Chapter 10: Rotation

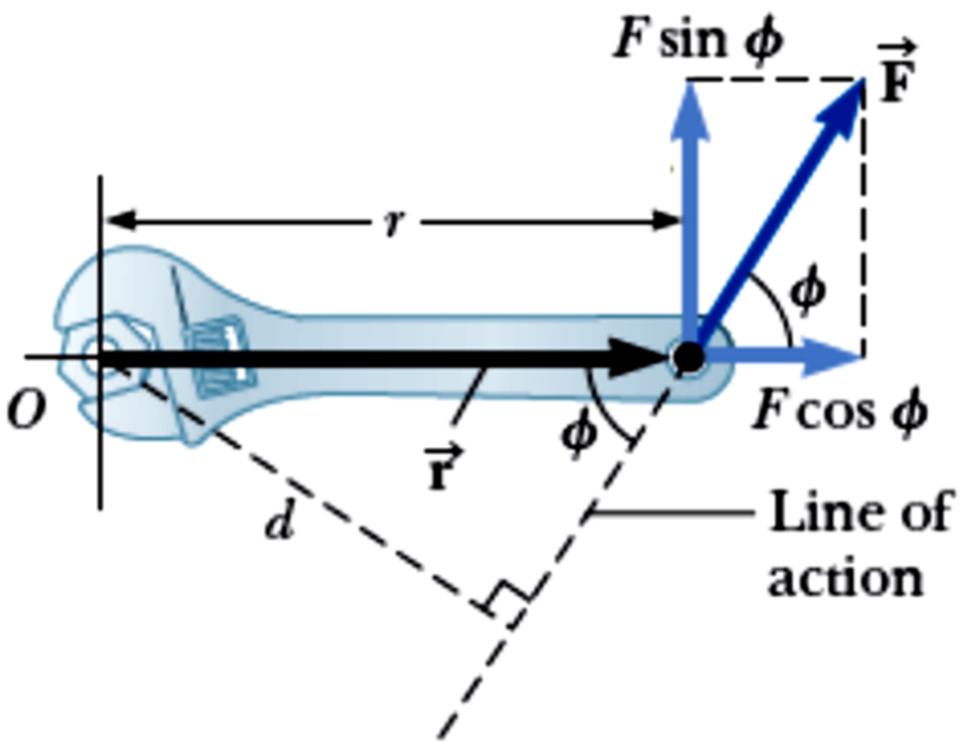
- ✓ **Angular Position, Velocity and Acceleration**
- ✓ **Rotational Kinematics**
- ✓ **Kinetic Energy of Rotation**
- ✓ **Rotational Inertia**
- ✓ **Torque**
- ✓ **Energy Consideration in Rotational Motion**

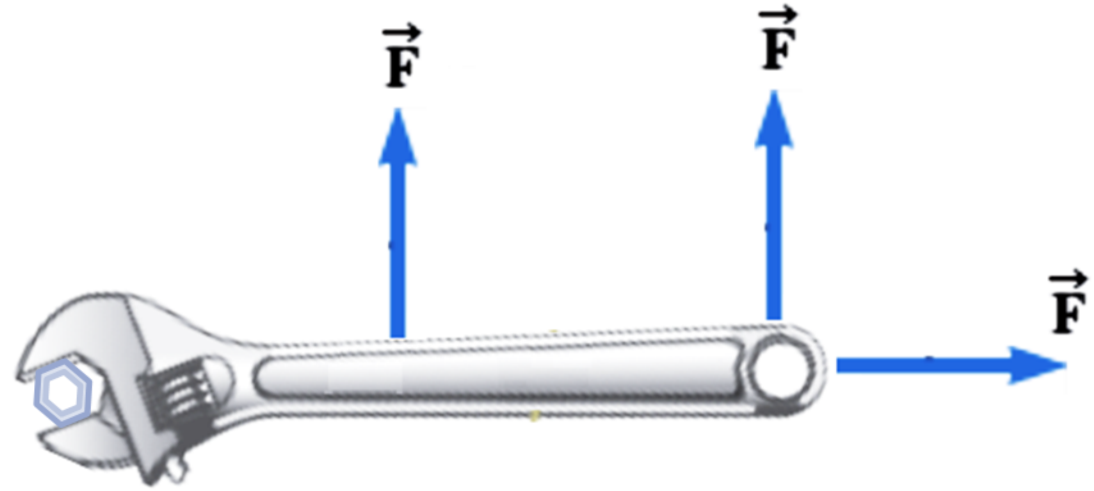
Session 22:

- ✓ Torque
- ✓ Energy Consideration in Rotational Motion
- ✓ Examples

Torque

Torque, τ **, is the tendency of a force to rotate an object about some axis.**



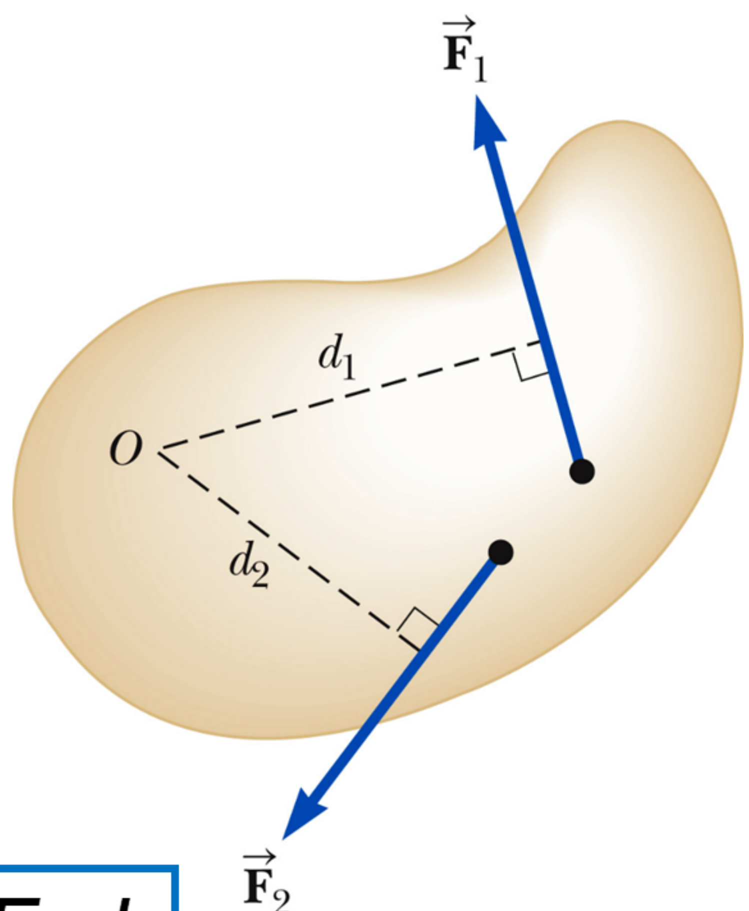


$$\boxed{\tau = r F \sin\varphi}$$

$$\boxed{\vec{\tau} = \vec{r} \times \vec{F}}$$

$$\begin{cases} \tau = r\,(F \sin\varphi) \\ \tau = F(r \sin\varphi) = F\,d \end{cases}$$

d is the ***moment arm*** of the force



$$\boxed{\tau_{net} = \tau_1 + \tau_2 = F_1 d_1 - F_2 d_2}$$

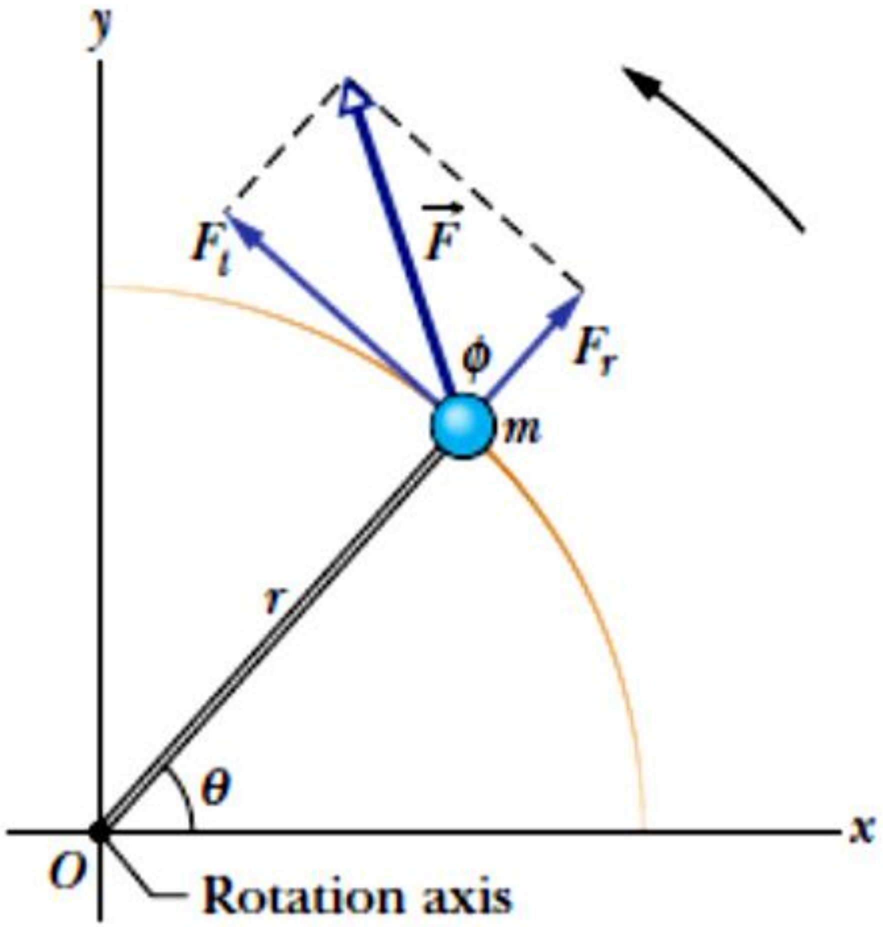
Newton's Second Law for Rotation

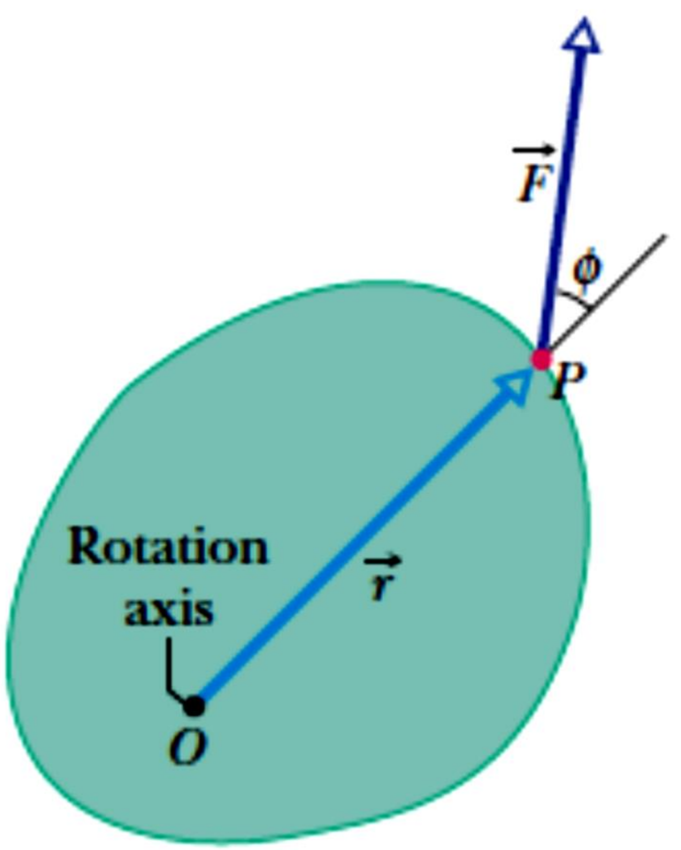
$$F_t = ma_t$$

$$r(F_t) = r(ma_t) \qquad a_t = r\alpha$$

$$\boxed{\tau = mr^2\alpha = I\alpha}$$

$$\boxed{\tau_{ext} = I\alpha}$$





Rigid body under a net torque

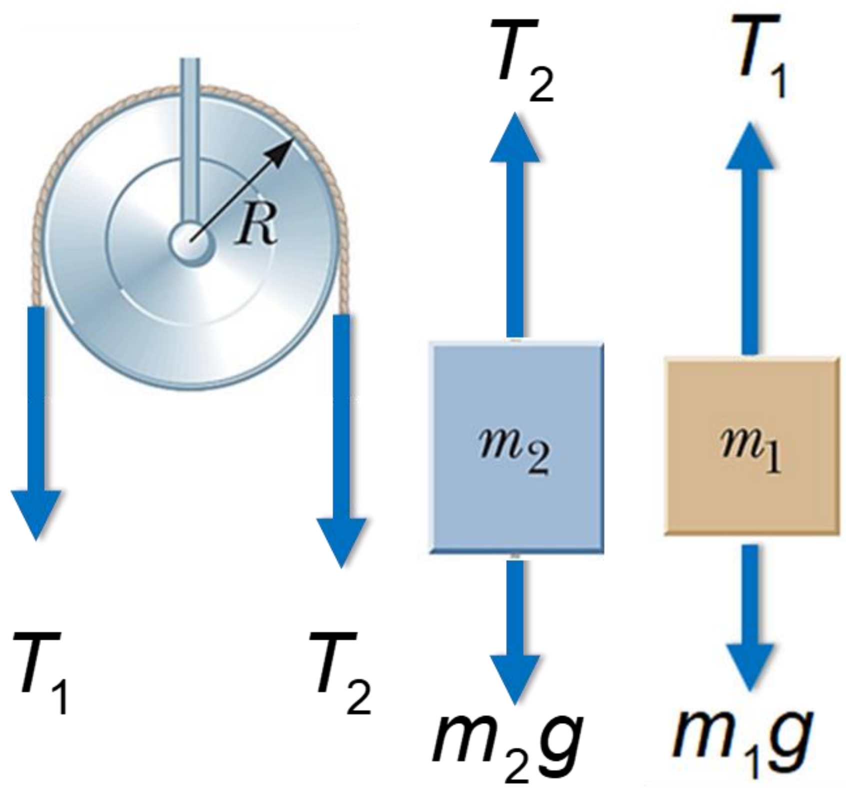
Ex 11: (Problem 10. 51 Halliday)

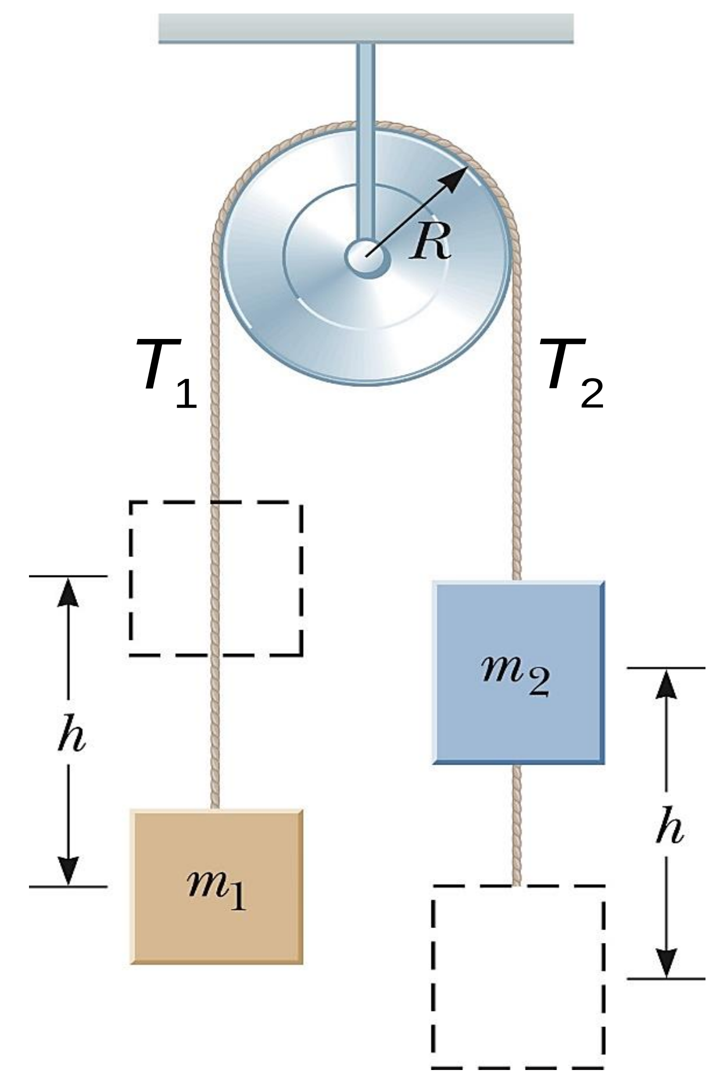
Block **1** has mass $\mathbf{m_1 = 460\ g}$ and block **2** has mass $\mathbf{m_2 = 500\ g}$, and the pulley, which is mounted on a horizontal axle with negligible friction, has radius **R = 5 cm**. When released from rest, block 2 falls **75 cm** in **5 s** without the cord slipping on the pulley. (a) What is the magnitude of the acceleration of the blocks? What are (b) tension $\mathbf{T_2}$ and (c) tension $\mathbf{T_1}$? (d) What is the magnitude of the pulley's angular acceleration? (e) What is its rotational inertia?

$$\boxed{a = \frac{2h}{t^2} = 0.06\ m/s^2}$$

$$\begin{cases} T_1 - m_1 g = m_1 a \\ T_2 - m_2 g = m_2(-a) \end{cases}$$

$$\begin{cases} \boxed{T_1 = m_1(g + a) = 4.54\ N} \\ \boxed{T_2 = m_2(g - a) = 4.87\ N} \end{cases}$$



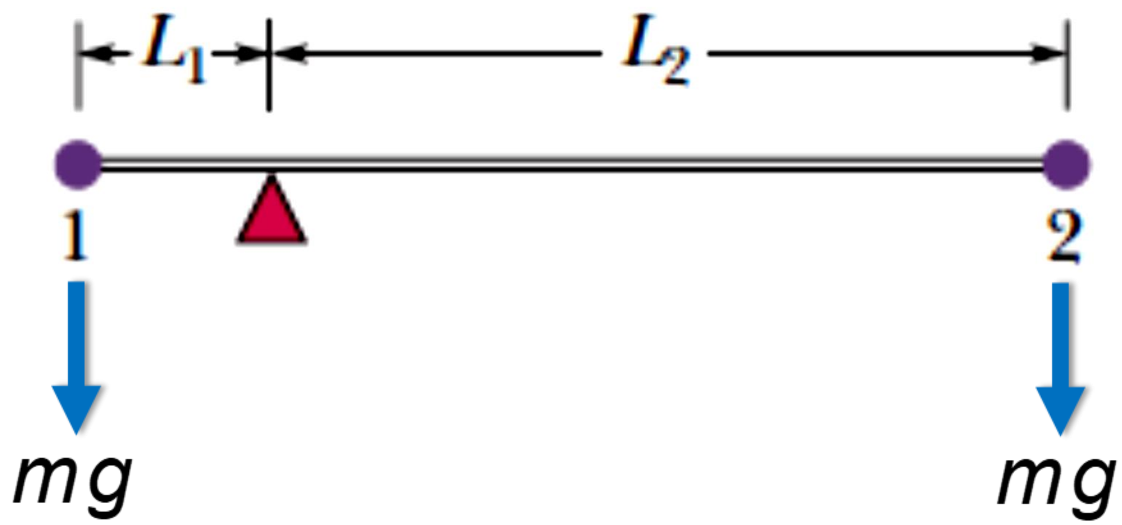


cord does not slip $\quad a = R\alpha \Rightarrow \boxed{\alpha = 1.2\ rad/s^2}$

$$\tau_{ext} = I\alpha \Rightarrow T_1 R - T_2 R = I(-\alpha) \Rightarrow \boxed{I = \frac{(T_2 - T_1)R}{\alpha} = 1.38 \times 10^{-2}\ kg.m^2}$$

Ex 12: (Problem 10. 56 Halliday)

Figure 10-46 shows **particles 1 and 2**, each of mass **m**, fixed to the ends of a rigid massless rod of length **$L_1 + L_2$**, with **$L_1 = 20$ cm** and **$L_2 = 80$ cm**. The rod is held horizontally on the fulcrum and then released. What are the magnitudes of the initial accelerations of (a) particle 1 and (b) particle 2 ?



$$\tau_{ext} = I\alpha$$

$$mg\,L_1 - mg\,L_2 = I(-\alpha)$$

$$I = mL_1^2 + mL_2^2$$

$$\boxed{\alpha = \frac{g(L_2 - L_1)}{L_1^2 + L_2^2} = 8.65\ rad/s^2}$$

$$a_t = r\alpha \qquad a_r = \frac{v^2}{r}$$

$$\boxed{t = 0;\ v = 0 \Rightarrow a_r = 0}$$

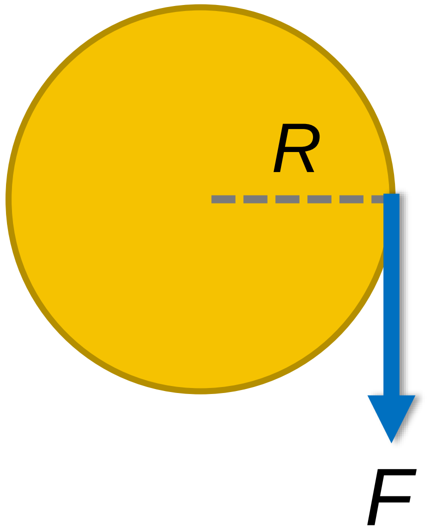
$$\boxed{|a_{1t}| = L_1\alpha = 1.73\ \ m/s^2}$$

$$\boxed{|a_{2t}| = L_2\alpha = 6.93\ \ m/s^2}$$

Ex 13: (Problem 10. 57 Halliday)

A pulley, with a rotational inertia of **1 ×10^{-3} kg m^2** about its axle and a radius of **10 cm**, is acted on by a force applied tangentially at its rim. The force magnitude varies in time as **F = 0.5 t + 0.3 t^2**,with F in newtons and t in seconds. The pulley is **initially at rest**. At **t = 3 s** what are its (a) angular acceleration and (b) angular speed?

$$\tau_{ext} = I\alpha$$



$$-FR = I(-\alpha) \Rightarrow \alpha = \frac{FR}{I} = \frac{(0.5t + 0.3t^2)(0.1)}{1\times 10^{-3}}$$

$$\boxed{\alpha = 50t + 30t^2} \xrightarrow{t=3s} \boxed{\alpha = 420\ rad/s^2}$$

$$\omega(t) - \underbrace{\omega_0}_{0} = \int_0^t \alpha\, dt = \int_0^t \left(50t + 30t^2\right) dt \Rightarrow \boxed{\omega(t) = 25t^2 + 10t^3}$$

$$\xrightarrow{t=3s} \boxed{\omega = 495\ rad/s}$$

Energy Consideration in Rotational Motion

Translational Motion:	Rotational Motion:
x	θ
F	τ
$W = \int F\, dx$	$W = \int \tau\, d\theta$
$P = \dfrac{dw}{dt} = F\, v$	$P = \dfrac{dw}{dt} = \tau\, \omega$
$k = \dfrac{1}{2} m v^2$	$k = \dfrac{1}{2} I\, \omega^2$

- For a **system of two or more objects** interacting with conservative forces:

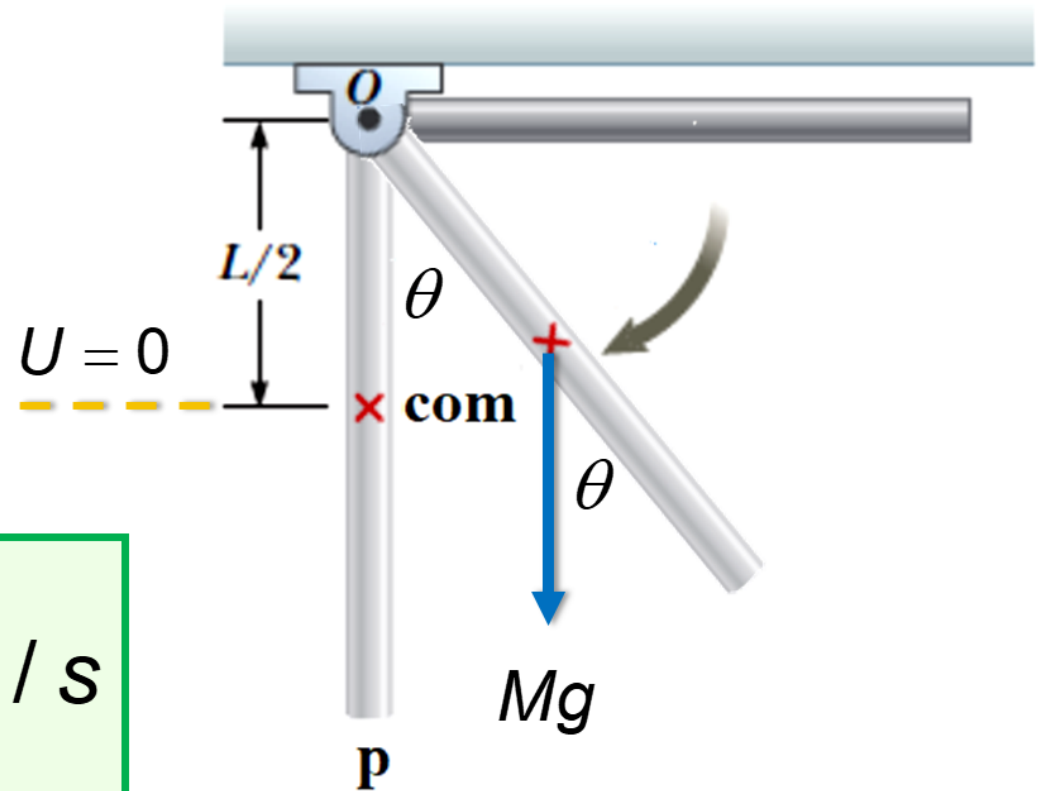
$$\boxed{W_{ext} = \Delta E_{mech} = \Delta K + \Delta U = \Delta K_{trans} + \Delta K_{rot} + \Delta U}$$

$$if\ W_{ext} = 0 \qquad \boxed{\Delta K_{trans} + \Delta K_{rot} + \Delta U = 0}$$

Ex 14: A uniform rod of length **L= 30 cm** and mass **M = 1 kg** is free to rotate on a **frictionless pin** passing through one end. The rod is **released from rest** in the horizontal position. (a) What is its angular speed when the rod reaches its lowest position? (b) Determine the tangential speed of the center of mass and the tangential speed of the lowest point on the rod when it is in the vertical position.

$$\boxed{\Delta K_{trans} + \Delta K_{rot} + \Delta U = 0}$$

$$0 + (\frac{1}{2} I \omega^2 - 0) + (0 - Mg\frac{L}{2}) = 0$$



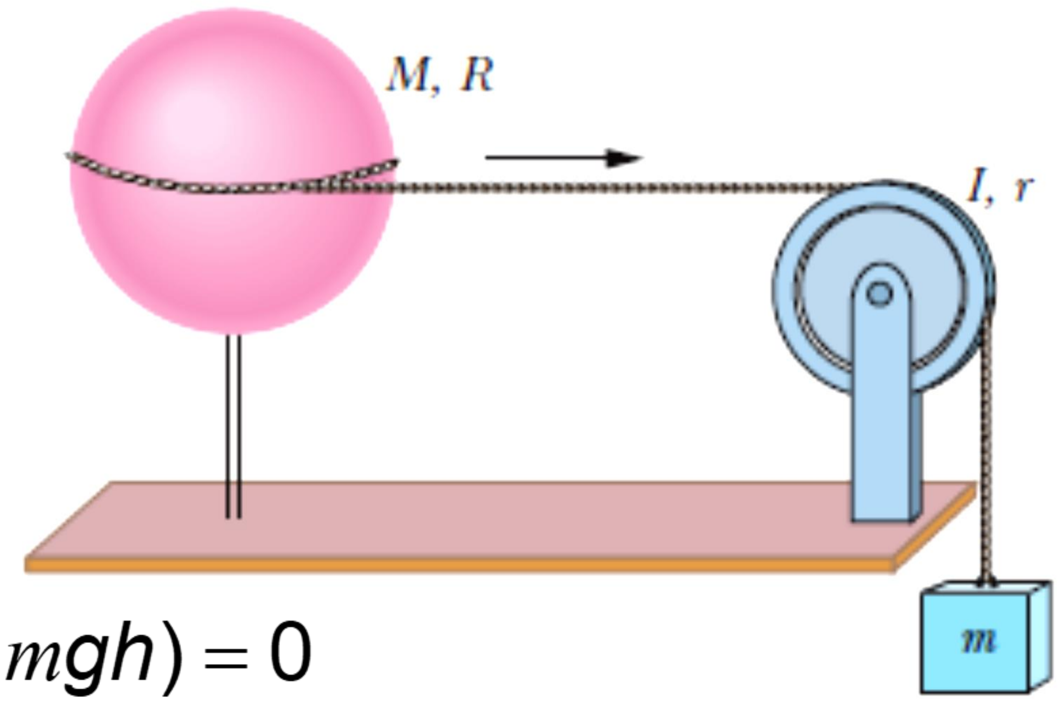
$$\boxed{\omega = \sqrt{\frac{MgL}{I}} = \sqrt{\frac{MgL}{\frac{1}{3}ML^2}} = \sqrt{\frac{3g}{L}} = 9.9\ rad/s}$$

$$v = r\,\omega \Rightarrow \boxed{v_{com} = \frac{L}{2}\omega = 1.48\ m/s} \qquad \boxed{v_p = L\omega = 2.96\ m/s}$$

$$W = \int \tau\, d\theta = \int_{\frac{\pi}{2}}^{0} (-Mg\frac{L}{2}\sin\theta)d\theta = Mg\frac{L}{2}\int_{\frac{\pi}{2}}^{0}(-\sin\theta)d\theta = Mg\frac{L}{2}(\cos(0) - \cos(\frac{\pi}{2})) = Mg\frac{L}{2}$$

Ex 15: (Problem 10. 66 Halliday)

A uniform spherical shell of mass **M = 4.5 kg** and radius **R = 8.5 cm** can rotate about a vertical axis on frictionless bearings. A massless cord passes around the equator of the shell, over a pulley of rotational inertia I = **3 × 10^{-3} kg m^2** and radius **r = 5 cm**, and is attached to a small object of mass **m = 0.60 kg**. There is no friction on the pulley's axle; the **cord does not slip on the pulley**. What is the speed of the object when it has fallen **82 cm** after being released from rest? Use energy considerations.



$$\boxed{\Delta K_{trans} + \Delta K_{rot} + \Delta U = 0}$$

$$(\frac{1}{2} m v^2 - 0) + (\frac{1}{2} I_{shell}\, \omega_1^2 + \frac{1}{2} I_{pulley}\, \omega_2^2 - 0) + (0 - mgh) = 0$$

$$\frac{1}{2} m v^2 + \left[\frac{1}{2} (\frac{2}{3} M R^2)(\frac{v}{R})^2 + \frac{1}{2} I_{pulley}\, (\frac{v}{r})^2 \right] - mgh = 0$$

$$\boxed{v = \sqrt{\frac{mgh}{\frac{1}{2} m + \frac{1}{3} M + \frac{1}{2r^2} I_{pulley}}} = 1.4 \;\; m/s}$$