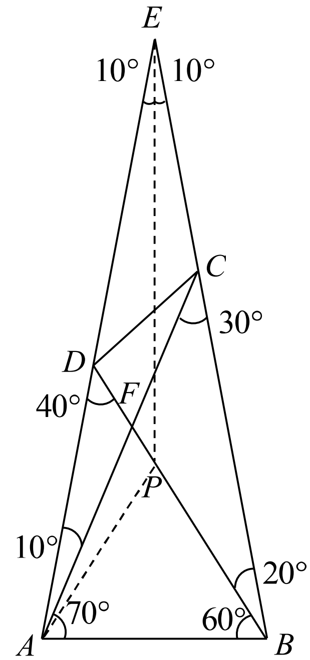
کتاب تکمیلی ص ۱۰۲ س ۲۲

<u>Method 3 (Purely Geometric Approach)</u>



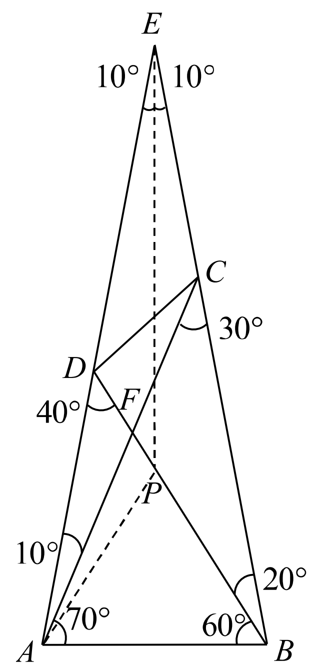
Let P be a point on DB such that EP is the angle bisector of $\angle AEB$.

Construct AP.

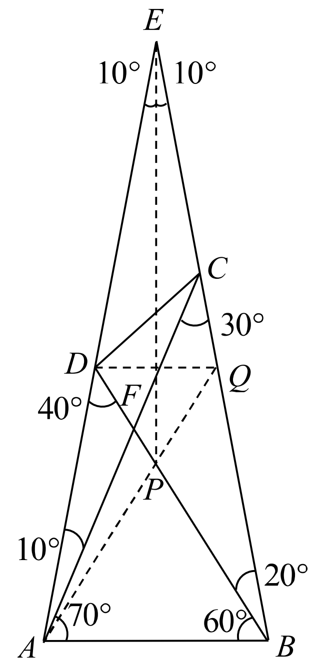
In ΔAEP & ΔBEP,

$EP = EP$	(common)
$\angle AEP = \angle BEP = 10°$	(by construction)
$\angle EAB = \angle EBA = 80°$	
$\therefore AE = BE$	(sides opp. eq. $\angle$ s)
$\therefore \Delta AEP \cong \Delta BEP$	(SAS)
$\therefore \angle EAP = \angle EBP = 20°$	(corr. $\angle$ s, $\cong \Delta$ s)
$\angle CAP = 20° - 10° = 10°$	
$\angle PAB = 70° - 10° = 60°$	
$\angle APB = 180° - 60° - 60° = 60°$	($\angle$ sum of Δ)
$\therefore \Delta APB$ is an equil. Δ	
$\therefore AP = PB = AB$	(def. of equil. Δ)

Let them be y.



In ΔAEC & ΔEBP,

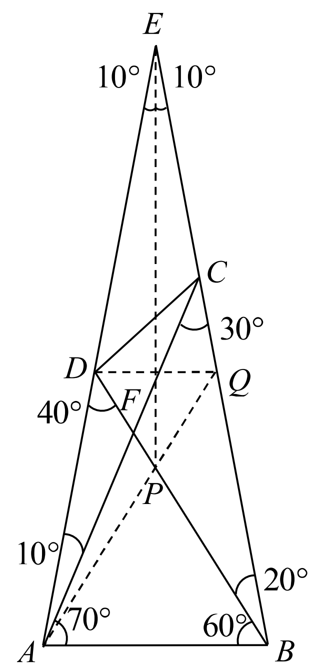


$\angle EAC = \angle BEP = 10°$	(proved)
$\angle AEC = \angle EBP = 20°$	(proved)
$AE = EB$	(proved)
$\therefore \Delta AEC \cong \Delta EBP$	(ASA)
$\therefore EC = BP = y$	(corr. sides, $\cong \Delta$ s)

Extend AP to meet BE at Q. Construct DQ.

$\angle DPQ = \angle APB = 60°$	(vert. opp. $\angle$ s)

In ΔDAB & ΔQBA,

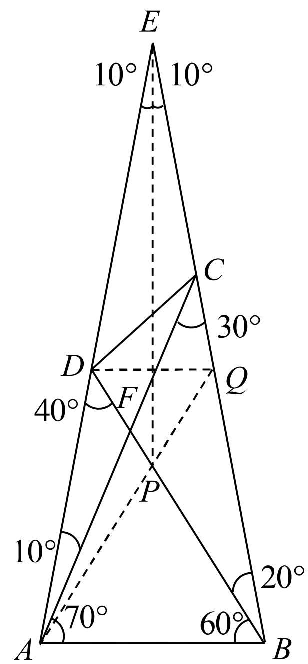


$\angle DAB = \angle QBA = 80°$	(given)
$AB = BA$	(common)
$\angle DBA = \angle QAB = 60°$	(proved)
$\therefore \Delta DAB \cong \Delta QBA$	(ASA)
$\therefore DB = QA$	(corr. sides, $\cong \Delta$ s)

Let them be x.

$DP = DB - PB = x - y = QA - PA = QP$

$\therefore \angle PDQ = \angle PQD$	(base $\angle$ s, isos. Δ)
$= \dfrac{180° - 60°}{2} = 60°$	($\angle$ sum of Δ)
$\therefore \Delta DPQ$ is an equil. Δ	
$\therefore DQ = QP = DP = x - y$	(def. of equil. Δ)



$\angle PQD = \angle QAB = 60°$

$\therefore DQ // AB$ (alt. $\angle$s eq.)

$\therefore \angle EQD = \angle EBA = 80°$ (corr. $\angle$s, $DQ // AB$)

$\angle EDQ = 180° - 20° - 80° = 80°$ ($\angle$ sum of Δ)

$\angle DEB = \angle DBE = 20°$

$\therefore ED = DB = x$ (sides opp. eq. $\angle$s)

$\angle EDQ = \angle EQD = 80°$

$\therefore EQ = ED = x$ (sides opp. eq. $\angle$s)

$\therefore CQ = EQ - EC = x - y = DQ$

$\therefore \angle QCD = \angle QDC$ (base $\angle$s, isos. Δ)

$= \dfrac{180° - 80°}{2} = 50°$ ($\angle$ sum of Δ)

$\therefore \angle ACD = 50° - 30° = 20°$

Remarks:

1. This is the most famous proof. All the other proofs that can be found on the web employ the same construction of straight lines.

2. It is a natural way to divide the isosceles triangle along the axis of symmetry. By doing so, we are lucky to obtain equilateral triangles and parallel lines.

3. As it is purely deductive geometric approach, the proof is long and complicated, but an elegant one.