Chapter 8: Potential Energy and Conservation of Energy

- ✓ **Potential Energy**
- ✓ **Conservative and Nonconservative Forces**
- ✓ **Mechanical Energy**
- ✓ **Conservation of Energy**

Chapter 8: Potential Energy and Conservation of Energy

Session 16:

- ✓ Mechanical Energy
- ✓ Conservation of Energy
- ✓ Examples

Mechanical energy

- **Energy**
  - **Kinetic**
  - **Potential**
  - **Internal**

$$E_{system} = K + U + E_{th}$$

$$E_{mech} = K + U$$

$$\Delta E_{system} = \Delta K + \Delta U + \Delta E_{th} = W + Q + T_{MW} + T_{MT} + T_{ET} + T_{ER}$$

Energy is transferred to the block by *work*.

Energy transfers to the handle of the spoon by *heat*.

Energy leaves the radio from the speaker by *mechanical waves*.

Energy enters the automobile gas tank by *matter transfer*.

Energy enters the hair dryer by *electrical transmission*.

Energy leaves the light-bulb by *electromagnetic radiation*.

Conservation of Energy

$$\boxed{\Delta E_{system} = \Delta E_{mech} + \Delta E_{th} = W_{ext}}$$

- For a **system of two or more objects** interacting with **conservative forces:**

$$\boxed{W_{ext} = \Delta E_{mech} = \Delta K + \Delta U}$$

$$if\ \ W_{ext} = 0 \Rightarrow \Delta E_{mech} = \Delta K + \Delta U = 0 \Rightarrow \boxed{K_i + U_i = K_f + U_f}$$

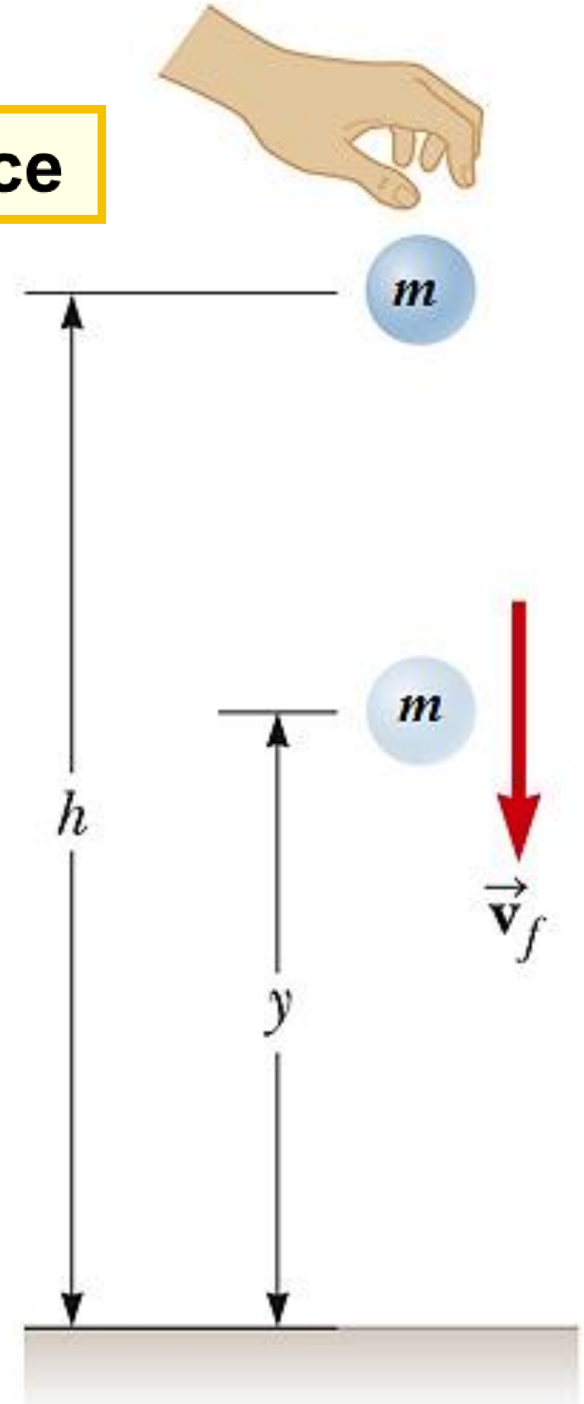
- If there is a **nonconservative force (f_k):**

$$\boxed{W_{ext} = \Delta E_{mech} + \Delta E_{th}}$$

$$if\ \ W_{ext} = 0 \Rightarrow \Delta E_{mech} = -\Delta E_{th} = W_{f_k} \Rightarrow \boxed{\Delta K + \Delta U = -f_k d}$$

Ex 3: Determine the **speed of the ball** at a height **y** above the ground (free fall).

mass-Earth system; gravity is conservative force



$$\begin{cases} y_i = h \\ U_{gi} = mgh \\ K_i = 0 \end{cases}$$

$$\begin{cases} y_f = y \\ U_{gf} = mgy \\ K_f = \frac{1}{2}mv_f^2 \end{cases}$$

$$\begin{cases} y = 0 \\ U_g = 0 \end{cases}$$

$$\boxed{K_i + U_{gi} = K_f + U_{gf}}$$

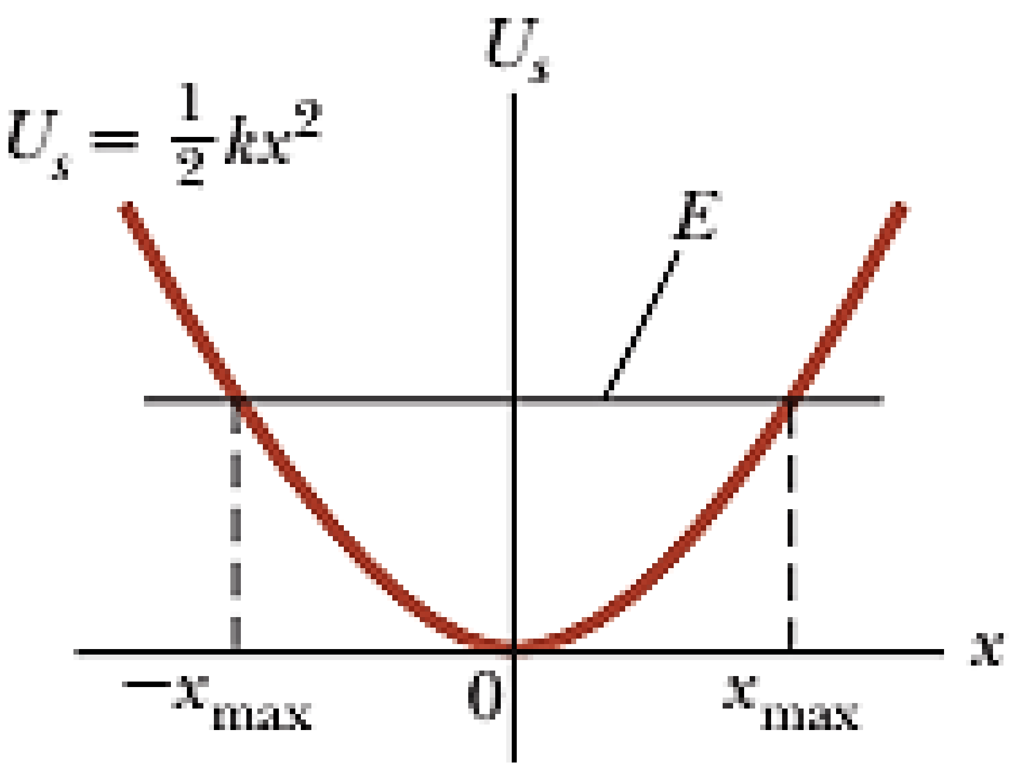
$$0 + mgh = \frac{1}{2}mv_f^{\ 2} + mgy$$

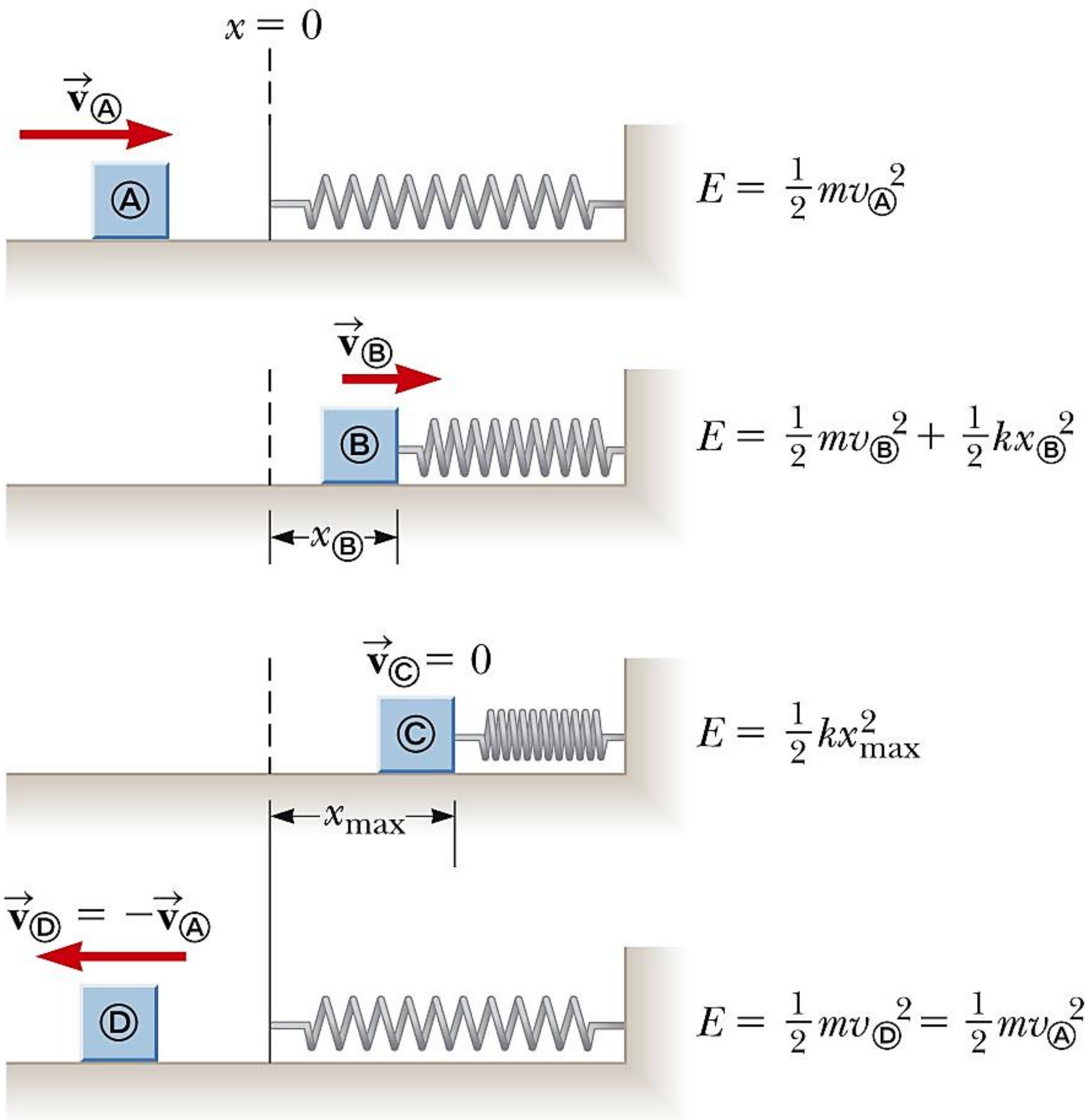
$$\boxed{v_f = \sqrt{2g(h-y)}}$$

$$if\ \ v_i \neq 0 \quad \Rightarrow \quad \boxed{v_f = \sqrt{v_i^{\ 2} + 2g(h-y)}}$$

Ex 4: Mechanical energy for a **system of mass-spring**

$$\boxed{E = K + U_s = \text{constant}}$$





Ex 5: (Problem 8.24 Halliday)

A block of mass **m = 2 kg** is dropped from height **h = 40 cm** onto a spring of spring constant **k = 1960 N/m**. Find the maximum distance the spring is compressed

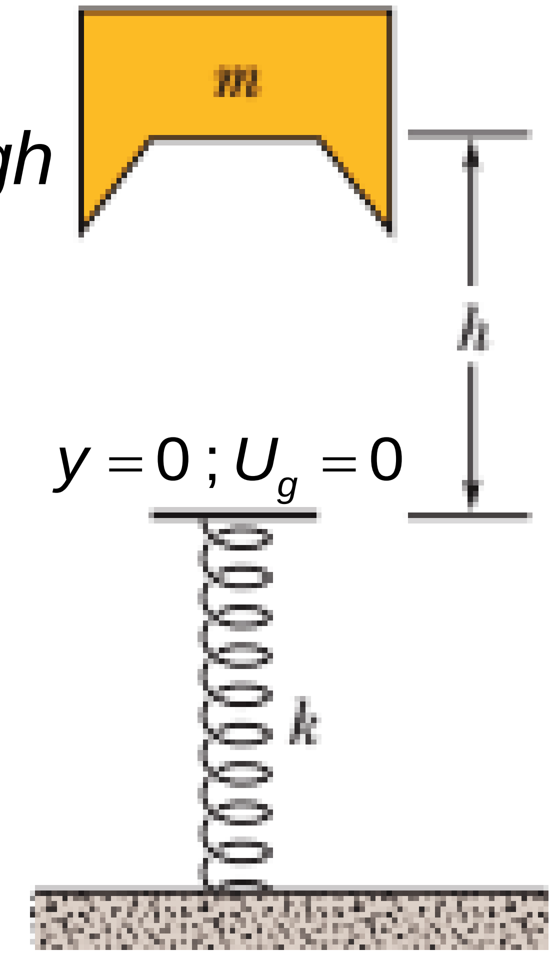
System: mass-Earth-spring

$$\boxed{K_i + U_i = K_f + U_f}$$

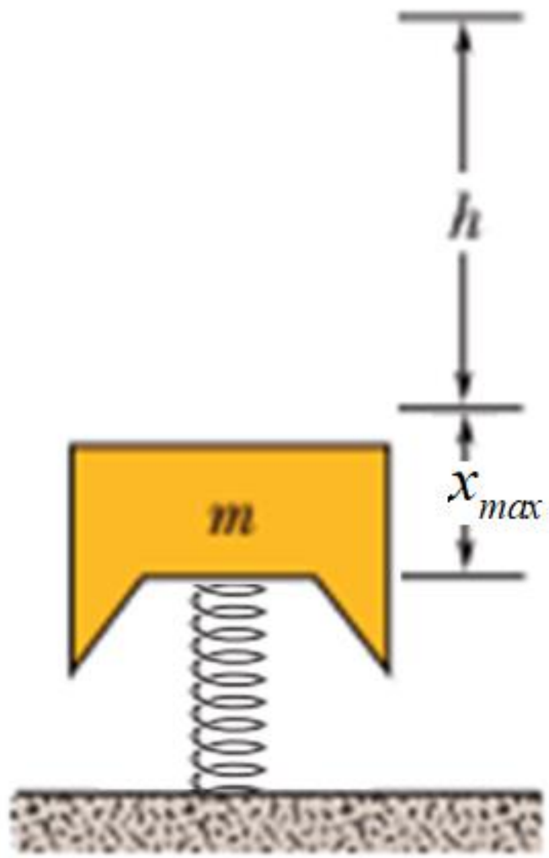
$y_i = h$

$U_{g_i} = mgh$

$K_i = 0$



$y = 0\,;\, U_g = 0$



$y_f = -x_{max}$

$U_{g_f} = -mgx_{max}$

$U_{s_f} = \frac{1}{2}kx_{max}^{\ 2}$

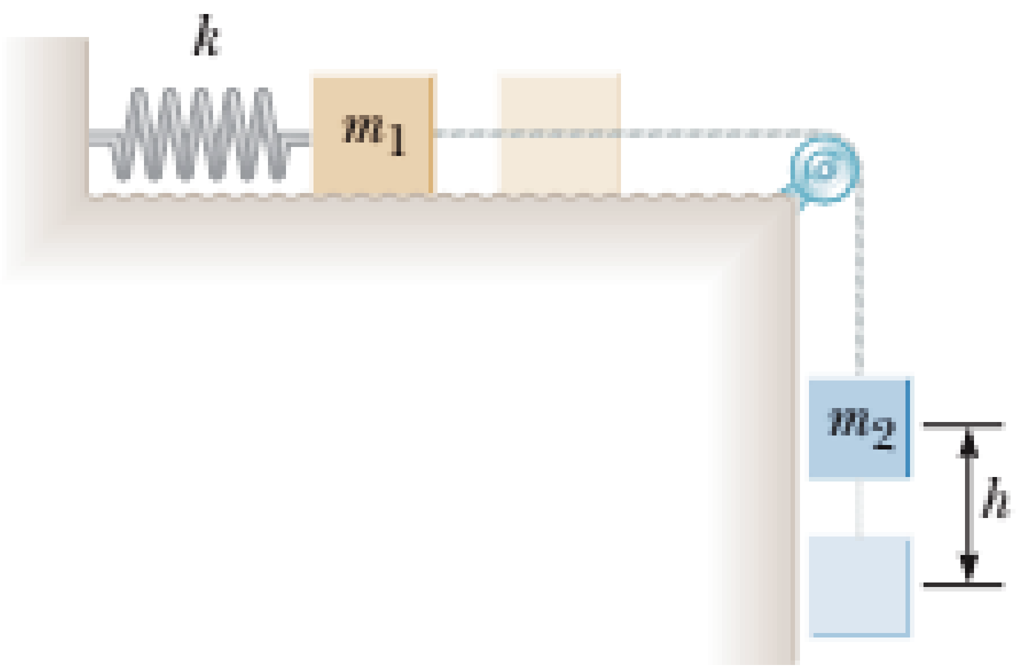
$K_f = 0$

$$0 + mgh = 0 - mgx_{max} + \frac{1}{2}kx_{max}^{\ 2} \quad \Rightarrow \quad \frac{1}{2}kx_{max}^{\ 2} - mgx_{max} - mgh = 0$$

$$x_{max} = \frac{mg \pm \sqrt{(mg)^2 + 2mghk}}{k} \quad \Rightarrow \quad \boxed{x_{max} = 0.10\ m}$$

Ex 6: Two blocks are connected by a light string that passes over a frictionless pulley as shown in Figure. The block of mass **m_1= 0.5 kg** lies on a horizontal surface and is connected to a spring of force constant **k = 100 N/m**. The system is **released from rest** when the spring is unstretched. If the hanging block of mass **m_2= 0.75 kg** falls a distance **h = 10 cm** before coming to rest, calculate the *coefficient of kinetic friction* between the block of mass **m_1** and the surface.

$$\boxed{W_{ext} = \Delta K + \Delta U + \Delta E_{th} = 0}$$



$$\boxed{\Delta K + \Delta U = -f_k d}$$

$$\Delta K = 0 \qquad \Delta U = \Delta U_g + \Delta U_s \qquad f_k = \mu_k N = \mu_k m_1 g$$

$$0 - m_2 g h + \frac{1}{2} k h^2 = -\mu_k m_1 g h$$

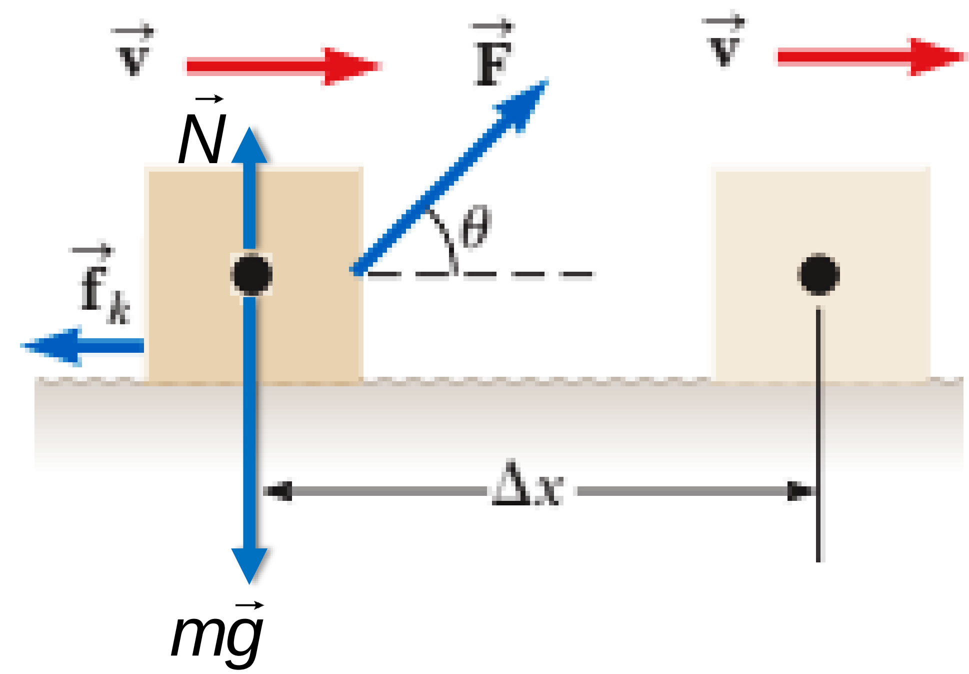
$$\mu_k = \frac{m_2 g - \frac{1}{2} k h}{m_1 g}$$

$$\boxed{\mu_k = 0.48}$$

Ex 7: (Problem 8.45 Halliday)

A rope is used to pull a **3.57 kg block,** at constant speed, **4.06 m** along a horizontal floor. The force on the block from the rope is **7.68 N** and directed **15°** above the horizontal. What are (a) the work done by the rope's force, (b) the increase in thermal energy of the block–floor system, and (c) the coefficient of kinetic friction between the block and floor?

$$W_F = W_{ext} = F\Delta x \cos\theta$$



$$\boxed{W_F = W_{ext} = 30.12 \text{ J}}$$

$$W_{ext} = \underbrace{\Delta K}_{0} + \underbrace{\Delta U}_{0} + \Delta E_{th}$$

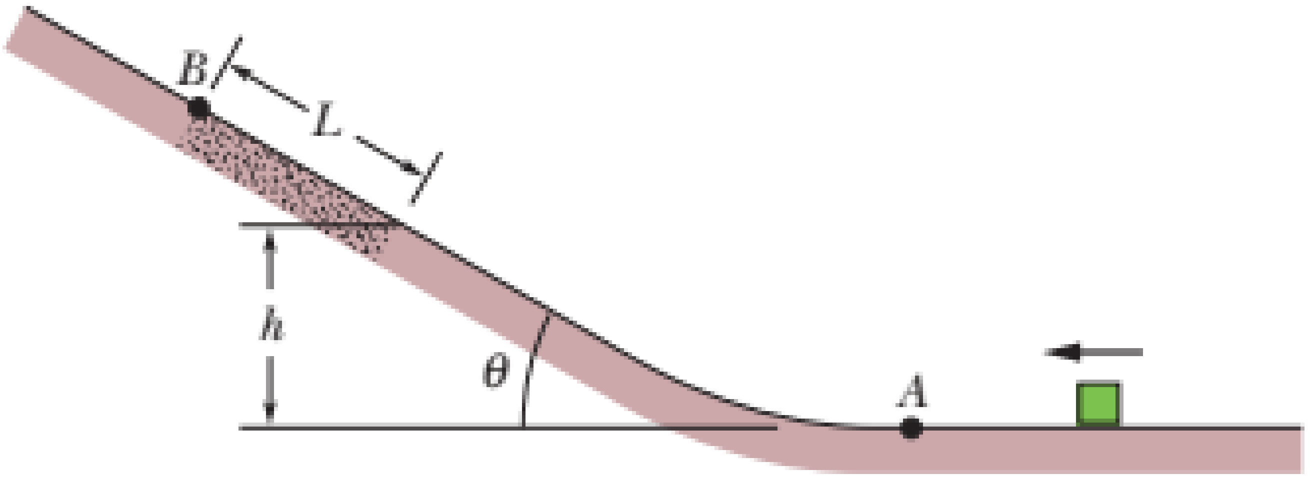
$$\boxed{\Delta E_{th} = W_{ext} = 30.12 \text{ J}}$$

$$\Delta E_{th} = -W_{f_k} = f_k d \Rightarrow \mu_k N d = \mu_k (mg - F\sin\theta)d = \Delta E_{th}$$

$$\boxed{\mu_k = \frac{\Delta E_{th}}{(mg - F\sin\theta)d} = 0.225}$$

Ex 8: **(Problem 8.62 Halliday)**

A block slides along a path that is without friction until the block reaches the section of length **L = 0.75 m**, which begins at height **h = 2 m** on a ramp of angle **θ= 30°** . In that section, the coefficient of kinetic friction is **0.40**. The block passes through point **A** with a speed of **8 m/s**. If the block can reach point **B** (where the friction ends), what is its speed there, and if it cannot, what is its greatest height above **A**?



$$\boxed{\Delta K + \Delta U = -f_k d}$$

$$\left(\frac{1}{2}mv_B^{\,2} - \frac{1}{2}mv_A^{\,2}\right) + mg(\underbrace{h + L\sin\theta}_{h_B}) = -\mu_k(\underbrace{mg\cos\theta}_{N})L$$

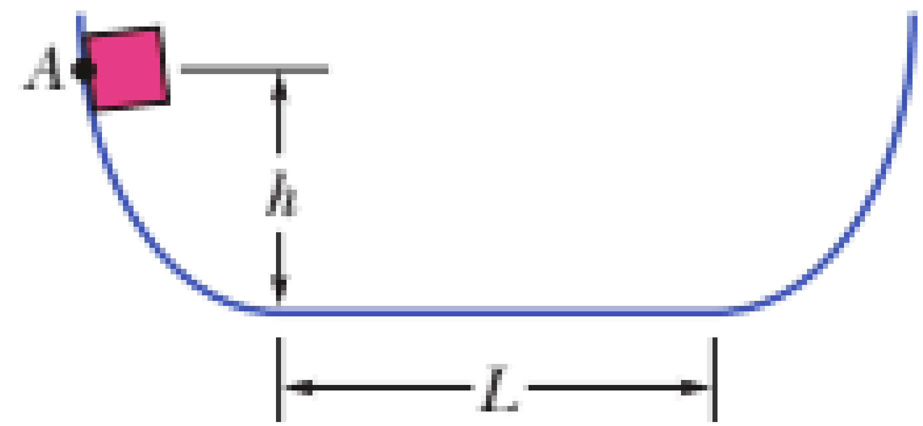
$$v_B^{\,2} = v_A^{\,2} - 2g(h + L\sin\theta) - 2\mu_k g\cos\theta L \Rightarrow \boxed{v_B^{\,2} = 12.35 \Rightarrow v_B = 3.5 \quad m/s}$$

$$\left(0 - \frac{1}{2}mv_A^{\,2}\right) + mg(h + d\sin\theta) = -\mu_k(mg\cos\theta)L$$

$$\boxed{d = 1.5\ m} \qquad \boxed{h_{max} = h + d\sin\theta = 2.75\ m}$$

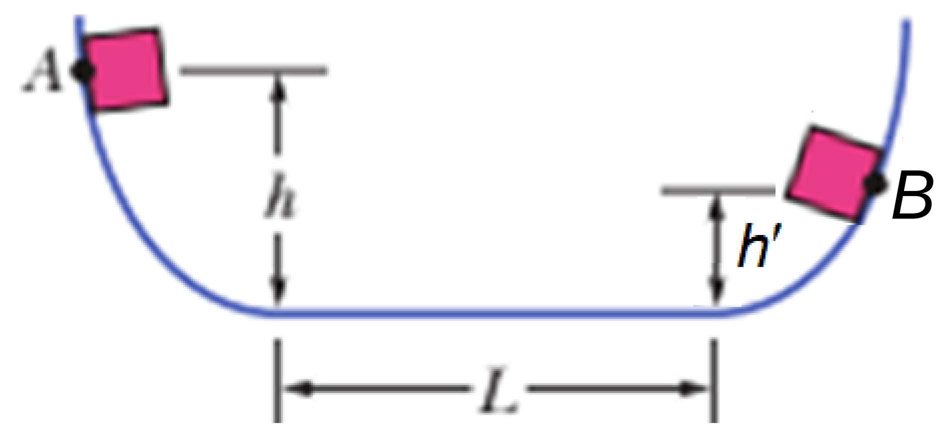
Ex 9: (Problem 8.65 Halliday)

A particle can slide along a track with elevated ends and a flat central part, as shown. The flat part has length **L = 40 cm**. The curved portions of the track are frictionless, but for the flat part the coefficient of kinetic friction is **μ_k = 0.20**. The particle is released from rest at point **A**, which is at height **h = L/2**. How far from the left edge of the flat part does the particle finally stop?



$$\boxed{\Delta K + \Delta U = -f_k d}$$

$$\left(\frac{1}{2}m\underbrace{v_B}_{0}{}^2 - \frac{1}{2}m\underbrace{v_A}_{0}{}^2\right) - mg(h-h') = -\mu_k \underbrace{mg}_{N} L$$



$$(h-h') = \mu_k L = 0.08\ m$$

$$-mgh = -\mu_k mg\, d \quad \Rightarrow \quad d = \frac{h}{\mu_k} = \frac{0.2}{0.2} = 1\ m$$

$$n = \frac{d}{L} = 2.5 \quad \Rightarrow \quad \boxed{D = 0.5L = 0.2\ \text{m}}$$