Chapter 3: Gauss's Law

- ✓ **Electric Flux**
- ✓ **Gauss's Law**
- ✓ **Applying Gauss's Law**

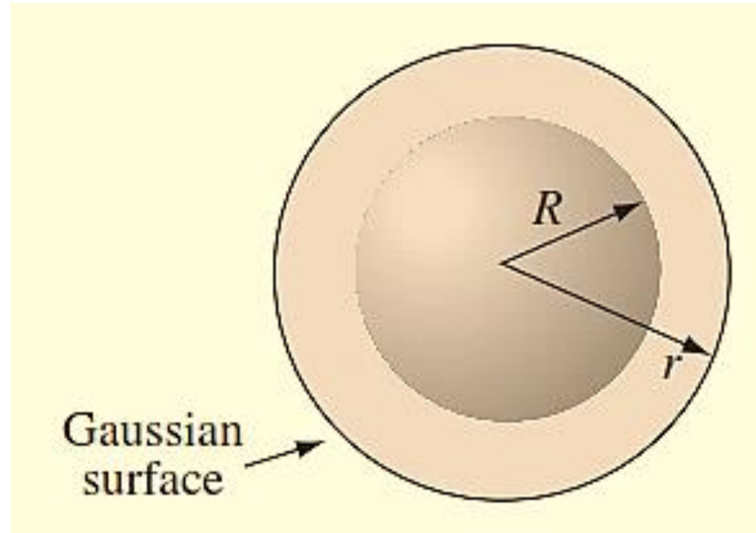
Session 8:

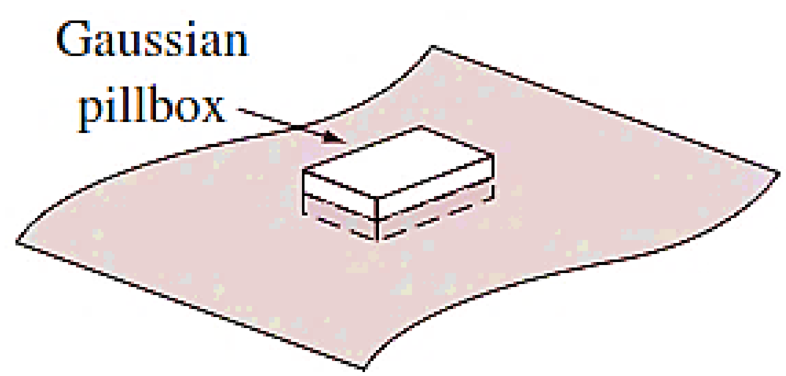
- ✓ **Applying Gauss’s Law**
- ✓ **Examples**

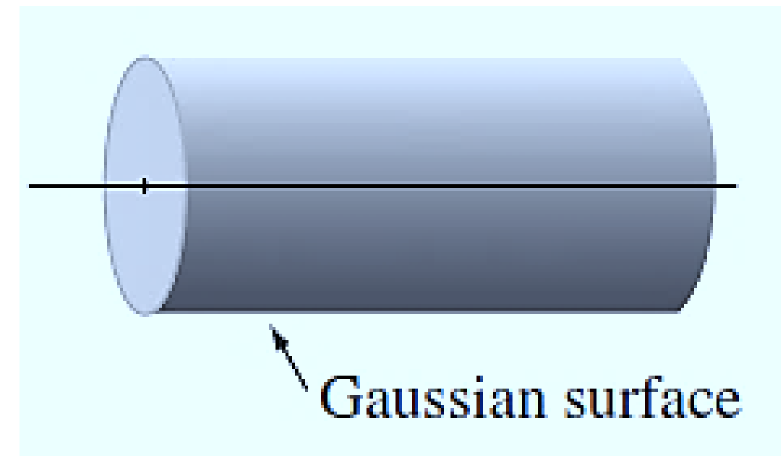
Applying Gauss's Law

❖ **Calculating Electric Field of Highly Symmetric Charge Distribution:**

- **Spherical Symmetry**
- **Cylindrical Symmetry**
- **Planar Symmetry**







Choose a **Gaussian surface** that satisfies one or more of these conditions:

$|\vec{\mathbf{E}}|$ constant over the surface.

$$\vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = |\vec{\mathbf{E}}|\, dA\,, \quad \vec{\mathbf{E}} \parallel d\vec{\mathbf{A}}$$

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{in}}{\varepsilon_o}$$

$\vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = 0$ over the portion of the surface.

Cylindrical Symmetry, Line Charge

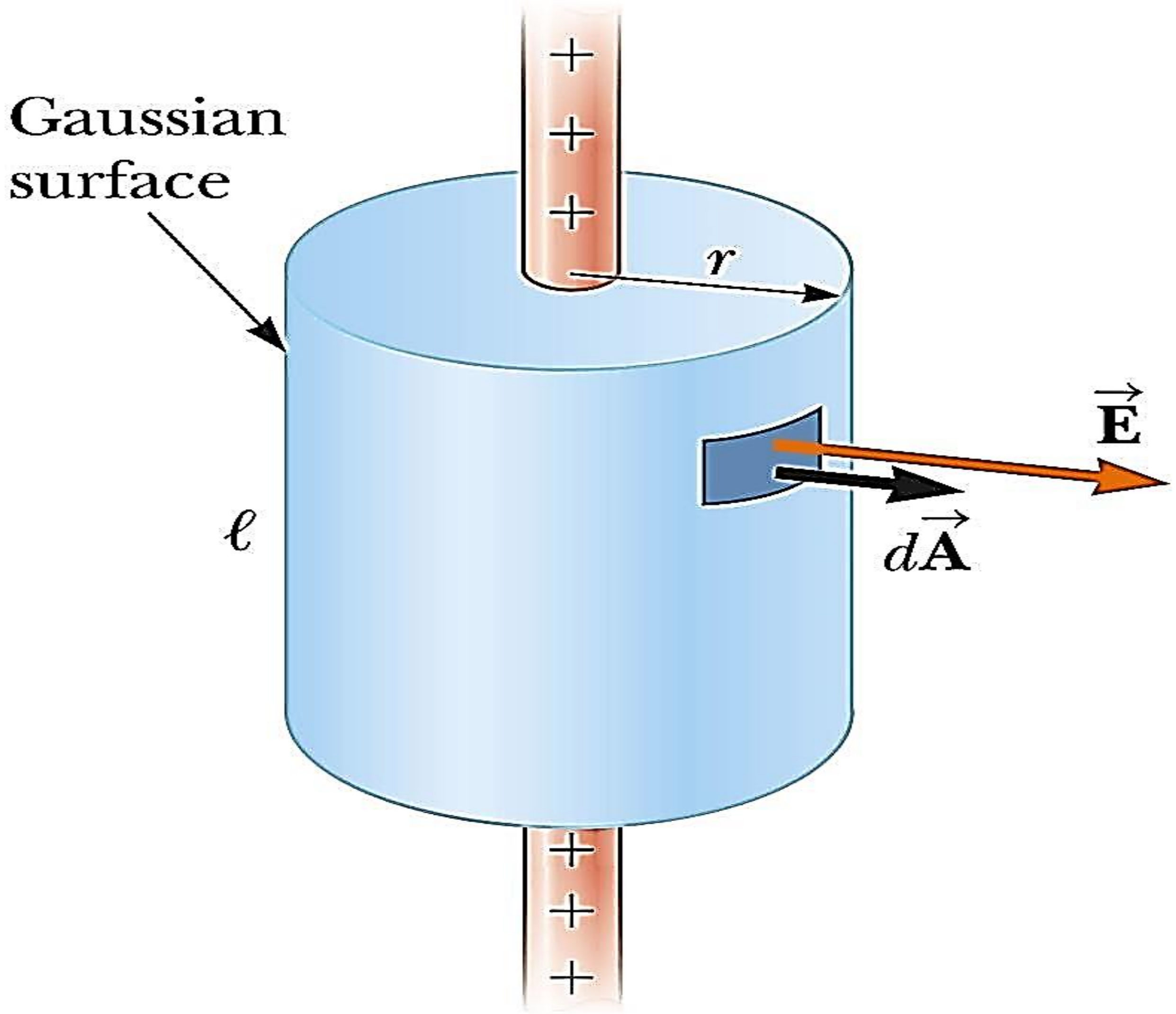
Ex 6. Find the electric field at distance ***r*** from a line of positive charge of infinite length and constant charge per unit length ***λ***.

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{in}}{\varepsilon_o}$$

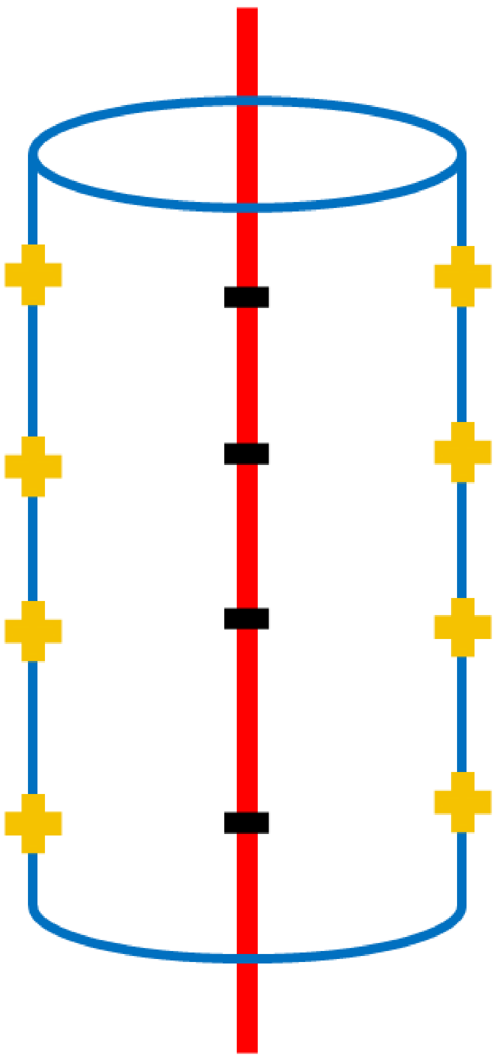
$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \oint E\,dA = \frac{q_{in}}{\varepsilon_o}$$

$$E\left(2\pi r \ell\right) = \frac{\lambda \ell}{\varepsilon_o}$$

$$E = \frac{\lambda}{2\pi\varepsilon_o r} = 2k_e\frac{\lambda}{r}$$



Ex 7. (Prob 23.27) A **long, straight wire** has fixed negative charge with a linear charge density of magnitude **3.6 nC/m**. The wire is to be enclosed by a **coaxial, thin-walled nonconducting cylindrical shell** of radius **1.5 cm**. The shell is to have positive charge on its outside surface with a **surface charge density σ** that makes the **net external electric field zero**. Calculate **σ**.

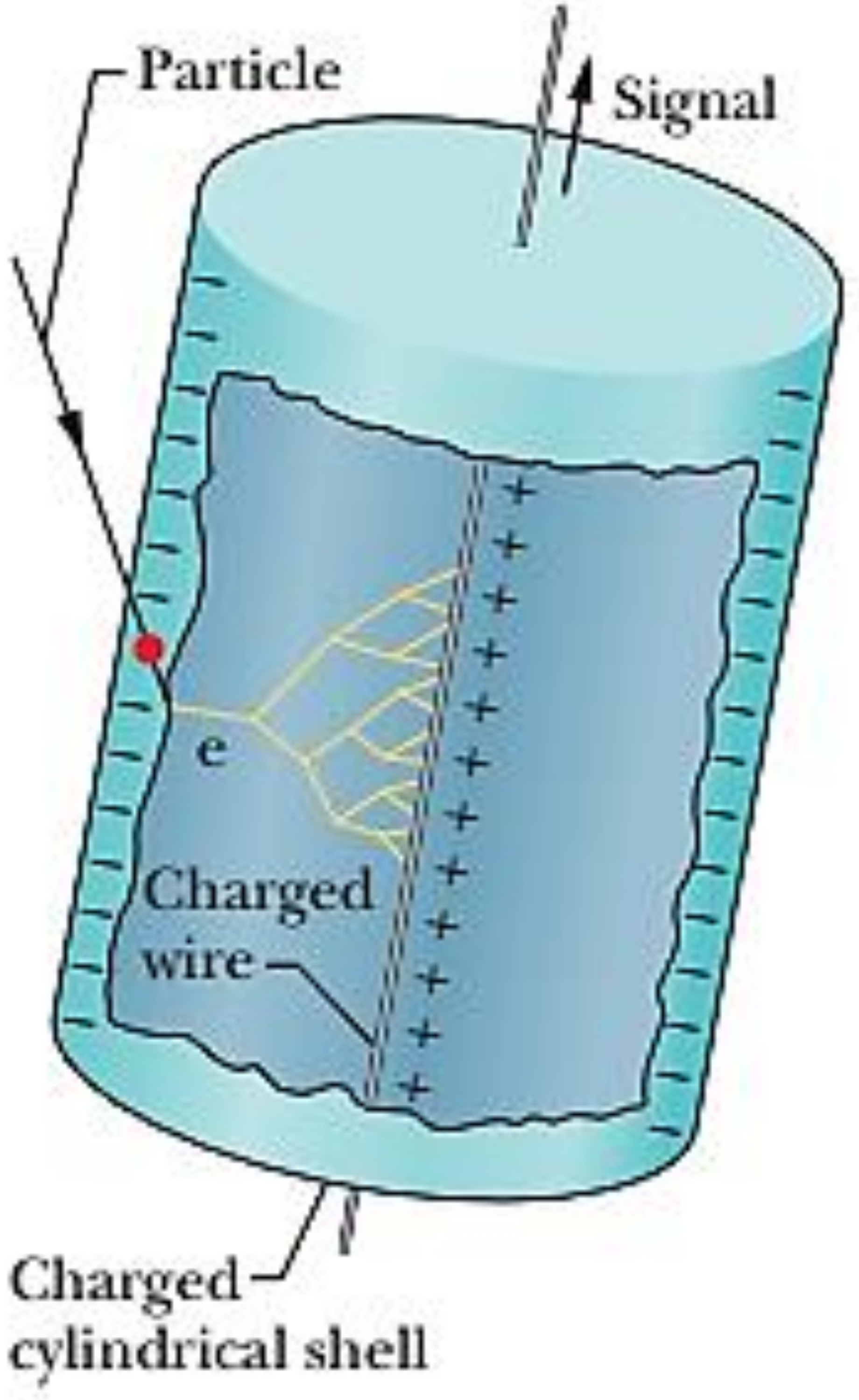


$$\vec{\mathbf{E}}_{\text{wire}} = -\frac{\lambda}{2\pi\varepsilon_o r}\hat{r} \qquad \vec{\mathbf{E}}_{\text{shell}} = \frac{\lambda'}{2\pi\varepsilon_o r}\hat{r} \quad (r > R)$$

$$\vec{\mathbf{E}} = \vec{\mathbf{E}}_{\text{wire}} + \vec{\mathbf{E}}_{\text{shell}} = 0 \;\Rightarrow\; \boxed{\lambda = \lambda'}$$

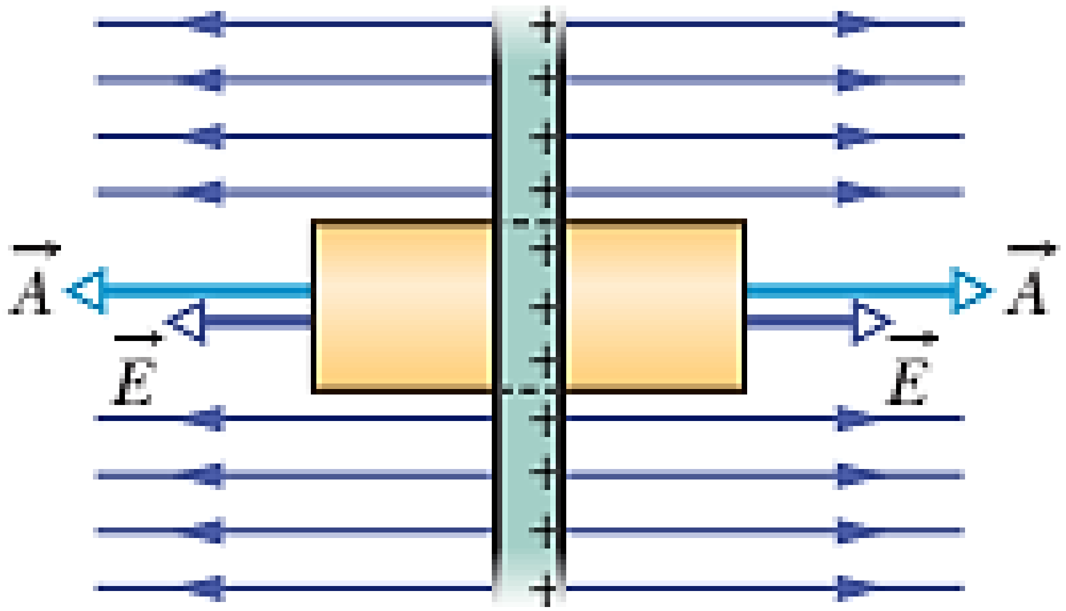
$$\sigma = \frac{Q}{2\pi RL} = \frac{\lambda'}{2\pi R} = \frac{3.6\times 10^{-9}}{2\pi(1.5\times 10^{-2})} = 3.8\times 10^{-8}\ \frac{C}{m^2}$$

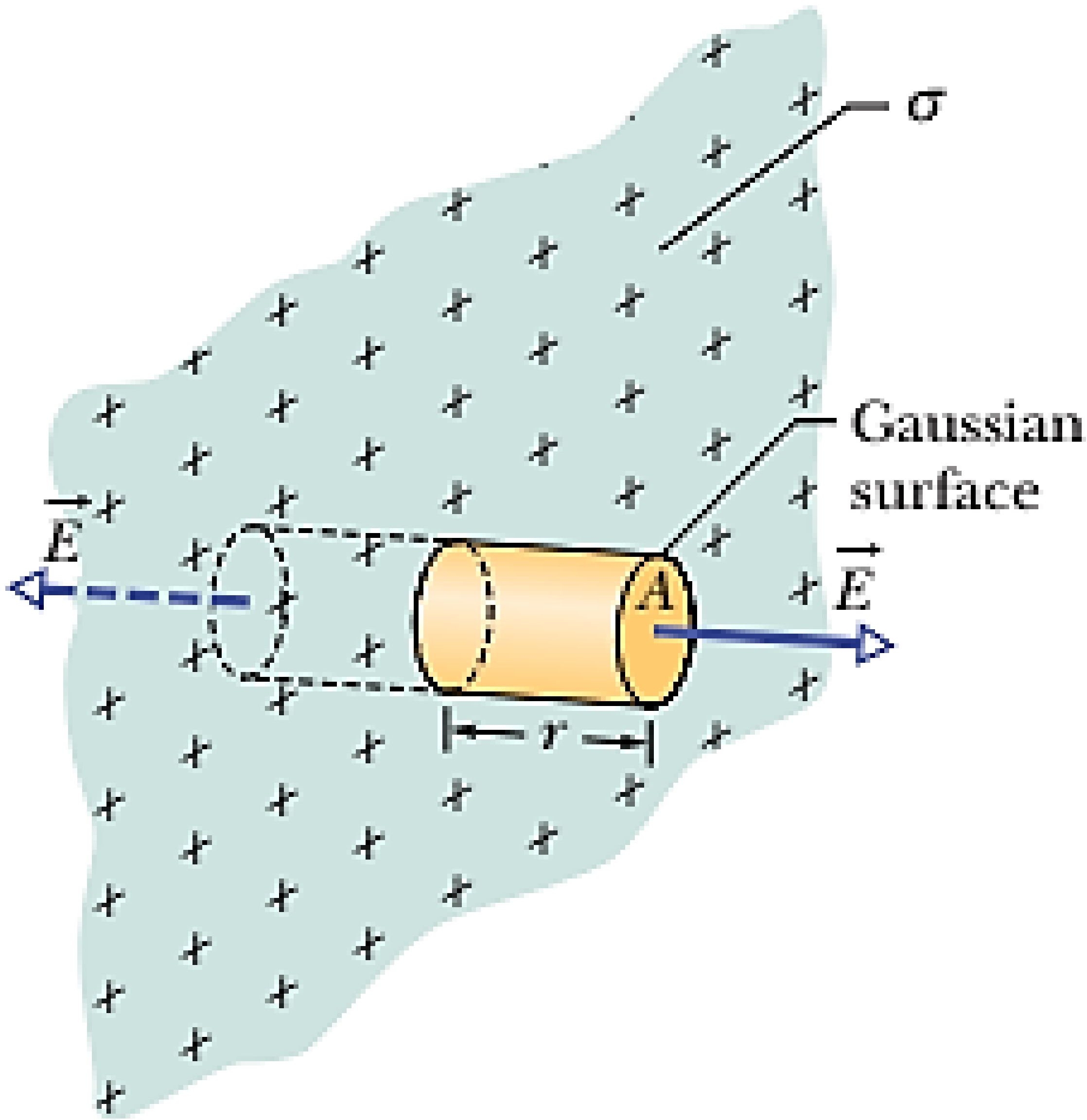
Geiger counter, a device used to detect ionizing radiation



Planar Symmetry, Plane of Charge

Ex 8. Find the electric field due to an **infinite plane** of positive charge with **uniform surface charge density σ**.



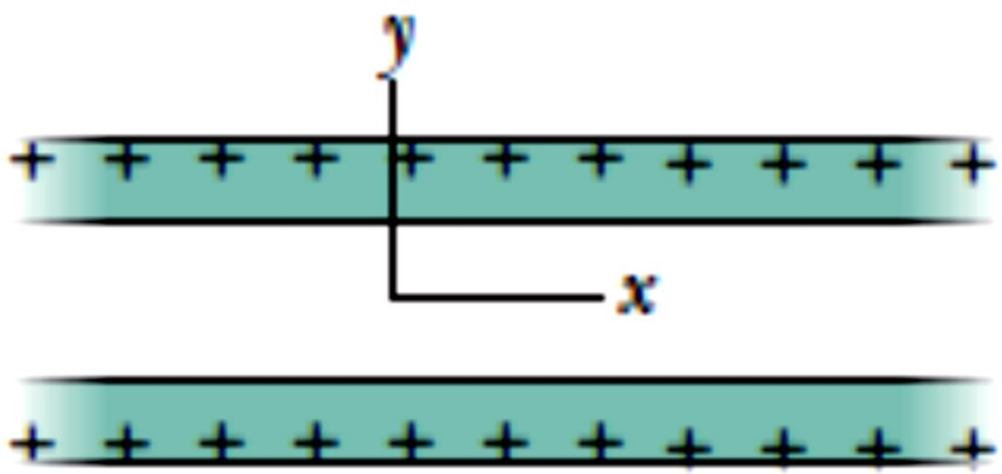


$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \oint E\,dA = \frac{q_{in}}{\varepsilon_o}$$

$$2EA = \frac{\sigma A}{\varepsilon_o}$$

$$E = \frac{\sigma}{2\varepsilon_o}$$

Ex 9. (Prob 23.36) Figure 23-47 shows cross sections through two large, parallel, **nonconducting sheets** with identical distributions of positive charge with **surface charge density** **$\sigma = 1.77 \times 10^{-22}$ C/m^2** . In unit-vector notation, what is **E** at points (a) above the sheets, (b) between them, and (c) below them?



$$\vec{\mathbf{E}} = \vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2$$

$$(a): \vec{\mathbf{E}}_1 = \frac{\sigma}{2\varepsilon_o}\hat{\mathbf{j}}, \vec{\mathbf{E}}_2 = \frac{\sigma}{2\varepsilon_o}\hat{\mathbf{j}} \Rightarrow \boxed{\vec{\mathbf{E}} = \frac{\sigma}{\varepsilon_o}\hat{\mathbf{j}} = 2\times 10^{-11}\,\hat{\mathbf{j}} \quad (N/C)}$$

$$(b): \vec{\mathbf{E}}_1 = -\frac{\sigma}{2\varepsilon_o}\hat{\mathbf{j}}, \vec{\mathbf{E}}_2 = \frac{\sigma}{2\varepsilon_o}\hat{\mathbf{j}} \Rightarrow \boxed{\vec{\mathbf{E}} = 0}$$

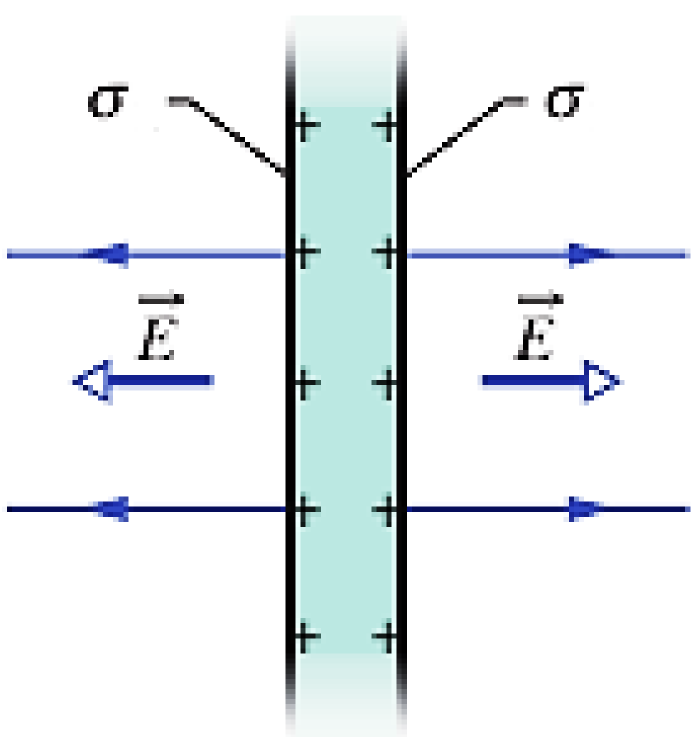
$$(c): \vec{\mathbf{E}}_1 = -\frac{\sigma}{2\varepsilon_o}\hat{\mathbf{j}}, \vec{\mathbf{E}}_2 = -\frac{\sigma}{2\varepsilon_o}\hat{\mathbf{j}} \Rightarrow \boxed{\vec{\mathbf{E}} = -\frac{\sigma}{\varepsilon_o}\hat{\mathbf{j}} = -2\times 10^{-11}\,\hat{\mathbf{j}} \quad (N/C)}$$

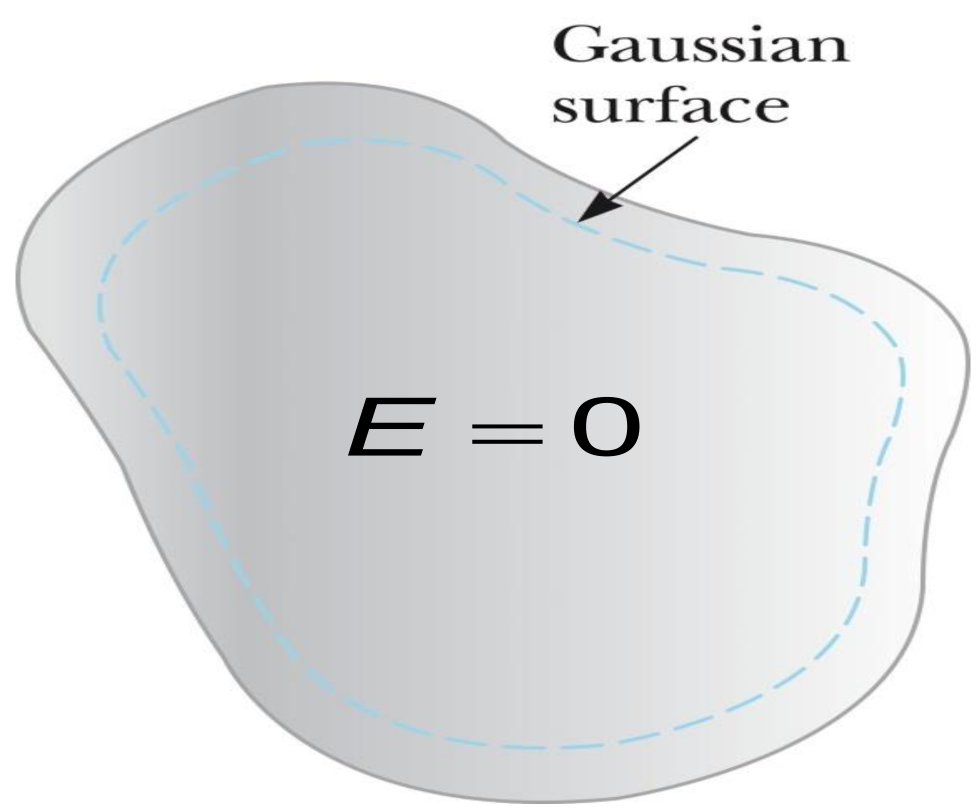
Properties of a Conductor in Electrostatic Equilibrium

When there is **no net motion of charge within a conductor**, the conductor is said to be in **electrostatic equilibrium**.

- **The electric field is zero everywhere inside the conductor.**
- **If the conductor is isolated and carries a charge, the charge resides on its surface.**
- **The electric field at a point just outside a charged conductor is perpendicular to the surface and has a magnitude of σ/ε_o.**

$$\left|\vec{\mathbf{E}}\right| = \frac{\sigma}{\varepsilon_o}$$





Ex 10. A solid insulating sphere of radius ***a*** carries a net positive charge ***Q*** uniformly distributed throughout its volume. A **conducting spherical shell** of inner radius ***b*** and outer radius ***c*** is concentric with the solid sphere and carries net charge ***-2Q***. Find the electric field in the regions labeled and the charge distribution on the shell when the entire system is in **electrostatic equilibrium**.

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{in}}{\varepsilon_o}$$

$$E_1 = k_e \frac{Q}{a^3} r \quad (\text{for } r < a)$$

$$E_2 = k_e \frac{Q}{r^2} \quad (\text{for } a < r < b)$$

$$E_3 = 0 \quad (\text{for } b < r < c)$$

$$E_4 = -k_e \frac{Q}{r^2} \quad (\text{for } r > c)$$

$r = b : -Q \quad ; \quad r = c : -Q$