Photogrammetric Block Adjustment

Chapter 3
Auxiliary Data

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Content

- Introduction
- Statoscope
  - Example for Statoscope data (M3)
  - Observation Equations
  - Number of Observation Equations
  - Number of Unknowns
  - Observation Equation Matrix
  - Normal Equation Matrix
  - Reduced Normal Equations
- APR
  - M3 with APR Data
  - Observation Equation M3 with APR Data
- Height Information from Shorelines of Lakes
  - Observation Equations
- Application of GPS for Aerial Triangulation
  - GPS observation used in the precise …
  - Problems
  - Combined GPS Photogrammetric B.A.
  - Geometric Stability of Block

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Introduction

The auxiliary equipments are usually used to determine the six degrees of freedom of each exposure station namely:

$$\omega, \varphi, \kappa, X_0, Y_0, Z_0$$

In order to reduce the necessary ground control points for aero triangulation, so called "auxiliary data" obtained from such devices as the statoscope, Air borne profile radar (APR), GPS, INS and …

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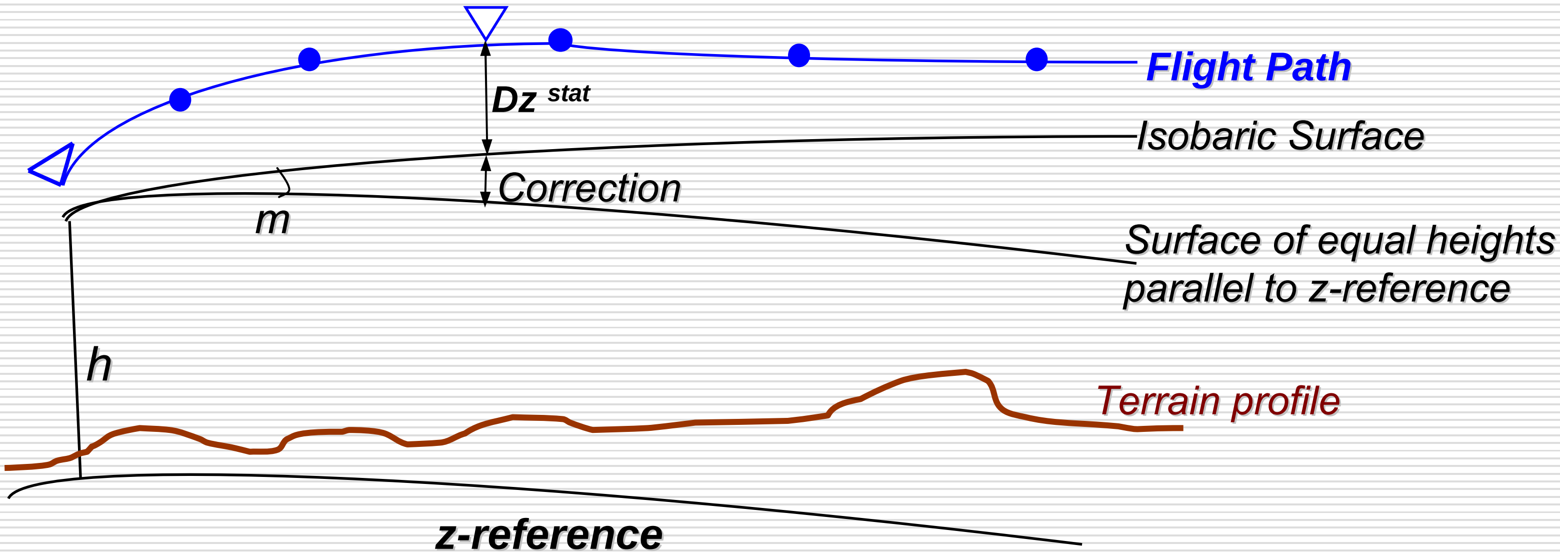
Statoscope

The Statoscope is used to give the deviation in the height of each air station from a certain isobaric surface (surface of equal air pressure).

This instrument is based on the principle of a liquid manometer which measured all variation in flights by the corresponding change of air pressure.

The statoscope is a simple and rather cheap piece of equipment, which can be used in all sorts of aeroplanes.



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Statoscope

It is obvious that the statoscope heights:

$$Z^{stat} = d_z^{stat} + \overline{h_0}$$

should have an additional correction which represents the deviation of the isobaric surface from the surface of equal heights.

Due to the fact that the shape and location of the isobaric surface are unknown, one may add a 1st or a 2nd order polynomial as a correction to the isobaric surface.

Experience has shown that it is sufficient to take only constant and linear terms in to account

$$Z^{pc} = \big(\underbrace{h}_{\text{constant}} + \underbrace{m \cdot t}_{\text{linear}}\big) + Z^{stat} + V_z^{stat}$$

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Statoscope

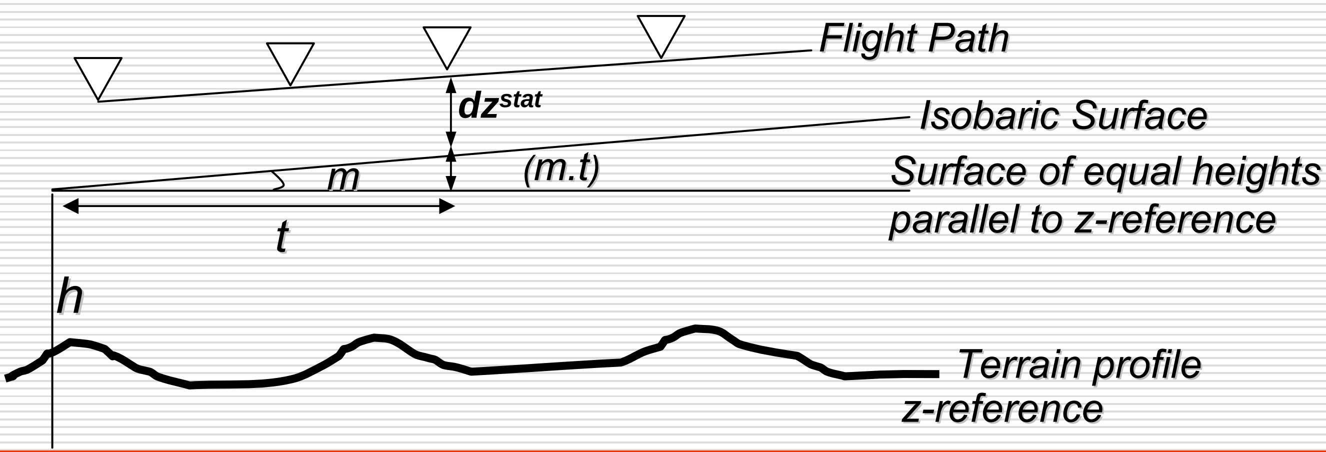
Correction for slop of isobaric surface (Henry correction) is only linear over sections of equal drift (A 2nd order term may be needed for long strips).

Therefore, additional unknown parameters (SP) are needed for each strip separately:

(h_k, m_k) , k=strip number. For an exposure station i in strip k to the observation equations.

$$Z_{ik}^{pc} = (h_k + m_k t_{ik}) + Z_{ik}^{stat} + (V_z)_{ik}^{stat}$$

$Z_{ik}^{stat} = dz_{ik}^{stat}, t_{ik} =$ **time for exposure** i **in strip** k



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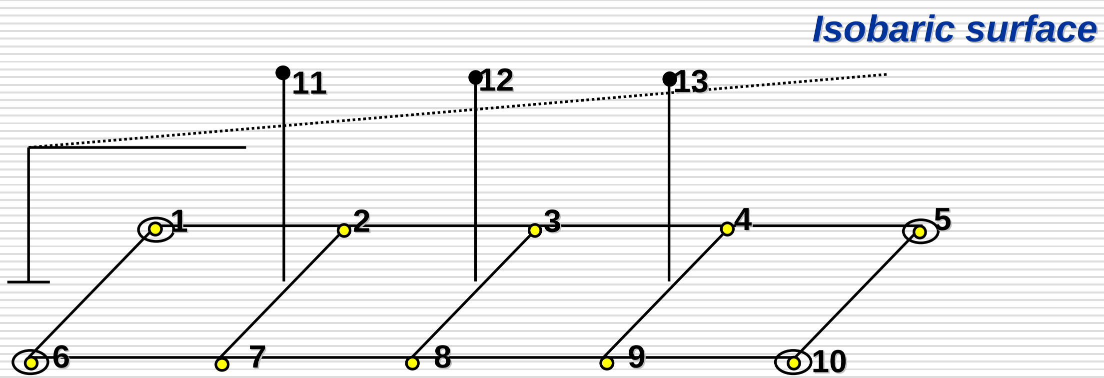
Statoscope

Example for statoscope data (M3)

Given the following strip of 4 models to be adjusted using the data below.



1- Model cords: x_{ij}, y_{ij}, z_{ij} **including those of the projection centers:**

model 1: *coords of pts 1,2,6,7*

model 2: *coords of pts 2,3,7,8*

model 3: *coords of pts 3,4,8,9*

model 4: *coords of pts 4,5,9,10*

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Statoscope

Example for statoscope data (M3)

2- Projection centers and coordinates of points

model 1: *coords of 11pc*

model 2: *coords of 11pc,12pc*

model 3: *coords 12pc,13pc*

model 4: *coords 13pc*

3- Terrestrial cords of height control points: H_1, H_5, H_6, H_{10}

4- Statoscope data

PN	Z^{st}	Time
11	Z_{11}^{st}	t_{11}
12	Z_{12}^{st}	t_{12}
13	Z_{13}^{st}	t_{13}

Time should be in metric (decimal) system not hours, minutes and seconds.

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Statoscope

Example for statoscope data (M3)

5- Weights

$g_{(1\times1)}$: weight number of model height Z_{ij}

$P_{(3\times3)}$: weight matrix for the projection center coordinates X_{PC}, Y_{PC}, Z_{PC}
in most cases this is a diagonal matrix

$G_{(1\times1)}$: weight of terrestrial heights H_i

$S_{(1\times1)}$: weight of statoscope heights Z_{PC}^{St}

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Statoscope

Example for statoscope data (M3)

The Unknowns

P: Unknown model parameter 3 / model: Ω_j, Φ_j, Z_{0j}

C: Unknown coords: Z_i of tie and control Pts and X_{PC}, Y_{PC}, Z_{PC} of projection centers

$\overline{\mathbf{P}}$**:** Unknown parameters of isobaric surface: m, h

Total Number of Unknowns:

P: 3 Par × 4 Models =12

C: 10 points × 1 coord (z) =10

+3 projection center × 3 coords (X,Y,Z) =9

$\overline{\mathbf{P}}$**:** m, h =2

Total = 33

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Statoscope

Example for statoscope data (M3)

Observation equations

1- model points

$$-\underbrace{\begin{bmatrix} y & -x & 1 \end{bmatrix}}_{A_{ij}} + \begin{bmatrix} \Delta\Omega \\ \Delta\Phi \\ \Delta Z_0 \end{bmatrix}_j + [Z]_i = [z]_{ij}$$

2- Projection center

$$-\underbrace{\begin{bmatrix} 0 & z & 0 \\ -z & 0 & 0 \\ y & -x & 1 \end{bmatrix}_{ij}}_{\bar{A}_{ij}} \begin{bmatrix} \Delta\Omega \\ \Delta\Phi \\ \Delta Z_0 \end{bmatrix}_j + \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_i = \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{ij}$$

3- Terrestrial point

$$[Z]_i = [H]_i$$

4- Statoscope height

$$-\begin{bmatrix} 1 & t \end{bmatrix}_{ik} \begin{bmatrix} h \\ m \end{bmatrix}_k + [Z]_i^{pc} = [Z]_{ik}^{stat}$$

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Statoscope

Example for statoscope data (M3)

Number of observation equations.

for models

model 1,4 : 4 points × 1= 4 × 2 + 1 p.c × 3 = 3 × 2

model 2,3 : 4 points × 1= 4 × 2 + 2 p.c × 3 = 6 × 2

for terrestrial points: 4 × 1 = 4

for statoscopic data: 3 × 1 = 3

Total= 41

Number of unknowns.

1- Parameters: P=3parameters model = 12

2- Coordinates: C=10 points

3- Statoscope parameters: m,h

Total=33

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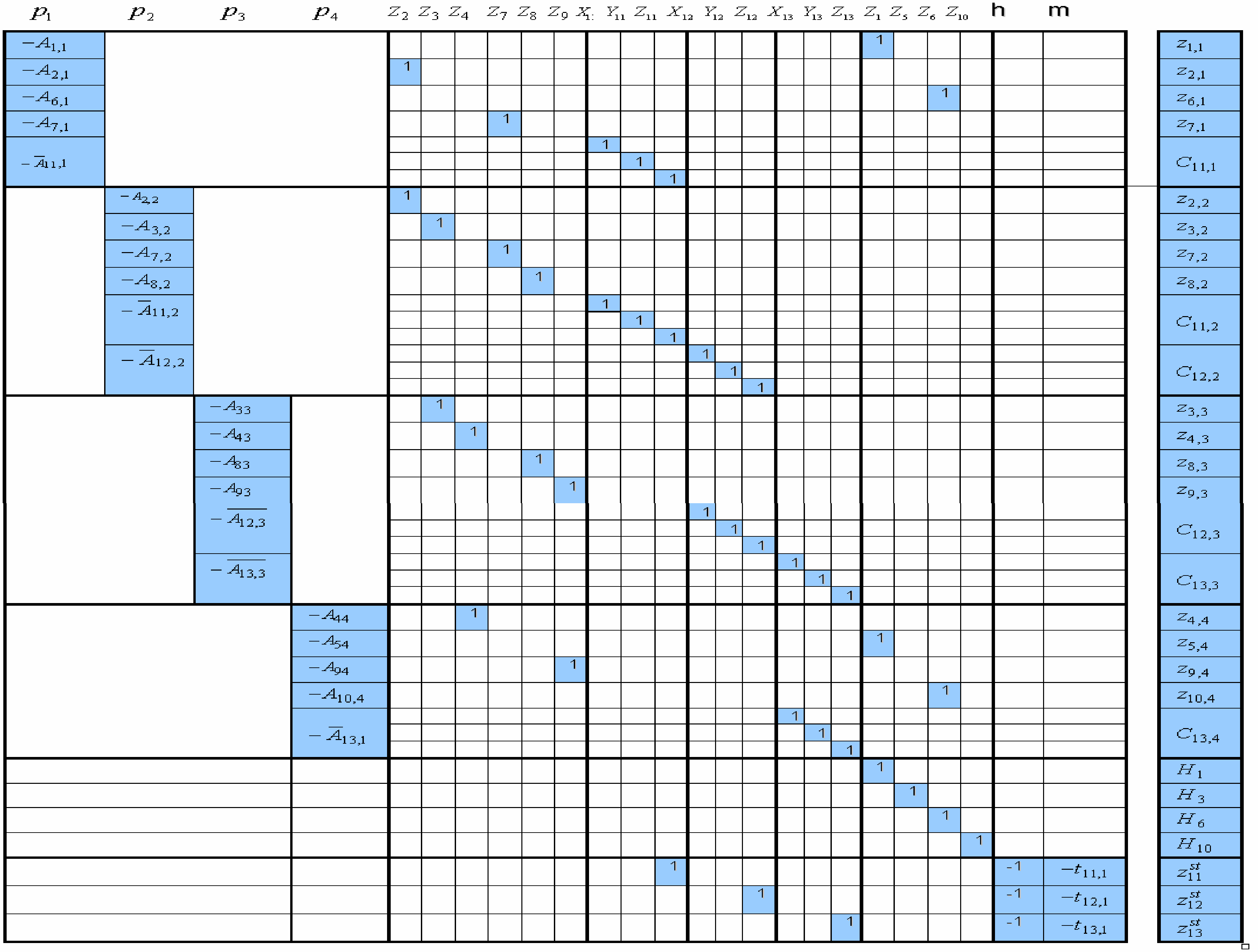

p_1 p_2 p_3 p_4
Z_2 Z_3 Z_4 Z_7 Z_8 Z_9 X_{11} Y_{11} Z_{11} X_{12} Y_{12} Z_{12} X_{13} Y_{13} Z_{13} Z_1 Z_5 Z_6 Z_{10} h m
$-A_{1,1}$
$-A_{2,1}$
$-A_{6,1}$
$-A_{7,1}$
$-\bar{A}_{11,1}$
$-A_{2,2}$
$-A_{3,2}$
$-A_{7,2}$
$-A_{8,2}$
$-\bar{A}_{11,2}$
$-\bar{A}_{12,2}$
$-A_{33}$
$-A_{43}$
$-A_{83}$
$-A_{93}$
$-\bar{A}_{12,3}$
$-\bar{A}_{13,3}$
$-A_{44}$
$-A_{54}$
$-A_{94}$
$-A_{10,4}$
$-\bar{A}_{13,1}$
1
-1
$-t_{11,1}$
$-t_{12,1}$
$-t_{13,1}$
$z_{1,1}$
$z_{2,1}$
$z_{6,1}$
$z_{7,1}$
$C_{11,1}$
$z_{2,2}$
$z_{3,2}$
$z_{7,2}$
$z_{8,2}$
$C_{11,2}$
$C_{12,2}$
$z_{3,3}$
$z_{4,3}$
$z_{8,3}$
$z_{9,3}$
$C_{12,3}$
$C_{13,3}$
$z_{4,4}$
$z_{5,4}$
$z_{9,4}$
$z_{10,4}$
$C_{13,4}$
H_1
H_3
H_6
H_{10}
z_{11}^{st}
z_{12}^{st}
z_{13}^{st}

p_1	p_2	p_3	p_4	Z_2 Z_3	Z_4	Z_7	Z_8	Z_9	X_{11} Y_{11}	Z_{11} X_{12}	Y_{12} Z_{12} X_{13}	Y_{13} Z_{13} Z_1	Z_5	Z_6	Z_{10}	**h**	**m**	
$\sum A^t gA$ $\sum \bar{A} p \bar{A}$																		
	$\sum A^t gA$ $\sum \bar{A} p \bar{A}$																	
		$\sum A^t gA$ $\sum \bar{A} p \bar{A}$																
			$\sum A^t gA$ $\sum \bar{A} p \bar{A}$															
$-gA_{2,1}$	$-gA_{2,2}$			2g														
	$-gA_{3,2}$	$-gA_{33}$			2g													
		$-gA_{43}$	$-gA_{4,4}$			2g												
$-gA_{7,1}$	$-gA_{7,2}$						2g											
	$-gA_{8,2}$	$-gA_{9,3}$						2g										
		$-gA_{93}$	$-gA_{9,4}$						2g									
$-P.\overline{A_{11,1}}$	$-P.\overline{A_{11,2}}$									2p 2p 2p+s								
	$-P.\overline{A_{12,2}}$	$-P.\overline{A_{12,3}}$									2p 2p 2p+s							
		$-P.\overline{A_{13,3}}$	$-P.\overline{A_{13,4}}$									2p 2p 2p+s						
$-gA_{2,1}$													g+ G					
			$-gA_{2,1}$											g+ G				
$-gA_{2,1}$															g+ G			
			$-gA_{2,1}$													g+ G		
										-s	-s	-s					3.s	$\sum$st
										-s.t_{11}	-s.t_{12}	-s.t_{13}					$\sum$st	$\sum st^2$

Statoscope

Right Hand (matrix)

$\Sigma A^t.g.z$ $+\bar{A}^t.P.C$	$\Sigma A^t.g.z$ $+\bar{A}^t.P.C$	$\Sigma A^t.g.z$ $+\bar{A}^t.P.C$	$\Sigma A^t.g.z$ $+\bar{A}^t.P.C$	$\Sigma_z^{g.}$	$\Sigma_z^{g.}$	$\Sigma_z^{g.}$	$\Sigma_z^{g.}$	$\Sigma_z^{g.}$	$\Sigma_z^{g.}$	$\Sigma_x^{g.}$ $\Sigma_y^{g.}$ $\Sigma_x^{g.}+\Sigma_y^{g.}$ $+\Sigma_x^{g.}$ $+s.Z^{st}$	$\Sigma_x^{g.}$ $\Sigma_y^{g.}$ $\Sigma_x^{g.}+\Sigma_y^{g.}$ $+\Sigma_x^{g.}$ $+s.Z^{st}$	$\Sigma_x^{g.}$ $\Sigma_y^{g.}$ $\Sigma_x^{g.}+\Sigma_y^{g.}$ $+\Sigma_x^{g.}$ $+s.Z^{st}$	$G.H_1$	$G.H_5$	$G.H_6$	$G.H_{10}$	$\Sigma -s.z^{st}$	$-\Sigma t.s.z^{st}$

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Statoscope

Reduced normal equations

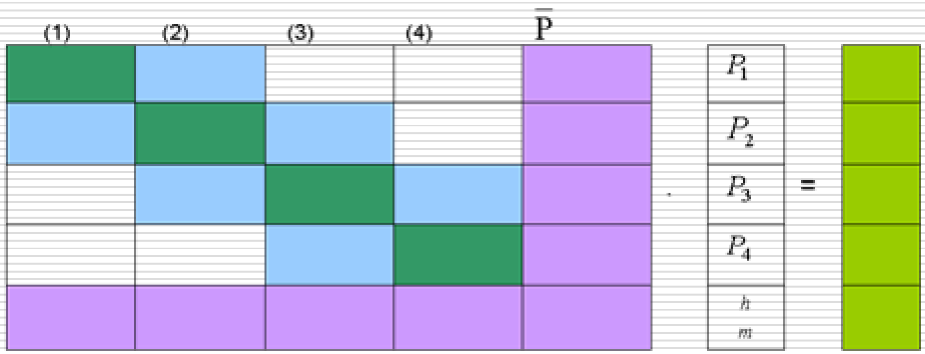
The normal equations can be expressed in terms of three sets of unknowns:

- P: Parameters for models $\Delta\Omega_j, \Delta\Phi_j, \Delta Z_{0j}$
- C: Coordinates
- Z: tie points and control
- X,Y,Z: projection centers
- $\overline{P}$: Parameters of isobaric surface (h,m)

$$
\begin{array}{llllll}
N_{1,1}.P & +N^T_{2,1}.C & & = F_1 & & (1) \\
N_{2,1}.P & N_{2,2}.P & +N^T_{3,2}.C & = F_1 & & (2) \\
 & N_{3,2}.P & +N_{3,3}.P & = F_1 & & (3)
\end{array}
$$

From (2) $C = N_{22}^{-1}.F_1 - N_{22}^{-1}.N_{21}.P - N_{22}^{-1}.N_{32}^T.\overline{P}$

Substituting in (1) and (3) we get the R.N.E which will have a banded bordered coefficient matrix.



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APR

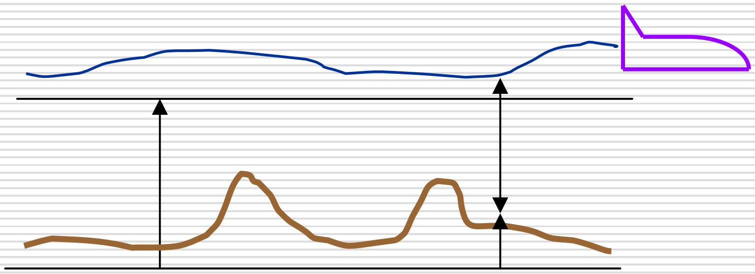
The APR is a very expensive equipment, which consists of three different sets of instruments.

1- a Statoscope, which records the deviations of the air-path from the isobaric surface (dz)

2- a radar (or laser) altimeters which records continuously the clearance (S) between the air-path and the terrain

3- A 35mm spotting camera

$$Z^{APR} = h_0 + d_z^{stat} - s$$



Airborne Profile Recorder data consist essentially of two different sets of continuously record measurements:

1- S = Distance measured by radar from the air station in vertical direction down to the terrain.

2- dz by Statoscope

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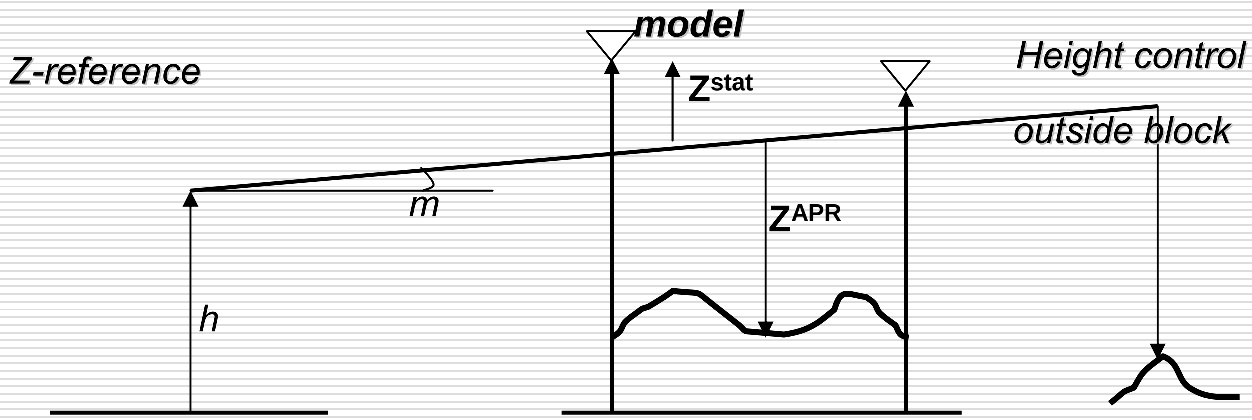
APR

M-3 with APR data

It is similar to the use of statoscope data with the following differences:

1- The APR heights refer to the points in the model while the Statoscope heights control the projection centers.



2- The APR height can be checked by control points (water surface) outside the block.

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APR

M-3 with APR data

Observation equation (M-3) with APR data

– Model APR points

$$-\begin{bmatrix} y & -x & 1 \end{bmatrix}_{ij} \begin{bmatrix} \Delta\Omega \\ \Delta\Phi \\ \Delta Z_0 \end{bmatrix}_j + [Z]_i = [Z]_{ij}$$

– APR Height

$$-\begin{bmatrix} 1 & t \end{bmatrix}_{ik} \begin{bmatrix} h \\ m \end{bmatrix}_k + [Z]_i = [Z]_i^{APR}$$

– Control points

$$[Z]_i = [H]_i$$

This control can be outside block.

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Height Information from Shorelines of Lakes

If arbitrary points on the shoreline of a lake are measured in one or several models, they can be used advantageously for the block adjustment even if the absolute height of the water level is not known.

Observation equations

1- *Each lake point will give such observation equation.*

$$-\begin{bmatrix} y & -x & 1 \end{bmatrix}_{ijL} \begin{bmatrix} \Delta\Omega \\ \Delta\Phi \\ \Delta Z_0 \end{bmatrix}_j + \left[Z\right]_L = \left[Z\right]_{ijL}$$

2- *If the absolute height of the water in the lake is known then we get another observation equation.*

$$\left[Z\right]_L = \left[H\right]_L$$

As we may have several lakes (or sub-lakes) the index L is used to refer to the lake measured.

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GPS Observation

Used in the precise photogrammetric Application

Air borne kinematic GPS can determine the position of the aerial camera at the instant of exposure. The combined photogrammetric GPS block adjustment takes advantage of GPS weighted observations which significantly reduces the number of ground control points needed in the conventional block adjustment with GPS, position of the aircraft at the individual exposure moments can be precisely determined.

There are three types of positioning information which can be extracted from the GPS satellite signals, Pseudo range (code), carrier phase and phase rate (Doppler frequency). Due to the high accuracy required for an aerotriangulation, GPS phase measurements are needed to meet the accuracy requirement. In order to eliminate the effects of timing error (receiver and satellite clock) double difference GPS phase measurement is applied. Observation equation for DGPS:

$$\nabla\Delta\Phi = \nabla\Delta\rho + \nabla\Delta d_{\rho} + \lambda\nabla\Delta \mathrm{N} - \nabla\Delta d_{ion} + \nabla\Delta d_{trop} + \varepsilon\nabla\Delta\Phi$$

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GPS Observation

Used in the precise photogrammetric Application

$$\nabla\Delta\Phi = \nabla\Delta\rho + \nabla\Delta d_{\rho} + \lambda\nabla\Delta N - \nabla\Delta d_{ion} + \nabla\Delta d_{trop} + \varepsilon\nabla\Delta\Phi$$

Air borne kinematic GPS can determine the position of the aerial camera

- $\nabla\Delta$: The double difference notation
- Φ : The carrier beat phase measurement in cycles.
- ρ : The distance from satellite to the receiver.
- d_{ρ} : The orbital error
- λ : The carrier wave length
- N : The integer carrier beat phase ambiguity
- d_{ion} : The ionospheric error
- d_{trop} : The tropospheric error
- ε : The receiver noise and multipath

The terms,and $\nabla\Delta \mathbf{d}_{\rho}$, $\nabla\Delta \mathbf{d}_{\mathbf{ion}}$ and $\nabla\Delta \mathbf{d}_{\mathbf{trop}}$ are generally small or negligible for short. (e.g.<10-20 km) distances (monitor-remote distances).

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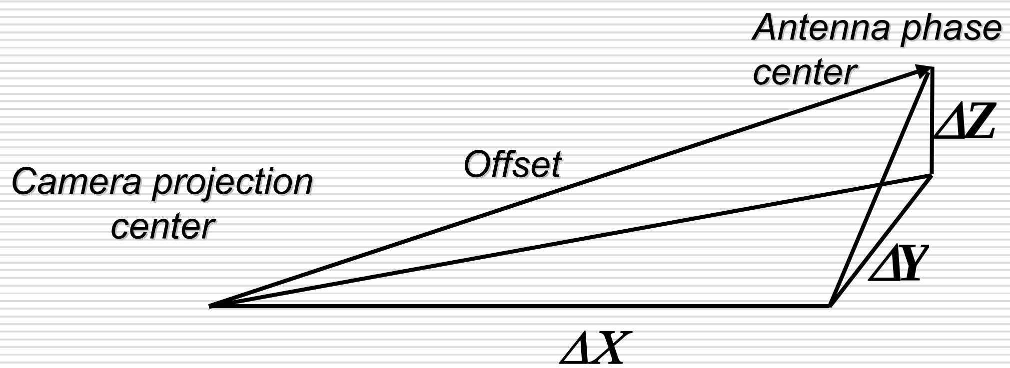
GPS Observation

Used in the precise photogrammetric Application

Problems

Antenna – Aerial camera offset

GPS given the coordinates of the antenna phase center and not as desired of the projection center of camera.



This offset distance can be surveyed using geodetic methods or treated as an unknown quantity and solved together with other unknowns in a block adjustment.

Time offset between exposure and GPS time.

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Problems

Synchronization between exposure and GPS time

To interpolate the GPS positions on to the exposure stations, the instants of exposure must be recorded using the receiver time scale with precise synchronization to GPS time. The measuring rate for GPS observation must be as high as 1Hz or more as the speed of survey aircraft is in order of 50m to 100m per second. The photogrammetric camera should be equipped with a shutter electronic signal, providing an accuracy better than 1ms.

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GPS Observation

Used in the precise photogrammetric Application

Problems

Geodetic data

GPS provides coordinates in the WGS-84 which a geocentric coordinate system centered at the mass center of the earth. However, the reference systems usually used in according triangulation are the local coordinates systems referring to the local ellipsoids. Planimetric coordinates may be obtained by transforming from WGS-84 to the local coordinate system meaning that there is still a need for minimum ground control points to carry out the transformation. The transformation of heights requires knowledge of specific geoid and its undulation.

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Problems

Ambiguity (N)

This problem can handled in a number of ways:

- It can be approximately determined from the pseudo ranges observed in the C/A or P cods.
- Calibrating N by going over a known point at the airport.
- Using OTF ambiguity resolution methods

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Problems

Cycle slip

Cycle slips are discontinuities in the time series of a carrier phase as measured in the GPS receiver.

It is occurred when:

- *Parts of aircraft obstruct the intervisibility between the antenna and satellites,*
- *Multipath from refraction of some parts of the aircraft ,*
- *Receiver power failure,*
- *Low signal strength due to the high ionospheric activity.*

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Problems

Cycle slip

Approaches for cycle slip correction are:

- Using receiver with more than 4 channels to obtain redundant observation,
- Using dual frequency receivers,
- Integrating GPS with INS or other sensors.
- Applying OTF ambiguity resolution techniques,
- new coarse GPS positions from C/A code pseudoranges after signal interruption.

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GPS Observation

Used in the precise photogrammetric Application

Combined GPS photogrammetric block adjustment.

The observation equations have the from:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_j^{GPS} + \begin{bmatrix} V_X \\ V_Y \\ V_Z \end{bmatrix}_j^{GPS} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_j^{PC} + R(\omega,\varphi,\kappa)\cdot \begin{bmatrix} d_x \\ d_y \\ d_z \end{bmatrix} + \left(\begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} + \begin{bmatrix} b_x \\ b_y \\ b_z \end{bmatrix} \times (t_i - t_o) \right)_k$$

$(X,Y,Z)^{GPS}$	: The observed GPS camera station coordinates
$(V_x,V_y,V_z)^{GPS}$	: The residuals
$(X,Y,Z)^{PC}$	: The unknown coordinates of the projection centers
$R(\omega,\varphi,\kappa)$	: The rotation matrix
d_x,d_y,d_z	: The offset vector
$(a_x,a_y,a_z,b_x,b_y,b_z)$	: Unknown drift corrections which are common for all observation equation of each strip k
t_o,t_i	: The GPS time of each exposure and time reference

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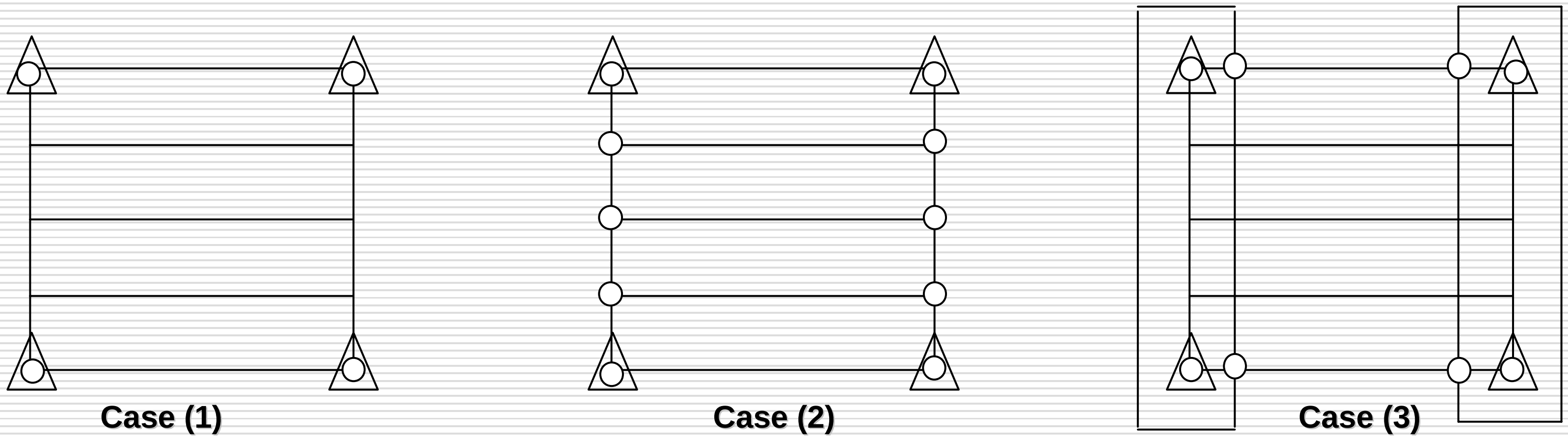
Geometric stability of blocks

The geometry of the combined GPS photogrammetric block is determined as in the conventional sense (standard overlap and standard tie point distribution). Three different ground control configurations are recommended by Ackermann:

1. *Block with 60% side lap*
2. *Block with 2 chains of vertical control points across the block.*
3. *Block with cross strips at both the start and end of the block.*

Case 3 with 2 cross strips is usually recommended for GPS aero triangulation due to its economic efficiency.

It may be required to have more than 2 cross strips and 4 ground control points where the blocks have irregular shape.



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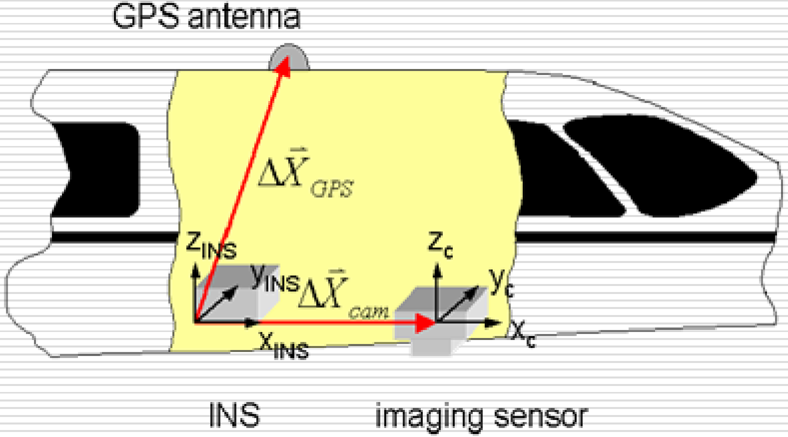
Combining GPS and INS with Aerial Triangulation

To take different error terms into account, the standard colinearity equation where:

$$\vec{X}^{m} = \vec{X}_0^{m} + \lambda \cdot R_p^m \cdot \vec{x}^{p}$$



$\vec{X}^{m} = \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$ coordinates of object points in the mapping frame m,

$\vec{X}_0^{m} = \begin{pmatrix} X_0 \\ Y_0 \\ Z_0 \end{pmatrix}$ coordinates of perspective center denoted in the mapping frame m,

$R_p^m = R_p^m(\omega,\varphi,\kappa)$ rotation matrix from image p to mapping frame m,

$\vec{x}^{p} = \begin{pmatrix} x' \\ y' \\ -c \end{pmatrix}$ coordinates of image points given in image frame p, λ scaling factor,

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GPS Observation

Used in the precise photogrammetric Application

Combining GPS and INS with Aerial Triangulation

has to be modified as follows:

$$\vec{X}^m = \vec{X}_0^m + R_b^m \cdot \left(\lambda \cdot R_p^m \cdot \vec{x}^p + \Delta\vec{X}_{cam}^b - \Delta\vec{X}_{GPS}^b \right)$$

$\vec{X}^m = \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$ coordinates of object points in the mapping frame m,

$\vec{X}_0^m = \begin{pmatrix} X_0 \\ Y_0 \\ Z_0 \end{pmatrix}$ coordinates of phase center of GPS antenna denoted in the mapping frame m,

$\Delta\vec{X}_{cam}^b = \begin{pmatrix} \Delta X_{cam} \\ \Delta Y_{cam} \\ \Delta Z_{cam} \end{pmatrix}$ offset between INS and the perspective center of imaging sensor given in body frame b

$\Delta\vec{X}_{GPS}^b = \begin{pmatrix} \Delta X_{GPS} \\ \Delta Y_{GPS} \\ \Delta Z_{GPS} \end{pmatrix}$ offset between GPS antenna and the perspective center of imaging sensor given in body frame b

$R_b^m = R_b^m(\omega, \varphi, \kappa)$ rotation matrix from INS body b to mapping frame m,

$R_p^b = R_p^b(\omega, \varphi, \kappa)$ rotation matrix from camera body p to INS body frame b,

$\vec{x}^p = \begin{pmatrix} x' \\ y' \\ -c \end{pmatrix}$ coordinates of image points given in image frame p , λ scaling factor.

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