8.1

$$V_{in} = I_{in}(Z_{in} + G_{22}) + G_{21} V_{out}$$

$$\frac{V_{out} - A_o I_{in} Z_{in}}{Z_{out}} = -G_{11} V_{out} - G_{12} I_{in}$$

$$\Rightarrow \quad I_{in}\left(\frac{A_o Z_{in}}{Z_{out}} - G_{12}\right) = V_{out}\left(\frac{1}{Z_{out}} + G_{11}\right)$$

$$V_{in} = V_{out}\left[G_{21} + \frac{(Z_{in} + G_{22})\left(\frac{1}{Z_{out}} + G_{11}\right)}{\frac{A_o Z_{in}}{Z_{out}} - G_{12}}\right]$$

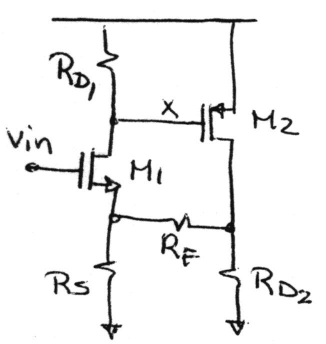
$$\frac{V_{out}}{V_{in}} = \frac{\frac{A_o Z_{in} - G_{12} Z_{out}}{Z_{out}}}{G_{21}\left(\frac{A_o Z_{in}}{Z_{out}} - G_{12}\right) + (Z_{in} + G_{22})\left(\frac{1}{Z_{out}} + G_{11}\right)}$$

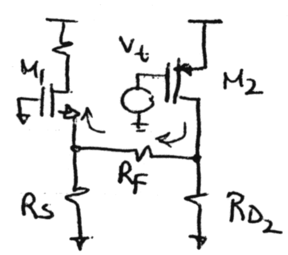
$$A_{v\,\text{open loop}} = \frac{1}{Z_{out}}(A_o Z_{in} - G_{12} Z_{out}) \frac{1}{Z_{in} + G_{22}} \cdot \frac{1}{\frac{1}{Z_{out}} + G_{11}}$$

$$= \frac{G_{11}^{-1}}{G_{11}^{-1} + Z_{out}} \cdot \frac{1}{Z_{in} + G_{22}} (A_o Z_{in} - G_{12} Z_{out})$$

if $G_{12} \ll A_o Z_{in}/Z_{out}$ then the second term can be neglected

8.2




The current through $R_S$:

$$-\frac{g_{m2} \cdot V_t R_{D2}}{R_{D2} + R_F + (R_S \| \frac{1}{g_{m2}})}$$

The current through $M_1$:

$$-\frac{g_{m2} V_t R_{D2}}{R_{D2} + R_F + (R_S \| \frac{1}{g_{m2}})} \cdot \frac{R_S}{R_S + \frac{1}{g_{m2}}}$$

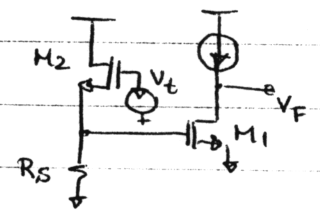
This current is multiplied by $R_{D1}$ to produce $V_F$

loop gain: $\dfrac{g_{m2} R_{D2} R_S R_{D1}}{(R_{D2}+R_F)(R_S+\frac{1}{g_{m2}}) + R_S \cdot \frac{1}{g_{m2}}}$

This result is accurate, whereas $G_{21} A_{v\,open}$ is approximate because it neglects the signal propagating thru the feedback network from the input to the output.

8.3

voltage - current




loop gain: $\dfrac{R_S}{R_S + \frac{1}{g_{m2}}} \cdot g_{m1} r_{o1}$

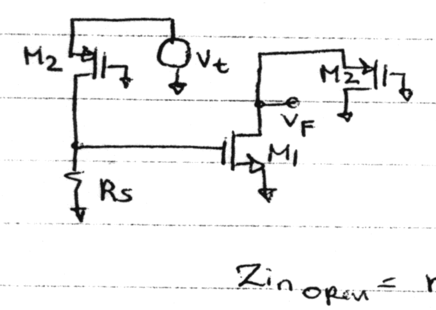
$\dfrac{V_{in}}{R_S} \times (R_S \| \frac{1}{g_{m2}})\, g_{m1} r_{o1} = V$

$\Rightarrow \left.\dfrac{V_{out}}{\frac{V_{in}}{R_S}}\right|_{open} = g_{m1} r_{o1} (R_S \| \frac{1}{g_{m2}})$ $\quad Z_{in\,open} = \dfrac{1}{g_{m2}}$

$Z_{out\,open} = r_{o1}$

$\left.\dfrac{V_{out}}{\frac{V_{in}}{R_S}}\right|_{closed} \underset{r_{o1}\to\infty}{=} \dfrac{1}{g_{m2}} \Rightarrow A_{v\,closed} = \dfrac{1}{g_{m2} R_S}$

$Z_{in\,closed} \underset{r_{o1}\to\infty}{=} 0$ $\quad Z_{out\,closed} = \dfrac{R_S + \frac{1}{g_{m2}}}{g_{m1} R_S} = \dfrac{1}{g_{m1}} + \dfrac{1}{g_{m1} g_{m2} R_S}$

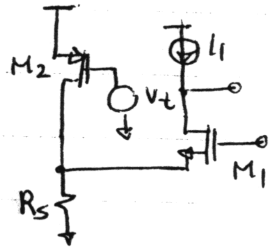



$\dfrac{V_F}{V_t} = g_{m2} R_S \dfrac{g_{m1}}{g_{m2}} = g_{m1} R_S$

$Z_{in\,open} = r_{o2}$ $\quad Z_{out\,open} = \dfrac{1}{g_{m2}}$

$\left.\dfrac{V_{out}}{\frac{V_{in}}{R_S}}\right|_{open} = R_S \cdot \dfrac{g_{m1}}{g_{m2}}$

$A_{v\,closed} = \dfrac{g_{m1}/g_{m2}}{1 + g_{m1} R_S}$ $\quad Z_{in\,closed} = \infty$ $\quad Z_{out\,closed} = \dfrac{1}{g_{m2}(1+g_{m1}R_S)}$



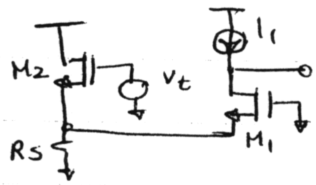

$$R_S\, g_{m2} V_t \frac{r_{o1}}{R_S + \frac{1}{g_{m1}}} = V_F \Rightarrow \text{loop gain: } \frac{R_S\, g_{m2}\, r_{o1}}{R_S + \frac{1}{g_{m1}}}$$

$$R_{out_{open}} = g_{m1} r_{o1} R_S + r_{o1} + R_S \approx g_{m1} r_{o1} R_S$$

$$R_{in_{open}} = \frac{1}{g_{m1}}$$

$$\left.\frac{V_{out}}{V_{in}}\right|_{open} = \frac{r_{o1}}{R_S + \frac{1}{g_{m1}}}$$

$r_{o1} \to \infty$ $\quad A_v = \frac{1}{R_S\, g_{m2}} \qquad R_{in} = 0 \qquad R_{out} = \frac{g_{m1}\left(R_S + \frac{1}{g_{m1}}\right)}{g_{m2}}$




$$\text{loop gain: } V_t \cdot \frac{R_S \| \frac{1}{g_{m1}}}{\frac{1}{g_{m2}} + R_S \| \frac{1}{g_{m1}}} \cdot g_{m1} r_{o1} = V_F$$

$$R_{in_{open}} = \frac{1}{g_{m1} + g_{m2}} \qquad R_{out_{open}} = g_{m1} r_{o1} \left(R_S \| \frac{1}{g_{m2}}\right)$$

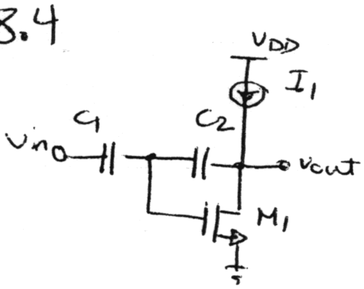
$$A_{v_{open}} \Rightarrow \frac{V_{in}}{R_S}\left(R_S \| \frac{1}{g_{m2}}\right) \times g_{m1} r_{o1} = V_{out} \Rightarrow A_{v_{open}} = \frac{g_{m1} r_{o1}\left(R_S \| \frac{1}{g_{m2}}\right)}{R_S}$$

$r_o \to \infty$

$$A_{v_{closed}} = \frac{\left(R_S \| \frac{1}{g_{m2}}\right)\left(\frac{1}{g_{m2}} + R_S \| \frac{1}{g_{m1}}\right)}{R_S\left(R_S \| \frac{1}{g_{m1}}\right)}$$

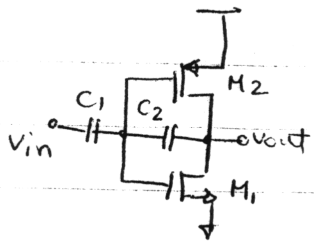
$$R_{in_{closed}} = 0 \qquad R_{out_{closed}} = \frac{\left(R_S \| \frac{1}{g_{m2}}\right)\left(\frac{1}{g_{m2}} + R_S \| \frac{1}{g_{m1}}\right)}{R_S \| \frac{1}{g_{m1}}}$$

## 8.4




$$R_{in} = \frac{1}{C_1 s} + \frac{1}{g_m}$$

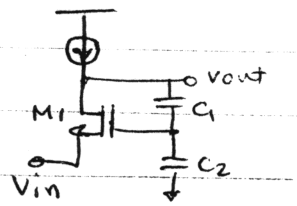
$$R_{out} = \frac{r_o}{1 + \frac{C_2}{C_1 + C_2} g_{m1} r_o} = \frac{C_1 + C_2}{g_{m1} C_2}$$




using the results in part (a)

$$R_{in} = \frac{1}{C_1 s} + \frac{1}{g_{m1} + g_{m2}}$$

$$R_{out} = \frac{C_1 + C_2}{(g_{m1} + g_{m2})C_2}$$




$$\text{loop gain} = g_{m1} r_o \frac{C_1}{C_1 + C_2}$$

$$R_{in\,closed} = \frac{1}{g_{m1}} + \frac{1}{C_2 s}$$

$$R_{out\,closed} = \frac{r_o}{1 + g_{m1} r_o \frac{C_1}{C_1 + C_2}} = \frac{C_1 + C_2}{g_{m1} C_1}$$

8.5

$g_{m1} r_{o1} = 50$

$$-\frac{1}{\left(1+\frac{1}{g_{m1} r_{o1}}\right)\frac{C_2}{C_1} + \frac{1}{g_{m1} r_{o1}}} = -0.95\frac{C_1}{C_2} \Rightarrow \frac{C_1}{C_2} = 1.63$$

Open loop output impedance : $r_o$

loop gain : $\frac{C_2}{C_1 + C_2} g_m r_o$

closed loop $R_{out} = \frac{r_o}{1 + \frac{C_2}{C_1+C_2} g_m r_o} = 0.49 r_o$

8.6

$$R_{in\,closed} = \frac{1}{g_{m1}} \cdot \frac{1}{1 + g_{m2} R_D \frac{C_1}{C_1 + C_2}}$$

$I_1 = I_2 \Rightarrow g_{m2} = \sqrt{2}\, g_{m1}$

$$= \frac{1}{g_{m1}} \cdot \frac{1}{1 + 1000\sqrt{2}\, g_{m1}} = 50 \Rightarrow g_{m1} = 3.42 \text{ m℧}$$

$$g_m = \sqrt{2\mu_n C_{ox} \frac{W}{L} I_D} \Rightarrow I_D = \frac{(3.42 \times 10^{-3})^2}{2 \times 1.342 \times 10^{-4} \times 100} = 435 \mu A$$

8.7

$$\frac{V_x}{I_x} = \frac{R_D}{1 + \frac{g_{m2} R_S (g_{m1} + g_{mb1}) R_D}{(g_{m1} + g_{mb1}) R_S + 1} \cdot \frac{C_1}{C_1 + C_2}} \overset{R_D \to \infty}{=} \frac{(g_{m1} + g_{mb1}) R_S + 1}{g_{m2} R_S (g_{m1} + g_{mb1})} \cdot \frac{C_1 + C_2}{C_1}$$

if $(g_{m1} + g_{mb1}) R_S \gg 1 \Rightarrow \frac{V_x}{I_x} = \frac{1}{g_{m2}} \cdot \frac{C_1 + C_2}{C_1}$

8.8

If $f_{-3dB}$ of each stage is $\omega_0$

$$\left| \frac{1}{(1 + \frac{s}{\omega_0})^n} \right| = \frac{1}{\sqrt{2}} \Rightarrow \left(1 + \left(\frac{\omega}{\omega_0}\right)^2\right)^n = 2$$

if we indicate the Gain x $f_{-3dB}$ as $K$ = const

$$\Rightarrow \left(\frac{K}{\omega_0}\right)^n = 500$$

$$\Rightarrow \frac{\ln 2}{\ln\left(1+\left(\frac{\omega}{\omega_0}\right)^2\right)} = \frac{\ln 500}{\ln\left(\frac{K}{\omega_0}\right)} \Rightarrow 1+\left(\frac{\omega}{\omega_0}\right)^2 = \left(\frac{K}{\omega_0}\right)^{\frac{\ln 2}{\ln 500}}$$

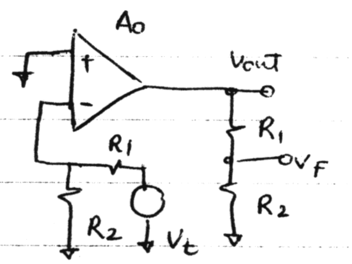
$$\Rightarrow \omega = \omega_0 \sqrt{\left(\frac{K}{\omega_0}\right)^{\frac{\ln 2}{\ln 500}} - 1}$$

$$\frac{d\omega}{d\omega_0} = 0 \Rightarrow \sqrt{\left(\frac{K}{\omega_0}\right)^{\frac{\ln 2}{\ln 500}} - 1} + \frac{\omega_0}{2} \cdot \frac{1}{\sqrt{\left(\frac{K}{\omega_0}\right)^{\frac{\ln 2}{\ln 500}} - 1}} \cdot \left(-\frac{\ln 2}{\ln 500} \cdot \frac{1}{\omega_0} \cdot \left(\frac{K}{\omega_0}\right)^{\frac{\ln 2}{\ln 500}}\right.$$

$$\Rightarrow \left(\frac{K}{\omega_0}\right)^{\frac{\ln 2}{\ln 500}} - 1 - \frac{1}{2}\frac{\ln 2}{\ln 500}\left(\frac{K}{\omega_0}\right)^{\frac{\ln 2}{\ln 500}} = 0 \Rightarrow \frac{K}{\omega_0} = 1.67$$

$\Rightarrow$ Gain per stage = 1.67 Stage BW = 598 MHz

8.9




$$Av_{open} = A_o \frac{R_2}{R_1+R_2+R_o}$$

$$Rout_{open} = R_o \| (R_1+R_2)$$

Loop gain=

$$\left(\frac{R_2}{R_1+R_2}\right) A_o \left(\frac{R_2}{R_o+R_1+R_2}\right)$$

$$Av_{closed} = \frac{A_o \dfrac{R_2}{R_o+R_1+R_2}}{1+\left(\dfrac{R_2}{R_1+R_2}\right) A_o \left(\dfrac{R_2}{R_o+R_1+R_2}\right)}$$

$$Rout_{closed} = \frac{R_o \| (R_1+R_2)}{1+\left(\dfrac{R_2}{R_1+R_2}\right) A_o \left(\dfrac{R_2}{R_o+R_1+R_2}\right)}$$

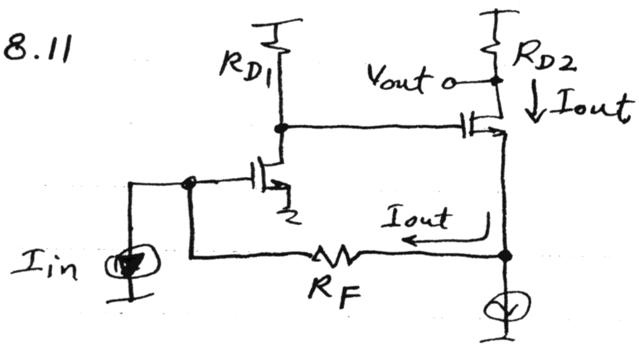
8.10 $$\frac{1+\frac{C_2}{C_1}}{\left(1+\frac{C_2}{C_1}\right)\frac{1}{g_{m1}(r_{o2}\|r_{o4})}+1} = 0.95\left(1+\frac{C_2}{C_1}\right)$$

$\uparrow$ 5% gain error

$$g_{m1}(r_{o2}\|r_{o4}) \approx 24.4$$

$$\Rightarrow \quad 1+\frac{C_2}{C_1} \leq 1.28$$

8.11




$I_{out}$ fully flows through $R_F$ $\therefore \Rightarrow$ $I_{out} = I_{in}$ and $V_{out} = I_{out} \cdot R_{D2}$.

Thus, the transimpedance is equal to $R_{D2}$.

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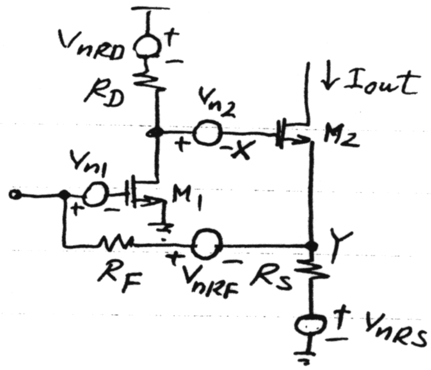



8.11 (cn'td)

$$-(\underbrace{I_{out}R_S + V_{nRS}}_{V_Y} + V_{nRF} + V_{n1})\, g_{m1}R_D + V_{nRD} + V_{n2} = V_X$$

$$(V_X - V_Y)\, g_{m2} = I_{out}$$

$$\Rightarrow g_{m2}\left[(I_{out}R_S + V_{nRS} + V_{nRF} + V_{n1})(-g_{m1}R_D) + V_{nRD} + V_{n2} - (I_{out}R_S + V_{nRS})\right] = I_{out}$$

$$\Rightarrow I_{out}\left[1 + g_{m2}R_S(g_{m1}R_D + 1)\right] = g_{m2}\left[(-g_{m1}R_D - 1)V_{nRS} - g_{m1}R_D V_{nRF} - g_{m1}R_D V_{n1} + V_{nRD} + V_{n2}\right]$$

$$\Rightarrow I_{out} = \frac{g_{m2}\left[-(1+g_{m1}R_D)V_{nRS} - g_{m1}R_D V_{nRF} - g_{m1}R_D V_{n1} + V_{nRD} + V_{n2}\right]}{1 + g_{m2}R_S(1+g_{m1}R_D)}$$

If we apply a current of $I_{in}$ to the input, the resulting output current is obtained as :

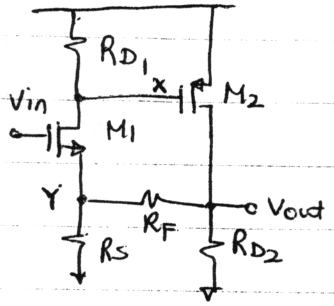
$$\left\{\underbrace{\left[(I_{in}+I_{out})R_S + I_{in}R_F\right](-g_{m1}R_D)}_{V_X} - \underbrace{(I_{in}+I_{out})R_S}_{V_Y}\right\} g_{m2} = I_{out}$$

$$\frac{I_{out}}{I_{in}} = \frac{\left[-g_{m1}R_D(R_S+R_F) - R_S\right] g_{m2}}{1 + g_{m2}R_S(1+g_{m1}R_D)}$$

Dividing the output noise current by the gain yields the input-referred noise current:

$$I_{n,in} = \frac{-(1+g_{m1}R_D)V_{nRS} - g_{m1}R_D V_{nRF} - g_{m1}R_D V_{n1} + V_{nRD} + V_{n2}}{-g_{m1}R_D(R_S+R_F) - R_S}$$

8.12




$I_{D_2} = 1\,mA \Rightarrow \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 (V_{DD} - V_X - |V_{TP}|)^2 = 1\,mA$

$\Rightarrow V_X = 1.478$

$I_{D_1} = 761\,\mu A$

$$\begin{cases} \dfrac{V_Y}{2k} + \dfrac{V_Y - V_{out}}{2k} = 761\,\mu A & \Rightarrow V_{out} = 1.841 \\ \dfrac{V_{out}}{2k} + \dfrac{V_{out} - V_Y}{2k} = 1\,mA & \Rightarrow V_Y = 1.681 \end{cases}$$

$I_{D_1} = 0.761\,mA \Rightarrow \frac{1}{2} \times 1.342\times10^{-4} \times 100\,(V_{in} - V_Y - V_{Tn})^2 = 0.761\,mA$

$V_{in} - V_Y = 1.037 \Rightarrow V_{in} = 2.717\,V$

$G_{21} = 0.5$

$$\begin{cases} g_{m_1} = \dfrac{2\times0.761m}{1.037-0.7} = 4.52\,m℧ \\ g_{m_2} = \dfrac{2\times1\,mA}{1.522-0.8} = 2.77\,m℧ \end{cases}$$

$Av_{open} = \dfrac{-2k}{1k + \dfrac{1}{4.52\,m℧}}\left[-2.77\,m℧\left[2k \,||\, 4k\right]\right] = 6.04$

$Av_{closed} = \dfrac{6.048}{1 + 3.024} = 1.503$

8.13

problem 8.9

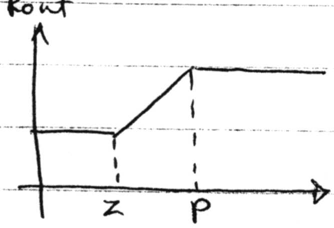
$$R_{out} = \frac{R_o(R_1+R_2)}{R_o + R_1 + R_2 + \dfrac{R_2^{\,2}}{R_1+R_2}\cdot\dfrac{A_o}{1+\dfrac{s}{\omega_o}}}$$

Zero: $\omega_o$

pole: $\omega_o + \dfrac{A_o R_2^2}{(R_1+R_2)(R_o+R_1+R_2)}\,\omega_o$

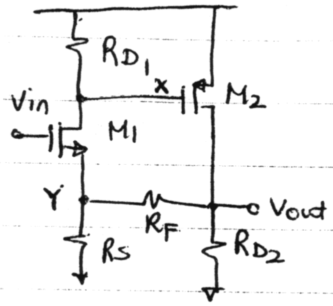
DC value: $\dfrac{R_o(R_1+R_2)}{R_o + R_1 + R_2 + A_o\, R_2^2/R_1+R_2}$

final value: $R_o \,||\, (R_1+R_2)$




The output impedance is less reduced, as the loop gain gets smaller.

8.12




$I_{D2} = 1\,mA \Rightarrow \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_2 (V_{DD} - V_X - |V_{TP}|)^2 = 1\,mA$

$\Rightarrow V_X = 1.478$

$I_{D1} = 761\,\mu A$

$$\begin{cases} \dfrac{V_Y}{2K} + \dfrac{V_Y - V_{out}}{2K} = 761\,\mu A & \Rightarrow V_{out} = 1.841 \\ \dfrac{V_{out}}{2K} + \dfrac{V_{out} - V_Y}{2K} = 1\,mA & \Rightarrow V_Y = 1.681 \end{cases}$$

$I_{D1} = 0.761\,mA \Rightarrow \frac{1}{2} \times 1.342\times10^{-4} \times 100\,(V_{in} - V_Y - V_{Tn})^2 = 0.761\,mA$

$V_{in} - V_Y = 1.037 \Longrightarrow V_{in} = 2.717\,V$

$G_{21} = 0.5$

$$\begin{cases} g_{m1} = \dfrac{2\times0.761m}{1.037-0.7} = 4.52\,m℧ \\ g_{m2} = \dfrac{2\times1mA}{1.522-0.8} = 2.77\,m℧ \end{cases}$$

$A_{V_{open}} = \dfrac{-2^K}{1^K + \dfrac{1}{4.52\,m℧}}\left[-2.77^{m℧}\left[2^K \| 4^K\right]\right] = 6.04$

$A_{V_{closed}} = \dfrac{6.048}{1+3.024} = 1.503$

8.13

problem 8.9

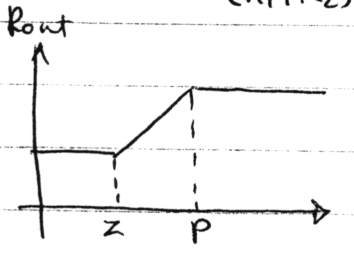
$$R_{out} = \frac{R_o(R_1+R_2)}{R_o + R_1 + R_2 + \dfrac{R_2^{\,2}}{R_1+R_2}\cdot\dfrac{A_o}{1+\frac{s}{\omega_o}}}$$

Zero: $\omega_o$

pole: $\omega_o + \dfrac{A_o R_2^2}{(R_1+R_2)(R_o+R_1+R_2)}\,\omega_o$

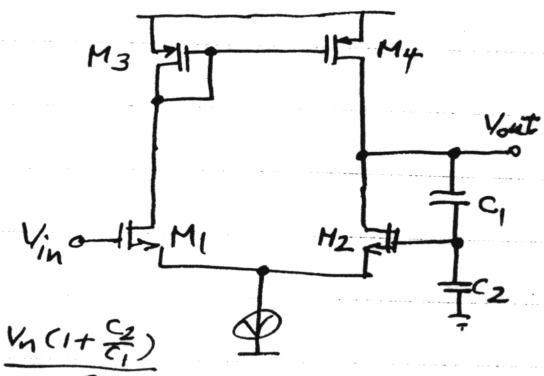
DC value: $\dfrac{R_o(R_1+R_2)}{R_o + R_1 + R_2 + A_o\, R_2^2/R_1+R_2}$

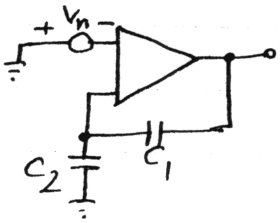
final value: $R_o \| (R_1+R_2)$




The output impedance is less reduced, as the loop gain gets smaller.

8.14 The input-referred noise voltage of the circuit is the same as that of the open-loop circuit:




$V_{out} = V_n\left(1+\frac{C_2}{C_1}\right)$

$\Rightarrow$ input-referred noise $= \dfrac{V_n\left(1+\frac{C_2}{C_1}\right)}{1+\frac{C_2}{C_1}}$

$= V_n$

The noise produced by $M_1 - M_4$ referred to the input is:

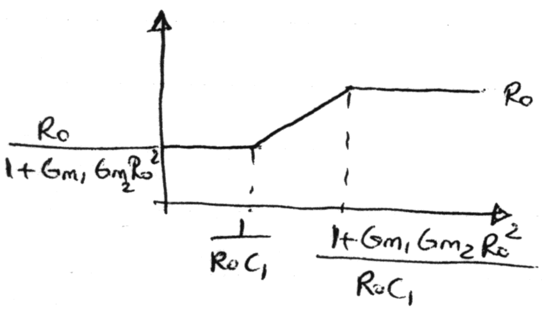
$$\overline{V_n^2} = 4kT\left(\frac{2}{3g_{m1,2}} + 2\,\frac{2g_{m3,4}}{3g_{m1,2}^2}\right) + 2\,\frac{K_N}{(WL)_{1,2}C_{ox}f} + 2\,\frac{K_P}{(WL)_{3,4}C_{ox}f} \times \frac{g_{m3,4}^2}{g_{m1,2}^2}$$

8.15

a) $Z_{in\ open} = R_o$

$$Z_{in\ closed} = \frac{R_o}{1 + G_{m1}G_{m2}R_o\dfrac{R_o}{1+R_oC_1s}} = \frac{R_o(1+R_oC_1s)}{R_oC_1s + 1 + G_{m1}G_{m2}R_o^2}$$

Zero: $\dfrac{1}{R_oC_1}$ Pole: $\dfrac{1+G_{m1}G_{m2}R_o^2}{R_oC_1}$ DC: $\dfrac{R_o}{1+G_{m1}G_{m2}R_o^2}$ final: $R_o$




b) Heavy feedback at lower frequency. As frequency increases, feedback weakens since the output impedance of the feedforward amplifier reduces

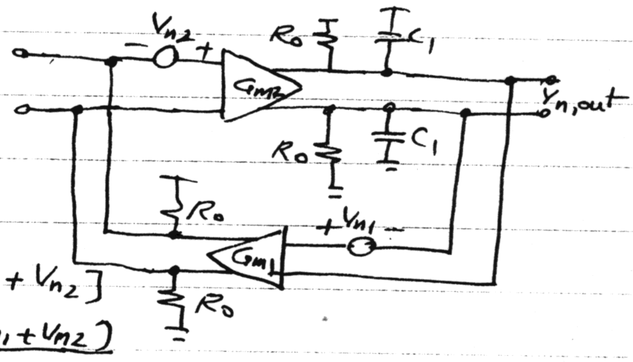
8.15 (c) For input-referred noise voltage, we short the input, hence $\overline{V_{n,out}^2} = 4kT \times 2\left(\frac{2}{3}g_m + \frac{1}{R_0}\right)\left(\left|R_o \| \frac{1}{C_1 s}\right|\right)$.

Dividing this by the voltage gain, $g_m^2\left(\left|R_o \| \frac{1}{C_1 s}\right|\right)^2$, we have

$$\overline{V_{n,in}^2} = 8kT\left(\frac{2}{3g_m} + \frac{1}{g_m^2 R_o}\right).$$

For the input noise current, we leave the input open.

Here, $V_{n1}$ and $V_{n2}$ represent the input noise of each differential pair (including the noise of resistors).




$$-V_{n,out} = G_{m2}\left(R_o \| \frac{1}{C_1 s}\right)\left[(V_{n,out} + V_{n1})G_{m1}R_o + V_{n2}\right]$$

$$\Rightarrow V_{n,out} = -\frac{G_{m2}\left(R_o \| \frac{1}{C_1 s}\right)(G_{m1}R_o V_{n1} + V_{n2})}{1 + G_{m2}\left(R_o \| \frac{1}{C_1 s}\right)G_{m1}R_o}$$

If we apply current between the two input terminals with value $I_{in}$, the output voltage is obtained as:

$$-V_{out} = (V_{out}G_{m1} + I_{in})R_o \cdot G_{m2}\left(R_o \| \frac{1}{C_1 s}\right)$$

$$\Rightarrow \frac{V_{out}}{I_{in}} = -\frac{G_{m2}R_o\left(R_o \| \frac{1}{C_1 s}\right)}{1 + G_{m1}G_{m2}R_o\left(R_o \| \frac{1}{C_1 s}\right)}$$

Dividning $V_{n,out}$ by this gain gives the input-referred noise current:

$$I_{n,in} = \frac{G_{m1}R_o V_{n1} + V_{n2}}{R_o} \Rightarrow \overline{I_{n,in}^2} = G_{m1}^2 V_{n1}^2 + \frac{V_{n2}^2}{R_o^2}$$

8.16

a) Due to symmetry of the $\pi$ network, no current flows through $R_F$.

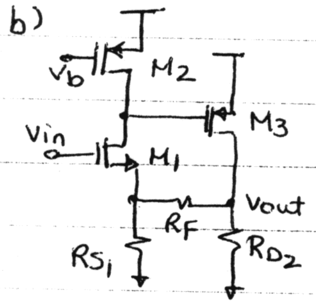
$$\frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)_1 (V_{in} - R_{S_1} \cdot I_{D_1} - V_{Tn})^2 = 0.5\ mA$$

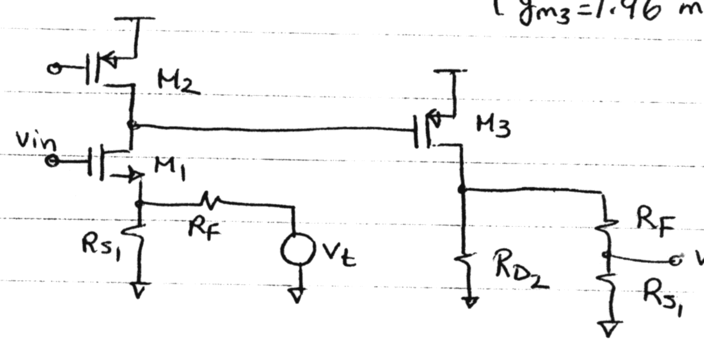
$$(V_{in} - 2.2)^2 = \frac{2 \times 0.5 \times 10^{-3}}{1.342 \times 10^{-4} \times 100} \Rightarrow V_{in} = 2.473\ V$$

$g_{m1} = 3.66\ m℧$, $g_{m2} = 1.96\ m℧$, $g_{m3} = 1.96\ m℧$

$r_{o1} = 20K$, $r_{o2} = 10K$, $r_{o3} = 10K$

b)




$$Av_{open} = \frac{r_{o2}}{R_{S_1} \| R_F + \frac{1}{g_{m1}}} \times g_{m3}\left[ r_{o3} \| R_{D2} \| (R_F + R_{S_1}) \right] = 18.42$$

$$Rout_{open} = r_{o3} \| R_{D2} \| (R_F + R_{S_1}) = 1.667\ K\Omega$$

Loop gain:

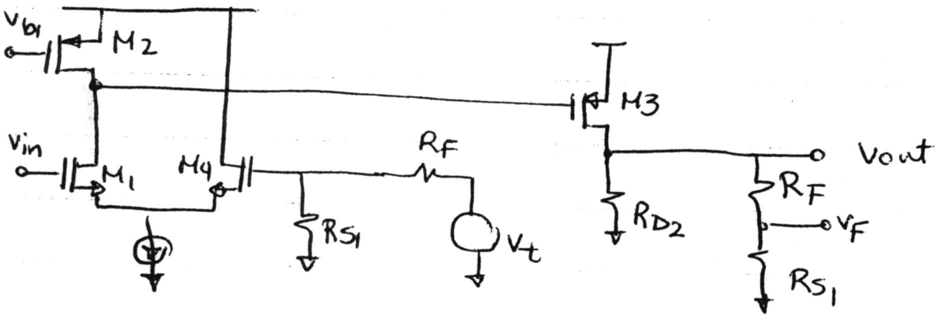
$$\frac{V_t}{R_F} \times (R_F \| R_{S_1}) \times \frac{r_{o2}}{R_{S_1} \| R_F + \frac{1}{g_{m1}}} \times g_{m3}\left[r_{o3} \| R_{D_2} \| (R_F + R_{S_1})\right] \times \frac{R_{S_1}}{R_F + R_{S_1}}$$

$$= V_F \Rightarrow \text{loop gain} = 4.605$$

$$A_v = \frac{18.42}{1 + 4.605} = 3.286$$

$$Rout = \frac{1.667^k}{1 + 4.605} = 297\ \Omega$$

8.17




$r_{o1} = r_{o4} = 20K$

$r_{o3} = 10K \quad r_{o2} = 10^K$

$g_{m1} = g_{m4} = 3.66\ m℧$

$g_{m2} = g_{m3} = 1.96\ m℧$

$$Av_{open} = \frac{1}{2} g_{m1} (r_{o1} \| r_{o2})\ g_{m3} \left( r_{o3} \| R_{D2} \| (R_F + R_{S1}) \right) = 39.85$$

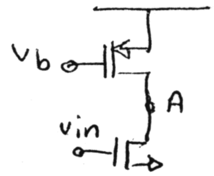
$$R_{out} = r_{o3} \| R_{D2} \| (R_F + R_{S1}) = 1.667\ k$$

$$V_t \quad \frac{R_{S1}}{R_{S1} + R_F} \times \frac{g_{m1}}{2} \times (r_{o1} \| r_{o2})\ g_{m3} \left( r_{o3} \| R_{D2} \| (R_F + R_{S1}) \right) \frac{R_{S1}}{R_{S1} + R_F} = V_F$$

loop gain = 9.96

$$A_v = \frac{39.85}{1 + 9.96} = 3.63 \qquad R_{out} = \frac{1.667^K}{1 + 9.96} = 153\,\Omega$$

Smaller output impedance compared to 8.16.




8.18

a) $I_{D_n} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS_n} - V_{TH_n})^2 \Rightarrow V_{GS_n} = 0.973\,V$

$V_{GS_p} = 1.311 \Rightarrow 3 - V_b = 1.311 \Rightarrow V_b = 1.6?$

$V_{in} = R_1 \times I + V_{GS_n}$

$M_3$: saturation $\Rightarrow -V_A + V_b > |V_{THp}| \Rightarrow V_A < 1.689 - 0.8 = 0.889$

$M_4$: saturation $\Rightarrow -V_{out} + V_b > V_{THP} \Rightarrow V_{out} < 0.889 \Rightarrow R_1 I < 0.889 \Rightarrow R_1 \leq 177\check{7}$

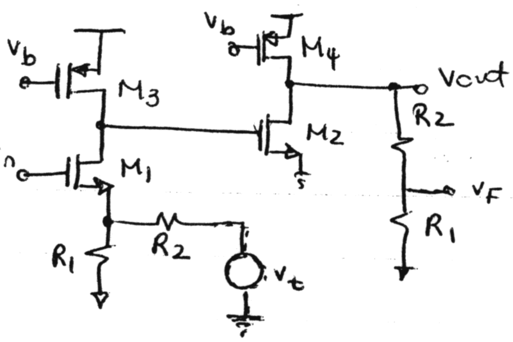
$M_1$: saturation $\Rightarrow V_A > R_1 I + (V_{GS_1} - V_{TH_n}) \Rightarrow 0.273 + R \times 0.5m < 0.889$

$\Rightarrow R_1 < 1232$

$M_2$: saturation: $V_{out} = R_1 I > V_A - V_{tn} \Rightarrow R_1 > \frac{0.889 - 0.7}{0.5\times 10^{-3}} = 378\,\Omega$

$378 \leq R_1 \leq 1232 \Rightarrow 1.162 \leq V_{in} \leq 1.589$

b)




$R_1 = 805\,\Omega$ $\qquad r_{o1} = r_{o2} = 20K$

$g_{m1} = 3.66\,m\mho$ $\qquad r_{o3} = r_{o4} = 10K$

$g_{m2} = 3.66\,m\mho$

open loop gain $= \frac{r_{o3}}{R_1 \| R_2 + \frac{1}{g_{m1}}} \times g_{m2}(r_{o4} \| (R_1 + R_2) \| r_{o2}) = 97.6$

Output impedance $= r_{o4} \| (R_1 + R_2) \| r_{o2} = 2422\,\Omega$

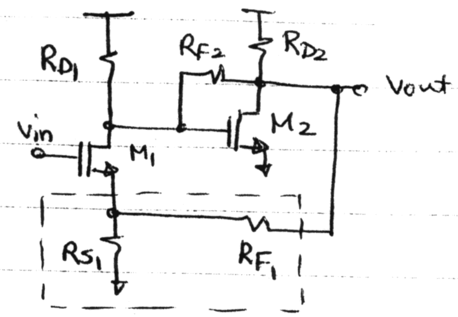
loop gain: $\frac{V_t}{R_2} \times (R_1 \| R_2) \times \frac{r_{o3}}{R_1 \| R_2 + \frac{1}{g_{m1}}} \times g_{m2}(r_{o4} \| (R_1 + R_2) \| r_{o2}) \frac{R_1}{R_1 + R_2} = V_F$

$\Rightarrow$ loop gain $= \frac{1}{3000} \times 635 \times 97.6 \times \frac{805}{3805} = 4.37$

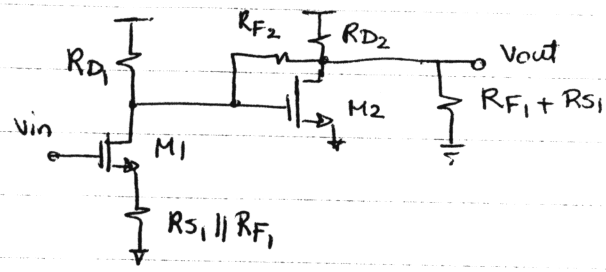
$A_v = \frac{97.6}{1 + 4.37} = 18.17$

$R_{out} = \frac{2422}{1 + 4.37} = 451\,\Omega$

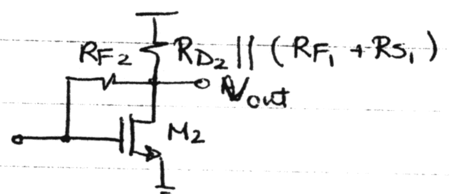
8.19




voltage - voltage




next we consider




$$R_{in_2} = \frac{R_{F2}}{1 + g_{m2}\left[R_{D2} \,||\, (R_{F1} + R_{S1})\right]} = 261\,\Omega$$

$$R_{out_2} = \frac{R_{D2} \,||\, (R_{F1} + R_{S1})}{1 + g_{m2}\left[R_{D2} \,||\, (R_{F1} + R_{S1})\right]} = 174\,\Omega$$

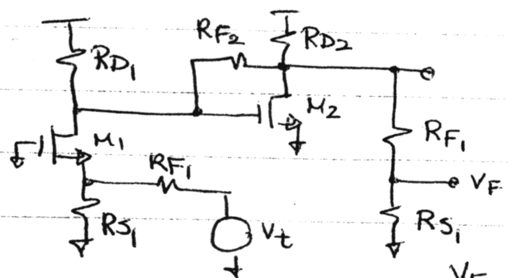
$$A_{V2} = \frac{R_D}{R_D + R_{F2}}\left(-g_{m2} R_{F2} + 1\right) = -3.6$$

$$R_D = R_{D2} \,||\, (R_{F1} + R_{S1}) = 1333$$

Open loop gain $= -\dfrac{R_{D1} \,||\, R_{in2}}{R_{S1} \,||\, R_{F1} + \frac{1}{g_{m1}}} \cdot A_{V2} = \dfrac{231}{1000 + 200} \times 3.6 = 0.69$

Open loop output impedance $= R_{out_2} = 174\,\Omega$

loop gain:




$$\frac{V_t}{R_{F1}} \times (R_{S1} \,||\, R_{F1}) \times \frac{R_{D1}}{R_{S1} \,||\, R_{F1} + \frac{1}{g_{m1}}} \times A_{V2} \times \frac{R_{S1}}{R_{S1} + R_{F1}} = V_F$$

$$\frac{V_F}{V_t} = \frac{1}{2000} \times 1000 \times \frac{1000}{1200} \times 3.6 \times \frac{1}{2} = 0.75$$

$$A_{V_{closed}} = \frac{0.69}{1 + 0.75} = 0.394 \qquad R_{out_{closed}} = \frac{174}{1 + 0.75} = 99.5\,\Omega$$

8.20 $\quad I_{D_1} = I_{D_2} \Rightarrow V_{in} = 1.2538 \Rightarrow I_D = 2.316\ mA$

$$g_{m_1} = \mu_n C_{ox} \left(\frac{W}{L}\right)_1 (V_{GS_1} - V_{THn}) = 1.342\times10^{-4} \times 100 \times (1.2538 - 0.7) = 7.432\ m℧$$

$$g_{m_2} = \mu_p C_{ox} \left(\frac{W}{L}\right)_2 (V_{GS_2} - V_{THp}) = 3.835\times10^{-5} \times 100\,(3 - 1.2538 - 0.8) = 3.628\ m℧$$

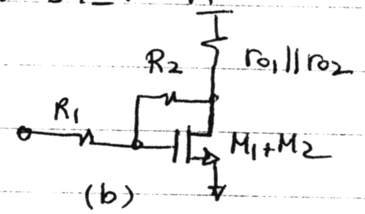
$$r_{o1} = \frac{1}{\lambda_1 I_1} = \frac{1}{0.1\times 2.316\times10^{-3}} = 4.317\ k$$

$$r_{o2} = \frac{1}{\lambda_2 I_2} = \frac{1}{0.2\times 2.316\times10^{-3}} = 2.159\ k$$

(a)

a) $A_V = -(g_{m1}+g_{m2})(r_{o1}||r_{o2}) = -(7.432+3.628)\,1.439 = 15.91$

$R_{out} = r_{o1}||r_{o2} = 1439\ \Omega$




b) $A_V = \frac{1}{R_1} \cdot \frac{-(R_1||R_2)(g_{m1}+g_{m2})(R_2||r_{o1}||r_{o2})}{1+(g_{m1}+g_{m2})(R_2||r_{o1}||r_{o2})\frac{R_1}{R_1+R_2}}$

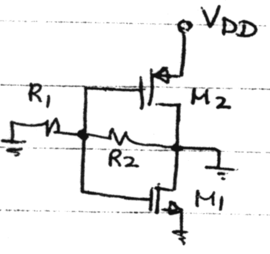
(eq. 8.70)

$$= \frac{1}{1} \times \frac{0.909\,(7.432+3.628)\,1.258}{1+(7.432+3.628)1.258\times\frac{1}{11}} = 5.58$$

$$R_{out} = \frac{R_2||r_{o1}||r_{o2}}{1+(g_{m1}+g_{m2})(R_2||r_{o1}||r_{o2})\frac{R_1}{R_1+R_2}}$$

$R_{out} = 1.258^k / 2.26 = 556\Omega$

(b) We figure out sensitivity for (b), (a) is a special case where $R_1 = 0 \quad R_2 = \infty$




$G_m = g_{m2}$

$R_{out}$ = same as before

$$A_v = \frac{g_{m2}(R_2||r_{o1}||r_{o2})}{1+(g_{m1}+g_{m2})(R_2||r_{o1}||r_{o2})\frac{R_1}{R_1+R_2}} = \frac{3.628\times1.258}{1+13.913\times\frac{1.258}{11}} = 1.76$$

if $R_1 = 0$ and $R_2 = \infty$

$A_v = g_{m2}(r_{o1}||r_{o2}) = 3.628 \times 1.439 = 5.217$

8.21

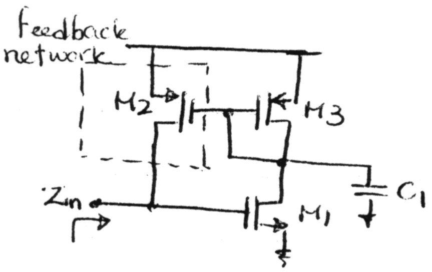
a) $\overline{v_{n\,out}^2} = 4KT \frac{2}{3}(g_{m1}+g_{m2})\,(r_{o1}||r_{o2})^2$

$\overline{v_{n\,in}^2} = \dfrac{4KT\frac{2}{3}}{g_{m1}+g_{m2}} =$

b) $\overline{v_{n\,in}^2} = 4KT\,R_1 + \dfrac{\left[\frac{4KT}{R_2} + 4KT\frac{2}{3}(g_{m1}+g_{m2})\right]R_o^2}{A_v^2} = 4KT\,R_1 + \dfrac{\frac{4KT}{R_2} + 4KT\frac{2}{3}(g_{m1}+g_{m}}{\left(\frac{R_2}{R_1+R_2}\right)^2(g_{m1}+g_{m2})}$

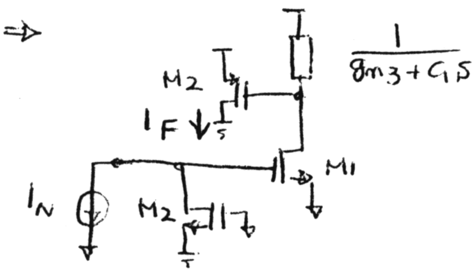
$= 4KT\,R_1 + \dfrac{4KT\,\frac{2}{3}}{g_{m1}+g_{m2}} + \dfrac{4KT/R_2}{\left(\frac{R_2}{R_1+R_2}\right)^2(g_{m1}+g_{m2})^2}$

## 8.22




Voltage - Current

⇒




$$\left|\frac{I_F}{I_N}\right| = r_{o2}\cdot g_{m1}\cdot\frac{1}{g_{m3}+C_1 s}\cdot g_{m2}$$

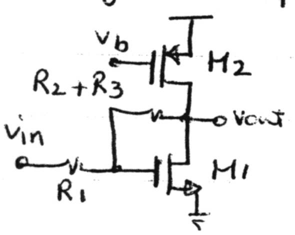
$$Z_{in_{open}} = r_{o2}$$

$$Z_{in_{closed}} = \frac{r_{o2}}{1 - r_{o2}\, g_{m1} g_{m2}\,\frac{1}{g_{m3}+C_1 s}} \overset{r_{o2}\to\infty}{=} -\frac{g_{m3}+C_1 s}{g_{m1} g_{m2}}$$

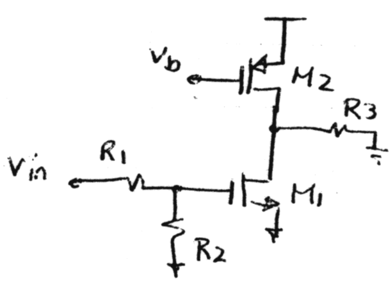
$$= -\frac{g_{m3}}{g_{m1} g_{m2}} - \underbrace{\frac{C_1}{g_{m1} g_{m2}}}\cdot s$$

## 8.23

Very low freq.




Very high freq.




eq. (8.70)

$$A_v = \frac{1}{R_1}\cdot\frac{-(R_1\,\|\,(R_2+R_3))\,g_{m1}\,(R_2+R_3)}{1+g_{m1}(R_2+R_3)\,\frac{R_1}{R_1+R_2+R_3}}$$

$$= \frac{-1}{2^k}\;\frac{(2\|4)^{k\,k}\,\frac{1}{200}\times 4^k}{1+\frac{1}{200}\times 4^k\times\frac{1}{3}}$$

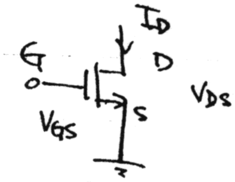
$$= -1.739$$

$$A_v = \frac{R_2}{R_1+R_2}(-g_{m1}R_3)$$

$$= \frac{1}{2}\left(-\frac{1}{200}\times 2000\right) = -5$$

# Chapter 9

## Problem 9.1




(a) For a MOSFET in triode region,

$$I_D = \mu_n C_{ox} \left(\frac{W}{L}\right)\left( (V_{GS} - V_T) V_{DS} - \frac{V_{DS}^2}{2}\right) \quad \text{— } (*)$$

Transconductance, $g_m = \frac{\partial I_D}{\partial V_{GS}}$ (from definition)

$$\frac{\partial I_D}{\partial V_{GS}} = \mu_n C_{ox} \frac{W}{L} V_{DS}$$

$$\boxed{g_m = \mu_n C_{ox} \frac{W}{L} V_{DS}}$$

Output resistance, $r_o = \frac{\partial V_{DS}}{\partial I_D}$

Take derivative of $(*)$ on both sides

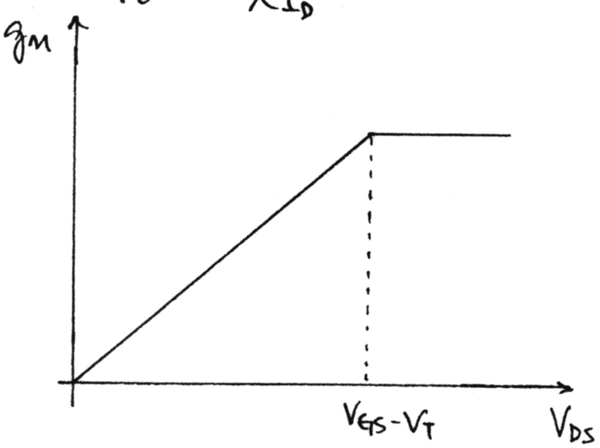
$$1 = \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_T - V_{DS}) \frac{\partial V_{DS}}{\partial I_D}$$

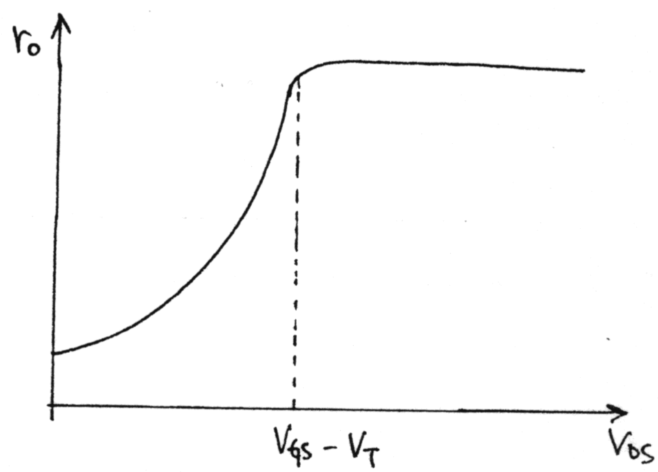
$$\boxed{r_o = \frac{\partial V_{DS}}{\partial I_D} = \frac{1}{\mu_n C_{ox} \frac{W}{L} (V_{GS} - V_T - V_{DS})}}$$

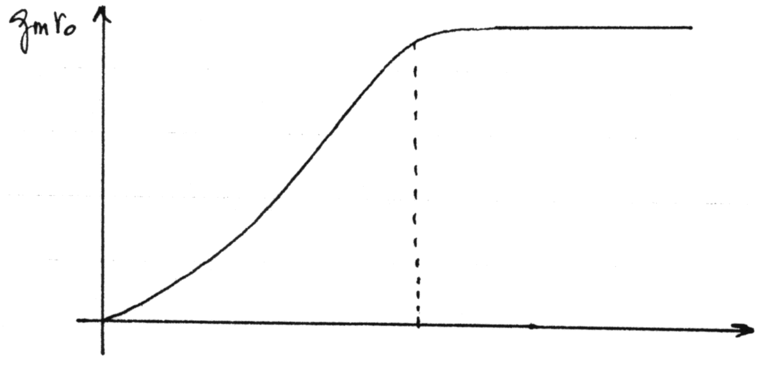
we know in saturation,

$g_m = \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_T)$ &

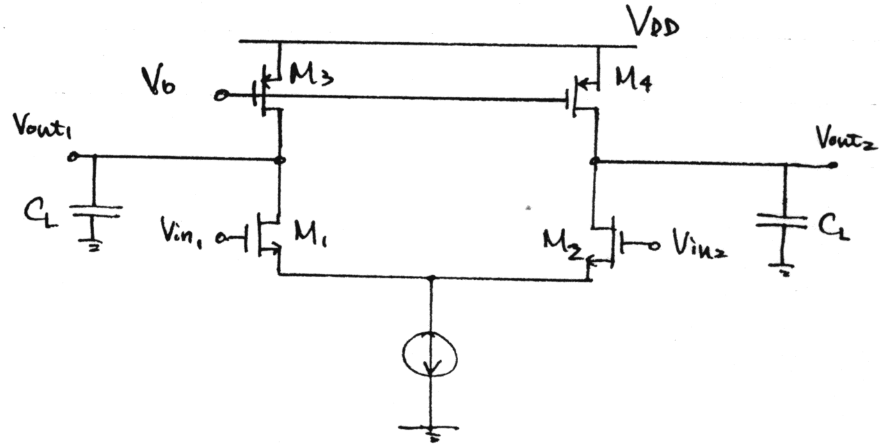
$r_o = \frac{1}{\lambda I_D}$



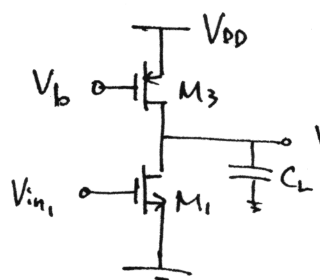






9.1 (b)




Use half-circuit concept:




$$A_V = g_{m1}(r_{o1} \,//\, r_{o2})$$

$$g_{m1} = \sqrt{2\mu_n C_{ox} \frac{W}{L_{eff}} I_D} \qquad \text{where } I_D = \frac{I_{SS}}{2} = 0.5\text{mA}$$

$$C_{ox}(@\, t_{ox} = 400\,\text{Å}) = 0.863\,\text{fF}/\mu\text{m}^2 = 86.3\times10^{-9}\,\text{F/cm}^2$$

$$C_{ox}(@\, t_{ox} = 9\times10^{-9}\text{m}) = \frac{86.3\times10^{-9}}{9\times10^{-9}} \times 400\times10^{-10} = 383.56\,\text{nF/cm}^2$$

$$g_{m1} = \left[2\times(350\,\text{cm}^2/\text{V·sec})(383.56\times10^{-9}\,\text{F/cm}^2)\left(\frac{50}{0.5-0.08\times2}\right)(0.5\times10^{-3}\,\text{A})\right]^{\frac{1}{2}}$$

$$= 4.443\,\text{m}\Omega^{-1}$$

$$r_{o1} = \frac{1}{\lambda_1 I_{D1}} = \frac{1}{(0.1)(1/\text{V})(0.5\,\text{mA})}$$

$$= 20\text{k}\Omega.$$

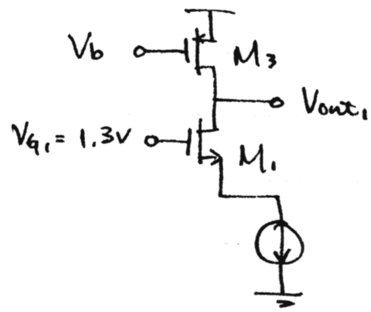
$$r_{o3} = \frac{1}{\lambda_3 I_{D3}} = \frac{1}{(0.2(1/\text{V}))(0.5\,\text{mA})}$$

$$= 10\text{k}\Omega.$$

$$A_V = g_{m1}(r_{o1} \,//\, r_{o3})$$

$$= (4.443\times10^{-3}\,\Omega^{-1})\left[\frac{20\text{k}\cdot 10\text{k}}{20\text{k}+10\text{k}}\right](\Omega)$$

$$= \boxed{29.6 = A_V}$$




To find maximum output swing
If we require both transistors are in saturation,

$V_{DS1} \geq V_{GS1} - V_T$

$V_{D1} - V_{S1} \geq V_{G1} - V_{S1} - V_T$

$V_{D1} \geq V_{G1} - V_T = 1.3 - 0.7 = 0.6\,V.$

$\Rightarrow V_{out1\,min} = 0.6V$

For $M_3$: $I_{D3} = \frac{1}{2}\mu_p C_{ox} \frac{W}{L_3} (V_{GS3} - V_{Tp})^2 (1 + \lambda V_{DS})$

For simplicity, assume channel length modulation is negligible, $\lambda \to 0$.

$$k_p' = \mu_p C_{ox}$$
$$= (100\ cm^2/V\cdot sec)(3.836\times10^{-7}\ F/cm^2)$$
$$= 3.836\times10^{-5}\ A/V^2$$

$$I_{D3} = 0.5mA = \frac{1}{2} k_p' \left(\frac{W}{L_{eff}}\right)(V_{GS3} - V_{Tp})^2$$

$$V_{GS3} - V_{Tp} = \left[\frac{(0.5mA)(2)}{(3.836\times10^{-5}\ A/V^2)\left(\frac{50}{0.5 - 0.09\times2}\right)}\right]^{\frac{1}{2}} = 0.408$$

$$V_{DS3} \geq V_{GS3} - V_{Tp} = 0.408\,V.$$

$$V_{out1\,max} = V_{DD} - V_{DS3} = 3V - 0.408\,V$$
$$= 2.59\,V.$$

$$0.6V \leq V_{out1} \leq 2.59V$$

$\therefore$ One sided output swing $= 2.59 - 0.6 = 1.99V.$

$$\boxed{\begin{aligned}\text{Differential output swing} &= (1.99\times2)V \\ &= 3.98\,V\end{aligned}}$$

9.1 (c). From part (b). $M_3$ will enter the triode region when $V_{DS3} < V_{GS3} - V_{TP} = 0.408\,V$.

At the peak of the output swing $V_{DS3} = 0.408 - 50mV$

$= 0.358\,V$

$$r_{o3} = \frac{1}{\mu_p C_{ox} \frac{W}{L_{eff}} (V_{GS_3} - V_{T_p} - V_{DS})} \quad \text{(from part a)}.$$

$$= \left[ (3.836 \times 10^{-5} A/V^2) \left(\frac{50}{0.32}\right) (0.408 - 0.358) \right]^{-1}$$

$$= 3.337\,k\Omega.$$

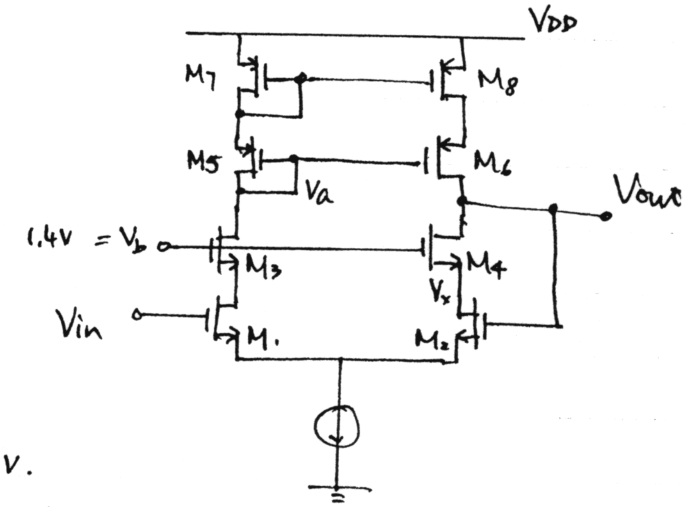
$$A_v = g_{m1} (r_{o1} \,//\, r_{o3})$$

$$= (4.443 \times 10^{-3}\,\Omega^{-1}) \left(\frac{20 \times 3.337}{20 + 3.337}\,k\Omega\right)$$

$$\boxed{A_v = 12.7}$$

9 P.5

# Problem 9.2




(A) $V_b = 1.4V \quad I_{SS} = 1mA \quad (\frac{W}{L})_{1-4} = \frac{100}{0.5}$

To keep $M_3$ in saturation,

$V_a > V_b - V_{THn} = 1.4V - 0.7V$

$= 0.7V$.

Assume $M_5 - M_8$ are identical,

so $V_{GS5} = V_{GS7}$

$$V_{GS5} = \frac{V_{DD} - V_a}{2} = \frac{3-0.7}{2} = 1.15V.$$

$$I_{D5} = \frac{1}{2}\mu_p C_{ox} (\frac{W}{L})_5 (V_{GS5} - V_{THp})^2 (1 + \lambda V_{DS5})$$

$$(\frac{W}{L_{eff}})_5 = \frac{2 I_{D5}}{\mu_p C_{ox} (V_{GS5} - V_{THp})^2 (1 + \lambda V_{DS})}$$

$$= \frac{2(0.5mA)}{(100)(3.836 \times 10^{-7})(1.15 - 0.8)^2 (1 + 0.2(1.15))}$$

$$= 173.$$

$W_5 = 173 \times L_{eff} = 173 \times (0.5 - 0.09 \times 2)$.

$W_5 \cong 56 \mu m$

$$\boxed{W_{5-8} = 56 \mu m}$$

b. Max. output swing $= V_{TH4} - (V_{GS4} - V_{TH2}) = V_{TH4} + V_{TH2} - V_{GS4}$

$I_{D4} = \frac{1}{2}\mu_n C_{ox} (\frac{W}{L})_4 (V_{GS4} - V_{TH4})^2 (1 + \lambda V_{DS4})$ assume $\lambda \to 0$. for simplicity

$$V_{GS4} - V_{TH4} \cong \left[\frac{2 I_D}{\mu_n C_{ox} (\frac{W}{L})_4}\right]^{\frac{1}{2}}$$

$$= \left[\frac{2(0.5mA)}{(350)(3.836 \times 10^{-7})(\frac{100}{0.34})}\right]^{\frac{1}{2}}$$

$$= 0.159V$$

$V_{GS4} = V_{TH4} + 0.159 = 0.859V$

Max. Output swing $= 0.7 + 0.7 - 0.859$

$= 0.541 V$

9.2 (c). $A_{v\,\text{open loop}} = g_{m1}(g_{m4} r_{o4} r_{o2} \,//\, g_{m6} r_{o6} r_{o8})$

$$g_{m1} = \sqrt{2\mu_n C_{ox} \left(\frac{W}{L_{eff}}\right) I_D}$$

$$= \sqrt{2(350)(383.6\times10^{-9})\left(\frac{100}{0.34}\right)(0.5\,mA)}$$

$$= 6.28\,m\Omega^{-1}$$

$$g_{m4} = g_{m1} = 6.28\,m\Omega^{-1}$$

$$g_{m6} = \sqrt{2\mu_n C_{ox}\left(\frac{W}{L_{eff}}\right) I_D}$$

$$= \sqrt{2(100)(383.6\times10^{-9})\left(\frac{56}{0.5-0.09\times2}\right)(0.5\,mA)}$$

$$= 2.59\,m\Omega^{-1}$$

$$r_{o2} = \frac{1}{\lambda I_D} = \frac{1}{(0.1)(0.5\,mA)} = 20\,k\Omega.$$

$$r_{o4} = r_{o2} = 20\,k\Omega.$$

$$r_{o6} = r_{o8} = \frac{1}{(0.2)(0.5\,mA)} = 10\,k\Omega.$$

$$A_v = (6.28\,m)\left[(6.28\,m\times 20k\times 20k) \,//\, (2.59\,m\times 10k\times 10k)\right]$$

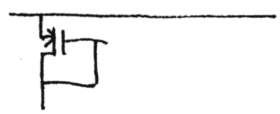
$$\boxed{A_v = 1474}$$

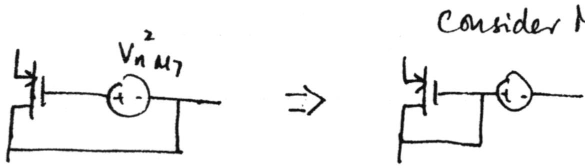
(d) Since this is a cascode configuration, the noise due to $M_{3,4,5,6}$ can be neglected.

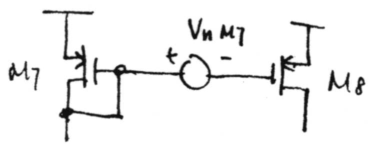
$\overline{V_{n\,input}^2}$ due to $M_{1,2}$ = $4kT\gamma \frac{1}{g_{m1,2}}$

$\overline{V_{n\,input}^2}$ due to $M_8$ = $4kT\gamma \frac{g_{m8}}{g_{m1,2}^2}$ same as in cascode.

$\overline{V_{n\,input}^2}$ due to $M_7$

Consider $M_7$: M7 




∴ 


$M_7$ will induce the same noise as $M_8$.

∴ input-referred noise voltage

$$\overline{V_n^2} = \left[4kT\gamma \frac{1}{g_{m1,2}} + 4kT\gamma \frac{g_{m7,8}}{g_{m1,2}^2}\right]\times 2 \quad \text{where } \gamma = \frac{2}{3}$$

9.2 (d) cont.

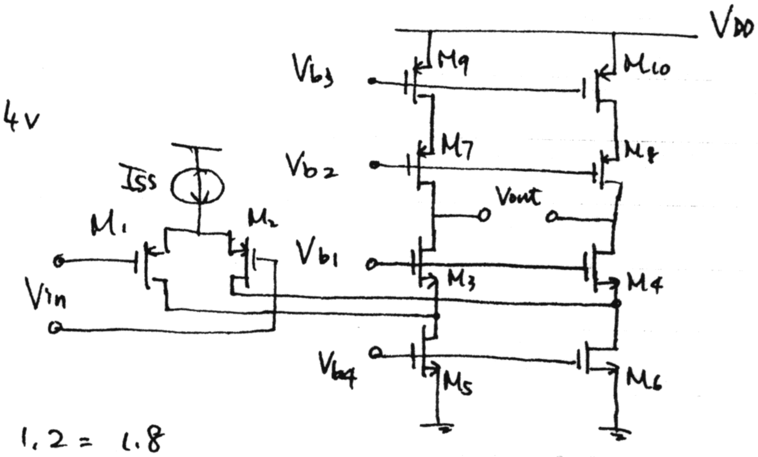
$$\overline{V_n^2} = 4 \times 1.38 \times 10^{-23} \times 300 \times \frac{2}{3} \times 2 \left[ \frac{1}{6.28m} + \frac{2.59m}{(6.28m)^2} \right]$$

$$\boxed{\begin{aligned} \overline{V_n^2} &= 4.966 \times 10^{-18} \ V^2/Hz \\ \text{or } &= 2.23 \times 10^{-9} \ V/\sqrt{Hz} \end{aligned}}$$

# Problem 9.3

Requirements: Max. diff. swing 2.4V

$P_{total} = 6mW$




Max diff swing $= 2[V_{DD} - (V_{OD3} + V_{OD5} + |V_{OD7}| + |V_{OD9}|)]$

$= 2.4V$

$V_{OD3} + V_{OD5} + |V_{OD7}| + |V_{OD9}| = V_{DD} - 1.2 = 1.8$

In general, assign $|V_{OD7}|$ & $|V_{OD9}|$ to be large than $V_{OD3}$ as PMOS has a smaller $\mu_p$. Also M5 need large $V_{OD}$ as $I_{D5}$ is larger.

Let the followings:

$V_{OD3} = 0.3V$, $V_{OD5} = 0.44V$ $|V_{OD7}| = |V_{OD9}| = 0.53V$ $|V_{OD,1}| = 0.53V$

$P_{total} = 6mW = V_{DD} \times (I_{D5} + I_{D6})$.

$I_{D5} = I_{D6} = \frac{6mW}{(3)(2)} = 1mA$.

$I_D = \frac{1}{2}\mu C_{ox}\left(\frac{W}{L}\right)(V_{GS} - V_{TH})^2(1 + \lambda V_{DS})$

$$\left(\frac{W}{L}\right) = \frac{2I_D}{\mu C_{ox}(V_{GS} - V_{TH})^2(1 + \lambda V_{DS})}$$

$$\left(\frac{W}{L_{eff}}\right)_{5,6} = \frac{2(1mA)}{(350)(383.6\times10^{-9})(0.44)^2(1 + (0.1)(0.44))} = 74$$

(Assume $V_{DS} \simeq V_{GS} - V_{TH}$, Since $\lambda$ is small, the result will not be affected too much.)

$W_{5,6} = (74)(0.34\mu m) = 25.16\mu m$.

Let $I_{D1,2} = 0.5mA$., $I_{D3,4} = 0.5mA$

$$\left(\frac{W}{L_{eff}}\right)_{1,2} = \frac{2(0.5mA)}{(100)(383.6\times10^{-9})(0.53)^2(1 + (0.2)(0.53))} = 84$$

$W_{1,2} = 84(0.32\mu m) = 26.88\mu m$

$$\left(\frac{W}{L_{eff}}\right)_{3,4} = \frac{2(0.5mA)}{(350)(383.6\times10^{-9})(0.3)^2(1 + (0.1)(0.3))} = 80.$$

$W_{3,4} = 80 \times 0.34\mu m = 27.2\mu m$.

$$\left(\frac{W}{L_{eff}}\right)_{7,9} = \frac{2(0.5mA)}{(100)(383.6\times10^{-9})(0.53)^2(1 + (0.2)(0.53))} = 84$$

9.3 cont.

$W_{7,9} = 84 \times 0.32\mu m = 26.88\mu m.$

$V_{b4} = V_{GS5} = V_{OD5} + V_{TH5} = 0.44 + 0.7$

$V_{b4} = 1.14V$

$V_{b1} = V_{OD5} + V_{GS3} = V_{OD5} + V_{OD3} + V_{TH3} = V_{OD5} + V_{OD3} + V_{TH0} + \gamma\left(\sqrt{|-2\phi_F + V_{SB}|} - \sqrt{|2\phi_F|}\right)$

$= 0.44 + 0.3 + 0.7 + 0.45\left(\sqrt{0.9 + 0.44} - \sqrt{0.9}\right)$

$= 1.53V$

$V_{b3} = V_{DD} - |V_{GS9}| = V_{DD} - \left[|V_{OD9}| + |V_{TH9}|\right] = 3 - 0.53 - 0.8$

$V_{b3} = 1.67V$

$V_{b2} = V_{DD} - |V_{OD9}| - |V_{GS7}| = V_{DD} - |V_{OD9}| - |V_{OD7}| - |V_{TH7}|..$

$= 3 - 0.53 - 0.53 - \left[0.8 + 0.4\left(\sqrt{0.8 + 0.53} - \sqrt{0.8}\right)\right]$

$V_{b2} = 1.04V$

$V_{in,\text{common mode}} \leq V_{DD} - V_{I_{SS}} - V_{GS1} = 3 - 0.3 - 0.8 - 0.53 = 1.37V$ } $V_{in}$ common mode can be zero ($V_{in_{CM}} = 0$)

$\geq V_{OD5} - V_{TH1} = 0.44 - 0.8 = -0.36V.$

$A_V = g_{m1}\left[(g_{m7} r_{o7} r_{o9}) \,//\, (g_{m3} r_{o3} (r_{o1} // r_{o5}))\right]$

$g_{m1} = \dfrac{2I_{D1}}{V_{GS} - V_{TH}} = \dfrac{2(0.5mA)}{0.53} = 1.89 m\Omega^{-1}$

$g_{m7} = g_{m1} = 1.89m\Omega^{-1}$

$g_{m3} = \dfrac{2I_{D1}}{V_{GS} - V_{TH}} = \dfrac{2(0.5mA)}{0.3} = 3.33m\Omega^{-1}$

$r_{o7} = r_{o9} = \dfrac{1}{\lambda I_D} = \dfrac{1}{(0.2)(0.5mA)} = 10k\Omega = r_{o1}$

$r_{o3} = \dfrac{1}{\lambda I_D} = \dfrac{1}{(0.1)(0.5mA)} = 20k\Omega.$

$r_{o5} = \dfrac{1}{\lambda I_D} = \dfrac{1}{(0.1)(1mA)} = 10k\Omega$

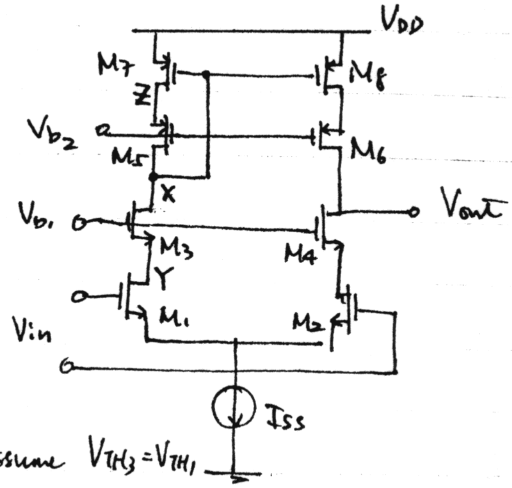
$A_V = (1.89m)\left[(1.89m\,(10k)^2) \,//\, (3.33m\,(20k)(10k/2))\right]$

$= 228$

| | | |
|---|---|---|
| $W_{1,2} = 26.88\mu m$ | $V_{b1} = 1.53V$ | $A_V = 228.$ |
| $W_{3,4} = 27.2\mu m$ | $V_{b2} = 1.04V$ | |
| $W_{5,6} = 25.16\mu m$ | $V_{b3} = 1.67V$ | |
| $W_{7,8,9,10} = 26.88\mu m$ | $V_{b4} = 1.14V$ | |

$-0.36 \leq V_{in,CM} \leq 1.37V$

# Problem 9.4




$(\frac{W}{L})_{1-8} = \frac{100}{0.5}$, $I_{SS} = 1mA$, $V_{b1} = 1.7V$, $\gamma = 0$

(A) $V_{in,CM,min} = V_{ISS} + V_{GS1}$

$= V_{ISS} + V_{THn} + V_{OD1}$

where $V_{ISS}$ is the voltage across $I_{SS}$.

$V_{in,CM,max} = V_Y + V_{TH1}$ ;

$V_Y = V_{b1} - V_{GS3} = V_{b1} - V_{TH3} - V_{OD3}$

$V_{in,CM,max} = V_{b1} - V_{TH3} - V_{OD3} + V_{TH1}$ Assume $V_{TH3} = V_{TH1}$

$\boxed{V_{in,CM,max} = V_{b1} - V_{OD3}}$

To calculate $V_{OD3}$, $I_{D3} = \frac{1}{2}\mu_n C_{ox} (\frac{W}{L})_3 (V_{GS} - V_{TH})^2 (1 + \lambda V_{DS})$ Assume $\lambda \to 0$

$V_{OD3} = V_{GS3} - V_{TH} = \left[ \frac{2 I_{D3}}{\mu_n C_{ox} (\frac{W}{L_{eff}})} \right]^{\frac{1}{2}}$

$= \left[ \frac{2(0.5mA)}{350(383.6 \times 10^{-9})(\frac{100}{0.34})} \right]^{\frac{1}{2}} = 0.159V.$

$\therefore$ $\boxed{V_{in,CM,max} = 1.7 - 0.159 = 1.541V}$

(b) $V_X = ?$ To find $V_X$, we can find $V_{GS7}$

$V_{GS7} - V_{THp} = \left[ \frac{2 I_{D7}}{\mu_p C_{ox} (\frac{W}{L_{eff}})_7} \right]^{\frac{1}{2}}$

$= \left[ \frac{2(0.5mA)}{(100)(383.6 \times 10^{-9})(\frac{100}{0.32})} \right]^{\frac{1}{2}} = 0.289$

$V_{GS7} = 0.289 + V_{THp} = 1.089V$

$V_X = V_{DD} - V_{GS7} = 3 - 1.089V$

$\boxed{V_X = 1.911V}$

(c) For details, please see page 284 (chapter 9).

Max. output swing $= V_{TH4} - (V_{GS4} - V_{TH2})$

$V_{GS3} = V_{GS4}$ by symmetry, $V_{GS3} - V_{THn} = V_{GS4} - V_{THn} = 0.159V$

$V_{GS4} = 0.7 + 0.159 = 0.859V$

Max output swing $= 0.7 - (0.859 - 0.7) =$

$\boxed{\text{Max output Swing} = 0.541V.}$

9.4 cont.

(d). We know $V_X = 1.911V$, $V_{GS5} = V_{GS7} = 1.089V$

To keep $M_7$ in saturation,

$V_Z < V_X + V_{THP}$ ; $V_{b2} = V_Z - |V_{GS5}|$

$V_{b2} < V_X + V_{THP} - |V_{GS5}| = 1.911V + 0.8 - 1.089$

$V_{b2} < 1.622V$

$V_{b2} > V_X - V_{TH5} = 1.911 - 0.8 \Rightarrow V_{b2} > 1.111V.$

$\therefore \boxed{1.11V < V_{b2} < 1.62V}$

e. As this is a cascode configuration, $M_3, M_4, M_5, M_6$ have negligible.

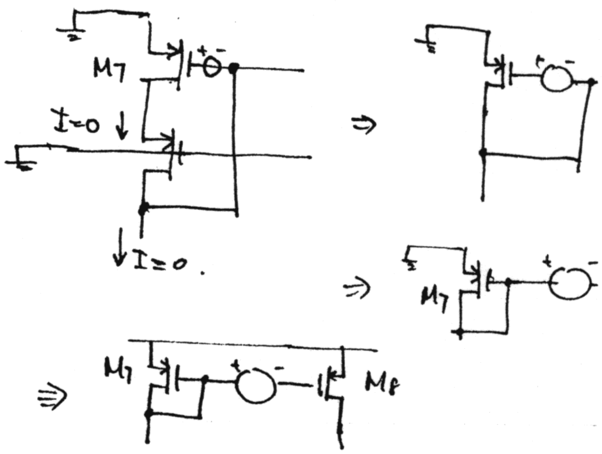
Input referred noise voltage due to $M_1, M_2$

$\overline{V_n^2}_{input} = 4kT\gamma \frac{1}{g_{m1,2}}$

$\overline{V_n^2}_{input\ M_8} = 4kT\gamma \frac{g_{m8}}{g_{m1,2}^2}$

$\overline{V_n^2}_{input\ M_7} = \overline{V_n^2}$ input due to $M_8$

$= 4kT\gamma \frac{g_{m7}}{g_{m1,2}^2}$




$\therefore$ Input referred noise voltage

$= \left[4kT\gamma\left(\frac{1}{g_{m1,2}} + \frac{g_{m7,8}}{g_{m1,2}^2}\right)\right] \times 2$

$g_{m1,2} = \sqrt{2\mu_n C_{ox}\left(\frac{W}{L_{eff}}\right) I_D}$

$= \left[2(350)(383.6\times10^{-9})\left(\frac{100}{0.34}\right)(0.5mA)\right]^{\frac{1}{2}}$

$= 6.28\times10^{-3}\ \Omega^{-1}$

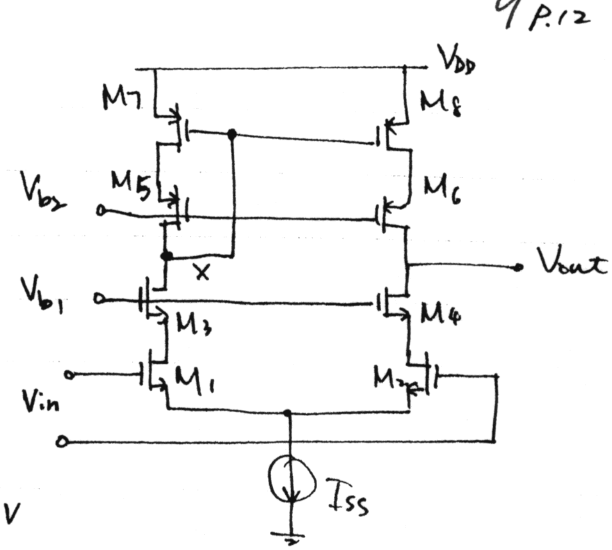
$g_{m7,8} = \left[2(100)(383.6\times10^{-9})\left(\frac{100}{0.32}\right)(0.5mA)\right]^{\frac{1}{2}}$

$= 3.46m\Omega^{-1}$

Input refered noise voltage $= \left[4\times1.38\times10^{-23}\times300\times\frac{2}{3}\times\left(\frac{1}{6.28m} + \frac{3.46m}{6.28m^2}\right)\right]\times2$

$$\boxed{\begin{aligned} &= 5.45\times10^{-18}\ V^2/Hz \\ \text{or} &= 2.34\times10^{-9}\ V/\sqrt{Hz} \end{aligned}}$$

Problem 9.5.




Requirement : Max. diff. swing = 2.4V

Power$_{max}$ = 6mW.

$V_{DD} \cdot I_{SS} = \text{Power}_{max} = 6mW.$

$I_{SS} = \frac{6mW}{3V} = 2mA$

$|V_{GS7}| - |V_{THp}| = \left[\frac{2 I_{D7}}{\mu_p C_{ox} \left(\frac{W}{L_{eff}}\right)_7}\right]^{\frac{1}{2}}$ assume $W_{7,8} = 100\mu m$

$= \left[\frac{2(1mA)}{(100)(383.6\times10^{-9})\left(\frac{100}{0.32}\right)}\right]^{\frac{1}{2}} = 0.408V$

$|V_{GS7}| = 0.408 + |V_{THP}| = 1.208V$

$V_X = V_{DD} - |V_{GS7}| = 3 - 1.208 = 1.79V.$

$V_X - V_{TH5} < V_{b2} < V_X + V_{THp} - |V_{GS5}|$ assume $W_{5,6} = 100\mu m$

$0.99V < V_{b2} < 1.382V$

For larger output swing, choose $V_{b2} = 1.3V$.

$V_{out\,max} = V_{b2} + V_{THp} = 1.3 + 0.8 = 2.1V.$

We need one sided output swing = 1.2V, so $V_{out\,min} = 0.9V$.

$|V_{OD1}| = |V_{OD3}| = 0.3 = V_{ISS}$

$I_D = \frac{1}{2}\mu C_{ox}\left(\frac{W}{L_{eff}}\right)(V_{GS} - V_{TH})^2(1 + \lambda V_{DS})$ Assume $V_{DS} \simeq V_{GS} - V_{TH}$

$\left(\frac{W}{L_{eff}}\right) = \frac{2I_D}{\mu_n C_{ox}(V_{GS} - V_{TH})^2(1 + \lambda V_{DS})} = \frac{2(1mA)}{(350)(383.6\times10^{-9})(0.3)^2(1+(0.1)(0.3))}$

$\left(\frac{W}{L_{eff}}\right) = 160 \Rightarrow W_{1,2,3,4} = (160)(L_{eff}) = (160)(0.34\mu m)$ let $L = 0.5\mu m$

$W_{1-4} = 55\mu m.$

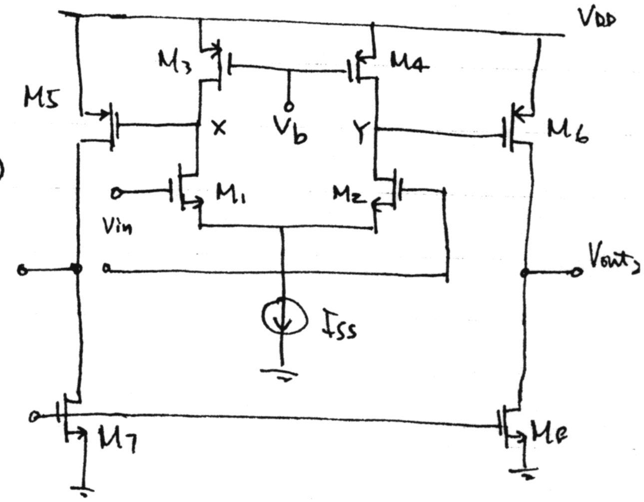
$V_{in,CM} = V_{ISS} + V_{GS1,2} = V_{ISS} + V_{THn} + V_{OD1} = 0.3 + 0.7 + 0.3V$

$= 1.3V$

$V_{b1} = V_{in,CM} + V_{OD1} = 1.3 + 0.3V = 1.6V.$

Summary

| | | |
|---|---|---|
| $\left(\frac{W}{L}\right)_{1-4} = \frac{55}{0.5}$ | $V_{inCM} = 1.3V$ | $I_{SS} = 2mA.$ |
| $\left(\frac{W}{L}\right)_{5-8} = \frac{100}{0.5}$ | $V_{b1} = 1.6V$ | |
| | $V_{b2} = 1.3V$ | |




Problem 9.6

(a) Given: $\left(\frac{W}{L}\right)_{1-8} = \frac{100}{0.5}$ $\quad I_{SS} = 1mA$

$I_{D5,6} = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_{5,6}(V_{GS5,6} - V_{THp})^2(1+\lambda V_{DS5,6})$

$V_{GS5,6} - V_{THp} = \left[\frac{2I_{D,5,6}}{\mu_p C_{ox}\left(\frac{W}{L}\right)_{5,6}}\right]^{\frac{1}{2}}$

$= \left[\frac{2(1mA)}{(100)(383.6\times10^{-9})\left(\frac{100}{0.32}\right)}\right]^{\frac{1}{2}} = 0.408v$

$V_{GS5,6} = 0.408 + 0.8 = 1.208v$

$V_{X,Y} = V_{DD} - V_{GS5,6} = 3 - 1.208v$

$\boxed{V_{X,Y} = 1.792v}$

In order to keep $M_1, M_2$ in saturation,

$V_{in,CM} < V_{X,Y} + V_{TH} = 1.792 + 0.7 = 2.492v$

$\boxed{\therefore V_{in,CM,max} = 2.49v}$

(b) $A_v$ of 1st stage $= g_{m1}[r_{o1} \| r_{o3}]$

$A_v$ of 2nd stage $= g_{m5}(r_{o5} \| r_{o7})$

$A_{v,tot} = g_{m1}(r_{o1} \| r_{o3})\, g_{m5}(r_{o5} \| r_{o7})$.

$g_{m1} = \sqrt{2\mu_n C_{ox}\left(\frac{W}{L_{eff}}\right)_1 (I_{D1})} = \left[2(350)(383.6\times10^{-9})\left(\frac{100}{0.34}\right)(0.5mA)\right]^{\frac{1}{2}}$

$= 6.28\ m\Omega^{-1}$

$g_{m5} = \frac{2I_D}{V_{GS} - V_{TH}} = \frac{2(1mA)}{0.408}$

$g_{m5} = 4.90\ m\Omega^{-1}$

$r_{o1} = \frac{1}{\lambda I_D} = \frac{1}{(0.1)(0.5mA)} = 20k\Omega$.

$r_{o3} = \frac{1}{\lambda I_D} = \frac{1}{(0.2)(.5mA)} = 10k\Omega$

$r_{o5} = \frac{1}{\lambda I_D} = \frac{1}{(0.2)(1mA)} = 5k\Omega$

$r_{o7} = \frac{1}{\lambda I_D} = \frac{1}{(0.1)(1mA)} = 10k\Omega$

$A_v = (6.28m)[20k \| 10k](4.90m)[5k \| 10k]$

$\boxed{A_v = 684}$

Max output swing $= 2(V_{DD} - |V_{OD5}| - V_{OD7})$

$$|V_{OD5}| = |V_{GS5}| - |V_{THp}| = 0.408V$$

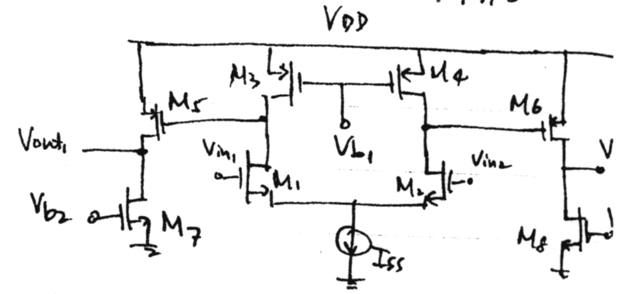
$$V_{OD7} = V_{GS7} - V_{THn} = \left[\frac{2\,I_D}{(\mu_n)\,C_{ox}\,\left(\frac{W}{L}\right)_7}\right]^{\frac{1}{2}}$$

$$= \left[\frac{2(1mA)}{(350)(383.6\times10^{-9})\left(\frac{100}{0.34}\right)}\right]^{\frac{1}{2}} = 0.225$$

$$\boxed{\text{Max output swing} = 2(3 - 0.408 - 0.225) = 4.734V.}$$

Problem 9.7

Design the op amp of fig. 9.21




Max diff. swing = 4V

total Power = 6mW $\qquad I_{SS} = 0.5mA$

Total current driven by $V_{DD} = \dfrac{6mW}{3V} = 2mA$

$I_{D5} + I_{D6} = 2mA - I_{SS} = 1.5mA \Rightarrow I_{D5} = I_{D6} = 0.75mA$

Max diff swing $= 2[V_{DD} - |V_{OD5}| - V_{OD7}] = 4V$

$\Rightarrow |V_{OD5}| + |V_{OD7}| = 1$ $\qquad$ choose $V_{OD5} = 0.6V$, $V_{OD7} = 0.4V$

$I_D = \frac{1}{2}\mu C_{ox}(\frac{W}{L_{eff}})(V_{GS} - V_{TH})^2(1+\lambda V_{DS})$

$(\frac{W}{L_{eff}})_5 = \dfrac{2I_D}{\mu_p C_{ox}(V_{GS} - V_{TH})^2(1+\lambda V_{DS})} = \dfrac{2(0.75mA)}{(100)(383.6\times10^{-9})(0.6)^2(1+0.2\times0.6)}$

$(\frac{W}{L_{eff}})_5 = 97 \Rightarrow W_{5,6} = 97\times0.32\mu = 31\mu m$

$(\frac{W}{L_{eff}})_7 = \dfrac{2(0.75m)}{(350)(383.6\times10^{-9})(0.4)^2(1+0.1(0.4))}$

$= 67 \Rightarrow W_{7,8} = 67\times0.34\mu m = 23\mu m$

We are generally not worried about the swing of 1st stage,

assume $|V_{OD3}| = 1V$, $V_{OD1} = 1V$.

$(\frac{W}{L_{eff}})_3 = \dfrac{2(0.25m)}{(100)(383.6\times10^{-9})(1)^2(1+0.2(1))} = 10.86$

$W_{3,4} = 3.5\mu m$

$(\frac{W}{L_{eff}})_1 = \dfrac{2(0.25m)}{(350)(383.6\times10^{-9})(1)^2(1+0.1)} = 3.4$

$W_{1,2} = 1.2\mu m$

$V_{b1} = V_{DD} - |V_{OD3}| - V_{TH3} = 3 - 1 - 0.8 = 1.2V.$

$V_{in,CM} = V_{ISS} + V_{TH1} + V_{OD1} = 0.3 + 0.7 + \cancel{0.} 1.0 = 2V.$

$V_{b2} = V_{TH7} + V_{OD7} = 0.7 + 0.4 = 1.1V.$

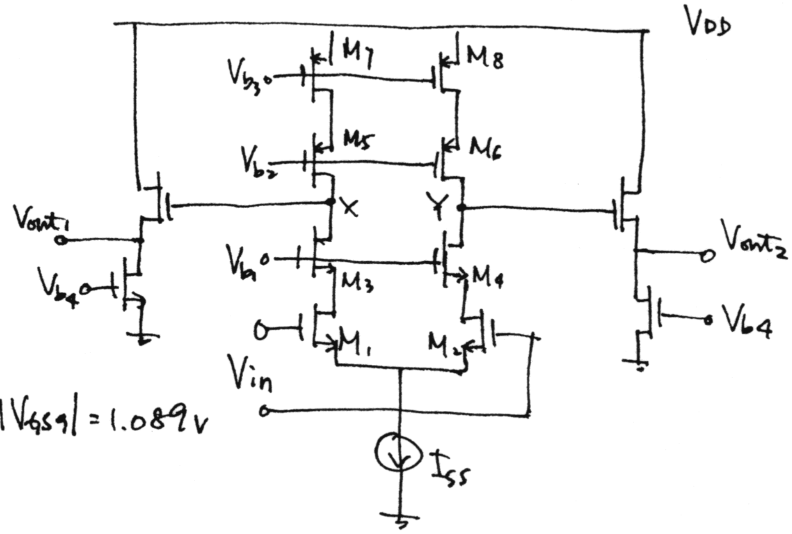
Summary $\qquad L = 0.5\mu m$

| | |
|---|---|
| $W_1 = W_2 = 1.2\mu m$ | $V_{b1} = 1.2V$ |
| $W_{3,4} = 3.5\mu m$ | $V_{b2} = 1.1V$ |
| $W_{5,6} = 31\mu m$ | $V_{in,CM} = 2V$ |
| $W_{7,8} = 23\mu m$ | |

# Problem 9.8

Given $I_{SS} = 1mA$, $I_{D9} - I_{D12} = 0.5mA$

$(\frac{W}{L})_{9-12} = \frac{100}{0.5}$




(a) $V_{X,Y,CM} = ?$

$$|V_{GS9}| - |V_{THp}| = \left[\frac{2I_{D9}}{\mu_p C_{ox} (\frac{W}{L})_9}\right]^{\frac{1}{2}}$$

$$= \left[\frac{2(0.5mA)}{(100)(383.6n)(\frac{100}{0.32})}\right]^{\frac{1}{2}}$$

$$= 0.289V, \Rightarrow \quad |V_{GS9}| = 1.089V$$

$$\boxed{V_{XY,CM} = V_{DD} - |V_{GS9}| = 1.911V}$$

(b) $V_X$ swing $= 0.2V$, $V_{X,CM} = 1.911V$

$V_{X\,max} = 2.011V$

$V_{X\,min} = 1.811V$

$$V_{OD7} = V_{OD5} = \frac{V_{DD} - V_{X\,max}}{2} = \frac{3 - 2.011}{2} = 0.495$$

$$V_{OD1} = V_{OD3} = \frac{V_{X\,min} - V_{ISS}}{2} = \frac{1.811 - 0.4}{2} = 0.7055$$

$$(\frac{W}{L_{eff}})_{5-8} = \frac{2I_D}{\mu_p C_{ox} (V_{GS} - V_{TH})^2 (1 + \lambda V_{DS})}$$

$$= \frac{2(0.5mA)}{(100)(383.6n)(0.495)(1 + 0.2 \times 0.495)}$$

$$= 97.02$$

$W_{5-8} = 97.02 \times L_{eff} = 31.05\mu m. \simeq 31.1\mu m$

$$(\frac{W}{L_{eff}})_{1-4} = \frac{2I_D}{\mu_n C_{ox} (V_{GS} - V_{TH})^2 (1 + \lambda V_{DS})}$$

$$= \frac{2(0.5mA)}{(350)(383.6n)(0.7055)(1 + 0.1 \times 0.7055)}$$

$$= 14$$

$W_{1-4} \cong 4.8\mu m$

9 P.17

1c) $A_v = g_{m1}(g_{m3} r_{o3} r_{o1} \,//\, g_{m5} r_{o5} r_{o7})\, g_{m9}(r_{o9} \,//\, r_{o11})$

$g_{m1} = \dfrac{2 I_D}{V_{GS1} - V_{TH}} = \dfrac{2(0.5mA)}{0.7055} = 1.417\, m\Omega^{-1}$

$g_{m3} = g_{m1} = 1.417 m\Omega^{-1}$

$g_{m5} = \dfrac{2 I_D}{V_{GS5} - V_{THp}} = \dfrac{2(0.5mA)}{0.495} = 2.022 m\Omega^{-1}$

$g_{m9} = \dfrac{2(0.5mA)}{0.289} = 3.46 m\Omega^{-1}$

$r_{oN} = r_{o1} = r_{o3} = r_{o11} = \dfrac{1}{\lambda I_D} = \dfrac{1}{(0.1)(0.5mA)}$

$= 20 k\Omega$

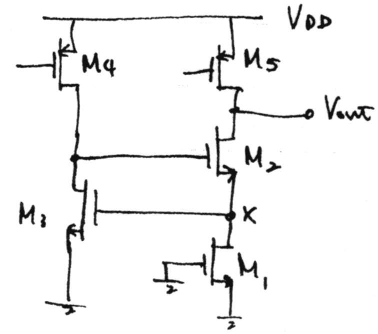
$r_{op} = r_{o5} = r_{o7} = r_{o9} = \dfrac{1}{\lambda I_D} = \dfrac{1}{(0.2)(0.5mA)}$

$= 10 k\Omega$.

$A_v = (1.417m)[1.417m \times 20k \times 20k \,//\, 2.022m \times 10k \times 10k] \times 3.46m \times (10k \,//\, 20k)$

$\boxed{A_v = 4871}$

Problem 9.9




$\overline{V_n^2}, input|_{M_1} = 4kT\gamma \frac{1}{g_{m1}}$

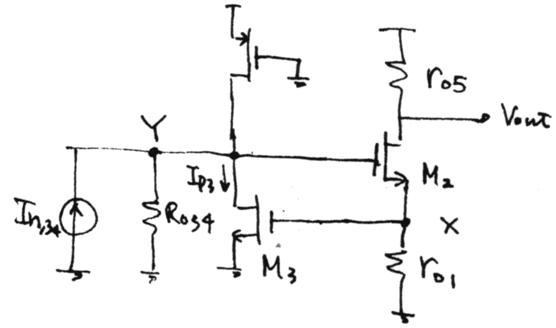
$\overline{V_n^2}, input|_{M_2} \simeq 0$

$\overline{V_n^2}, out|_{M5} = 4kT\gamma g_{m5} R_{out}^2$

$\overline{V_n^2}, out|_{M5} = \frac{4kT\gamma g_{m5} R_{out}^2}{(g_{m1} R_{out})^2}$

$= 4kT\gamma \frac{g_{m5}}{g_{m1}^2}$

Noise due to M3, M4 :




$I_{n,34} = 4kT\gamma(g_{m3}+g_{m4})$

$R_{o34} = r_{o3} // r_{o4}$

$V_X = r_{o1}\left(-\frac{V_{out}}{r_{o5}}\right)$

$I_{D3} = g_{m3} V_X = -\frac{g_{m3} r_{o1} V_{out}}{r_{o5}}$

$V_Y = R_{o34}\left(I_{n,34} + \frac{g_{m3} r_{o1}}{r_{o5}} V_{out}\right)$

neglect the $r_{o2}$ to approximate the result,

$V_{out} = \frac{-r_{o5}}{\frac{1}{g_{m2}} + r_{o1}} V_Y = \frac{-r_{o5}}{\frac{1}{g_{m2}} + r_{o1}} R_{o34}\left(I_{n,34} + \frac{g_{m3} r_{o1}}{r_{o5}} V_{out}\right).$

$V_{out}\left(\frac{\frac{1}{g_{m2}} + r_{o2}}{+ r_{o5} R_{o34}} + \frac{g_{m3} r_{o1}}{r_{o5}}\right) = -I_n$

$V_{out} = \frac{-I_n r_{o5} R_{o34}}{\frac{1}{g_{m2}} + r_{o2} + g_{m3} r_{o1} R_{o34}} \simeq \frac{-I_n r_{o5} R_{o34}}{g_{m3} r_{o1} R_{o34}} = -\frac{r_{o5}}{r_{o1} g_{m3}} I_n$

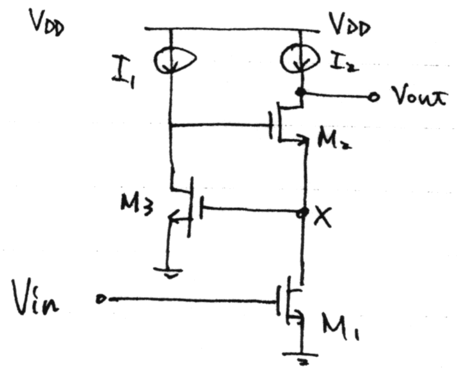
$\overline{V_n^2}, input|_{M3,M4} = \frac{\left(\frac{r_{o5}}{r_{o1} g_{m3}}\right)^2 I_n}{g_{m1}^2 \left(r_{o5} // [g_{m2} r_{o2} r_{o1} g_{m3} (r_{o3} // r_{o4})]\right)^2} \simeq \frac{I_n \left(\frac{\cancel{r_{o5}}}{g_{m3} r_{o1}}\right)^2}{g_{m1}^2 \cancel{r_{o5}^2}}$

$= 4kT\gamma (g_{m3} + g_{m4})\left[\frac{1}{g_{m1}^2 g_{m3}^2 r_{o1}^2}\right]$

This is negligible compared with the noise due to M1, M5

$$\boxed{\therefore \overline{V_n^2}, in\ total = 4kT\gamma\left[\frac{1}{g_{m1}} + \frac{g_{m5}}{g_{m1}^2}\right]}$$

## Problem 9.10




(a). $I_1 = 100\mu A$, $I_2 = 0.5mA$, $(\frac{W}{L})_{1-3} = \frac{100}{0.5}$

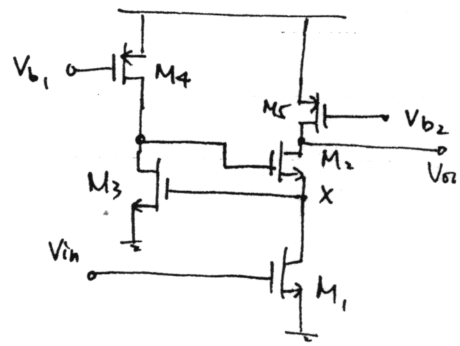
$(\frac{W}{L})_p = \frac{50}{0.5}$

$I_{D3} = I_1 = \frac{1}{2}\mu_n C_{ox} (\frac{W}{L})_3 (V_{GS} - V_{TH})^2 (1+\lambda V_{DS})$

$V_{GS3} - V_{TH} = \left[\frac{2I_1}{\mu_n C_{ox} (\frac{W}{L_{eff}})_3}\right]^{\frac{1}{2}}$

$= \left[\frac{2(100\mu A)}{350 \times (383.6n)(\frac{100}{0.34})}\right]^{\frac{1}{2}} = 0.0712$

$\boxed{V_{GS3} = 0.7712V = V_{G3} = V_X}$




$V_{GS2} - V_{THn} = \left[\frac{2I_2}{\mu_n C_{ox} (\frac{W}{L_{eff}})_2}\right]^{\frac{1}{2}}$

$= \left[\frac{2(0.5mA)}{(350)(383.6n)(\frac{100}{0.34})}\right]^{\frac{1}{2}} = 0.159V$

$V_{GS2} = 0.859V$

$\boxed{V_{G2} = V_{GS2} + V_X = 1.630V.}$

(b) Max output swing:

$V_{out\,max} = V_{DD} - |V_{OD5}|$

$V_{out\,min} = V_X + V_{OD2} = V_{GS3} + V_{OD2}$

Max output swing $= V_{DD} - |V_{OD5}| - V_{OD2} - V_{GS3}$

$|V_{GS5}| - |V_{TH}| = \left[\frac{2I_D}{\mu_p C_{ox} (\frac{W}{L_{eff}})_5}\right]^{\frac{1}{2}} = \left[\frac{2(0.5mA)}{(100)(383.6n)(\frac{50}{0.32})}\right]^{\frac{1}{2}}$

$= 0.408V = |V_{OD5}|$

$$\boxed{\begin{aligned}\text{Max output swing} &= 3 - 0.408 - 0.159 - 0.7712 \\ &= 1.6618V\end{aligned}}$$

(c) $A_v = g_{m1}[r_{o5} // (g_{m2} r_{o2} r_{o1} g_{m3} (r_{o3} // r_{o4}))]$ Note: $r_{o5}$ is limiting the gain.

$\simeq g_{m1} r_{o5}$

$g_{m1} = \sqrt{2\mu_n C_{ox} (\frac{W}{L})_1 I_{D1}} = \sqrt{2(350)(383.6n)(\frac{100}{0.34})(0.5mA)}$

$= 6.28 m\Omega^{-1}$

$r_{o5} = \frac{1}{\lambda I_D} = \frac{1}{(0.2)(0.5mA)} = 10k\Omega$

$\boxed{A_v = 62.8}$

9.10 cont.

(c) $\overline{V_{n,in}^2} = 4kT\gamma\left[\frac{1}{g_{m1}} + \frac{g_{m5}}{g_{m1}^2}\right] + 4kT\gamma\left(\frac{1}{g_{m3}} + \frac{g_{m4}}{g_{m3}^2}\right)\left[\frac{g_{m2} r_{o2} g_{m3} (r_{o3} \| r_{o4})}{g_{m1}(r_{o1} + r_{o2} + r_{o5})}\right]^2$ (see

$g_{m1} = g_{m2} = 6.28 m\Omega^{-1}$

$g_{m5} = \left[2(100)(383.6n)\left(\frac{50}{0.32}\right)(0.5m)\right]^{\frac{1}{2}} = 2.45 m\Omega^{-1}$

$g_{m3} = \left[2(350)(383.6n)\left(\frac{100}{.34}\right)(100\mu)\right]^{\frac{1}{2}} = 2.81 m\Omega^{-1}$

$g_{m4} = \left[2(100)(383.6n)\left(\frac{50}{.32}\right)(100\mu)\right]^{\frac{1}{2}} = 1.09 m\Omega^{-1}$

$r_{o1} = r_{o2} = \frac{1}{\lambda I_D} = \frac{1}{(0.1)(0.5m)} = 20k\Omega$

$r_{o3} = \frac{1}{(0.1)(100\mu)} = 100k\Omega$

$r_{o4} = \frac{1}{(0.2)(100\mu)} = 50k\Omega$

$r_{o5} = \frac{1}{(0.2)(0.5m)} = 10k\Omega.$

$\overline{V_{n,in}^2} = 4(1.38\times10^{-23})(300)\left(\frac{2}{3}\right)\left[\frac{1}{6.28m} + \frac{2.45m}{6.28m^2}\right]$

$= 2.444\times10^{-18}\ V^2/Hz$

$\overline{V_{n,in}} = 1.56\times10^{-9}\ V/\sqrt{Hz}$

9 P.21

Problem 9.11

$V_P = 100mV$

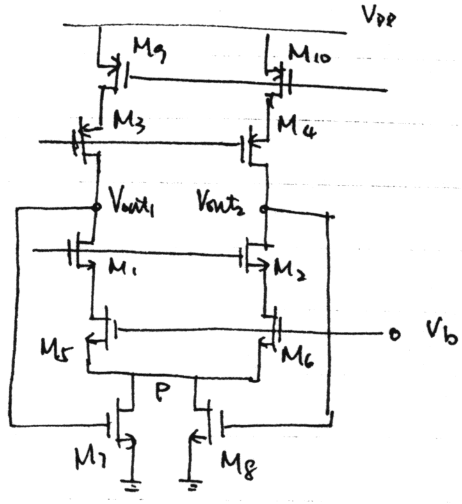
$V_{out,CM} = 1.5V$ , $I_{D7,8} = 0.5mA$

$I_D = \frac{1}{2}\mu_n C_{ox} \left(\frac{W}{L}\right)_7 \left[(V_{GS}-V_T)V_{DS} - \frac{V_{DS}^2}{2}\right]$

$\left(\frac{W}{L_{eff}}\right)_7 = \frac{2I_D}{\mu_n C_{ox}\left[(V_{GS}-V_T)V_{DS} - \frac{V_{DS}^2}{2}\right]}$

$= \frac{2(0.5mA)}{(350)(383.6n)\left[(1.5-0.7)(0.1) - \frac{0.1^2}{2}\right]}$

$= 99.3$

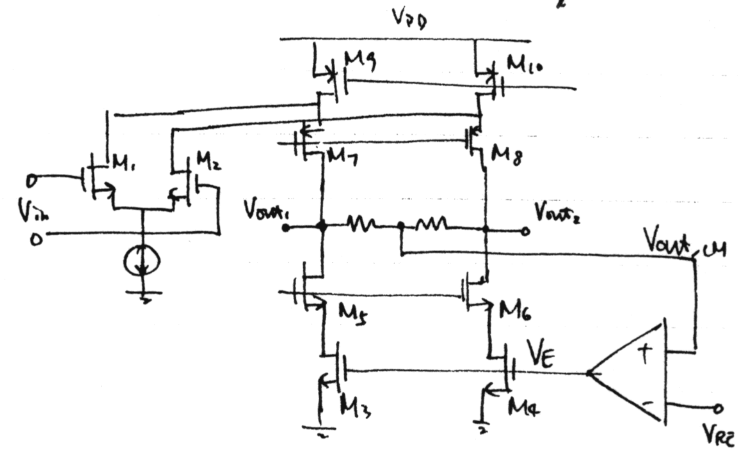



$W_{7,8} = 99.3 \times 0.34\mu m = 33.762\mu m \simeq 34\mu m$

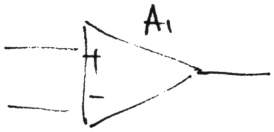
$\boxed{\left(\frac{W}{L}\right)_{7,8} = \frac{34}{0.5}}$

Problem 9.12

(a) PMOS devices should be used.
Since $V_{out,CM}$ is in the middle voltage range, (around 1.5V), and $V_{GS3,4}$ are in low voltage range, (around 0.7 – 0.8V), we should use PMOS to bring down the voltage.




b)

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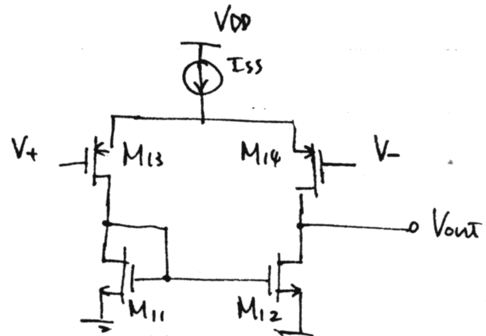


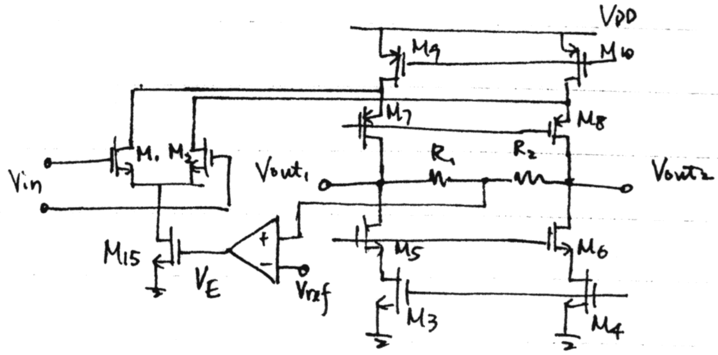

$A_1 = g_{m13}(r_{o12} \| r_{o14})$

$$\frac{V_{out,CM}}{V_E} = -g_{m3,4}\left(g_{m5} r_{o5} r_{o3} \| g_{m7} r_{o7} (r_{o1} \| r_{o9})\right)$$

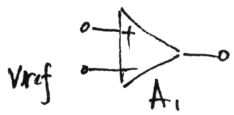
$$\boxed{\text{Loop gain} = -g_{m3,4}\left[(g_{m5} r_{o5} r_{o3}) \| (g_{m7} r_{o7} (r_{o1} \| r_{o9}))\right] g_{m13}(r_{o12} \| r_{o14})}$$

# Problem 9.13

(a) Since we need to bring down $V_{out,CM}$ to fit the bias voltage of NMOS, which is relatively low, we should use PMOS for the input pair of amplifier.




(b)

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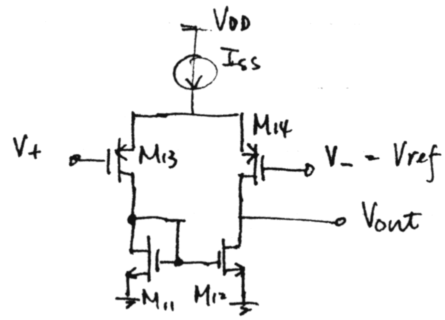
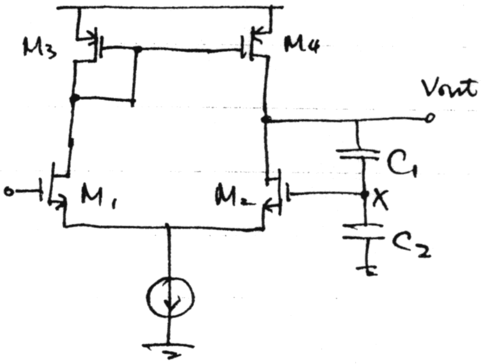



$$A_1 = g_{m13}(r_{o12} \parallel r_{o14})$$

$$\frac{V_{out,CM}}{V_E} = -g_{m15}\left[(g_{m5} r_{o5} r_{o3}) \parallel (g_{m7} r_{o7}(r_{o9} \parallel g_{m1} r_{o1} r_{o15}))\right]$$

$$\text{loop gain} = -g_{m15}\left[(g_{m5} r_{o5} r_{o3}) \parallel (g_{m7} r_{o7}(r_{o9} \parallel g_{m1} r_{o1} r_{o15}))\right] g_{m13}(r_{o12} \parallel r_{o14})$$

Problem 9.14



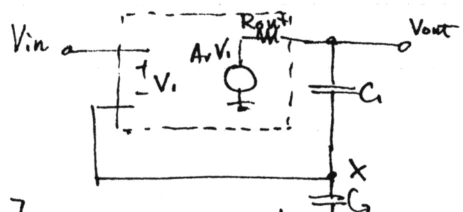

(a) $\left(\frac{W}{L}\right)_{1-4} = \frac{100}{0.5}$ , $C_1 = C_2 = 0.5pF$ , $I_{SS} = 1mA$

$A_v = g_{m1}(r_{o2} \| r_{o4})$

$R_{out} = r_{o2} \| r_{o4}$

$V_{in} = V_1 + V_x$

$V_x = V_{out} \frac{C_1}{C_1 + C_2}$




$V_{out} = A_v V_1 \left[ \frac{\frac{1}{C_1 \| C_2 s}}{R_{out} + \frac{1}{C_1 \| C_2 s}} \right] = A_v V_1 \frac{1}{(C_1 \| C_2) R_{out} s + 1}$

$= \frac{A_v}{1 + (C_1 \| C_2) R_{out} s} \left( V_{in} - V_{out} \frac{C_1}{C_1 + C_2} \right)$

$\left(1 + (C_1 \| C_2) R_{out} s + A_v \frac{C_1}{C_1 + C_2}\right) V_{out} = A_v V_{in}$

$\frac{V_{out}}{V_{in}} = \frac{A_v}{1 + A_v \frac{C_1}{C_1 + C_2} + (C_1 \| C_2) R_{out} s} = \frac{\frac{A_v}{1 + A_v \frac{C_1}{C_1 + C_2}}}{1 + \frac{(C_1 \| C_2) R_{out}}{1 + A_v \frac{C_1}{C_1 + C_2}} s}$

$$\boxed{\tau = \frac{\frac{C_1 C_2}{C_1 + C_2} R_{out}}{1 + A_v \frac{C_1}{C_1 + C_2}}}$$

(b) $I_{D2} = 0.1 I_{SS}$ ,

Since $I_{D2}$ is still small, we can solve this problem by assuming the current through $C_1$ & $C_2$ roughly equal to $I_{SS}$

$V_X(t) - V_X(0) = \frac{I_{SS}}{C_2} t$

At $t = 0^-$ $\quad I_{D1} = I_{D2} = 0.5mA$

$V_{GS1,2} - V_{TH} = \left[ \frac{2 I_D}{\mu_n C_{ox} \left(\frac{W}{L_{eff}}\right)} \right]^{\frac{1}{2}} = \left[ \frac{2(0.5mA)}{(350)(383.6n)\left(\frac{100}{0.34}\right)} \right]^{\frac{1}{2}}$

$= 0.159V$

At time $= t$, when $I_{D2} = 0.1 I_{SS} \Rightarrow I_{D1} = 0.9 I_{SS}$

$I_D = \frac{1}{2} k_n C_{ox} \left(\frac{W}{L}\right)(V_{GS} - V_T)^2$

$\frac{I_{D1}}{I_{D2}} = \frac{0.9 I_{SS}}{0.1 I_{SS}} = \frac{(V_{GS1} - V_T)^2}{(V_{GS2} - V_T)^2} \Rightarrow \frac{(V_{GS1} - V_T)}{(V_{GS2} - V_T)} = 3$

$V_{GS1}(t) - V_T = 3(V_{GS2} - V_T)$

$(V_{GS1}(t) - V_T) = (V_{GS1}(0) - V_T) + 1V = 0.159 + 1V = 1.159V = 3(V_{GS2} - V_T)$

$V_{GS2}(t) - V_T = 0.386V$

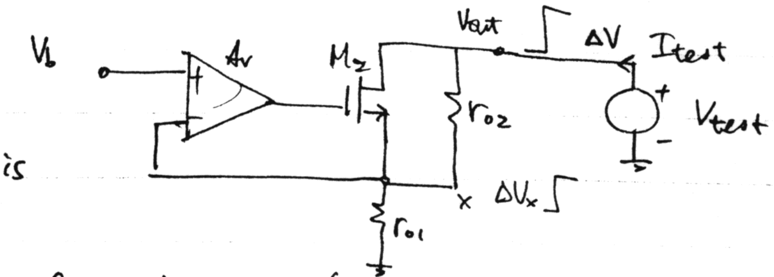
9.14 cont.

$$[V_{GS_2}(t) - V_T] - [V_{GS_2}(0) - V_T] = V_X(t) - V_X(0) = \frac{I_{SS}}{C_L} t$$

$$0.386 - 0.159 = 0.227 = \frac{1mA}{0.5pF}(t)$$

$$\boxed{t = 113.5ps.}$$

Problem 9.15



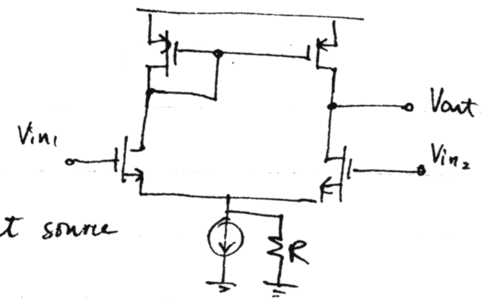

The mistake is made when we say the current from Vtest is equal to $\Delta V / r_{o2}$.

We can see it when we start from the amplifier.

If we ~~a~~ assume current from $V_-$ is very small or negligible, the current through $r_{o1}$ is equal to $I_{test}$, the current driven from $V_{test}$. The current through $r_{o1}$ is $\frac{\Delta V_x}{r_{o1}}$, which is a much smaller value than $\Delta V / r_{o2}$.

The mistake is made because the current through $r_{o2}$ is actually equal to $\Delta V / r_{o2}$ or $\approx \frac{\Delta V - \Delta V_x}{r_{o2}}$. This current is larger ~~since~~ than $I_{test}$ since some extra current from $M_2$ makes the current through $r_{o2}$ larger. As a result, $\Delta V_{ro2}$ ($\Delta$ voltage across $r_{o2}$) increases by about $\Delta V$, but the current from $V_{test}$ ~~only~~ increases only by $\frac{\Delta V_x}{r_{o1}}$.

Problem 9.16

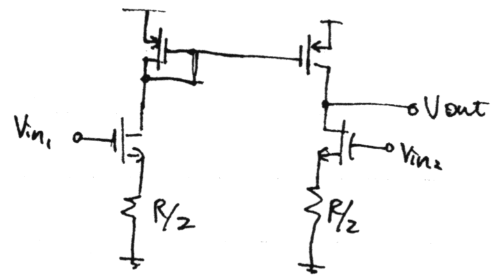



$$CMRR = \frac{\text{diff. gain}}{\text{CM. gain}}$$

diff. gain $= g_{m1}(r_{o2} \| r_{o4})$

CM gain: let R is the resistance of current source

$\Delta V_{in1} = \Delta V_{in2} = V_{CM}$




$$\frac{V_{out}}{\Delta V_{in1}} = \frac{-\frac{1}{g_{m3}}}{\frac{R}{2}+\frac{1}{g_{m1}}} g_{m4}(R_{out})$$

$$\frac{V_{out}}{\Delta V_{in2}} = \frac{-R_{out}}{\frac{R}{2}+\frac{1}{g_{m2}}}$$

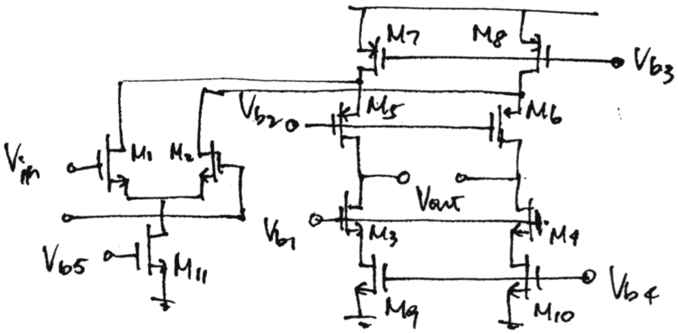
$$\left|\frac{V_{out}}{V_{CM}}\right| = \frac{2R_{out}}{\frac{R}{2}+\frac{1}{g_{m1}}} \approx \frac{4R_{out}}{R}$$

where $R_{out} = r_{o4} \| r_{o2}$

$$\text{CM gain} = \frac{4(r_{o4} \| r_{o2})}{R}$$

$$CMRR = \frac{g_{m1}(r_{o2} \| r_{o4})}{\frac{4(r_{o2} \| r_{o4})}{R}} = \boxed{\frac{g_{m1}R}{4} = CMRR}$$

Problem 9.17




Neglect the noise due to $M_{11}, M_3, M_4, M_5, M_6$.

Input-referred flicker noise due to $M_{7,8} = 2\left[\dfrac{\overline{V_{n,7,8}^2} \cdot g_{m7,8}^2 R_{out}^2}{A_v}\right]$

where $A_v = g_{m1}(R_{out})$, $\overline{V_{n,7,8}^2} = \dfrac{K_P}{C_{ox}(WL)_{7,8}} \cdot \dfrac{1}{f}$

$\overline{V_{n,input}^2}\Big|_{M9,10} = \dfrac{\overline{V_{n,9,10}^2} \cdot g_{m9,10}^2 R_{out}^2}{g_{m1}^2 R_{out}^2} = 2\left[\overline{V_{n\,9,10}^2} \dfrac{g_{m9,10}^2}{g_{m1,2}^2}\right]$

Total Input-referred flicker noise

$$= \frac{2K_N}{C_{ox} f}\left[\frac{1}{(WL)_{1,2}} + \frac{1}{(WL)_{9,10}} \frac{g_{m9,10}^2}{g_{m1,2}^2}\right] + \frac{2K_P}{C_{ox} f} \frac{1}{(WL)_{7,8}} \frac{g_{m7,8}^2}{g_{m1,2}^2}$$

Problem 9.18

$P = 6mW$, output swing $= 2.5V$

$L_{eff} = 0.5\mu m$

(a) $I_{D5,6} = 1mA$. $V_{OD5} \simeq V_{OD6} = \frac{V_{DD} - \text{Output Swing}}{2} = \frac{3-2.5}{2} = 0.25V$

$I_D = \frac{1}{2}\mu C_{ox}(\frac{W}{L})(V_{GS}-V_{TH})^2(1+\lambda V_{DS})$

$$\left(\frac{W}{L}\right)_{eff} = \frac{2I_D}{\mu C_{ox}(V_{GS}-V_{TH})^2(1+\lambda V_{DS})}$$

$$\left(\frac{W}{L}\right)_5 = \frac{2(1mA)}{(350)(383.6n)(0.25)^2(1+(0.1)(0.25))}$$ $I_{D5} = I_{D6} = 1mA$

$= 233$

$$\left(\frac{W}{L}\right)_6 = \frac{2(1mA)}{(100)(383.6n)(0.25)^2(1+0.2(0.25))}$$

$= 795$

$$\boxed{\left(\frac{W}{L}\right)_5 = 233 \qquad \left(\frac{W}{L}\right)_6 = 795}$$

b. $A_v$ of 1st stage $= g_{m1}(r_{o2} \| r_{o4})$

$A_v$ of 2nd stage $= g_{m5}(r_{o5} \| r_{o6})$

$$g_{m5} = \frac{2I_D}{V_{GS}-V_{TH}} = \frac{2(1mA)}{0.25} = 8m\Omega^{-1}$$

$$r_{o5} = \frac{1}{\lambda I_D} = \frac{1}{(0.1)(1mA)} = 10k\Omega$$

$$r_{o6} = \frac{1}{\lambda I_D} = \frac{1}{(0.2)(1mA)} = 5k\Omega$$

$$\boxed{\begin{aligned} A_v \text{ of output stage} &= (8m)(10k \| 5k) \\ &= 26.67 \end{aligned}}$$

(c) $I_{D7} = 1mA \Rightarrow I_{D3} = I_{D4} = 0.5mA$

$V_{GS5} - V_{TH} = 0.25V \Rightarrow V_{GS5} = 0.25 + V_{TH} = 0.95V$

$V_{GS3} - V_{TH} = 0.25$

$$\left(\frac{W}{L}\right)_{3,4} = \frac{2(0.5mA)}{(350)(383.6n)(0.25)^2(1+0.1\times 0.25)}$$

$$\boxed{\left(\frac{W}{L}\right)_{3,4} = 116}$$

9.18

(a) $A_{v\,tot} = g_{m1}(r_{o2} \| r_{o4})\, g_{m5}(r_{o5} \| r_{o6})$

$r_{o2} = \frac{1}{\lambda I_D} = \frac{1}{(0.2)(0.5m)} = 10k\Omega.$

$r_{o4} = \frac{1}{(0.1)(0.5m)} = 20k\Omega.$

$r_{o2} \| r_{o4} = 6.67k\Omega$

$A_{v\,tot} = g_{m1}(6.67k)(26.7) = 500$

$g_{m1} = 2.81\ m\Omega^{-1}$

$g_{m1} = \sqrt{2\mu_p C_{ox}\left(\frac{W}{L}\right) I_D} \Rightarrow \left(\frac{W}{L}\right) = \frac{g_{m1}^2}{2\mu_p C_{ox} I_D}$

$\left(\frac{W}{L}\right)_{1,2} = \frac{(2.81\ m)^2}{2(100)(383.6n)(0.5m)}$

$\boxed{\left(\frac{W}{L}\right)_{1,2} = 206}$

Problem 9.19

$A_v$ of 2nd stage = 20 $\quad I_{D5,6} = 1mA$

(a) $V_{GD5} = V_{GD6}$

$A_v$ of 2nd stage = $g_{m5}(r_{o5} \parallel r_{o6})$

$$= \frac{2I_D}{V_{GS5} - V_{TH}} \cdot \left[ \frac{1}{\lambda_n I_{D5}} \parallel \frac{1}{\lambda_p I_{D6}} \right] = 20.$$

$r_{o5} = \frac{1}{(0.1)(1mA)} = 10k\Omega \quad r_{o6} = \frac{1}{(0.2)(1mA)} = 5k\Omega. \quad r_{o5} \parallel r_{o6} = 3.33k\Omega$

$V_{GS5} - V_{TH} = \frac{2I_D(r_{o5} \parallel r_{o6})}{A_v} = \frac{2(1mA)(3.33k\Omega)}{20} = 0.333V$

$$\left(\frac{W}{L}\right)_5 = \frac{2(1mA)}{(350)(383.6n)(0.33)^2(1+0.1\times0.33)} = \boxed{132. = (W/L)_5}$$

$$\left(\frac{W}{L}\right)_6 = \frac{2(1mA)}{(100)(383.6n)(0.33)^2(1+0.2\times0.33)} = \boxed{449 = (W/L)_6}$$

(b) $$r_{o6} = \frac{1}{\mu_p C_{ox}\left(\frac{W}{L}\right)(V_{GS6} - V_{THp} - V_{DS})}$$

$V_{GS6} - V_{THp} - V_{DS} = 50mV$

$$= \frac{1}{(100)(383.6n)(449)(50m)} = 1.16k\Omega$$

$$A_v \text{ of 2nd stage} = \left[\frac{2(1mA)}{0.333}\right]\left[10k \parallel 1.16k\right]$$

$$= \boxed{6.24 = A_v}$$

Problem 9.20

(a) $|V_{GS7} - V_{TH7}| = 0.4\,V = V_{OD7}$

$V_{in\,max} = V_{DD} - |V_{OD7}| - |V_{OD1}| - |V_{TH1}|$

In part (d), Prob 9.18. $g_{m1} = 2.81\,m\Omega^{-1} = \frac{2I_D}{V_{GS} - V_{TH}} = \frac{2(0.5m)}{V_{OD1}}$

$|V_{OD1}| = 0.356\,V$

$V_{in\,max} = 3 - 0.4V - 0.356V - 0.8 = 1.444V$

$V_{in,min} = |V_{OD3}| = 0.25$ from Prob 9.18 (c)

Allowable input voltage range : $\boxed{0.25 \leq V_{in} \leq 1.44V}$

(b) At $V_{in} = V_{out}$, $V_{in1} = V_{in2}$ since $V_{in2}$ is connect to $V_{out}$

Since $V_{in1} = V_{in2}$, $I_{D1} = I_{D2} \Rightarrow V_X = V_Y$.

$\Rightarrow I_{D5} = 1mA \Rightarrow I_{D1} = I_{D2} = 0.5mA$.

$I_D = \frac{1}{2}\mu C_{ox}\left(\frac{W}{L}\right)(V_{GS} - V_{TH})^2(1 + \lambda V_{DS})$

$V_{GS3} - V_{TH} = 0.25 \Rightarrow V_{GS3} = 0.95$

$\Rightarrow V_{DS} \cong V_{DD} - V_{DS7} - V_{GS3} \cong 3 - 0.7 - 0.95 \approx 1.3V$

$V_{GS1} - V_{TH} = \sqrt{\frac{2I_D}{\mu C_{ox}\left(\frac{W}{L}\right)(1 + \lambda V_{DS})}}$

$= \left[\frac{2(0.5m)}{(100)(383.6n)(206)(1 + 0.2 \times 1.3)}\right]^{\frac{1}{2}}$

$= 0.317V$

$V_{GS1} = 0.317 + 0.7 = 1.017\,V$

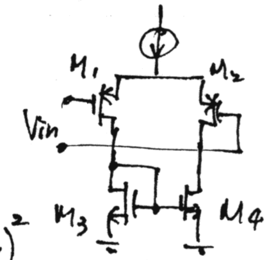
$V_{in} = V_{GS3} + V_{GS1} = 0.95 + 1.017V$

$V_{in} = 1.97V$

Problem 9.21

Noise due to $M_7$ is negligible since induce common mode gain, which is very small.

Consider 1st stage:




$$\overline{V_{n,input}^2}\Big|_{1st\ stage} = \left[4kT\gamma\left(\frac{1}{g_{m1,2}} + \frac{g_{m3,4}}{g_{m1,2}^2}\right)\right]\times 2$$

Consider 2nd stage

$$\overline{V_n^2}_{\ output}\Big|_{2nd\ stage} = \left[4kT\gamma(g_{m5} + g_{m6})\right](r_{o5} \,\|\, r_{o6})^2$$

Overall:

$$\overline{V_{n,input}^2} = \left[4kT\gamma\left(\frac{1}{g_{m1,2}} + \frac{g_{m3,4}}{g_{m1,2}^2}\right)\right]\times 2 + \frac{\left[4kT\gamma\left(\frac{1}{g_{m5}} + \frac{g_{m6}}{g_{m5}^2}\right)\right]}{\left[g_{m1}(r_{o2} \,\|\, r_{o4})\right]^2}$$

From Prob. 9.18

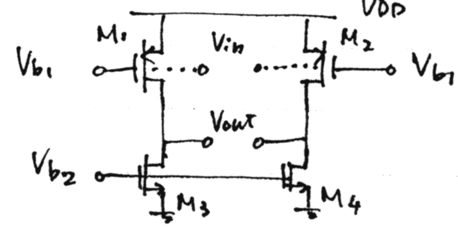
$g_{m1,2} = 2.81\,m\Omega^{-1}$, $g_{m3,4} = \dfrac{2I_D}{V_{GS3}-V_{TH}} = \dfrac{2(0.5m)}{0.25} = 4m\Omega^{-1}$

$g_{m5} = 8m\Omega^{-1}$ $g_{m6} = 8m\Omega^{-1}$, $r_{o2} = 10k$ $r_{o4} = 20k$ $r_{o2} \| r_{o4} = 6.67k\Omega$

$$\overline{V_{n,input}^2} = \left[4\times 1.38\times 10^{-23}\times 300\times \frac{2}{3}\times\left(\frac{1}{2.81m} + \frac{4m}{2.81m^2}\right)\right]\times 2 + \frac{4\times 1.38\times 10^{-23}\times 300\times \frac{2}{3}\times\left(\frac{1}{8m} + \frac{1}{8m}\right)}{\left[2.81m(6.67k)\right]^2}$$

$$\boxed{\begin{aligned} &= 1.905\times 10^{-17}\ V^2/Hz \\ \overline{V_{n,input}^2} &= 4.36\times 10^{-9}\ V/\sqrt{Hz} \end{aligned}}$$

9.22.




(a) $A_v = g_{mb1,2}\,(r_{o1} \parallel r_{o3})$

b) $V_{in} > V_{DD} - V_{D1}$ where $V_{D1}$ is the diode junction voltage of the diode between source and body.

(c) $g_{mb} = g_m \dfrac{\gamma}{2\sqrt{|2\phi_F| + |V_{SB}|}}$

As $V_{in,CM}$ decreases, $|V_{SB}| \uparrow$, $g_{mb}$ decreases.
More accurately, $g_{mb} \propto \dfrac{1}{\sqrt{|2\phi_F| + |V_{SB}|}}$
As a result, $A_v$ decreases.

(d)

$$\overline{V^2_{n,out}} = \left[4kT\gamma\,(g_{m1} + g_{m3})\,R_{out}^2\right] \times 2$$

$$\overline{V^2_{n,in}} = \frac{4kT\gamma\,(g_{m1} + g_{m3})\,R_{out}^2 \times 2}{\left[g_{mb1,2}\,(R_{out})\right]^2}$$

$$= \left[4kT\gamma\,\frac{g_{m1} + g_{m3}}{(g_{mb1,2})^2}\right] \times 2$$

Problem 9.23

a. $A_v$ of 1st stage $= g_{m1,2}(r_{o1} // r_{o3})$

$A_v$ of 2nd stage $= [g_{m5,9}(r_{o7} // r_{o5})] \times 2$

$A_{v\text{-}tot} = g_{m1,2}(r_{o1} // r_{o3})\, g_{m5,9}(r_{o5} // r_{o7}) \times 2$

b. 1st major pole :

$$\omega_1 = \frac{1}{(r_{o9} // r_{o11})[C_{DG9} + C_{DB9} + C_{GS11} + C_{DB11} + C_{GS7} + C_{GD7}(1 + g_{m7}(r_{o5} // r_{o7}))]}$$

2nd major pole: node X, Y

$$\omega_X = \frac{1}{(r_{o1} // r_{o3})\left[\begin{array}{l} C_{DG1} + C_{DB1} + C_{DG3} + C_{DB3} + C_{GS10} + C_{GD10}(1 + \frac{g_{m10}}{g_{m12}}) + C_{GS5} \\ + C_{GD5}(1 + g_{m5}(r_{o5} // r_{o7}) \end{array}\right]}$$

3rd major pole: node output

$$\omega_{out} = \frac{1}{(r_{o5} // r_{o7})(C_{GD5} + C_{DB5} + C_{GD7} + C_{DB7})}$$

Prob. 9.24.

$A_v$ of fast path: $g_{m1}'(r_{o5} \parallel r_{o7})$

$A_v$ of slow path: $g_{m1}(r_{o1} \parallel r_{o3}) g_{m5}(r_{o5} \parallel r_{o7})$.

Overall gain $A_{v\,tot} = \left[\dfrac{g_{m1}' + g_{m1} g_{m5}(r_{o1} \parallel r_{o3})}{2}\right](r_{o5} \parallel r_{o7})$

The output swing is usually limited by $M_{5-8}$, i.e.
$V_{DD} - |V_{OD7}| - V_{OD5}$.

Problem 9.25

Noise due to $M_{1,2}'$

$$\overline{V_n^2}, \text{input} \Big| M_{1,2}' = 4kT\gamma\left(\frac{1}{g_{m1,2}'}\right) \times 2$$

$$\overline{V_n^2}, \text{input} \Big| M_{1,2} = 4kT\gamma\left(\frac{1}{g_{m1,2}}\right) \times 2$$

$$\overline{V_n^2} \text{ input} \Big| M_{3,4} = 4kT\gamma\left(\frac{g_{m3,4}}{g_{m1,2}^2}\right) \times 2$$

$$\overline{V_n^2} \text{ output} \Big| M_{5,6} = 4kT\gamma\, g_{m5,6} R_{out} \times 2$$

$$\overline{V_n^2} \text{ output} \Big| M_{7,8} = 4kT\gamma\, g_{m7,8} R_{out} \times 2$$

$$\overline{V_n^2}, \text{input} \Big| M_{5\text{-}8} = \frac{4kT\gamma\,(g_{m5,6} + g_{m7,8}) \times 2}{\left(\dfrac{g_{m1}' + g_{m1} g_{m5} (r_{o1} \| r_{o3})}{2}\right)^2}$$

$$\overline{V_n^2} \text{ input} \Big|_{tot} = 2\left[4kT\gamma\left(\frac{1}{g_{m1,2}'} + \frac{1}{g_{m1,2}} + \frac{g_{m3,4}}{g_{m1,2}^2} + \frac{4(g_{m5,6} + g_{m7,8})}{(g_{m1}' + g_{m1} g_{m5} (r_{o1} \| r_{o3}))^2}\right]\right.$$

# CHAPTER 10



**10.1** Two poles $\omega_{P_1} = 10\,MHz$ $\omega_{P_2} = 500\,MHz$

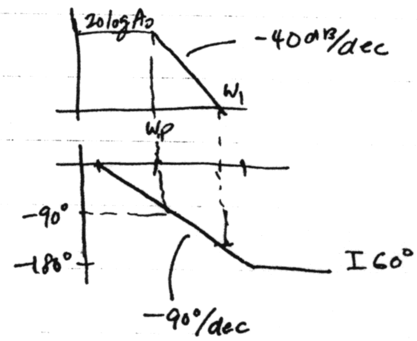
First find $\omega_1$ ($= GX$) that gives phase $= -120°$ (P.M. of 60°)

$$-120° = -\tan^{-1}\frac{\omega_1}{\omega_{P_1}} - \tan^{-1}\frac{\omega_1}{\omega_{P_2}} \longrightarrow \omega_1 \cong 311\,MHz$$

$$A_o = \left(\log\frac{\omega_1}{\omega_{P_1}}\right)(20\,dB/dec) = \left(\log\frac{311}{10}\right)(20) = \underline{\underline{29.9\,dB}}$$

**10.2** $\omega_{P_1} = \omega_{P_2} = \omega_P$

a) $60° \cdot \frac{1}{90°/dec} = 0.67$ decade $\qquad (\beta = 1)$




$\log\frac{10\omega_P}{\omega_1} = 0.67$ dec $\quad$ ($\omega_1$ is GX)

$\Rightarrow \omega_1 = 2.14\,\omega_P$

$$A_o = \left(\log\frac{2.14\,\omega_P}{\omega_P}\right)(40\,dB/dec) = \underline{\underline{13.2\,dB}}$$

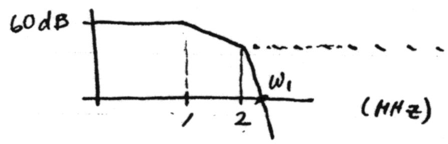
b) For closed-loop gain $= 4 \Longrightarrow \beta \approx \frac{1}{4}$

Thus $A_o$ can increase by a factor of 4 to maintain 60° P.M.

$$\Rightarrow A_o' = 13.2\,dB + 20\log 4 = \underline{\underline{25.2\,dB}}$$

**10.3** $A_o = 1000$ $\quad \omega_{P_1} = 1\,MHz$

a)




$$= 60\,dB - \left(\log\frac{2\,MHz}{1\,MHz}\right)\left(20\,\frac{dB}{dec}\right) = \underline{54\,dB}$$

$$\log\frac{\omega_1}{2\,MHz} = 54\,dB\,\frac{1}{40\,dB/dec} = 1.35\,dec$$

$$\underline{\omega_1 = 44.8\,MHz}$$

$$\angle H(j\omega_1) = -\tan^{-1}\frac{\omega_1}{1\,MHz} - \tan^{-1}\frac{\omega_1}{2\,MHz} = -176.2° \Longrightarrow P.M. = 180° - 176.2° = \underline{\underline{3.8°}}$$

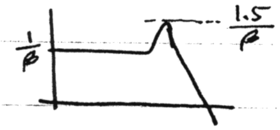
b) $\omega_{P_2}' = 4\,MHz$

$$\log\frac{\omega_1'}{4\,MHz} = \left[60\,dB - \left(\log\frac{4\,MHz}{1\,MHz}\right)(20\,dB/dec)\right]\frac{1}{40\,dB/dec} = 1.199\,dec$$

$\Rightarrow \omega_1' = 63.2\,MHz$

$$\angle H(j\omega_1') = -175.5° \Longrightarrow \underline{\underline{P.M. = 4.5°}}$$

$\boxed{10.4}$




$\beta = 1$

At GX, $H(j\omega_1) = 1 \cdot e^{j\theta_1}$

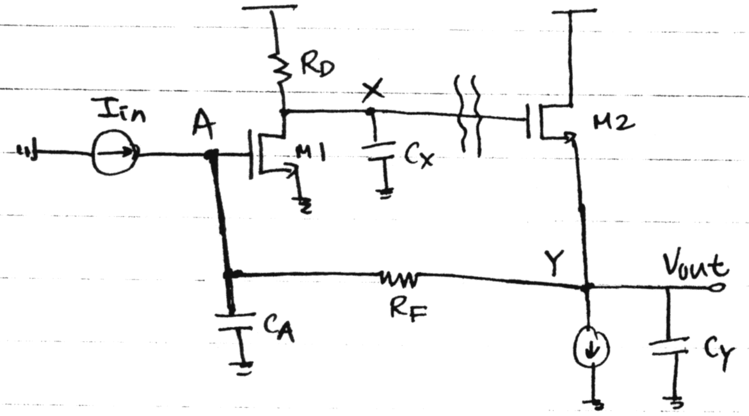
Closed loop: $\left| \frac{Y}{X}(j\omega_1) \right| = \left| \frac{H(j\omega_1)}{1 + H(j\omega_1)} \right| = 1.5$

$$\left| \frac{1}{1 + e^{j\theta_1}} \right| = 1.5$$

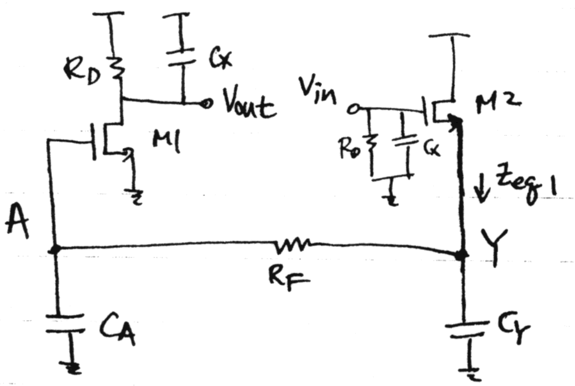
$$\rightarrow \frac{1}{\sqrt{1 + 2\cos\theta_1 + 1}} \rightarrow \frac{1}{2 + 2\cos\theta_1} = 1.5^2 \rightarrow \theta_1 = -141.1^\circ$$

$\underline{\underline{P.M. = 38.9^\circ}}$

$\boxed{10.5}$




Breaking the loop at node X as shown by }} and replacing each end by the impedance each sees, we get the next circuit :




Next, calculate the loop gain

$$\frac{V_{out}}{V_{in}}(s) = \frac{V_Y}{V_{in}} \cdot \frac{V_A}{V_Y} \cdot \frac{V_{out}}{V_A}$$

$$= A_{v1} \cdot A_{v2} \cdot A_{v3}$$

$Z_{eq1} \cong \frac{1}{sC_Y}$ since $R_F = 10\,k\Omega >> \frac{1}{g_{m2}}$

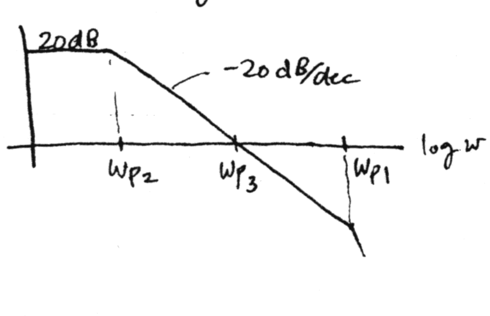
$$A_{v1} = \frac{g_{m2} Z_{eq1}}{1 + g_{m2} Z_{eq1}} \cong \frac{g_{m2} \frac{1}{sC_Y}}{1 + g_{m2}\frac{1}{sC_Y}} = \frac{1}{s\left(\frac{C_Y}{g_{m2}}\right) + 1}$$

$$A_{v2} = \frac{\frac{1}{sC_A}}{R_F + \frac{1}{sC_A}} = \frac{1}{sC_A R_F + 1}$$

$$A_{v3} = -g_{m1}\left(R_o \,\|\, \frac{1}{sC_X}\right) = \frac{-g_{m1} R_D}{1 + sC_X R_D}$$

Hence $\omega_{P1} = \frac{g_{m2}}{C_Y} = 1\times10^{11}\ \text{rad/s}$, $\omega_{P2} = \frac{1}{C_A R_F} = 1\times10^{9}\ \frac{\text{rad}}{s}$, $\omega_{P3} = \frac{1}{C_X R_D} = 1\times10^{10}\ \frac{\text{rad}}{s}$

and $g_{m1} R_D = 10 \rightarrow 20\ dB$




$\Rightarrow$ $\underline{\underline{P.M. = 45°}}$

$\boxed{10.6}$ $R_D' = 2\,k\Omega$

$\rightarrow g_{m1} R_D' = 20 \Rightarrow \underline{26.0\,dB}$ , $\omega_{P3}' = \frac{1}{C_X R_D'} = \underline{5\times10^9\ rad/s}$ .

$\omega_1' \Rightarrow 26\,dB - \left(\log \frac{\omega_{P3}'}{\omega_{P2}}\right)(20\,dB/dec) - \left(\log \frac{\omega_1}{\omega_{P3}'}\right)(40\,\frac{dB}{dec}) = 0\,dB$

$\underline{\omega_1' = 9.99\times10^9\ rad/s}$

$\phi = -\tan^{-1}\frac{\omega_1'}{1\times10^9} - \tan^{-1}\frac{\omega_1'}{5\times10^9} - \tan^{-1}\frac{\omega_1'}{1\times10^{11}} = -153.4°$

$\underline{\underline{P.M. = 26.6°}}$

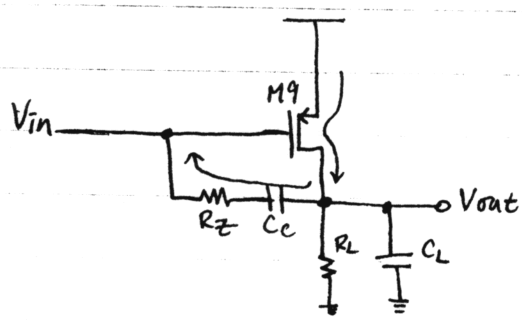
$\boxed{10.7}$ From 10.5 $\omega_{P1} = \frac{g_{m2}}{C_Y}$ $\omega_{P2} = \frac{1}{C_A R_F}$ $\omega_{P3} = \frac{1}{C_X R_D}$

a) Increasing $C_Y$ causes $\omega_{P1}$ to move towards $\omega_{P3}$ and will be less than 1 decade from $\omega_{P3}$. This will reduce the already 45°-phase margin. Hence $\underline{\underline{C_{Y\,max} = 100fF}}$.

b) Increasing $C_A$ will increase phase margin.
Hence $\underline{\underline{C_{A\,max} = 100fF}}$.

c) $\underline{\underline{C_{X\,max} = 100fF}}$ since increasing $C_X$ will reduce phase margin.

$\boxed{10.8}$ The approximation can be derived from the ideal case in which the circuit looks like the following:




At the zero, $V_{out} = 0$. and

$$\frac{-V_{in}}{R_z + \frac{1}{S_z C_c}} = -g_{m9} V_{in}$$

$$\left(\frac{1}{g_{m9}} - R_z\right)^{-1} = S_z C_c$$

$$\therefore \; S_z = \frac{1}{C_c(g_{m9}^{-1} - R_z)}$$

$\boxed{10.9}$ $\left(\frac{W}{L}\right)_{1-4} = \frac{50}{0.5}$ $I_{SS} = I_1 = 0.5mA$ $C_X = C_Y = 0.5pF$

a)

$$\omega_{px} \cong \frac{1}{C_X(r_{op3} \| r_{op2})} \qquad \omega_{py} \cong \frac{1}{C_Y \cdot (g_{m4}^{-1})}$$

In saturation :

$$r_o = \frac{1}{\lambda I_D} \qquad \lambda_p = 0.2 \quad \lambda_n = 0.1 \text{ from Table 2.1}$$

$$r_{op3} = \frac{1}{0.2(0.25mA)} = \underline{20k\Omega}$$

$$r_{on2} = 2r_{op3} = \underline{40k\Omega}$$

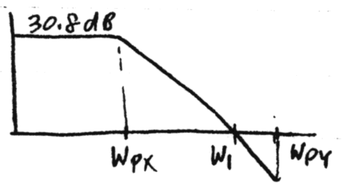
$$g_{m4} = \sqrt{2I_D\mu_n C_{ox}\tfrac{W}{L}} = \sqrt{2(0.5mA)(1.34\times10^{-4})(\tfrac{50}{0.5})} = \frac{1}{273} \text{ A/V}$$

$$g_{m4}^{-1} = 273.0\ \Omega$$

$\Rightarrow$ $\omega_{px} = 150\times10^6$ rad/s , $\omega_{py} = 7.33\times10^9$ rad/s //

$$|\text{Low frequency gain}| \cong g_{m2}(r_{on2} \| r_{op3})\cdot(1) \qquad \left(g_{m2} = \frac{g_{m4}}{\sqrt{2}}\right)$$

$$\cong 34.5 \text{ V/V} \Longrightarrow 30.8\text{ dB} //$$




$$30.8\text{dB} - \left[\log\left(\frac{7.33\times10^9}{150\times10^6}\right)\right](20\text{dB/dec}) = -3.78\text{dB}$$

$$\left(\log\frac{\omega_1}{150\times10^6}\right)(20\text{dB/dec}) = 30.8\text{ dB}$$

$$\Rightarrow \omega_1 = 5.20\times10^9 \text{ rad/s} //$$

$$\phi = -\tan^{-1}\frac{\omega_1}{\omega_{px}} - \tan^{-1}\frac{\omega_1}{\omega_{py}} = -123.7°$$

$\Rightarrow$ $\underline{\underline{P.M. = 56.3°}}$

b)

$$\phi = -\tan^{-1}\frac{\omega_1}{150\times10^6} - \tan^{-1}\frac{\omega_1}{\omega_{py}} = -120°$$

If $\omega_1$ is same as (a), then $\omega_{py} = 8.43\times10^9$ rad/s

$$= \frac{1}{C_{Ymax}\, g_{m4}^{-1}}$$

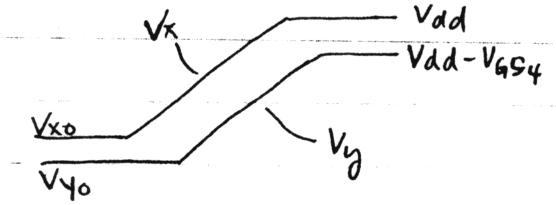
$$\underline{\underline{C_{Ymax} = 434\text{ fF}}}.$$

## 10.10

For large positive step in Vin:

$M_2$ turns off. $M_3$ charges $C_X$ and $M_4$ charges $C_Y$.

($M_4$ also provides $I_1$.)




The slew rates of Vx and Vy (or Vout) must be exactly the same — regardless of Cx or Cy.

Hence slew rate due to positive step input $\cong \frac{I_{D3}}{C_X}$ for both parts (a) and (b) of 10.9.

$$\text{slew rate} \cong \frac{I_{D3}}{C_X} \cong \frac{0.25mA}{0.5pF} = 5.00 \times 10^8 \ V/s \ //$$

For large negative step in Vin:

Again, Vout tracks Vx — as Vx drops, Vout drops at the same rate.

$$\text{Slew rate} \cong \frac{-I_{CX}}{C_X}, \quad I_{D3} \cong 0.25\,mA, \ I_{CX} \cong 0.5mA - I_{CX} = 0.25mA$$

For both $C_Y$'s, slew rate $\cong -\frac{0.25mA}{0.5pF} = -5.00 \times 10^8 \ V/s$ //

$\boxed{10.11}$ $\left(\frac{W}{L}\right)_{5,6} = \frac{60}{0.5}$ $I_{SS} = 0.25mA.$

a) CM level $V_x = V_y = V_{DD} - V_{GS5} = V_{DD} - V_{GS6}$

$I = 1mA = \frac{1}{2}\mu_p C_{ox} \left(\frac{60}{0.5}\right)(|V_{gs6}| - |V_{tp}|)^2$

$\underline{V_{gs6} = 1.46\ V} \longrightarrow \underline{V_x = V_y = 3 - 1.46 = 1.54V.}$

b) Max. output swing :

$V_{out,max} = V_{dd} - V_{overdrive6} = 3 - (1.46 - 0.8) = \underline{\underline{2.34\ V}}$

$V_{out,min} = V_{overdrive8} = V_b - V_{tn} = \underline{\underline{0.39V}}$

(since $V_b = 1.09V$ from $1mA = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_b - 0.7)^2$).

Total max. swing $= 2.34 - 0.39 = \underline{\underline{1.95V}}$

c) $r_{on2} = \frac{1}{(0.1)(0.125mA)} = \underline{80\,k\Omega}$

$r_{op4} = \underline{40\ k\Omega}$ $\Big\} \rightarrow r_{on2} \| r_{op4} = \underline{26.67\ k\Omega}$

$r_{on8} = 10\ k\Omega,\ r_{op6} = 5k\Omega \Big\} \rightarrow r_{on8} \| r_{op6} = \underline{3.33\ k\Omega}$

$g_{m2} = \sqrt{2(0.125mA)(1.34\times10^{-4})\left(\frac{50}{0.5}\right)} = \underline{1.83\times10^{-3}\ A/V}$

$g_{m6} = \mu_p C_{ox}\frac{W}{L}(V_{GS6} - V_{th}) = (3.83\times10^{-5})\left(\frac{60}{0.5}\right)(1.46 - 0.8) = \underline{3.03\times10^{-3}\ A/V}$

$A_{v2} = g_{m6}(3.33\ k\Omega) = \underline{-10.09\ V/V}$

$A_{v1} = g_{m2}(26.67\ k\Omega) = \underline{-48.8\ V/V}$ $\Big\} \rightarrow A_{v1}A_{v2} = 492.4 \rightarrow \underline{\underline{53.8\,dB}}$

$C_{out} = C_L + \left[C_{db6} + \left(1 + \frac{1}{|A_{v2}|}\right)C_{gd6}\right] + (C_{db8} + C_{gd8})$

$\cong 1pF + 52.1 + \left(1 + \frac{1}{10.09}\right)0.18 + 23.4 + 0.2 = \underline{\underline{1.076pF}}$

$C_Y = C_{gd4} + C_{db4} + C_{gd2} + C_{db2} + C_{gs6} + C_{gd6}(1 + |A_{v2}|)$

$\cong 0.15 + 43.8 + 0.2 + 23.4 + 76.6 + 0.18(11.09) = \underline{\underline{146.1\ fF}}$

Using:

$C_{overlap} = CGDO\cdot W,\ C_{db} = \frac{(CJ)(W\cdot 1.5\mu m)}{\left[1 - \frac{V_D}{PB}\right]^{MJ}} + \frac{CJSW\,(2W + 3\mu m)}{\left[1 - \frac{V_D}{PB}\right]^{MJSW}}$

($V_D$ = reverse bias junction voltage.)

10.11 c) cont. $C_{gs} = \frac{2}{3} C_{ox} W \cdot L$, Values from Table 2.1.

Before Compensation:

Dominant Pole: $\omega_y = \frac{1}{C_y R_y} = \frac{1}{(146.1\ fF)(26.67\ k\Omega)} = \underline{2.57 \times 10^8\ rad/s}$

2nd Pole: $\omega_{out} = \frac{1}{C_{out} R_{out}} = \frac{1}{(1.076\ pF)(3.33\ k\Omega)} = \underline{2.79 \times 10^8\ rad/s}$

☆ After compensation:

2nd Pole: $\omega_{out}' \cong \frac{g_{m6}}{C_y + C_{out}} = \frac{3.03 \times 10^{-3}}{146.1\ fF + 1.076\ pF} = \underline{\underline{2.48 \times 10^9\ rad/s}}$

Dominant Pole: $\omega_y' = \frac{1}{[C_y + (1+|A_{v2}|)C_c]\ R_y}$

(For 60° P.M.) $90° + \tan^{-1} \frac{\omega_1'}{\omega_{out}'} = 120° \rightarrow \omega_1' = \omega_{out}' \tan 30° = \underline{1.43 \times 10^9\ rad/s}$

$\log \frac{\omega_1'}{\omega_y'} = \frac{53.8\ dB}{20\ dB/dec} \longrightarrow \omega_y' = \frac{1}{10^{53.8/20}} \cdot \omega_1' = \underline{\underline{2.91 \times 10^6\ rad/s}}$

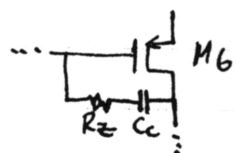
$C_c = \frac{[(2.91 \times 10^6)(26.67\ k\Omega)]^{-1} - 146.1\ fF}{1 + 10.08} = \underline{\underline{1.15\ pF}}$

(So $C_c >> C_y$)

Zero: $\omega_z' = \frac{g_{m6}}{C_c + C_{gd6}} = \frac{3.03 \times 10^{-3}}{1.15\ pF + 0.18\ fF} = \underline{\underline{2.63 \times 10^9\ rad/s}}$ ($> \omega_y', \omega_{out}'$)

d) $\omega_z = \frac{1}{C_c(\frac{1}{g_{m6}} - R_z)} = -|\omega_{out}'|$

$\hookrightarrow R_z = \frac{1}{g_{m6}} + \frac{1}{|\omega_{out}'| \cdot C_c} = \underline{\underline{680.7\ \Omega}}$



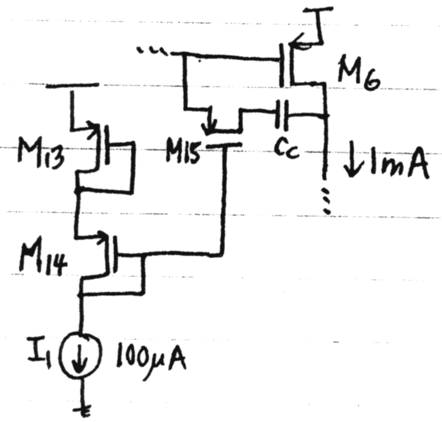

e) Slew rate: Symmetrical for large positive $V_{in}$ or large negative $V_{in}$.

Large $+V_{in}$:

Slew rate of $V_{out2} \cong -\frac{I_{D4}}{C_c} \cong -\frac{0.125\ mA}{1.15\ pF} = \underline{\underline{-1.09 \times 10^8\ V/s}}$

Slew rate of $V_{out1} = -(\text{slew rate of } V_{out2}) = \underline{\underline{1.09 \times 10^8\ V/s}}$

10.12




Want $|V_{GS13}| = |V_{GS6}| = 1.46V$ (from 10.11a)

$100\mu A = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_{13}(|V_{GS6}| - |V_{tp}|)^2$

$\hookrightarrow \left(\frac{W}{L}\right)_{13} = \frac{2(100\mu A)}{(3.83\times10^{-5})(1.46-0.8)^2} = \underline{\underline{12.0}}$ e.g. $\left(\frac{W}{L}\right)_{13} = \frac{6}{0.5}$

Allowing 0.5V across $I_1$ and maximizing $V_{gs14} = V_{gs15}$,
we get $V_{gs14} = V_{gs15} = V_{xy} - 0.5 = 1.54 - 0.5 = \underline{\underline{1.04V}}$

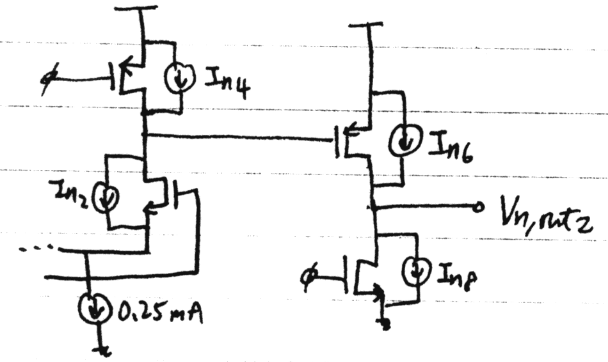
$R_{on15} = \frac{1}{\mu_p C_{ox}\left(\frac{W}{L}\right)_{15}(1.04-0.8)} = 680.7\,\Omega$

$\hookrightarrow \left(\frac{W}{L}\right)_{15} = \underline{\underline{384}}$ e.g. $\frac{192}{0.5}$

$I_{D14} = 100\mu A = \frac{1}{2}\mu_p C_{ox}\left(\frac{W}{L}\right)_{14}(1.04-0.8)^2$

$\hookrightarrow \left(\frac{W}{L}\right)_{14} = \underline{\underline{90.7}}$ e.g. $\frac{45.5}{0.5}$

$\boxed{10.13}$




$$\overline{I_n^2} = 4KT\tfrac{2}{3}g_m$$

$$V_{n,out_2} = (I_{n_2} + I_{n_4})(r_{o2} \| r_{o4}) \cdot A_{v_2} + (I_{n_6} + I_{n_8})(r_{o6} \| r_{o8})$$

$$\overline{V_{n,out_2}^2} = (\overline{I_{n_2}^2} + \overline{I_{n_4}^2})\left[(r_{o2} \| r_{o4}) A_{v2}\right]^2 + (\overline{I_{n_6}^2} + \overline{I_{n_8}^2})(r_{o6} \| r_{o8})^2$$

$$\overline{V_{n,out_1}^2} = (\overline{I_{n_1}^2} + \overline{I_{n_3}^2})\left[(r_{o2} \| r_{o4}) A_{v2}\right]^2 + (\overline{I_{n_5}^2} + \overline{I_{n_7}^2})(r_{o6} \| r_{o8})^2$$

$$\overline{V_{n,out}^2} = \overline{V_{n,out_2}^2} + \overline{V_{n,out_1}^2}$$

$$\overline{V_{n,in}^2} = \frac{\overline{V_{n,out}^2}}{(A_{v_1} A_{v_2})^2}$$

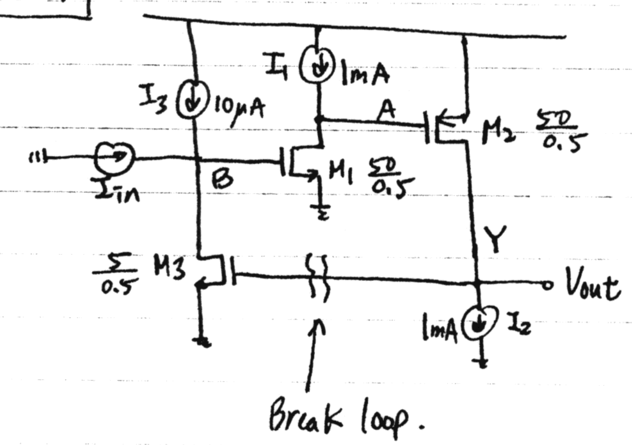
$$= \left(\frac{1}{A_{v_1} A_{v_2}}\right)^2 \left\{ \left[(r_{o2} \| r_{o4}) A_{v2}\right]^2 \left[\overline{I_{n_1}^2} + \overline{I_{n_2}^2} + \overline{I_{n_3}^2} + \overline{I_{n_4}^2}\right] + (r_{o6} \| r_{o8})^2 (\overline{I_{n_5}^2} + \overline{I_{n_6}^2} + \overline{I_{n_7}^2} + \overline{I_{n_8}^2}) \right.$$

Aside
$$\left\{\begin{array}{l}
g_{mn_1} = g_{m2} = 1.83 \times 10^{-3}\ A/V \\
g_{mp_1} = g_{m4} = \sqrt{2 I \mu_p C_{ox} \frac{W}{L}} = \sqrt{2(0.125mA)(3.83\times10^{-5})\left(\frac{50}{0.5}\right)} = 9.79\times10^{-4}\ A/V \\
g_{mn_2} = g_{m8} = 5.18\times10^{-3}\ A/V\ ; \qquad A_{v_1} = 48.8\ ,\ A_{v_2} = 10.09 \\
g_{mp_2} = g_{m6} = 3.03\times10^{-3}\ A/V\ ; \qquad r_{o2} \| r_{o4} = 26.67 K\Omega,\ r_{o6} \| r_{o8} = 3.33 K\Omega
\end{array}\right.$$

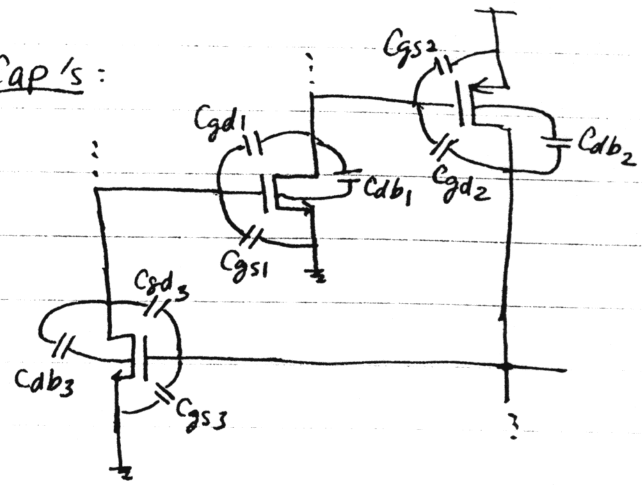
$$\overline{V_{n,in}^2} = 4KT\tfrac{2}{3}\left[1678.1 + 0.7511\right] = 4KT\tfrac{2}{3}\left[1678.85\right] \qquad (4KT = 1.658\times10^{-20}$$

$$= \underline{\underline{1.86\times10^{-17}\ V^2/Hz}}$$

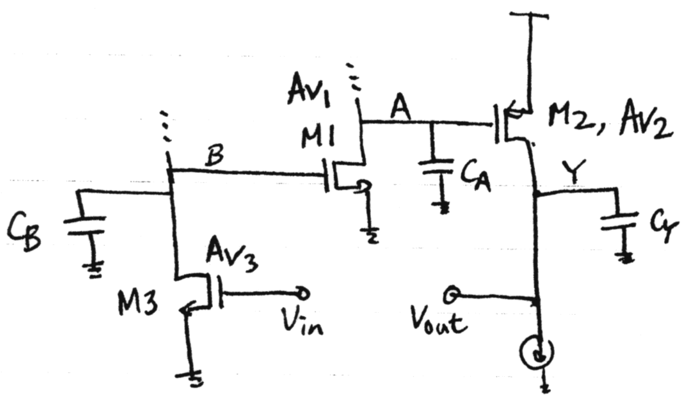
$\boxed{10.14}$




Cap's:




a)




$C_B = C_{db3} + \left(1 + \frac{1}{|A_{v3}|}\right) C_{gd3} + C_{gs1} + (1 + |A_{v1}|)\, C_{gd1}$

$C_A = \left(1 + \frac{1}{|A_{v1}|}\right) C_{gd1} + C_{gs2} + (1 + |A_{v2}|)\, C_{gd2} + C_{db1}$

$C_Y = C_{db2} + \left(1 + \frac{1}{|A_{v2}|}\right) C_{gd2} + C_{gs3} + (1 + |A_{v3}|)\, C_{gd3}$

$r_{o1} = \frac{1}{(0.1)(1mA)} = 10\,k\Omega$

$r_{o2} = 5\,k\Omega$

$r_{o3} = 1\,M\Omega$

Using capacitance formulas of (10.11), we get:

$C_A = 80.4\ fF$ $\qquad V_A = 1.48V$

$C_B = 77.1\ fF$ $\qquad V_B = 1.09V$

$C_Y = 105.6\ fF$ $\qquad V_y = 0.822V$

Also,

$g_{m1} = \sqrt{2(1mA)(1.34\times10^{-4})(50/0.5)} = 5.18\times10^{-3}\ A/V$

$g_{m2} = 2.77\times10^{-3}\ A/V$

$g_{m3} = 1.64\times10^{-4}\ A/V$

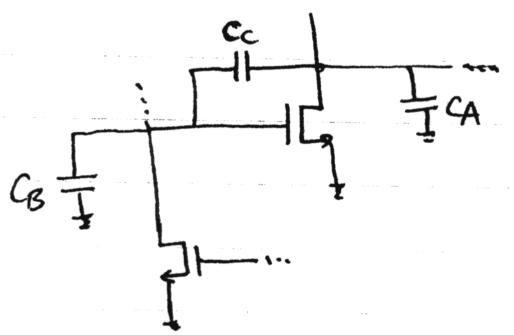
$\left.\begin{aligned} A_{v1} &= g_{m1} r_{o1} = -51.8\ V/V \\ A_{v2} &= g_{m2} r_{o2} = -13.85\ V/V \\ A_{v3} &= g_{m3} r_{o3} = -164.0\ V/V \end{aligned}\right\} \rightarrow |A_{v1} A_{v2} A_{v3}| = 1.18\times10^{5} \rightarrow \underline{\underline{101.4\,dB}}$

10.14 a) cont.

$$\boxed{\begin{aligned} W_A &= \frac{1}{C_A r_{o1}} = 1.24\times10^{9}\ \text{rad/s} && \leftarrow \text{2nd} \\ W_B &= \frac{1}{C_B r_{o3}} = 1.30\times10^{7}\ \text{rad/s} && \leftarrow \text{Dominant} \\ W_Y &= \frac{1}{C_Y r_{o2}} = 1.89\times10^{9}\ \text{rad/s} && \leftarrow \text{3rd} \end{aligned}}$$

(Phase Margin is $-78.3°$ → unstable system)

b) Compensate by adding $C_c$ across G and D of $M_1$.




$$\rightarrow W_A' \cong \frac{g_{m1}}{C_B + C_A} = \frac{5.18\times10^{-3}}{77.1\,fF + 80.4\,fF} = \underline{\underline{3.3\times10^{10}\ \text{rad/s}}}$$

$$\rightarrow W_y \text{ unchanged} = \underline{\underline{1.89\times10^{9}\ \text{rad/s}}}$$

$$90° + \tan^{-1}\frac{W_1}{W_y} = 120° \quad (\text{For } 60° \text{ P.M.})$$

$$\tan^{-1}\frac{W_1}{W_y} = 30°$$

$$\underline{W_1 = 1.09\times10^{9}\ \text{rad/s}}$$

$$\left(\log\frac{W_1}{W_B'}\right)\frac{20\,dB}{dec} = 101.4\,dB$$

$$\rightarrow \underline{\underline{W_B' = 9.28\times10^{3}\ \text{rad/s}}}$$

Dominant: $W_B'$, 2nd: $W_y$, 3rd: $W_A'$

$$W_B' = \frac{1}{[C_B + (1+|A_{v1}|)C_c]\,r_{o3}} \rightarrow C_c = \frac{\frac{1}{W_B'(1\times10^{6})} - 77.1\,fF}{52.8} = \underline{\underline{2.04\,pF}}$$

$$W_z' \cong \frac{g_{m1}}{C_c} = \frac{5.18\times10^{-3}}{2.04\,pF} = \underline{2.54\times10^{9}\ \text{rad/s}} \quad (> W_y.)$$

10.14 c)

$$W_z = \frac{1}{C_c(g_{m_1}^{-1} - R_z)} = -|W_y| = -1.89 \times 10^9$$

$$\hookrightarrow R_z = \frac{1}{g_{m_1}} + \frac{1}{|W_y| C_c} = \underline{\underline{452.4\,\Omega}}$$

$\boxed{10.15}$ a) Before compensation:

$C_A = 80.4\ fF$

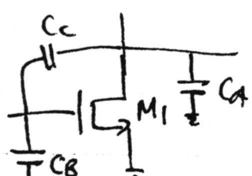
$C_B = 77.1\ fF$

$C_Y = 105.6\ fF + 0.5\ pF = 605.6\ fF$

$$\boxed{\begin{array}{ll} W_A = 1.24\times10^9\ \text{rad/s} & \leftarrow \text{3rd} \\ W_B = 1.30\times10^7\ \text{rad/s} & \leftarrow \text{Dominant} \\ W_Y = \dfrac{1}{(605.6\,fF)(5\times10^3)} = 3.30\times10^8\ \text{rad/s} & \leftarrow \text{2nd} \end{array}}$$

b) Two choices: We can put $C_c$ across G & D of $M_1$ OR just add a C from G of $M_1$ to ground. Cannot take advantage of splitting 1st and 2nd pole here since 1st pole is at B and 2nd pole is at Y and the gain between those two nodes is >0.

Choose to put $C_c$ across $M_1$: Splits 1st and 3rd poles, 2nd pole unchanged.




3rd: $W_A' \cong \dfrac{g_{m1}}{C_A + C_B} = \underline{3.30\times10^{10}\ \text{rad/s}}$

2nd: $W_Y = \underline{3.30\times10^8\ \text{rad/s}}$ (unchanged).

$W_1' = W_Y \cdot \tan 30° = \underline{1.91\times10^8\ \text{rad/s}}$

Dominant: $W_B' = \dfrac{1}{10^{(101.4/20)}} \cdot W_1' = \underline{1.63\times10^3\ \text{rad/s}}$

$$C_c = \frac{\dfrac{1}{(1.63\times10^3)(1\times10^6)} - 77.1\,fF}{52.8} = \underline{\underline{11.6\ pF}}$$

10.15 c)

$$W_z = \frac{1}{C_c(g_{m1}^{-1} - R_z)} = -|W_g| = -3.30\times10^8 \text{ rad/s}$$

$$\hookrightarrow R_z = \frac{1}{g_{m1}} + \frac{1}{|W_g|\cdot C_c}$$

$$R_z = \frac{1}{5.18\times10^{-3}} + \frac{1}{(3.3\times10^8)(11.6pF)}$$

$$\underline{\underline{R_z = 454.3\,\Omega}}$$

$\boxed{10.16}$ If $M_1$ turns off momentarily, $I_1$ causes a positive jump in voltage at node A. This causes $M_2$ to shut off momentarily so the slew rate is determined by $I_2$ and $C_Y$.

i) $$\text{Slew rate} = -\frac{I_2}{C_Y} = -\frac{1mA}{105.6fF} = \underline{\underline{-9.47 \times 10^9 \, V/s}}$$

(If unloaded)

ii) $$\text{Slew rate} = -\frac{I_2}{C_Y + C_L} = -\frac{1mA}{105.6fF + 0.5pF} = \underline{\underline{-1.65 \times 10^9 \, V/s}}$$

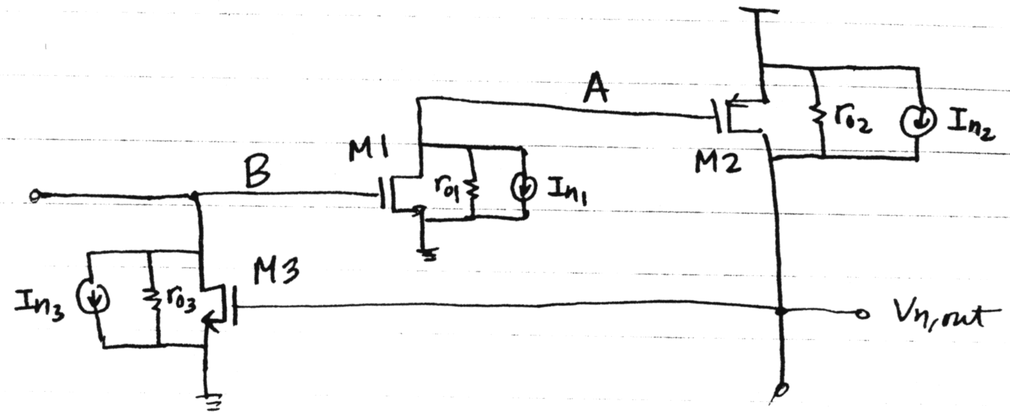
(loaded)

$\boxed{10.17}$ For problem 10.14, $C_c$ should not be placed "across" $M_2$ or $M_3$ because of the location of the poles. Since the dominant pole was at node B, the 2nd pole at node A, and the 3rd pole at node Y, we need to split the 1st two poles by placing $C_c$ across $M_1$.

Putting $C_c$ "across" $M2$ only splits the 2nd and 3rd pole keeping the dominant pole unchanged. It moves the 2nd pole toward the dominant pole and the 3rd pole away. That cannot give a 60° phase margin.

Putting $C_c$ "across" $M3$ only affects the dominant pole and we cannot take advantage of pole-splitting to widen the bandwidth.

$\boxed{10.18}$




$$V_B = -r_{o3}(g_{m3} V_{n,out} + I_{n3})$$

$$V_A = -r_{o1}(g_{m1} V_B + I_{n1}) = -r_{o1}(-g_{m1} g_{m3} r_{o3} V_{n,out} - g_{m1} r_{o3} I_{n3} + I_{n1})$$

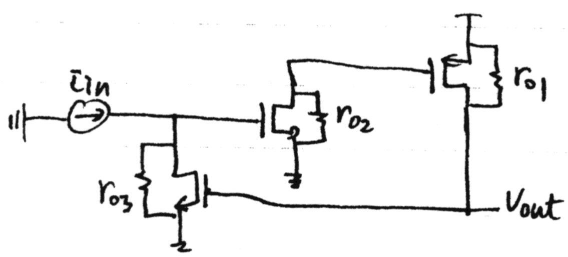
$$V_{n,out} = -r_{o2}(g_{m2} V_A + I_{n2})$$

$$= -r_{o2}[I_{n2} + g_{m1} g_{m2} g_{m3} r_{o1} r_{o3} V_{n,out} + g_{m1} g_{m2} r_{o1} r_{o3} I_{n3} - g_{m2} r_{o1} I_{n1}]$$

$$\overline{V_{n,out}^2} = \frac{\overline{I_{n2}^2} + (g_{m1} g_{m2} r_{o1} r_{o3})^2 \overline{I_{n3}^2} + (g_{m2} r_{o1})^2 \overline{I_{n1}^2}}{\left(\frac{1}{r_{o2}} + g_{m1} g_{m2} g_{m3} r_{o1} r_{o3}\right)^2}$$

//

Transresistance of the circuit is :




$$\frac{V_{out}}{i_{in}} = \frac{1}{g_{m3} + [g_{m1} g_{m2} r_{o1} r_{o2} r_{o3}]^{-1}}$$

//

★ Input referred noise current: $\underline{\underline{\overline{i_{n,in}^2}}} = \dfrac{\overline{V_{n,out}^2}}{\left(\frac{V_{out}}{i_{in}}\right)^2} = \underline{\underline{7.25 \times 10^{-24}\ A^2/Hz}}$

Aside: Since from (10.14): $g_{m1} = 5.18 \times 10^{-3}$ A/V, $g_{m2} = 2.77 \times 10^{-3}$ A/V, $g_{m3} = 1.64 \times 10^{-4}$ A/V.
$r_{o1} = 10\,k\Omega$, $r_{o2} = 5\,k\Omega$, $r_{o3} = 1\,M\Omega$.
$\overline{I_{n2}^2} = 4kT\frac{2}{3}g_{m2} = 3.062 \times 10^{-23}$ $A^2/Hz$, $\overline{I_{n3}^2} = 1.813 \times 10^{-24}$ $A^2/Hz$,
$\overline{I_{n1}^2} = 5.726 \times 10^{-23}$ $A^2/Hz$.

$\boxed{10.19}$ $H_{open}(s) = \dfrac{A_o\left(1+\frac{s}{w_z}\right)}{\left(1+\frac{s}{w_{p1}}\right)\left(1+\frac{s}{w_{p2}}\right)}$ $\qquad w_z \cong w_{p2}$

a)

$$H_{closed}(s) = \frac{A}{1+A\beta} = \frac{A_o\left(1+\frac{s}{w_z}\right)}{\left(1+\frac{s}{w_{p1}}\right)\left(1+\frac{s}{w_{p2}}\right)+A_o\left(1+\frac{s}{w_z}\right)}$$

$$= \frac{A_o\left(1+\frac{s}{w_z}\right)}{\frac{s^2}{w_{p1}w_{p2}} + s\left(\frac{1}{w_{p1}}+\frac{1}{w_{p2}}+\frac{A_o}{w_z}\right)+A_o+1} \qquad \text{Q.E.D.}$$

b) $D(s) = \left(1+\frac{s}{w_{pA}}\right)\left(1+\frac{s}{w_{pB}}\right) \cong 1+\frac{s}{w_{pB}}+\frac{s^2}{w_{pA}w_{pB}}$ $\quad (w_{pB} << w_{pA})$

$$w_{pB} = \frac{A_o+1}{\frac{1}{w_{p1}}+\frac{1}{w_{p2}}+\frac{A_o}{w_z}} \quad //$$

$$w_{pA} = (1+A_o)\,w_{p1}w_{p2}\cdot\frac{1}{w_{pB}} = w_{p2}+w_{p1}+\frac{A_o}{w_z}w_{p1}w_{p2} \quad //$$

c) Using $w_z \cong w_{p2}$, $w_{p2} << (1+A_o)w_{p1}$ or $\frac{1}{w_{p1}} << \frac{A_o+1}{w_{p2}}$

$$w_{pB} = \frac{A_o+1}{\frac{1}{w_{p1}}+\frac{1}{w_{p2}}+\frac{A_o}{w_z}} \cong \frac{A_o+1}{\frac{1}{w_{p1}}+\frac{A_o+1}{w_{p2}}} \cong \underline{\underline{w_{p2}}}$$

$$w_{pA} = w_{p2}+w_{p1}+\frac{A_o}{w_z}w_{p1}w_{p2} \cong w_{p2}+(A_o+1)w_{p1} \cong \underline{\underline{(A_o+1)\,w_{p1}}}$$

$$H_{closed}(s) \cong \frac{\frac{A_o}{A_o+1}\left(1+\frac{s}{w_z}\right)}{\left(1+\frac{s}{(A_o+1)w_{p1}}\right)\left(1+\frac{s}{w_{p2}}\right)} \quad //$$

10.19 d) Step response: $Y(s)$

$$Y(s) = \frac{1}{s}\,\frac{A(1+\frac{s}{w_z})}{(1+\frac{s}{w_{pA}})(1+\frac{s}{w_{pB}})}$$

$A = \frac{A_0}{1+A_0}$, $w_{pA} = (1+A_0)\,w_{p1}$, $w_{pB} = w_{p2}$.

$$= \frac{K_1}{s} + \frac{K_2}{(1+\frac{s}{w_{pA}})} + \frac{K_3}{(1+\frac{s}{w_{pB}})}$$

$$= \frac{s^2\left(\frac{K_1}{w_{pA}w_{pB}} + \frac{K_2}{w_{pB}} + \frac{K_3}{w_{pA}}\right) + s\left(\frac{K_1}{w_{pA}} + \frac{K_1}{w_{pB}} + K_2 + K_3\right) + K_1}{s\,(1+\frac{s}{w_{pA}})(1+\frac{s}{w_{pB}})}$$

$$\boxed{K_1 = A}$$

$$\frac{A}{w_{pA}} + \frac{A}{w_{pB}} + K_2 + K_3 = \frac{A}{w_z} \longrightarrow K_2 + K_3 = A\left(\frac{1}{w_z} - \frac{1}{w_{pA}} - \frac{1}{w_{pB}}\right) \quad \ldots (1)$$

$$\frac{A}{w_{pA}w_{pB}} + \frac{K_2}{w_{pB}} + \frac{K_3}{w_{pA}} = 0 \longrightarrow w_{pA}K_2 + w_{pB}K_3 = -A \quad \ldots (2)$$

$$-(1)\times w_{pA} + (2) \longrightarrow -\left[w_{pA}K_2 + w_{pA}K_3 = A\left(\frac{w_{pA}}{w_z} - 1 - \frac{w_{pA}}{w_{pB}}\right)\right]$$

$$+\quad w_{pA}K_2 + w_{pB}K_3 = -A$$

$$K_3(w_{pB} - w_{pA}) = A\left(\frac{w_{pA}}{w_{pB}} - \frac{w_{pA}}{w_{pz}}\right)$$

$$K_3 = \frac{A\left(\frac{w_{pA}}{w_B} - \frac{w_{pA}}{w_z}\right)}{w_{pB} - w_{pA}}$$

Plug back into (2) to get $K_2$:

$$w_{pA}K_2 + w_{pB}\left[\frac{A\,w_{pA}\left(\frac{1}{w_{pB}} - \frac{1}{w_z}\right)}{w_{pB} - w_{pA}}\right] = -A$$

$$w_{pA}K_2 = -A\left[1 + \frac{w_{pA} - \frac{w_{pA}w_{pB}}{w_z}}{w_{pB} - w_{pA}}\right] = -A\cdot\frac{w_{pB} - \frac{w_{pA}w_{pB}}{w_z}}{w_{pB} - w_{pA}}$$

$$K_2 = -A\,\frac{w_{pB}\left(1 - \frac{w_{pA}}{w_z}\right)}{w_{pA}(w_{pB} - w_{pA})}$$

10.19d) (cont)

can simplify:

$$K_3 = \frac{A\,W_{PA}\left(\frac{1}{W_{PB}} - \frac{1}{W_z}\right)}{W_{PB} - W_{PA}} \cong \frac{\frac{A_o}{A_o+1}(A_o+1)W_{P1}\left(\frac{1}{W_{P2}} - \frac{1}{W_z}\right)}{W_{P2} - (A_o+1)W_{P1}}$$

$$\cong -\frac{A_o}{A_o+1}\left(\frac{1}{W_{P2}} - \frac{1}{W_{Pz}}\right) = K_3$$

$$K_2 = -A\,\frac{W_{P2}\left(1 - \frac{(A_o+1)W_{P1}}{W_z}\right)}{(A_o+1)W_{P1}\left(W_{P2} - (A_o+1)W_{P1}\right)} \cong \frac{-A_o}{A_o+1}\,\frac{-(A_o+1)W_{P1}}{-(A_o+1)^2 W_{P1}^2}$$

$$\cong \frac{-A_o}{(A_o+1)^2 W_{P1}} = K_2$$

$$Y(s) = \frac{A}{s} + \frac{\frac{-A_o}{(A_o+1)^2 W_{P1}}\cdot W_{PA}}{W_{PA}+s} + \frac{-\frac{A_o}{A_o+1}\left(\frac{1}{W_{P2}} - \frac{1}{W_z}\right)\cdot W_{PB}}{s + W_{PB}}$$

$$\cong \frac{A}{s} + \frac{-\frac{A_o}{A_o+1}}{s+(A_o+1)W_{P1}} + \frac{\frac{-A_o}{A_o+1}\left(1 - \frac{W_{P2}}{W_z}\right)}{s + W_{P2}}$$

★ $$y(t) = \frac{A_o}{1+A_o}\left[1 - \left(1 - \frac{W_{P2}}{W_z}\right)e^{-W_{P2}t} - e^{-(A_o+1)W_{P1}t}\right]u(t)$$

= small signal step response.

★ $y(t) \cong \frac{A_o}{1+A_o}\left[1 - \left(1 - \frac{W_{P2}}{W_z}\right)e^{-W_{P2}t}\right]u(t)$ since $(1+A_o)W_{P1} \gg W_{P2}$.

Hence if $W_z$ and $W_{P2}$ do not exactly cancel, there is an exponential term $\left(1 - \frac{W_{P2}}{W_z}\right)e^{-W_{P2}t}$ with a time constant $\frac{1}{W_{P2}} \approx \frac{1}{W_z}$. Q.E.D.

$\boxed{10.20}$

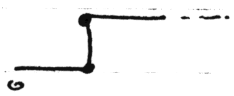
a) Perfect pole-zero cancellation.

Then

$$y(t) \cong \frac{A_o}{1+A_o}\left[1 - 0 - e^{-\omega_{p1}(A_o+1)t}\right]u(t)$$

$$\cong \frac{A_o}{1+A_o}\,u(t)$$

$\Rightarrow$ Step.



$\frac{A_o}{1+A_o}$

b) 10% mismatch.

$$y(t) \cong \frac{A_o}{1+A_o}\left[1 - 0.9\,e^{-\omega_{p2}t}\right]u(t)$$

$\Rightarrow$

$\tau = \frac{1}{\omega_{p2}}.$