difference\ \ in\ pressure \\ N: Rotational\ speed \\ A = C: \ Capacitance \end{cases}$$

$$q = \Delta Q = (\frac{\partial Q}{\partial P})_0 \ \ \Delta P + (\frac{\partial Q}{\partial N})_0 \ \ \Delta N$$

The terms within brackets in the above equation can be obtained from Figure 2.5.

$$(\frac{\partial Q}{\partial P})_0 = \beta, \qquad (\frac{\partial Q}{\partial N})_0 = \delta$$

$$P = P_1 - P_2 = \alpha h - 0 = \alpha h \qquad \Rightarrow q = \beta \alpha h + \delta n$$

$$\Rightarrow A\frac{dh}{dt} = q_i - \alpha\beta\, h - \delta\, n$$

$$\rightarrow (As + \alpha\beta)\, h(s) = q_i(s) - \delta\, n(s)$$

$$\frac{h(s)}{n(s)} = \frac{-\delta}{As + \alpha\beta} \qquad , \qquad \frac{h(s)}{q_i(s)} = \frac{1}{As + \alpha\beta}$$

## 2.2 THERMAL SYSTEMS

Thermal systems are those that involve the transfer of heat from one substance to another.

Thermal systems can be analyzed in terms of resistance and capacitance, although the thermal capacitance and thermal resistance may not be represented accurately as lumped parameters since they are usually distributed through out the substance. However, to simplify the analysis, we shall assume that the thermal system can be represented by a lumped-parameter model.

There are three different ways heat can flow from one substance to another: conduction, convection, and radiation. Here we consider only conduction and convection (radiation heat transfer is appreciable only if the temperature of the emitter is very high compared to that of the receiver).

For conduction and convection heat transfer

$$q = K\Delta\theta \;\; where \; \begin{cases} q = heat\; flowrate, Kcal/\sec \\ \Delta\theta = Temperature\; difference,\; {}^\circ c \\ K = coefficient\;,\;\; Kcal/\sec\, {}^\circ c \end{cases}$$

The coefficient K is given by

$$\begin{aligned} K &= \frac{kA}{\Delta X} \qquad && for\; conduction \\ &= HA && for\; convection \end{aligned}$$

$where$

$k = Thermal\; conductivity,\; Kcal/m.\sec.{}^\circ c$

$A = Area\; for\; heat\; flow,\; m^2$

$\Delta X = Thickness\; of\; conductor,\; m$

$H = Convection\; coeeficient,\; Kcal/m^2.\sec.{}^\circ c$

The thermal resistance R for the heat transfer between two substances may be defined as follows:

$$R = \frac{Change\ in\ temperature\ difference,\ ^\circ c}{Change\ in\ heat\ flow rate,\ Kcal/\sec}$$

Thus the thermal resistance for conduction or convection heat transfer is given by

$$R = \frac{d(\Delta\theta)}{dq} = \frac{1}{K} = \frac{\Delta X}{kA} \quad or \quad \frac{1}{HA}$$

The thermal capacitance is defined by

$$C = \frac{Change\ in\ \ heat\ stored,\ Kcal}{Change\ \ in\ \ temperature,\ \ ^\circ c}$$

Or

$$C = mc$$

$$where \qquad m = mass\ of\ \ subs\tan ce,\ kg$$

$$c = specific\ heat\ of\ \ subs\tan ce, Kcal/kg^\circ c$$

### *2.2.1. A Typical Thermal System*

Consider the system shown in Figure 2.5.

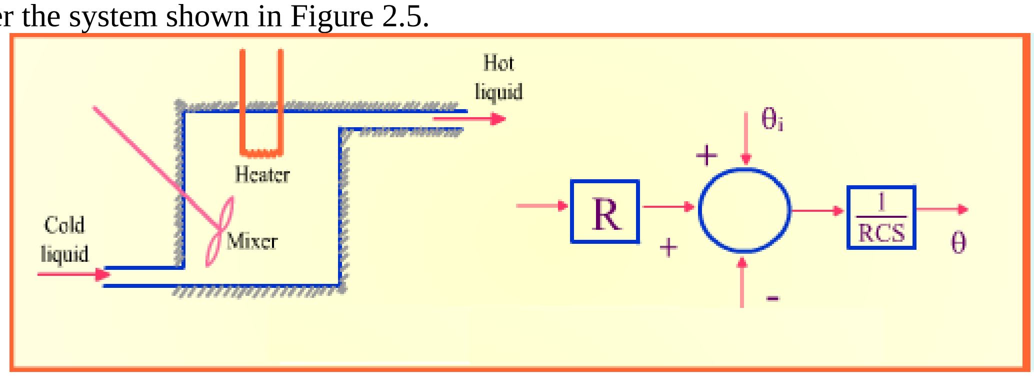



**Figure 2.5** A typical thermal system.

It is assumed that the tank is insulated to eliminate heat loss to the surrounding. It is also assumed that there is no heat storage in the insulation and that liquid in the tank is perfectly mixed so that it is at a uniformed temperature.

Let us define:

$\overline{\theta_i}$ = Steady state temperature of inflowing liquid, °c

$\overline{\theta_o}$ = Steady state temperature of outflowing liquid, °c

$G$ = Steady state liquid flow rate, kg/sec

$M$ = Mass of liquid in the tank, kg

$c$ = Specific heat of liquid, Kcal/kg°c

$R$ = Thermal resistance, °c

$C$ = Thermal capacitance, Kcal/°c

$\overline{H}$ = Steady state heat input rate, Kcal/sec

Assume the temperature of the inflowing liquid is kept constant and that the heat input rate to the system is suddenly changed from $\overline{H}$ to $\overline{H} + h_i$ , where $h_i$ represent a small change in the heat input rate. The heat outflow will gradually change from $\overline{H}$ to $\overline{H} + h_o$ .

The temperature of the outflowing liquid will also be changed from $\overline{\theta_o}$ to $\overline{\theta_o} + \theta$ . We have:

$$h_o = Gc\theta$$
$$C = Mc$$
$$R = \frac{\theta}{h_o} = \frac{1}{Gc}$$

The differential equation for this system is

$$C\frac{d\theta}{dt} = h_i - h_o$$

which may be written as

$$C\frac{d\theta}{dt} = Gc\theta_i - h_o$$
$$RC\frac{d\theta}{dt} + \theta = \theta_i$$
$$\frac{\theta(s)}{\theta_i(s)} = \frac{1}{RCs + 1}$$

## **2.3** PRESSURE **SYSTEMS**

Many industrial processes and pneumatic controllers involve the flow of a gas or air through connected pipelines and pressure vessels. A typical pressure system is shown in Figure 2.6.

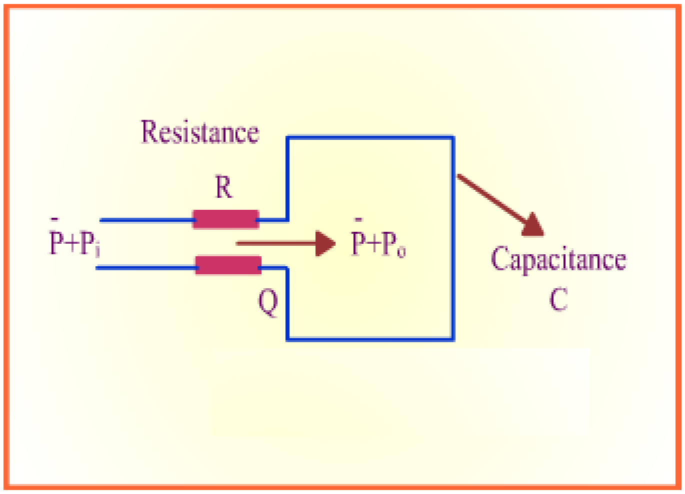



**Figure 2.6** A typical pressure system.

In the above system, the gas flow through the restriction is a function of gas pressure difference $P_i - P_o$. Such a pressure system may be characterized in terms of a resistance and capacitance,

$$R = \frac{Change\ in\ gas\ pressure\ difference,\ kg/m^2}{Change\ in\ gas\ flow\ rate,\ kg/\sec} = \frac{d(\Delta P)}{dQ}$$

Computation of the value of the gas flow resistance R may be quite time consuming. Experimentally, however, it can be easily determined from a plot of the pressure difference versus flow curve by calculating the slope of the curve at a given operating condition as shown in Figure 2.7.

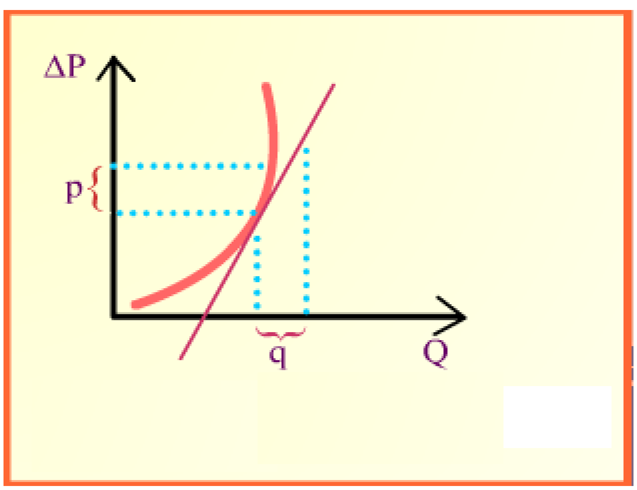



**Figure 2.7** Experimental calculation of resistance around an equilibrium point.

The capacitance of the pressure vessel may be defined by

$$C = \frac{Change\ in\ gas\ stored,\ kg}{Change\ in\ gas\ pressure,\ kg/m^3} = \frac{dW}{dP} = V\frac{dL}{dP}$$

where

$$C = Capacitance, m^3$$
$$W = Weight\ of\ gas\ in\ vessel,\ kg$$
$$P = Gas\ pressure,\ kg/m^2$$
$$V = Volume\ of\ vessel, m^3$$
$$L = Specific\ gas\ weight,\ kg/m^3$$

The capacitance of the pressure systems depends on the type of expansion process involved. For ideal gas we have

$$C = \frac{V}{nR_{gas}T}$$

where

$$V = Volume\ of\ vessel, m^3$$
$$n = A\,constant\ of\ the\ gas\ \exp ansion \approx 1 - 1.2$$
$$R_{gas} = Universal\ gas\ constant$$
$$T = Absolute\ temprerature,\ {}^\circ K$$

The capacitance of a given system is constant if the temperature stays constant.

### *2.3.1. A Typical Pressure System*

For the system shown in the Figure 2.6, assuming only small deviations in the variables from their respective steady state values, then the system may be considered linear (see Figure 2.7).

Since the pressure change $dP_o$ times the capacitance $C$ is equal to the gas added to the vessel during $dt$ seconds, we obtain

$$CdP_o = q\ dt$$
$$C\frac{dP_o}{dt} = \frac{P_i - P_o}{R}$$
$$RC\frac{dP_o}{dt} + P_o = P_i \rightarrow \frac{P_o(s)}{P_i(s)} = \frac{1}{RCs + 1}$$

# 3. LINEARIZATION OF NONLINEAR SYSTEMS

The differential equation representing a system is often seen to be non-linear. It is common on the other hand that the control system is to operate near an equilibrium point (or operating point), and thus a linearization around that point will be done. The linearization result in a much simpler model, but one which is still adequate for control design.

Besides the linearization, some parameters of the original equations representing a system could be quite large or small numbers. Using some scaling methods, and/or change of variables, one may be able to simplify the equations to a further degree.

One general approach for linearization is to use the Taylor series expansion of non-linear terms and then ignore the high order terms of the expansion.

Consider the water tank example in Figure 3.1.

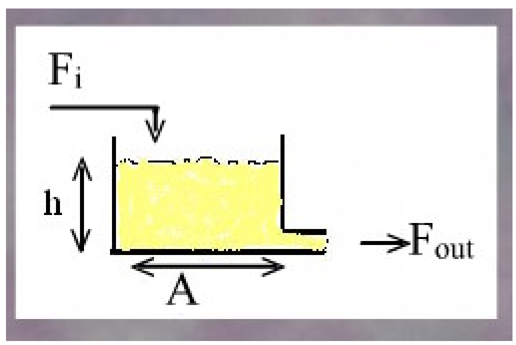



**Figure 3.1 . A typical liquid level system.**

The total mass balance for the system is

$$\frac{d(\rho V)}{dt} = \rho F_i - \rho F_{out} \rightarrow \quad \frac{d(\rho A h)}{dt} = \rho F_i - \rho F_{out}$$

Where

$V$ = Volume of the liquid in the tank

$F_i$ = Inlet volumetric flow rate

$F_{out}$ = Outlet volumetric flow rate

$A$ = Cross sectional area of the tank

$\rho$ = Density of water

$h$ = Height of water

$m$ = Mass of water in the tank

Assuming $\rho$ is constant and expressing the output flow rate as $F_{out} = \alpha\sqrt{h}$ , we will have

$$A\frac{dh}{dt} = F_i - \alpha\sqrt{h} \tag{3.0}$$

Let us develop a linear approximation for this non-linear system around an operating point, namely ($h_0$ , $F_0$). Here we have

$$F_{out} = \alpha\sqrt{h}$$

As you see in Figure 3.2, the relation between Output Flow and input h is nonlinear and we are looking for a linear approximation around an operating (or steady state point).

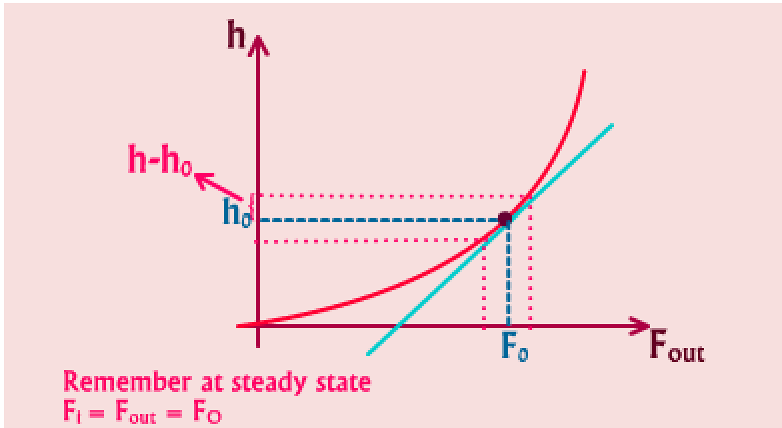



**Figure 3.2: Linear approximation of the nonlinear function for the Tank example.**

To start we should write the Taylor series expansion of the non-linear term around point $h_{0}$,. In general, for a function f(x) at point $x=x_o$ we write :

$$f(x) = f(x_0) + \frac{df}{dx}|_{x=xo}\ (x - x_0) + \frac{d^2 f}{dx^2}|_{x=xo}\ \frac{(x - x_0)^2}{2!} + \cdots$$

and thus <u>for the linear approximation we only consider the first two terms</u>. In this case we have,

$$\alpha\sqrt{\mathbf{h}} = \alpha\sqrt{\mathbf{h_0}} + \left[\frac{\mathbf{d}(\alpha\sqrt{\mathbf{h}})}{\mathbf{dh}}\right]_{\mathbf{h}=\mathbf{h_0}} (\mathbf{h} - \mathbf{h_0}) + \cdots$$

$$\alpha\sqrt{\mathbf{h}} \approx \alpha\sqrt{\mathbf{h_0}} + \frac{\alpha}{\mathbf{2}\sqrt{\mathbf{h_0}}}(\mathbf{h} - \mathbf{h_0})$$

and thus the system equation becomes

$$A\frac{dh}{dt} = F_i - \alpha\sqrt{h_0} - \frac{\alpha}{2\sqrt{h_0}}(h - h_0) \qquad (3.1)$$

This is now a linearized equation but with an offset. To overcome this offset and derive a simplified linear equation, we need further work.

The above equation should also satisfy in the operating point which means:

$$A\frac{dh_0}{dt} = F_0 - \alpha\sqrt{h_0} - 0 \qquad (3.2)$$

Subtracting Equation (3.2) from (3.1) yields:

$$A\frac{d(h - h_0)}{dt} = (F_i - F_0) - \frac{\alpha}{2\sqrt{h_0}}(h - h_0) \qquad (3.3)$$

Now we introduce the **<u>deviation variables</u>** (i.e. the variables showing only the changes from the operating point) defining as

$$\overline{h} = h - h_0 \quad , \quad \overline{F_i} = F_i - F_0 \quad , \quad \overline{F_{out}} = F_{out} - F_0$$

Now Equation (3.3) will be simplified as

$$A\frac{d\overline{h}}{dt} = \overline{F_i} + \beta\,\overline{h} \quad , \quad \beta = -\frac{\alpha}{2\sqrt{h_0}} \tag{3.4}$$

This equation is now representing a simple linear system as a linear approximation of the original non-linear system shown by equation (3.0) around an operating point. It should be noted that this approximation is only valid in the neighborhood of $h_0$ ( or the operating point). <u>It is also seen that the dependence of this linear equation on the operating point is only through parameter β.</u> **Therefore, for different operating points, we can still use this equation but with different values for β.**

If the non-linear term is of higher dimensions, we can follow the same procedure but using the Taylor expansion for higher order functions. For instance, for a second order term we have

$$f(x,y) \approx f(x_0,y_0) + \frac{\partial f}{\partial x}\Big|_{\substack{x=xo\\ y=yo}} (x - x_0) + \frac{\partial f}{\partial y}\Big|_{\substack{x=xo\\ y=yo}} (y - y_0)$$

The following example presents the linearization of a nonlinear function of two variables.

Consider the conical tank system shown in Figure 3.3. The general features of the dynamic behaviour of this tank is similar to the the tank in Figure 3.1 except the fact that the cross sectional area changes with any change in the height.

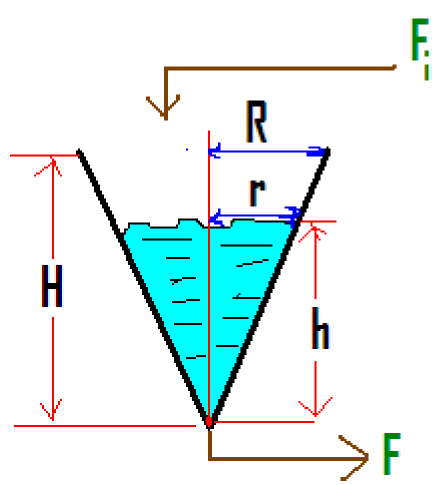



**Figure 3.3: Linear approximation of the nonlinear function for the Tank example.**

For this tank, we have the cross section of the top of the liquid as:

$$A = \pi r^2 = \pi \left( \frac{Rh}{H} \right)^2 \text{ since } \frac{R}{r} = \frac{H}{h}$$

The tank's model becomes:

$$\frac{d(\rho V)}{dt} = \rho F_i - \rho F \quad , \qquad V = \frac{1}{3} A h, \qquad F = \alpha \sqrt{h}$$

$$\rightarrow \frac{dh}{dt} = \frac{\gamma F_i}{h^2} - \beta h^{-3/2} \quad , \quad where \;\; \gamma = \frac{1}{\pi}\left(\frac{H}{R}\right)^2 \quad , \quad \beta = \alpha \gamma$$

In this new nonlinear diferential equation there are two nonlinear terms: $F_i \, h^{-2}$, which is a product of two functions, and $h^{-3/2}$. Therefore, we need to linearize each nonlinear term separately around the steady state $(h_s, F_{is})$. We proceed as below:

For $F_i \, h^{-2}$ :

$$f(h, F_i) \approx f(h_s, F_{is}) + \left( \frac{\partial f}{\partial h} \right)_{(h_s, F_{is})} (h - h_s) + \left( \frac{\partial f}{\partial F_i} \right)_{(h_s, F_{is})} (F_i - F_{is})$$

$$\rightarrow F_i \, h^{-2} \approx F_{is} h_s^{-2} - 2 F_{is} h_s^{-3} (h - h_s) + h_s^{-2} (F_i - F_{is}) \tag{3.5}$$

And for $h^{-3/2}$ :

$$h^{-3/2} \approx h_s^{-3/2} - \frac{3}{2} h_s^{-5/2} (h - h_s) \tag{3.6}$$

Introducing the deviation variables $y = (h - h_s)$ and $u = (F_i - F_{is})$, and substituting the linear approximations (3.5) and (3.6) into the original nonlinear differential equation leads to this simple linear equation

$$\tau \frac{dy}{dt} + y = Ku \tag{3.7}$$

Where

$$K = \frac{2\gamma}{\beta} h_s^{1/2} = \frac{2 h_s^{1/2}}{\alpha} \quad , \quad \tau = \frac{2 h_s^{5/2}}{\beta}$$

# 4. EMPIRICAL PROCESS MODELING

The mathematical (or mechanistic) modeling approaches introduced in the previous chapters leads to one of the most explicit and useful set of models to describe plant or process behaviour. Nevertheless, such approaches are difficult and very time consuming.

Therefore various empirical modelling approaches are oftenly used in practice. These methods could be very easy, fast, and accurate, though they give a black box representation of the input-output behaviour.

Selecting a structure for the model and then finding the best parameters of that structure to fit the input-output data we have already collected from real experiments or simulations is called **empirical modeling**.

In the following, a few examples of such metods are introduced. More useful tools for developing such methods through collected input-output data from the plant based on available software are also introduced in other parts of this course.

## 4.1 FIRST ORDER PLUS TIME DELAY MODELS

This method is based on the step response of the process. The step response of most industrial processes has the general S-shaped curve shown in Figure 5.7 which is also called the *process reaction curve* and can be generated experimentally or from a dynamic simulation of the plant.

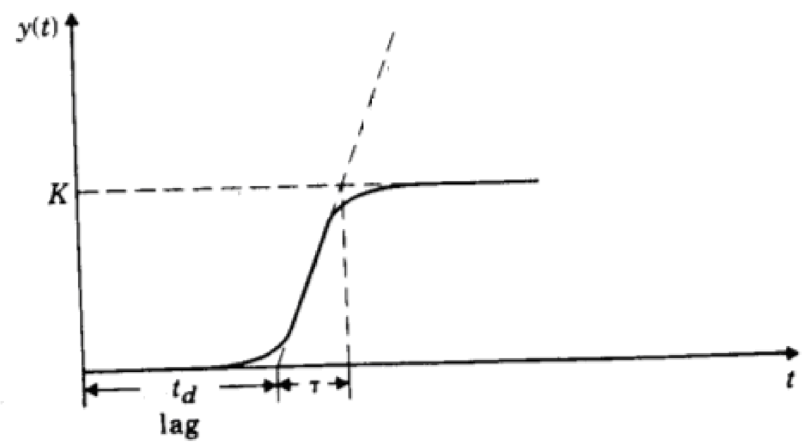



**Figure 4.1. Process reaction curve (i.e a simple step response) .**

The shape of this curve is characteristic of high order systems, and the plant input-output behavior may be approximated by

$$G(s) = \frac{Y(s)}{U(s)} = \frac{K e^{-t_d s}}{\tau s + 1},$$

which is simply a first order system plus a transportation delay (i.e. or lag. It should be noted that time delay in s-domain is equal to time shift in time-domain).

The constants parameters of the above equation can be determined from the unit step response of the process shown above.

If a tangent is drawn from the inflection point of the reaction curve, then the time constant $\tau$, and the time delay $t_d$ can be approximated as shown on the Figure.

Though the above equation often provides an adequate model for the plant, if the actual plant output does not fit this simple model, other structures may be selected as is described below.

## 4.2 HIGHER ORDER MODELS

Using the step response data, the following approach can be invoked to develop models without any pre-assumption on the order of the best model to fit data in a hierarchical way.

We may assume that the transient unit step response of the plant is a combination of some elementary transient responses as

$$y(t) = y(\infty) + Ae^{-\alpha t} + Be^{-\beta t} + Ce^{-\gamma t} + \cdots \qquad (4.1)$$

Figure 4.2 depicts a typical step response of this system.

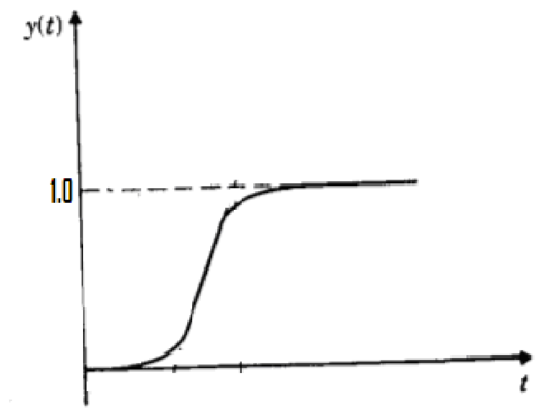



**Figure 4.2. A typical step response.**

Subtracting off the final value $y(\infty)$ and assuming $-\alpha$ as smallest (the slowest) root, we may write

$$\begin{aligned} y(t) - y(\infty) &= Ae^{-\alpha t} + \cdots \\ \log_{10}[y(t) - y(\infty)] &\cong \log_{10} A - \alpha t \log_{10} e \\ &\cong \log_{10} A - 0.4343\alpha t \end{aligned} \qquad (4.2)$$

If $[y(t)-y(\infty)]$ is negative then we multiply both sides by -1 and we will have

$$\log_{10}[y(\infty) - y(t)] \cong \log_{10}(-A) - 0.4343\alpha t$$

This is the equation of a line.

If we fit a line to the plot of $\log_{10}[y(\infty)-y(t)]$, or $\log_{10}[y(t)-y(\infty)]$, then we can estimate A and $\alpha$ . Once these two were estimated we follow the same procedure except that we have

$$\log_{10}[y(\infty)-y(t)+Ae^{-\alpha t}] \cong \log_{10}(-Be^{-\beta t}) \qquad , \qquad (4.3)$$

to estimate B and $\beta$. This procedure will be continued until a model with appropriate order is obtained.

As an example this method is appled to the data which has been collected in Table 4.1.

**Table 4.1. A set of step response data.**

| $t$ | $y(t)$ | $\hat{y}(t)$ | $t$ | $y(t)$ | $\hat{y}(t)$ |
|---|---|---|---|---|---|
| 0.0 | 0.000 | 0.00 | 1.0 | 0.510 | 0.507 |
| 0.1 | 0.005 | −0.022 | 1.5 | 0.700 | 0.701 |
| 0.2 | 0.034 | •0.01 | 2.0 | 0.817 | 0.819 |
| 0.3 | 0.085 | 0.068 | 2.5 | 0.890 | 0.89 |
| 0.4 | 0.140 | 0.134 | 3.0 | 0.932 | 0.933 |
| 0.5 | 0.215 | 0.206 | 4.0 | 0.975 | 0.975 |
| | | | | 1.000 | 1.000 |

Following the procedure described above and using the data given in Table 4.1 we will come up with the graph of Figure 4.3.

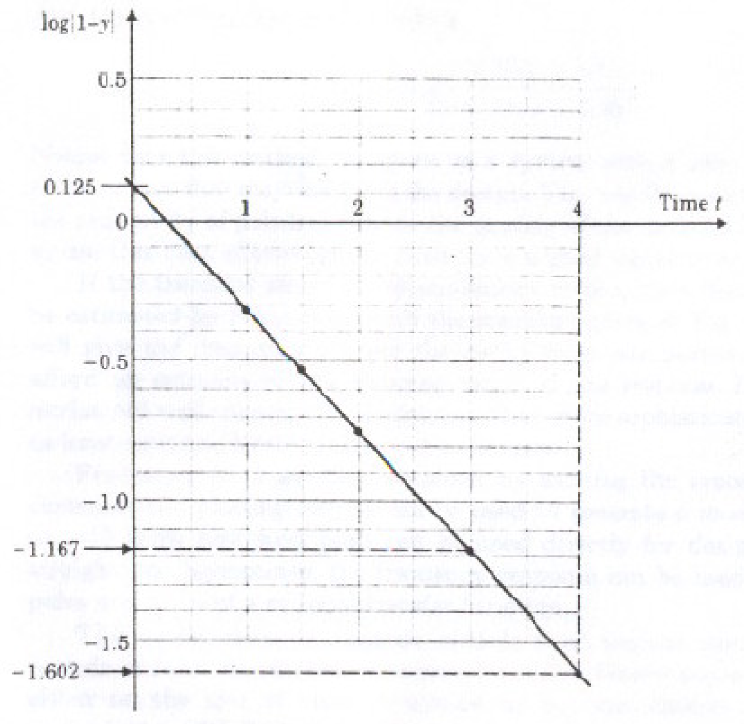



**Figure 4.3. Plot of step response data based on the procedure explained.**

Now from the line fitted by eye on this figure the values are

$$\log_{10}(-A) = 0.125 \qquad \Rightarrow \qquad A = -1.33$$

$$0.4343\alpha = \frac{1.602 - 1.167}{\Delta t} = \frac{0.435}{1} \Rightarrow \qquad \alpha = 1$$

Now, if we subtract this line from the previous log plot (following Equation (4.3)), we will obtain Figure 4.4.

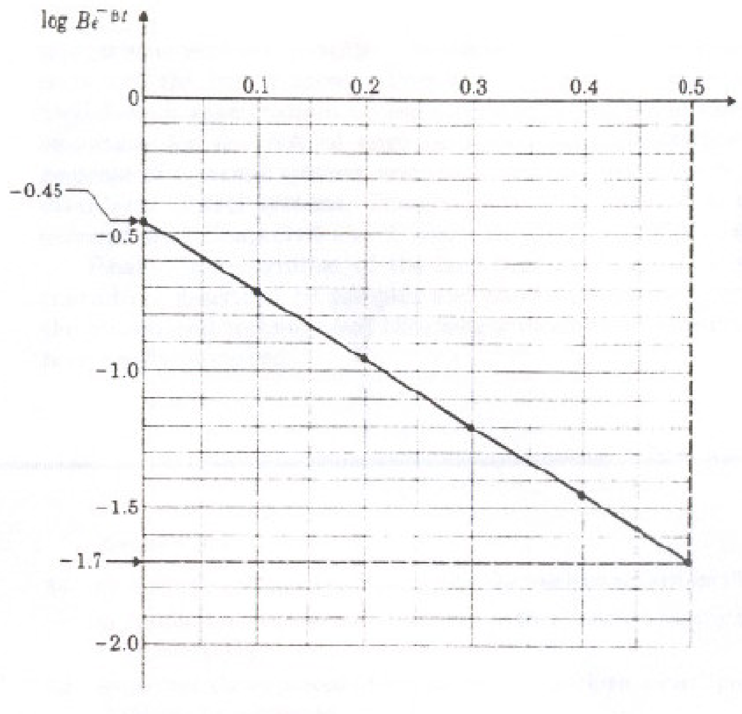



**Figure 4.4. Plot of** $\log_{10}[y(\infty) - y(t) + Ae^{-\alpha t}]$ **for the step response data.**

From Figure 4.4 we estimate B and β,

$$\log_{10}(-B) = -.45 \qquad \Rightarrow \qquad B = -0.35$$

$$0.4343\beta = \frac{-0.45 - (-1.7)}{0.5} = 0.25 \Rightarrow \qquad \beta = 5.8$$

Combining these results, we estimate output

$$\hat{y}(t) = 1 - 1.33e^{-t} + 0.35e^{-5.8t}$$

Where its corresponding Laplace form is

$$\hat{Y}(s) = \frac{1}{s} - \frac{1.33}{s+1} + \frac{0.35}{s+5.8} = \frac{0.02s^2 - 0.56s + 5.8}{(s+1)(s+5.8)} \cdot \frac{1}{s}$$

$$\Rightarrow G(s) \approx \frac{-0.56s + 5.8}{(s+1)(s+5.8)}$$

If this model is not accurate enough, we may continue to obtain higher orders of this model. But it does not seem necessary here.

Experimetal modelling approaches presented in this section and the previous section are only two simple examples of experimental modelling. However, since you will be familiar with MATLAB and other modelling and control software, you will have access to various modelling tools and there is no need to learn hand-driven methods anymore.

# 5. CLASSICAL CONTROLLERS AND TUNING

The typical control system has the feature that some output quantity is measured and then compared with the desired value, and the resulting error is used to correct the system's output. This concept is called *feedback* control or *closed loop* control. This is in contrast to the case when no output measurement is used to decide on the control signal which is called *open loop* control (i.e. Two common examples of open-loop control systems are an electric toaster in the kitchen, and traffic lights in in the streets). Figure 5.1 illustrates these structures.

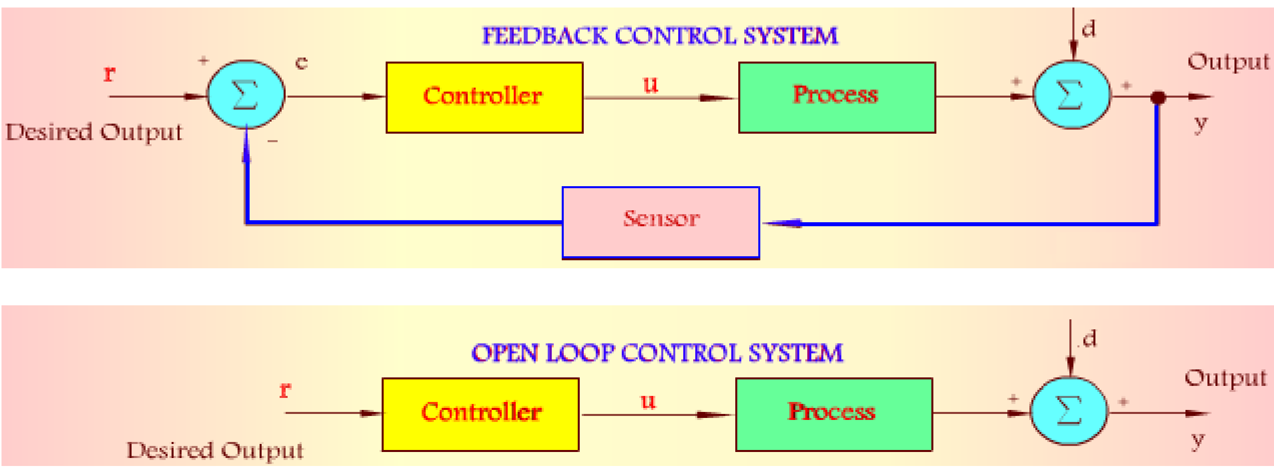



**Figure 5.1. Open loop and feedback (closed loop) control systems.**

## 5.1. General Features of Feedback

Feedback control has several interesting features which will be discussed here. However, such a control system will also bring various issues into the design problem and makes the design and analysis tasks more complicated than open loop control systems. These concepts are explained in the following.

Lets consider the topology of the systems shown in Figure 5.2.

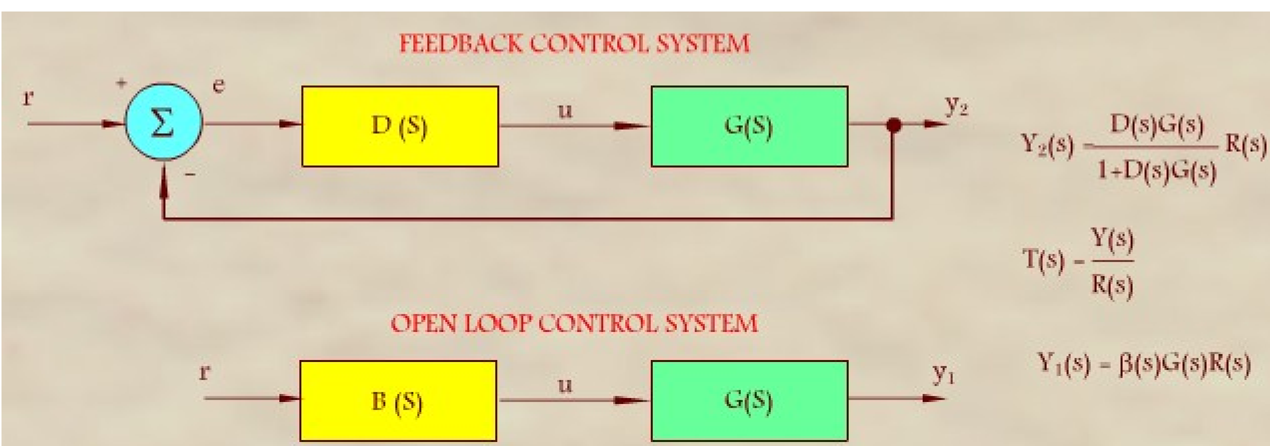



**Figure 5.2. Control system topology (open loop and closed loop (feedback)).**

The outputs of these systems are

$$Y_2(s) = \frac{D(s)G(s)}{1 + D(s)G(s)} R(s) \quad , \qquad Y_1(s) = \beta(s)G(s)R(s)$$

It is readily apparent that any change in $\beta$ or $G$ for the open loop case will cause proportionate errors in $y_1$. In contrast, changes in $D$ or $G$ in the feedback case will be attenuated if $|DG|$ is made bigger than unity. In fact, if $|DG|>>1$, from the above transfer functions one can see that $y_2$ will be essentially independent of the values of $D$ and $G$. This reduced ***sensitivity*** is one of the key reasons for using feedback.

Though the most important reason to use feedback is to reduce sensitivity, there are other useful features when using feedback which are:

- **Disturbance rejection** – since in closed loop system we notice the effect of disturbances through output measurement, the controller will compensate for that.
- **Model uncertainty** – since models are always an approximation to the true behavior of the plant, we need to take care of *uncertainty* of the plant, feedback easily deals with that with the reduced sensitivity concept (this is somehow similar to plant parameter variations concept).
- **Controlling unstable plants** – if the plant to be controlled is unstable, then open loop control cannot easily handle that while feedback can.

On the other hand, feedback control design is usually more costy, and more complicated and the system gain will be reduced (i.e. compare open loop and closed loop transfer functions from the input-output gain point of view).

## On-off controllers (or two positions controllers)

Though feedback or closed loop control is more powerful than open loop controllers, its quality highly depends on the type of controller used in closed loop.

On-off (or two position) controller algorithm is as the simplest type of feedback controller defined as:

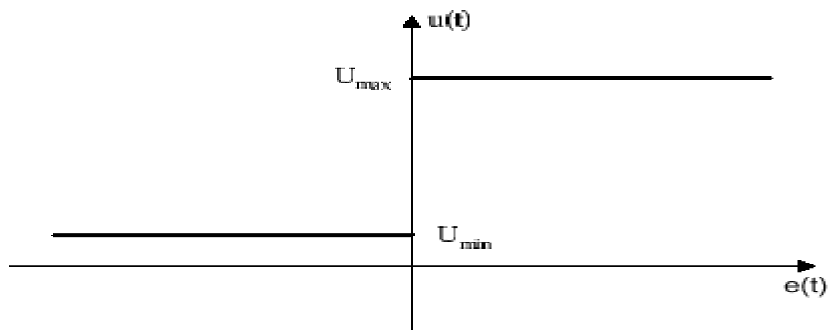
$$\begin{cases} U_{\max}; & \forall e(t) > 0 \\ U_{\min}; & \forall e(t) < 0 \end{cases}$$

Where:

e(t) – control error (for unit feedback)

u(t) – control signal (controller output).

Static characteristic of on-off controller is given in as.




Control signal *u(t)* can have only two possible values, high $U_{max}$ or low level $U_{min}$, depending if error is positive or negative.

Assuming that process (controlled plant) has a positive static gain, high-level control signal will cause increase in controlled variable value. The main idea in this way of control, with only two control levels is achieve desired value of the controlled variable in shortest time possible.
An inadequacy in this way of control is that control signal oscillates which may cause control variable to oscillate around desired value. Sometimes there is no remedy for this problem. For example, if level of liquid in tank is controlled using valve with only two possible states (open or closed) the level will always oscillates around desired value.

On-off controller is very simple since there are only two possible control signal values, no matter what is the value of control error. Process is forced to oscillate since *u(t)* is never zero (it is either $U_{max}$ or $U_{min}$). The only way to avoid these forced oscillations is to diminish gain for small values of control error *e(t)*. That can be achieved by introducing a proportional mode that will be active for certain values of control error.

Though this type of controller often used for very simple processes in industry, it is not even a continuous controller. Therefore, not much attention is paid to this type of feedback controllers.

**The simplest but continous and effective controllers which are used in industry are the PID family of controllers, also known as classical controllers. This group of controllers are the main subjects of our course and are explained in the next section.**

## 5.2. Types of Feedback Control

- While there are various types of feedback control systems, here the most common types which are still widely used in industries will be introduced.

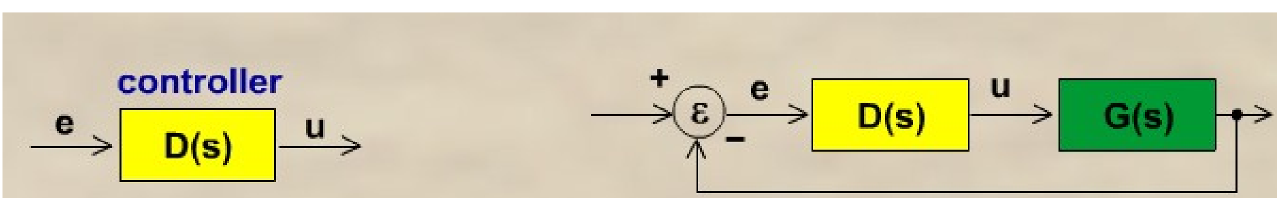



**Figure 5.3. Controller block in a feedback control system.**

### 5.2.1 *Proportional Control* ( **P** )

- In the proportional feedback, the feedback control signal is made to be linearly proportional to the error in the measured output (i.e. comparing to the desired output, see Figure 5.1 and 5.3). The general form of proportional control is

$$u(t) = K_p e(t) \quad ---> \; U(S) = K_p E(s) \quad ---> \; D(s) = K_p$$

and thus $D(s)=K_p$. This means the input error is multiplied by a constant $K_p$ to generate the controller output u(t). But the value of this parameter $K_p$ can be tuned (i.e. changed occasionally).

- To gain a better understanding, consider the Table 5.1 where it shows a very rough idea of how proportional controller works in the closed loop (see also Figures 5.2. and 5.3). It is clear that the most powerful control command (max. of u(t)) happens when the output is zero and far from desired value. The weakest command appears when the output reaches the desired value, and thus (i.e. unfortunately) since the error is zero the control command stops! Thus the output start moving away from its desired value! As you see **P** control acts only on the basis of *current status of the error signal*

**Table 5.1. A very rough explanation of P-Controller operation.**

Assume $K_P = 5$

| t | r | y | e | u |
|---|---|---|---|---|
| 0 | 10 | 0 | 10 | 50 |
| 2 | 10 | 3 | 7 | 35 |
| 4 | 10 | 7 | 3 | 15 |
| 6 | 10 | 10 | 0 | 0 |
| 8 | 10 | 8 | 2 | 10 |

- To understand how the proportional controller changes the closed loop poles of the system consider G(s) as an arbitrary second order plant,

$$G(s) = \frac{1}{s^2 + as + b}$$

then with a proportional feedback the closed loop transfer function becomes

$$H(s) = \frac{D(s)G(s)}{1 + D(s)G(s)} = \frac{K_p}{s^2 + as + b + K_p}$$

The *characteristic equation* of the closed loop system is

$$s^2 + as + b + K_p = 0$$

and the roots of this equation (i.e. poles of the closed loop system) are

$$s = -\frac{a}{2} \pm \sqrt{\left(\frac{a}{2}\right)^2 - (b + K_p)}\ .$$

- **Now as a design task**, one may choose the proportional gain $K_p$ in such a way that poles of the closed loop system be at the left half plane to assure system stability (assume $a=3$ and $b=2$).
- Though the proportional feedback is simple and useful for some applications, it is most often not capable enough to provide a desirable control (see Figure 5.5).

### **5.2.2.** *Integral Control* **( I )**

- Integral control has the form

$$u(t) = \frac{K_p}{T_i}\int_0^t e\,dt \;\rightarrow U(s) = \frac{K_p}{T_i \cdot s}E(s) \;\rightarrow\; D(s) = \frac{K_p}{T_i s}$$

where $T_i$ is called the *integral or reset time* and $1/T_i$ is referred to as the *reset rate*. One beauty of this feedback type is that it can provide a finite value of *u* while the error signal is zero. This is because *u* is a function of the past values of *e* rather than only of the current value. This is an important feature which can eliminate the steady state error of the system.

- Considering Figure 5.4, we see that **I** controller calculates the area under the error signal and this area is almost zero at the beginning and become larger and larger as time goes on. Besides, even if the error becomes zero, the area calculated could be a finite value and if error remains zero, the area will remain constant, and consequently output stays at its desired value! Therefore, we could easily say that **I** control decides based on the history of the error (i.e. or roughly based on past experiences obtained) not the current situation or future direction of the error.

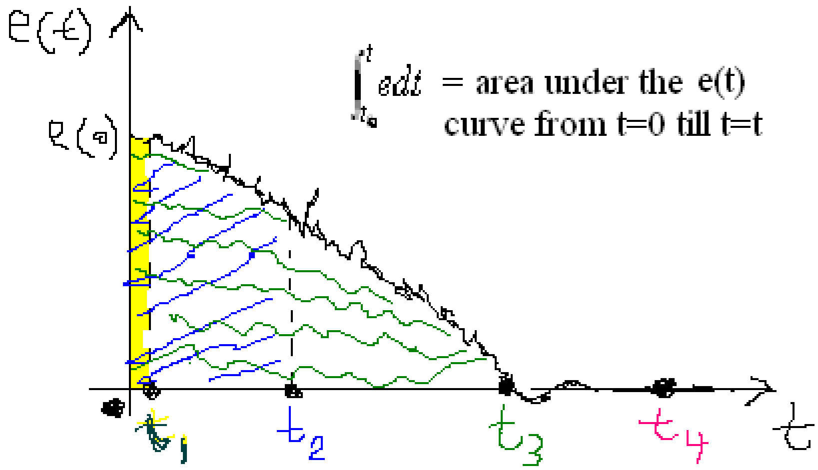


**Figure 5.4. Integral control action.**

- While the integral control can improve the performance of the system and help the output to reach its desired value, it will make the system *less stable* as it is discussed in the following.

### 5.2.3. *Derivative Control* **( D )**

- The form of derivative feedback is as follows,

$$u(t) = K_p T_d \frac{de(t)}{dt} \rightarrow U(s) = K_p T_d \cdot s \ \ E(s) \rightarrow D(s) = K_p T_d s$$

here $T_d$ is called the derivative time.

- Considering Figure 5.4, we can realize that the **D** controller calculates the derivative or the slope of the changes in error. So the largest command appears when a sudden changes happens in error and the smallest (i.e. zero) command happens when error is almost constant (i.e. not necessarily zero). Besides, if there are noises in error as you see in Figure 5.4, then the **D** action will be terrible and cannot make correct decisions! Therefore, we can say, **D** control make decision based on the future direction of the error (i.e. one step ahead prediction), not on the past or current situation of the error. But, it cannot be used when there is noise in the error and also does almost nothing if the error is not changing much or is just constant.
- The derivative feedback is typically used in conjunction with proportional and/or integral control in order to increase the damping and generally to improve the stability of a system.

### 5.2.4. *PID Control (Proportional + Integral + Derivative Control)*

- Table 5.2 summarizes some of the features of each of the controllers introduced above. These are important features and it is because of these features that in practice we use a combination of these controllers more often to reduce their weaknesses and benefit from their positive features.

**Table 5.2. A brief comparison of the P, I, and D controllers.**

| Features / Control Type | advantages | disadvantages |
|---|---|---|
| P Control | fast, simple, cheap, | cannot take us to desired output (there will be steady state error) , decides only on current status of error |
| I Control | can take us to the desired output , it is not sensetive to noise or sudden changes , act based on history of the error | very slow, make system less stable, not cheap and simple |
| D Control | very fast, make system more stable | cannot take us to desired output very sensitive to noise not cheap and simple |

- A feedback control signal which is the sum of proportional plus integral plus derivative control is referred to as *PID* control. Such a control signal aims at using the advantages of all three types of control actions.
- The control signal and its Laplace transform are expressed as below,

$$u(t) = K_p(1 + \frac{1}{T_i}\int_0^t e(t)dt + T_d\frac{de(t)}{dt}) \rightarrow U(s) = K_p(1 + \frac{1}{T_i s} + T_d s)E(s)$$

$$D(s) = K_p(1 + \frac{1}{T_i s} + T_d s) \iff D(s) = K_p\frac{T_i T_d s^2 + T_i s + 1}{T_i s}$$

$$D(s) = K_p + \frac{K_i}{s} + K_d s \iff D(s) = \frac{K_d s^2 + K_p s + K_i}{s}$$

- This combination is often used to provide an acceptable degree of error reduction simultaneously with acceptable stability and damping. Many of commercially available controllers used in industries have this form and the designer only needs to *tune* the controller by finding appropriate values of the parameters $K_p$ , $T_i$ and $T_d$ (i.e. or $K_p$ , $K_i = K_p / T_i$ , and $K_d = K_p T_d$).
- Figure 5.5 depicts the closed loop response of a typical second order system to a unit change in the reference input for different control combinations and their tuning values. MATLAB codes are shown in Figure 5.6.

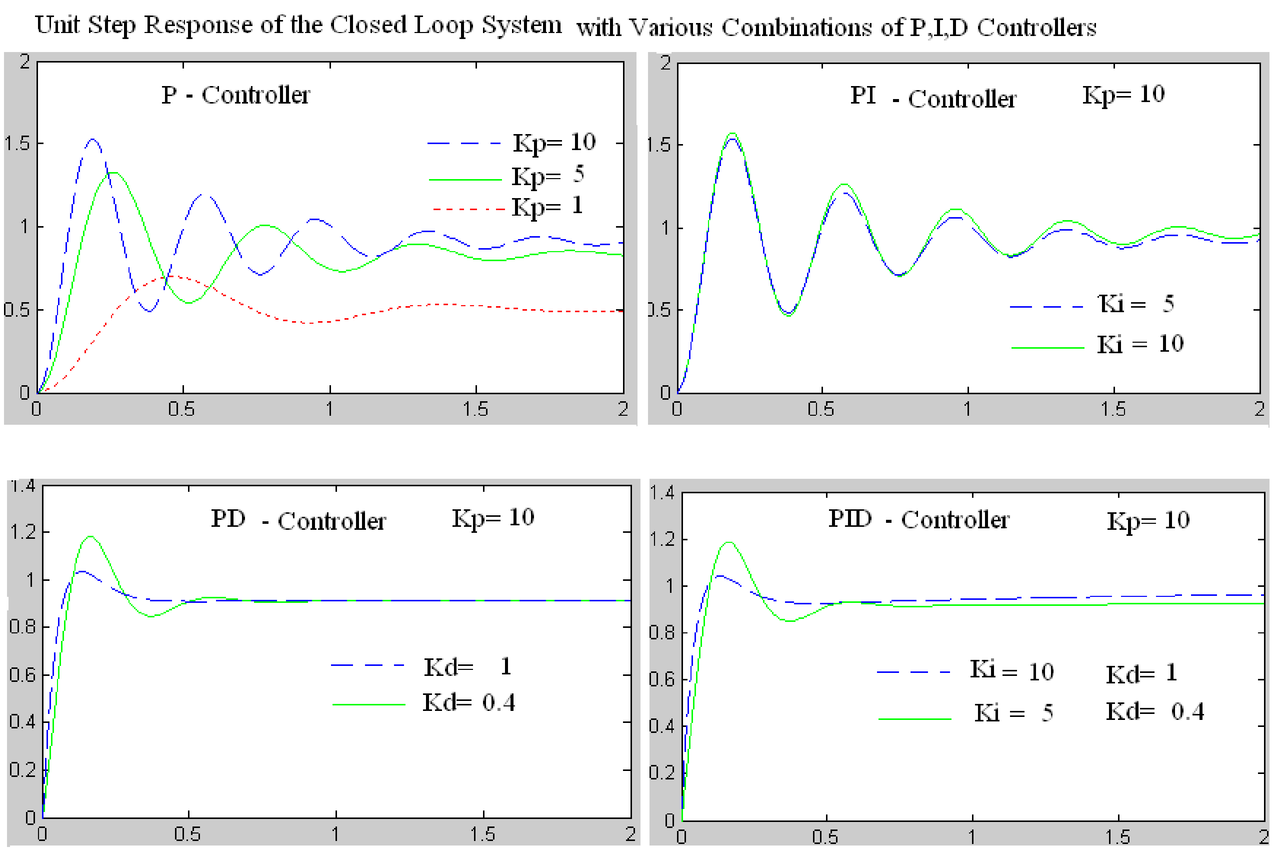



**Figure 5.5. Applications of the PID controller family to a second order plant (step response at reference input). Here $G(s)= 25/(S^2 + 4S + 25)$ . In this figure, all the responses with contribution of " I " controller will reach the final desired value after some time.**

- The role of tuning parameters $K_p$ , $T_i$ and $T_d$ (i.e. or $K_p$ , $K_i = K_p / T_i$ , and $K_d = K_p T_d$), can be seen as a tuning parameters to change the location of the closed loop poles resulting in various type of response. They may also be seen as the weights to contribution of each controller type (i.e. P, I, or D) in making final control decision. Therefore, for the family of PID controllers, the major decision is just to find the proper tuning values!
- Table 5.3 shows some guidelines for understanding the effect of each tuning parameter. Investigate these tuning guidelines with results in Figure 5.5.

**Table 5.3. Effects of PID tuning parameters on closed loop (S_S means Steady State) .**

| CL RESPONSE | RISE TIME | OVERSHOOT | SETTLING TIME | S-S ERROR |
|---|---|---|---|---|
| **Kp** | Decrease | Increase | Small Change | Decrease |
| **Ki** | Small Change | Increase | Increase | Eliminate |
| **Kd** | Decrease | Decrease | Decrease | Small Change |

```
% MATLAB Code for creating Figure 5.5

Kp=10;   %% Kp= 1 5 10, Ki=  0 1 5; Kd= 0 .4 1;
Ki=1;
Kd=.4;
num_Control=[Kd Kp Ki];
den_Control=[1 0];
disp(' Controller')
sys_controller=tf(num_Control,den_Control)
disp('Plant')
num_plant=[25];
den_plant=[1 4  25];
sys_plant=tf(num_plant,den_plant)


disp(' Closed Loop System')
sys_closed_loop=feedback(series(sys_controller,sys_plant),1)
t=0:.01:2;

plot(t, step(sys_closed_loop,t),'g')
```

**Figure 5.6. MATLAB codes to create each part of Figure 5.5. By changing tuning parameters and plant transfer function various studies can be done. Try it!**

## More Comments on The Controller-Type Decision

- Even though we could advance various reasons and criteria for selecting which controller type will be adequate for which application, it is generally agreed that selections made on the basis of the general characteristics of the different feedback controllers are the most practical.
- Based on detailed theoretical analyses of closed-loop transient responses, the following is a summary of the most salient characteristics of the classical feedback controllers:
  1. Proportional Controller: Accelerates the control system response but leaves a nonzero steady-state offset for all processes except for the pure capacity process.
  2. Proportional + Integral Controller: Eliminates offsets but the system response becomes more oscillatory; the added integral action tends to increase the propensity towards instability as Kc increases.
  3. Proportional + Derivative Controller: Enjoys the anticipatory and stabilizing effect of derivative action but still leaves a nonzero steady-state offset except for the pure capacity process.
  4. Proportional + Integral + Derivative Controller: Integral action eliminates offsets, and the oscillations normally introduced as a result may be curbed somewhat by the derivative action; the presence of derivative action tends to amplify the noise components in noisy signals.
- In light of these characteristics, the following guidelines may be used in selecting the most suitable controller type:
  1. When steady-state offsets are unimportant and can therefore be tolerated, or when the process possesses a natural integrator (as is the case with pure capacity systems), use a P controller. Many liquid level control loops, for example, are on P control.
  2. When offsets cannot be tolerated, use a PI controller. A large proportion of feedback controllers in a typical plant are of the PI type.

3. When it is important to compensate for some natural sluggishness in the overall system, and the process signals are relatively noise-free, use a PID controller. For example, temperature control loops are sometimes under PID control; the effect of the "lag" usually introduced by the measuring devices are compensated for by the derivative action. In contrast, flow loops are seldom on PID control; the signals are more susceptible to noise, and the processes are usually not sluggish.
4. PD controllers are seldom used; however, when used, they make it possible for the control system to withstand higher controller gain values and still remain stable, so that smaller steady-state offsets are therefore achievable with PD controllers than are normally possible with P controllers.

### 5.2.5 *Tuning of PID Controllers*

• As mentioned above, once a PID controller is available, the design task is to appropriately find the parameters $K_p$ , $T_i$ and $T_d$ so that a desirable response is obtained. **This is called the tuning of PID controllers.**

• Though there are many sophisticated design methods for this task, some very simple rules developed in some decades ago are still very useful. The most famous one was developed by **Ziegler and Nichols**.

**Ziegler and Nichols Tuning Methods**

• Ziegler and Nichols recognized that the step response of most process control systems has the general S-shaped curve shown in Figure 5.7 which is called the *process reaction curve* and can be generated experimentally or from a dynamic simulation of the plant.

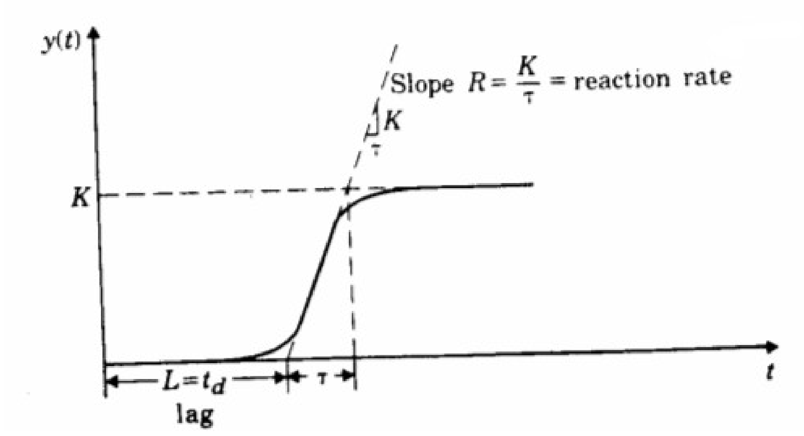



**Figure 5.7. Process reaction curve.**

• The shape of this curve is characteristic of high order systems, and the plant input-output behavior may be approximated by

$$G(s) = \frac{Y(s)}{U(s)} = \frac{K e^{-t_d s}}{\tau s + 1},$$

which is simply a first order system plus a transportation delay (i.e. or lag. It should be noted that time delay in s-domain is equal to time shift in time-domain).

- Modeling a system using step response data based on the above structure was already mentioned in Chapter 4. Zeigler and Nichols used this model and Figure 5.7 to find tuning parameters of PID controllers for one of their tuning methods.

- The constants (i.e. parameters) of the above equation can be determined from the unit step response of the process. If a tangent is drawn from the inflection point of the reaction curve, then the slope of the line is approximately $R=K/\tau$, and the intersection of the tangent line with the time axis identifies the time delay $L=t_d$ .
- Though the above equation often provides an adequate model for the first control design attempt, if the actual plant output does not fit this simple model, other poles may be added to the equation, but this is not our concern here.
- The experimental modeling stage is finished here and we now use it for tuning PID controllers.
- **Ziegler and Nichols gave two methods for tuning the PID controllers** as are discussed below.
- **The first method chooses the controller parameters based on a decay ratio of approximately 0.25** which means that a dominant transient decays to a quarter of its value after one period of oscillation as shown in Figure 5.8. This could provide a good compromise between quick response and adequate stability margins. Such controller parameters suggested by Ziegler and Nichols are seen in Table 5.4 for **the most often used combinations of P, I , and D controllers.**

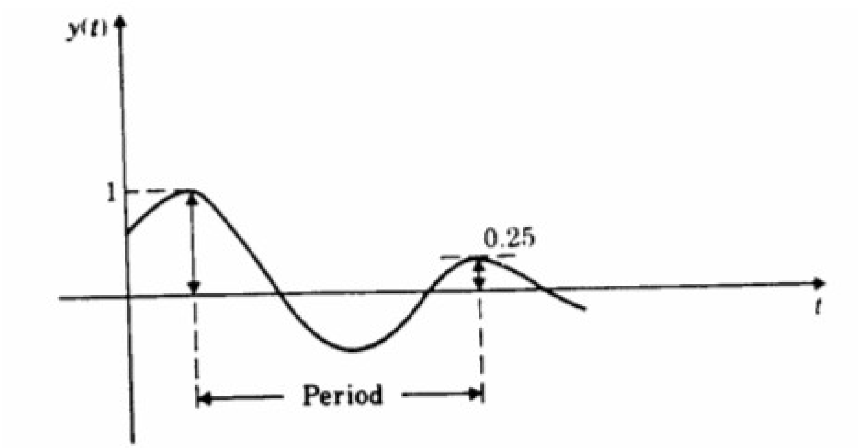



**Figure 5.8. A Quarter decay ratio.**

| Ttpe of Controller | Optimum Gain |
|---|---|
| P | $K_P = \frac{1}{RL}$ |
| PI | $K_P = \frac{0.9}{RL}$ , $T_i = \frac{L}{0.3}$ |
| PID | $K_P = \frac{1.2}{RL}$ , $T_i = 2L$, $T_D = 0.5L$ |

**Table 5.4. Zeigler-Nichols tuning for the regulator,** $D(s) = K_p(1 + \frac{1}{T_i s} + T_d s)$ **,**

**for a decay ratio of 0.25 .**

- **In the second method, the criteria for adjusting the regulator parameters are based on evaluating the system at the limit of stability.** The proportional gain is increased until continuous oscillation is observed, that is, until the system becomes marginally stable. The corresponding gain $K_u$ (also called the ***ultimate gain***) and the period of oscillation $P_u$ (also called the ***ultimate period***) are determined as shown in Figures 5.9 and 5.10. The associated tuning parameters are also given in Table 5.5.

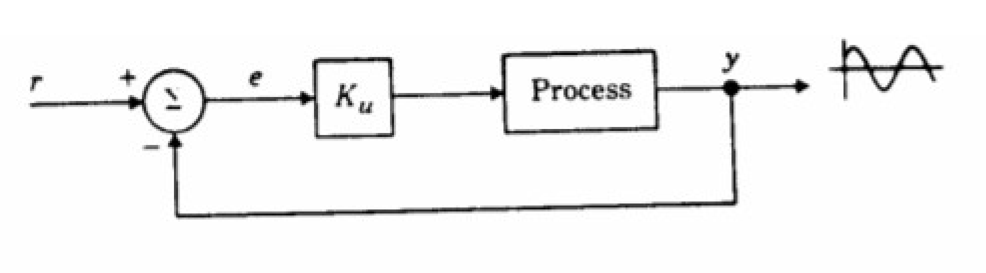



**Figure 5.9. Determination of the ultimate gain and the ultimate period.**

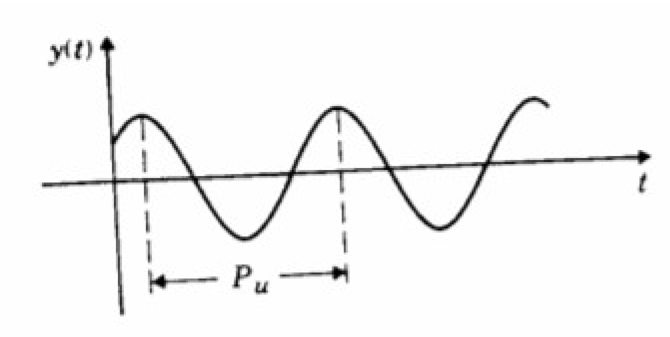



**Figure 5.10. Marginally stable system.**

| Ttpe of Controller | Optimum Gain |
|---|---|
| P | $K_P = 0.5\ K_u$ |
| PI | $K_P = 0.45\ K_u$<br>$T_i = \frac{1}{1.2} P_u$ |
| PID | $K_P = 0.6\ K_u$<br>$T_i = \frac{1}{2} P_u$<br>$T_D = \frac{1}{6} P_u$ |

**Table 5.5. Zeigler-Nichols tuning for the regulator ,** $D(s) = K_p(1 + \frac{1}{T_i s} + T_d s)$ **, based on a stability boundary .**

- Experience has shown that the controller setting based on the above two methods often provide a good closed loop response for many systems. However, the final tuning of the controller can be done manually by the process operator to yield the "best" control.

## Tuning Based on the Optimization of Time-Integral Criteria

- There are also "time-integral" performance criteria that can be used to optimize the choice of controller parameters. The most common of these involve minimizing time-integral functions of e(t) = r(t) – y(t); for example, the minimization of the following:

1. **Integral Absolute Error (IAE)**

$$IAE = \int_0^\infty |e(t)|dt$$

2. **Integral Squared Error (ISE)**

$$ISE = \int_0^\infty |e(t)|^2 dt$$

which selectively penalizes large errors.

3. **Integral Time-weighted Absolute Error (ITAE)**

$$ITAE = \int_0^\infty t.|e(t)|dt$$

which more heavily penalizes errors at long times.

4. **Integral Time-weighted Squared Error (ITSE)**

$$ITSE = \int_0^\infty t.|e(t)|^2 dt$$

which more heavily penalizes large errors at long times.

- Typical responses resulting from applying these criteria will be illustrated with the next example.

- It is possible, given any arbitrary process model, to use standard computer programs to find the optimal tuning parameters $K_p$ , $T_i$ and $T_d$ for the given criteria above.
- The procedure simply involves optimizing over the controller tuning parameters in order to minimize the chosen error criterion.
- To illustrate, the following example shows the results of this type of design for a three-tank system under feedback level control.

- Example: CONTROLLER DESIGN BASED ON TIME-INTEGRAL OBJECTIVES.
- For a system with the following transfer function

$$G(s) = \frac{6}{(2s+1)(4s+1)(6s+1)}$$

determine the optimal PI controller settings and the optimal closed-loop responses for each of the integral-time criteria given before.

Solution:

By using the MATLAB program to perform the required optimization, the following parameters were determined to minimize the indicated time-integral criteria:

- IAE: Kc = 0.314; l/ $T_i$, = 0.769;
- ISE: Kc = 0.537; 1 / $T_i$, = 0.0432;
- ITAE: Kc = 0.207; 1/ $T_i$; = 0.0996;
- 1TSE: Kc = 0.343; 1 / $T_i$, = 0.0708;

• The various closed-loop responses obtained using these settings are shown in Figure 15.11. The results indicate that ISE and 1TSE produce shorter rise times than IAE and ITAE but larger overshoot. Also the time weighted objectives ITSE and ITAE have longer rise times because deviations at short times are not penalized heavily. All criteria provide rather good settling times.

• The ITAE criterion is usually more desirable.

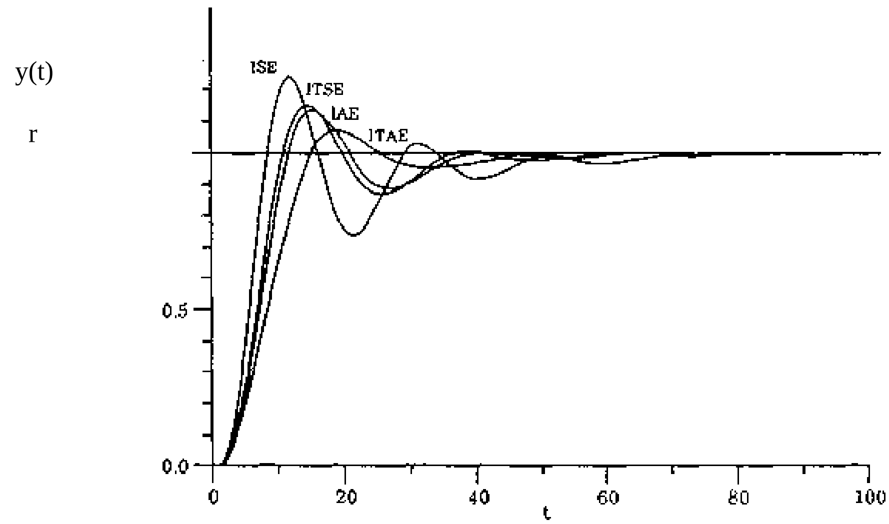



**Figure 5.11. Closed-loop responses for the given transfer function with a PI controller that results from minimizing the time integral objectives.**

## Model Following PID Controller Designs: Direct Synthesis Tuning

• There are a number of model following tuning procedures that work on the principle of finding the PID controller parameters that cause the actual closed-loop system to behave in a prescribed fashion.

• In some cases, what is prescribed is an entire closed-loop trajectory which the actual process is required to track; in others, it is the location of the closed-loop poles, rather than the entire trajectory, which is prescribed.

• The direct synthesis approach seeks to find the feedback controller $G_c$ required to produce a prespecified closed-loop response. Recall that the closed-loop transfer function for a process under feedback control is:

$$Y(s) = \frac{G(s)\cdot G_c(s)}{1+G(s)\cdot G_c(s)} R(s)$$

• If we require that the closed-loop response follow a desired trajectory represented by the transfer function $G_{desired}(s)$, i.e.:

$$Y(s) = G_{desired}(s)\, R(s)$$

then by combining above equations we obtain:

$$G_c = \frac{1}{G}\left(\frac{G_{\text{desired}}}{1-G_{\text{desired}}}\right) = \frac{1}{G}\left[\frac{1}{\left(\frac{1}{G_{\text{desired}}}\right)-1}\right]$$

- With this choice, the feedback controller will result in a closed-loop behavior represented as desired. Thus, given G(s), the process model, and $G_{desired}(s)$ the specified desired trajectory, one can determine the feedback controller $G_c$.
- We note here that in general, the resulting $G_c$ may have a non-PID structure; in fact it is only for certain classes of process transfer functions and certain desired closed-loop trajectories that the controller $G_c$ calculated as above has the PID structure. In the cases which lead to non-PID structures, one may use whatever structure naturally arises or employ approximations to bring the controller into the PID form.

- For the purpose of illustration, suppose the desired approach to a new set-point is modeled by a reference trajectory:

$$\frac{Y(s)}{R(s)} = G_{\text{desired}}(s) = \frac{1}{\tau_r^2 s^2 + 2\tau_r \zeta_r s + 1} \tag{5.1}$$

- Let us find the controller tuning parameters that bring the actual process response as close as possible to the specified response by Eq. (5.1).
- Recall that Eq. (5.1) is the general model for a second-order system that can have either overdamped (no overshoot) or underdamped response. Thus Eq. (5.1) can approximate most of the actual closed-loop responses possible.
- Let us now illustrate the design of direct synthesis controllers using as an example a third-order process with G(s) of the form:

$$G(s) = \frac{K}{(\tau_1 s + 1)(\tau_2 s + 1)(\tau_3 s + 1)}$$

- Then $G_c(s)$ from G(s) and $G_{\text{desired}}(s)$ is derived as

$$G_c(s) = \frac{(\tau_1 s + 1)(\tau_2 s + 1)(\tau_3 s + 1)}{K\tau_r s \ (\tau_r s + 2\zeta_r)}$$

Which can be rearranged to

$$G_c(s) = \frac{(\tau_1 s + 1)(\tau_2 s + 1)(\tau_3 s + 1)}{\beta s(\phi s + 1)} \quad \text{where } \phi = \frac{\tau_r}{2\zeta_r}, \quad \beta = 2K\tau_r\zeta_r = 4K\zeta_r^2\phi$$

- Now if we choose $\phi$ to be equal to one of the time constants (say, $\tau_3$), we obtain:

$$G_c(s) = \frac{(\tau_1 s + 1)(\tau_2 s + 1)}{\beta s}$$

which when compared to the PID controller transfer function

$$G_c^{PID}(s) = K_p(1 + \frac{1}{T_i s} + T_d s) = \frac{K_p(1 + T_i s + T_i T_d s^2)}{T_i s}$$

yields the PID tuning parameters

$$K_p = \frac{(\tau_1 + \tau_2)}{\beta}, \ T_i = (\tau_1 + \tau_2), \ \ T_d = \frac{\tau_1 \tau_2}{(\tau_1 + \tau_2)}$$

## 5.4 More Industrial Perspective of PID Controllers

- Many industrial processes are nonlinear and thus complicate to describe mathematically. However, it is known that a good many nonlinear processes can be satisfactorily controlled using PID controllers providing that controller parameters are tuned well.
- Practical experience shows that this type of control has a lot of sense since it is simple and based on 3 basic behavior types: proportional (P), integrative (I) and derivative (D). Instead of using a small number of complex controllers, a larger number of simple PID controllers are used to control simpler processes in an industrial assembly in order to automate certain more complex processes.
- PID controller and its different types such as P, PI and PD controllers are today basic building blocks in

## Proportional Band (PB)

Characterizes the range over which the error must change in order to drive the actuating signal of the controller over its full range. It is defined as:

$$PB = \frac{Maximum\ range\ of\ measured\ variable}{Maximum\ range\ of\ controller\ output} \times \left( {}^{100}\!/_{K_c} \right)$$

The proportional band is the range of deviations, in percent of scale, that corresponds to the full range of valve opening. This terminology most often used in industry instead of Kc.

## Proportional Gain ($K_c$)

The larger the gain $K_c$, or equivalently, the smaller the proportional band, the higher the sensitivity of controller's actuating signal to deviations will be.

# Basic controller types

PID controllers use a 3 basic behavior types or modes: P-proportional, I-integrative and D-derivative. While proportional and integrative modes are also used as single control modes, a derivative mode is rarely used on it's own in control systems.

Combinations such as PI and PD control are very often in practical systems.

## PI controller

PI controller forms control signal in the following way:

$$u(t) = K\left[ e(t) + \frac{1}{T_i}\int_0^t e(\tau)d\tau \right]$$

where: $T_i$ – integral time constant of PI controller

Constant $K_i = \frac{K}{T_i}$ is called "reset mode". Integral control is also sometimes called reset control.

The name comes from the term "manual reset" which marks a manual change of operating point or of "bias" $u_0$ in order to eliminate error. PI controller performs this function automatically.

PI controllers are very often used in industry, especially when speed of the response is not an issue. Thus, PI controller will not increase the speed of response. It can be expected since PI controller does not have means to predict what will happen with the error in near future. This problem can be solved by introducing derivative mode which has ability to predict what will happen with the error in near future and thus to decrease a reaction time of the controller.

Integral action can occur in the controller only on purpose, by design. Integral action can be noted on the other parts of the control system (actuators, plant etc.). These components may help in diminishing steady state error, but control system designer generally cannot tune these components.

## PID controller

$$u(t) = Ke(t) + K_i \int_0^t e(\tau)d(\tau) + K_d \frac{de(t)}{d(t)}$$

where:

- $K_i = \frac{K}{T_i}$ - gain (reset) of integral part of the controller,
- $K_d = KT_d$ - gain of derivative part of the controller.

Derivative part of PID controller is proportional to the prognosis of error signal at time $t + T_d$ where $T_d$ is derivative time constant of the controller.

A transfer function of PID controller is obtained as sum of transfer functions of individual P, I and D elements.

It can be concluded that PID controller has all the necessary dynamics: fast reaction on change of the controller input (D mode), increase in control signal to lead error towards zero (I mode) and suitable action inside control error area $e(t) < |e_0|$ eliminate oscillations (P mode).

### PD controller

D mode is used when prediction of the error can improve control or when it is necessary to stabilize the system

Often derivative is not taken from the error signal but from the system output variable. This is done to avoid effects of the sudden change of the reference input that will cause sudden change in the value of error signal. Sudden change in error signal will cause sudden change in control output. To avoid that it is suitable to design D mode to be proportional to the change of the output variable *y(t)*.

Another issue is that the D part of the PID controller in the upper form is not proper. To overcome this, it is commonly implemented as:

$$ST_d \approx \frac{ST_d}{1 + s\frac{T_d}{N}}$$

This is often referred to as 'using a real deviator'. N is used to limit derivative gain on higher frequencies, which becomes another parameter of the PID that has to be selected. Is worth noting that a high N makes the implementation of the D action similar to a true derivative but it also increases the high frequency gain, thus increasing noise sensitivity.

Processes that usually require control error prediction are thermal processes with big inertia. Speed of reaction in this case improves temperature control.

When dealing with systems with transport delay it is also important to have a good error prediction. However, D mode will not be able to give a reliable prediction in the case of transport delay, so in those cases one should use Otto-Smith predictor (controller), not PID controller. If Otto-Smith predictor is not available it is better to use PI controller.

## Limitation of PID controllers

Problem of topology (structure) of controller arises when:

- designing control system (defining structure and controller parameters)
- tuning parameters of the given controller

There are a number of different PID controller structures. Different manufacturers design controllers in different manner. However, two topologies are the most often case:

- parallel (non-interactive)
- serial (interactive)

Parallel structure is most often in textbooks, so it is often called "ideal" or "textbook type". In this non-interactive structure proportional, integral and derivative mode are independent on each other. Parallel structure is still very rare in the market. The reason for that is mostly historical.

First controllers were pneumatic and it was very difficult to build parallel structure using pneumatic components. Due to certain conservatism in process industry most of the controller used there are still in serial structure, although it is relatively simple to realize parallel structure controller using electronics. In other areas, where tradition is not so strong, parallel structure can be found more often.

## Parallel PID topology

A parallel connection of proportional, derivative and integral element is called parallel or non-interactive structure of PID controller. Parallel structure is shown in Figure 5.12.

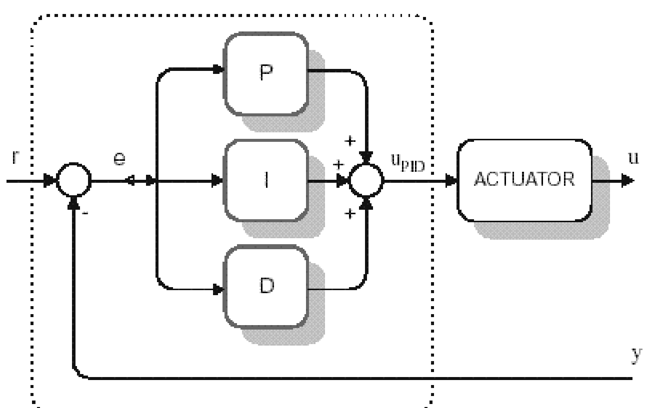



***Figure 5.12:*** Parallel structure of PID controller

PID controller algorithm is given by:

$$u(t) = K\left[e(t) + \frac{1}{T_i}\int_0^t e(\tau)d(\tau) + T_d\frac{de(t)}{d(t)}\right]$$

or:

$$u(t) = Ke(t) + K_i\int_0^t e(\tau)d(\tau) + K_d\frac{de(t)}{d(t)}$$

It can be seen that P, I and D channels react on the error signal and that they are unbundled. This is basic structure of PID controller most often found in textbooks. There are other non-interactive structures.

It is even more suitable controller structure if there exist sensors that give that information, such as tachometers in electromechanical servo systems or "rate gyro" in mobile objects control. If PI-D structure (Figure 5.13) is used, discontinuity in *r(t)* will be still transferred through proportional into control signal uPI-D, but it will not have so strong effect as if it was amplified by derivative element.

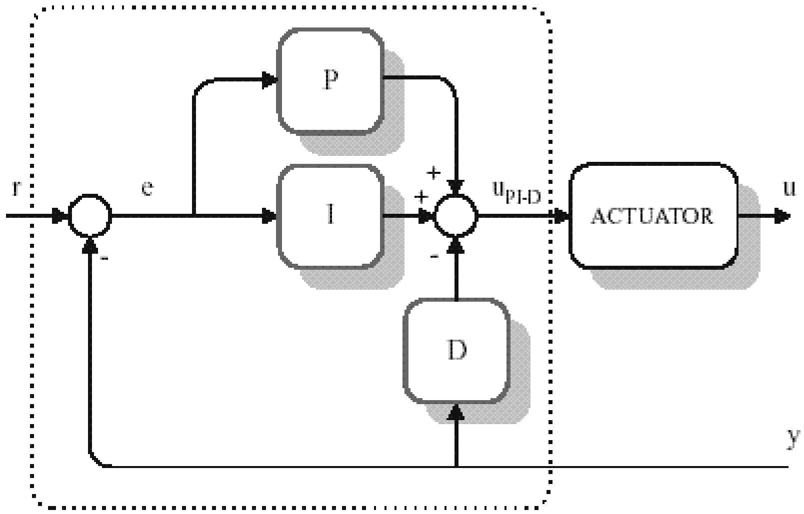



***Figure 5.13:*** Derivative of output controller form (PI-D form)

PI_D controller algorithm is given by:

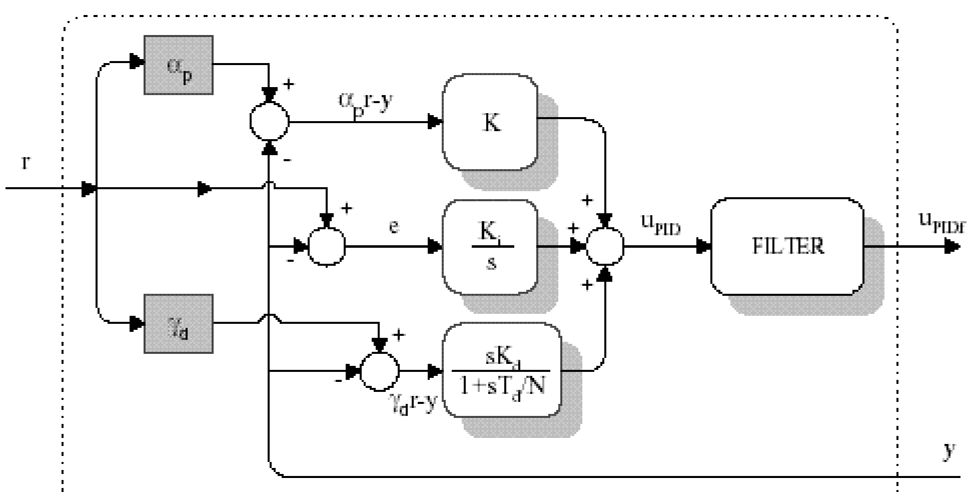
$$u(t) = K\left[e(t) + \frac{1}{T_i}\int_0^t e(\tau)d(\tau) + T_d\frac{dy(t)}{d(t)}\right]$$

or:

$$u(t) = Ke(t) + K_i\int_0^t e(\tau)d(\tau) + K_d\frac{dy(t)}{d(t)}$$

## Standard form (ISA form)

Standard form takes care of possible discontinuity transfer through proportional and derivative channel. A weighting factor is used to limit transferred discontinuity. Also, instead ideal derivate a real derivate is used (casual). ISA form is shown in Figure 5.14.



or

$$U_{ISA} = K\left[\alpha_p R(s) - Y(s)\right] + K_i \frac{1}{s} E(s) + \frac{sT_d}{1 + s\frac{T_d}{N}}\left[\gamma_d R(s) - Y(s)\right]$$

Filter is usually used to filter out high frequency component form the controller output in order to spare actuator from unwarranted action. If sensor gives signals that cannot be followed by system, often a dead zone or notch filter is used instead of lowpass filter to spare actuator of the actions that will be of no use anyway.

Filter can be used with each of PID structures shown if it will improve control system performance. Type of the filter depends on actual case.

## Set-point-on-I-only controller (I-PD form)

This structure of PID controller is not so often as PI-D structure, but it has certain advantages. Control law for this structure is given as:

$$u(t) = K\left[-y(t) + \frac{1}{T_i}\int_0^t e(\tau)d\tau - T_d \frac{dy(t)}{dt}\right]$$

or

$$u(t) = -Ky(t) + K_i \int_0^t e(\tau)d\tau - K_d \frac{dy(t)}{dt}$$

Block diagram for I-PD form is shown in Figure 5.15:

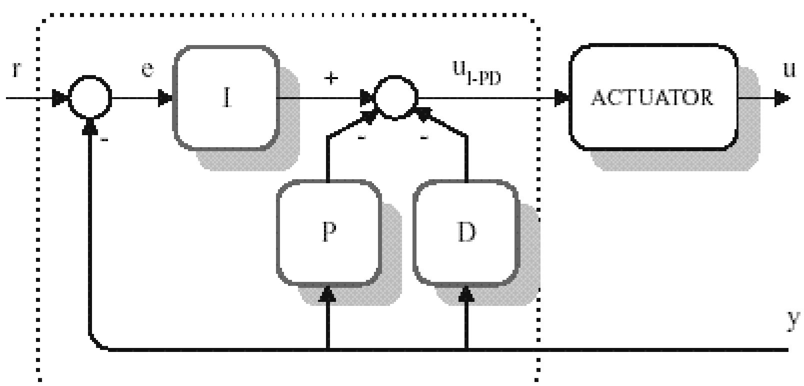



***Figure 5.15:*** I-PD form of PID controller

With this structure transfer of reference value discontinuities to control signal is completely avoided. Control signal has less sharp changes than with other structures.

## General structure of parallel PID controller

After the previous analysis a structure that can perform as any of the previously described controllers can be synthesized. A so called general structure of parallel PID controller is shown in. Figure 5.16. By defining different weighting factors different controller action could be realized.

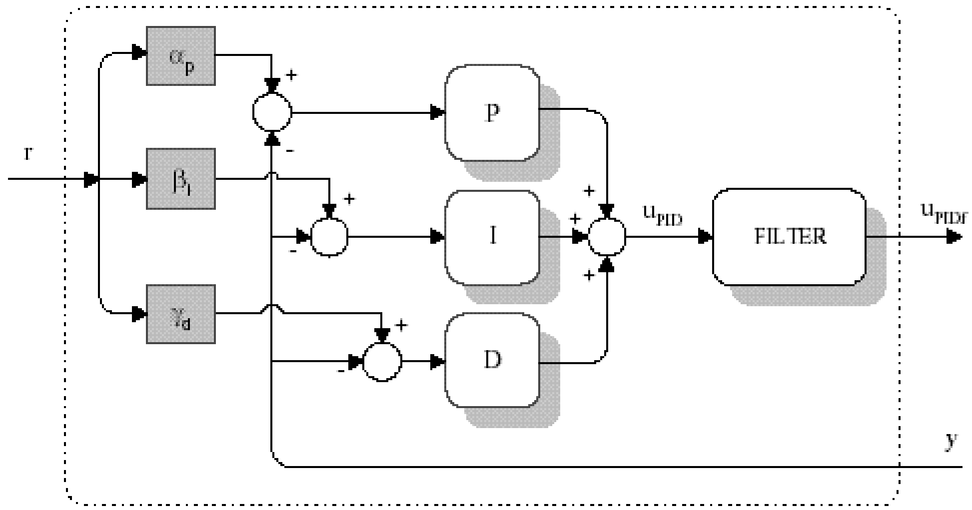



*Figure 5.16:* General structure of PID controller

Weighting factors most often have the following values:

- $\alpha_p = 0$ or 1
- $\beta_i = 1$
- $\gamma_d = 0$

## Serial (interactive) structure (PD*PI form)

This structure is very often in process industry. I channel uses both the error signal *e(t)* and derivative of the error signal $\frac{de(t)}{d(t)}$. It is realized as serial connection of PD and PI controller. Control algorithm is given as:

$$u_{PD*PI}(t) = K^s \left[ e_1(t) + \frac{1}{T_i^s} \int_0^t e_1(\tau) d\tau \right]$$

where:

$$e_1(t) = e(t) + T_d^s \frac{de(t)}{d(t)}$$

Block diagram is given in Figure 5.17.

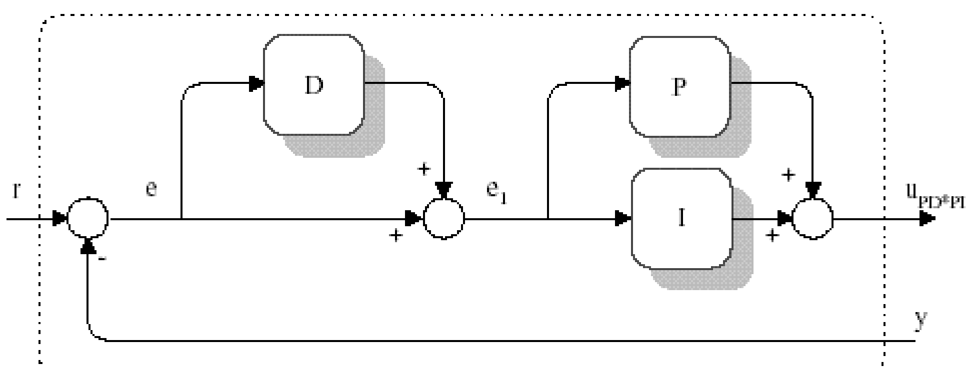



*Figure 5.17*: Serial (interactive) control structure

## Interconnection between parallel and serial structure

However, if parameters of the parallel structure are known, it is not always possible to compute corresponding serial structure. It will be possible to do that only if:

$$T_i > 4T_d$$

The fact that this condition exists shows that the parallel structure is more general than serial structure. In most cases condition $T_i > 4T_d$ is satisfied and in this case serial structure parameters can be computed from:

$$K^s = \frac{K}{2}\left(1+\sqrt{1-\frac{4T_d}{T_i}}\right) \quad T_i^s = \frac{T_i}{2}\left(1+\sqrt{1-\frac{4T_d}{T_i}}\right) \quad T_d^s = \frac{T_i}{2}\left(1-\sqrt{1-\frac{4T_d}{T_i}}\right)$$

**Serial and parallel structures are different only in PID controller case. For P, PI and PD controllers both structures are identical.**

# 6. ADVANCED CONTROL LOOPS

*Up to here, we have discussed the so called* "conventional" *feedback control strategy in which a single output variable is controlled by manipulating a single input variable, to eliminate the effect of a disturbance or to allow for a set point tracking.*

*In the following we will discuss some other strategies which involve more sophisticated schemes (manipulation of more than one input, model based, etc.) referred as* "advanced" control.

*Among them we will discuss the following control schemes:*

- *Feedforward Control*
- *FeedForwad/Feedback Control*
- *Cascade Control*
- *Delay Compensation*
- *Decoupling Control*

## *6.1 Feedforward Control*

*Unlike the feedback system, a feedforward control configuration measures the disturbance (load) directly and takes control action to eliminate its impact.*
*Schematically, this is shown in Figure 6.1.*

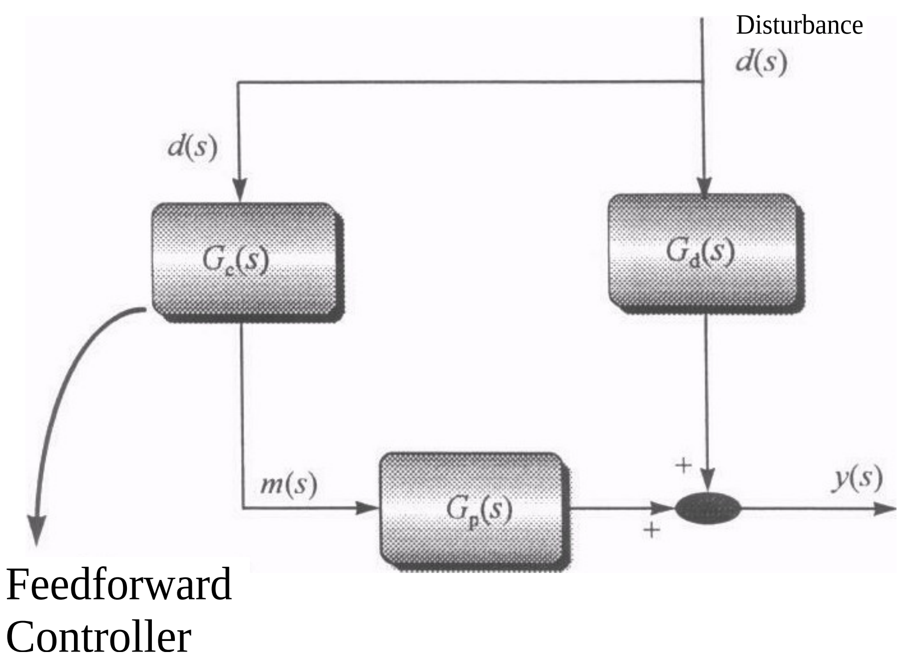



***Figure 6.1: The basic concept of a feedforward control strategy.***

**But how** *do we design a feedforward controller? Let us consider the block diagram of the process shown in Figure 6.2.*

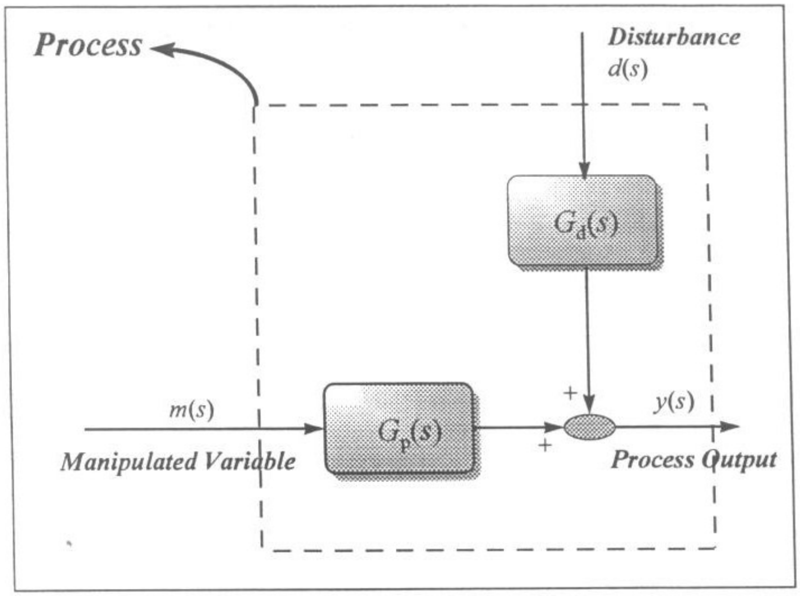



**Figure 6.2: A typical process.**

*For this system*

$$y(s) = G_p(s)m(s) + G_d(s)d(s)$$

*we want y(s)=$y_{sp}$ (s), thus*

$$y_{sp}(s) = G_p(s)m(s) + G_d(s)d(s)$$

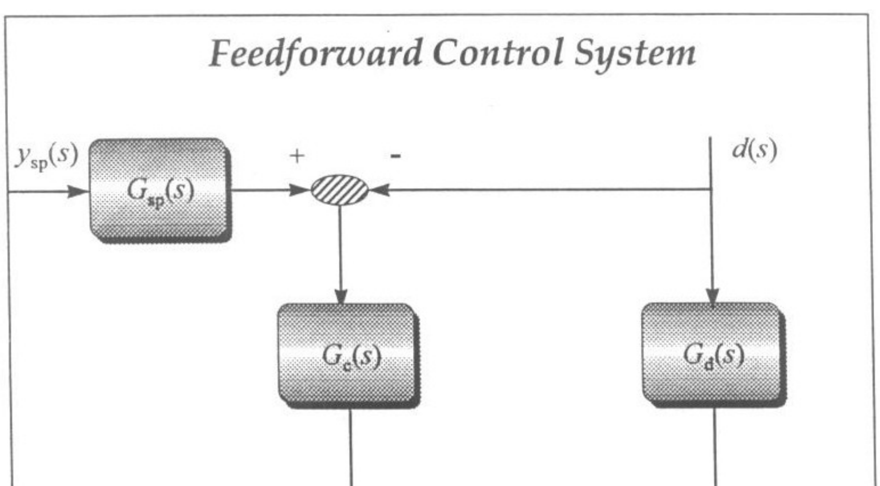
*Consequently*

$$m(s) = \left[\frac{1}{G_d} y_{sp} - d\right]\frac{G_d}{G_p}$$

*Since $y_{sp}$ is given and* **d** *is measured,*

**'We can evaluate m(s) that will keep y(s)=$y_{sp}$(s) in the presence of disturbances and set-point changes"**

*Schematically,*



*Where*

$$G_c(s) = \frac{G_d(s)}{G_p(s)}, \quad G_{sp}(s) = \frac{1}{G_d(s)}$$

***Remarks:***

1. *The feedforward controller cannot be a conventional controller (P,PI or PID). We need a special purpose computer machine.*
2. *Feedforward control depends heavily on a good knowledge of the process models ($G_p$ and $G_d$).*

3. *Feedforward control can be developed for more than one disturbance and can be easily extended to systems with multiple controlled variables.*

4. *With the exception of the controller, all the other hardware elements in the loop are the same as for feedback control.*

## *6.2 Feedforward-Feedback Control Strategy*

*Feedforward has the potential for perfect control but,*

- *Requires the identification of all possible disturbances and their measurements, which may not be possible*
- *Changes in system parameters (e.g. deactivation of a catalyst, heat transfer coefficients, etc.) cannot be compensated.*
- *Requires very good model of the process*

*On the other hand, Feedback controller,*

* *Is insensitive to all previous drawbacks of feedforward but,*

**"Has poorer performance "**

*Remedy,*

" Use a combination of Feedforward-Feedback Control"

*

*Schematically, the idea is depicted in Figure 6.4.*

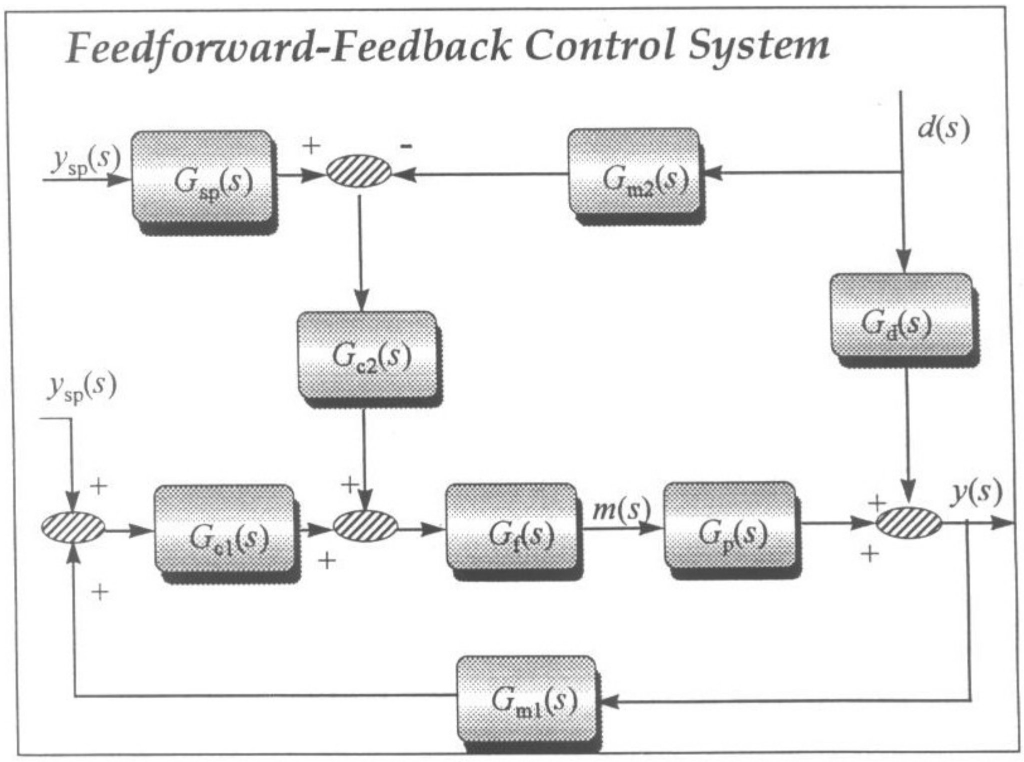



*Figure 6.4: The general feedforward-feedback control structure.*

*For this system the closed-loop transfer function looks like this*

$$y(s)=\frac{G_pG_f(G_{c1}+G_{c2}G_{sp})}{1+G_pG_fG_{c1}G_{m1}}y_{sp}(s)+\frac{G_d-G_pG_fG_{c2}G_{m2}}{1+G_pG_fG_{c1}G_{m1}}d\ (s)$$

*NOTE:*

\. 1. *Characteristic equation for the closed-loop system*

$$1+G_pG_fG_{c1}G_{m1}=0$$

**"The stability characteristics of a feedback control system will not change with the addition of a feedforward loop"**

2. *In this case*

$$G_{c2}=\frac{G_d}{G_pG_fG_{m2}} \quad \text{and} \quad G_{sp}=\frac{G_{m2}}{G_d}$$

## *6.3 Cascade Control*

*In the cascade control configuration we have*

- *One manipulated variable*
- *More than one measurement ,*

*Consider as an example the CSTR given in Fig. 6.5 with an exothermic reaction and with a coolant jacket around the tank. Also shown in the Figure is the implementation of a conventional control strategy. For this specific example we have:*

Control Objective: *Keep* T *at a desired value* $T_{sp}$
Manipulated variable: *Coolant flowrate* $F_c$
Possible Disturbances: $T_i$ and $T_c$

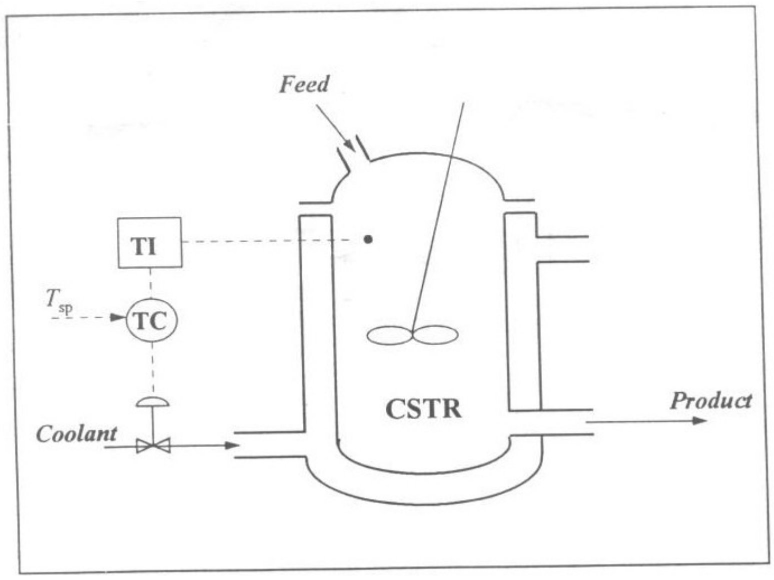



***Figure 6.5:  A typical CSTR process with temperature  control loop.***

*The conventional feedback control is:*

- *Effective for changes in* $T_i$
- *Less effective for changes in $T_c$ (slow response for changes in* $T_c$)

*The question now is:*

*Is there any way that we can improve the response of the simple feedback system to changes in $T_c$?*

*The answer is* yes *we can, by implementing a* Cascade **Control Configuration.** *Consider the new*

*arrangement shown in Fig. 6.6.  It uses:*

- *Two different measurements* T *and* $T_c$
- *Only one manipulated variable $F_c$*

*As net result, this new configuration improves the response   under changes in* $T_c$.

NOTE:
1.*The loop that measures* T *is the dominant or* primary *or* master control loop. *The master loop uses the set-point provided by the operator.*

*2. The loop that measures Tc is called secondary loop or slave loop. It uses the output from the primary loop as set-point.*

***Schematically, this is represented in Fig. 6.6***

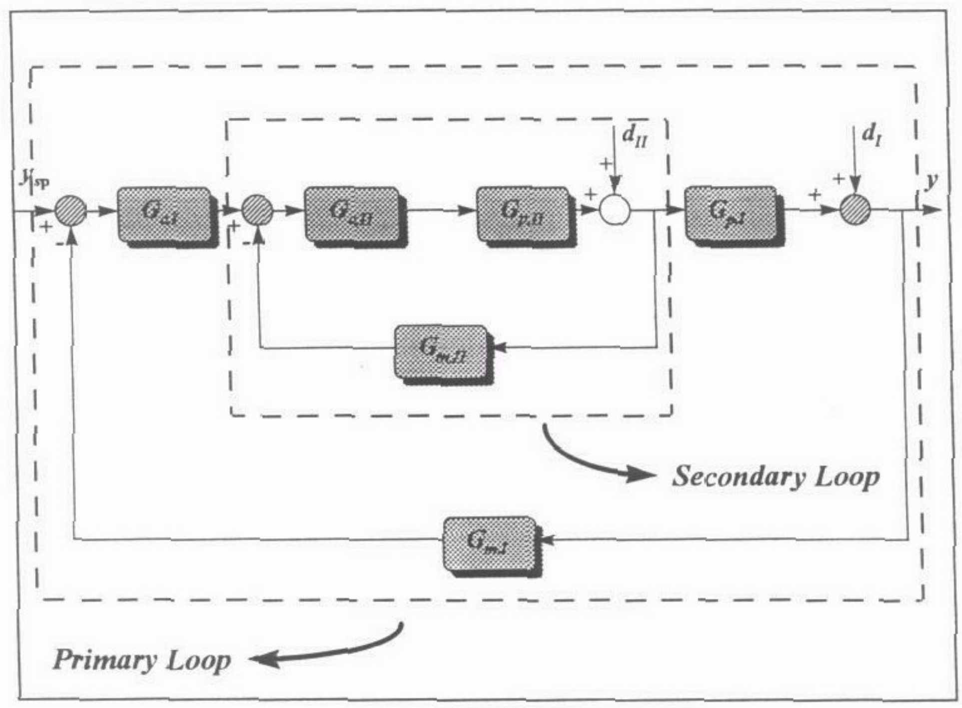



***Figure 6.6: A cascade control configuration.***

"Disturbances **arising within the secondary loop are corrected by the secondary controller before they can affect the values of the primary controlled** output"

**"In chemical processes, flowrate control loops are almost always cascaded with other control loops"**

*The open-loop transfer function for the secondary controller ($G_{OL\,II}$) is given by (assuming the transfer functions for measuring elements are equal to 1),*

$$G_{OL\,II} = G_{C\,II} G_{P\,II}$$

*and the closed-loop equation is*

$$G_{CL\,II} = \frac{G_{OL\,II}}{1 + G_{OL\,II}} = \frac{G_{C\,II} G_{P\,II}}{1 + G_{C\,II} G_{P\,II}}$$

*For the primary loop we have*

$$G_{OL\,I} = G_{C\,I} \left( \frac{G_{C\,II} G_{P\,II}}{1 + G_{C\,II} G_{P\,II}} \right) G_{P\,I}$$

*with a closed-loop equation*

$$G_{CL\,I} = \frac{G_{C\,I} \left( \frac{G_{C\,II} G_{P\,II}}{1 + G_{C\,II} G_{P\,II}} \right) G_{P\,I}}{1 + G_{C\,I} \left( \frac{G_{C\,II} G_{P\,II}}{1 + G_{C\,II} G_{P\,II}} \right) G_{P\,I}}$$

***Accordingly the diagram for a cascade control scheme can be redrawn as in Fig. 6.7.***

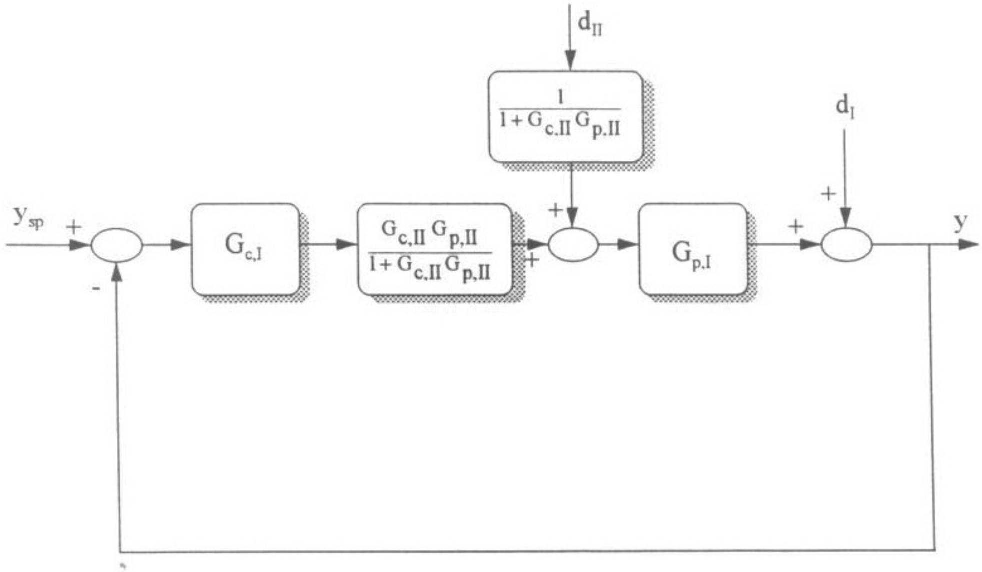



***Figure 6.7: The simplified cascade configuration.***

REMARKS:

- ***Offset in Loop II is not important, we are not interested in controlling the output of the secondary process.***
- ***Controllers*** $G_{C\,I}$ ***and*** $G_{C\,II}$ ***are usually standard feedback controllers (P, PI or PID). Generally a P controller is used for*** $G_{C\,II}$.

*For tuning a cascade control loop we have a two step procedure:*

Step 1: *Determine the settings for loop II using conventional techniques.*

Step 2: *Using the setting above, determine the settings for* GQ *using again conventional tuning techniques*

**Tuning a cascade control system involves two steps;**

- First the secondary controller is tuned; then the primary controller is tuned.
- Conventional initial tuning guidelines and fine-tuning heuristics apply.

### *6.4 Delay Compensation (Smith Predictor)*

*For systems with large dead time conventional controllers (P, PI, PID) may not be sufficient, consequently we need more sophisticated control schemes. Consider the feedback control system shown in Fig. 6.8.*

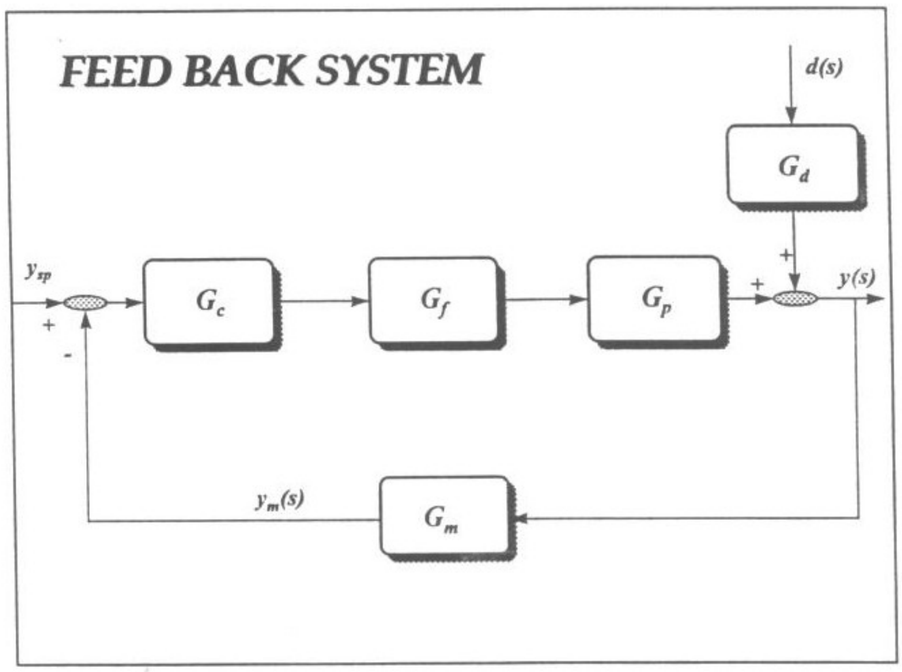



***Figure 6.8: A typical feedback control system.***

*Each dynamic component of the loop may exhibit significant time delays in their response, thus:*

- ***A disturbance entering the process will not be detected after some period of time.***
- ***The control action based on the delayed information will be inadequate.***
- ***The control action may take some time to make its effect felt by the process.***

***From all the above we can conclude that dead time is a source of instability.***

***Consider for example a system represented by the following open-loop transfer function,***

$$G_{OL} = \frac{K_c}{0.5s+1}$$

***The bode plots, the ultimate gains and the crossover frequencies, for different values of time delay are given in Fig. 6.9 .***

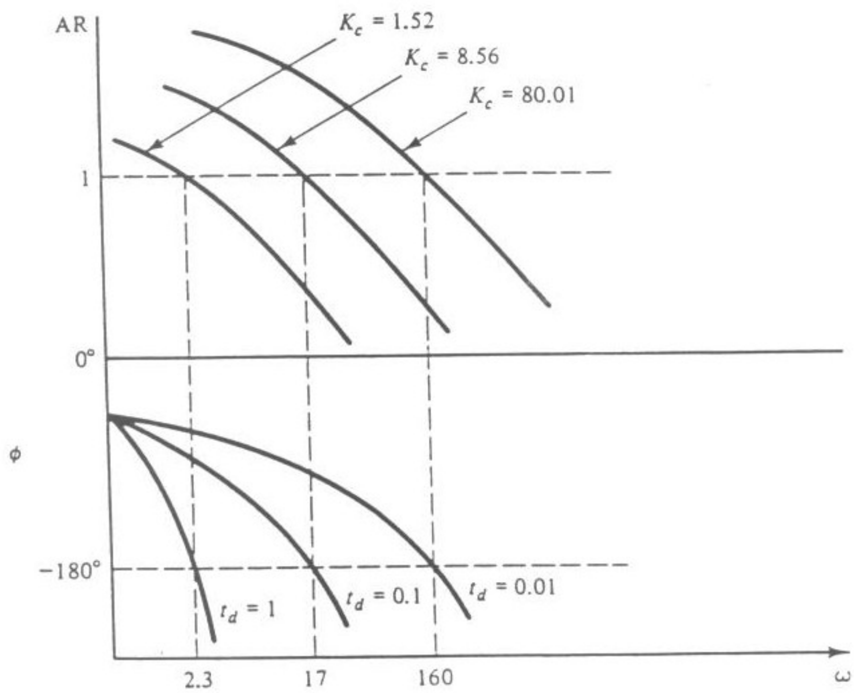



*Figure 6.9: The changes in Bode plots due to time delay.*

*From the figure we can conclude that as the dead time of the process increases*

- **The crossover frequency decreases.**
- **The ultimate gain (Ku) decreases, so we must reduce the gain of the controller.**

**As a final result the amount of feedback control is reduced.**

**" Need to compensate for the negative effect of delays"** ⇒ Delay compensation scheme

**Dead Time Compensation**

*Consider the feedback loop given in Fig. 6.10.*

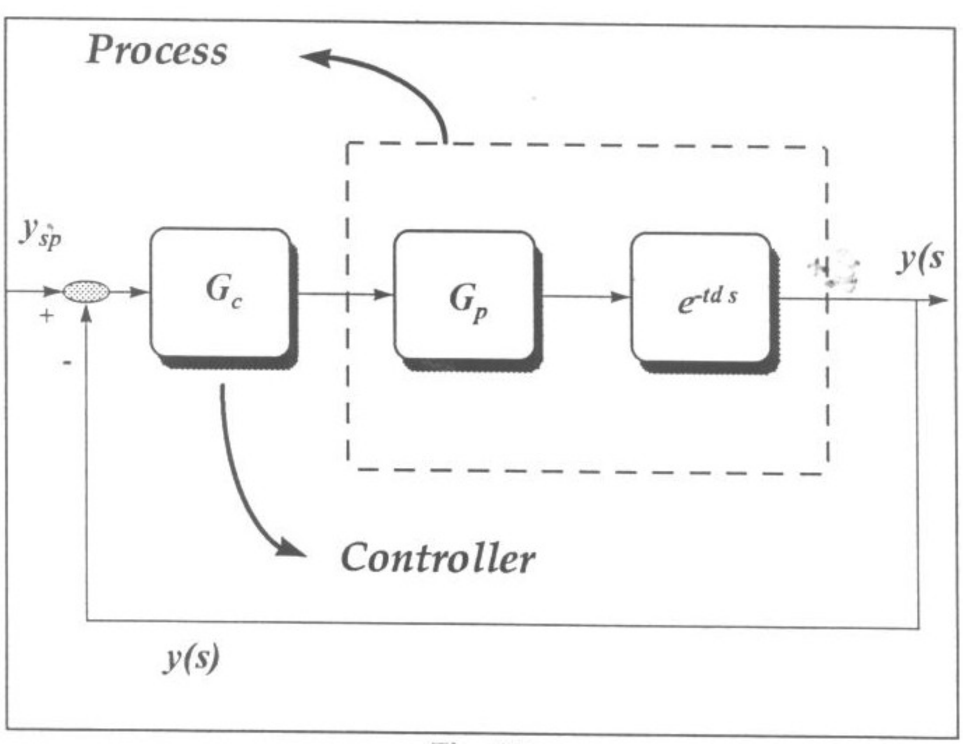


*Figure 6.10: A typical existence of time delay in a feedback loop.*

*Open-loop response to a set point change is*

$$y(s) = G_c(s)\left[G(s)e^{-t_d s}\right]y_{sp}$$

"We would like to have current and not delayed information"

*That is, we would like to have*

$$y^*(s) = G_c(s)G(s)y_{sp}$$

**Remedy:** *Add to the signal y(s) another signal y'(s) where*

$$y'(s) = \left[1 - e^{-t_d s}\right]G_c(s)G(s)y_{sp}$$

*Then*

$$y'(s) + y(s) = y^*(s)$$

*This is schematically represented in Figure 6.11.*

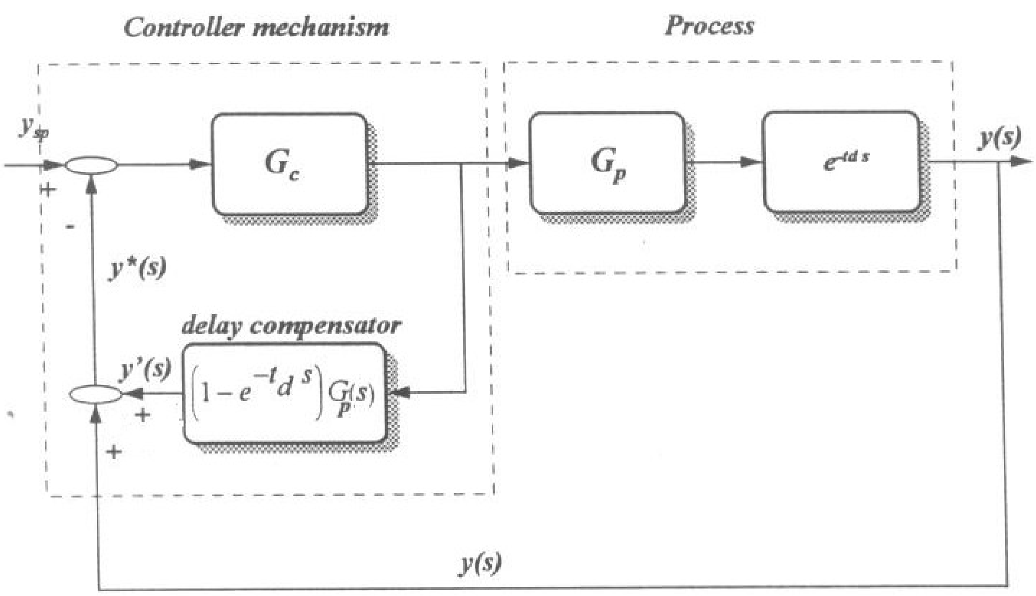



*Figure 6.11: Dead time compensation structure.*

*The **net** result of the dead time **compensator** can be visualized as Figure 6.12.*

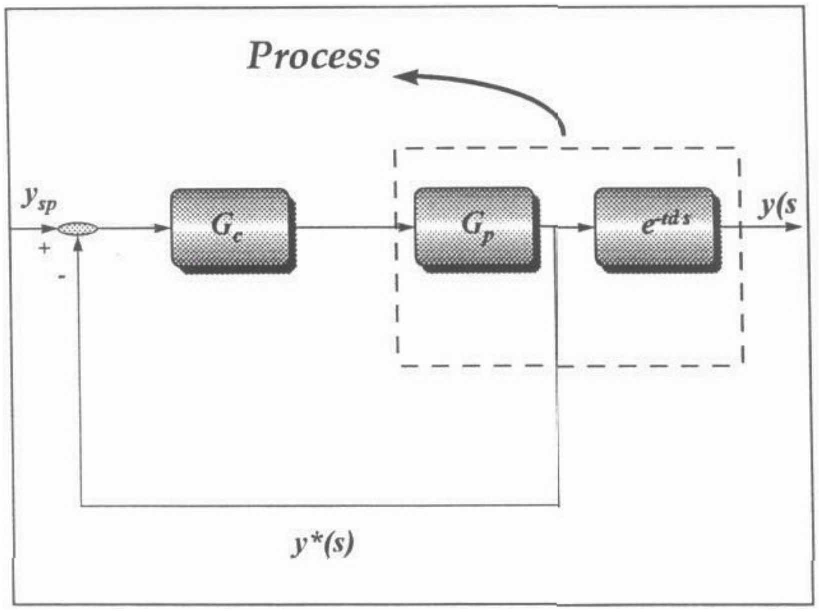



***Figure 6.12: Visualization of the dead time compensation results.***

"Compensator moves the dead-time out of the loop"*:*

*Remarks:*

- *The compensator "predicts" the delayed effect that the manipulated variable will have on the process output (Smith Predictor).*
- *In most process control problems*

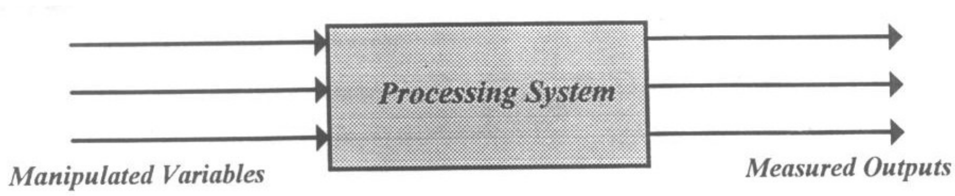
**real process ≠ model => *mo*delling error**

**Consequently, compensation is not complete.**

### *6.5 Decoupling Control for MIMO Systems*

*In this part we continue the trend of addressing increasingly complex process control systems. Most of the control systems we considered in the previous chapters were single-variable controllers because they had the ultimate objective of maintaining only one variable near its set point.*

*By contrast,* multivariable control *involves the objective of maintaining several controlled variables at independent set points.*
*Consider the following process with several inputs and outputs:*



In designing controllers for MIMO systems, a typical starting point is the use of multiple independent single loop controllers.

For MIMO **systems there is** a large number of alternative **control loops** $\Rightarrow$

**The selection of the most appropriate control configuration is the central and critical task to be done.**

***Control of multivariable systems requires more analysis than that of single-variable systems. In multivariable systems new characteristics due to* interaction *must be considered.***

## Characteristics unique to multivariable systems

1. ***Interaction between variables influences control stability and performance.***
2. ***Feasibility of control depends on overall process.***
3. ***The pairing of measured and manipulated variables via control is a design decision.***
4. ***Some processes have unequal number of controlled and manipulated variables.***
5. ***Some multivariable control designs are very sensitive to modeling errors.***

## Transfer function **matrix for MIMO systems**

***In general for a MIMO system we have***

$$\mathbf{y = G(s)\ m}$$

***where G(s) is the matrix of the plant transfer functions. Schematically this is shown in Figure 6.13.***

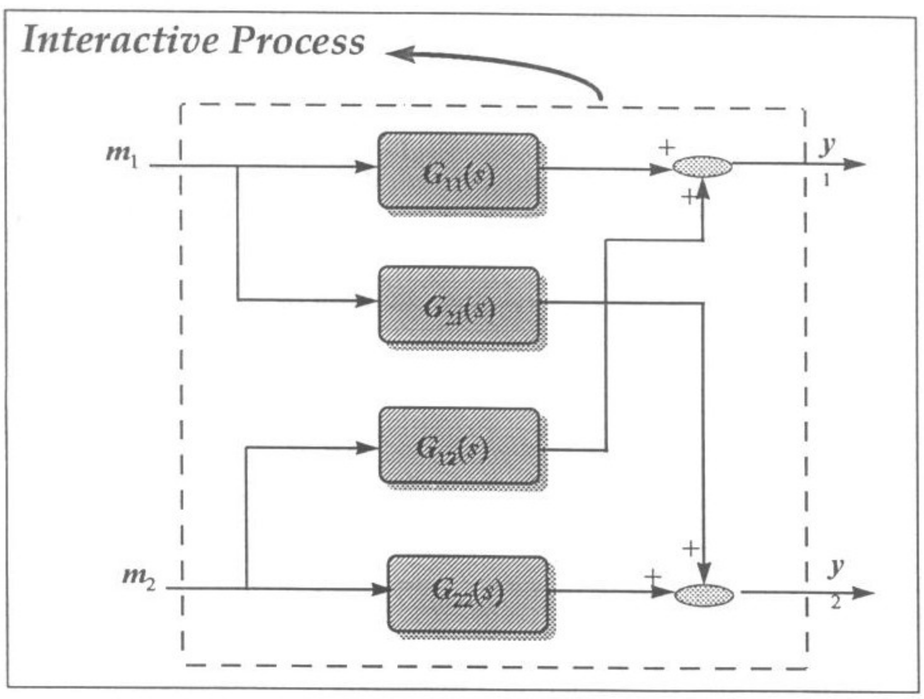



***Figure 6.13: A MIMO system.***

**Remarks:**

- *In general* G(s) *is an (n×l) matrix where n is the number of outputs and l the number of inputs.*
- *For complex systems, usually the input-output relationships are obtained experimentally.*

*Consider the following distillation column:*

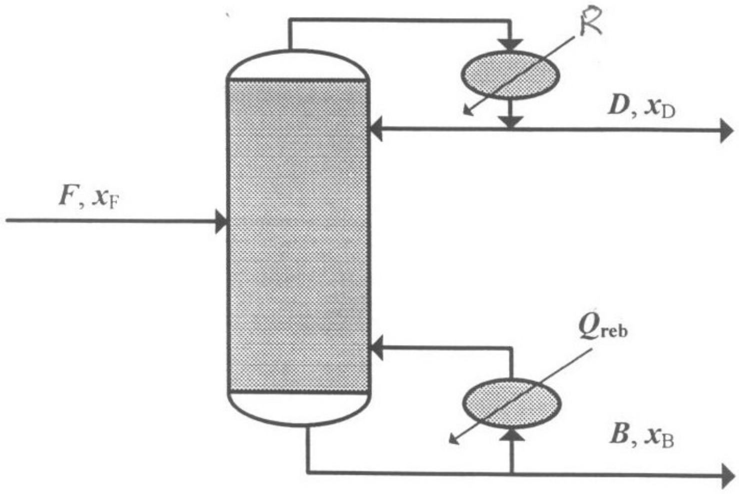



***Figure 6.14: A Distillation Column (A typical MIMO system).***

*The objective in this column is to:*

Regulate the composition of distillate and bottom products using reflux rate R and reboiler duty $Q_{reb}$ *as* manipulated variables.

*The following input output relationships have been determined experimentally*

$$\begin{bmatrix} x_D \\ x_B \end{bmatrix} = \begin{bmatrix} \dfrac{360\,e^{-0.3s}}{1+3s} & \dfrac{130\,e^{-0.4s}}{1+2.5s} \\ \dfrac{130}{1+1.5s} & \dfrac{-16}{1+s} \end{bmatrix} \begin{bmatrix} R \\ Q_{reb} \end{bmatrix}$$

Another example for MIMO systems is the mixing tank in Figure 1.16 we presented in the first chapter of this course.

## Design of MIMO Feedback Controllers

*Let us consider the above example as the case study. In designing the multivariable control system for this system two questions arise:*

1. Should we use R for controlling $X_D$ and $Q_{reb}$ for $X_B$ or vice versa?
   $\Rightarrow$ Pairing problem
2. Should we design the controllers separately as single loops or not?
   $\Rightarrow$ Interaction problem

*In this example both off-diagonal elements of the plant transfer function matrix are non-zero. This means that changing one of the manipulated variables will affect the other loop. Therefore there is interaction between the two control loops (Fig. 6.15).*

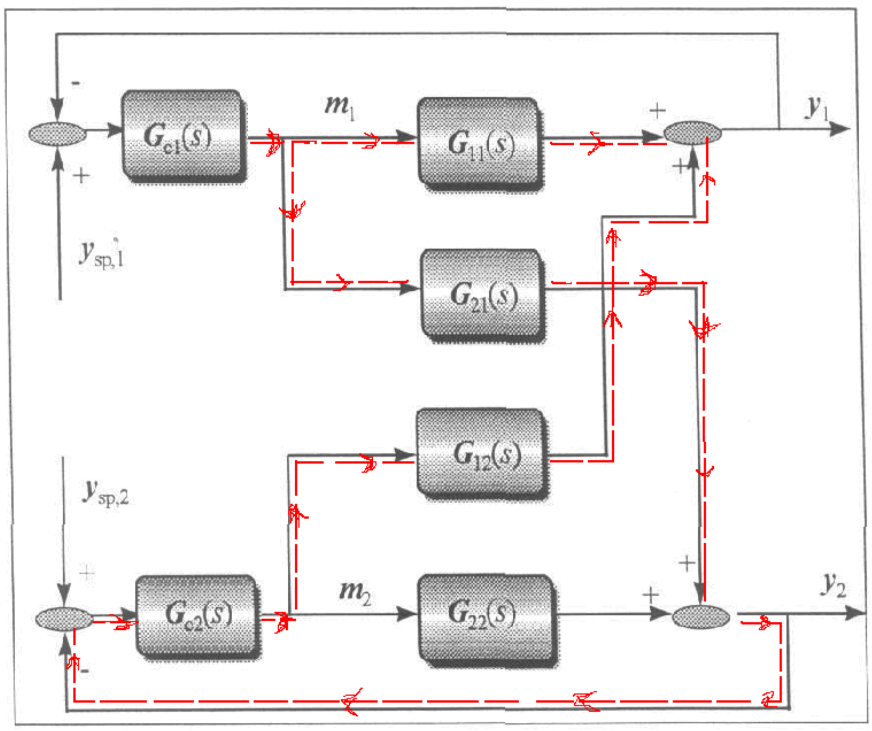



***Figure 6.15: A Multi-Loop Control System. The red arrows show how a change by $m_1$ (for example) can influence the first output through two path of the first loop and the second loop.***

*If the feedback controllers of the individual loops are tuned separately, then*
"We can not guarantee stability and performance for the overall control system, where both loops are closed."

*Therefore for MIMO systems we then need:*

- ***A way to measure the amount of interaction among the loops***
  - ⇨ Relative Gain Array (RGA)
- ***A way to cancel the interaction effects between the loops***
  - ⇨ Decoupling

## Interaction Measure

*For the system to be non-interactive,* G(s) *should be diagonal but this is not usually the case for MIMO systems.*

*The first step before attempting to make some corrective action is to measure the amount of interaction of a given process. For that purpose we will use the so called* "Relative Gain Array (RGA)" *or* "Bristol Array" *that can be obtained from the gains of the matrix of plant transfer functions.*

*To measure the control loop interactions and define RGA we proceed as below (**see Figure 6.15**):*

***Experiment 1***:*Apply a unit step change in* $\mathbf{m_1}$ *with all loops open. After steady state has been achieved, define*

$\Delta y_{1m1} = k_{11}$: *The change in $y_1$ due to the change in $m_1$.*

***Experiment 2***:*Apply a unit step change in* $\mathbf{m_1}$ *with loop 2 closed but loop 1 open. After steady state the following happen:*

- *A change in* $\boldsymbol{y_1}$ *because of* $G_{11}$, *and also* $\boldsymbol{y_2}$ *because of* $G_{21}$.

- *By manipulating* $\boldsymbol{m_2}$, $\boldsymbol{y_2}$ *is restored to its initial value.*

- *The change in* $\boldsymbol{m_2}$ *now return to affect* $\boldsymbol{y_1}$ *because of* $G_{12}$.

*Therefore, the changes in* $\boldsymbol{y_1}$ *come from two different sources:*

$$\Delta y_1^* = \Delta y_{1m1} + \Delta y_{1r}$$

*Observe now that a good measure of how well the process can be controlled if* $\boldsymbol{m_1}$ *is used to control* $\boldsymbol{y_1}$ *is:*

$$\lambda_{11} = \frac{\Delta y_{1m1}}{\Delta y_{1m1} + \Delta y_{1r}}$$

*The same experiments can be performed to investigate the effects of a change in* $\boldsymbol{m_2}$ *on* $\boldsymbol{y_1}$ *(and similar study for* $\boldsymbol{m_2}$ *on* $\boldsymbol{y_2}$ *and* $\boldsymbol{m_1}$ *on* $\boldsymbol{y_2}$ *).*

***Loop pairing on basis of interaction analysis:***

1. *If* $\lambda_{11} = 1$:

   - $\boldsymbol{m_1}$ *does not affect* $\boldsymbol{y_2}$, *or it does affect* $\boldsymbol{y_2}$ *but* $\boldsymbol{m_2}$ *has no affect on* $\boldsymbol{y_1}$. *Thus,* $\boldsymbol{m_1}$ *is perfect to control* $\boldsymbol{y_1}$.

2. *If* $\lambda_{11} = 0$:

   - $\boldsymbol{m_1}$ *has no effect on* $\boldsymbol{y_1}$. *Thus,* $\boldsymbol{m_1}$ *is not good for controlling* $\boldsymbol{y_1}$.

3. *If* $0 < \lambda_{11} < 1$:

   - *The direction of interaction is the same as that of the main effect.*

     - $\lambda_{11} > 0.5$: *Main effect more than interaction: reasonable*

     - $\lambda_{11} < 0.5$: *Interaction more than main effect: not reasonable*

4. *If* $\lambda_{11} > 1$:

   - *The direction of interaction is opposite to the main effect but smaller in absolute value than the main. Thus, for* $\lambda_{11}$ *very large it is very difficult to control* $\boldsymbol{y_1}$ *using* $\boldsymbol{m_1}$.

5. *If* $\lambda_{11} < 0$:

   - *The direction of interaction is opposite to the main effect, and larger in absolute value than the main. Thus, this is a catasrophy!*

## *The relative gain array (RGA):*

*The quantity $\lambda_{11}$ introduced just before is known as the relative gain between output **$y_1$** and input **$m_1$**, and it provides a measure of the extent of the influence of the process interactions.*

*In general we define $\lambda_{ii}$, the relative gain between output **$y_i$** and input **$m_j$**, as the ratio of two steady state gains:*

$$\lambda_{ij} = \frac{\left(\dfrac{\partial y_i}{\partial m_j}\right)_{\text{all loops open}}}{\left(\dfrac{\partial y_i}{\partial m_j}\right)_{\text{all loops closed except for the } m_j \text{ loop}}}$$

*Calculating the relative gain for all input-output combinations of a multivariable system, the results can be written as RGA*

$$\Lambda = \begin{bmatrix} \lambda_{11}\, \lambda_{12} \cdots \lambda_{1n} \\ \lambda_{21}\, \lambda_{22} \cdots \lambda_{2n} \\ \vdots \quad\quad \vdots \quad \vdots \\ \lambda_{n1}\, \lambda_{n2} \cdots \lambda_{nn} \end{bmatrix}$$

*On the other hand, suppose:*

$$\lim_{s\to 0} G(s) = K = \begin{bmatrix} k_{11}\, k_{12} \cdots k_{1n} \\ \vdots \quad\quad \vdots \quad \vdots \\ k_{n1}\; k_{n2} \cdots k_{nn} \end{bmatrix}$$

***Then, RGA can be calculated as***

$$\Lambda(K) = K \times (K^{-1})^T \quad (\times \text{ is element by element multiplication})$$

***Example:***

$$G(0) = K = \begin{bmatrix} 12.8 & -18.9 \\ 6.6 & -19.4 \end{bmatrix} \rightarrow (K^{-1})^T = \begin{bmatrix} 0.157 & 0.053 \\ -0.153 & -0.104 \end{bmatrix} \rightarrow \Lambda = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

"A system is interactive if the magnitude of the off-diagonal elements of $\Lambda$ are larger than those of the diagonal ones"

## Selection of Loops

*For a process with N controlled outputs and N manipulated variables, there are N! different ways to form the control loops, i.e., N! control configurations.*

Question:

Which configuration is the best?

Answer:

One way is to consider the interaction among the loops and select the one with minimum interaction.

“RGA provides a systematic methodology for screening among the alternative loop searching for minimum interaction”

**Rule:**

*Select the control loop pairing the controlled outputs* $y_i$ *with the manipulated variables* $m_j$ *in such a way that the relative gains* $\lambda_{ij}$ *are positive and as close as possible to unity (*$\lambda_{ij}$ *: elements of* $\Lambda$*).*

*Remarks:*

The relative gain array method provides a measure of interaction based on steady state considerations. This does not guarantee that the dynamic interaction between loops will be also minimal.

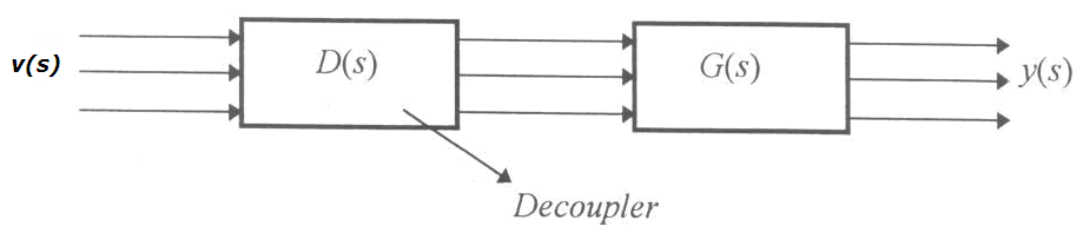
## Design of noninteracting control loops: Decouplers

*The relative gain array indicates how the inputs should be coupled with the outputs to form control loops with the smallest amount of interaction.*

*However, the persisting interaction, although it is the smallest possible, may not be small enough.*

**Remedy:** **Decoupling**

*Consider now the system represented by*




*The purpose is to find a* D(s) *such that*

$$G(s)D(s)=Q(s) \quad \text{Diagonal matrix}$$

*For a 2 x 2 system as described above and shown in Figure 6.16 we proceed as below:*

$$m_1 = v_1 + d_{12}(s).v_2 \qquad , \quad m_2 = v_2 + d_{21}(s).v_1$$

$$y_1 = G_{11}(s)\,m_1 + G_{12}(s)\,m_2 \qquad , \quad y_2 = G_{21}(s)\,m_1 + G_{22}(s)\,m_2$$

*or*

$$y_1 = [G_{11}(s) + G_{12}(s)\,d_{21}(s)]v_1 + [G_{11}(s)d_{12}(s) + G_{12}(s)]v_2$$

$$y_2 = [G_{21}(s) + G_{22}(s)\,d_{12}(s)]v_1 + [G_{22}(s)d_{21}(s) + G_{12}(s)]v_2$$

*Now in order to eliminate effect of* $v_2$ *on* $y_1$ *and* $v_1$ *on* $y_2$ *, we choose decouplers as:*

$$[G_{11}(s)d_{21}(s) + G_{12}(s)] = 0 \rightarrow d_{12}(s) = -\frac{G_{12}(s)}{G_{11}(s)}$$

$$[G_{21}(s) + G_{22}(s)\,d_{21}(s)] = 0 \rightarrow d_{21}(s) = -\frac{G_{21}(s)}{G_{22}(s)}$$

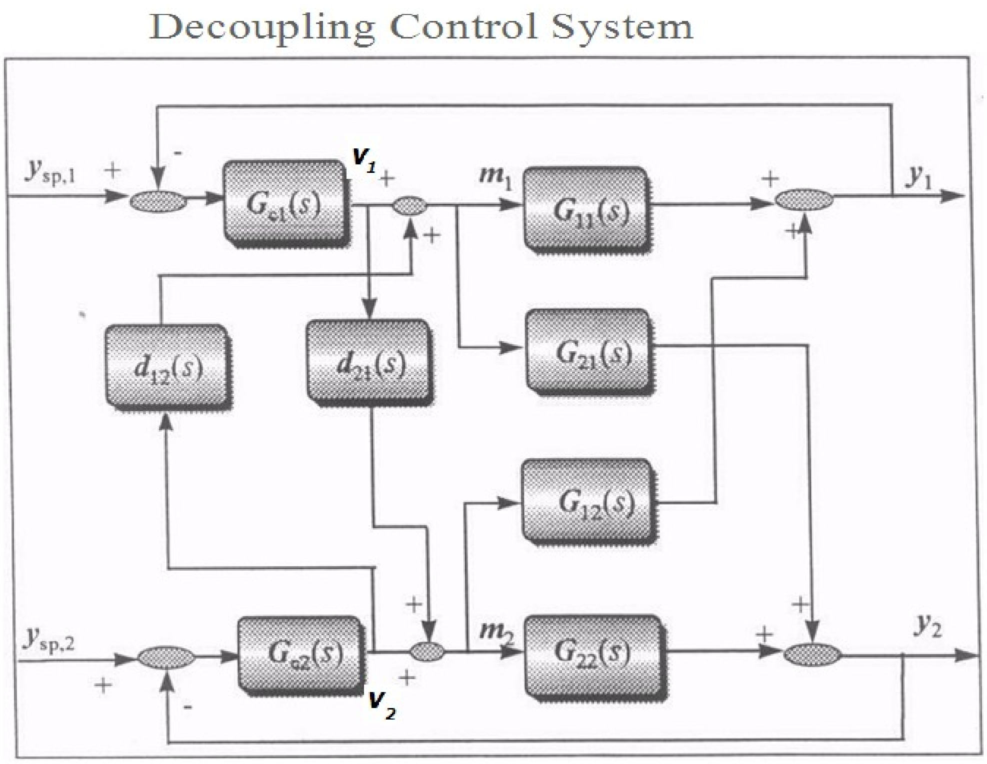



***Figure 6.16: Decoupling control system.***

<u>Remarks:</u>

- *The above compensator is a dynamic decoupler that will decouple the loops perfectly.*
- *The decoupling technique is heavily model dependent.*
- *This dynamic decoupler is physically realisable if both* $d_{12}(s)$ *and* $d_{21}(s)$ *are realisable.*
- *The controllers are now design simply based on* $G_{11}(s)$ *and* $G_{22}(s)$ *as usuall.*

## 7. VARIOUS TYPES OF PID CONTROLLERS

In this section, three basic methods of constructing PID controllers using Pneumatic, Hydraulic, and Electronic signals and components are introduced. We will also mention how within gas or liquid environment signals and power are transmitted.

### 7.1 Pneumatic Controllers

As the most versatile medium for transmitting signals and power, fluids, either as liquids or gases, have wide usage in industry. The term pneumatics refers to fluid systems that use air or gas and hydraulic applies to those using oil.

**A brief comparison of pneumatics systems and hydraulic systems**

1. Air and gas are compressible, whereas oil is not.

2. Air lacks lubricating property and always contains water vapour. Oil functions as a hydraulic fluid as well as a lubricator.

3. The normal operating pressure of pneumatic systems is very much lower than of hydraulic systems.

4. Output powers of pneumatic systems are considerably less than those of hydraulic systems.

5. Accuracy of pneumatic actuators is poor at low velocities, whereas accuracy of hydraulic actuators may be made satisfactory at all velocities.

6. In pneumatic systems, external leakage is permissible to a certain extent, but internal leakage must be avoided because the effective pressure difference is rather small. In hydraulic systems internal is permissible to certain extent, but external leakage must be avoided.

7. No return pipes are required in pneumatic systems when air is used, whereas they are always needed in hydraulic system.

8. Normal operating temperature for pneumatic systems is $5^{\circ}$ to $60^{\circ C}$. The pneumatic systems, however, can be operated in $0^{\circ}$ to $200^{\circ C}$ ranges. Pneumatic systems are insensitive to temperature changes, in constant to hydraulic systems, in which fluid friction due to viscosity depends greatly on temperature. Normal operating temperature for hydraulic systems is $20^{\circ}$ to $70^{\circ C}$.

9. Pneumatic systems are fire- and explosion –proof, whereas hydraulic systems are not.

**Basic principles governing the pneumatic systems**

Many industrial processes and pneumatic controllers involve the flow of a gas or air through connected pipelines and pressure vessels. As we thoroughly discussed in Section 2, a typical pressure system is shown in Figure 7.1.

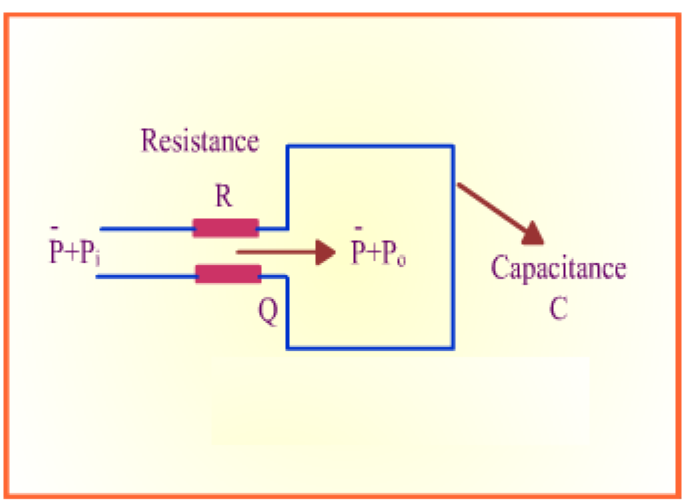



**Figure 7.1** A typical pressure system.

In the above system, the gas flow through the restriction is a function of gas pressure difference $P_i - P_o$. Such a pressure system may be characterized in terms of a resistance R and capacitance C,

$$\frac{P_o(s)}{P_i(s)} = \frac{1}{RCs+1}$$

Now this concepts and formulation is used within the simplest component of a pneumatic system in the following.

## Pneumatic nozzle-flapper amplifiers

A schematic diagram of a pneumatic nozzle-flapper amplifier as the basic component of pneumatics systems is shown in Figure 7.2 (a). A typical curve relating the nozzle back pressure $P_b$ to nozzle-flapper distance is shown in Figure 7.2(b).

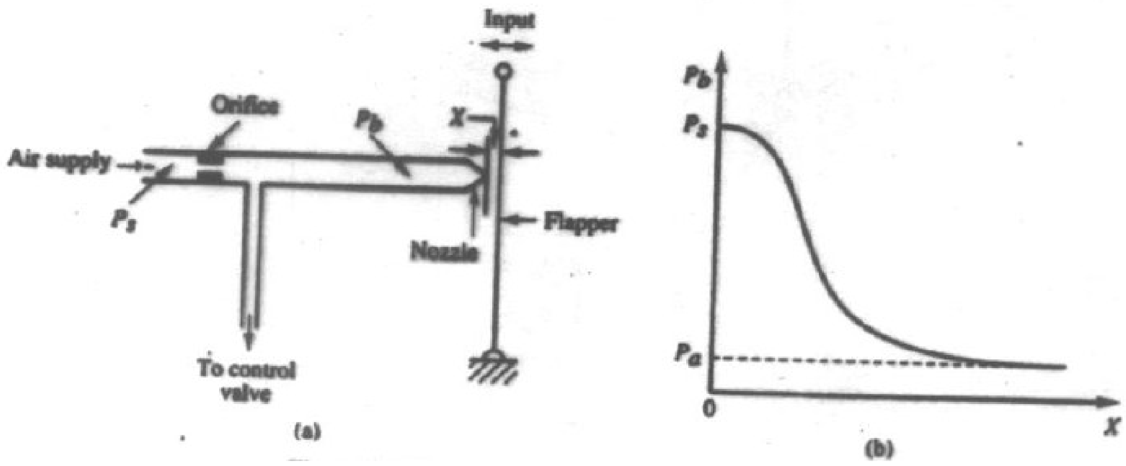



**Figure 7.2 . Schematics of nozzle-flapper (a) and its action (b).**

The nozzle-flapper amplifier converts displacement into a pressure signal.

Since industrial process control systems require large output power to operate large pneumatic actuating valves, the power amplification of the nozzle-flapper amplifier is usually insufficient. Consequently, a pneumatic relay often serves as a power amplifier in connection with the nozzle-flapper amplifier as is explained in the following.

## Pneumatic Relays

In practice, a nozzle-flapper amplifier acts as the first-stage amplifier and a pneumatic relay as the second-stage amplifier. The pneumatic rely is capable of handling a large quantity of airflow. The schematic diagrams of two types of relays are depicted in Figure 7.3.

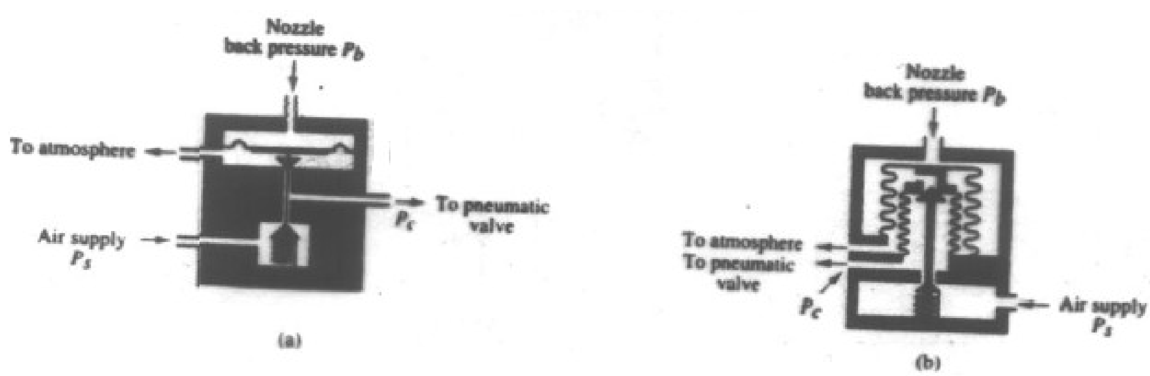



**Figure 7.3. Schematics of a bleed-type (a) and a nonbleed-type (b) relay.**

It is noted that some pneumatic relays are reverse acting. For example, the relay shown in Figure 7..4. is a reverse-acting relay.

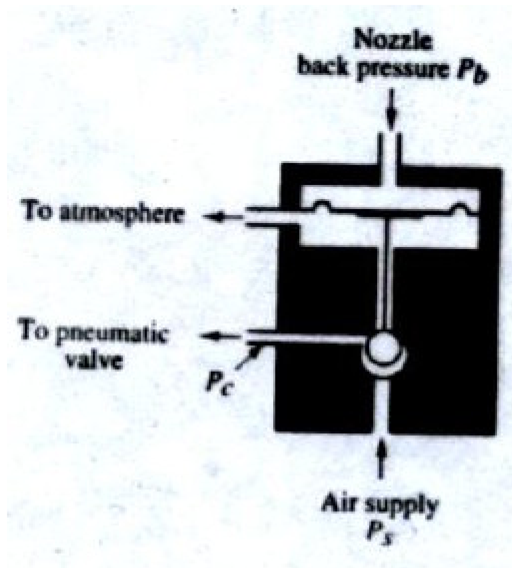



**Figure 7.4. A reverse acting relay.**

## Pneumatic proportional controllers (force-distance type).

Two types of pneumatic controllers, one called the force-distance type and the other the force-balance type, are used extensively in industry. Regardless of how differently industrial pneumatic controllers may appear, careful study will show the close similarity in the functions of the pneumatic circuit. Here we shall consider the force-distance type of pneumatic controllers. Figure 7.5(a) shows a schematic diagram of such a proportional controller.

In most pneumatic controllers, some type of pneumatic feedback is employed. Feedback of the pneumatic output reduces the amount of actual movement of the flapper. See Figure 7.5(b)

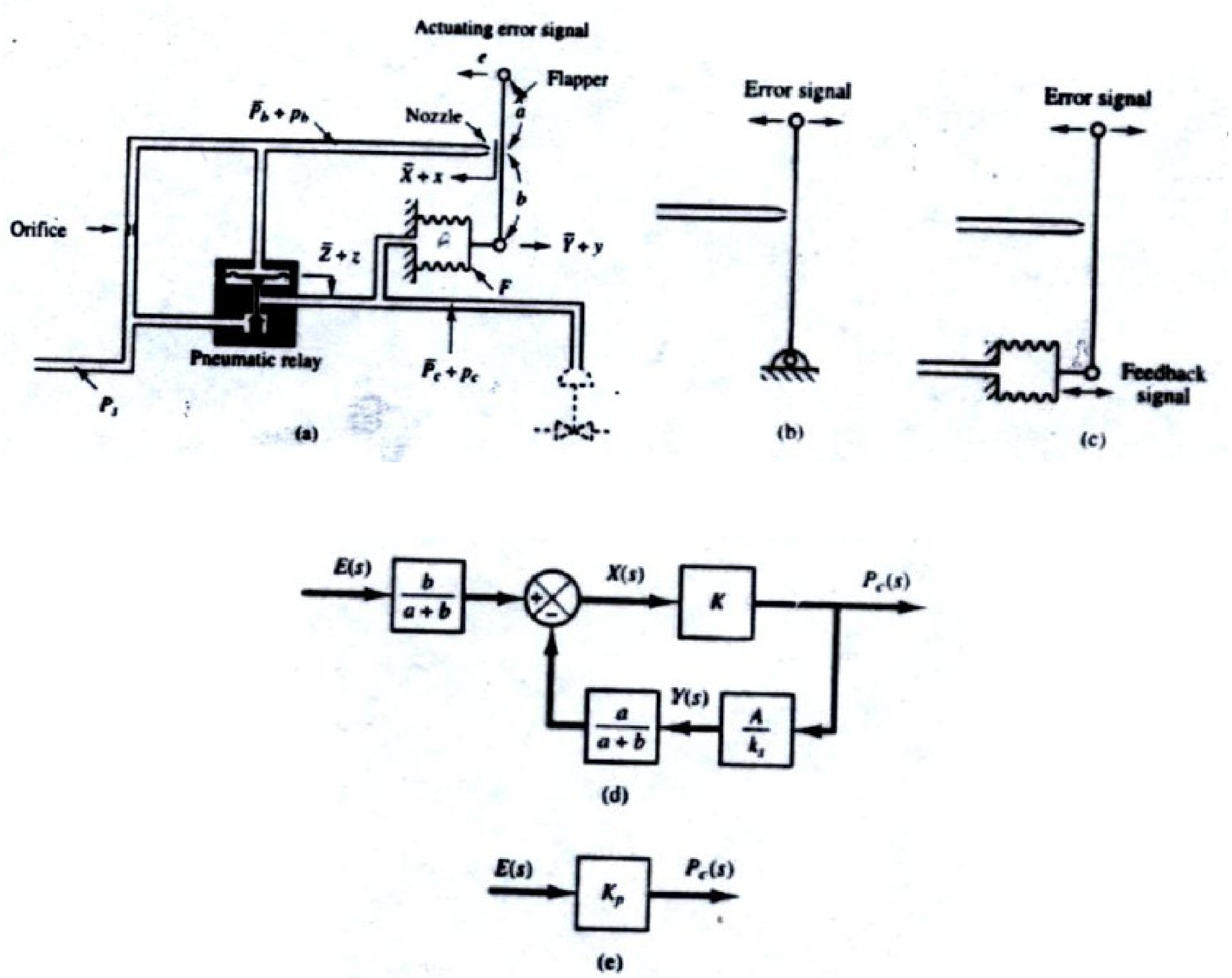



**Figure 7.5. Schematic diagram of such a force-distance type of proportional controller.**

Equations for this controller can be derived as follows. When the actuating error is zero, or $e = 0$, an equilibrium state exists with the nozzle-flapper distance equal to $\bar{X}$, the displacement of bellows equal to $\bar{Y}$, the displacement of the diaphragm equal to $\bar{Z}$, the nozzle back pressure equal to $\bar{P}_b$, and the control pressure equal to $\bar{P}_c$.

When an actuating error exists, the nozzle-flapper distance, the displacement of the bellows, the displacement of the diaphragm, the nozzle back pressure, and the control pressure deviate from their respective equilibrium values. Let these deviations be $x, y, z, p_b, \text{ and } p_c$, respectively. (The positive direction for each displacement variable is indicated by an arrowhead in the diagram.)

Assuming that the relationship between the variation in the nozzle back pressure and the variation in the nozzle-flapper distance is linear, we have

$$p_b = K_1 x$$

where $K_1$ is a positive constant. For the diaphragm valve,

$$p_b = K_2 z$$

where $K_2$ is a positive constant. The position of the diaphragm valve determines the control pressure. If the diaphragm valve is such that the relationship between pc and z is linear, then

$$p_c = K_3 z$$

where $K_3$ is a positive constant. From above equations, we obtain

$$p_c = \frac{K_3}{K_2} p_b = Kx$$

where $K = K_1 K_3 / K_2$ is a positive constant. For the flapper movement, we have

$$x = \frac{b}{a+b} e - \frac{a}{a+b} y$$

The bellows acts like a spring, and the following equation holds true:

$$A p_c = k_s y$$

where A is the effective area of the bellows and $k_s$ is the equivalent spring constant, that is, the stiffness due to the action of the corrugated side of the bellows.

Assuming that all variations in the variables are within a linear range, we can obtain a block diagram for this system as shown in Figure 7.5(d). From Figure 7.5(d), it can be clearly seen that the pneumatic controller shown in Figure 7.5(a) itself is a feedback system. The transfer function between pc and e is given by

$$\frac{P_c(s)}{E(s)} = \frac{\frac{b}{a+b} K}{1 + K \frac{a}{a+b} \frac{A}{k_s}} = K_p$$

A simplified block diagram is shown in Figure 7.5( e). Since $p_c$ and e are proportional, the pneumatic controller shown in Figure 7.5(a) is called *a pneumatic proportional controller*

Pneumatic controllers that do not have feedback mechanisms [which means that one end of the flapper is fixed, as shown in Figure 7.6(a)] have high sensitivity and are called *pneumatic two-position controllers* or *pneumatic on-off controllers.* In such a controller, only a small motion between the nozzle and the flapper is required to give a complete change from the maximum to the minimum control pressure. The curves relating *Pb* to *X* and *Pc* to *X* are shown in Figure 7.6(b). Notice that a small change in *X* can cause a large change in *Pb*, which causes the diaphragm valve to be completely open or completely closed.

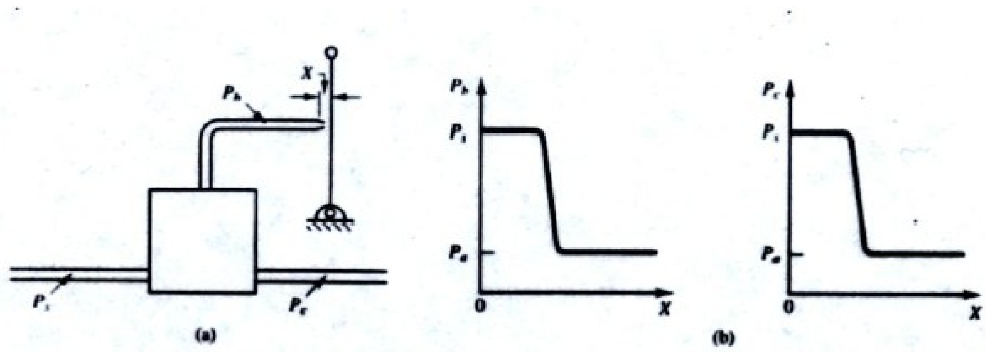



**Figure 7.6. a) Pneumatic controller without a feedback mechanism; (b) curves Pb versus X and Pc versus X.**

**Pneumatic proportional controllers (force-balance type).**

Figure 7.7 shows a schematic diagram of a force-balance pneumatic proportional controller. Force balance controllers are in extensive use in industry. Such controllers are called stack controllers. The basic principle of operation does not differ from that of the force distance controller. The main advantage of the force-balance controller is that it eliminates many mechanical linkages and pivot joints, thereby reducing the effects of friction.

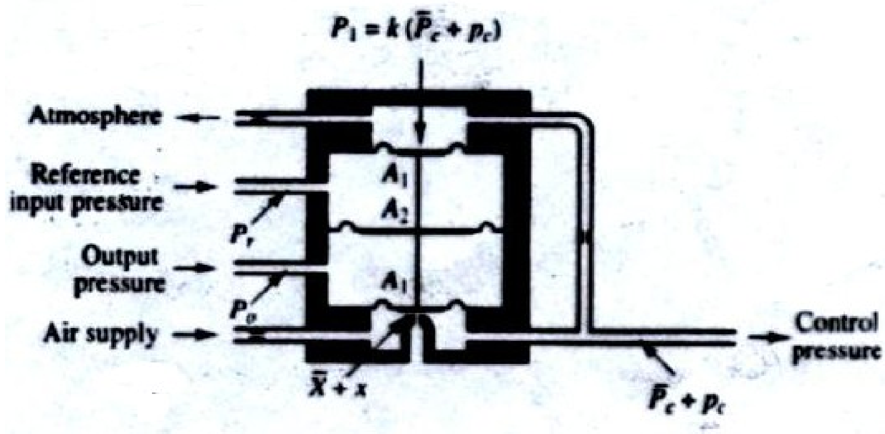



**Figure 7.7 Schematic diagram of a force-balance type of pneumatic proportional controller.**

**Pneumatic actuating valves.**

One characteristic of pneumatic controls is that they almost exclusively employ pneumatic actuating valves. A pneumatic actuating valve can provide a large power output. (Since a pneumatic actuator requires a large power input to produce a large power output, it is necessary that a sufficient quantity of pressurized air be available.)

In practical pneumatic actuating valves, the valve characteristics may not be linear; that is, the flow may not be directly proportional to the valve stem position, and also there may be other nonlinear effects, such as hysteresis.

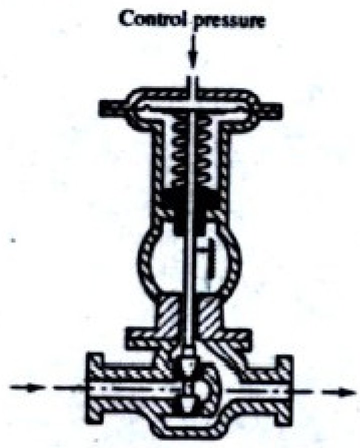



**Figure 7.8 Schematic diagram of a pneumatic actuating valve.**

Consider the schematic diagram of a pneumatic actuating valve shown in Figure 7.8. Assume that the area of the diaphragm is *A*. Assume also that when the actuating error is zero the control pressure is equal to *Pc* and the valve displacement is equal to *x* .

In the following analysis, we shall consider small variations in the variables and linearize the pneumatic actuating valve. Let us define the small variation in the control pressure and the corresponding valve displacement to be *pc* and *x,* respectively. Since a small change in the pneumatic pressure force applied to the diaphragm repositions the load, consisting of the spring, viscous friction, and mass, the force balance equation becomes

$$Ap_c = m\ddot{x} + b\dot{x} + kx$$

where *m* = mass of the valve and valve stem
*b* = viscous-friction coefficient
*k* = spring constant

If the force due to the mass and viscous friction are negligibly small, then above equation can be simplified to:

$$Ap_c = kx$$

Transfer function between *x* and $p_e$ thus becomes

$$\frac{X(s)}{P_c(s)} = \frac{A}{k} = K_c$$

Where $X(s) = \mathcal{L}[x]$ and $P_c(s) = \mathcal{L}[p_c]$.

The change in flow through the pneumatic actuating valve, is proportional to *x*, the change in the valve-stem displacement, then

$$\frac{Q_i(s)}{X(s)} = K_q$$

Where $Q_i(s) = \mathcal{L}[q_i]$ and $K_q$ is a constant.

The transfer function between $q_i$ and $p_c$ becomes

$$\frac{Q_i(s)}{P_c(s)} = K_cK_q = K_v$$

**Basic principle for obtaining derivative control action**

We shall now present methods for obtaining derivative control action. We shall again place the emphasis on the principle and not on the details of the actual mechanisms.

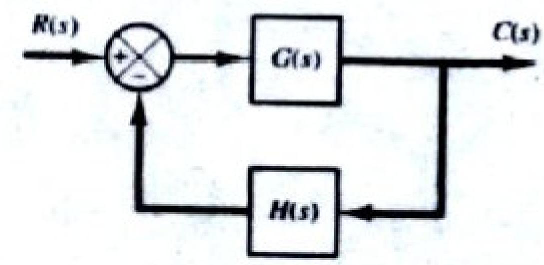



**Figure 7.9 Block diagram of a control system.**

The basic principle for generating a desired control action is to insert the inverse of the desired transfer function in the feedback path. For the system shown in Figure 7.9, the closed-loop transfer function is

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$$

If $|G(s)H(s)| >> 1$ then C(s)/R(s) can be simplified to:

$$\frac{C(s)}{R(s)} = \frac{1}{H(s)}$$

Thus, if proportional-plus-derivative control action is desired, we insert an element having the transfer function *1/(Ts* + 1) in the feedback path.

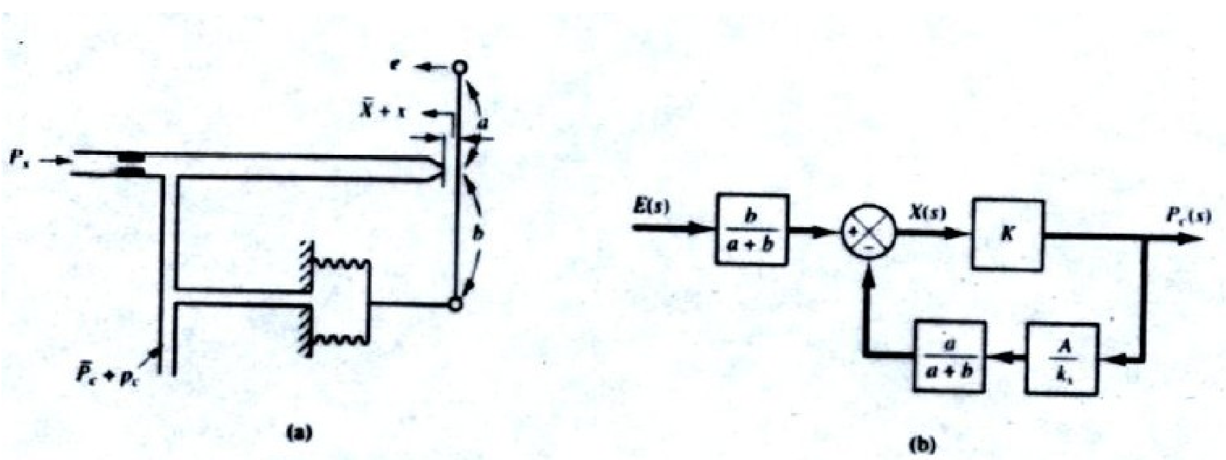



**Figure 7.10 Schematic diagram of a pneumatic PD controller.**

Consider the pneumatic controller shown in Figure 7.10(a). Considering small changes in the variables, we can draw a block diagram of this controller as shown in Figure 7.10(b). From the block diagram we see that the controller is of proportional type.

We shall now show that the addition of a restriction in the negative feedback path will modify the proportional controller to a proportional-plus-derivative controller, commonly called a PD controller.

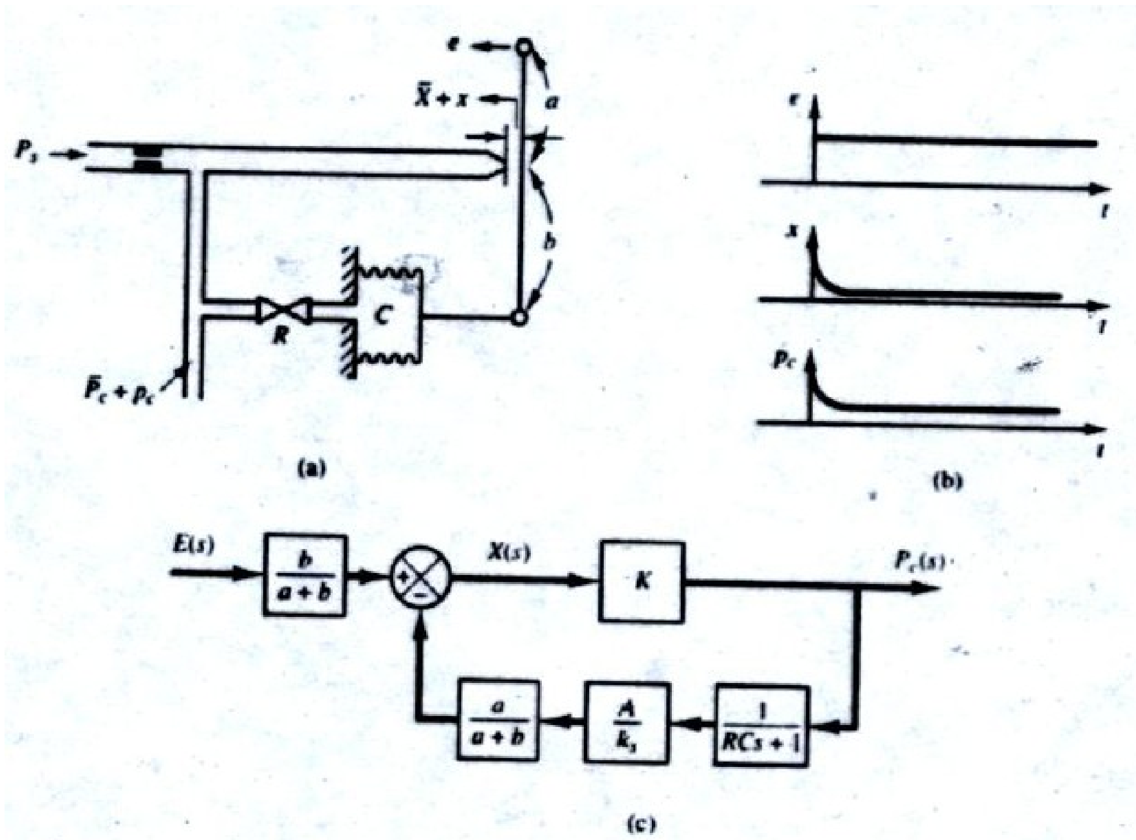



**Figure 7.11. (a) Pneumatic proportional-plus-derivative controller; (b) step change in e and the corresponding changes in x and pc plotted versus t; ( c) block diagram of the controller.**

Consider the pneumatic controller shown in Figure 7.11(a).

Assuming again small changes in the actuating error, nozzle-flapper distance, and control pressure, the change in the nozzle-flapper distance x and the change in the control pressure $p_c$ can be plotted against time t, as shown in Figure ٧.١١(b). At steady state, the feedback bellows acts like an ordinary feedback mechanism. The curve $p_c$ versus t clearly shows that this controller is of the proportional-plus-derivative type.

A block diagram corresponding to this pneumatic controller is shown in Figure ٧.١١(c). In the block diagram, K is a constant, A is the area of the bellows, and k. is the equivalent spring constant of the bellows. The transfer function between $p_c$ and e can be obtained from the block diagram as follows:

$$\frac{P_c(s)}{E(s)} = \frac{\frac{b}{a+b}K}{1 + \frac{Ka}{a+b}\frac{A}{k_s}\frac{1}{RCs+1}}$$

In such a controller the loop gain

$$\left| \frac{Ka}{a+b}\frac{A}{k_s}\frac{1}{RCs+1} \right|$$

is normally very much greater than unity. Thus the transfer function $P_c(s)/E(s)$ can be simplified to give

$$\frac{P_c(s)}{E(s)} = K_p(1 + T_d s)$$

where

$$K_p = \frac{bk_s}{aA}, \qquad T_d = RC$$

Thus, delayed negative feedback, or the transfer function *1/(RCs* + 1) in the feedback path, modifies the proportional controller to a proportional-plus-derivative controller.

Note that if the feedback valve is fully opened the control action becomes proportional. If the feedback valve is fully closed, the control action becomes narrow-band proportional (on-off).

**Obtaining pneumatic proportional-plus-integral control action**.

Consider the proportional controller shown in Figure 7.11 (a). Considering small changes in the variables, we can show that the addition of delayed positive feedback will modify this proportional controller to a proportional-plus-integral controller, commonly called a PI controller.

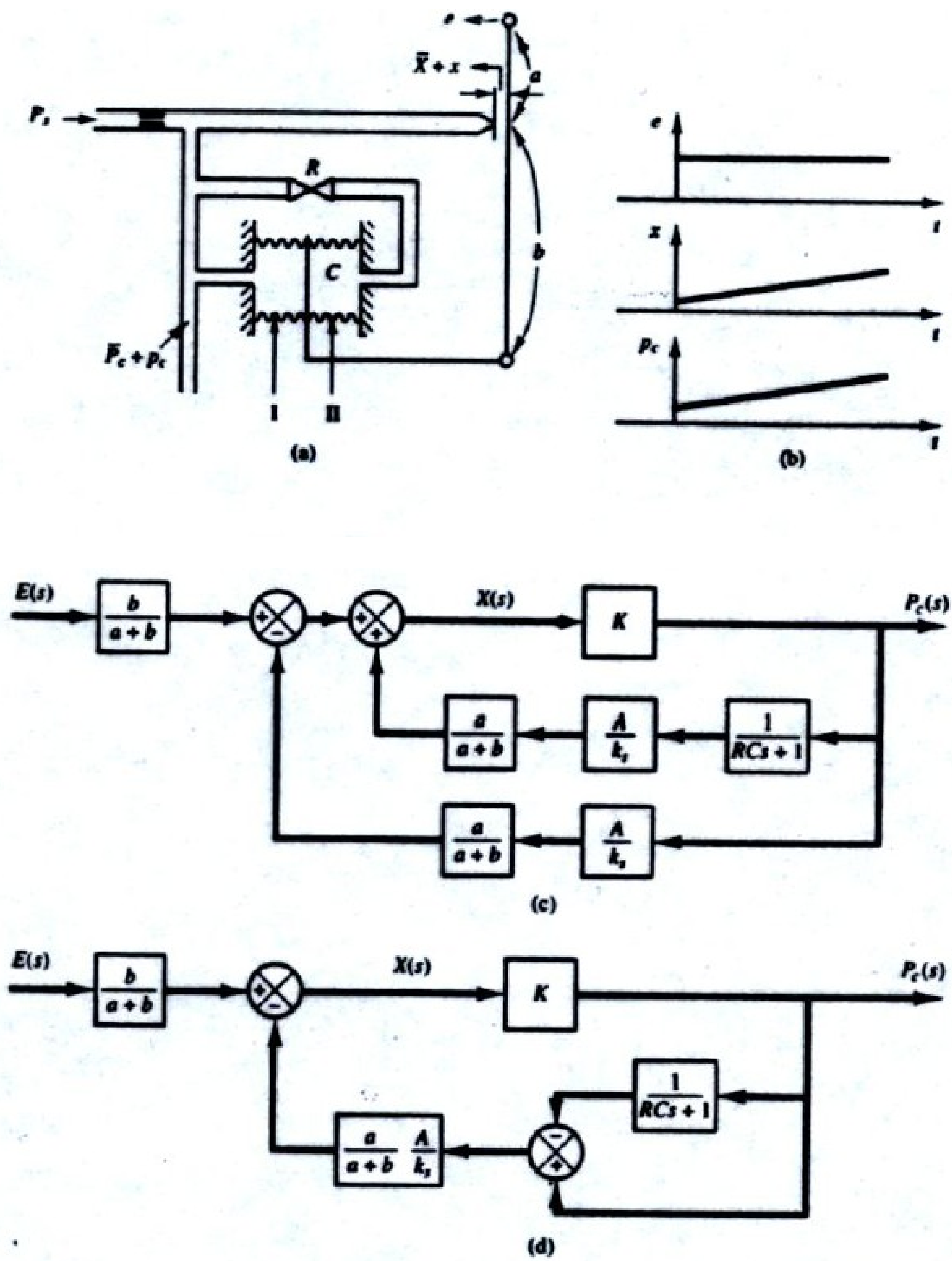



**Figure 7.12. (a) Pneumatic proportional-plus-integral controller; (b) step change in e and the corresponding changes in ;c and p. plotted versus t; (c) block diagram of the controller; (d) simplified block diagram.**

Consider the pneumatic controller shown in Figure 7.12(a).

Let us assume a small step change in the actuating error. This will cause the back pressure *pe* in the nozzle to change continuously, as shown in Figure 7.12 (b).

Note that the integral control action in the controller takes the form of slowly cancelling the feedback that the proportional control originally provided.

A block diagram of this controller under the assumption of small variations in the variables is shown in Figure 7.12(c). A simplification of this block diagram yields Figure 7.12(d). The transfer function of this controller is

$$\frac{P_c(s)}{E(s)} = \frac{\dfrac{b}{a+b}K}{1 + \dfrac{Ka}{a+b}\dfrac{A}{k_s}\left(1 - \dfrac{1}{RCs+1}\right)}$$

where K is a constant, A is the area of the bellows, and ks is the equivalent spring constant of the combined bellows.If $\left|KaARCs/[(a+b)k_s(RCs+1)]\right| >> 1$ which is usually the case, the transfer function can be simplified to

$$\frac{P_c(s)}{E(s)} = K_p\left(1 + \frac{1}{T_i s}\right)$$

Where

$$K_p = \frac{bk_s}{aA}, \qquad T_i = RC$$

**Obtaining pneumatic proportional-plus-integral-plus-derivative control action.**

A combination of the pneumatic controllers shown in Figures 7.11(a) and 7.12( a) yields a proportional-plus-integral-plus-derivative controller, commonly called a PID controller. Figure 7.13(a) shows a schematic diagram of such a controller. Figure 7.13(b) shows a block diagram of this controller under the assumption of small variations in the variables.

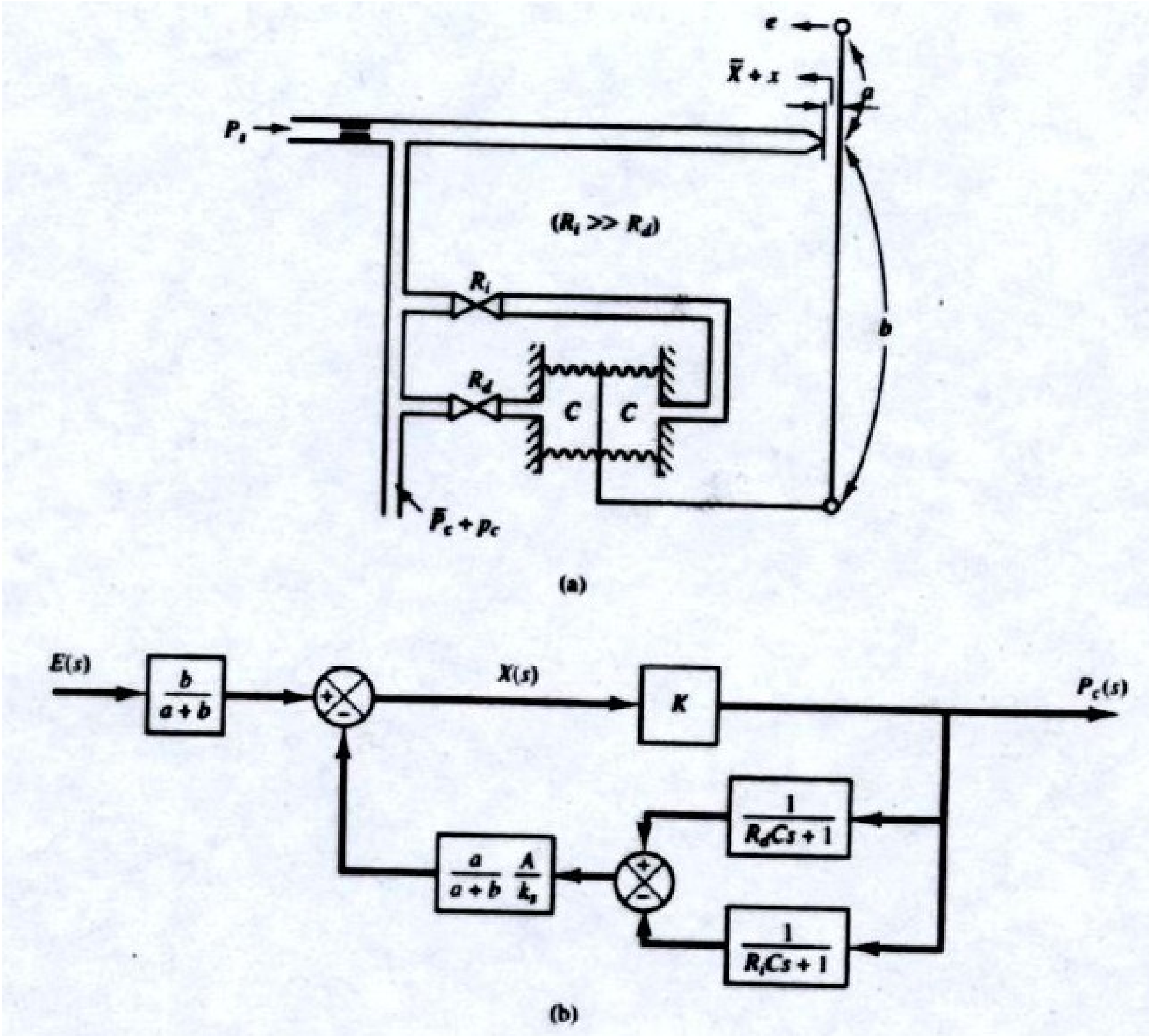



**Figure 7.13. a) Pneumatic proportional-plus-integral-plus derivative controller; (b) block diagram of the controller.**

## 7.2 Hydraulic Controllers

Except for low-pressure pneumatic controllers, compressed air has seldom been used for the continuous control of the motion of devices having significant mass under external load forces. For such a case, hydraulic controllers are generally preferred.

### Hydraulic systems.

The widespread use of hydraulic circuitry in machine tool applications, aircraft control systems, and similar operations occurs because of such factors as positiveness, accuracy, flexibility, high horsepower-to-weight ratio, fast starting, stopping, and reversal with smoothness and precision, and simplicity of operations.

The operating pressure in hydraulic systems is somewhere between 145 and 5000 lb/in$^2$ (between 1 and 35 MPa). In some special applications, the operating pressure may go up to 10,000 lb/in$^2$ (70 MPa). For the same power requirement, the weight and size of the hydraulic unit can be made smaller by increasing the supply pressure.

With high-pressure hydraulic systems, very large force can be obtained. Rapid-acting, accurate positioning of heavy loads is possible with hydraulic systems. A combination of electronic and hydraulic systems is widely used because it combines the advantages of both electronic control and hydraulic power.

## Advantages and disadvantages of hydraulic systems.

There are certain advantages and disadvantages in using hydraulic systems rather than other systems. Some of the advantages are the following:

1. Hydraulic fluid acts as a lubricant, in addition to carrying away heat generated in the system to a convenient heat exchanger.
2. Comparatively smaIl sized hydraulic actuators can develop large forces or torques.
3. Hydraulic actuators have a higher speed of response with fast starts, stops, and speed reversals.
4. Hydraulic actuators can be operated under continuous, intermittent, reversing, and stalled conditions without damage. .
5. Availability of both linear and rotary actuators gives flexibility in design.
6. Because of low leakages in hydraulic actuators, speed drop when loads are applied is small.

On the other hand, several disadvantages tend to limit their use.

1 Hydraulic power is not readily available compared to electric power.
2. Cost of a hydraulic system may be higher than a comparable electrical system performing a similar function.
3. Fire and explosion hazards exist unless fire-resistant fluids are used.
4. Because it is difficult to maintain a hydraulic system that is free from leaks, the system tends to be mess. .
5. Contaminated oil may cause failure in the proper functioning of a hydraulic system.
6. As a result of the nonlinear and other complex characteristics involved, the design of sophisticated hydraulic systems is quite involved.
7. Hydraulic circuits have generally poor damping characteristics. If a hydraulic circuit is not designed properly, some unstable phenomena may occur or disappear, depending on the operating condition.

## Hydraulic integral controllers.

The hydraulic servomotor shown in Figure 7.14 is essentially a pilot-valve-controlled hydraulic power amplifier and actuator. The pilot valve is a balanced valve in the sense that the pressure forces acting on it are all balanced. A very large power output can be controlled by a pilot valve, which can be positioned with very little power.

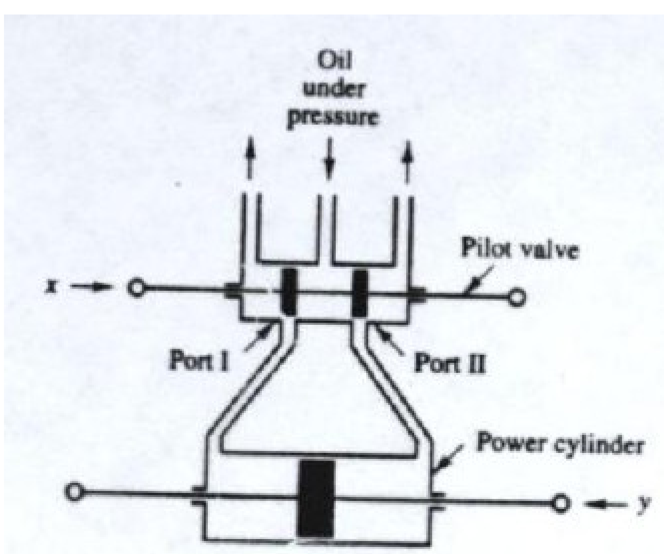



**Figure 7.14. Hydraulic servomotor.**

It will be shown in the following that for negligibly small load mass the servomotor shown in Figure 7.14 acts as an integrator or an integral controller. Such a servomotor constitutes the basis of the hydraulic control circuit.

In the present analysis, we assume that hydraulic fluid is incompressible and that the inertia force of the power piston and load is negligible compared to the hydraulic force at the power piston. We also assume that the pilot valve is a zero-lapped valve, and the oil flow rate is proportional to the pilot valve displacement.

Note that the rate of flow of oil $q$ (kg/sec) times $dt$ (sec) is equal to the power piston displacement $dy(m)$ times the piston area A(m$^2$) times the density of oil ρ (kg/m$^3$). Therefore,

$$A\rho\, dy = q\, dt$$

Because of the assumption that the oil flow rate $q$ is proportional to the pilot valve displacement $x$, we have

$$q = K_1 x$$

where $K_1$ is a positive constant. From above equations  we obtain

$$A\rho \frac{dy}{dt} = K_1 x$$

The Laplace transform of this last equation, assuming a zero initial condition, gives

$$\frac{Y(s)}{X(s)} = \frac{K_1}{A\rho s} = \frac{K}{s}$$

where $K = K_1/(A\rho)$. Thus the hydraulic servomotor shown in Figure 7.14 acts as an integral controller.

## Hydraulic proportional controllers.

It has been shown that the servomotor in Figure 7.14 acts as an integral controller. This servomotor can be modified to a proportional controller by means of a feedback link.

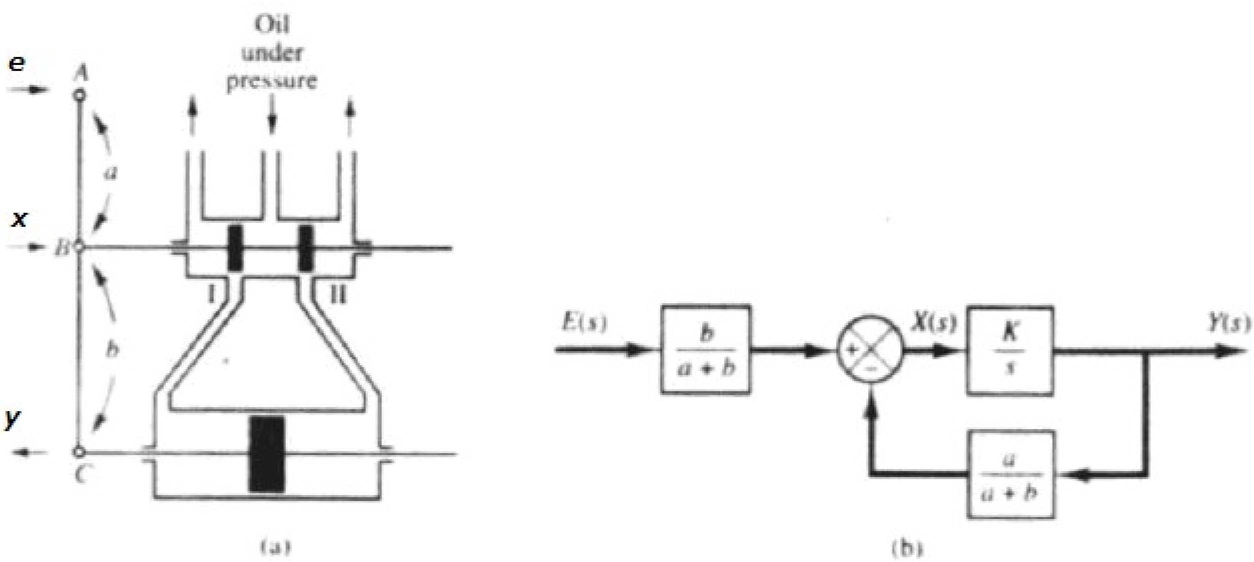



**Figure 7.15. (a) Servomotor that acts as a proportional controller; (b) block diagram of the servomotor.**

Consider the hydraulic controller shown in Figure 7.15(a). The left side of the pilot valve is joined to the left side of the power piston by a link *ABC*. This link is a floating link rather than one moving about a fixed pivot.

A block diagram of the system can be drawn as in Figure 7.15(b). The transfer function between *Y(s)* and *E(s)* is given by

$$\frac{Y(s)}{E(s)} = \frac{\frac{b}{a+b}\frac{K}{s}}{1+\frac{K}{s}\frac{a}{a+b}} = \frac{bK}{s(a+b)+Ka}$$

Noting that under the normal operating conditions we have $|Ka/[s(a+b)]| >> 1$, this last equation can be simplified to

$$\frac{Y(s)}{E(s)} = \frac{b}{a} = K_p$$

The transfer function between *y* and *e* becomes a constant. Thus, the hydraulic controller shown in Figure 7.15(a) acts as a proportional controller, the gain of which is *Kp*. This gain can be adjusted by effectively changing the lever ratio *b/a*. (The adjusting mechanism is not shown in the diagram.)

We have thus seen that the addition of a feedback link will cause the hydraulic servomotor to act as a proportional controller.

**Dashpots.** The dashpot (also called a damper) shown in Figure 7.16(a) acts as a differentiating element.

The curves *x* versus *t* and *y* versus *t* are shown in Figure 7.16(b).

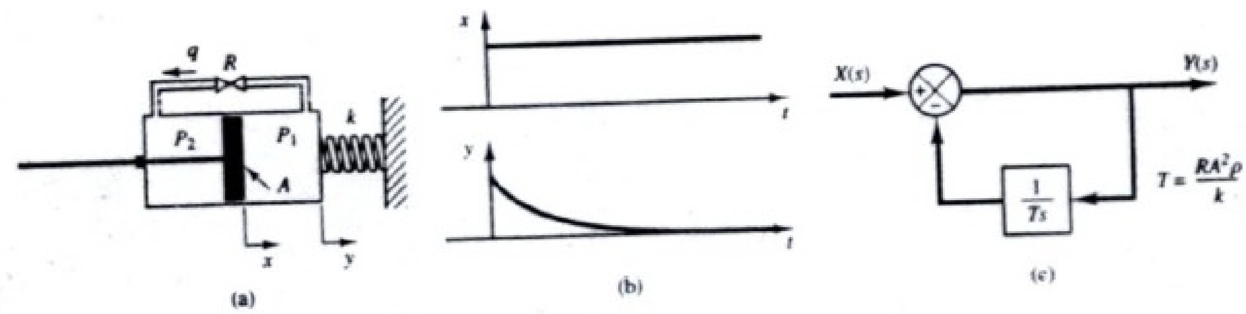



**Figure 7.16. (a) Dashpot; (b) step change in x and the corresponding change in y plotted versus t; (c) block diagram of the dashpot.**

Let us derive the transfer function between the displacement *y* and displacement *x*. Define the pressures existing on the right and left sides of the piston as P1 (lb/in$^2$) and *P2* (lb/in$^2$), respectively. Suppose that the inertia force involved is negligible. Then the force acting on the piston must balance the spring force. Thus

$$A(P_1 - P_2) = ky$$

Where

*A*=Piston Area in in$^2$

*k* = spring constant, lb/ in.

The flow rate *q* is given

$$q = \frac{P_1 - P_2}{R}$$

Where

*q* = flow rate through the restriction, lb/sec

*R* = resistance to flow at the restriction, lb-sec/in$^2$.lb

Since the flow through the restriction during *dt* seconds must equal the change in the mass of oil to the left of the piston during the same *dt* seconds, we obtain

$$q\,dt = A\rho(dx - dy)$$

where p = density, Ib/in$^3$. (We assume that the fluid is incompressible or p = constant.) This last equation can be rewritten as

$$\frac{dx}{dt} = \frac{dy}{dt} + \frac{ky}{RA^2\rho}$$

Taking the Laplace transforms of both sides of this last equation, assuming zero initial conditions, we obtain

$$sX(s) = sY(s) + \frac{k}{RA^2\rho}Y(s)$$

$$\frac{Y(s)}{X(s)} = \frac{s}{s + \frac{k}{RA^2\rho}}$$

$$\frac{Y(s)}{X(s)} = \frac{Ts}{Ts + 1} = \frac{1}{1 + \frac{1}{Ts}}$$

$$RA^2\rho/k = T$$

Figure 7.16(c) shows a block diagram representation for this system.

**Obtaining hydraulic proportional-plus-integral control action.**

Figure 7.17 shows a schematic diagram of a hydraulic proportional-plus-integral controller.

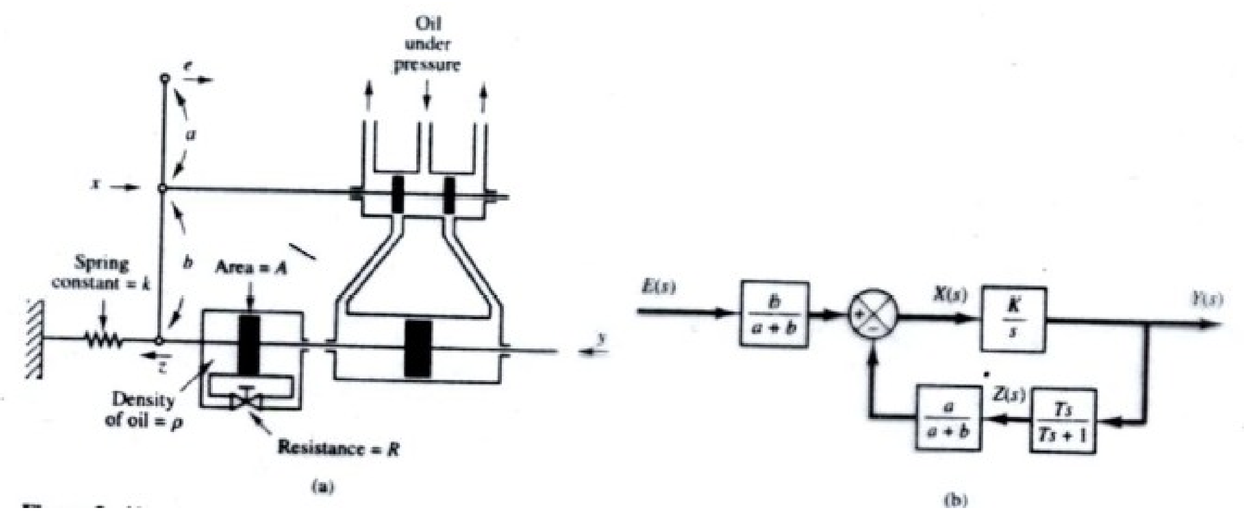



**Figure 7.17. (a) Schematic diagram of a hydraulic proportional-plus-integral controller; (b) block diagram of the controller.**

A block diagram of this controller is shown in Figure 7.17(b). The transfer function *Y(s)/ E(s)* is given by

$$\frac{Y(s)}{E(s)} = \frac{\frac{b}{a+b}\frac{K}{s}}{1 + \frac{Ka}{a+b}\frac{T}{Ts+1}}$$

Thus the controller shown in Figure 7.17(a) is a proportional-plus-integral controller (a PI controller.)

$$\frac{Y(s)}{E(s)} = K_p\left(1 + \frac{1}{T_i s}\right)$$

$$K_p = \frac{b}{a}, \qquad T_i = T = \frac{RA^2\rho}{k}$$

**Obtaining hydraulic proportional-plus-derivative control action.**

Figure 7.18(a) shows a schematic diagram of a hydraulic proportional-plus-derivative controller. The cylinders are fixed in space and the pistons can move.

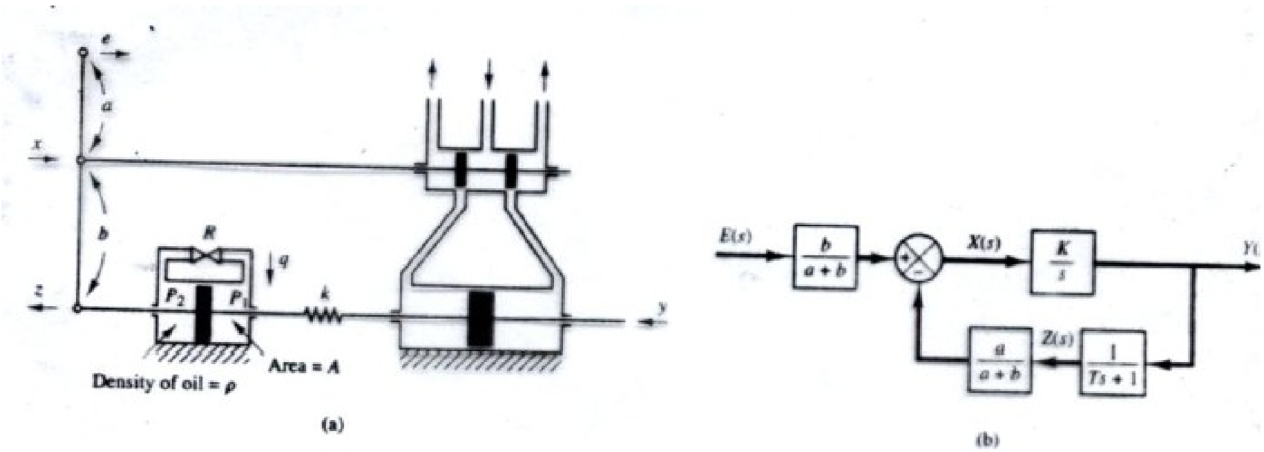



**Figure 7.18 . (a) Schematic diagram of a hydraulic proportional-plus-derivative controller; (b) block diagram of the controller.**

For this system, notice that

$$k(y - z) = A(P_2 - P_1)$$

$$q = \frac{P_2 - P_1}{R}$$

$$q\,dt = \rho A\,dz$$

Therefore we have

$$y = z + \frac{A}{k} qR = z + \frac{RA^2\rho}{k}\frac{dz}{dt}$$

$$\frac{Z(s)}{Y(s)} = \frac{1}{Ts + 1}$$

$$T = \frac{RA^2\rho}{k}$$

Thus the controller shown in Figure 7.18(a) is a proportional-plus-derivative-controller (a PD controller). A block diagram for this system is shown in Figure 7.18(b). From the block diagram the transfer function Y(s)/E(s) can be obtained as

$$\frac{Y(s)}{E(s)} = \frac{\frac{b}{a+b}\frac{K}{s}}{1 + \frac{a}{a+b}\frac{K}{s}\frac{1}{Ts+1}}$$

under normal operation

$$|aK/[(a + b)s(Ts + 1)]| \gg 1.$$

Hence,

$$\frac{Y(s)}{E(s)} = K_p(1 + Ts)$$

$$K_p = \frac{b}{a}, \qquad T = \frac{RA^2\rho}{k}$$

Thus the controller shown in Figure 7.18(a) is a proportional-plus-derivative-controller (a PD controller).

## 7.3 Electronic Controllers