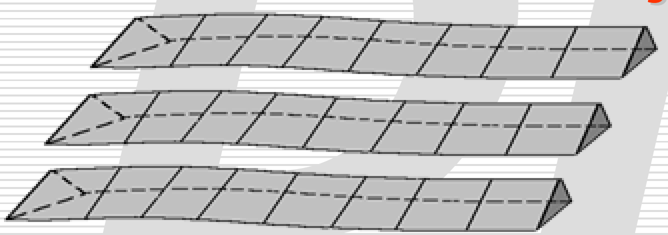
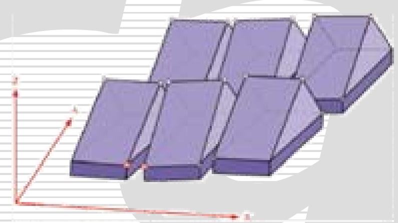
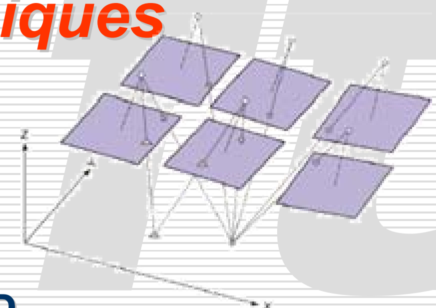
Photogrammetric Block Adjustment

Chapter 2
Block Adjustment Techniques







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Overview

Objective

Investigate the possibility of object space reconstruction from imagery using:

- *Single image.*
- *Stereo-pair.*
- *Single flight lines (Strip adjustment).*
- *Image blocks:*
  - *Block Adjustment of Independent Models (BAIM).*
  - *Bundle Block Adjustment.*
    - *Special cases (resection, intersection, and stereo-pair orientation).*

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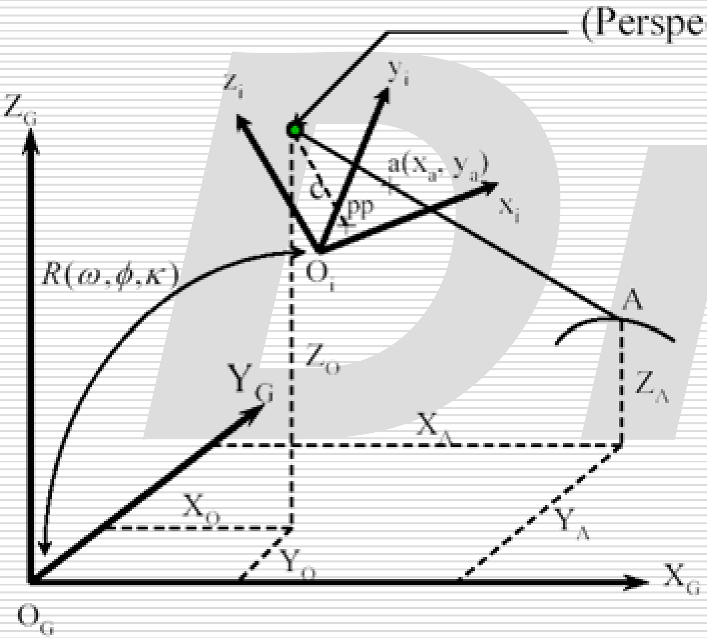
Overview

The Reconstruction Process

The main objective of photogrammetry is to derive ground coordinates of object points from imagery for 3D map production.

We would like to investigate the feasibility of performing this task using single image, stereo image pair, or more.

The General mathematical model we are going to use is the ***collinearity equations.***



$$x_a = x_p - c\frac{r_{11}(X_A - X_O) + r_{21}(Y_A - Y_O) + r_{31}(Z_A - Z_O)}{r_{13}(X_A - X_O) + r_{23}(Y_A - Y_O) + r_{33}(Z_A - Z_O)} + \Delta x$$

$$y_a = y_p - c\frac{r_{12}(X_A - X_O) + r_{22}(Y_A - Y_O) + r_{32}(Z_A - Z_O)}{r_{13}(X_A - X_O) + r_{23}(Y_A - Y_O) + r_{33}(Z_A - Z_O)} + \Delta y$$

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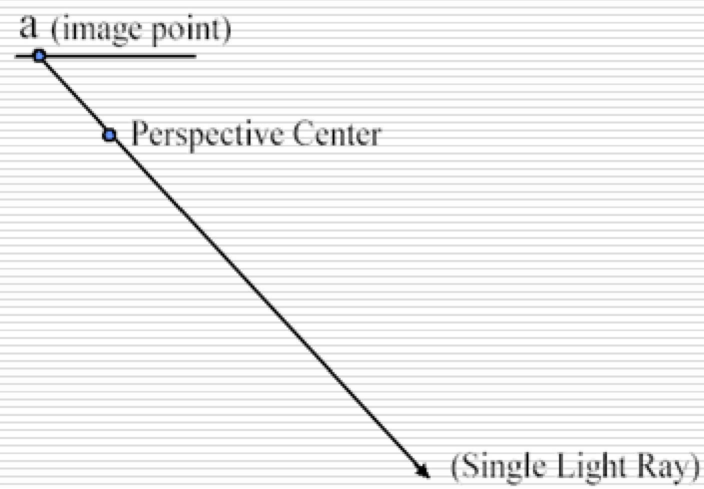
Basic Concepts

Single Image (Mono Situation)

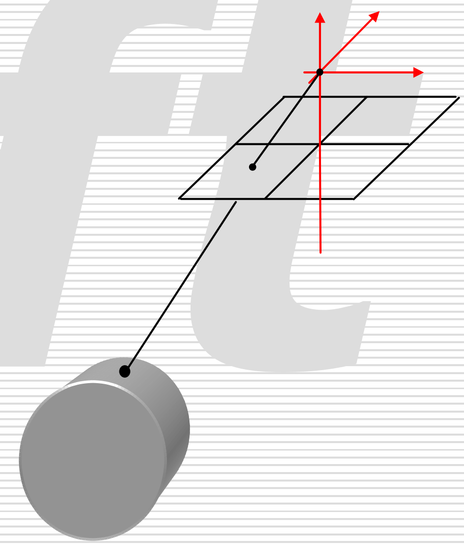
- *Cameral/Sensor Calibration*
- *Interior Orientation*
- *Space Resection*
  - *Perfect*
  - *Non Perfect*



We have two equations (collinearity equations) in three unknowns (ground coordinates of the corresponding object point - Best Case Scenario).

– IOP & EOP are available.

Consequently:
this problem is under determined, an image point will define a single (infinite) light ray. The object point can be anywhere along this light ray.



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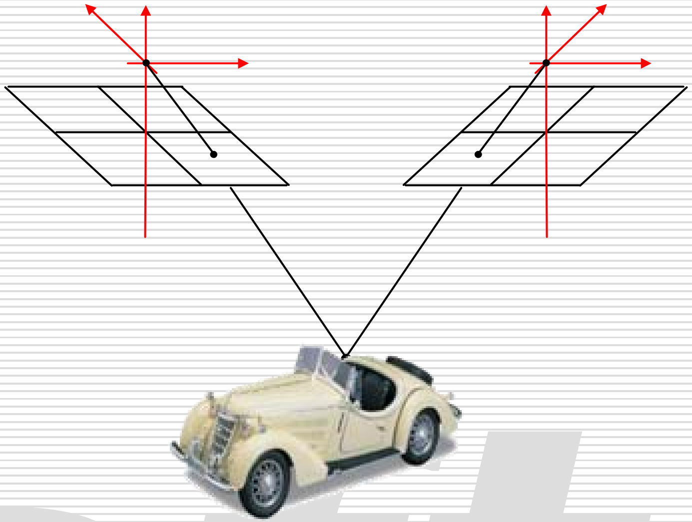
Basic Concepts

Two Images (Stereo Situation)

- **Classic Method**
  - Relative Orientation
  - Absolute Orientation

- **New Method**
  - Resection – Intersection
  - Resection/Intersection



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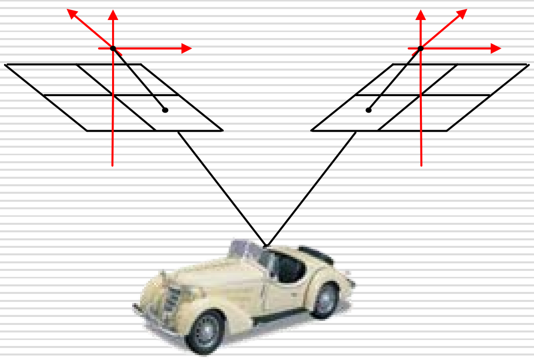
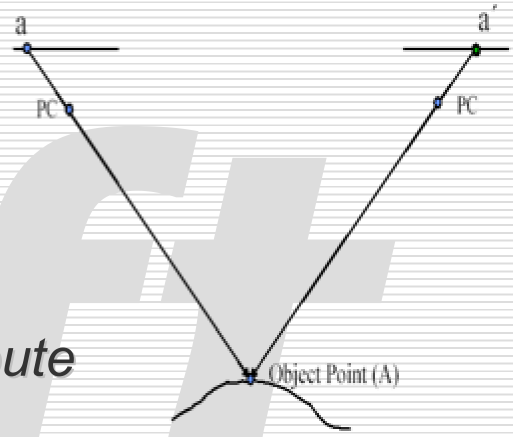
Basic Concepts

Two Images (Stereo Situation)



We have the same point appearing in two images.

- – Four equations (two collinearity equations in each image).
- – Three unknowns (ground coordinates of the corresponding object points - Best Case Scenario). IOP & EOP are available.



Thus, we have a redundancy of one (which will contribute towards the computation of the ROP).

Conceptually: the two image points define two light rays. The object point is the intersection of these light rays.

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Basic Concepts

Two Images (Stereo Situation)

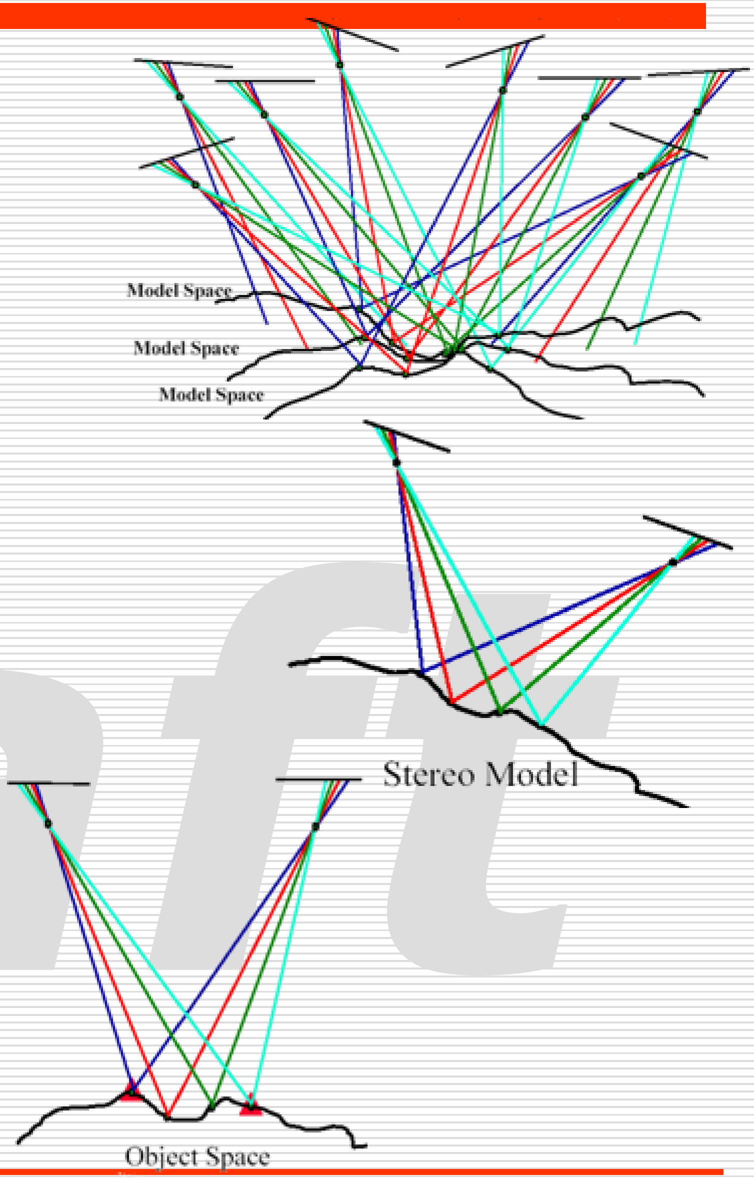
From Images to Object Space

For a stereo-pair, we can derive a 3-D representation of the object space covered by the overlap area as follows:

– Perform RO of the stereo-pair under consideration (using at least five conjugate points) . stereo-model.

– Using some GCP, we can rotate, scale and shift the stereo-model (AO) until it fits at the location of the ground control points (i.e. the residuals at the GCP are as small as possible).

*– For the absolute orientation, we need at least three ground control points **(Those points should not be collinear).***



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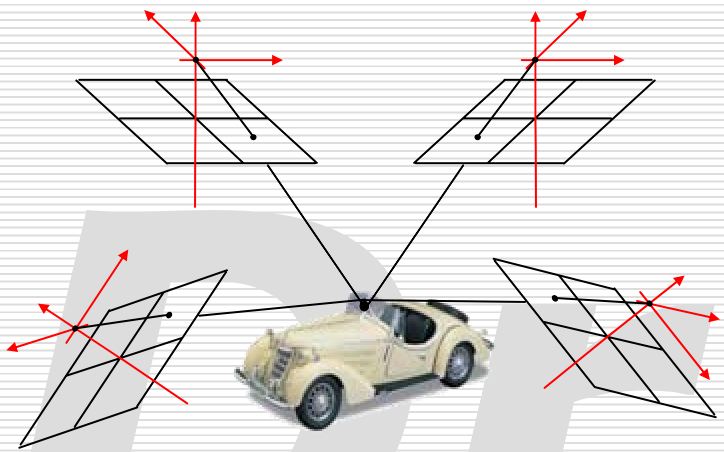
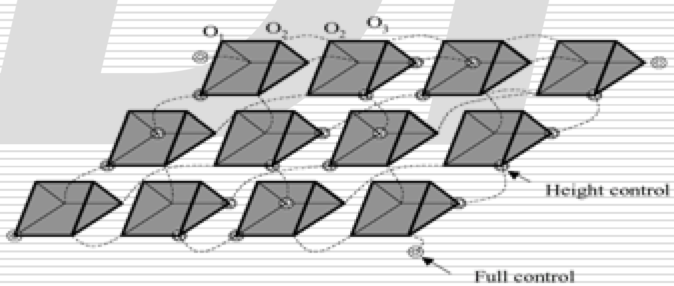
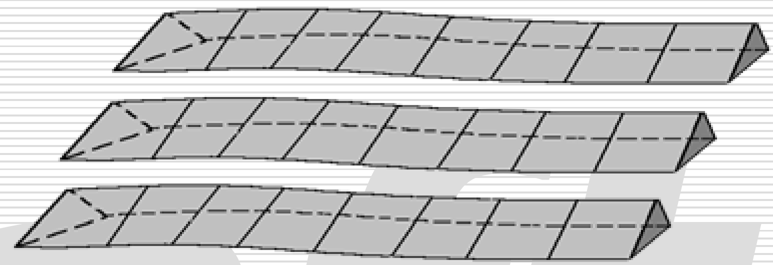
Basic Concepts

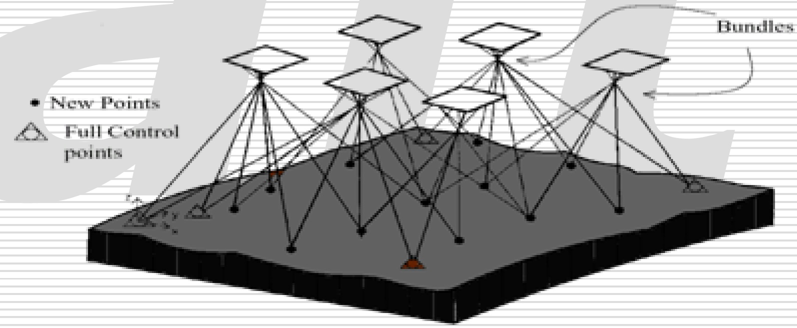
Block of Images

Block adjustment (Triangulation) is a general term for photogrammetric methods of coordinating points on the object using a series of overlapping photographs.









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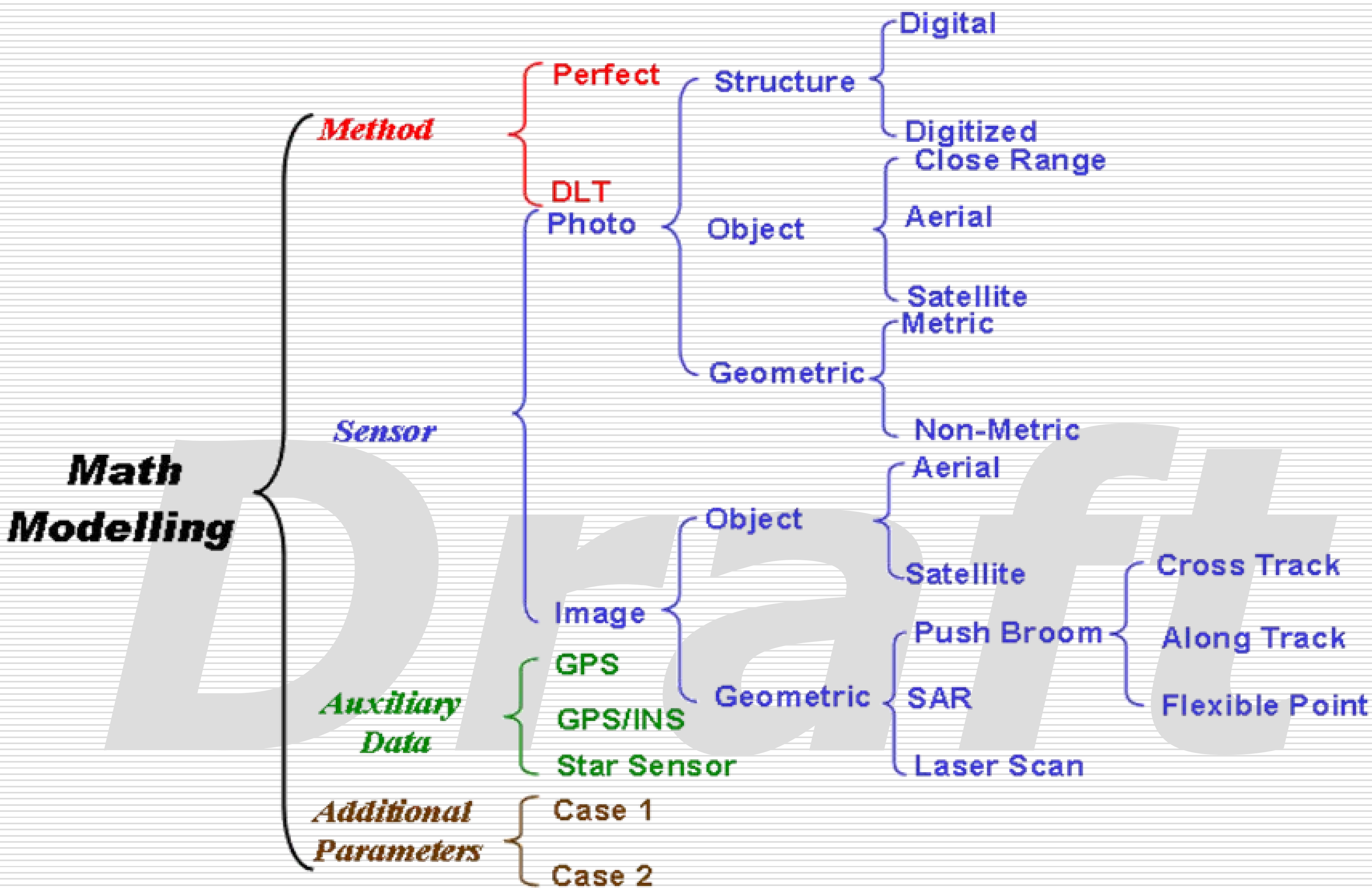
Basic Concepts



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Photogrammetric Block Adjustment

Objective

– How can we reconstruct the object space from imagery without the need for three ground control points in each stereo-model?

Alternatives

– Strip Adjustment .

– Block Adjustment of Independent Models (BAIM).

– Bundle Block Adjustment.

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What is B.A.?

Objectives

- Establish ground control points (GCP) for absolute orientation of stereo models
- Determine 3D points in object space
  - Cataster
  - Road design
- Determine exterior orientation of images and stereo models

Background

- Concept of models, strips, and blocks
- Methods
  - Analog
  - Analytical
    1. Polynomial strip adjustment
    2. Independent model method
    3. Bundle block adjustment

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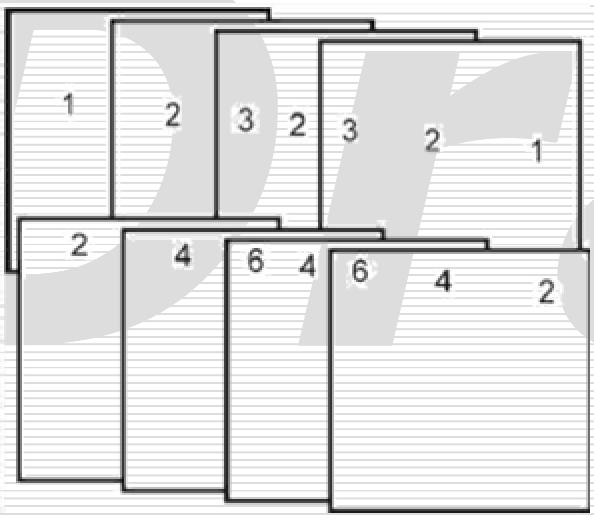
What is B.A.?

Concept of models, strips, and blocks

Ideal Overlap Configuration



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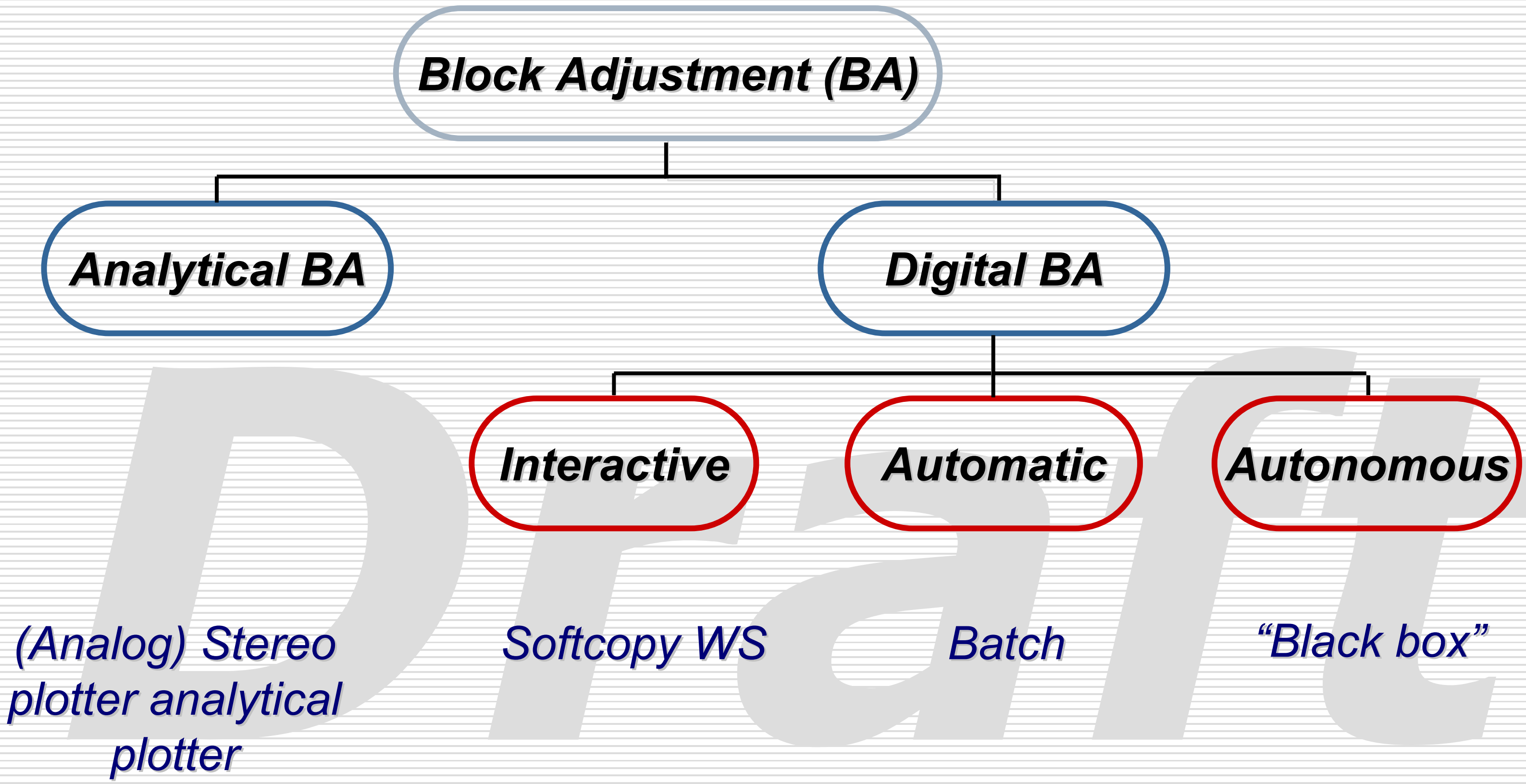
Taxonomy

Is it digital, automatic, autonomous, or classical B.A.?



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Applications

- **Point Densification**
  - For absolute orientation of stereo models
  - For engineering projects
    - – road design
  - For cadastre

- **Determine Exterior Orientation**
  - Import EXOR in analytical plotters for
  - Map compilation

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Is B.A. still necessary?

Direct Orientation

- Modern airborne data acquisition systems are equipped with precise navigation tools: GPS and INS
- GPS gives coordinates of projection center to an accuracy of ~ ??
- INS gives attitude (3 angles) to an accuracy of ~ ??
- **Isn't this good enough for orientation?**

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Is B.A. still necessary?

Some Considerations

- **Intricate relationship between interior and exterior orientation**
  - Errors of IO propagate to object space, e.g. error in focal length Δc causes elevation error: $\Delta h = s \times \Delta c$
  - Errors of IO are (almost) compensated by indirect orientation
- **Direct orientation gives no information about object space**

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Typical Workflow of B.A. Project

- Planning
- Establish Control Points
- Preparation
- Point Transfer
- Measurements
- Block Adjustment
- Analysis

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Typical Workflow of B.A. Project

Planning

- **Optimal design of flight lines to**
  - Assure overlap (60% forward, 20% strip overlap
  - Minimize flying time
  - Consider across flight lines
  - Take existing GCPs into account
- **Flight time Restrictions**
- **Weather**

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Typical Workflow of B.A. Project

Ground Control Points (GCP)

- **Select GCPs based on**
  - *Accuracy considerations (e.g. along block boundaries)*
  - *Minimum number of points (economical reasons)*
  - *Measuring techniques*
  - *Marking GCPs semi-permanently*
- **Timing**
  - *Before or after data acquisition*

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Typical Workflow of B.A. Project

Establishing Control Points

- Distribution and quality of control points greatly affects accuracy
- Planimetric control points along periphery
- Elevation control points regularly distributed within block
- GPS on airplane reduces number of ground control points

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Typical Workflow of B.A. Project

Preparations

- Check data acquisition (quality, overlap, etc.)
- Prepare photo mosaic
- Identify GCPs
- Select suitable tie points
- Uniquely annotate points
  - Same point in object space appears on several images → should have same label

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Typical Workflow of B.A. Project

Point Transfer

- **Selected tie points must be measured on all images where tie point occurs**
  - *Tie point may not be identifiable on all images*
  - *On stereo measuring systems tie point must be identified on one image only*
- **Most precise solution: identify natural points and make sketches to aid ientification**
- **Most economical solution: mark 3 points in the center of image**

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Typical Workflow of B.A. Project

Measurements

- **Photo coordinates on**
  - *comparators (mono or stereo)*
  - *analytical plotters*

- **Model coordinates on**
  - *(analog) stereo plotters*
  - *analytical plotters*
  - *compuationally from photo coordinates by relative orientation*

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Typical Workflow of B.A. Project

Adjustment

- **Strip adjustment**
  - strip coordinates measured on stereo plotter (base in/base out capability) or on analytical plotters
  - strip adjusted by polynomials

- **Block adjustment**
  - independent models
  - bundle block adjustment

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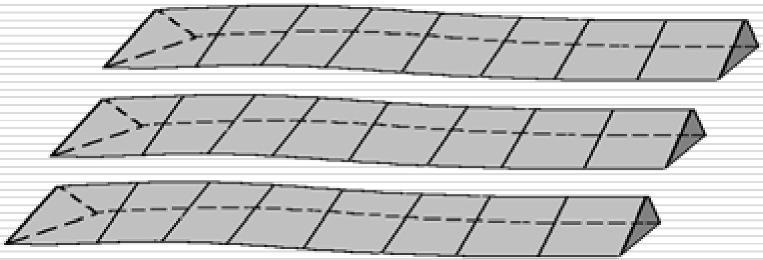
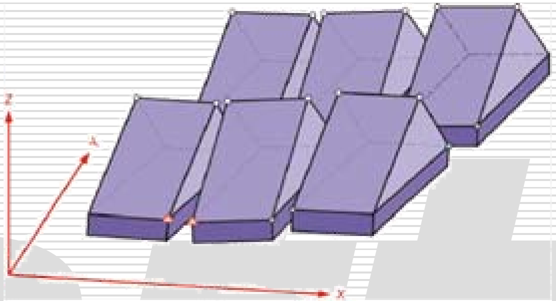
Typical Workflow of B.A. Project

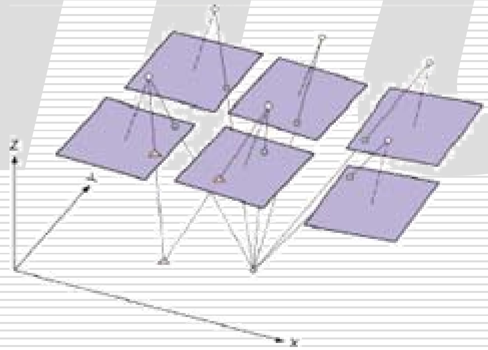
Concept of Strip Adjustment



Concept of Independent Models



Concept of Bundle Block Adjustment



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Typical Workflow of B.A. Project

Analysis

- **Check block adjustment**
  - blunders
  - connection of models or images

- **Error analysis**
  - variance component (so)
  - errors on control points
  - variance/covariance matrix

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What factors determine outcome of B.A.?

Quality Factors

- **Instruments**
  - data acquisition
    - Camera
    - Film
    - navigation aids
- **point marking and point transfer**
- **measurements of model- or photo coordinates**

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What factors determine outcome of B.A.?

Quality Factors Data Acquisition

- Metric camera (stable IO, distortions known → calibration)
- Image motion compensation
- Platform stabilizer
- Navigation aids
  - Improve flight accuracy
  - Use navigation data in adjustment

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What factors determine outcome of B.A.?

Quality Factors Data Acquisition

- **Film**
  - base material → geometric stability
  - emulsion → resolution (important for identifying GCPs!)

- **Digital cameras**
  - frame cameras
  - pushbroom cameras

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What factors determine outcome of B.A.?

Quality Factors

- **Control points**
  - Distribution
  - Accuracy
  - Identification
- **Methods**
  - Strip adjustment
  - Block adjustment
    - Independent models
    - Bundle block adjustment

Quality Factors

- Working procedures
- Experience
- Special expertise

If all goes well then single most critical factor are the tie points

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What factors determine outcome of B.A.?

Quality Control

- **point accuracy in object space depends on:**
  - Control points
    - Quality
    - Density
    - Distribution
  - Tie points
    - Degree of connection
    - Number of tie points
  - $\sigma_P = \sigma_o \; sqrt\,(Q_{xx})$

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What factors determine outcome of B.A.?

Quality Control

- **Point accuracy is measured by**
  - $\sigma_P = \sigma_o \ \mathrm{sqrt}\,(Q_{xx})$
  - σ_o : variance component
    - Reflects measuring error
      - 3 μ to 10 μ under good conditions
  - Q_{xx} : Cofactor
    - Depends on geometry of block
      - ~ 1 if control points along boundary at distance of one base (in everyn model)

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Summary

- **AT very effective method to**
  - Determine exterior orientation to set up models for map compilation
  - Establish points in object space at high accuracy and low cost

- **Cost per model: 30 to 50 $ by service organizations**

- **Time per model: 10 to 20 minutes**

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Flight Planning

- Optimal design of flight lines to
  - Assure overlap (60% forward, 20% strip overlap
  - Minimize flying time
  - Cconsider across flight lines
  - Take existing GCPs into account
- Flight time restrictions
- Weather

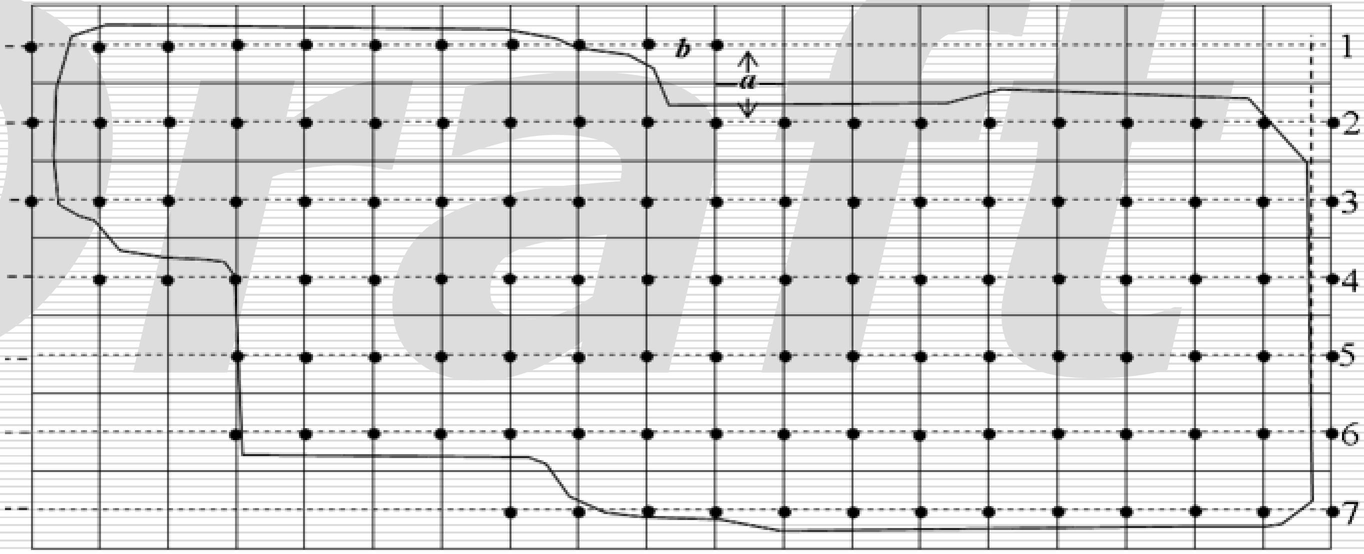

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Image Scale

The image or photo scale depends on the aim of the photo flight.

Photo Flight for Plotting of Topographic Maps

Accuracy as well as economy have to be taken into account.

If standard equipment is used, the image scale figure can be derived using the formula:

$$m_i = 200 \cdot \sqrt{m_m} \quad \textbf{(1)}$$

Where

m_i = image scale figure

m_m = map scale figure

If newly developed and modern equipment with forward motion compensation device is used the image scale figure can be calculated according to:

$$m_i = 300...350 \cdot \sqrt{m_m} \quad \textbf{(2)}$$

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Image Scale

Photo flight focusing on particular accuracies of the restitution

Good Planimetric Accuracy of the Photogrammetric Restitution

The planimetric accuracy depends on the image scale and on the accuracy with which points can be measured in the images. Normally an accuracy of

$$\sigma'_{xy} = \pm 0.005 \text{ mm}$$

can be achieved.

Related to the ground we then get

$$\sigma_{xy}[m] = \pm \sigma'_{xy} \ [mm] \cdot \frac{m_i}{1000} \qquad (3)$$

And for the image scale

$$m_i = \frac{\sigma_{xy}}{\sigma'_{xy}} \cdot 1000 \qquad (4)$$

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Image Scale

Photo flight focusing on particular accuracies of the restitution

Height Accuracy of the Photogrammetric Restitution

The height accuracy can be derived on the basis of the z-equation

$$z = -\frac{b_x}{X'_l - X'_r} \cdot c = -\frac{b_x}{P'_x} \cdot c \qquad (5)$$

assuming: the base b_x and the principal distance c as constants:

$$\frac{dz}{dp'_x} = \frac{b_x \cdot c}{p'^{\,2}_x} \Rightarrow dz = \frac{b_x \cdot c}{p'^{\,2}_x} \cdot dp'_x \qquad (6)$$

And from (5)

$$p'^{\,2}_x = \frac{b_x^2 \cdot c^2}{z^2} \qquad (7)$$

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Image Scale

Photo flight focusing on particular accuracies of the restitution

Height Accuracy of the Photogrammetric Restitution

Substitution of (7) into (6) and transition to the standard deviation yields

$$\sigma_z = \pm \frac{b_x \cdot c \cdot z^2}{b_x^{\,2} \cdot c^2} \cdot \sigma_{px'} \quad \Rightarrow \quad \sigma_z = \pm \frac{z^2}{b_x . c} \sigma_{px'} \qquad (8)$$

For the further derivation we replace z by the relative flight height above ground h_g b_x by the flight base b:

$$\boxed{\sigma_z = \pm \frac{{h_g}^2}{b \cdot c} \sigma_{px'}} \qquad (9)$$

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Image Scale

Photo flight focusing on particular accuracies of the restitution

Height Accuracy of the Photogrammetric Restitution

The relative height accuracy then reads

$$\frac{\sigma_z}{h_g} = \pm \frac{h_g}{b} \cdot \frac{1}{c} \sigma_{px'}$$

The flight base reads

$$b = d' \cdot \frac{h_g}{c} \left(1 - \frac{p}{100} \right) \qquad (10)$$

With p = 60 we get

$$b = d' \cdot 0.4 \cdot \frac{h_g}{c} \qquad \frac{b}{h_g} = d' \cdot 0.4 \cdot \frac{1}{c} \qquad (11)$$

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Image Scale

Photo flight focusing on particular accuracies of the restitution

Height Accuracy of the Photogrammetric Restitution

$\frac{b}{h_g}$ is named base-altitude ratio

$$\frac{h_g}{b} = \frac{c}{d' \cdot 0.4} \tag{12}$$

Substitution of (12) into (10) yields

$$\frac{\sigma_z}{h_g} = \frac{\sigma_{px'}}{d' \cdot 0.4} \tag{13}$$

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Image Scale

Photo flight focusing on particular accuracies of the restitution

Height Accuracy of the Photogrammetric Restitution

Setting for the standard deviation of the x'-parallax ±0.008 mm and for d'=230 mm we get as a result for the relative accuracy of the photogrammetric height measurement of well defined points

$$\frac{\sigma_z}{h_g} = \pm 0.00008 = \pm 0.87 \text{‰} \qquad (14)$$

$$\sigma_z = \pm 0.09 \text{‰} \cdot h_g \qquad (15)$$

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Image Scale

Photo flight focusing on particular accuracies of the restitution

Height Accuracy of the Photogrammetric Restitution

Therefore for a photo flight focusing on good height accuracy (e.g. for road construction purposes) the photo scale can be derived on the basis of (15):

$$\sigma_z = 0.09 \circ/\circ\circ . h_g \quad \Rightarrow \quad \boxed{h_g = \frac{\sigma_z \cdot 1000}{0.087} = \sigma_z \cdot 11500} \qquad \textbf{(16)}$$

and

$$\boxed{m_i = \frac{h_g[m] \cdot 1000}{c[mm]}} \qquad \textbf{(17)}$$

where

- ***σ_z**= required standard deviation of the z-co-ordinates*
- ***h_g** = flight height above ground*
- ***c** = calibrate focal length of the aerial camera*

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Image Scale

Photo flight focusing on particular accuracies of the restitution

Height Accuracy of the Photogrammetric Restitution

and

$$m_i = \frac{h_g[m] \cdot 1000}{c[mm]} \qquad (17)$$

where

σ_z= required standard deviation of the z-co-ordinates

h_g = flight height above ground

c = calibrate focal length of the aerial camera

Draft

The Flight Height above Ground h_g

$$h_g = \frac{c[mm] \cdot m_i}{1000} \qquad (18)$$

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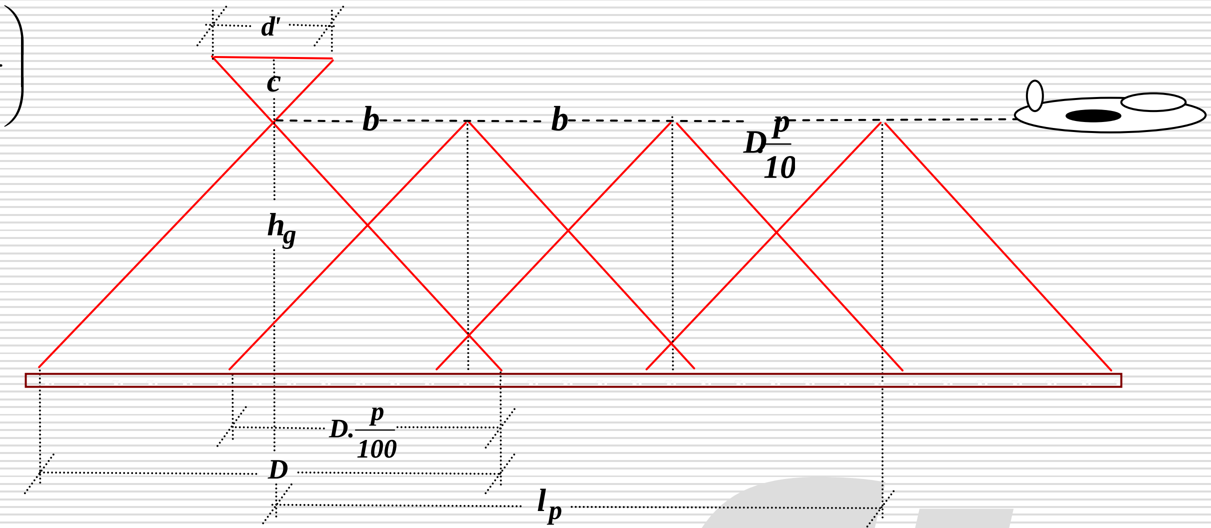
The Flight Based b

$$b = D - D \cdot \frac{p}{100} = D \cdot \left(1 - \frac{p}{100}\right)$$



with

$$D = d' \cdot m_i = d' \cdot \frac{h_g}{c}$$

$$\boxed{b = d' \cdot \frac{h_g}{c} \cdot \left(1 - \frac{p}{100}\right)} \quad (19)$$

where

- d' = Format of the photograph (230 mm)
- p = Percentage of end lap (for stereo restitution $p \approx 60$)
- h_g = Flight height above ground
- c = Calibrated focal length (principal distance)

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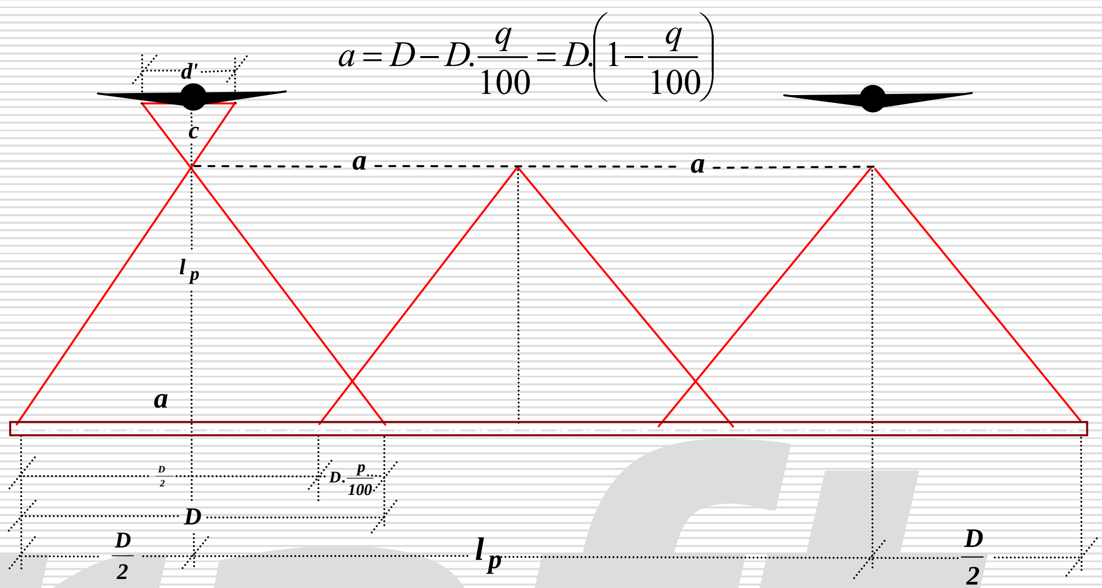
The Strip Distance

(distance between flight axes with q % side lap)

$$a = D - D.\frac{q}{100} = D.\left(1 - \frac{q}{100}\right)$$



with $D = d' \cdot m_i = d' \cdot \frac{h_g}{c}$

$$a = d'.\frac{h_g}{c}.\left(1 - \frac{p}{100}\right) \quad \textbf{(20)}$$

where

- d' = Format of the photograph (230 mm)
- p = Percentage of end lap (for stereo restitution $p \approx 60$
- h_g = Flight height above ground
- c = Calibrated focal length (principal distance)

Remark: In order to avoid gaps of the coverage flight base and strip distance have to be referred to the highest ground level

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Area Covered by a Single Photograph

$$A_g = d'^2 . m_i^2 \qquad (21)$$

Area of the Net Model (new stereoscopic area)

$$A_n = a \cdot b = D^2 \cdot \left(1 - \frac{p}{100}\right) \cdot \left(1 - \frac{p}{100}\right)$$

$$A_n = d'^2 \cdot m_i^2 \left(1 - \frac{p}{100}\right) \cdot \left(1 - \frac{q}{100}\right) \qquad (22)$$

Number of Photographs per Strip

$$n_p = \frac{l_p}{b} + 1 \qquad (23)$$

l_p = Length of Flight strip
b = Flight Base

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Number of Flight Strip

l_q = Width of Area to be covered
a = Distance between Flight Axes
D = Photo Format related to Ground

$$n_q = \frac{l_q - D}{a} + 1 \qquad (24)$$

Required Number of Photographs

Rectangular Shape of the Area to be covered

$$n_i = n_p \,.\; n_q \qquad (25)$$

In case of an irregular shape the number of photographs has to be determined stripwise

Approximation

$$n_i \approx \frac{A_{tot}}{A_n} \qquad (26)$$

A_{tot} = Total Area to be covered stereoscopically
A_n = Area of the Net Model

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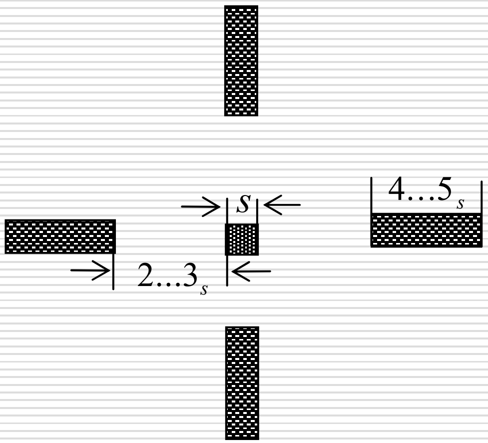
Signalization (Size s and Shape of Signals) of GCP

$$s \geq \frac{m_i}{400} \quad [cm] \qquad (27)$$



s = Signal Side
m_i = Photo scale

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Classification of Points

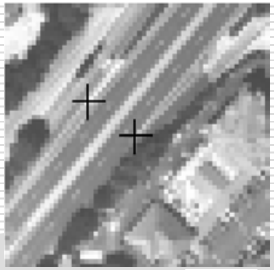
According to identification

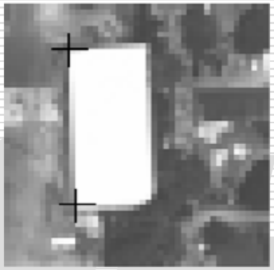
1- Signalized point

Example:

2- Natural point

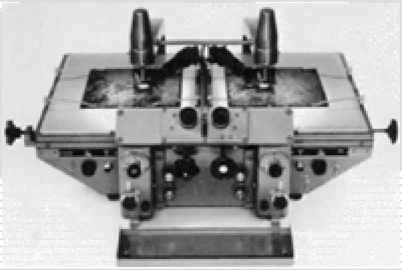
Example: Building corner





3- Artificial point

Example: tie points



Point Transfer Devices

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Classification of Points

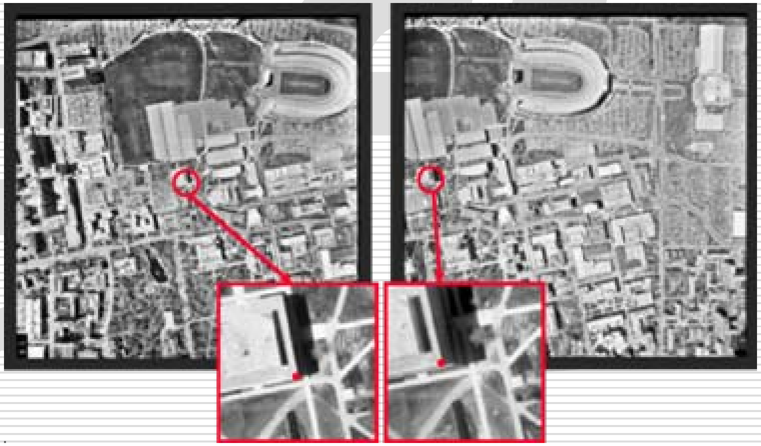
According to their use in adjustment

1- Control points as measured in strip, model or photo.

points whose ground coordinates are available from geodetic measurements (e.g. GPS). They are used to define the datum during the adjustment.

2- Tie points which appear at least in two units.

Their function is to tie together overlapping images.They should be well defined in the images. Their ground coordinates are determined through photogrammetric adjustment.



3- Single points which appear only in one unit.

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Photogrammetric Block Adjustment

Bundle Block Adjustment

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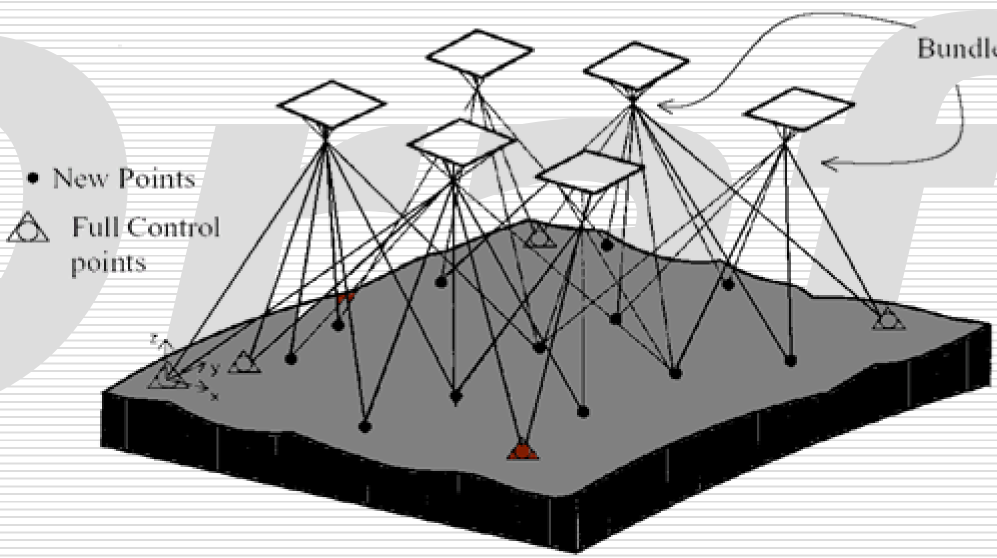
Bundle Adjustment - Introduction

In this method of block adjustment, the position (X_0 , Y_0 , Z_0) and rotations (ω, φ, κ) of all bundles in the block are adjusted in space simultaneously to achieve the best possible intersection of all conjugate rays of each point on the ground (X , Y , Z) and fit to the given control points.

Each light ray is assumed to be a straight line passing through the projection center, image point and terrain point



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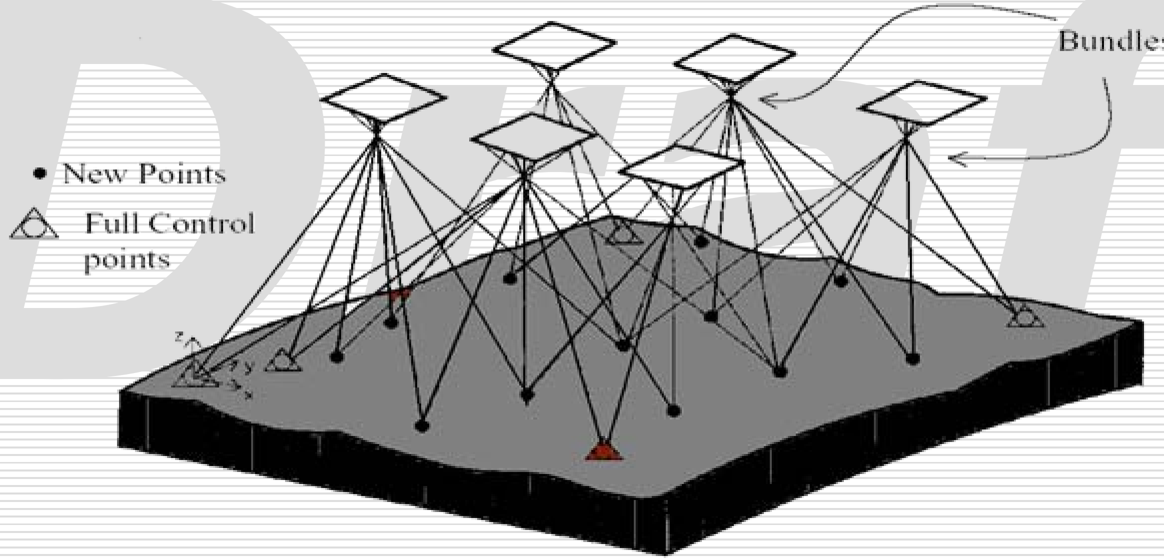
Bundle Adjustment - Relations Between Image and Ground Coordinates

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \lambda_p \cdot R \cdot \begin{bmatrix} x \\ y \\ -c \end{bmatrix} + \begin{bmatrix} X_0 \\ Y_0 \\ Z_0 \end{bmatrix}$$



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Bundle Adjustment Observation Equation

$$v_{x_{ij}} + x_{ij} = -c \cdot \left[\frac{a_1^j \left(X_i - X_o^j\right) + a_4^j \left(Y_i - Y_o^j\right) + a_7^j \left(Z_i - Z_o^j\right)}{a_3^j \left(X_i - X_o^j\right) + a_6^j \left(Y_i - Y_o^j\right) + a_9^j \left(Z_i - Z_o^j\right)} \right]$$

$$v_{y_{ij}} + y_{ij} = -c \cdot \left[\frac{a_2^j \left(X_i - X_o^j\right) + a_5^j \left(Y_i - Y_o^j\right) + a_8^j \left(Z_i - Z_o^j\right)}{a_3^j \left(X_i - X_o^j\right) + a_6^j \left(Y_i - Y_o^j\right) + a_9^j \left(Z_i - Z_o^j\right)} \right]$$

x_{ij}, y_{ij} : Photo coordinates of point I in photo (j)

X_0^j, Y_0^j, Z_0^j : Coordinates of projection center (j)

$a_1^j, \ldots, a_9^j$: are the elements of point (i)

X_i, Y_i, Z_i : Terrain coordinates of point (i)

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Bundle Adjustment - Linearization of the Observation Equations

According to Taylor expansion the linearized observation equations may be obtained as follows:

$$v_{x_{ij}} + x_{ij} = f_x^{(0)} + \left(\frac{\partial f_x}{\partial \omega_j}\right)^{(0)} \Delta\omega_j + \left(\frac{\partial f_x}{\partial \varphi_j}\right)^{(0)} \Delta\varphi_j + \left(\frac{\partial f_x}{\partial \kappa_j}\right)^{(0)} \Delta\kappa_j + \left(\frac{\partial f_x}{\partial X_{0j}}\right)^{(0)} \Delta X_{0j} + \left(\frac{\partial f_x}{\partial Y_{0j}}\right)^{(0)} \Delta Y_{0j} + \left(\frac{\partial f_x}{\partial Z_{0j}}\right)^{(0)} \Delta Z_{0j}$$
$$+ \left(\frac{\partial f_x}{\partial X_j}\right)^{(0)} \Delta X_j + \left(\frac{\partial f_x}{\partial Y_j}\right)^{(0)} \Delta Y_j + \left(\frac{\partial f_x}{\partial Z_j}\right)^{(0)} \Delta Z_j$$

$$v_{y_{ij}} + y_{ij} = f_y^{(0)} + \left(\frac{\partial f_y}{\partial \omega_j}\right)^{(0)} \Delta\omega_j + \left(\frac{\partial f_y}{\partial \varphi_j}\right)^{(0)} \Delta\varphi_j + \left(\frac{\partial f_y}{\partial \kappa_j}\right)^{(0)} \Delta\kappa_j + \left(\frac{\partial f_y}{\partial X_{0j}}\right)^{(0)} \Delta X_{0j} + \left(\frac{\partial f_y}{\partial Y_{0j}}\right)^{(0)} \Delta Y_{0j} + \left(\frac{\partial f_y}{\partial Z_{0j}}\right)^{(0)} \Delta Z_{0j}$$
$$+ \left(\frac{\partial f_y}{\partial X_j}\right)^{(0)} \Delta X_j + \left(\frac{\partial f_y}{\partial Y_j}\right)^{(0)} \Delta Y_j + \left(\frac{\partial f_y}{\partial Z_j}\right)^{(0)} \Delta Z_j$$

Notice that both equations have been differentiated with respect to the 6 orientation parameters of the bundle and 3 coordinates of the tie points

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Bundle Adjustment - Linearization of the observation equations

The linearized equations can be written in matrix form as follows:

$$\begin{bmatrix} Ax_1 & Ax_2 & Ax_3 & Ax_4 & Ax_5 & Ax_6 \\ Ay_1 & Ay_2 & Ay_3 & Ay_4 & Ay_5 & Ay_6 \end{bmatrix}_{ij} \cdot \begin{bmatrix} \Delta\omega \\ \Delta\varphi \\ \Delta\kappa \\ \Delta X_0 \\ \Delta Y_0 \\ \Delta Z_0 \end{bmatrix}_j + \begin{bmatrix} Bx_1 & Bx_2 & Bx_3 \\ By_1 & By_2 & By_3 \end{bmatrix}_{ij} \cdot \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix} = \begin{bmatrix} r_x \\ r_y \end{bmatrix}_{ij}$$

$$A_{ij} \cdot \Delta P_j + B_{ij} \cdot \Delta C_i = R_{ij}$$

- *The coefficients of matrix A_{ij} (2,6) are the partial derivatives of the 2 'colinearity' equations with respect to the 6 unknown parameters X_0, Y_0, Z_0, ω, φ, κ*

The coefficients of matrix A_{ij} (2,3) are the partial derivatives with respect to the 3 unknown Pt cords: X, Y, Z

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Approximate values of the unknowns

The bundle adjustment is an iterative procedure because we use linearized observation equations. The number of iterations depends on how good we know before hand the approximate value of the unknowns.

(i) $\overline{X}_0, \overline{Y}_0, \overline{Z}_0$, $\overline{\omega}, \overline{\phi}, \overline{\mathrm{K}}$ for each bundle.

(ii) $\overline{X}, \overline{Y}, \overline{Z}$ coordinates of points on the ground.

Ackermann uses M-4 adjustment.

(1) $\overline{X}, \overline{Y}$ all points obtained from the adjustment.

(2) For each photo 4 transformation parameters: (a, b, C_X, C_Y) and

$$\overline{X}_0 = C_x$$

$$\overline{Y}_0 = C_y$$

(3) $\overline{\mathrm{K}} = \tan^{-1} = \dfrac{b}{a}$

(4) $\overline{\omega} = \overline{\phi} = 0$

(5) $\overline{Z}_0 =$ Flying height

(6) $\overline{Z} =$ mean ground level

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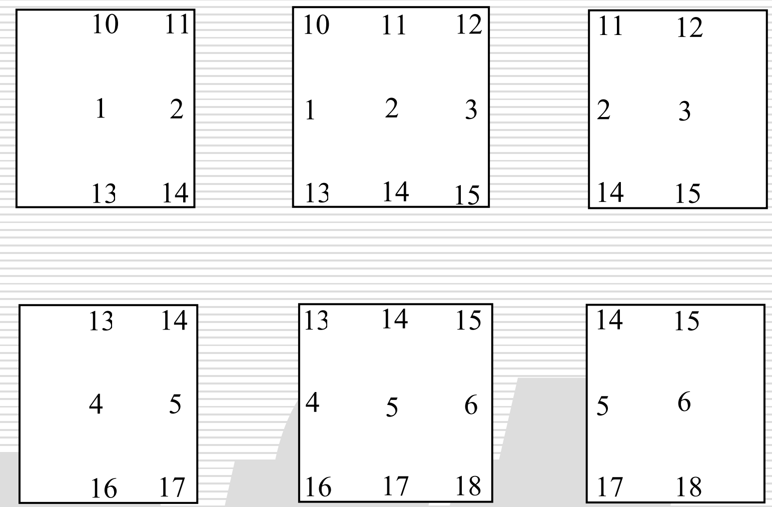
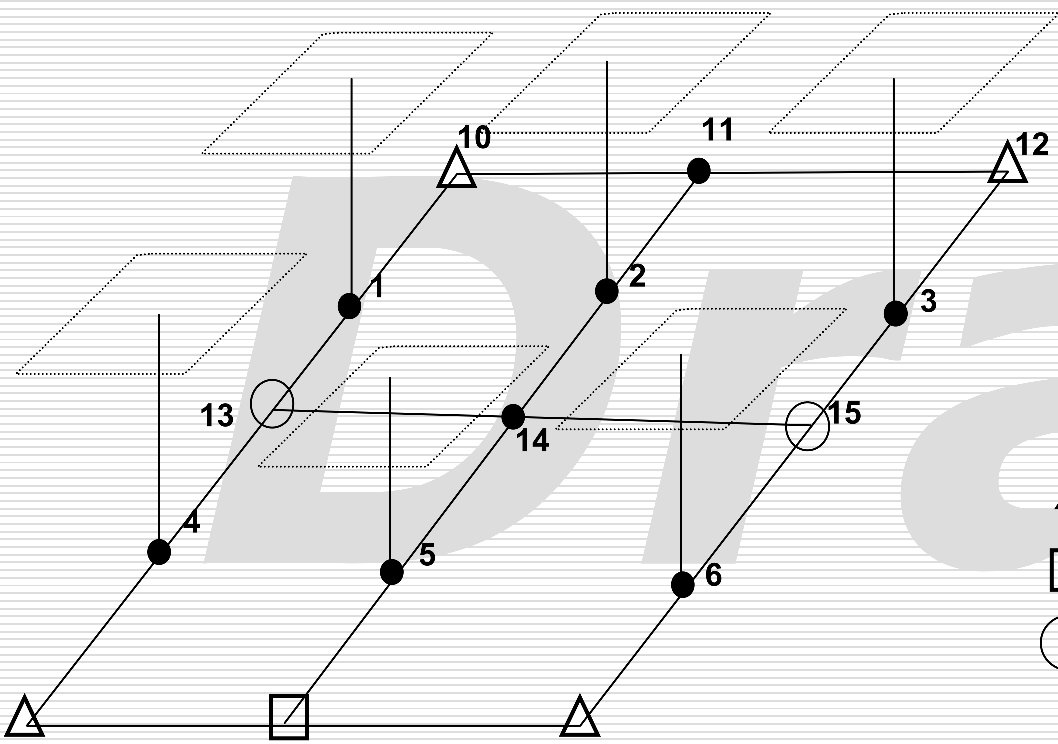
Example for Bundle Adjustment

Given the following small block of 6 bundles with ground control distribution as shown in the figure below: (Notice that terrestrial coordinates of control points are error free.)





△ ***Full Control Point (X,Y,Z)***

□ ***Planimetric Control Point (X,Y)***

○ ***Altimetric Control Point (Z)***

● ***Tie and Single Points***

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Example for Bundle Adjustment

The unknowns

(i) Parameters:

P= 6 bundle × parameters=36

(ii) Unknown coordinates

8 tie point × 3 coordinates= 24

2 height control × 2 = 4

1 plan control × 1 = 1

Total = 36 + 29 = 65

The number of observation equations

Observations equation= (6+9+6+6+9+6) × 2 =84

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Example for Bundle Adjustment

The observation equation

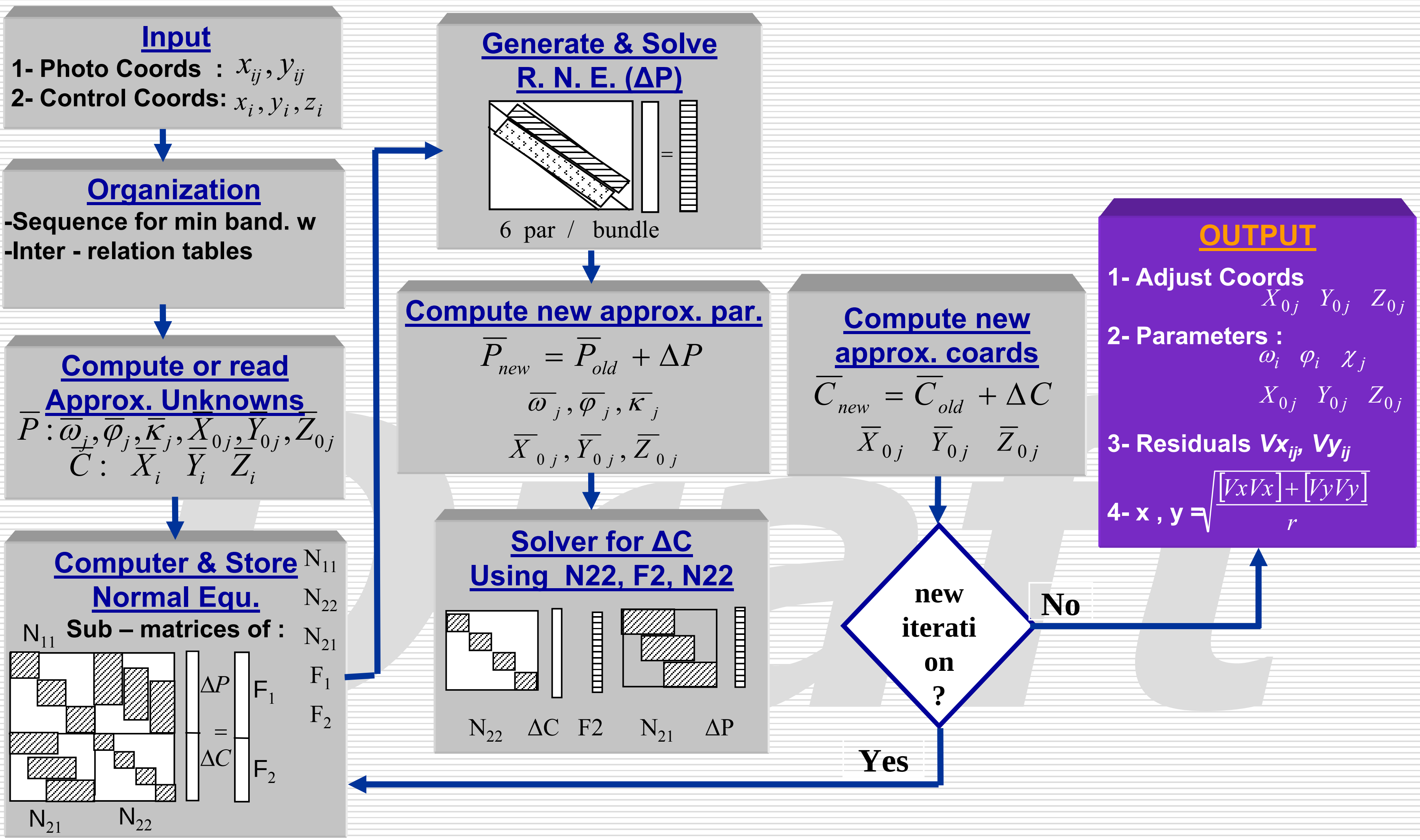
P^1	P^2	P^3	P^4	P^5	P^6	C_1	C_2	C_3	C_4	C_5	C_6	C_{11}	C_{14}	C_{13}	C_{15}	C_{17}	R.H.S.
A_{10}^1																	R_{10}^1
A_{11}^1												B_{11}^1					R_{11}^1
A_1^1						B_1^1											R_1^1
A_2^1							B_2^1										R_2^1
A_{13}^1														B_{13}^1			R_{13}^1
A_{14}^1													B_{14}^1				R_{14}^1
	A_{10}^2																R_{10}^2
	A_{11}^2											B_{11}^2					R_{11}^2
	A_{12}^2																R_{12}^2
	A_1^2					B_1^2											R_1^2
	A_2^2						B_2^2										R_2^2
	A_3^2							B_3^2									R_3^2
	A_{13}^2													B_{13}^2			R_{13}^2
	A_{14}^2												B_{14}^2				R_{14}^2
	A_{15}^2														B_{15}^2		R_{15}^2
		A_{11}^3										B_{11}^3					R_{11}^3
		A_{12}^3															R_{12}^3
		A_2^3					B_2^3										R_2^3
		A_3^3						B_3^3									R_3^3
		A_{14}^3											B_{14}^3				R_{14}^3
		A_{15}^3													B_{15}^3		R_{15}^3
			A_{13}^4											B_{13}^4			R_{13}^4
			A_{14}^4										B_{14}^4				R_{14}^4
			A_4^4						B_4^4								A_4^4
			A_5^4							B_5^4							R_5^4
			A_{16}^4														R_{16}^4
			A_{17}^4													B_{17}^4	R_{17}^4
				A_{13}^5										B_{13}^5			R_{13}^5
				A_{14}^5									B_{14}^5				R_{14}^5
				A_{15}^5											B_{15}^5		R_{15}^5
				A_4^5					B_4^5								R_4^5
				A_5^5						B_5^5							R_5^5
				A_6^5							B_6^5						R_6^5
				A_{16}^5													R_{16}^5
				A_{17}^5													R_{17}^5
				A_{18}^5												B_{17}^5	R_{18}^5
					A_{14}^6								B_{14}^6				R_{14}^6
					A_{15}^6										B_{15}^6		R_{15}^6
					A_5^6					B_5^6							R_5^6
					A_6^6						B_6^6						R_6^6
					A_{17}^6											B_{17}^6	R_{17}^6
					A_{18}^6												R_{18}^6

Example for Bundle Adjustment

The normal equations

p^1	p^2	p^3	p^4	p^5	p^6	C_1	C_2	C_3	C_4	C_5	C_6	C_{11}	C_{14}	C_{13}	C_{15}	C_{17}	R.H.S.
$\Sigma A_i^{1T} A_i^1$						$A_1^{1T} B_1^1$	$A_2^{1T} B_2^1$					$A_{11}^{1T} B_{11}^1$	$A_{14}^{1T} B_{14}^1$	$A_{13}^{1T} B_{13}^1$			$\Sigma A_i^{1T} R_i^1$
	$\Sigma A_i^{2T} A_i^2$					$A_1^{2T} B_1^2$	$A_2^{2T} B_2^2$	$A_3^{2T} B_3^2$				$A_{11}^{2T} B_{11}^2$	$A_{14}^{2T} B_{14}^2$	$A_{13}^{2T} B_{13}^2$	$A_{15}^{2T} B_{15}^2$		$\Sigma A_i^{2T} R_i^2$
		$\Sigma A_i^{3T} A_i^3$					$A_2^{3T} B_2^3$	$A_3^{3T} B_3^3$				$A_{11}^{3T} B_{11}^3$	$A_{14}^{3T} B_{14}^3$		$A_{15}^{3T} B_{15}^3$		$\Sigma A_i^{3T} R_i^3$
			$\Sigma A_i^{4T} A_i^4$						$A_4^{4T} B_4^4$	$A_5^{4T} B_5^4$			$A_{14}^{4T} B_{14}^4$	$A_{13}^{4T} B_{13}^4$		$A_{17}^{4T} B_{17}^4$	$\Sigma A_i^{4T} R_i^4$
				$\Sigma A_i^{5T} A_i^5$					$A_4^{5T} B_4^5$	$A_5^{5T} B_5^5$	$A_6^{5T} B_6^5$		$A_{14}^{5T} B_{14}^5$	$A_{13}^{5T} B_{13}^5$	$A_{15}^{5T} B_{15}^5$	$A_{17}^{5T} B_{17}^5$	$\Sigma A_i^{5T} R_i^5$
					$\Sigma A_i^{6T} A_i^6$					$A_5^{6T} B_5^6$	$A_6^{6T} B_6^6$		$A_{14}^{6T} B_{14}^6$		$A_{15}^{6T} B_{15}^6$	$A_{17}^{6T} B_{17}^6$	$\Sigma A_i^{6T} R_i^6$
$B_1^{1T} A_1^1$	$B_1^{2T} A_1^2$					$\Sigma B_1^{jT} B_1^j$											$\Sigma B_1^{jT} R_1^j$
$B_2^{1T} A_2^1$	$B_2^{2T} A_2^2$	$B_2^{3T} A_2^3$					$\Sigma B_2^{jT} B_2^j$										$\Sigma B_2^{jT} R_2^j$
	$B_3^{2T} A_3^2$	$B_3^{3T} A_3^3$						$\Sigma B_3^{jT} B_3^j$									$\Sigma B_3^{jT} R_3^j$
			$B_4^{4T} A_4^4$	$B_4^{5T} A_4^5$					$\Sigma B_4^{jT} B_4^j$								$\Sigma B_4^{jT} R_4^j$
			$B_5^{4T} A_5^4$	$B_5^{5T} A_5^5$	$B_5^{6T} A_5^6$					$\Sigma B_5^{jT} B_5^j$							$\Sigma B_5^{jT} R_5^j$
				$B_6^{5T} A_6^5$	$B_6^{6T} A_6^6$						$\Sigma B_6^{jT} B_6^j$						$\Sigma B_6^{jT} R_6^j$
$B_{11}^{1T} A_{11}^1$	$B_{11}^{2T} A_{11}^2$	$B_{11}^{3T} A_{11}^3$										$\Sigma B_{11}^{jT} B_{11}^j$					$\Sigma B_{11}^{jT} R_{11}^j$
$B_{14}^{1T} A_{14}^1$	$B_{14}^{2T} A_{14}^2$	$B_{14}^{3T} A_{14}^3$	$B_{14}^{4T} A_{14}^4$	$B_{14}^{5T} A_{14}^5$	$B_{14}^{6T} A_{14}^6$								$\Sigma B_{14}^{jT} B_{14}^j$				$\Sigma B_{14}^{jT} R_{14}^j$
$B_{13}^{1T} A_{13}^1$	$B_{13}^{2T} A_{13}^2$		$B_{13}^{4T} A_{13}^4$	$B_{13}^{5T} A_{13}^5$										$\Sigma B_{13}^{jT} B_{13}^j$			$\Sigma B_{13}^{jT} R_{13}^j$
	$B_{15}^{2T} A_{15}^2$	$B_{15}^{3T} A_{15}^3$		$B_{15}^{5T} A_{15}^5$	$B_{15}^{6T} A_{15}^6$										$\Sigma B_{15}^{jT} B_{15}^j$		$\Sigma B_{15}^{jT} R_{15}^j$
			$B_{17}^{4T} A_{17}^4$	$B_{17}^{5T} A_{17}^5$	$B_{17}^{6T} A_{17}^6$											$\Sigma B_{17}^{jT} B_{17}^j$	$\Sigma B_{17}^{jT} R_{17}^j$

Flow Diagram of Bundle I.M Block Adjustment



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Bundle Adjustment

Advantages of Bundle Block Adjustment

- Most accurate triangulation technique since we have direct transformation between image and ground coordinates.
- Straight forward to include parameters that compensate for various deviations from the collinearity model.
- Straight forward to include additional observations:
  - GPS observations at the exposure stations.
  - Object space distances.
- Can be used for normal, convergent, aerial, and close range imagery.
- After the adjustment, the EOP can be set of analogue and analytical plotters for compilation purposes.

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Bundle Adjustment

Disadvantages of Bundle Block Adjustment

- Model is non linear: approximations as well as partial derivatives are needed.
- Requires computer intensive computations.
- Analogue instruments can not be used (they cannot measure image coordinate measurements).
- The adjustment cannot be separated into planimetric and vertical adjustment.

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Photogrammetric Block Adjustment

Independent Models Block Adjustment

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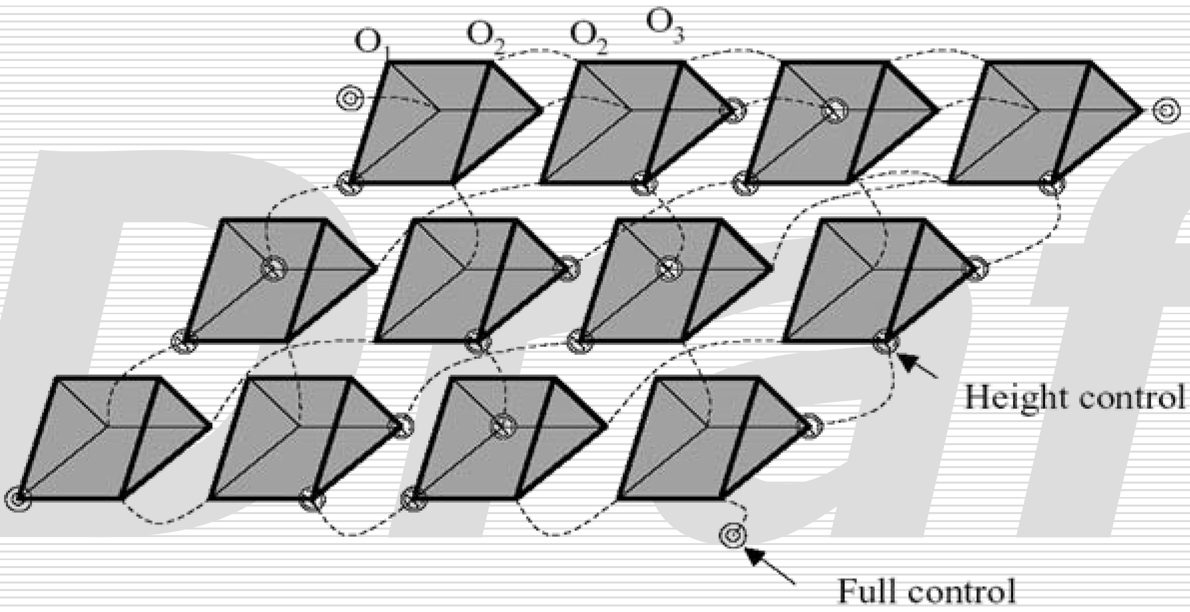
2006

Independent Models - Introduction

The basic computational unit is the photogrammetric model, the models maybe measured in stereo-instruments or analytically computed from comparator coordinates.



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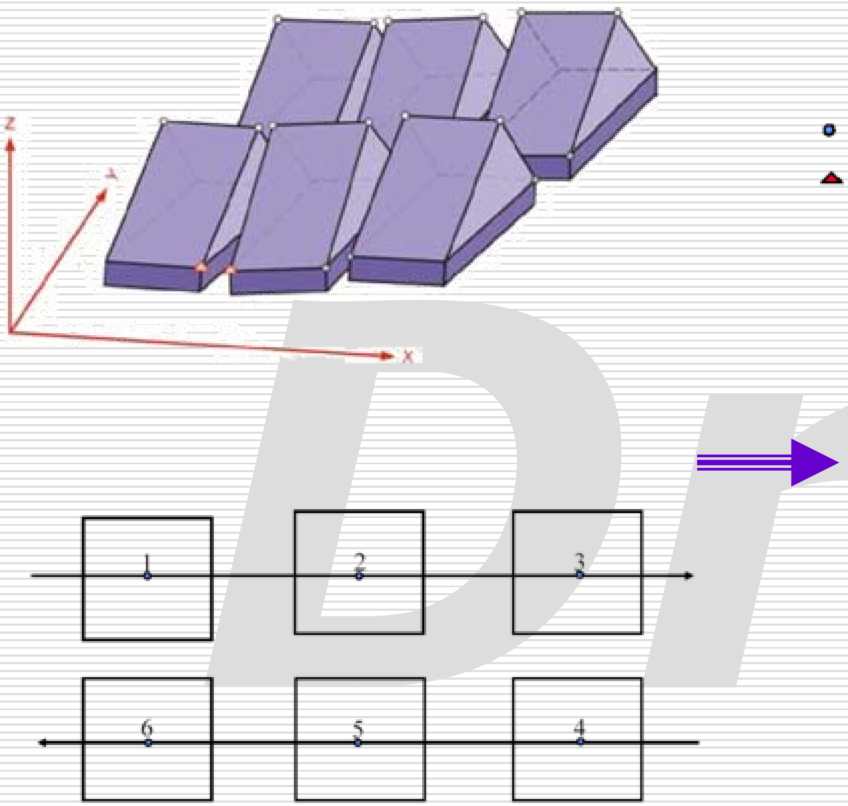
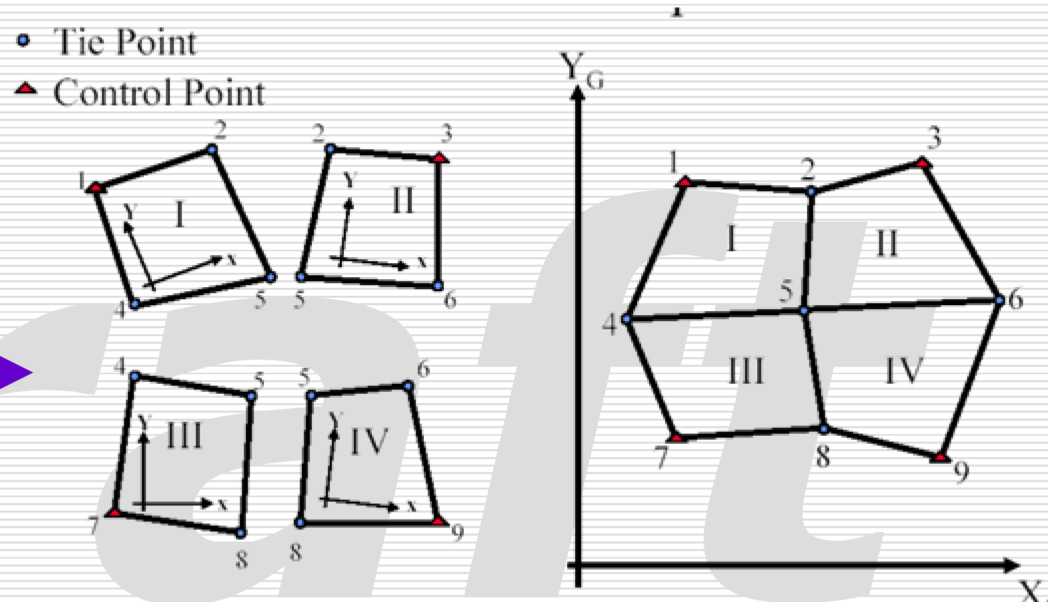
Independent Models - Introduction

Block Adjustment of Independent Models (BAIM)

starts with model coordinate measurements after relative orientation.

Dependent or independent relative orientation can be used.





Before BAIM

After BAIM

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Independent Models - Introduction

BAIM can be carried out in either 2-D or 3-D

Planimetric BAIM:

– **Given:** 2-D model coordinates (X, Y) measured in relatively oriented and leveled models.

– **Required:** 2-D ground coordinates of these points as well as the transformation parameters associated with the involved models.

Spatial BAIM:

– **Given:** 3-D model coordinates (X, Y, Z) measured in relatively oriented models.

– **Required:** 3-D ground coordinates of these points as well as the transformation parameters associated with the involved models.

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Independent Models - General Transformation Equation

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_i = \lambda_j . R_j \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{i,j} + \begin{bmatrix} X_0 \\ Y_0 \\ Z_0 \end{bmatrix}_j + \begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix}_{i,j}$$

X_i, Y_i, Z_i are the ground coordinates of point i

x_{ij}, y_{ij}, z_{ij} are the model coordinates of point (i) in model (j)

λ_j is scale factor of model (j)

R_j is rotation matrix of model (j), $R_j = f(\Omega_j, \Phi_j, A_j)$

$X_{oj}, Y_{oj} Z_{oj}$ are three shifts of model (j)

These equations are not linear in terms of the 4 transformation parameters $\lambda_j, \Omega_j, \Phi_j, A_j$

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Procedures of I.M. Adjustment

(i) M-7 (x,y,z)

Simultaneous 3-dimensional adjusted iteratively 7 parameters model.

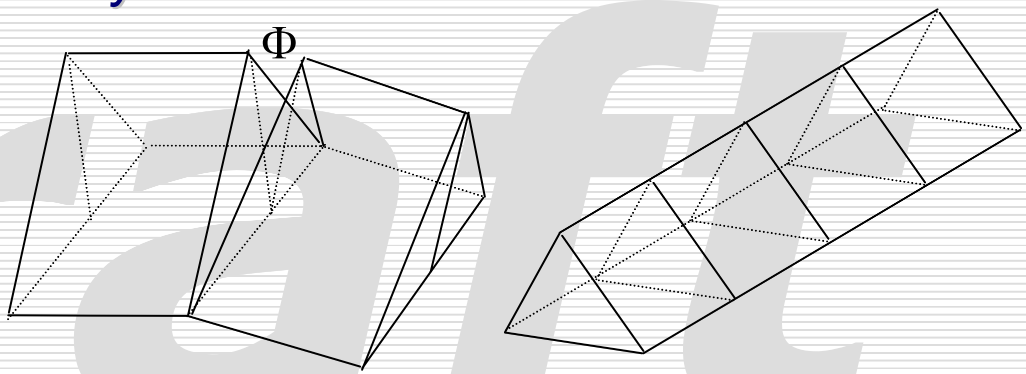
Usually Ω and Φ are small while the scale and azimuth can have large value. One of the main disadvantages of the M-7 is the need for having a good approximate values before a least squares solution is possible.

(ii) M-43 (x,y) (z)

Alternating plan and neight adjusted by iterative.

4 parameters model plan

3 parameters model height



N.B :

As soon as 'heights' are included in 'Independent-Model' block adjustment, the 'Projection Center' have to be used to allow the determination of the longitudinal tilt Φ

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Procedures of I.M. Adjustment

M-7 Independent Model Adjustment

(i) For a control point

$$A_{i,j} \cdot P_j = C_i$$

$$\begin{bmatrix} x & o & 3 & y & 1 & o & 0 \\ y & 3 & o & -x & o & 1 & 0 \\ 3 & -y & x & o & o & o & 1 \end{bmatrix}_{i,j} \begin{bmatrix} \lambda \\ \Delta\Omega \\ \Delta\Phi \\ \Delta A \\ X_0 \\ Y_0 \\ Z_0 \end{bmatrix}_j = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_i$$

P_j : The unknown 7 transformation parameters of model j.

C_j : The unknown coordinates of tie point i.

A_{ij} :.Transformation coefficient matrix.

(ii) For a tie and Projection Center point

$$A_{i,j} \cdot P_j - I \cdot C_i = O$$

$$\begin{bmatrix} x & o & z & y & 1 & o & 0 \\ y & z & o & -x & o & 1 & 0 \\ 3 & -y & x & o & o & o & 1 \end{bmatrix}_{i,j} \begin{bmatrix} \lambda \\ \Delta\Omega \\ \Delta\Phi \\ \Delta \\ X_0 \\ Y_0 \\ Z_0 \end{bmatrix}_j - \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

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Procedures of I.M. Adjustment

M-7 Independent Model Adjustment

Approximate Values for the Unknowns

The most Critical Unknowns are:

$$\lambda_j, \Omega_j, \Phi_j, A_j$$

Usually Ω and Φ are small while the scale λ and Azimuth A can have large values.

One of the main disadvantages of the (M-7) simultaneous (x,y,z) adjustment is the need for having a good approximate values before a least squares solution is possible.

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Procedures of I.M. Adjustment

M-43 Independent Model Adjustment

The M-43 procedure is more popular due to the following advantages:

- For blocks of about 200 models, experience showed that M-43 takes much shorter computation time than (M-7) (~50%: 73 and 43+33).
- Due to the fact that the "reduced normal equations" should be solved at a time are smaller than that of M-7, therefore, it is possible to adjust larger blocks using the same computer.
- The M-43 procedure does not require any approximate values for the unkown parameters and coordinates as needed in M-7

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Procedures of I.M. Adjustment

M-43 Independent Model Adjustment

M-43 Independent Model Adjustment Procedure

1- Start planimetric adjustment, the observation equations are nothing but the linear conformal similarity transformation equation:

$$\begin{bmatrix} x & -y & 1 & 0 \\ y & x & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ X_0 \\ Y_0 \end{bmatrix} = \begin{bmatrix} X \\ Y \end{bmatrix}$$

- Linear observation equation, therefore, no linearization and no approximate values for the unknowns

- The "Projection Center" are not used in the planimetric adjustment due to their disturbing effects

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Procedures of I.M. Adjustment

M-43 Independent Model Adjustment

M-43 Independent Model Adjustment Procedure

2- After the formation and solution of R.N.E for Plan, we get 4 parameters/model a, b, X_0, Y_0

- Using a, b, X_0, Y_0 a rigorous (4x4) rotation matrix R is formed for each model

- All Points in each model are transformed using the following rigorous 3-dimensional transformation equations

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{New} = \begin{bmatrix} a & -b & 0 \\ b & a & 0 \\ 0 & 0 & \sqrt{a^2 + b^2} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{Old} + \begin{bmatrix} X_0 \\ Y_0 \\ 0 \end{bmatrix}$$

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Procedures of I.M. Adjustment

M-43 Independent Model Adjustment

M-43 Independent Model Adjustment Procedure

3- *Height adjustment is started using the x,y,z coordinates obtained from plan adjustment. The following linearized observation equations are used*

$$\begin{bmatrix} X - x \\ Y - y \\ Z - z \end{bmatrix} = \begin{bmatrix} 0 & z & 0 \\ -z & 0 & 0 \\ y & -x & 1 \end{bmatrix} \begin{bmatrix} \Delta\Omega \\ \Delta\Phi \\ Z_0 \end{bmatrix}$$

- *For the "Projection Center" the 3 equations are used while for "Tie Points" and "Height Points" only the last equation is used*

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Procedures of I.M. Adjustment

M-43 Independent Model Adjustment

M-43 Independent Model Adjustment Procedure

4- After the formation and solution of R.N.E for heights, we get 3 parameters/model $\Delta\Omega, \Delta\Phi, Z_0$

- Using $\Delta\Omega, \Delta\Phi$ a rigorous (3x3) rotation matrix R is formed for each model
- All Points in each model are transformed using the following rigorous 3-dimensional transformation equations

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{New} = R_{(\Delta\Omega, \Delta\Phi)} \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{Old} + \begin{bmatrix} 0 \\ 0 \\ Z_0 \end{bmatrix}$$

5- Usually 2 or 3 iterations (P-> H, P->H,…) are sufficient.

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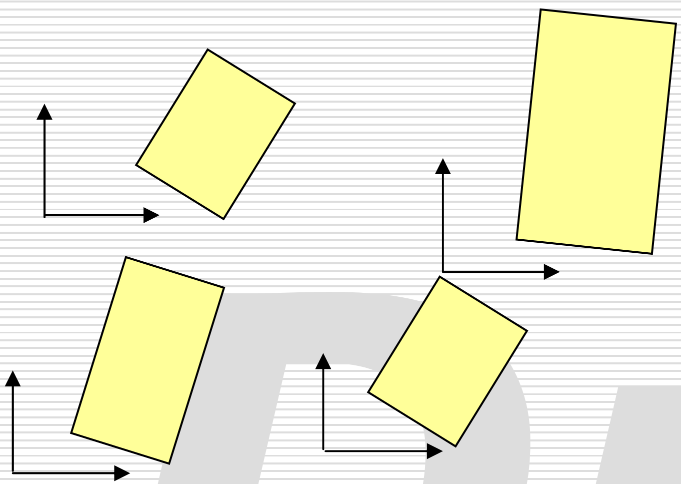
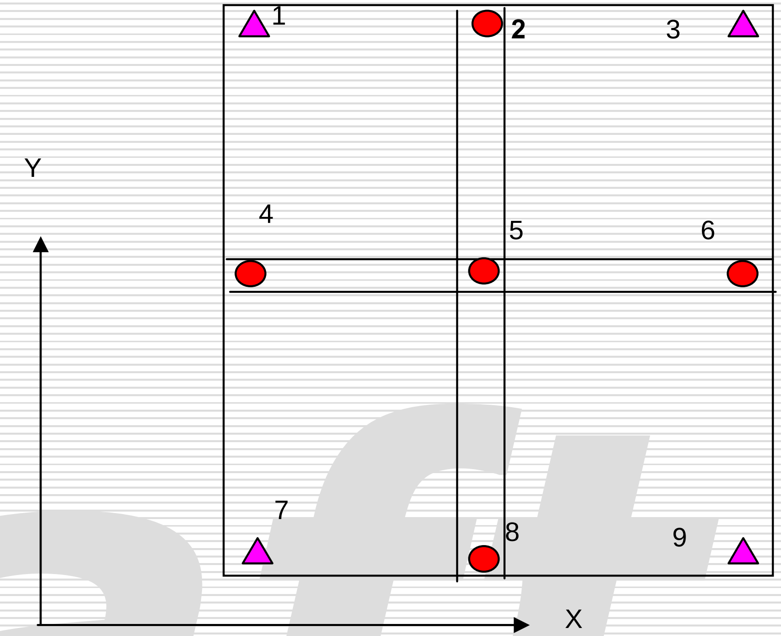
Example for I.M. Plan block adjustment (M-4)

Given the following small block of 4 models tobe adjusted in planimetry using the linear conformal transformation.

$$X = a.x - b.y + X_0$$

$$Y = b.x - a.y + Y_0$$





Points to be used in the least squares I.M adjustment are:

(i) Control Points (1,3,7,9)

(ii) Tie Points (2,4,5,6,8)

Single point 10 will not be used.

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Example for I.M. Plan block adjustment (M-4)

Inter-Relation Table

Pts	Models of Appearance				Freq	Type
1	M^1				-	Plan
2	M^1	M^2			2	Tie
3		M^2			-	Plan
4	M^1		M^3		2	Tie
5	M^1	M^2	M^3	M^4	4	Tie
6		M^2		M^4	2	Tie
7			M^3		-	Plan
8			M^3	M^4	2	Tie
9				M^4	-	Plan
10				S^3	1	Single

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Example for I.M. Plan block adjustment (M-4)

The unknowns

(i) Unknown parameters

P=4 model $\times$ 4 parameters=16

(ii) Unknown coordinates of the points

C= 5 points $\times$ 2 coordinates=10

U = P+C = 16 + 10 = 26

The observation equations

(i) For each control point in each model:

$$A_{i,j}.P_j = C_i \qquad \begin{bmatrix} x & -y & 1 & 0 \\ y & x & 0 & 1 \end{bmatrix}_{i,j} \cdot \begin{bmatrix} a \\ b \\ X_0 \\ Y_0 \end{bmatrix}_j = \begin{bmatrix} X \\ Y \end{bmatrix}_i$$

(ii) For each tie point in each model:

$$A_{i,j}.P_j - I \cdot C_i = O \qquad \begin{bmatrix} x & -y & 1 & 0 \\ y & x & 0 & 1 \end{bmatrix}_{i,j} \cdot \begin{bmatrix} a \\ b \\ X_0 \\ Y_0 \end{bmatrix}_j - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} X \\ Y \end{bmatrix}_i = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

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Example for I.M. Plan block adjustment (M-4)

The observation equation matrix

$A_{1,1}$								
$A_{2,1}$				$-I$				
$A_{4,1}$					$-I$			
$A_{5,1}$						$-I$		
	$A_{2,2}$			$-I$				
	$A_{3,2}$							
	$A_{5,2}$					$-I$		
	$A_{6,2}$						$-I$	
		$A_{4,3}$			$-I$			
		$A_{5,3}$				$-I$		
		$A_{7,3}$						
		$A_{8,3}$						$-I$
			$A_{5,4}$			$-I$		
			$A_{6,4}$				$-I$	
			$A_{8,4}$					$-I$
			$A_{9,4}$					

·

P_1
P_2
P_3
P_4
C_2
C_4
C_5
C_6
C_8

=

C_1
0
0
0
0
C_3
0
0
0
0
C_7
0
0
0
0
C_9

Example for I.M. Plan block adjustment (M-4)

The Normal equation in matrix notation

$$
\left[\begin{array}{cccc|ccccc}
\sum\limits_{i=1,2,4,5} A_{i,1}^T A_{i,1} & & & & -A_{2,1}^T & -A_{4,1}^T & -A_{5,1}^T & & \\
 & \sum\limits_{i=2,3,5,6} A_{i,2}^T A_{i,2} & & & -A_{2,2}^T & & -A_{5,2}^T & -A_{6,2}^T & \\
 & & \sum\limits_{i=4,5,7,8} A_{i,3}^T A_{i,3} & & & -A_{4,3}^T & -A_{5,3}^T & & -A_{8,3}^T \\
 & & & \sum\limits_{i=5,6,8,9} A_{i,4}^T A_{i,4} & & & -A_{5,4} & -A_{6,4} & -A_{8,4} \\
\hline
-A_{2,1} & -A_{2,2} & & & 2I & & & & \\
-A_{4,1} & & -A_{4,3} & & & 2I & & & \\
-A_{5,1} & -A_{5,2} & -A_{5,3} & -A_{5,4} & & & 4I & & \\
 & -A_{6,2} & & -A_{6,4} & & & & 2I & \\
 & & -A_{8,3} & -A_{8,4} & & & & & 2I
\end{array}\right]
\cdot
\left[\begin{array}{c}
P_1 \\ P_2 \\ P_3 \\ P_4 \\ C_2 \\ C_4 \\ C_5 \\ C_6 \\ C_8
\end{array}\right]
=
\left[\begin{array}{c}
A_{1,1}^T C_1 \\ A_{3,2}^T C_3 \\ A_{7,3}^T C_7 \\ A_{9,4}^T C_9 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0
\end{array}\right]
$$

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Reduced Normal Equations

The reduced normal equations 16 equations in 16 unknown parameters will be as follows Example

$$
\begin{bmatrix}
\Theta & \Xi & & \\
\Xi & \Theta & \Xi & \\
& \Xi & \Theta & \Xi \\
& & \Xi & \Theta
\end{bmatrix}
*
\begin{bmatrix} P_1 \\ P2 \\ P3 \\ P4 \end{bmatrix}
=
\begin{bmatrix} A^T_{1,1}.C_1 \\ A^T_{2,2}.C_3 \\ A^T_{7,3}.C_7 \\ A^T_{19,4}.C_9 \end{bmatrix}
$$

Direct generation of R.N.E.

Main diagonal sub-matrix

$$N^{RNE}_{j,j} = \sum A^T_{i,j} A_{i,j} - \sum \frac{1}{f_t} A^T_{i,j} A_{i,j}$$

control and tie points in model j

tie points in model j

f_t : frequency of tie point

$$\therefore \quad N^{RNE}_{1,1} = A^t_{1,1} \cdot A_{1,1} + A^t_{2,1} \cdot A_{2,1} + A^t_{4,1} \cdot A_{4,1} + A^t_{5,1} \cdot A_{5,1} - \frac{1}{2} A^t_{2,1} \cdot A_{2,1} - \frac{1}{2} A^t_{4,1} \cdot A_{4,1} - \frac{1}{4} A^t_{5,1} \cdot A_{5,1}$$

$$\therefore \quad N^{RNE}_{2,2} = A^t_{2,2} \cdot A_{2,2} + A^t_{3,2} \cdot A_{3,2} + A^t_{5,2} \cdot A_{5,2} + A^t_{6,2} \cdot A_{6,2} - \frac{1}{2} A^t_{2,2} \cdot A_{2,2} - \frac{1}{4} A^t_{5,2} \cdot A_{5,2} - \frac{1}{2} A^t_{6,2} \cdot A_{6,2}$$

Notice: $f = 2 \longrightarrow TiePts: 2,4,6,8 \quad , \quad f = 4 \longrightarrow TiePt: 5$

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Reduced Normal Equations

Direct generation of R.N.E.

Off- diagonal sub-matrix

$$\boxed{N_{j,l}^{RNE} = -\underbrace{\sum}_{\text{tie points common to model } j \text{ and } l} \frac{1}{f_t} A_{t,j}^T . A_{t,e}}$$

tie points common
to model j and l

Notice:

- *The above (4x4) off-diagonal matrix is not symmetrical*
- $N_{j,l}^{RNE} = 0$ *if no tie points exist between model j and l*

$$\therefore \quad N_{1,2}^{RNE} = -\frac{1}{2} A_{2,1}^t \cdot A_{2,2} - \frac{1}{4} A_{5,1}^t \cdot A_{5,2}$$

$$\therefore \quad N_{1,4}^{RNE} = -\frac{1}{4} A_{5,1}^t \cdot A_{5,4}$$

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Reduced Normal Equations

Direct generation of R.N.E.

Constant terms sub – matrix

$$T_j^{RNE} = \sum A_{k,j}^T . C_k$$

control points in model j

$$\begin{bmatrix} -\sum\left(x_{k,j}X_k + y_{k,j}Y_k\right) \\ -\sum\left(y_{k,j}X_k - x_{k,j}Y_k\right) \\ \sum\left(X_k\right) \\ \sum\left(Y_k\right) \end{bmatrix}$$

Control Points k appearing in model j

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Flow Diagram of M-4 (x,y)

Input of the Data:
- x , y Model Coordinates
- X , Y Ground Coordinates for the Control Points

Forming of Interrelation Table Required for the Generation of the Reduced Normal Equations

Direct Generation of the Reduced Normal Equations
- Main-Diagonal Sub-Matrixes
- Off-Diagonal Sub-Matrixes
- R.H.S Sub-Matrixes

Direct Solution of the RNE, i.e. Computation of Planimetric Transformation Parameters

Transformation of Each Model

Coordinate Computation
- Computation of Mean Coordinates of Tie Points
- Computation of Absolute & Relative Discrepancies and Standard Deviation

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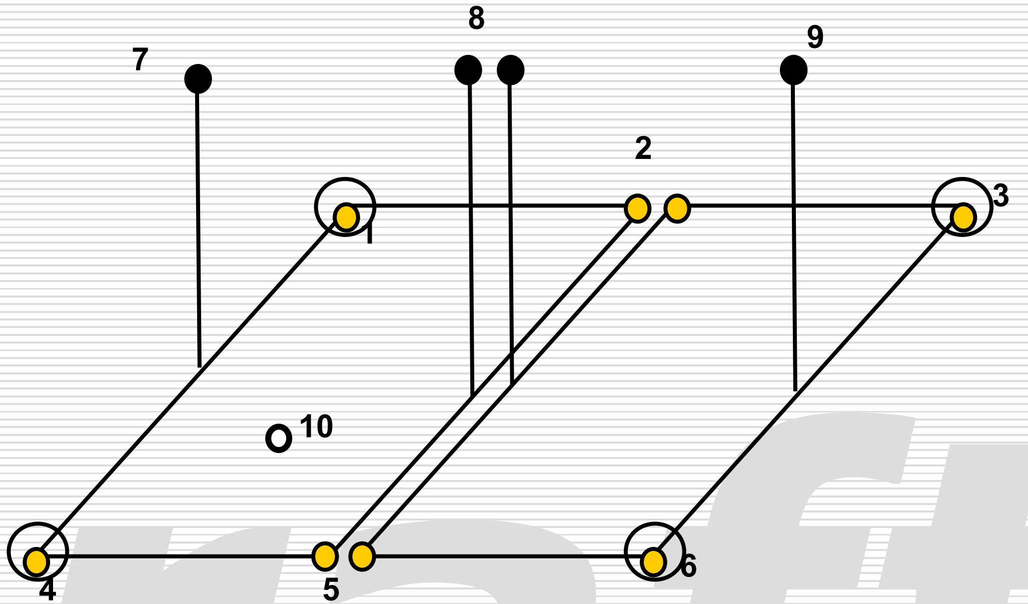
Example for I.M height adjustment (M-3)

Given the following small block of 2 models to be adjusted in height.



Notice that the projection centers 7,9 and the model point 10 will not be used in the adjustment, because single points where is no relative or absolute discrepanncies to be minimized.

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Example for I.M height adjustment (M-3)

Inter-Relation Table

Pts	Models of Appearance		Freq	Type
1	M^1		-	Alti
2	M^1	M^2	2	Tie
3		M^2	-	Alti
4	M^1		2	Tie
5	M^1	M^2	4	Tie
6		M^2	2	Tie
7	M^1		1	P.C
8	M^1	M^2	2	P.C
9		M^2	1	P.C
10			1	Single

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Example for I.M height adjustment (M-3)

The unknowns

(i) Unknown parameters

P =2 model × 3 parameters = 6

(ii) Unknown coordinates of the points

2 tie × 1= 2

1 projection center × 3 = 3

C=2+3=5

Total = P+C = 6 + 5 = 11

The observation equations (Continued…)

The rigorous transformation equations are non linear in terms of Φ, Ω

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = R_{(\Phi,\Omega)} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ Z_0 \end{bmatrix}$$

Assuming the rotations to be small ΔΦ, ΔΩ then linearized equation will be:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} 1 & O & \Delta\Phi \\ O & 1 & -\Delta\Omega \\ -\Delta\Phi & \Delta\Omega & 1 \end{bmatrix} . \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ Z_0 \end{bmatrix}$$

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Example for I.M height adjustment (M-3)

The observation equations (Continued...)

There are three types of observation equations

(i) For height control points

$$\begin{bmatrix} y_{ij} & x_{ij} & 1 \end{bmatrix} \begin{bmatrix} \Delta\Omega \\ \Delta\Phi \\ Z_0 \end{bmatrix}_j = \begin{bmatrix} Z_i - z_{ij} \end{bmatrix}$$

(ii) For tie points

$$\begin{bmatrix} y_{ij} & x_{ij} & 1 \end{bmatrix} \begin{bmatrix} \Delta\Omega \\ \Delta\Phi \\ Z_0 \end{bmatrix}_j - \begin{bmatrix} Z_i \end{bmatrix} = \begin{bmatrix} - z_{ij} \end{bmatrix}$$

(iii) For projection centers

$$\begin{bmatrix} o & z_{ij} & o \\ -z_{ij} & o & o \\ y_{ij} & -x_{ij} & 1 \end{bmatrix} \begin{bmatrix} \Delta\Omega \\ \Delta\Phi \\ Z_0 \end{bmatrix}_j - \begin{bmatrix} X_i \\ Y_i \\ Z_i \end{bmatrix} = \begin{bmatrix} -x_{ij} \\ -y_{ij} \\ -z_{ij} \end{bmatrix}$$

The number of observation equations

First model:

2 control $\times$ 1=2 , tie $\times$ 1= 2 , 1 Projection Center $\times$ 3 = 3

2+2+3=7

Second model:

7 equations

Redundancy= 14 – 11 = 3

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Example for I.M height adjustment (M-3)

The observation equation matrix

$$
\begin{array}{ccccccccccc}
\Delta\Omega_1 & \Delta\Phi_1 & Z\circ_1 & \Delta\Omega_2 & \Delta\Phi_2 & Z\circ_2 & Z_2 & Z_5 & X_8 & Y_8 & Z_8
\end{array}
$$

$$
\left[\begin{array}{ccc|ccc|c|c|c|c|c}
y_{1,1} & -x_{1,1} & 1 & & & & & & & & \\
y_{4,1} & -x_{4,1} & 1 & & & & & & & & \\
y_{2,1} & -x_{2,1} & 1 & & & & -1 & & & & \\
y_{5,1} & -x_{5,1} & 1 & & & & & -1 & & & \\
0 & z_{8,1} & 0 & & & & & & -1 & & \\
-z_{8,1} & 0 & 0 & & & & & & & -1 & \\
y_{8,1} & x_{8,1} & 1 & & & & & & & & -1 \\
\hline
 & & & y_{3,2} & -x_{3,2} & 1 & & & & & \\
 & & & y_{6,2} & -x_{6,2} & 1 & & & & & \\
 & & & y_{2,2} & -x_{2,2} & 1 & -1 & & & & \\
 & & & y_{5,2} & -x_{5,2} & 1 & & -1 & & & \\
 & & & 0 & z_{8,2} & 0 & & & -1 & & \\
 & & & -z_{8,2} & 0 & 0 & & & & -1 & \\
 & & & y_{8,2} & -x_{8,2} & 1 & & & & & -1
\end{array}\right]
\left[\begin{array}{c}
\Delta\Omega_1 \\ \Delta\Phi_1 \\ Z0_1 \\ \hline \Delta\Omega_2 \\ \Delta\Phi_2 \\ Z0_2 \\ \hline Z_2 \\ Z_5 \\ X_8 \\ Y_8 \\ Z_8
\end{array}\right]
\left[\begin{array}{c}
Z_1 - z_{1,1} \\ Z_4 - z_{4,1} \\ \hline -z_{2,1} \\ -z_{5,1} \\ \hline -x_{8,1} \\ -y_{8,1} \\ -z_{8,1} \\ \hline Z_3 - z_{3,2} \\ Z_6 - z_{6,2} \\ \hline -z_{2,2} \\ -z_{5,2} \\ \hline -x_{8,2} \\ -y_{8,2} \\ -z_{8,2}
\end{array}\right]
\begin{array}{l}
\text{cont pts} \\ \\ \text{tie pts} \\ \\ \\ \text{P.C.} \\ \\ \text{cont pts} \\ \\ \text{tie pts} \\ \\ \\ \text{P.C} \\ \\
\end{array}
$$

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Example for I.M height adjustment (M-3)

The Normal equation in matrix notation

$$
\left[\begin{array}{c|c|ccccc}
\begin{array}{c}\sum A_{all,1}^{T}\cdot A_{all,1} \\ \sum \bar{A}_{p,1}^{T}\cdot \bar{A}_{p,1}\end{array} & & -A^{T}_{2,1} & -A^{T}_{5,1} & \multicolumn{3}{c}{-\bar{A}^{T}_{8,2}} \\
\hline
 & \begin{array}{c}\sum A_{all,2}^{T}\cdot A_{all,2} \\ \sum \bar{A}_{p,2}^{T}\cdot \bar{A}_{p,2}\end{array} & -A^{T}_{2,2} & -A^{T}_{5,2} & \multicolumn{3}{c}{-\bar{A}^{T}_{8,2}} \\
\hline
-y_{2,1}\;\; x_{2,1}\;\; -1 & -y_{2,2}\;\; x_{2,2}\;\; -1 & 2 & & & & \\
-y_{5,1}\;\; x_{5,1}\;\; -1 & -y_{5,2}\;\; x_{5,2}\;\; -1 & & 2 & & & \\
0\;\; z_{8,1}\;\; 0 & 0\;\; z_{8,2}\;\; 0 & & & 2 & & \\
-z_{8,1}\;\; 0\;\; 0 & -z_{8,2}\;\; 0\;\; 0 & & & & 2 & \\
-y_{8,1}\;\; x_{8,1}\;\; -1 & -y_{8,2}\;\; x_{8,2}\;\; -1 & & & & & 2
\end{array}\right]
\left[\begin{array}{c} P_1 \\ \hline P_2 \\ \hline Z_2 \\ Z_5 \\ X_8 \\ Y_8 \\ Z_8 \end{array}\right]
=
\left[\begin{array}{c}
\begin{array}{ll} A_{k,1}^{T}\cdot \Delta Z_{k,1} & cont \\ -A_{t,1}^{T}\cdot z_{t,1} & tie \\ -\bar{A}_{p,1}^{T}\cdot C_{p,1} & P.C \end{array} \\
\hline
\begin{array}{ll} A_{k,2}^{T}\cdot \Delta Z_{k1} & cont \\ -A_{t,1}^{T}\cdot z_{t1} & tie \\ -\bar{A}_{p,2}^{T}\cdot C_{p,2} & P.C \end{array} \\
\hline
\sum z_2 \\ \sum z_5 \\ \sum x_8 \\ \sum y_8 \\ \sum z_8
\end{array}\right]
$$

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Flow Diagram of M-3 (z)

Input of the Data:
- z Model Coordinates of Tie Points
- x , y , z Model Cordinate of Projection Centers
- Z Ground Coordinates for the Control Pts

↓

Forming of Interrelation Table Required for the Generation of the Reduced Normal Equations

↓

Direct Generation of the Reduced Normal Equations
- Main-Diagonal Sub-Matrixes
- Off-Diagonal Sub-Matrixes
- R.H.S Sub-Matrixes

↓

Direct Solution of the RNE, i.e. Computation of Altimetric Transformation Parameters

↓

Transformation of Each Model

↓

Coordinate Computation
- Computation of Mean Coordinates of Tie Points
- Computation of Absolute & Relative Discrepancies and Standard Deviation

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Flow Diagram of M-43 (xy <-> z) I.M Block Adj.

Input of the Data:

- x , y , z Model Coord of Tie Points
- x , y , z Model Coord of P.C.
- X , Y ,Z Ground Coord of the GCPs

Forming of Interrelation Table Required for the Generation of the Reduced Normal Equations

Planimmetric Adjustment (x,y)

Direct Generation of the (x,y) RNE
Main-Diagonal, Off-Diagonal, R.H.S Sub-Matrixes

Direct Solution of the (x,y) RNE,
Computation of Planimetric Transformation Parameters

Transformation of Each Model

$$x^P_{New} = a \cdot x - b \cdot y + X_0$$

$$y^P_{New} = b \cdot x + a \cdot y + Y_0$$

$$z^P_{New} = \sqrt{a^2 + b^2} . z$$

Altimetric Adjustment (z)

Direct Generation of the (z) RNE
Main-Diagonal, Off-Diagonal, R.H.S Sub-Matrixes

Direct Solution of the (z) RNE,
Computation of Altimetric Transformation Parameters

Transformation of Each Model

$$\begin{bmatrix} x^A_{New} \\ y^A_{New} \\ z^A_{New} \end{bmatrix} = R_{(\Delta\Omega\Delta\Phi)} \begin{bmatrix} x^P_{New} \\ y^P_{New} \\ z^P_{New} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ Z_0 \end{bmatrix}$$

New Iteration ?

Coordinate Computation

- Computation of Mean Coordinates of Tie Points
- Computation of Absolute & Relative Discrepancies and Standard Deviation

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Photogrammetric Block Adjustment

Strip Adjustment

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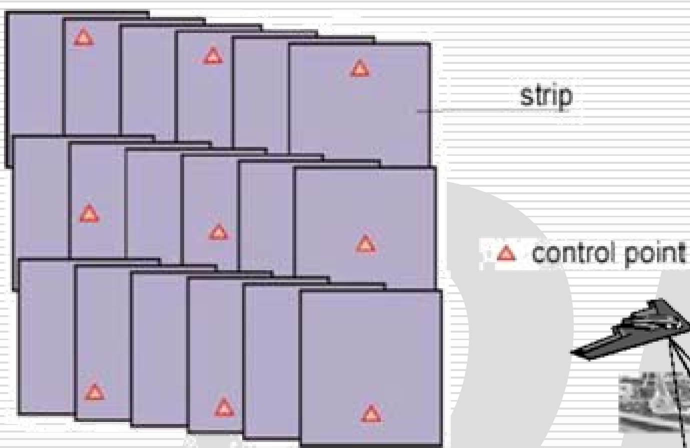
2006

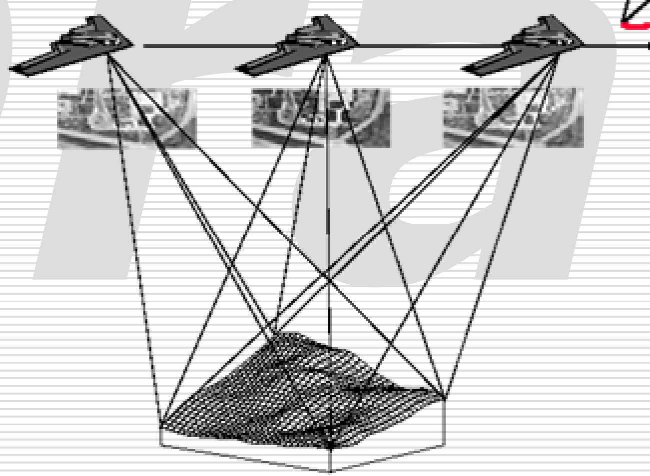
Introduction

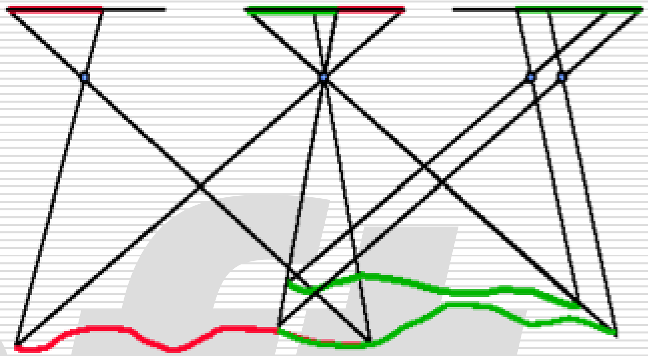
Strip Adjustment

Strip triangulation is used for mapping linear features (such as road and rail road network and pipelines).

***Given:** imagery along one strip with 60% overlap between successive imagery*







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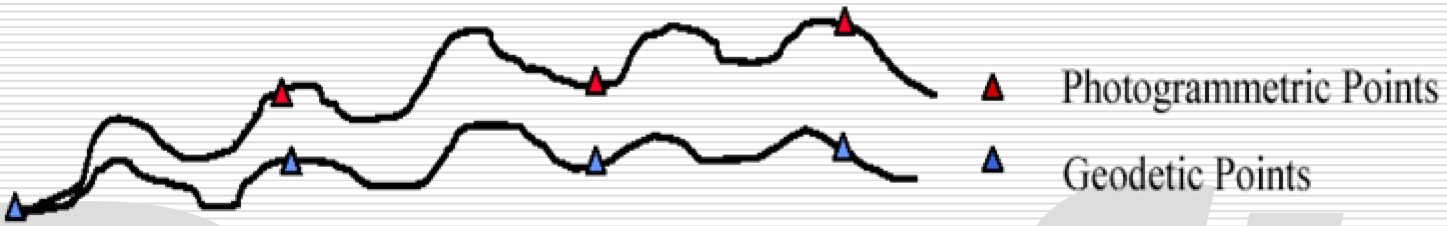
Introduction

Error Propagation

– Similar to an open traverse, errors will increase as the length of the strip increases.



Solution

– Implement **GCP** every three or four models.

– Apply **Corrections Polynomials**.

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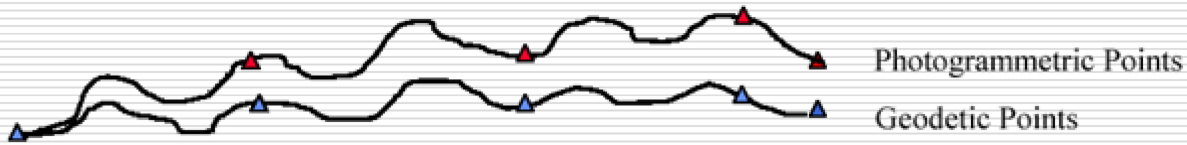
Introduction

Correction Polynomials:

– They are used to reduce the difference between the photogrammetric and geodetic coordinates of the control points

– Using the Ground Control Points (GCP), we can solve for the polynomial coefficients.



Precaution:

– Make sure that you have enough GCP to recover the polynomial coefficients.

– Use low order polynomials to avoid undulations/over parameterization.

– For long strips, separate them into shorter strips with low order polynomials and use constraints to ensure smooth transition between strips.

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Introduction

Which Polynomials Should be Used?

In order to answer the above question it was necessary to define the sources and types of errors affecting the strip triangulation.

The Source of Errors:

(1) The photograph

(2) The instrument

(3) The observer

The Types of Errors:

The actual magnitude and type of different errors made in forming a strip are often difficult to determine. The situation was over-simplified by assuming that there are only two types of errors:

(1) Constant (systematic) error

(2) Random error

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Strip Deformation

Vermier Theory

In order to study the effect of errors on triangulated strip further simplifications were assumed namely:

(1) The models are assumed to be internally error free (i.e. no model deformation – or its magnitude is negligible).

(2) The coordinate errors of triangulated points are due to errors in the absolute orientation of the model in which they appear.

In fact, the errors in the 7 absolute orientation elements of model (i), ΔS_i , ΔA_i , $\Delta \Phi_i$, $\Delta \Omega_i$, ΔCx_i , ΔCy_i , ΔCz_i are due to errors in the absolute orientation elements of the first model plus the errors in transferring the 7 elements from one model to next one (Δs_i ,Δa_i , $\Delta \varphi_i$, $\Delta \omega_i$, Δcx_i , Δcy_i , Δcz_i).

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Strip Deformation

Vermier Theory

The 7 elements of transfer are associated with errors, namely:

- ***Scale** transfer error: Δs_i (i=1,2…n-1) because of (n-1) connection for n- models*
- ***Azimuth** transfer error: Δa_i*
- ***Longitudinal** transfer error: $\Delta \varphi_i$*
- ***Lateral** tilt transfer error: $\Delta \omega_i$*
- ***x- shift** error: Δx_i*
- ***y- shift** error : Δy_i*
- ***z- shift** error : Δz_i*

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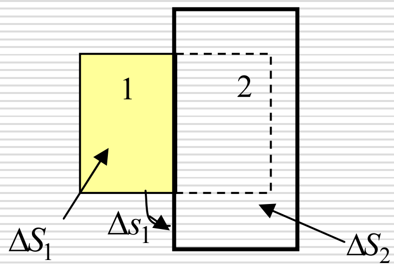
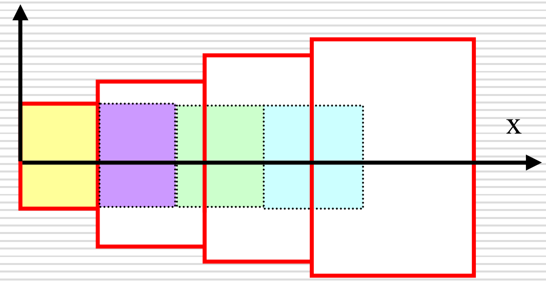
Strip Deformation

Group 1:

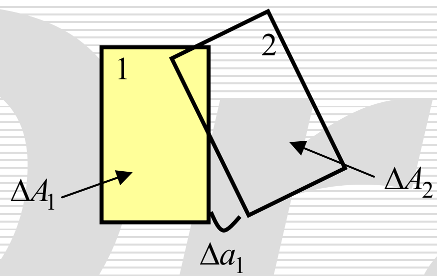
The errors in transfer of scale, azimuth and longitudinal cause appreciable deformation in the strip-axis.

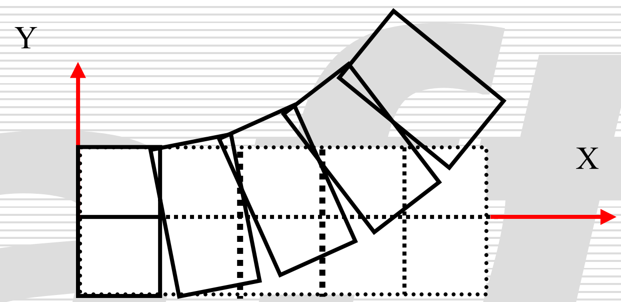
Scale



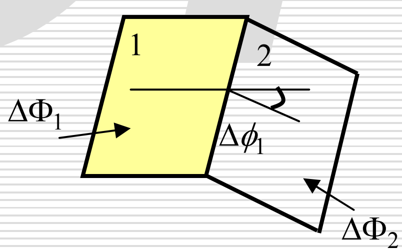


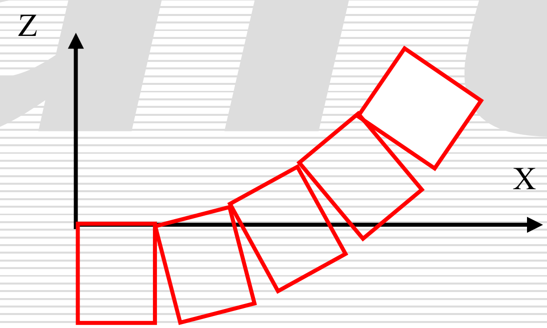
Azimuth





Φ





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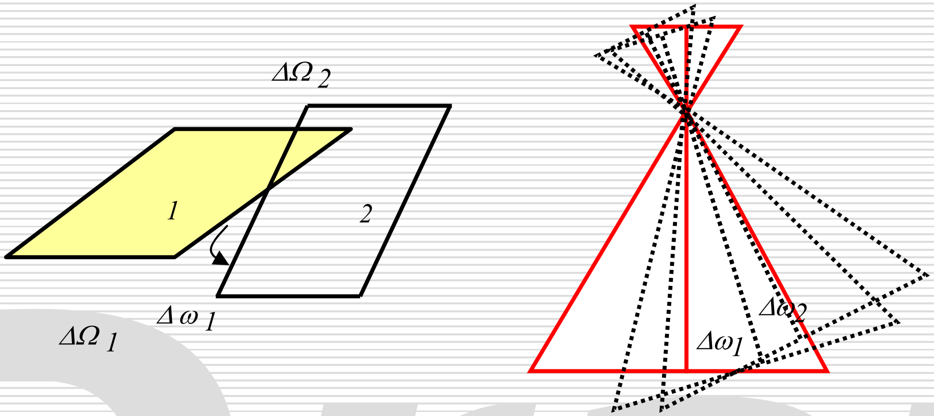
Strip Deformation

Group 2:

The errors in transfer of attitude does not affect the "Strip-axies"



Group 3:

Cx, Cy, Cz transfer errors have negligible effect on the strip deformation.

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Strip Deformation

Effect of *Azimuth* Transfer Errors

In order to investigate the strip deformation due to errors in transfer of orientation element we shall consider as an example the relations between:

- *Azimuth transfer errors Δa_i*
- *Absolute azimuth errors ΔA_i*
- *Y- coordinate errors along the strip axis ΔY_i*

Relation between transfer and absolute azimuth errors:

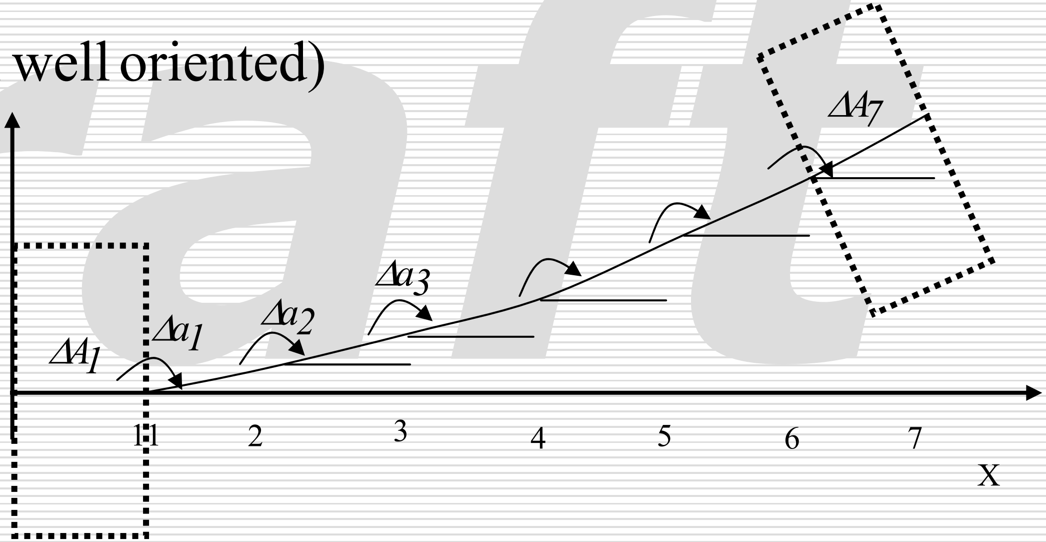
$$\Delta A_1 = 0 \qquad \text{(Assume 1st model well oriented)}$$

$$\Delta A_2 = 0 + \Delta a_1$$

$$\Delta A_3 = 0 + \Delta a_1 + \Delta a_2$$

$$\vdots$$

$$\Delta A_i = \sum_{k=1}^{i-1} \Delta a_k$$



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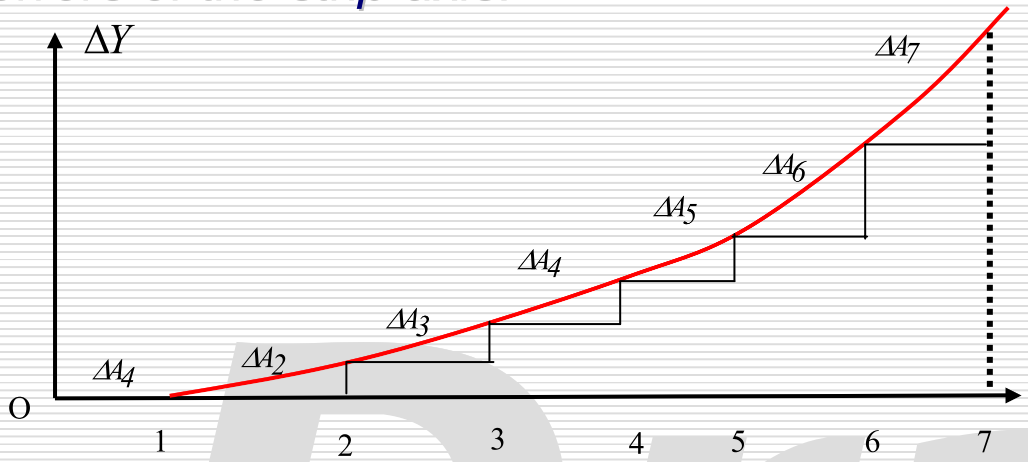
Strip Deformation

Effect of **Azimuth** Transfer Errors

Relations between absolute azimuth model errors ΔA_i and y coordinate errors of the strip axis.



$$\Delta Y_0 = 0$$
$$\Delta Y_1 = 0$$
$$\Delta Y_2 = b.\Delta A_2$$
$$\Delta Y_3 = b.\Delta A_2 + b.\Delta A_3$$
$$\vdots$$
$$\Delta Y_i = b.\sum_{k=1}^{i} \Delta A_k$$

But ΔA_i is obtained by single summation of transfer azimuth errors Δa_i

$$\Delta A_i = \sum \Delta a_k$$
$$\Delta Y_i = b.\sum \Delta A_k$$
$$\Delta Y_i = b.\sum \sum \Delta a_k$$

ΔY_i is obtained by double summation of Δa

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Strip Deformation

Effect of **Azimuth** Transfer Errors

Proof:

systematic azimuth transfer errors cause 2 order y-coordinate errors along strip axis:

(1) Assume for the sake of simplicity that all bases are equal to b.

As transfer errors are constant $(\Delta a_1 = \Delta a_2 = \Delta a_3 = \dots = \Delta a)$

$$\Delta Y_0 = 0$$

$$\Delta Y_1 = 0$$

$$\Delta Y_2 = b.\Delta a$$

$$\Delta Y_3 = b.\Delta a + 2b.\Delta a$$

$$\Delta Y_4 = b.\Delta a + 2b.\Delta a + 3b.\Delta a$$

$$\vdots$$

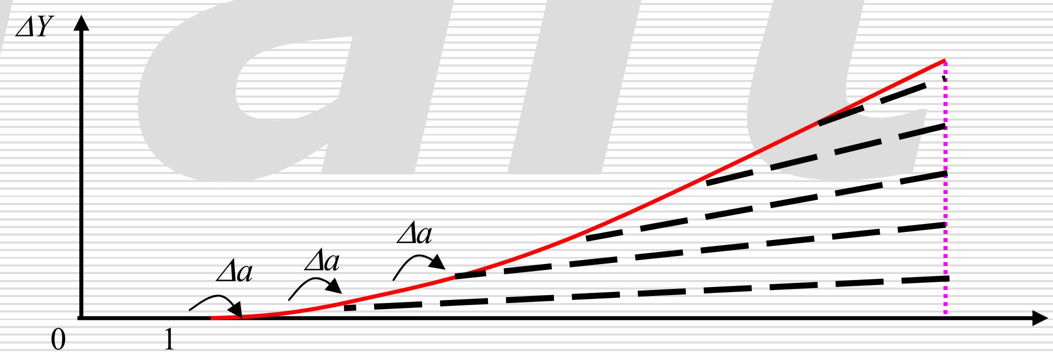
$$\Delta Y_i = b.\Delta a(1+2+3+\cdots+(i-1))$$

$$\Delta Y_i = b.\Delta a\frac{(i-1)i}{2} = b.\Delta a\frac{(i^2-i)}{2}$$

$$X_i = i.b$$

$$\Delta Y_i = \frac{b.\Delta a}{2b^2}.X^2 + \frac{\Delta a}{2}.X$$

$$\Delta Y_i = b_0 + b_1 X_i + b_2 X_i^2$$



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Strip Deformation

*Effect of **Azimuth** Transfer Errors*

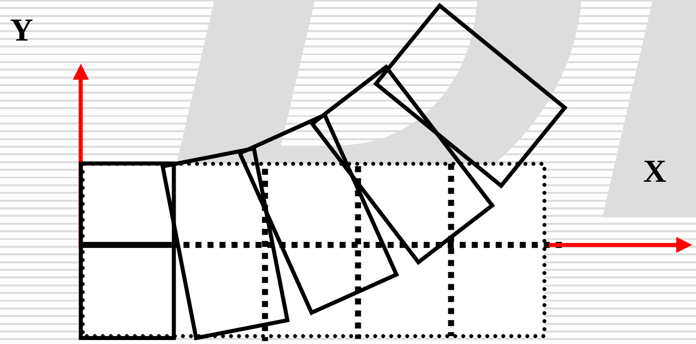
Proof:

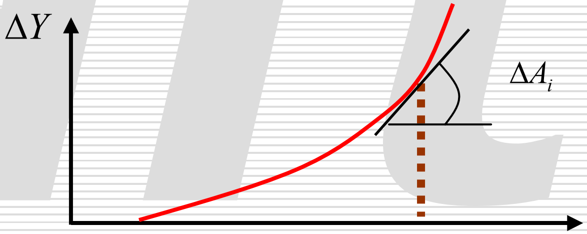
Constant azimuth transfer errors cause 2 ordercoordinate errors along the strip axis.

$$\Delta Y_A^{\Sigma\Sigma} = b_0 + b_1.X + b_2.X^2$$

Notice that if we differentiate the 2 order coordinate error curves we get linear absolute orientation error curve.

$$\Delta A_i = \frac{\partial(\Delta Y)}{\partial X} = b_1 + 2b_2.X$$





$$\Delta Y = b_0 + b_1 X + b_2 X^2$$

$$\Delta A_i = b_1 + 2b_2 X$$

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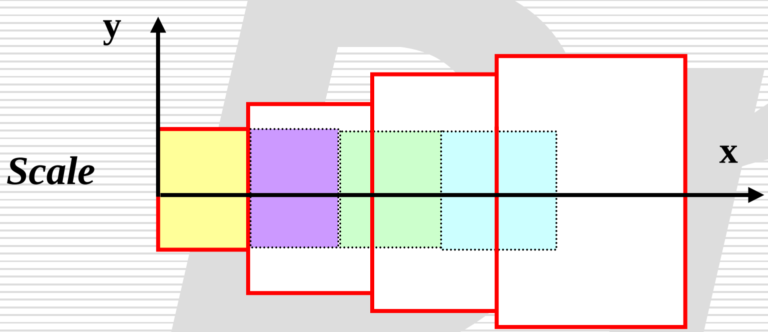
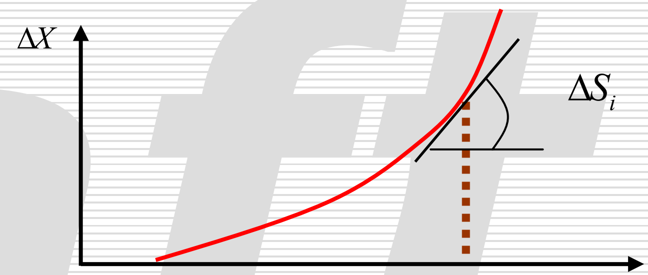
Strip Deformation

Effect of ***Scale*** *Transfer Errors*

We have shown briefly that constant scale transfer errors cause 2 order coordinate errors along the strip axis.

$$\Delta X_S^{\Sigma} = a_0 + a_1.X + a_2.X^2$$





$$\Delta S_i = \frac{\partial(\Delta X)}{\partial X} = a_1 + 2a_2.X$$

$$\Delta X = a_0 + a_1 X + a_2 X^2$$

$$\Delta S_i = a_1 + 2a_2 X$$

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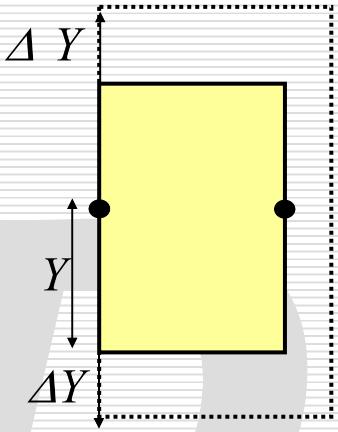
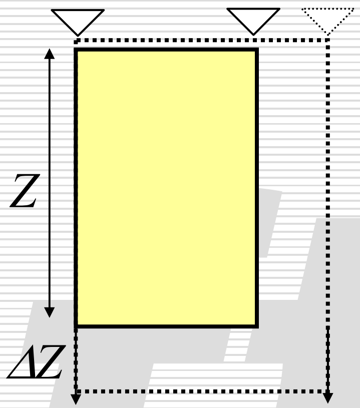
Strip Deformation

Effect of **Scale** Transfer Errors

The following diagrams show the total effect of scale transfer errors on points outside strip axis.





$$\Delta Y_S^{\Sigma} = Y \cdot \Delta S = Y \cdot (a_1 + 2a_2.X)$$

$$\Delta Z_S^{\Sigma} = Z \cdot \Delta S = Z \cdot (a_1 + a2_2.X)$$

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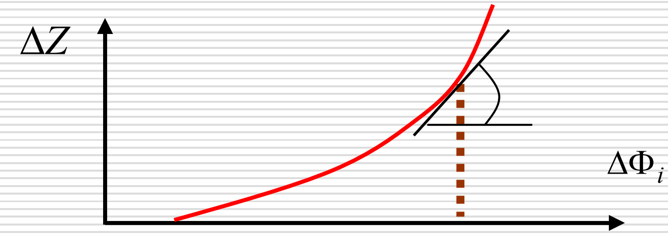
Strip Deformation

Effect of **Longitudinal tilt (Φ)** Transfer Errors

Constant φ transfer errors cause 2 order ΔZ coordinate errors along the strip axis.

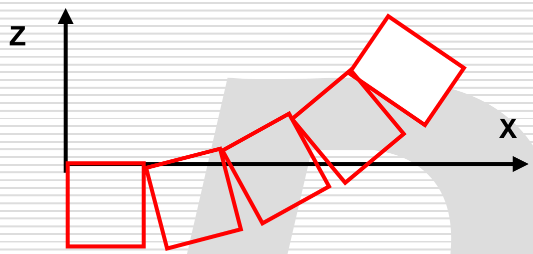
$$\Delta Z_{\Phi}^{\Sigma\Sigma} = c_0 + c_1.X + c_2.X^2$$

$$\Delta\phi_i = \frac{\partial(\Delta Z)}{\partial X} = c_1 + 2c_2.X$$



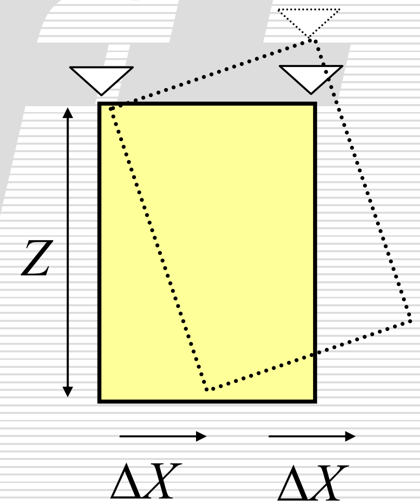
$$\Delta Z = c_0 + c_1 X + c_2 X^2$$

$$\Delta\Phi_i = c_1 + 2c_2 X$$



The following diagram shows the total effect of φ transfer error including the effect on points outside the strip axis.

$$\Delta X_{\Phi}^{\Sigma} = Z \cdot \Delta\Phi = Z \cdot (c_1 + 2c_2.X)$$



φ errors dose not affect y-coordinates

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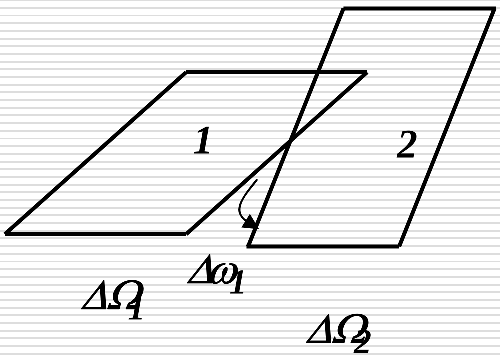
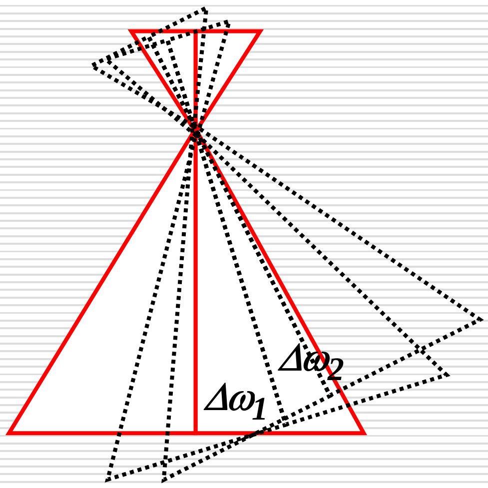
Strip Deformation

*Effect of **Lateral tilt (Ω)** Transfer Errors*





Δω transfer errors dose not affect the strip axis.

$$\Delta Z_{\Omega}^{\Sigma\Sigma} = 0$$

As $\Delta\Omega_i$ is obtained from single summation of Δω transfer errors, $\Delta\Omega_i$ ($\Delta\Omega_i = \Sigma\, \Delta\omega_i$) Can be represented by a linear function.

$$\Delta\Omega_i = d_1 + 2d_2.X$$

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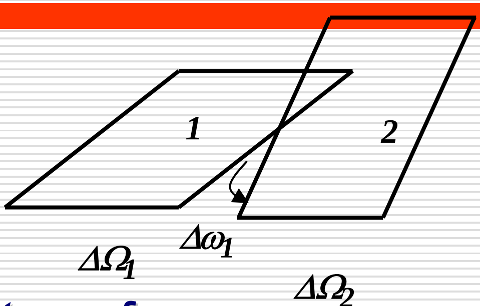
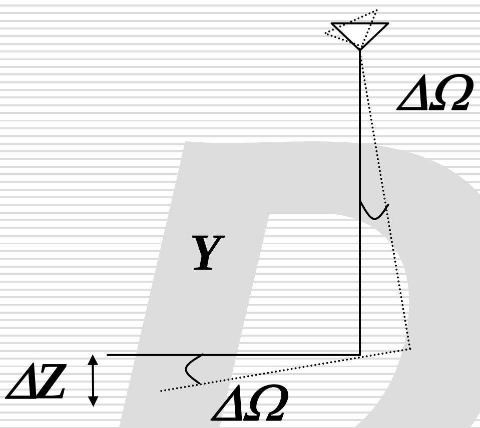
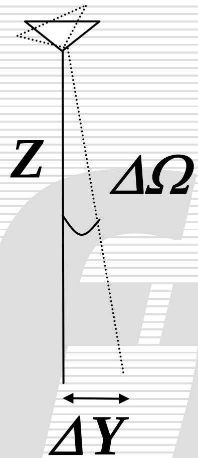
Strip Deformation

Effect of **Lateral tilt (Ω)** Transfer Errors



The following diagrams show the total effect of lateral transfer errors on points outside strip axis.





$$\Delta Z_{\Omega}^{\Sigma} = Y \cdot \Delta S = Y \cdot (d_1 + 2d_2.X)$$

$$\Delta Y_{\Omega}^{\Sigma} = -Z \cdot \Delta S = -Z \cdot (d_1 + 2d_2.X)$$

Ω affects only y, z strip coordinates.

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Strip Deformation

Summary

Polynomials to correct systematic strip triangulation errors.

$$\Delta X = a_0 + a_1.X + a_2.X^2 - Y(b_1 + 2b_2.X) + Z(c_1 + 2c_2.X)$$

$$\Delta Y = b_0 + b_1.X + b_2.X^2 + Y(a_1 + 2a_2.X) + Z(d_1 + 2d_2.X)$$

$$\Delta Z = c_0 + c_1.X + c_2.X^2 + Y(d_1 + 2d_2.X) + Z(a_1 + 2a_2.X)$$

Specialization of Formula for Flat Terrain

In case the terrain is relatively flat. i.e. all points have the same height (z=constant), then the above equations are reduced to the following.

Plan:

$$\Delta X = a_0 + a_1.X + a_2.X^2 - Y(b_1 + 2b_2.X)$$

$$\Delta Y = b_0 + b_1.X + b_2.X^2 + Y(a_1 + 2a_2.X)$$

Height:

$$\Delta Z = c_0 + c_1.X + c_2.X^2 + Y(d_1 + 2d_2.X)$$

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Strip Deformation

Specialization of Formula for Flat Terrain

Notice that height adjustment can be separated from planimetric adjustment.

For systematic errors we proved that 2 order polynomials can be used, but for random errors it is quite difficult to find the most convenient polynomial.

11 Transformation parameters have to be determined, for flat terrain one can see that the parameters are divided two groups.

Six parameters, a_0, a_1, a_2, b_0, b_1, b_2 which appear only in planimetric equations.

Five parameters , c_0, c_1, c_2, ,c_3, c_4 which appear only in height equations. (c_3 , c_4 define the change of Ω along the strip)

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Observation Equations

Observation Equations

(1) Two order (x, y, z) polynomials for flat terrain

$$\begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix} = \begin{bmatrix} 1 & x & x^2 & 0 & -y & -2xy & 0 & z & 2xz & 0 & 0 \\ 0 & y & 2xy & 1 & x & x^2 & 0 & 0 & 0 & -z & -2xz \\ 0 & z & 2xz & 0 & 0 & 0 & 1 & x & x^2 & y & 2xy \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ b_0 \\ b_1 \\ b_2 \\ c_0 \\ c_1 \\ c_2 \\ d_1 \\ d_2 \end{bmatrix}$$

(2) Two order (x, y) polynomials for flat terrain

$$\begin{bmatrix} \Delta X \\ \Delta Y \end{bmatrix} = \begin{bmatrix} 1 & x & x^2 & 0 & -y & -2xy \\ 0 & y & 2xy & 1 & x & x^2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ b_0 \\ b_1 \\ b_2 \end{bmatrix}$$

(3) Two order (z) polynomials for flat terrain

$$\begin{bmatrix} \Delta Z \end{bmatrix} = \begin{bmatrix} 1 & x & x^2 & y & 2xy \end{bmatrix} \begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix}$$

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The Unknowns

(1) Unknown coordinates (c)

The (x,y) and (z) coordinates of tie points.

(2) The unknown parameters (p)

The number of parameters strip depends on the type and order of polynomials chosen for adjustment.

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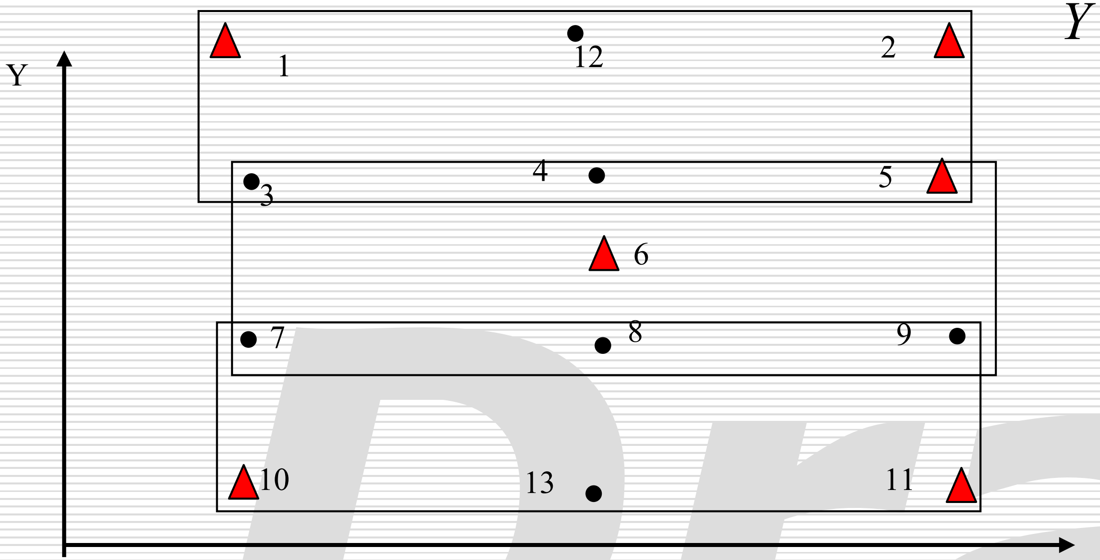
Example for Polynomial Adjustment

Given the following small block of 3 strips to be adjusted in planimetry using the 2 order polynomials.

These correction equation are linear.

$$X = a_0 + a_1.x + a_2.x^2 - y(b_1 + 2b_2.x)$$

$$Y = b_0 + b_1.x + b_2.x^2 + y(a_1 + 2a_2.x)$$



Notice that 12 and 13 are single points, therefore they will not be used in the adjustment and all control points (1,2,5,6,10,11) will be used where absolute discrepancies have to be minimized.

All tie points (3,4,7,8,9) will be used where relative discrepancies have to be minimized.

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Example for Polynomial Adjustment

Inter-Relation Table

Pts	Strips of Appearance			Freq	Type
1	S^1			-	Plan
2	S^1			-	Plan
3	S^1	S^2		2	Tie
4	S^1	S^2		2	Tie
5	S^1	S^2		2	Plan
6		S^2		-	Plan
7		S^2	S^3	2	Tie
8		S^2	S^3	2	Tie
9		S^2	S^3	2	Tie
10			S^3	-	Plan
11			S^3	-	Plan
12	S^1			1	Single
13			S^3	1	Single

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Example for Polynomial Adjustment

The unknowns

(1) Unknown parameters

$P = 3 \text{ strip} \times 6 \text{ parameters} = 18$

(2) Unknown coordinates

$C = 5 \text{ tie points} \times 2 \text{ coordinates} = 10$

$P + C = 10 + 18 = 28$

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Example for Polynomial Adjustment

The observation equations

(1) For each control points in each strip

$$A_{i,j}.P_j = C_i$$

$$\begin{bmatrix} 1 & x & x^2 & 0 & -y & -2xy \\ 0 & y & 2xy & 1 & x & x^2 \end{bmatrix}_{i,j} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ b_0 \\ b_1 \\ b_2 \end{bmatrix}_j = \begin{bmatrix} x \\ y \end{bmatrix}_i$$

(2) For each tie points

$$A_{i,j}.P_j - I \cdot C_i = 0$$

$$\begin{bmatrix} 1 & x & x^2 & 0 & -y & -2xy \\ 0 & y & 2xy & 1 & x & x^2 \end{bmatrix}_{i,j} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ b_0 \\ b_1 \\ b_2 \end{bmatrix}_j - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}_i = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

i *to indicate point code number,* ***j*** *to indicate strip code number,* ***A_{ij}*** *is the coefficient matrix,* ***P_j*** *is the vector of the given terrestrial coordinates of control points* ***I*** *.*

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Example for Polynomial Adjustment

The observation equations

Total number of obseirvation:

Strip1:

$3 \text{ control} \times 2 + 2 \text{ tie} \times 2 = 10$

Strip2:

$2 \text{ control} \times 2 + 5 \text{ tie} \times 2 = 14$

Strip3:

$2 \text{ control} \times 2 + 3 \text{ tie} \times 2 = 10$

Total = 10 + 14 + 10 = 34

Redundancy = 34 - 28 = 6

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Example for Polynomial Adjustment

The observation equation matrix

$$
\left[\begin{array}{ccc|ccccc}
A_{1,1} & & & & & & & \\
A_{2,1} & & & & & & & \\
A_{3,1} & & & -I & & & & \\
A_{4,1} & & & & -I & & & \\
A_{5,1} & & & & & & & \\
\hline
 & A_{3,2} & & -I & & & & \\
 & A_{4,2} & & & -I & & & \\
 & A_{5,2} & & & & & & \\
 & A_{6,2} & & & & & & \\
 & A_{7,2} & & & & -I & & \\
 & A_{8,,2} & & & & & -I & \\
 & A_{9,2} & & & & & & -I \\
\hline
 & & A_{7,3} & & & -I & & \\
 & & A_{8,3} & & & & -I & \\
 & & A_{9,3} & & & & & -I \\
 & & A_{10,3} & & & & & \\
 & & A_{11,3} & & & & &
\end{array}\right]
*
\left[\begin{array}{c}
P_1 \\ P_1 \\ P_1 \\ \hline C_3 \\ C_4 \\ C_7 \\ C_8 \\ C_9
\end{array}\right]
=
\left[\begin{array}{c}
C_1 \\ C_2 \\ 0 \\ 0 \\ C_5 \\ \hline 0 \\ 0 \\ C_5 \\ C_6 \\ 0 \\ 0 \\ 0 \\ \hline 0 \\ 0 \\ 0 \\ C_{10} \\ C_{11}
\end{array}\right]
$$

Example for Polynomial Adjustment

The Normal equation in matrix notation

$$
\left[\begin{array}{ccc|ccccc}
\sum\limits_{i=1,2,3,4,5} A_{i,1}^T \cdot A_{i,1} & & & -A_{3,1}^T & -A_{4,1}^T & & & \\
& \sum\limits_{i=3,4,5,6,7,8,9} A_{i,2}^T \cdot A_{i,2} & & -A_{3,2}^T & -A_{4,2}^T & -A_{7,2}^T & -A_{8,2}^T & -A_{9,2}^T \\
& & \sum\limits_{i=7,8,9,10,11} A_{i,3}^T \cdot A_{i,3} & & & -A_{7,3}^T & -A_{8,3}^T & -A_{9,3}^T \\
\hline
-A_{3,1} & -A_{3,2} & & 2I & & & & \\
-A_{4,1} & -A_{4,2} & & & 2I & & & \\
& -A_{7,2} & -A_{7,3} & & & 2I & & \\
& -A_{8,2} & -A_{8,3} & & & & 2I & \\
& -A_{9,2} & -A_{9,3} & & & & & 2I
\end{array}\right]
\cdot
\left[\begin{array}{c}
P_1 \\ P_2 \\ P_3 \\ \hline C_3 \\ C_4 \\ C_7 \\ C_8 \\ C_9
\end{array}\right]
=
\left[\begin{array}{c}
\sum\limits_{k=1,2,5} A_{k,1} C_k \\
\sum\limits_{k=5,6} A_{k,2} C_k \\
\sum\limits_{k=10,11} A_{k,3} C_k \\
\hline
0 \\ 0 \\ 0 \\ 0 \\ 0
\end{array}\right]
$$

Reduced Normal Equations

The normal equations are function of two sets of unknowns, P (Parameters) and C (adjusted coordinates of tie points).

Consequently the normal equation can be partitioned two sets of equations.

$$\begin{bmatrix} N_{11} & N_{21}^T \\ N_{21} & N_{22} \end{bmatrix} \cdot \begin{bmatrix} P \\ C \end{bmatrix} = \begin{bmatrix} F_1 \\ 0 \end{bmatrix}$$

Usually the set of unknown coordinates C is eliminated such that we get the reduced normal equations function of the set of parameters P.

Why the reduced normal equation?

(1) *The number of unknowns in the R.N.E is much less than in the normal equations.*

(2) *The coefficient matrix of the R.N.E is usually banded therefore*

- ***(i)*** *It requires less memory*
- ***(ii)*** *It takes shorter computation time*

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Reduced Normal Equations

Usually the R.N.E in terms of the unknown parameters P is formed and solved because:

(a) In most blocks the number of unknown coordinates C is much larger than that of parameters.

(b) The number of parameters P is proportional to the number of units to be adjusted. Therefore, it is easy to predict the size of R.N.E and memory required.

$$N_{11} \cdot P + N_{21}^T \cdot C = F_1 \qquad (1)$$

$$N_{21} \cdot P + N_{22} \cdot C = 0 \qquad (2)$$

To eliminate C multiply equations (2) by $\left[N_{22}^{-1}\right]$, $\left[N_{22}\right]$ *is square and non-singular.*

$$N_{22}^{-1} \cdot N_{21} \cdot P + \frac{N_{22}^{-1}.N_{22}}{I} \cdot C = O$$

Substituting (3) in (1) we get

$$C = -N_{22}^{-1} \cdot N_{21}.P \qquad (3)$$

$$\left[N_{11} - N_{21}^T \cdot N_{22}^{-1} \cdot N_{21}\right] \cdot P = F_1$$

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Reduced Normal Equations

The reduced normal equations, which correspond to our example, will then contain 18 equations in 18 unknown parameters.

$$\begin{bmatrix} N_{11}^R & N_{21}^R & \\ N_{12}^R & N_{22}^R & N_{23}^R \\ & N_{32}^R & N_{33}^R \end{bmatrix} \cdot \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix} = \begin{bmatrix} T_1 = \sum A_{K,1}^T \cdot C_K \\ T_2 = \sum A_{K,2}^T \cdot C_K \\ T_3 = \sum A_{K,3}^T \cdot C_K \end{bmatrix} \begin{matrix} k = 1,2,5 \\ k = 5,6 \\ k = 10,11 \end{matrix}$$

Notice that the R.N.E has a banded coefficient matrix.

Direct generation of R.N.E

The 3 types of sub matrices in the R.N.E are

(1) Main-diagonal sub matrix $N_{j,j}^R$

(2) Off-diagonal sub matrix $N_{j,l}^R$

(3) Constant terms sub matrix T_j^R

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Reduced Normal Equations

Direct generation of R.N.E

Main diagonal sub matrix $N^R_{j,j}$

$$\boxed{N^R_{j,j} = \underbrace{\sum}_{\text{all points in strip } j \text{ eexcluding single point}} A^T_{i,j} \cdot A_{i,j} - \underbrace{\sum}_{\text{all tie point in strip } j} \frac{1}{f_t} A^T_{i,j} \cdot A_{i,j}}$$

all points in strip j eexcluding single point

all tie point in strip j

f_t =Frequency of tie point (number of units in which the tie point appear)

$$\therefore \quad N^R_{1,1} = A^t_{1,1} \cdot A_{1,1} + A^t_{2,1} \cdot A_{2,1} + A^t_{3,1} \cdot A_{3,1} + A^t_{4,1} \cdot A_{4,1} + A^t_{5,1} \cdot A_{5,1} - \frac{1}{2} A^t_{3,1} \cdot A_{3,1} - \frac{1}{2} A^t_{4,1} \cdot A_{4,1}$$

$$Where \quad A_{3,1} = \begin{bmatrix} 1 & x_{3,1} & x^2_{3,1} & 0 & -y_{3,1} & -2x_{3,1}y_{3,1} \\ 0 & y_{3,1} & 2x_{3,1}y_{3,1} & 1 & x_{3,1} & x^2_{3,1} \end{bmatrix}$$

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Reduced Normal Equations

Direct generation of R.N.E

Off- diagonal sub matrix $N^R_{j,l}$

$$N^R_{j,l} = -\sum \frac{1}{f_t} A^T_{t,j} \cdot A_{t,l}$$

Tie points common to strip j, l

The Coefficient of the off-diagonal sub-matrix $N_{j,l}$ are computed from the cords of tie points common to unit j and l

$A_{t,j}$: Coefficient matrix of point t as measured in strip j.

$Z_{t,l}$: Confident matrix of tie point t as measured in strip l

$N^R_{j,l} = 0$, if there are no tie points common to strips j and l

$$\therefore \quad N^R_{1,2} = -\frac{1}{2} A^t_{3,1} \cdot A_{3,2} - \frac{1}{2} A^t_{4,1} \cdot A_{4,2}$$

$$\therefore \quad N^R_{1,3} = 0, \qquad \text{No tie points common between strip 1 and 3}$$

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Reduced Normal Equations

Direct generation of R.N.E

Right hand side sub matrix T_j^R

The coefficients of R.H.S sub-matrix T_j are computed from the coordinates of control points which appear in strip j

$$\boxed{T_j^R = \sum A_{k,j}^T \cdot C_k}$$

Where

$A_{k,j}$: coefficient matrix of control point k with coordinates measured in unit j

C_k : ground coordinates of control points.

$$T_2^R = A_{5,2}^t \cdot C_5 + A_{6,2}^t \cdot C_6$$

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Flow Diagram of Polynomial Block Adjustment

Choice of the Transformation Equations to be Used:

- 2nd or 3nd order Polynomial
- Conformal or Non-Conformal
- (x,y,z) Simultaneously
- (x,y) Separated From (Z)

↓

Input of the Data:

- Strip Coordinates, one Strip after the Other
- Ground Coordinates for the Control Points

↓

Forming of Interrelation Table Required for the Generation of the Reduced Normal Equations

↓

Direct Generation of the Reduced Normal Equations

- Main-Diagonal Sub-Matrixes
- Off-Diagonal Sub-Matrixes
- R.H.S Sub-Matrixes

↓

Direct Solution of the RNE, i.e. Computation of Transformation Parameters

↓

Transformation of Each Strip

↓

Coordinate Computation

- Computation of Mean Coordinates of Tie Points
- Computation of Absolute & Relative Discrepancies and Standard Deviation

Photogrammetric Block Adjustment

Terrestrial Coordinates

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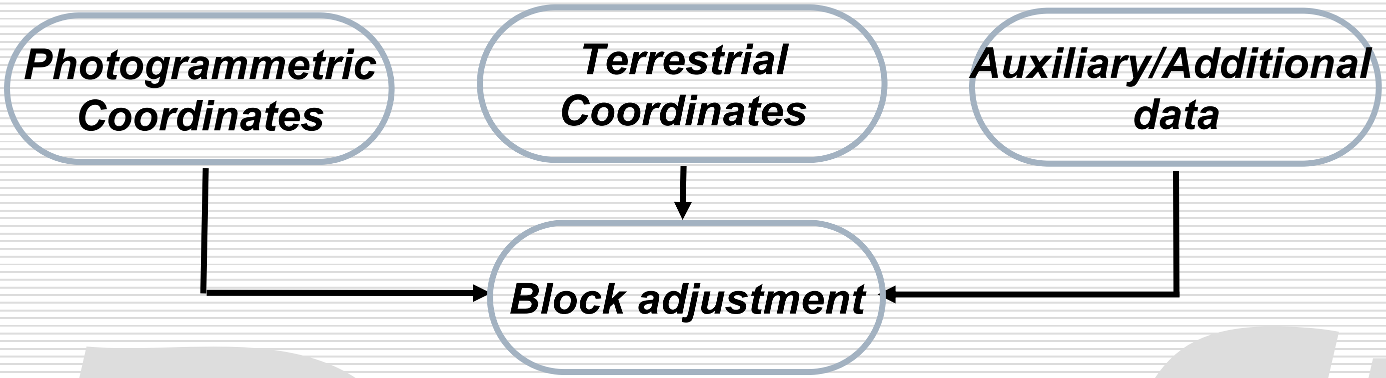
Introduction

In block adjustment we have three main groups of observations with different stochastical properties



It is obvious that such an adjustment can best be done if the proper weights are associated to each group of similar observation.

The terrestrial coordinates E,N,H should not be treat as error-free quantities or as observations of weight infinity since they are not in reality free from errors.

E,N,H : Terrestrial coordinates of ground control.

x,y,z : Photogrammetric model coordinates.

X,Y,Z : Unknown coordinates

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Observation equation

1- For full Control

$$\begin{bmatrix} V_E \\ V_N \\ V_H \end{bmatrix}_i + \begin{bmatrix} E \\ N \\ H \end{bmatrix}_i = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_i$$

2- For Planimetric Control

$$\begin{bmatrix} V_E \\ V_N \end{bmatrix}_i + \begin{bmatrix} E \\ N \end{bmatrix}_i = \begin{bmatrix} X \\ Y \end{bmatrix}_i$$

3- For Altimetry Control

$$[VH]_i + [H]_i = [Z]_i$$

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Example for M-4

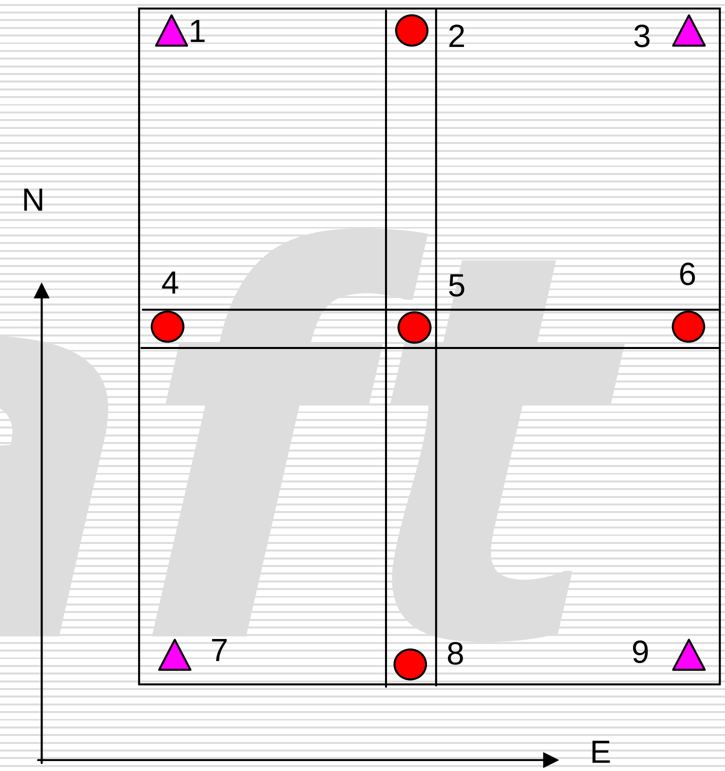
Given the following small block of 4 models to be adjusted taking into account that the:

1- Weight matrix for **transformed model coordinates**:

$$g = \begin{bmatrix} g_{xx} & g_{xy} \\ g_{xy} & g_{yy} \end{bmatrix}$$

2- Weight matrix for **terrestrial coordinates:**

$$G = \begin{bmatrix} G_{EE} & G_{EN} \\ G_{EN} & G_{NN} \end{bmatrix}$$



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Example for M-4

Observation equations:

1- from **Photogrammetric** data

$$\begin{bmatrix} V_X \\ V_Y \end{bmatrix}_{ij} + \begin{bmatrix} X & -Y & I & O \\ Y & X & O & I \end{bmatrix}_{ij} \begin{bmatrix} a \\ b \\ x_o \\ y_o \end{bmatrix}_j = \begin{bmatrix} x \\ y \end{bmatrix}_i$$

2- from **Terrestrial** data

$$\begin{bmatrix} V_E \\ V_N \end{bmatrix}_i + \begin{bmatrix} E \\ N \end{bmatrix}_i = \begin{bmatrix} X \\ Y \end{bmatrix}_i$$

Number of observation

1- for model

model 1: 4 points x 2 = 8, model 2: 4 points x 2 = 8,
model 3: 4 points x 2 = 8, model 2: 4 points x 2 = 8.

2- for control points

4 x 2 = 8

Total = 8 + 4 x 8 = 40

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Example for M-4

The unknowns:

Parameters

P=4 model x 4 parameteres = 16

Unknown coordinates

C= 9 points x 2 coordinates = 18

Total = P+C = 18+16 = 34

Redundancy: 40 – 34 = 6

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Observation equation (matrix)

T: *Stands for E, N* **I :** *Unit Matrix* **C:** *Stands for X , Y*

PARAMETER				TIE					CONTROL							
$-A_{11}$									I					P_1		0
$-A_{21}$				I										P_2		0
$-A_{41}$					I									P_3		0
$-A_{51}$						I								P_4		0
	$-A_{22}$			I										C_2		0
	$-A_{32}$									I				C_4		0
	$-A_{52}$				I									C_5		0
	$-A_{62}$					I							·	C_6	=	0
		$-A_{43}$			I									C_8		0
		$-A_{53}$				I								C_1		0
		$-A_{73}$									I			C_7		0
		$-A_{83}$						I						C_8		0
			$-A_{54}$			I										0
			$-A_{64}$				I									0
			$-A_{84}$					I								0
			$-A_{94}$									I				0
									I							T_1
										I						T_3
											I					T_4
												I				T_9

Normal equation (matrix)

$$N_{11} \cdot P + N_{21}^{T} \cdot C + N_{31}^{T} \cdot C_{T} = 0$$

$$N_{21} \cdot P + N_{22} \cdot C = 0$$

$$N_{31} \cdot P + N_{33} \cdot C_{T} = F$$

$N_{22} = f \cdot g$ f = frequency, (For Tie points)

$N_{33} = f \cdot g + G$ (Control points)

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Normal equation (matrix)

$\sum A^t_{i1} g A_{i1}$ 1,2,4,5				N^T_{21}					N^T_{31}					P_1		0
	$\sum A^t_{i2} g A_{i2}$ 2,3,5,6													P_2		0
		$\sum A^t_{i3} g A_{i3}$ 4,5,7,8												P_3		0
			$\sum A^t_{i4} g A_{i4}$ 5,6,8,9										*	P_4	=	0
$-gA_{21}$	$-gA_{22}$			2g										C_2		
$-gA_{41}$		$-gA_{43}$			2g									C_4		
$-gA_{51}$	$-gA_{52}$	$-gA_{53}$	$-gA_{54}$			4g								C_5		0
	$-gA_{62}$		$-gA_{64}$				2g							C_6		
		$-gA_{83}$	$-gA_{84}$					2g						C_8		
$-gA_{11}$									g+G					C_1		$G.T_1$
	$-gA_{32}$									g+G				C_3		$G.T_3$
		$-gA_{73}$									g+G			C_7		$G.T_7$
			$-gA_{94}$									g+G		C_9		$G.T_9$

Reduced normal equations

The reduced normal equation are obtained by eliminating the two sets of unknown coordinates.

1. C Unknown coordinates of tie points.
2. C_T Unknown coordinates of terrestrial.

$$C = -N_{22}^{-1} \,.\, N_{21} \,.\, P$$

$$C_T = -N_{33}^{-1} \,.\, N_{31} \,.\, P + N_{33}^{-1} \,.\, F$$

$$\left(N_{11} - N_{21}^{T} \; N_{22}^{-1} \; N_{21} - N_{31}^{T} \; N_{33}^{-1} \; N_{31}\right) \cdot P = -N_{31}^{T} \cdot N_{33}^{-1} \cdot F$$

All points tie points control points control points

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Reduced normal equations

General of R.N.E sub-matrix :

(1) Main- diagonal sub-matrix

$$N_{jj} = \sum A_{ij}^{T}.g.A_{ij} - \sum \frac{1}{f} A_{ij}^{T}.g.A_{ij} - \sum A_{ij}^{T}.g.(f \cdot g + G)^{-1}.g.A_{ij}$$

All points in model j

Tie points in model j

control point in model j

(2) Off – diagonal sub matrix

$$N_{jl} = - \sum \frac{1}{f}.A_{ij}^{T}.g.A_{il}$$

Tie points common to Models j & l

(3) Constant terms sub- matrix

$$R_{j} = \sum A_{ij}^{T} \cdot g \cdot (f \cdot g + G)^{-1} \cdot G \cdot T_{i}$$

Control points in model j

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Photogrammetric Block Adjustment

Accuracy Assessment

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Factors Affecting Accuracy of B.A.

1. The scale of photography and flying height.
2. The type of used camera
3. Stability of photographic material and type of image corrections used.
4. The type of photographic coverage, number and distribution of tie points.
5. Type of triangulation and instruments used.
6. Identification and transfer of points.
7. Distribution and accuracy of control points.
8. The Use of auxiliary data.
9. The procedure of adjustment.
10. Possibility of screening gross errors.

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Factors Affecting Accuracy of B.A.

1- The scale of photography and flying height

- The smallest photo scale can be obtain by super wide angle camera from high altitudes. (Jet aeroplanes). Due to haze the image quality at the corners photographs can drop remarkably.
- The largest scale can be obtained by long focal length cameras from low altitudes.

Problems

a- Image movement

The movement of aeroplane during exposure will result in image movement especially at photo scale larger than 1:2000. Moreover lower air layers are usually more turbulent and angular movements of the camera during exposure can result in poor sharpness.

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Factors Affecting Accuracy of B.A.

1- The scale of photography and flying height

Problems

b- The camera cycle

Moving and flattening the film against the vacuum plate for a new exposure takes about 2.5 seconds.

Suppose average speed of aeroplane is 200 km/hour. One can see that 200km/h corresponds to 55m/sec. The smallest possible air-base (B) is 2.5×55= 140m. Accuracy a format of (24× 24) and 60% forward overlap then b=0.4×24= 9.2 cm. Consequently the largest photo scale will be:

$$\frac{b}{B} = \frac{9.2}{14000} \cong \frac{1}{1500}$$

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Factors Affecting Accuracy of B.A.

2- The type of used camera

a- Narrow angle

b- Normal angle

c- Wide angle

d- Supper wide angle

- Although S.W.A cameras are good from the geometrical point of view, however, experiences have shown that they are not as good as anticipated from the physical point of view.

- The main trouble is the poor image quality at the corners which is highly affected by haze especially from high altitudes.

- Some cameras are provided with reseau while others are provided with 8 fiducial marks. The 4 standard fiducial marks maybe located in different ways.

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Factors Affecting Accuracy of B.A.

3- Stability of photographic material and type of image corrections used

- Film is nowadays used for both negative and diapositives. Special test fields and self-calibrating techniques maybe used to check the camera under actual aerial photogrammetry conditions.

4- The type of photographic coverage

- Under normal circumstance usually 60% forward and 20% side overlap of parallel strips are used. Cross strips maybe added to minimize the vertical control requirements.
- For high procession work such as geodetic network densification or for cadastral survey 60% forward and 60% side overlap maybe applied.

5- Type of triangulation and instruments used

The instrumental errors:

a- comparators 2-3 μm

b- analogue instruments 5-7 μm

c- analytical plotters

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Factors Affecting Accuracy of B.A.

6- Identification of A.T points

- Accuracy depends to a great extend on the type of point measured (signalized, natural, artificial)
- For high precision A.T. (cadastral survey) signalized points have to be used.

7- Distribution and accuracy of control points

- The accuracy of terrestrial coordinates of control points depends on the method their determination.
- In cadastral survey the accuracy of terrestrial coordinates plays an important role.

8- The use of auxiliary data

- The use of auxiliary data such as the APR and statoscope can improve the result of block adjustment for topographic mapping.
- The most important thing is the simultaneous block adjustment using both photogrammetric, terrestrial and auxiliary data with appropriate weighting.

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Factors Affecting Accuracy of B.A.

9- The procedure of adjustment

The accuracy of adjustment can be affected by the procedure used:

- *a- Polynomials*
- *b- Independent models*
- *c- Bundles*

Other important factors are:

- *a- Self calibration technique*
- *b- Using auxiliary data with proper weighting*

10- The possibility of screening gross errors

The obtained the accuracy depends on the complete elimination of gross errors which is still a big challenge.

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Accuracy of Adjusted Blocks

Theoretical accuracy investigations are mainly concerned with the influence of:

- **a-** Size and shape of blocks
- **b-** Overlap (20% - 60% side overlap)
- **c-** Location of ground control points
- **d-** The adjustment method

Theoretical accuracy studies are mostly based on simplified assumptions. For example ground control coordinates are assumed to be uncorrelated and of weight 1.

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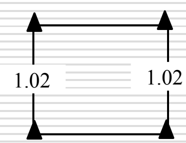
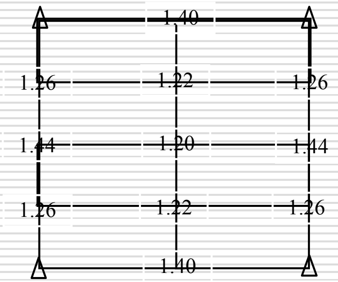
Investigation of Horizontal Ground Control Location

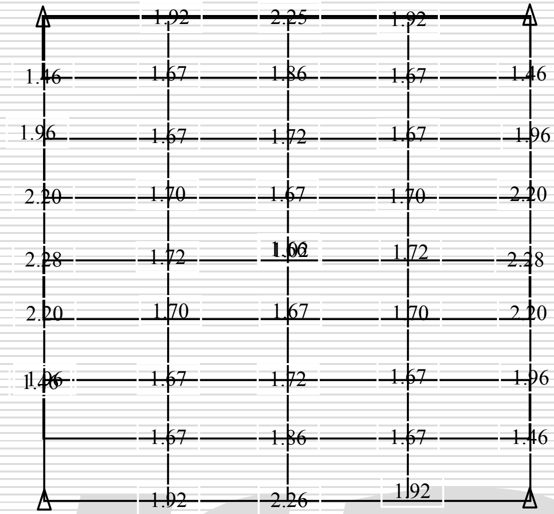
Square blocks with 4 corner control points



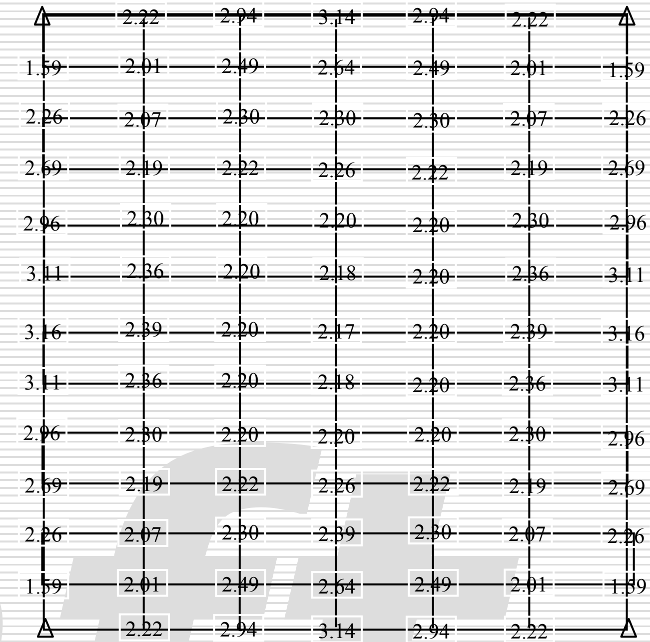
$\mu = 1.02 \cdot \sigma_0$



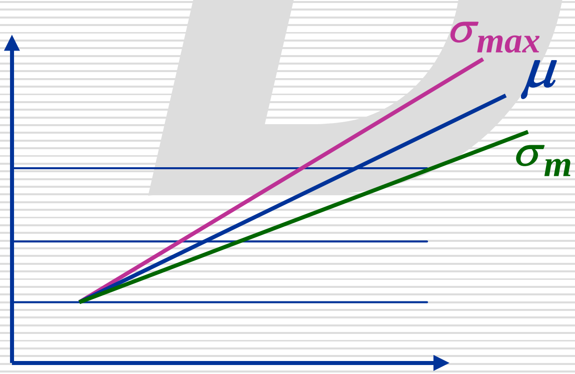
$\mu = 1.31 \cdot \sigma_0$



$\mu = 1.85 \cdot \sigma_0$



$\mu = 2.41 \cdot \sigma_0$



σ_{max} : *Mamimum Standard Deviation*

μ : *average of the whole block*

σ_m : *standard Deviation at the middle of the block*

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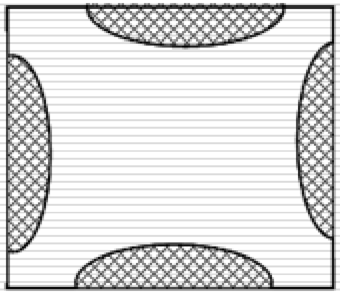
Investigation of Horizontal Ground Control Location

Square blocks with 4 corner control points



Notice that:

- *The maximum errors occur on the perimeter on the block.*
- *The middle of the block has relatively good accuracy.*

Conclusion:

- *The steep slope of all curves indicate that the accuracy deteriorate with increase of block size.*
- *The large separation between the σ_{max} and σ_m indicate that the adjusted block dose not have a homogeneous accuracy*

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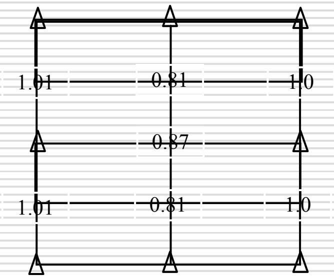
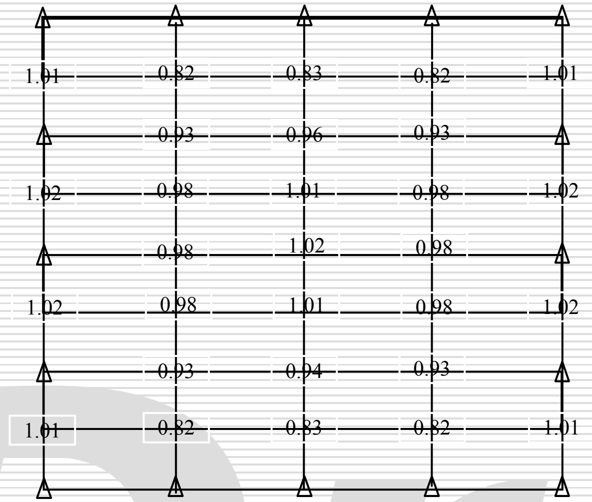
Investigation of Horizontal Ground Control Location

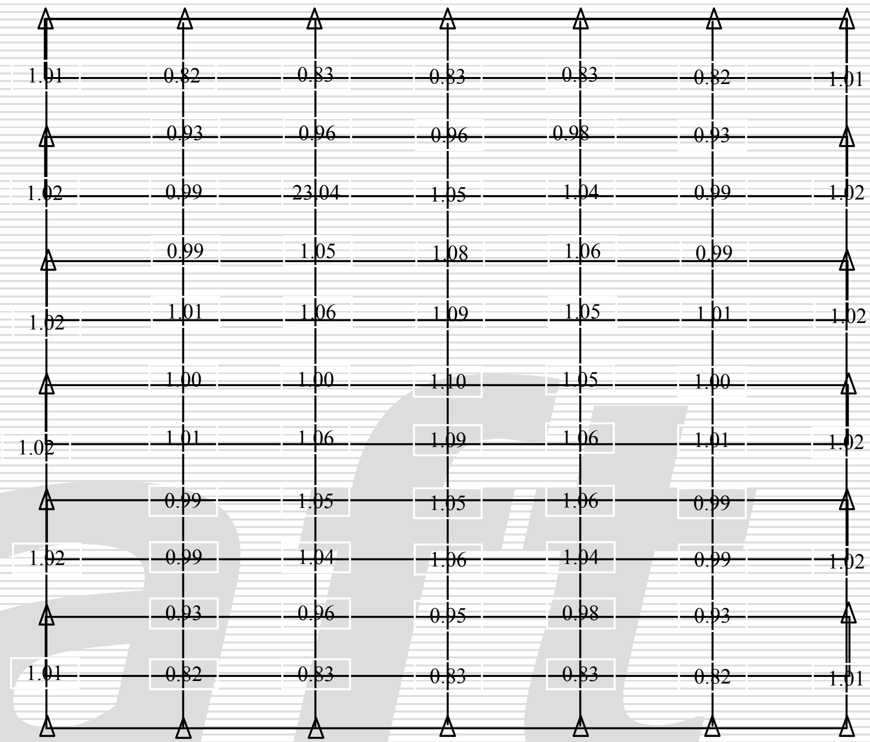
Square block with full perimeter control (l=2b)



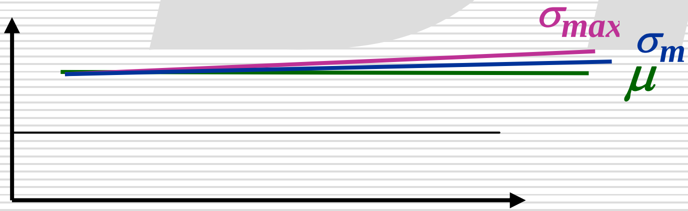
$\mu = 0.94 \cdot \sigma_0$



$\mu = 0.95 \cdot \sigma_0$



$\mu = 1.00 \cdot \sigma_0$



σ_{max} : Mamimum Standard Deviation

μ : average of the whole block

σ_m : standard Deviation at the middle of the block

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Investigation of Horizontal Ground Control Location

Square block with full perimeter control (l=2b)

Conclusions:

i- All curves are almost parallel to the x-axis, i.e. Block size has almost no effect on accuracy.

ii- σ_{max} curve is very close to σ_{mean} curve i.e. Homogenous accuracy over the whole block.

iii- All curves almost coincide with $\frac{\sigma}{\sigma_0} = 1$ Very high accuracy is obtained ($\frac{\sigma}{\sigma_0} = 1$)

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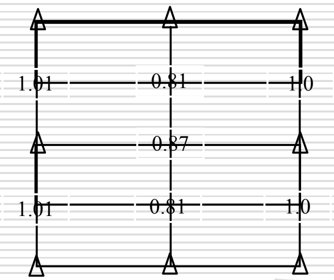
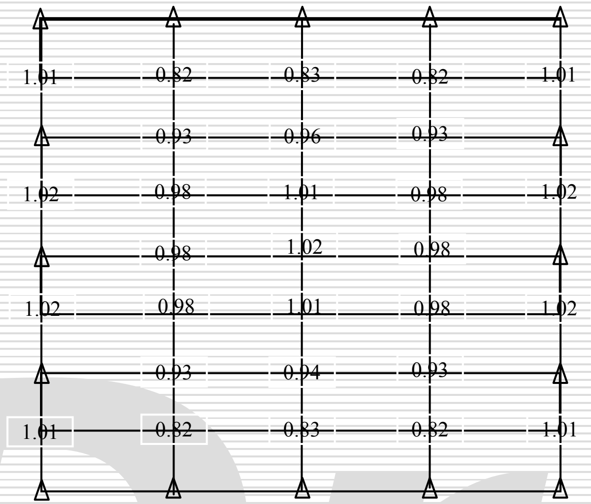
Investigation of Horizontal Ground Control Location

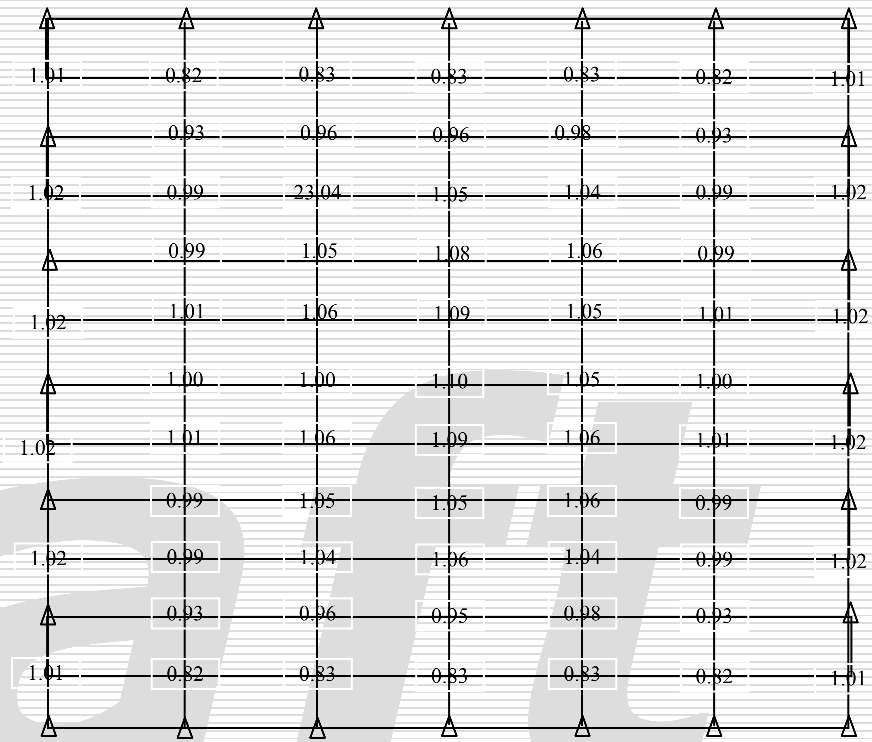
Square block with full perimeter control (l=2b)



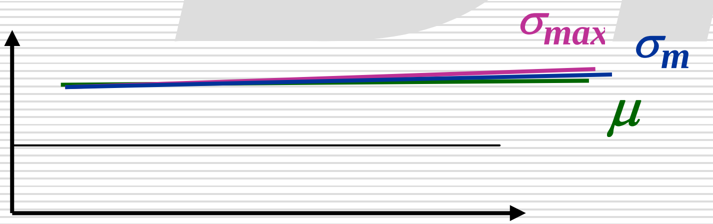
$\mu = 0.94 \cdot \sigma_0$



$\mu = 0.95 \cdot \sigma_0$



$\mu = 1.00 \cdot \sigma_0$



σ_{max} : Mamimum Standard Deviation

μ : average of the whole block

σ_m : standard Deviation at the middle of the block

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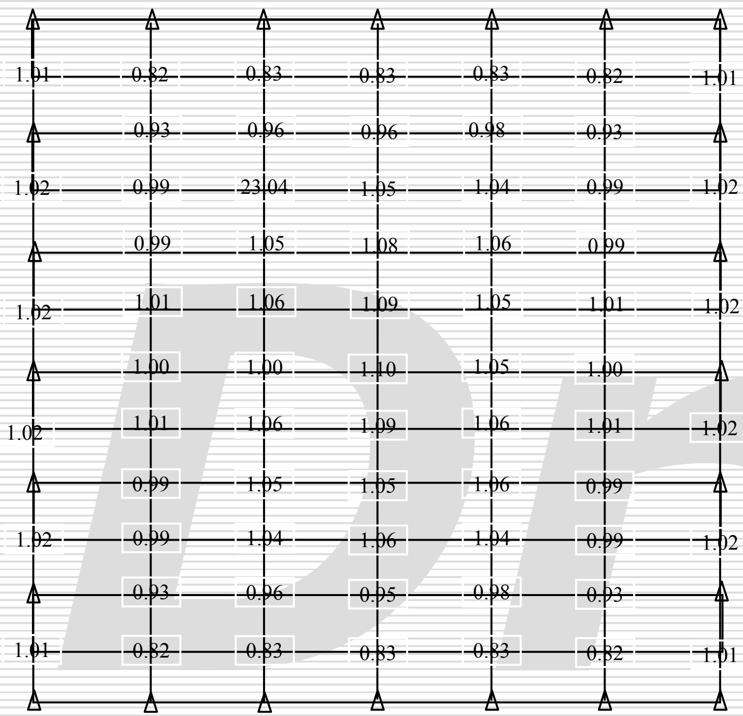
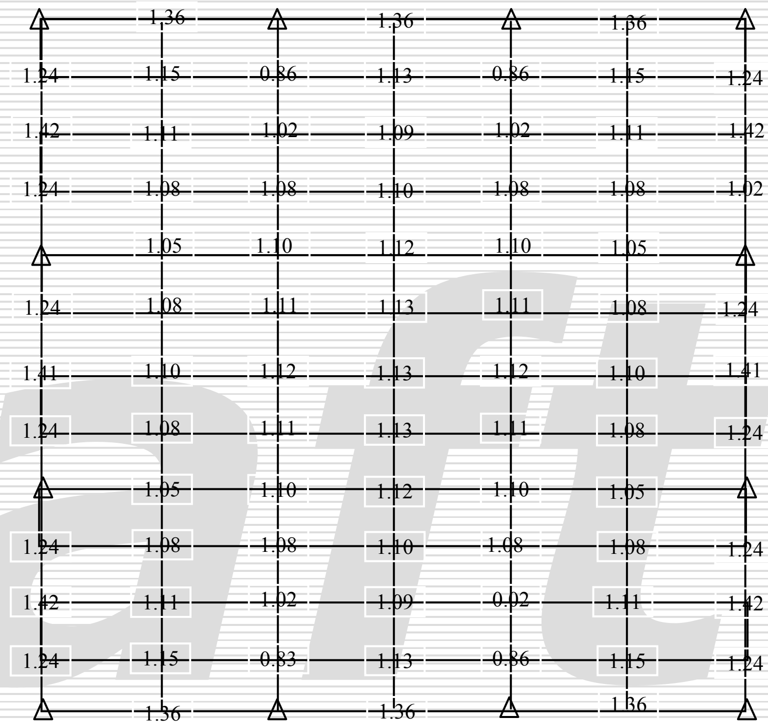
Investigation of Horizontal Ground Control Location

Relaxed Control along the Perimeter



$\mu = 1.00 \cdot \sigma_0$



$\mu = 1.16 \cdot \sigma_0$

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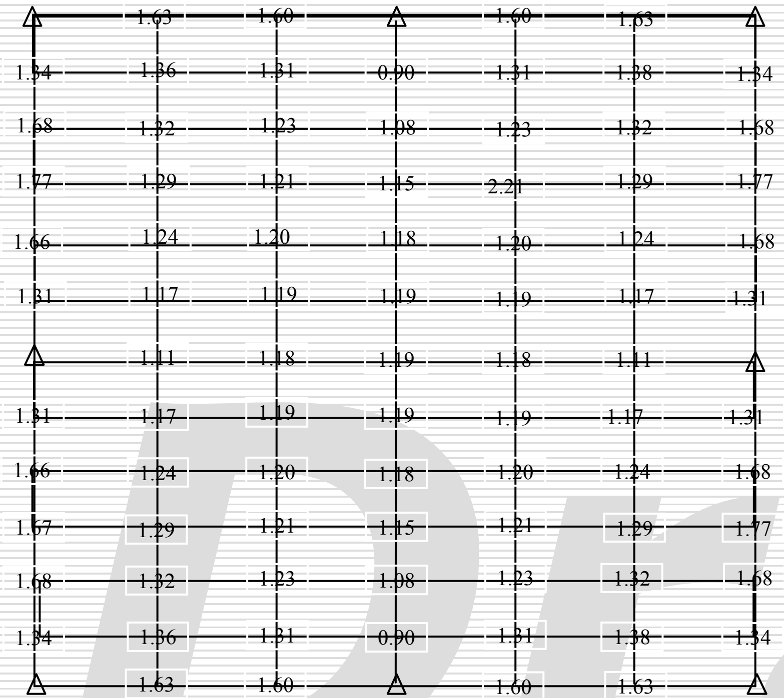
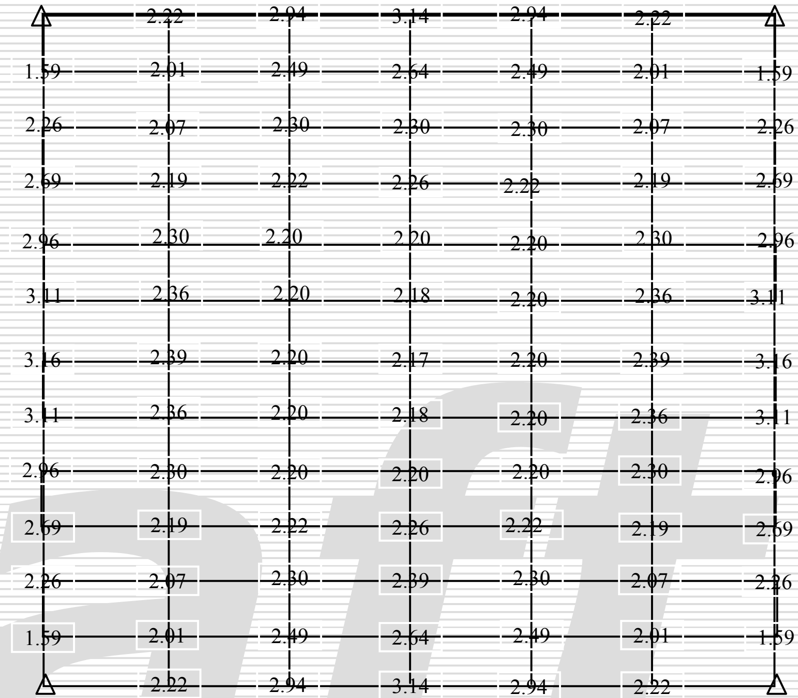
Investigation of Horizontal Ground Control Location

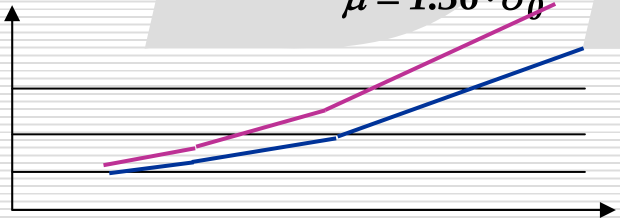
Relaxed Control along the Perimeter



$\mu = 1.36 \cdot \sigma_0$



$\mu = 2.41 \cdot \sigma_0$



Relaxation of control on perimeter cause deterioration of both relative and absolute accuracy

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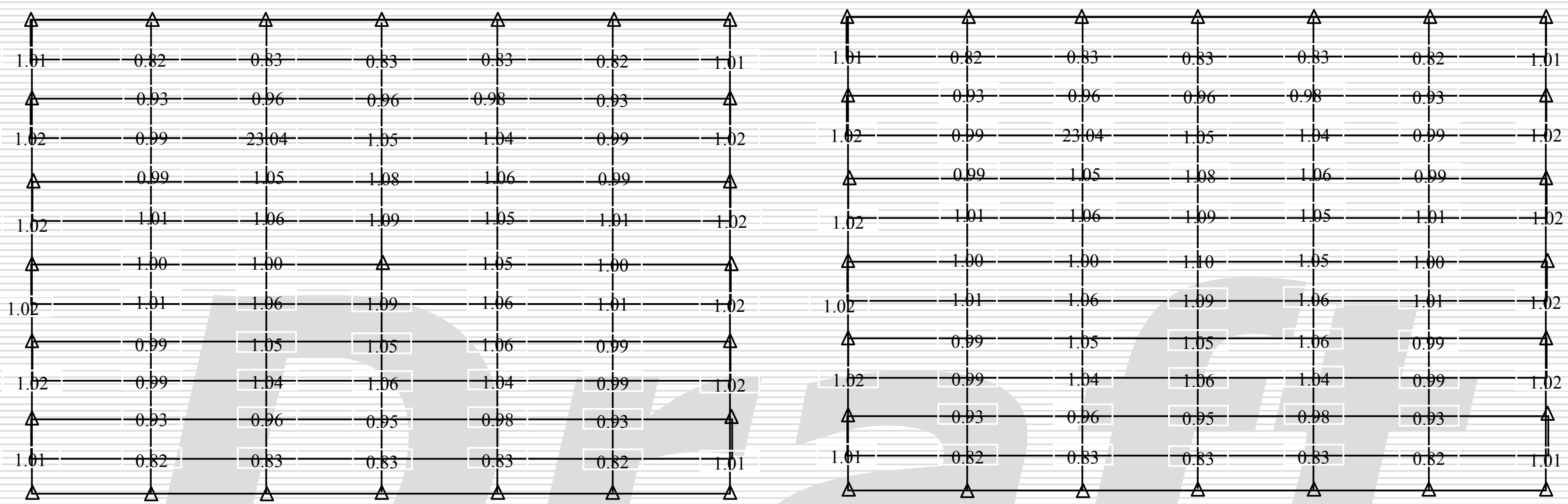
Investigation of Horizontal Ground Control Location

The Effect of additional 'Plan' Control Point in the Center of Block



$\mu = 0.96 \cdot \sigma_0$ $\mu = 1.00 \cdot \sigma_0$

Conclusions:

The above two figures show that the overall accuracy of the block did not improve significantly, i.e. 'Plan' Control in the center of block is not very useful.

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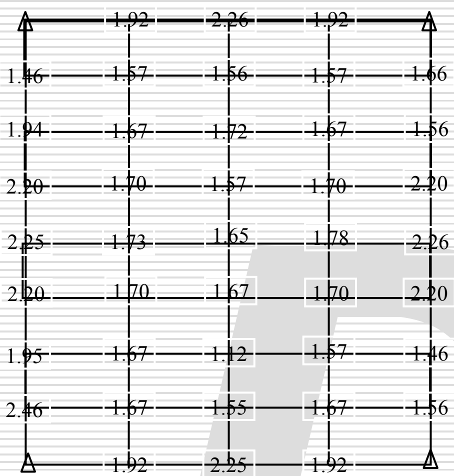
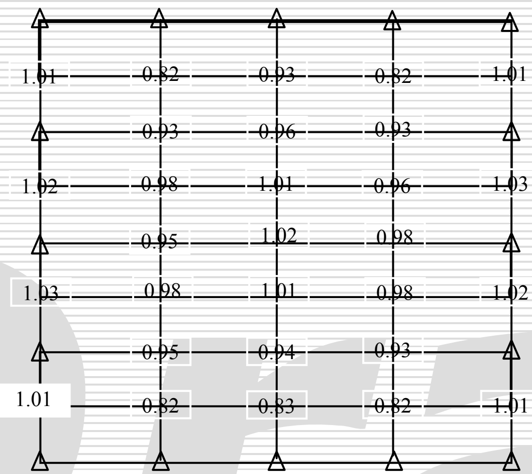
Investigation of Horizontal Ground Control Location

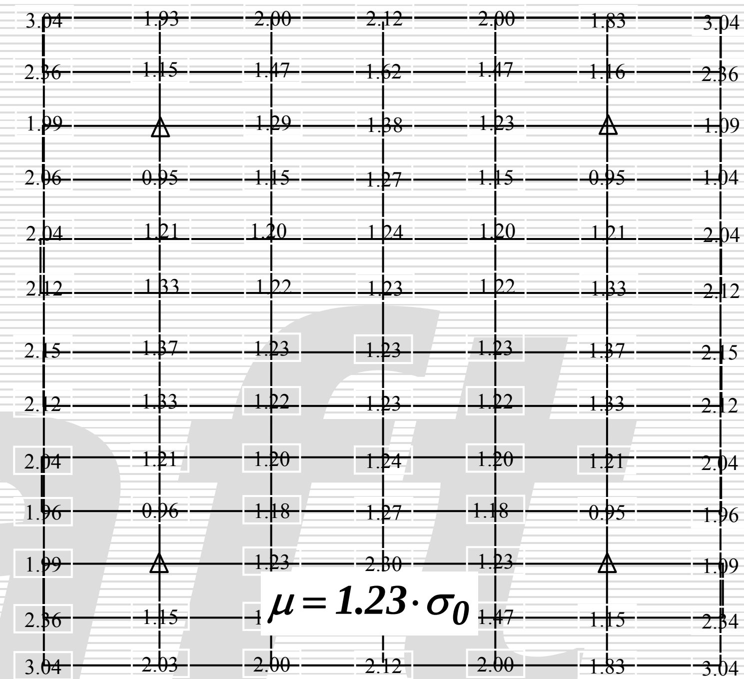
Bordering of Blocks Beyond control Points



$\mu = 1.85 \cdot \sigma_0$



$\mu = 0.97 \cdot \sigma_0$



$\mu = 1.23 \cdot \sigma_0$

$\mu = 1.71 \cdot \sigma_0$

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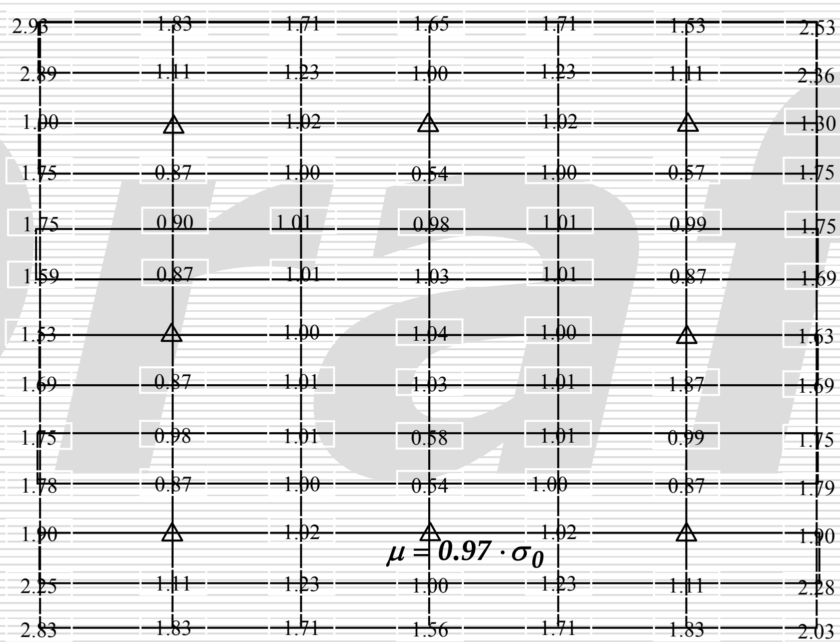
Investigation of Horizontal Ground Control Location

Bordering of Blocks Beyond control Points

Conclusions:

Comparison between I and II shows that extending the block beyond control will increase the accuracy in the inside area. Comparisons between III and IV shows that full perimeter control remains superior.

$\mu = 1.51 \cdot \sigma_0$



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Location of Height Control Points

(1) 20% side overlap

At least 3 chains of height control points are needed that to determine at the beginning, middle, end of each strip.

(2) 60% side overlap

Two chains of height control points in a grid distribution have been proposed,

however using 3 chains of height control is still recommended.

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Location of Height Control Points

Comparison between height accuracy of independent model and bundle adjustment

(1) Planimetry

(σ_{0p}) independent model $\approx 1.5(\sigma_0)$ bundle

(2) Altimetry

(σ_{0H}) independent model $\approx 2.4(\sigma_0)$ bundle

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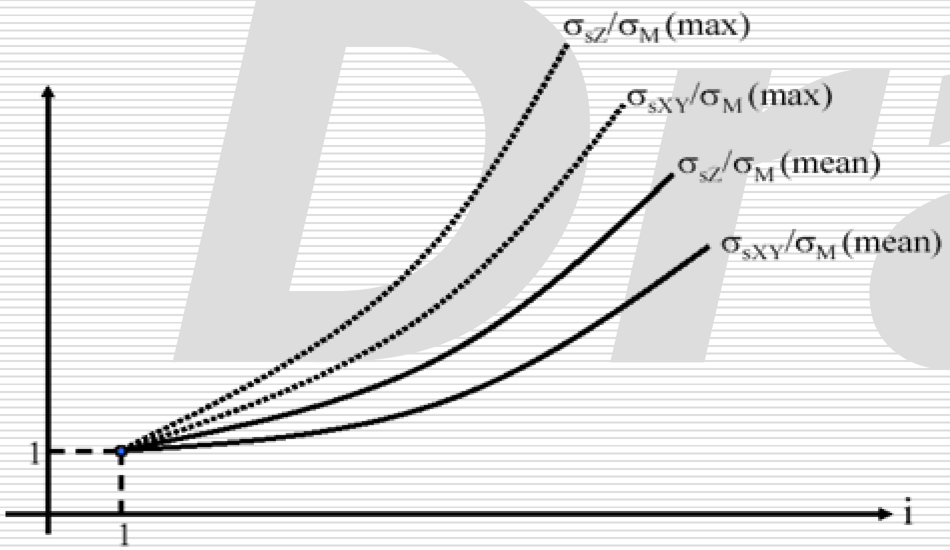
Accuracy of Strip Triangulation

The accuracy of points measured in the strip after applying the correction polynomial depends on the number of models bridged together without ground control points.

As a rule of thumb:

- *A group of ground control points should be used every four models.*
- *12 models: 3 groups of ground control points.*
- *16 models: 4 groups of ground control points.*



i number of models bridged together.

Maximum (worst) accuracy takes place at the center between the control points.

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Accuracy of BAIM

Due to the reduction in the involved GCP, we should expect a degradation in the accuracy when compared with this associated with a single model.

Planimetric accuracy is not affected by the accuracy of the measured model heights or by the layout/distribution of vertical control points.

Height accuracy is not affected by the accuracy of measured model planimetric coordinates or the the distribution of horizontal control points.

Planimetric and height accuracies can be treated separately.

We can derive the **accuracy** of the parameters **after the adjustment** as follows:

$$D\{\hat{x}\} = \sigma_o^2 \, Q\{\hat{x}\}$$

$$where:$$

$$Q\{\hat{x}\} = N^{-1}$$

$$\sigma_o^2 \equiv \text{Variance of observation of unit weight (variance componenet)}$$

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Accuracy of BAIM

Planimetric Accuracy of BAIM

– The accuracy deteriorates as the size of the block increases.

– The worst accuracy occurs at the center of the edges of the block.

Vertical Accuracy of BAIM

– Height accuracy is weak across the strips.

– Therefore, chains of vertical control points across the strips are recommended.

– We use chains of vertical control points every i models.

– To improve the height accuracy along the upper and lower edges of the block, we implement a vertical control point every i/2 models.

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Accuracy of Bundle Block Adjustment

The accuracy of the estimated EOP as well as the ground coordinates of tie points can be obtained by the product of:

- *The estimated variance component, and*
- *The inverse of the normal equation matrix (cofactor matrix).*

For planning purposes, we can use the same rules applied for BAIM.

However, the accuracy of a single model has to be modified.

Accuracy of Bundel =Accuracy of a single model: If we have

- *Bundle block adjustment with additional parameters that compensate for various distortions.*
- *Regular blocks with 60%overlap and 20% side lap.*
- *Signalized targets.*

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Photogrammetric Block Adjustment

Blunder Detection

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2006

Blunder Detection Based on Least squares

Introduction

After performing the least squares adjustment it is appropriate to evaluate whether the residuals are satisfactory or not.

There may be gross errors present among observations. If such errors do exist one does not know how many and where they are located nor what their magnitudes are. Besides gross errors, there may exist other causes can affect the results of adjustment (Uotila,1975). Considering only the problems of gross error detection/location, the assumption is that the other causes are not presents or have no significant effect on adjustment results.

There are a variety of statistical testing procedures based on least squares for gross error detection and location. One of these methods which has extensively been used in photogrammetry is the "data-snooping" technique. Other procedures which fall in this category are, in fact, variations of the data-snooping technique among these are the procedures of pope (1976), Stefanovic (1978), and the "premium/protection" method of Anscombe (1960).

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Blunder Detection Based on Least squares

Data-snooping Technique

The method starts with the assumption of a certain distribution for the true errors. Since the true errors e of the observations are not known, residuals v and/or parameters x are used in the test for the expectation of errors. Usually, expectations for the whole vector of parameters are not known; therefore, observations can only be tested for residuals under the null hypothesis,

$$H_0^v : \quad \sigma^2 = \sigma_0^2 \qquad (1)$$

The null hypothesis H_0 is accepted if the test statistic

$$F_{stat} = \frac{\hat{\sigma}_0^2}{\sigma_0^2} \qquad (2)$$

is less than the critical value of F (l-α,r, ∞) with α significance level and r,∞ degrees of freedom. Otherwise, the F_{star} is distributed as the non-central f-distribution f '(r,∞ ,λ), with the non-centrality parameter λ . The non-centrality parameter λ is equal to the square root of δ in Eq. (2.17).

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Blunder Detection Based on Least squares

Data-snooping Technique

There are two problems with this test. In the first place, the variance-covariance matrix of observations is not really known. In the second place, even if F_{star} is rejected, one does not know which observations caused the F_{star} to fail. Hence, one should continue by testing (n) individual residuals,

$$H_0^v: \qquad E(v_i) = 0: \qquad i = 1, 2, \ldots, n \qquad n = \text{number of observations} \qquad (3)$$

Suppose there is only on blunder in the i-th observation and the weight matrix P is diagonal. A test value w_i is obtained by

$$w_i = \frac{|v_i|}{\sigma_0 (q_{v_i v_i})^{1/2}} = \frac{|v_i|}{\sigma_{v_i}} \qquad (4)$$

where $q_{v_i v_i}$ is the i-th diagonal element of the cofactor matrix of the residuals.

The acceptance interval for w_i is given by $w_i > F^{1/2}_{1-a,r,\infty}$ *(5)*

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Blunder Detection Based on Least squares

Data-snooping Technique

where α is the significance level of type-I error. α can be related to the global significance level $\overline{\alpha}$ by

$$l-\overline{\alpha}=(l-\alpha)^{n} \qquad (6)$$

$$\alpha \stackrel{\approx}{=} \frac{\overline{\alpha}}{n}$$

provided that the individual tests are independent.

As mentioned earlier, usually the variance-covariance of observations is unknown, consequently σ_{v_i} is also unknown. In this case the estimated variances of residuals

$\hat{\sigma}_{V_i}$ are used:

$$t_i=\frac{|v_i|}{\hat{\sigma}_{V_i}} \qquad (7)$$

are used:
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Blunder Detection Based on Least squares

Pope's Method

The above shortcomings of the data-snooping technique motivated pope (1976) to develop a more practical procedure for blunder detection. His method differs from the data snooping technique in two respects:

1. No test on reference variance σ_0^2 is required.
2. The critical value for the test statistic t_i in Eq.(7) is determined from the so-called Tau-distribution, because v_i and σ_0 are computed from the same sample and they therefore not independent.

In this case t_i is distributed as a Tau-distribution

$$t_i \sim \tau_{1-\alpha,r} \qquad (8)$$

The Tau-distribution can be related to the t-distribution witch (r-l) degrees of freedom by

$$\tau^2(r) = \frac{r.t_{r-1}^2}{(r-1)+t_{r-1}^2} \qquad (9)$$

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Blunder Detection Based on Least squares

Pope's Method

Problems with this method includes:

1. Difficulty in exact computation of τ . Iterative procedures are used.
2. Difficulty in the derivation of τ under the alternative hypothesis.
3. some of the erroneous observations may not be detected, since t_i decreases as $\hat{\sigma}_0$ increases.

In general redundancy of the adjustment in photogram metric triangulation is large (say r>50) then the Tau-distribution could be replaced by the t-distribution. An approximate critical value for Eq. (8) may de obtained from the student t density function (Gruen, 1982),

$$t_i \sim \tau_{1-\alpha,r} \qquad (10)$$

which produces nearly the same critical value as Baarda's method.

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Blunder Detection Based on Least squares

Statistical Tests on Total and partial values of $v^t p v$

The method of the total and partial quadratic forms is used for locating the blunders and deals with groups of observations rather than individual ones. This test consists of two parts:

I- Test on Total Quadratic Form (Global Test).

II- Test on partial Quadratic Form.

If the first test $v^t p v$ is accepted (is compared to the critical value obtained a chi-square table), no blunders are assumed to be present in the data set. On the other hand, if the first test is rejected, blunders are present in the data, but one does not know which observations are erroneous. To determine the erroneous observations, select a subset of (b) residuals, v_2 (b has to be less then or equal to r (redundancy of the system)) and compute the change in total quadratic from $v^t p v$ (Stefanovic, 1978),

$$\Delta v^t p v = v_2^t (Q_{v_2 v_2})^{-1} v_2 \qquad (11)$$

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Blunder Detection Based on Least squares

Statistical Tests on Total and partial values of $v^t p v$

where $Q_{v_2 v_2}$ is the corresponding cofactor matrix of the residuals, v_2 . The following two tests can be used to determine if the subset of residuals, v_2 did, in fact, cause the first test to fail:

$$\phi' < \sigma_0^2 \cdot \chi^2_{1-\alpha, r-b} \qquad (12)$$

$$\Delta v^t p v < \sigma_0^2 x^2_{1-\alpha, b} \qquad (13)$$

ϕ' denotes the quadratic from after the contribution from v_2 has been removed and α is the significant level.

If a particular set of residuals v_2 has been selected, such that ϕ' passes the χ^2 test (inequality (12) is satisfied) and $\Delta v^t p v$ fails the test (inequality (13) is not satisfied), the observations corresponding to these residuals are presumed to contain blunders.

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Blunder Detection Based on Least squares

Statistical Tests on Total and partial values of $v^t p v$

This test has two major drawbacks:

1- It requires the a priori variance-covariance matrix of observations.

2- It requires the inversions of $\binom{n}{b}$ combinations of $Q_{v_2 v_2}$.

In many cases, $Q_{v_2 v_2}$ the inverse of the matrix dose not exist. Ellious (1982) studied this problem for blocks of 4 and 9 photographs using synthetic data. He provided a list of rules in which this matrix becomes singular.

The only advantage of this method over pope's method is, that the test appears to be robust against the presence of many blunderers in the data set.

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Blunder Detection Based on Robust Estimation

If the observations have a normal propagation, the multivariate probability density function will be trustful for them. So we will have:

$$f(\underline{l}) = \frac{1}{(2\pi)^{n/2}\sqrt{|\Sigma|}} e^{-\frac{1}{2}\left[\underline{l-E(l)}\right]^t \Sigma_{\underline{l}}^{-1} \left[\underline{l-E(l)}\right]} \quad \underline{\underline{\underline{l}+\underline{v}=A\underline{X}}} \quad \frac{1}{(2\pi)^{n/2}\sqrt{|\Sigma|}} e^{-\frac{1}{2}v^t \Sigma_{\underline{l}}^{-1} v} \qquad \text{(14)}$$

If we want, our estimate be a maximum likelihood estimation, $\varphi = \frac{1}{2}\underline{v}^t \sum_{\underline{l}}^{-1} \underline{v}$ must be minimum. If $\sum_{\underline{l}} = I$, we will have: $\varphi = \frac{1}{2}\sum_i v_i^2$.

As it mentioned above, the least squares method uses: $\sum v^2 \longrightarrow Min$, which makes it rather difficult to find and locate possible gross errors. This is because through the least squares adjustment, a gross error existing somewhere will often extend its influence to other measurements, and hence become undetectable. So, because appearance of gross errors in the digital photogrammetric points, which are generated, based on matching techniques are very possible, a more efficient method for adjustment is required.

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Blunder Detection Based on Robust Estimation

A good replacement for usual least squares adjustment is adjustment based on robust estimation. The term “Robustness” refer to a certain kind of property of a statistical method which generally has two implications; first: when there is not too big difference between the model in question and the theoretical model, the function of the method is not seriously affected; second: when a sample contain a small number of “outliers”, the function of the method is not seriously affected either.

The basic concept of the iterative method of weight selection in gross error testing is: Adjustment still starts with the customarily used least squares method. However, after each adjustment, the weight of each observed value in the next iteration is computed according to the weight function selected based on residual and other pertinent parameters. If the weight functions are properly selected, and gross errors can be located in this way, then the weights of the observed values containing gross errors will become smaller until they finally approach zero. When iterations stop, the results of adjustment will not affected by gross errors.

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Blunder Detection Based on Robust Estimation

The Computational process of the iterative method of weight functions is:

$$\sum \left(P \cdot v^2\right)^k \longrightarrow Min \qquad (15)$$

$$P^{(k+1)} = P^{(k)} \cdot f(v) \quad k = 0,1,2,\ldots$$

Where, k represents the number of iterations; v are the residuals; f(v) is called the weight function, and is the function of residuals. The fact that in Equation (15), the weight P is changed from a constant in least squares method to a variable which varies with the value v can be regarded as an adjustment which makes the total sum of another kind of functions of v such as: $\sum \rho(v) \longrightarrow Min$.

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Blunder Detection Based on Robust Estimation

The structure of Robust estimation is similar to the usual Least square method. The difference is the nature of repetitively nature of the process. This repetition is not a simple repetition (e.g. Least squares), but in this repetition the weight matrix is changed in each iteration. Changes of weight matrix is based on a function of normalized residuals, by name, Weight Function. This function is related to ρ function by the following formula:

$$W(v) = \frac{1}{v} \partial\rho(v) \Big/ \partial v \qquad (16)$$

and the weight matrix in each iteration:

$$P^{(k+1)} = P^{(0)} \times W^{(k)}(v_n) \qquad (17)$$

where, n: number of iteration, $P^{(0)}$: primary weight matrix, $P^{(k+1)}$: weight matrix in n-th iteration, v normalized residuals.

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Blunder Detection Based on Robust Estimation

The design of ρ and W must satisfy the requirements of math model. In Robust estimation, usually two-weight function is used. The first one has a smooth and monotonous changes and the second one, has a rapid and non-monotonous changes. As it mentioned before, the weight of each observations, based on its normalized residuals, is changed. The form of weighting functions dictated inverse relation to the value of residuals. So how many the residual of an observation be grater, the weight of it and proportionally it's effect in final results decreased or even eliminated.

$$P^{(k+1)} \langle T \longrightarrow = P^{(k+1)} = 0 \qquad (18)$$

In practice for improving the process of the detection and elimination of blunders, a threshold is defined. The weights of observations those are smaller than threshold is set to zero.

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The decision about threshold is related to confidence interval that we considered for accept or reject an observation.

There has been considerable work in this area and several classes of robust estimators have been proposed. See for example, Hampel (1974) and Huber (1981). One class used in geodetic science referred to as the 'Danish Method', wherein krarup et al (1980) proposed a searching routine for eliminating blunders in large geodetic networks. Since then, this idea has been extended and used as a standard computational at the Geodetic Institute of Denmark and other tasks at the Aalborg University center (Kubik, 1982).

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Photogrammetric Block Adjustment

Combined Adjustment of Geodetic Observations and Photogrammetric Data

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Introduction

Historically, the adjustment of photogrammetric and geodetic measurements has been treated as two separate problems. In such a two- step solution, the terrestrial surveys are first adjusted to give a unique set of coordinates plus perhaps a variance- matrix for the ground control points which are then used as input data into the photogrammetric solutions. This, of course, requires that all points can be coordinated from surveying data alone.

In the previous session it was illustrated how APR observation were adjusted simultaneously with photogrammetric observations, thus replacing a two step solution with a rigorous simultaneous approach. The same principle can be applied to conventional surveying observations. This also provides a more realistic approach to error analysis and weighting of the observations.

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Mathematical Modeling

The first major development in this area was the program SAPGO (Simultaneous Adjustment of Photogrammetric and Geodetic Observations), developed at the University of Illinois (Wong and Elphingstone, 1972). Laplace azimuths, straight- line distances, geodetic azimuths, horizontal angles, elevation differences, latitudes, longitudes and elevations are accepted as input into SAPGO, together with photo coordinates. The photogrammetric model is the bundle adjustment without additional parameters, while classical geodesy forms the basis of the geodetic model, treating horizontal and vertical adjustments separately. The horizontal adjustment is performed on the surface of a reference ellipsoid as a function of longitude and latitude, while the vertical adjustment is based on mean sea level elevations.

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Mathematical Modeling

In order to combine these geodetic models with the photogrammetric one (based on a spatial rectangular coordinate system) the geodetic observation equation has to be modified. This means that the terms $d\phi$, $d\lambda$ and dh of the classical geodetic equations need to be replaced by dX, dY, dZ as :

$$d\Phi = \left(\frac{\delta F_\Phi}{\delta X}\right) dX + \left(\frac{\delta F_\Phi}{\delta Y}\right) dY + \left(\frac{\delta F_\Phi}{\delta Z}\right) dZ$$

and similarly for $d\lambda$ and dh.

Unfortunately, there is no explicit functional relationship between ϕ, λ, H and X, Y, Z. Therefore the partial derivatives were obtained in SAPGO by a numerical method, using the following approach:

If $g = f(X,Y)$ **and** $g_{\Delta x} = f(X + \Delta X, Y)$ **then** $\frac{\delta g}{\delta X} = \frac{g - g_{\Delta X}}{\Delta X}$ **and similarly** $\frac{\delta g}{\delta Y} = \frac{g - g_{\Delta Y}}{\Delta Y}$

This does not provide a rigorous solution, and the accuracy depends on the values chosen for X and Y.

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Mathematical Modeling

At the University of New Brunswick, the program system GEBAT (General Bundle Adjustment Triangulation) was developed (El Hakim, 1979; El Hakim and Faig, 1981). This system is a bundle adjustment with additional parameters using a harmonic function to compensate for the effect of systematic errors. The geodetic observations (slope distances, vertical angles, horizontal directions, astronomic azimuths, elevation differences, astronomic latitude and longitude) are based on modern three dimensional geodesy and rigorously combined with the photogrammetric adjustment. The stochastic model can be further improved by taking the correlation between the observations into account and applying least squares collection.

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Mathematical Modeling

After formulating observation equations for each type of observation, the following combined system was obtained for the least squares adjustment:

- *For* ***Photogrammetric*** *observations*

$$F_P\left(\hat{X}_1,\ \ \hat{X}_2,\ \ L_P\right)=0$$

- *For* ***Geodetic*** *observations*

$$F_g\left(\hat{X}_2,\ \ \hat{X}_3,\ \ L_P\right)=0$$

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These are linearized to

$$W_P + A_{P1}\hat{X}_1 + A_{P2}\hat{X}_2 + B_P\hat{V}_P = 0$$

$$W_g + A_{g1}\hat{X}_2 + A_{g2}\hat{X}_3 + B_g\hat{V}_g = 0$$

where

$\hat{X}_1$ is the vectored of photo orientation elements and additional parameters

$\hat{X}_2$ is the vector of object coordinates

$\hat{X}_3$ is the vector of orientation, refraction unknowns and astronomic coordinates

$W_p = F_p(X_1^0, X_2^0, L_p)$ is the photogrammetric misclosure vector for the initial value X_1^0 and X_2^0

$W_g = F_g(X_1^0, X_2^0, L_g)$ is the geodetic misclosure vector

v_p, v_g are the vectors of residual for photo ordinates and geodetic observations

$A_{p_1}, A_{p_2}, B_p, A_{g_1}, A_{g_2}, B_g$ are the design matrices, obtained for instance as :

$$A_{P1} = \left.\frac{\delta F_P}{\delta \hat{X}_1}\right|_{X_1^0,\ X_2^0,\ L_P}$$

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Mathematical Modeling

The variation function is then:

$$\Phi = \hat{V}_p^T P_p \hat{V}_p + \hat{V}_g^T P_g \hat{V}_g + \hat{X}_1^T P_1 \hat{X}_1 + \hat{X}_2^T P_2 \hat{X}_2 + \hat{X}_3^T P_3 \hat{X}_3 + 2\hat{K}_p^T (W_p$$
$$+ A_{p1}\hat{X}_1 + A_{p2}\hat{X}_2 + B_p \hat{V}_p) + 2\hat{K}_g^T (W_p + A_{g1}\hat{X}_1 + A_{g2}\hat{X}_2 + B_g \hat{V}_g) = min$$

Where P_p and P_g are the weight matrices for the observations ; P_1, P_2, P_3 the weight matrices for the unknowns, and K_p and K_g are estimates for the vectors of Lagrange multipliers :

The normal equations become:

$$\begin{bmatrix} p_p & 0 & B_P^T & 0 & 0 & 0 & 0 \\ 0 & P_g & 0 & B_g^T & 0 & 0 & 0 \\ B_P & 0 & 0 & 0 & A_{P1} & A_{P2} & 0 \\ 0 & B_g & 0 & 0 & 0 & A_{g1} & A_{g2} \\ 0 & 0 & A_{P_1}^T & 0 & P_1 & 0 & 0 \\ 0 & 0 & A_{P_2}^T & A_{g_1}^T & 0 & P_2 & 0 \\ 0 & 0 & 0 & A_{g_2}^T & 0 & 0 & P_3 \end{bmatrix} \begin{bmatrix} V_P \\ V_g \\ K_P \\ K_g \\ X_1 \\ X_2 \\ X_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ w_P \\ w_g \\ 0 \\ 0 \\ 0 \end{bmatrix} = 0$$

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Mathematical Modeling

The normal equations provide the following results :

$$\hat{X}_2 = -\left[P_2 + N_{P22} + N_{g11} - N_{g12}\left(P_3 + N_{g22}\right)^{-1} N_{g21} - N_{P21}\left(P_1 + N_{P11}\right)^{-1} N_{P12}\right]^{-1}$$

$$\cdot \left[U_{P2} + U_{g1} - \quad - N_{g12}\left(P_3 + N_{g22}\right)^{-1} U_{g2} - N_{P21}\left(P_1 + N_{P11}\right)^{-1} U_{P1}\right]$$

and

$$\sum_{XX} = -\hat{\sigma}_0^2 \left[P_2 + N_{P22} + N_{g11} - N_{g12}\left(P_3 + N_{g22}\right)^{-1} N_{g21} - N_{P21}\left(P_1 + N_{P11}\right)^{-1} N_{P11}\right]^{-1}$$

Where

$$\hat{\sigma}_0{}^2 = \frac{\hat{V}^T P \hat{V} + \hat{X}_1^T P_1 \hat{X}_1 + \hat{X}_2^T P_2 \hat{X}_2 + \hat{X}_3^T P_3 \hat{X}_3}{df}$$

The degree of freedom (df) equals the number of observations, since all unknowns are weighted.

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The following abbreviations were used:

$$N_{pij} = A_{pi}^T \left[B_p \; P_p^{-1} \; B_p^T \right]^{-1} A_{pj}$$

$$N_{gij} = A_{gi}^T \left[B_g \; P_g^{-1} \; B_g^T \right]^{-1} A_{gj} = A_{gi}^T \; P_g \; A_{gi}$$

$$U_{pi} = A_{pi}^T \left[B_p \; P_p^{-1} \; B_p^T \right]^{-1} w_p$$

$$U_{gi} = A_{gi}^T \left[B_g \; P_g^{-1} \; B_g^T \right]^{-1} w_g$$

At the National Research Council of Canada, this basic program was subsequently optimized and expanded to include gross- error detection (data snooping) (El Hakim, 1984).

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