Reliability Engineering and System Safety 204 (2020) 107139

Contents lists available at ScienceDirect

Reliability Engineering and System Safety

journal homepage: www.elsevier.com/locate/ress

Reliability evaluation of power systems with multi-state warm standby and multi-state performance sharing mechanism

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ARTICLE INFO

Keywords:
Reliability
Power systems
Multi-state warm standby systems
Multi-state decision diagram
Performance sharing mechanism

ABSTRACT

With the increasing interconnection of the power grids, the imbalanced distribution between power generation and demand in different areas has been effectively alleviated. In practical power systems, the subsystems in different areas need to meet the load requirements of each subsystem, and the performance sharing among different subsystems is one way to increase system reliability. Moreover, each subsystem can be configured with redundancy techniques especially warm standby, which consumes less energy than hot standby and has a shorter recovery time than cold standby. Furthermore, both the generating units and the performance sharing mechanism may have more than binary states in practice. Therefore, in this paper, the reliability evaluation of power systems with multi-state warm standby and multi-state performance sharing mechanism is proposed. Arbitrary state transition time distributions are allowed, and the successful activation probabilities for warm standby generating units are also embedded in the proposed model. The multi-state decision diagram (MSDD) technique is developed for system reliability evaluation. Time-dependent reliability is presented in illustrative examples to validate the proposed model and technique.

1. Introduction

With the development of larger scale and increasing interconnection of the power grids, the imbalanced distribution between power generation and demand in different areas has been effectively alleviated [1]. In practical power systems, the subsystems in different areas need to satisfy the load requirements of each subsystem [2]. When outages of generating units or load fluctuations occur in a certain area, necessary standby generation can be provided by the systems in other areas [3]. Performance sharing is a mechanism for improving system reliability in power systems, where power deficiency of a subsystem can be compensated by the surplus power from another subsystem through transmission lines. However, the shared generation will be limited by the transmission capacity of the transmission lines between different subsystems.

Moreover, in subsystems of the entire power systems, warm standby has been widely applied to power generation systems to ensure safe and reliable operation with lower energy consumption and shorter leading time compared with the case of hot standby and cold standby, respectively [4]. Considering different operating environmental pressure, generating units in warm standby modes may have lower state transition rates compared with units in online modes [5]. Furthermore, because of aging or degradation [6], system components may present multi-state characteristics during the operation period, which can accurately demonstrate features of system components than binary-state models [7,8]. Therefore, multi-state characteristics for generating units and transmission lines are considered in this paper.

Considerable research has also been devoted to the reliability analysis of warm standby systems. In reference [9], the reliability function of a k-out-of-n system with only one warm standby component was obtained using an explicit expression. Analytical availability models for k-out-of-n: G warm standby repairable systems were proposed in [10] utilizing the performance evaluation process algebra. In [11], the reliability model of a warm standby system with two identical sets of units was developed with analytical solution. The reliability of a warm standby system was investigated in [12] where individual components were repairable but the entire system was subject to non-repairable failures. In [13], the reliability of a warm standby repairable system with one unit being priority in use was analyzed based on Markov process and Laplace transform. In [14], optimal component order

Abbreviations: MSDD, Multi-State Decision Diagram; PDF, Probability Density Function; CDF, Cumulative Distribution Function; MCS, Monte Carlo simulation

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https://doi.org/10.1016/j.ress.2020.107139
Received 3 January 2020; Received in revised form 18 July 2020; Accepted 24 July 2020
Available online 27 July 2020

Nomenclature

N	number of generating units in the power system
M	number of subsystems in the power system
Di	maximal load requirement of the i th subsystem
Ai,j	the j-th generating unit in subsystem i
$C^i_{j,k}$	capacity of the j-th generating unit with state k in subsystem i
ni	number of generating units in subsystem i
mi	number of initially online generating units in subsystem i
p^i_j	successful activation probability of the generating unit Ai,j
Y	number of states for a multi-state transmission line
By	capacity of a transmission line in the y-th ($y = 1,\ldots,$Y) state
qy	probability of a transmission line in the y-th ($y = 1,\ldots,$Y) state
W	surplus power of the power system
X	power deficiency of the power system
E	amount of generation that can be transmitted in the entire system
W'	updated surplus power of the system after redistribution
X'	updated power deficiency of the system after redistribution
$\boldsymbol{L}^s_x$	a vector representing the state indices of units of the x-th branch for the s-th state transition in the MSDD
$\boldsymbol{G}^s_x$	a vector indicating different modes of units of the x-th branch for the s-th state transition
$E^{s'}_x$	updated transmitted generation of the x-th branch for the s-th state transition
$X^{s'}_x$	updated power deficiency of the x-th branch for the s-th state transition
H^s_x	a vector indicating the transmitted generation $E^v_u$and the updated power deficiency $X^{'v}_u$
s	index of a state transition of the system
S	number of state transitions of the system
$Path^y_b(t)$	occurrence probability of the b-th path when a transmission line is in the y-th state
t	system operation time
τ_s	the time when the s-th system state transition occurs which is an integral variable and located in (τ_{s-1}, t)
$f^{i,o}_{j,k}(t)$	PDF of the state transition time of the j-th generating unit with online mode in the i th subsystem from state k to state $(k + 1)$
$F^{i,o}_{j,k}(t)$	CDF of the state transition time of the j-th generating unit with online mode in the i th subsystem from state k to state $(k + 1)$
$f^{i,w}_{j,k}(t)$	PDF of the state transition time of the j-th generating unit with warm standby mode in the i th subsystem from state k to state $(k + 1)$
$F^{i,w}_{j,k}(t)$	CDF of the state transition time of the j-th generating unit with warm standby mode in the i th subsystem from state k to state $(k + 1)$
$R^{i,o}_{j,k}(t)$	reliability function of the state transition time of the j-th generating unit with online mode in the i th subsystem from state k to state $(k + 1)$
$R^{i,w}_{j,k}(t)$	reliability function of the state transition time of the j-th generating unit with warm standby mode in subsystem i from state k to state $(k + 1)$
R(t)	reliability of the power system

strategies were proposed for a general 1-out-of-n warm standby system in terms of the system reliability. A solution methodology for determining combined optimal design configuration for heterogeneous warm standby series-parallel systems was presented using numerical reliability evaluation algorithm in [15]. In [4], the reliability of multi-state power systems with warm standby was evaluated without the consideration of performance sharing.

Furthermore, the performance sharing characteristic has also been investigated in previous studies. A system with performance sharing was first proposed in [16], where the surplus performance can be only transmitted from a standby unit to an online unit and the amount of transmitted capacity is limited by the capacity of the common bus. Extended from a simple series system in [16], the system was complicated by modifying as a series-parallel system in [17]. Moreover, the common bus performance sharing systems with arbitrary number of multi-state components were developed in [18] where traditional steady-state reliability evaluation was investigated. Availability of systems with performance sharing considering loading and protection was analyzed in [19]. Reliability of non-repairable phased-mission systems with performance sharing was evaluated where the state of a producer was binary in [20]. A phased-mission system with performance sharing considering common cause failures was analyzed in [21]. The reliabilities of multi-state systems with two performance sharing groups [22] and limited groups [23] instead of only a single performance sharing group were respectively assessed. In [24], the reliability of a system with performance sharing was investigated where the system worked if the summed weighted deficiency was smaller than the reliable threshold. For achieving time-varying system availability, an instantaneous availability model for a multi-state system with common bus performance sharing was proposed in [25]. The reliability evaluation for multi-state balanced systems with state transitions caused by external shocks was presented in [26]. The optimal design of a linear sliding window system with performance sharing was proposed in [27] based on the extended universal generating function technique. In [28], the optimal trade-off between protection and maintenance for multi-state performance sharing systems with transmission loss was analyzed. However, redundancy techniques were not embedded in the reliability evaluation in the previous studies. The reliability evaluation of a multi-state k-out-of-n: G system with performance sharing was conducted in [29] without the consideration of warm standby components. In [30], the reliability of a system with binary-state components considering performance sharing was analyzed where components with multi-state characteristics were not involved.

This paper considers power systems with both multi-state generating units and multi-state performance sharing. The power generation can be shared among different subsystems, where surplus power from a subsystem can be transmitted to arbitrary subsystem suffering from power deficiency through multi-state transmission lines [31,32]. Different state transition distributions of generating units in warm standby modes and operating modes are allowed. Moreover, considering the fact that when warm standby units are activated, start failures or imperfect switches may occur [4], constant successful activation probabilities for generating units in warm standby modes are generally adopted in this paper.

The method for evaluating steady-state reliability of systems with performance sharing in [17–19] was universal generating function technique. The combined stochastic processes methods and universal generating function or *Lz* transform techniques were utilized for time-varying reliability evaluation in [16,25]. A recursive algorithm was adopted in references [20,21], for evaluating reliability of phased-mission common bus systems. However, in these studies, components in different modes with distinct state transition rates were not considered, where these techniques might not be directly applied to the proposed power systems with both warm standby and performance sharing. Furthermore, the state probabilities of multi-state components are usually considered as known [18] or obtained from stochastic process models, especially Markov process models [25] which require exponential state transition time distributions [33] and restricts the application ranges.

Decision diagram techniques have been proven to be effective ways to deal with reliability evaluation of complex engineering systems, for example, standby systems [34–36] and phased mission systems [37,38]. Decision diagram approaches have great advantages in reliability assessment of systems following arbitrary time-to-failure distributions, including the commonly used exponential distributions [35]. Multi-valued decision diagram techniques can be applied to multi-state systems [39], while traditional binary decision diagram methods are usually utilized for systems with binary-state components [40]. Therefore, in this paper, the multi-state decision diagram (MSDD) approach is developed to achieve the reliability analysis of the proposed power systems.

In this paper, the system model for a power system consisting of multi-state generating units and performance sharing is first proposed without the consideration of reparation, where the power deficiency of a subsystem can be compensated by the surplus power from another subsystem. The redistributed generation is restricted by the transmission capacity of multi-state transmission lines. Successful activation probabilities of warm standby units are embedded in the proposed model and technique. A MSDD-based method is developed to achieve the reliability evaluation of the proposed power systems, which allow generating units with arbitrary state transition time distributions besides the commonly utilized exponential distributions. Moreover, time-dependent reliability rather than stead-state reliability of the proposed is presented.

The contributions of this paper can be elaborated as follows. (1) The power systems with multi-state warm standby and multi-state performance sharing are modeled, where surplus power from a subsystem can be transmitted to another subsystem suffering from power deficiency under the constraint of multi-state transmission capacities. (2) To involve imperfect switches of standby components when activated, the constant successful activation probability for warm standby is embedded in the proposed model. (3) The time-dependent reliability evaluation of the proposed power systems is conducted utilizing the developed MSDD method which allows arbitrary state transition time distributions for generating units.

The remainder of the paper is organized as follows. The model for the power systems with multi-state warm standby and multi-state performance sharing mechanism is elaborated in Section 2. Section 3 presents the reliability assessment for power systems utilizing the developed MSDD technique. Illustrative examples are provided in Section 4 to validate the proposed model and technique. Conclusions and future research directions are presented in Section 5.

2. Model description for the power systems with multi-state warm standby and multi-state performance sharing mechanism

A power system consists of M subsystems and N multi-state generating units where each subsystem should satisfy its individual load requirement. The maximal load requirement of the i *th* subsystem is D_i. In subsystem i, generating unit j can be in K_{ij} states, where the corresponding capacity is $C^i_{j,k}$ when it is in the state k. In the initial state, the unit j is in the state 1 with the rated capacity $C^i_{j,1}$. The capacities are presented in a descending order, i.e., for the unit j we have $C^i_{j,1} > C^i_{j,2} > \cdots > C^i_{j,K_{ij}}$. In the proposed model, generating units are assumed to be non-repairable during the system operation [41].

The number of generating units allocated in subsystem i are

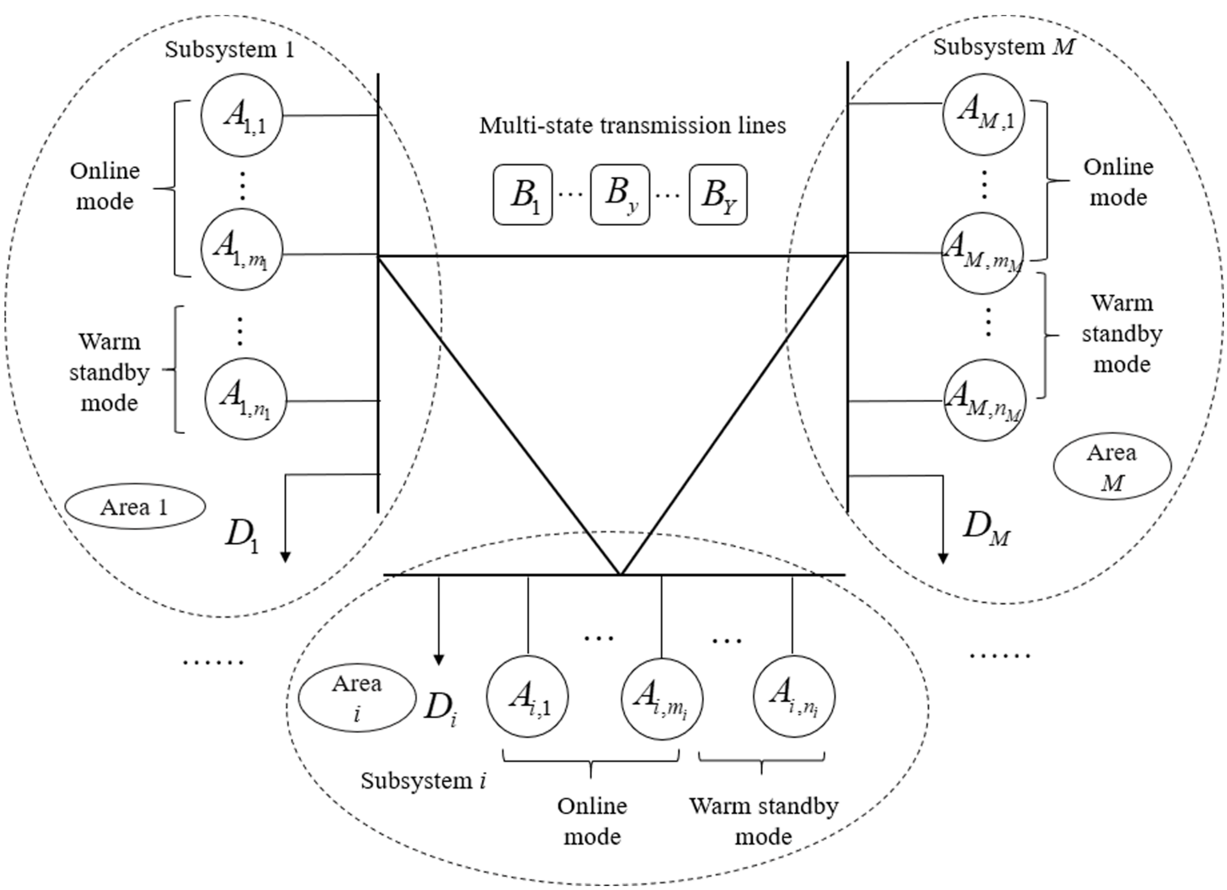

Fig. 1. The structure of the power system with multi-state generating units and performance sharing.

predesigned as n_i. Then, we have $\sum_{i=1}^{M} n_i = N$. The numbers of units in online modes and warm standby modes are determined by the capacities of different units and load requirement in each subsystem. Specifically, in subsystem i, if the former m_i units can satisfy the load D_i, then, these units are in the online modes, while the remaining (n_i-m_i) units are in the warm standby modes. The sets for units in online modes and warm standby modes for subsystem i are respectively presented as Ω_i^o and Ω_i^s. Therefore, initially, $\Omega_i^o = \{1, 2, \cdots, m_i\}$and $\Omega_i^s = \{m_i + 1, \cdots, n_i\}$. The probability density function (PDF) of the state transition time distribution for an online unit j from state k to state $(k+1)$ is $f_{j,k}^{i,o}(t)$ while the PDF of the state transition time distribution for a warm standby unit j from state k to state $(k+1)$ is $f_{j,k}^{i,w}(t)$. The structure of the proposed power system is illustrated in Fig. 1, where A_{i,m_i} indicates the m_i-*th* generating unit in subsystem i.

When a state transition occurs to arbitrary generating unit, it may be needed to adopt performance sharing among different subsystems or activation of warm standby units to online mode in order to satisfy the system loads. In this way, the power deficiency caused by the state transition is offset by either the shared power from other subsystems or the capacities of activated warm standby units. Moreover, considering the possible imperfect switches of warm standby units, successful activation probability p_j^i of unit $A_{i,j}$ is embedded in the proposed model.

Since the cost for transmission of surplus power from another subsystem is usually lower than that for activating warm standby units [42], no warm standby units will be activated if the current surplus performance of a subsystem can balance the deficiency of another subsystem. In other words, transmission of surplus performance is supposed to be preferentially adopted for ensuring system reliability. When power deficiency occurs in a subsystem, surplus power in another subsystem i with generation capacity exceeding the maximal load requirement D_i, can be transmitted to the subsystems with power deficiency through the multi-state common bus. There are Y states for the transmission lines where the capacity of the y-*th* ($y = 1,\ldots,Y$) state is B_y with a probability q_y. The transmission limitation for the surplus power is restricted by the capacity B_y of the transmission lines.

If the power deficiency of a subsystem cannot be balanced by the performance sharing, then, warm standby units in the subsystem with deficiency have priorities to be activated than standby units in other subsystems. Apart from this priority, warm standby units are activated depending on their indexes of the warm standby components [43]. Specifically, in subsystem i, the warm standby unit $A_{i,\ k}$ is activated after the start-up of warm standby unit $A_{i,\ j}$ where $m_i + 1 \le j < k \le n_i$. The power system fails when at least one of the subsystems cannot satisfy its load.

In the subsystem i, when the component j is in the state k with the capacity $C_{j,k}^i$, the surplus power and the power deficiency in the subsystem i are presented as $W_i$and X_i in Eqs. (1) and (2), respectively.

$$W_i = \max\left(\sum_{j\in\Omega_i^o} C_{j,k}^i - D_i, 0\right) \tag{1}$$

$$X_i = \max\left(D_i - \sum_{j\in\Omega_i^o} C_{j,k}^i, 0\right) \tag{2}$$

Thus, we can respectively obtain the total surplus power W and the total power deficiency X of the entire power system by summing up all surplus power and deficiency in each subsystem, which are formulated as Eqs. (3) and (4).

$$W = \sum_{i=1}^{M} W_i = \sum_{i=1}^{M} \max\left(\sum_{j\in\Omega_i^o} C_{j,k}^i - D_i, 0\right) \tag{3}$$

$$X = \sum_{i=1}^{M} X_i = \sum_{i=1}^{M} \max\left(D_i - \sum_{j\in\Omega_i^o} C_{j,k}^i, 0\right) \tag{4}$$

With generation sharing mechanism, the surplus power of subsystems can be shared with subsystems suffering from power deficiency, where no more than W amount of generation can be transmitted. Therefore, the amount of generation which is supposed to be transmitted is $\min\{W, X\}$.

Moreover, when the transmission line is in state y with capacity B_y, due to the constraint of the transmission capacity, the amount of generation that can be transmitted in the entire system E can be formulated as Eq. (5).

$$\begin{aligned} E &= \min(W, X, B_y) \\ &= \min\left(\sum_{i=1}^{M} \max\left(\sum_{j\in\Omega_i^o} C_{j,k}^i - D_i, 0\right), \sum_{i=1}^{M} \max\left(D_i - \sum_{j\in\Omega_i^o} C_{j,k}^i, 0\right), B_y\right) \end{aligned} \tag{5}$$

It should be noted that W and X are statistically dependent, where W and B_y as well as X and B_y are statistically independent.

The power deficiency of the total system after the redistribution is represented as X' in Eq. (6).

$$\begin{aligned} X' &= X - E = X - \min(W, X, B_y) = \max(0, X - \min(W, B_y)) \\ &= \max\left(0, \sum_{i=1}^{M} \max\left(D_i - \sum_{j\in\Omega_i^o} C_{j,k}^i, 0\right) - \min\left(\sum_{i=1}^{M} \max\left(\sum_{j\in\Omega_i^o} C_{j,k}^i - D_i, 0\right), B_y\right)\right) \end{aligned} \tag{6}$$

The remaining unused surplus power W' can be formulated as Eq. (7).

$$\begin{aligned} W' &= W - E = W - \min(W, X, B_y) = \max(0, W - \min(X, B_y)) \\ &= \max\left(0, \sum_{i=1}^{M} \max\left(\sum_{j\in\Omega_i^o} C_{j,k}^i - D_i, 0\right) - \min\left(\sum_{i=1}^{M} \max\left(D_i - \sum_{j\in\Omega_i^o} C_{j,k}^i, 0\right), B_y\right)\right) \end{aligned} \tag{7}$$

The system is reliable if there is no power deficiency in all subsystems after the generation sharing. Therefore, the reliability R of the proposed power system can be presented as Eq. (8).

$$\begin{aligned} R &= \Pr\{X' = 0\} = \Pr\{X - \min(W, B_y) \le 0\} \\ &= \Pr\left\{\sum_{i=1}^{M} \max\left(D_i - \sum_{j\in\Omega_i^o} C_{j,k}^i, 0\right) - \min\left(\sum_{i=1}^{M} \max\left(\sum_{j\in\Omega_i^o} C_{j,k}^i - D_i, 0\right), B_y\right) \le 0\right\} \end{aligned} \tag{8}$$

Particularly, if the transmission line is in a state with complete failure, i.e., $B_y = 0$, which indicates no surplus power can be transmitted through the transmission line, the power system can be regarded as a series-parallel system. When $B_y = 0$, the updated power deficiency after generation sharing X' can be demonstrated as Eq. (9), while system reliability R is formulated as Eq. (10).

$$\begin{aligned} X' &= X - \min(W, X, B_y) = X - \min(W, X, 0) = X \\ &= \sum_{i=1}^{M} \max\left(D_i - \sum_{j\in\Omega_i^o} C_{j,k}^i, 0\right) \end{aligned} \tag{9}$$

$$R = \Pr\{X' = 0\} = \Pr\left\{\sum_{i=1}^{M} \max\left(D_i - \sum_{j\in\Omega_i^o} C_{j,k}^i, 0\right) = 0\right\} = \prod_{i=1}^{M} \Pr\left\{D_i \le \sum_{j\in\Omega_i^o} C_{j,k}^i\right\} \tag{10}$$

In another specific case, if the transmission line is in a state with unconstrained transmission capacity, i.e., $B_y = \infty$, the proposed system becomes a parallel system. Thus, the updated performance deficiency X'takes the form of Eq. (11), while system reliability R is presented as Eq. (12).

$$\begin{aligned} X' &= X - \min(W, X, \infty) = \max(0, X - W) \\ &= \max\left(0, \sum_{i=1}^{M} \max\left(D_i - \sum_{j\in\Omega_i^o} C_{j,k}^i, 0\right) - \sum_{i=1}^{M} \max\left(\sum_{j\in\Omega_i^o} C_{j,k}^i - D_i, 0\right)\right) \\ &= \max\left(0, \sum_{i=1}^{M} D_i - \sum_{i=1}^{M} \sum_{j\in\Omega_i^o} C_{j,k}^i\right) \end{aligned} \tag{11}$$

$$R = \Pr\{X' = 0\} = \Pr\left\{\max\left(0, \sum_{i=1}^{M} D_i - \sum_{i=1}^{M} \sum_{j\in\Omega_i^o} C_{j,k}^i\right) = 0\right\} = \Pr\left\{\sum_{i=1}^{M} D_i \le \sum_{i=1}^{M} \sum_{j\in\Omega_i^o} C_{j,k}^i\right\} \tag{12}$$

In order to better demonstrate the proposed system model, an illustrative example is provided as depicted in Fig. 2. The system is composed of 2 subsystems, each of which consists of 2 generating units. The capacities of the multi-state generating units [44] and maximal loads of different subsystems are listed in Table 1. There are three states for the transmission line with capacities 100, 50 and 0 (MW).

Initially, all generating units are in state 1 with maximal generation capacities. Based on the provided capacities and load in each subsystem, units $A_{1,1}$ and $A_{1,2}$ in the subsystem 1 are in the online modes; unit $A_{2,1}$ is in the online mode while unit $A_{2,2}$ is in the warm standby mode in the subsystem 2. If state transitions occur to the unit $A_{1,1}$ or the unit $A_{1,2}$, which leads to power deficiency of subsystem 1, the unit $A_{2,2}$ is supposed to be activated to the online mode and the surplus power of the subsystem 2 is transmitted to subsystem 1 through the transmission line with the restriction of transmission capacity.

3. Reliability analysis of the power system based on MSDD technique

Decision diagram approaches have been extensively applied to reliability analysis of warm standby systems. In this paper, a MSDD technique is developed for achieving reliability evaluation of the proposed power systems.

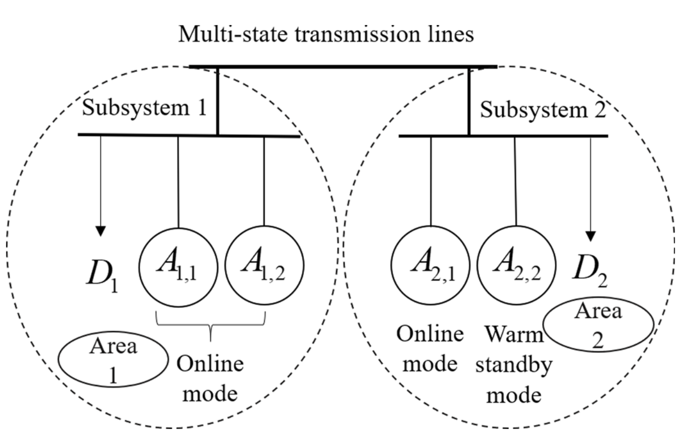

Fig. 2. The system structure of the illustrative example.

Table 1
The capacities and demands of a system in the illustrative example.

Generating units/ State	Capacity/demand (MW)		
	1	2	3
$A_{1,1}$, $A_{1,2}$	80	40	0
$A_{2,1}$, $A_{2,2}$	100	50	0
D_1		120	
D_2		100	

3.1. Construction of system MSDD

Based on the model description of the proposed system, three aspects are embedded when creating MSDD for a proposed power system. First, the vector $\boldsymbol{L}_x^s$represents the state indices of generating units at the *x-th* branch for the *s-th* state transition in the MSDD. In the vector $\boldsymbol{G}_x^s$ which presents different modes of units at the *x-th* branch for the *s-th* state transition, "1" indicates a unit in an online mode while "0" denotes a unit in a warm standby mode. Furthermore, the vector $\boldsymbol{H}_x^s = [E_x^{s\prime}, X_x^{s\prime}]$ indicates the updated transmitted generation and the updated power deficiency of the *x-th* branch for the *s-th* state transition in the system MSDD. The system is regarded as unreliable if the updated power deficiency is larger than 0, denoting power deficiency occurs in the system after the generation capacity redistribution and warm standby activation. Embedded with three aspects mentioned above, a MSDD can be extended from traditional binary decision diagram techniques. Moreover, the node value in the system MSDD is demonstrated by a ternary $\{\boldsymbol{L}_x^s, \boldsymbol{G}_x^s, \boldsymbol{H}_x^s\}$, which is composed of the state and mode of each generating unit as well as the system generation capacity.

The system MSDD can be iteratively constructed from the top node with the increasing number of state transitions of generating units. Let an unlimited transmission capacity of transmission lines, then, the creation of system MSDD is clarified as follows.

Step 1: Construct the top node of the system MSDD.

In the system MSDD, the top node denotes that all units are in the perfect functioning states with the largest generation capacities. Different modes of units are dependent on their capacities and subsystem loads. The value of the top node is presented in Eq. (13).

$$\begin{cases} \boldsymbol{L}_1^0 = \left[\underbrace{1, \cdots, 1}_{N}\right] \\ \boldsymbol{G}_1^0 = \left[\cdots, \underbrace{1, \cdots, 1}_{m_i}, \underbrace{0, \cdots, 0}_{n_i - m_i}, \cdots\right] \\ \boldsymbol{H}_1^0 = [E_1^{0\prime}, X_1^{0\prime}] \end{cases} \tag{13}$$

In Eq. (13), the vector $\boldsymbol{G}_1^0$ represents different modes of units including online modes or warm standby modes. Initially, the former m_i units in subsystem i can satisfy the subsystem load D_i, the remaining (n_i-m_i) units are in warm standby modes. The updated transmitted generation $E_1^{0\prime}$ and the updated power deficiency $X_1^{0\prime}$in vector $\boldsymbol{H}_1^0$are formulated in Eq. (14).

$$\begin{cases} E_1^{0\prime} = \min(W_1^0, X_1^0, \infty) \\ = \min\left(\sum_{i=1}^{M} \max\left(\sum_{j=1}^{n_i} C_{j,1}^i - D_i, 0\right), \sum_{i=1}^{M} \max\left(D_i - \sum_{j=1}^{n_i} C_{j,1}^i, 0\right)\right) \\ X_1^{0\prime} = X_1^0 - E_1^0 = \\ \max\left(0, \sum_{i=1}^{M} \max\left(D_i - \sum_{j=1}^{n_i} C_{j,1}^i, 0\right) - \min\left(\sum_{i=1}^{M} \max\left(\sum_{j=1}^{n_i} C_{j,1}^i - D_i, 0\right)\right)\right) \end{cases} \tag{14}$$

Considering the example presented in Fig. 2, the values in the top node are formulated in Eq. (15).

$$\begin{cases} \boldsymbol{L}_1^0 = [1,\ 1,\ 1,\ 1] \\ \boldsymbol{G}_1^0 = [1,\ 1,\ 1,\ 0] \\ \boldsymbol{H}_1^0 = [0,\ 0] \end{cases} \tag{15}$$

Step 2: Construct the MSDD for the first state transition of an arbitrary unit with an unlimited transmission capacity.

Based on the top node, the system MSDD for the first possible state transition of an arbitrary unit can be constructed. The system MSDD for the first state transition is demonstrated in Fig. 3. The first state transition can occur to anyone of these N units, leading to N branches for the top node, where the x-*th* ($x = 1,\ldots, N$) branch denotes that the first state transition occurs to the x-*th* unit. Since the state transition might cause power deficiency in some sub-systems, performance sharing and activation of warm standby units are sequentially conducted for ensuring system reliability, which might cause the updated sets $\Omega_i^{o'}$ and $\Omega_i^{s'}$ for units in online modes and warm standby modes.

In Fig. 3, the node value of the x-*th* ($x = 1,\ldots, N$) branch for the first state transition is formulated in Eq. (16), which denotes the x-*th* unit transits from state 1 to state 2. If the x-*th* unit belongs to subsystem k, then we have $\sum_{i=1}^{k} n_i \le x \le \sum_{i=1}^{k+1} n_i$.

$$\begin{cases} \boldsymbol{L}_x^1 = \left[\underbrace{1,\cdots,1}_{x-1},\ 2,\ \underbrace{1,\cdots,1}_{N-x} \right] \quad (x = 1,\cdots,N) \\ \boldsymbol{H}_x^1 = [E_x^{1'},\ X_x^{1'}] \end{cases} \tag{16}$$

where the parameters of $\boldsymbol{H}_x^1 = [E_x^{1'}, X_x^{1'}]$ are formulated in Eq. (17).

$$\begin{cases} E_x^{1'} = \min(W_x^{1'},\ X_x^{1'},\ \infty) \\ = \min\left(\begin{array}{l} \sum_{i=1,i\neq k}^{M} \max\left(\sum_{j\in\Omega_i^{o'}} C_{j,1}^i - D_i,\ 0\right) + \max \\ \left(\sum_{j\in\Omega_i^{o'}, j\neq x-\sum_{i=1}^{k-1} n_i} C_{j,1}^k + C_{x-\sum_{i=1}^{k-1} n_i,2}^k - D_k,\ 0\right), \\ \sum_{i=1,i\neq k}^{M} \max\left(D_i - \sum_{j\in\Omega_i^{o'}} C_{j,1}^i,\ 0\right) + \max \\ \left(D_k - \sum_{j\in\Omega_i^{o'}, j\neq x-\sum_{i=1}^{k-1} n_i} C_{j,1}^k - C_{x-\sum_{i=1}^{k-1} n_i,2}^k,\ 0\right) \end{array} \right) \\ X_x^{1'} = \max(0,\ X_x^1 - \min(W_x^1,\ \infty)) \\ = \max\left(\begin{array}{l} 0,\ \sum_{i=1,i\neq k}^{M} \max\left(D_i - \sum_{j\in\Omega_i^{o'}} C_{j,1}^i,\ 0\right) + \max \\ \left(D_k - \sum_{j\in\Omega_i^{o'}, j\neq x-\sum_{i=1}^{k-1} n_i} C_{j,1}^k - C_{x-\sum_{i=1}^{k-1} n_i,2}^k,\ 0\right) - \\ \min\left(\sum_{i=1,i\neq k}^{M} \max\left(\sum_{j\in\Omega_i^{o'}} C_{j,1}^i - D_i,\ 0\right) + \max \right. \\ \left. \left(\sum_{j\in\Omega_i^{o'}, j\neq x-\sum_{i=1}^{k-1} n_i} C_{j,1}^k + C_{x-\sum_{i=1}^{k-1} n_i,2}^k - D_k,\ 0\right)\right) \end{array} \right) \end{cases} \tag{17}$$

Moreover, $\boldsymbol{G}_x^1$ is dependent on the unit capacities and subsystem loads. If there is power deficiency for the first state transition for the x-*th* ($x = 1,\ldots, N$) branch, performance sharing takes precedence for providing surplus power for a subsystem with deficiency. If the deficiency cannot be balanced by performance sharing, r warm standby units are activated to online modes until the deficiency is made up. The updated sets $\Omega_i^{o'}$ for online generating units can be obtained using the elements in $\boldsymbol{G}_x^1$ which are equal to 1.

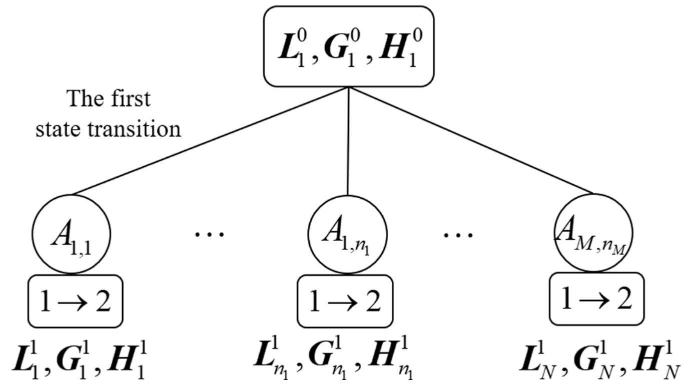

Fig. 3. The system MSDD for the first state transition.

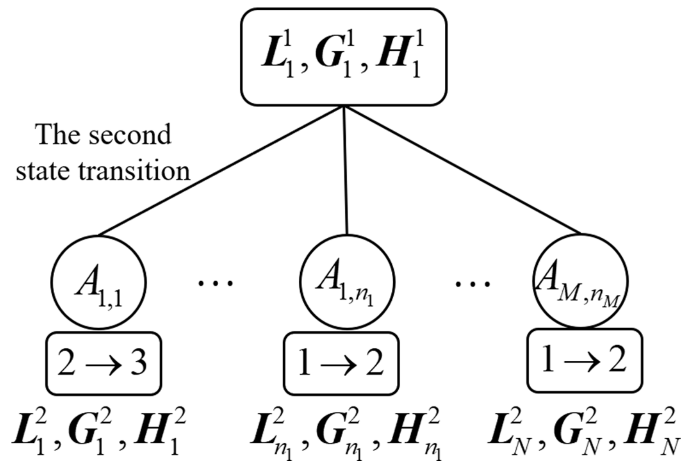

Fig. 4. System MSDD for the second state transition while the first state transition occurs to the first unit.

Step 3: Construct the system MSDD for the second state transition.

Based on the MSDD construction in Step 2, the system MSDD for the second state transition can be developed. The x-*th* terminal node in Fig. 3 is assumed to develop $(N - q_x^1)$ branches, where q_x^1 indicates the number of units in complete failure states for the x-*th* branch after the first state transition. q_x^1 can be obtained by the unit state $\boldsymbol{L}_x^1$. In other words, if a unit has reached its complete failure state, state transition cannot occur to this unit any more. The l-*th* $(l = 1,\cdots,N - q_x^1)$branch indicates the second state transition occurs to a unit.

Taking the leftmost node in Fig. 3 as an example where the first state transition occurs to the first unit, the system MSDD for the second state transition is presented in Fig. 4. In Fig. 4, assuming that there is no unit in complete failure state after the first state transition, i.e., $q_x^1 = 0$. The first branch denotes this state transition occurs to the first unit from the second state to the third state. The l-*th* $(l = 2,\cdots,N)$branch indicates the l-*th* unit suffers from the state transition from the first state to the second state. The terminal values of nodes in the MSDD are updated resulting from the second possible state transition. Eqs. (18)-(19) present the corresponding updated values.

$$\boldsymbol{L}_l^2 = \begin{cases} \left[3,\ \underbrace{1,\cdots,1}_{N-1} \right],\ l = 1 \\ \left[2,\ \underbrace{1,\cdots,1}_{l-2},\ 2,\ \underbrace{1,\cdots,1}_{N-l} \right],\ l = 2,\cdots,N \end{cases} \tag{18}$$

$$E_x^{2'} = \min(W_x^{2'}, X_x^{2'}, \infty)$$

$$= \begin{cases} \min\begin{pmatrix} \max\left(\sum_{j\in\Omega_1^{o'}, j\neq 1} C_{j,1}^{i} + C_{1,3}^{1} - D_1, 0\right) + \\ \sum_{i=2}^{M} \max\left(\sum_{j\in\Omega_i^{o'}} C_{j,1}^{i} - D_i, 0\right), \\ \max\left(D_1 - \sum_{j\in\Omega_1^{o'}, j\neq 1} C_{j,1}^{i} - C_{1,3}^{1}, 0\right) + \\ \sum_{i=2}^{M} \max\left(D_i - \sum_{j\in\Omega_i^{o'}} C_{j,1}^{i}, 0\right) \end{pmatrix}, l = 1 \\ \min\begin{pmatrix} \max\left(\sum_{j\in\Omega_1^{o'}, j\neq 1} C_{j,1}^{i} + C_{1,2}^{1} - D_1, 0\right) + \\ \sum_{i=2, i\neq k}^{M} \max\left(\sum_{j\in\Omega_i^{o'}} C_{j,1}^{i} - D_i, 0\right) \\ +\max\left(\sum_{j\in\Omega_1^{o'}, j\neq l-\sum_{j=1}^{k-1} n_j} C_{j,1}^{k} + C_{l-\sum_{i=1}^{k-1} n_i, 2}^{k} - D_k, 0\right), \\ \max\left(D_1 - \sum_{j\in\Omega_1^{o'}, j\neq 1} C_{j,1}^{i} - C_{1,2}^{1}, 0\right) + \\ \sum_{i=2, i\neq k}^{M} \max\left(D_i - \sum_{j\in\Omega_i^{o'}} C_{j,1}^{i}, 0\right) \\ +\max\left(D_k - \sum_{j\in\Omega_1^{o'}, j\neq l-\sum_{j=1}^{k-1} n_j} C_{j,1}^{k} + C_{l-\sum_{i=1}^{k-1} n_i, 2}^{k}, 0\right) \end{pmatrix}, \begin{matrix} l = 2, \cdots, N, \\ k \neq 1 \end{matrix} \\ \min\begin{pmatrix} \max\left(\sum_{j\in\Omega_1^{o'}, j\neq l} C_{j,1}^{k} + C_{l,2}^{k} + C_{1,2}^{1} - D_1, 0\right) + \\ \sum_{i=2}^{M} \max\left(\sum_{j\in\Omega_i^{o'}} C_{j,1}^{i} - D_i, 0\right), \\ \max\left(D_1 - \sum_{j\in\Omega_1^{o'}, j\neq l} C_{j,1}^{k} - C_{l,2}^{k} - C_{1,2}^{1}, 0\right) + \\ \sum_{i=2}^{M} \max\left(D_i - \sum_{j\in\Omega_i^{o'}} C_{j,1}^{i}, 0\right) \end{pmatrix}, \begin{matrix} l = 2, \cdots, N, \\ k = 1 \end{matrix} \end{cases} \tag{19}$$

Step 4: Iteratively construct the system MSDD for the *s-th* ($s = 3, \cdots, S$)possible state transition based on the MSDD representation for the (s-1)-*th* state transition. Moreover, the node values are updated in accordance with the capacities of units in different states and load of each subsystem. The overall MSDD for a proposed power system is presented in Fig. 5.

Step 5: if the updated performance deficiency is larger than zero, the MSDD construction is terminated since the load of at least one subsystem cannot be satisfied. In addition, the MSDD creation is stopped if no more state transition can occur. Therefore, the stopping criterion is the maximal number of state transitions$S = \max s(X_x^{s'} = 0)$ *or* $S = \sum_{i=1}^{M}\sum_{j=1}^{n_i}(K_{ij} - 1)$.

Step 6: set the transmission lines in the state y with capacity *By,* and neglect the state transitions where the updated transmitted capacity is more than *By*.

Step 7: obtain the system MSDDs with different transmission capacities *By* ($y = 1, \ldots, Y$).

The previous procedure for the algorithm of the MSDD generation can be illustrated as Fig. 6.

Finally, based on the MSDD construction algorithm above, the system MSDD representation can be developed. Fig. 7 illustrates the MSDD construction for the example in Fig. 2 where the first state transition occurs to the first unit (assuming the transmission line is in a state with capacity 100MW).

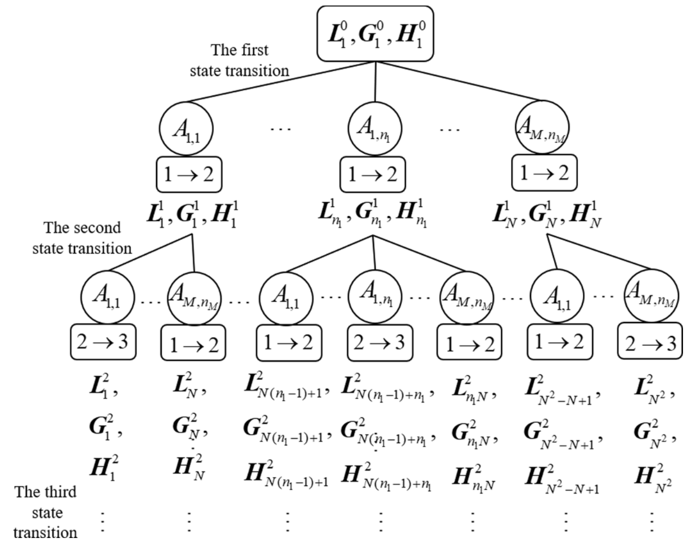

Fig. 5. The overall MSDD construction of a proposed power system.

3.2. Reliability assessment based on the proposed MSDD method

Taking the leftmost path in Fig. 7 which indicates the state transitions occur to components $A_{1,1}$ and $A_{1,2}$ for example, the path can be presented as: the unit $A_{1,1}$ suffers from the first state transition from state 1 to state 2; then, the second state transition occurs to unit $A_{1,1}$ from state 2 to state 3, while unit $A_{2,2}$ in subsystem 2 are activated from a warm standby mode to an online mode for providing generation capacity for subsystem 1 through the transmission line; unit $A_{1,2}$ transits from state 1 to state 2 for the third system state transition, and the power deficiency in subsystem 1 is compensated by surplus power in subsystem 2 through the multi-state transmission line. The occurrence probability of this path where the transmission line is in the *y-th* state is formulated in Eq. (20).

$$Path_1^{y} = p_2^2 \cdot \int_{\tau_0}^{t}\int_{\tau_1}^{t}\int_{\tau_2}^{t} \frac{f_{1,1}^{1,o}(\tau_1) f_{1,2}^{1,o}(\tau_2 - \tau_1) f_{2,1}^{1,o}(\tau_3)}{R_{2,2}^{1,o}(t - \tau_3) R_{1,1}^{2,o}(t) R_{2,1}^{2,w}(\tau_2) R_{2,1}^{2,o}(t - \tau_2)} d\tau_1 d\tau_2 d\tau_3 \tag{20}$$

where t is the system operation time, and τ_0 is the start time for the system operation. $f_{j,k}^{i,o}(t)$and $F_{j,k}^{i,o}(t)$ are the PDF and the cumulative distribution function (CDF) of the state transition time of the *j-th* unit with online mode in subsystem i from state k to state ($k + 1$), respectively. $f_{j,k}^{i,w}(t)$ and$F_{j,k}^{i,w}(t)$are the PDF and CDF of the state transition time of the *j-th* unit with warm standby mode in subsystem i from state k to state ($k + 1$), respectively. Correspondingly, $R_{j,k}^{i,o}(t) = 1 - F_{j,k}^{i,o}(t)$ and $R_{j,k}^{i,w}(t) = 1 - F_{j,k}^{i,w}(t)$are the reliability functions of generating units.$\tau_s$is the time when the *s-th* system state transition occurs. It is an integral variable and located in (τ_{s-1}, t). p_j^{i} is the successful activation probability of unit $A_{i,j}$ when activated from warm standby mode to the online mode.

After obtaining occurrence probabilities of paths leading to a reliable system, we can get the system reliability where the transmission line is in the *y-th* state with a capacity B_y and a probability q_y. Finally, the system reliability can be achieved by summing up all the occurrence probabilities of the paths in different states as presented in Eq. (21).

$$R(t) = \sum_{y=1}^{Y} q_y \cdot \sum_{b} Path_b^{y}(t) \tag{21}$$

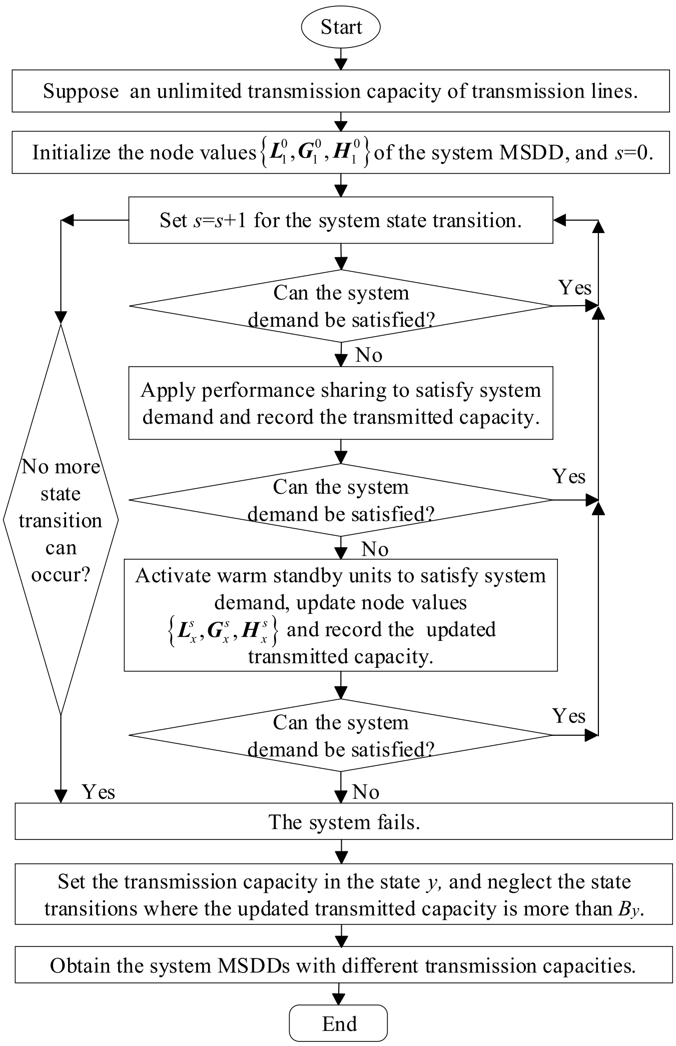

Fig. 6. The algorithm for the MSDD generation.

3.3. Complexity analysis of the proposed technique

The complexity of the proposed technique lies in the MSDD construction and the reliability assessment. For the MSDD construction, the complexity mainly comes from the number of nodes in the MSDD representation. For the reliability assessment based on MSDD, the complexity mainly lies in the integration calculations for obtaining the occurrence probabilities. Therefore, with the consideration of the number of nodes in the MSDD construction and the integration for occurrence probabilities, we can analyze the computational complexity

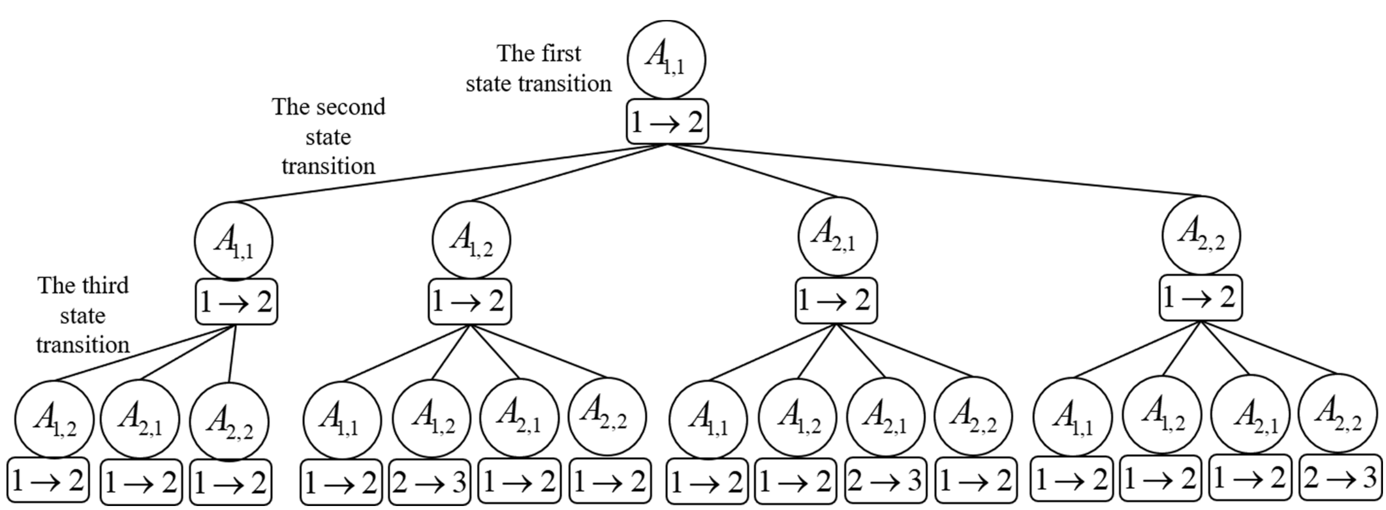

Fig. 7. The system MSDD where the first state transition occurs to the first unit for the example in Fig. 2.

of the proposed technique.

According to steps for MSDD construction above, for the system MSDD when the transmission line is in the state y, the number of nodes is less than Eq. (22). Therefore, the number of nodes of all system MSDDs for different states of a transmission line is less than Eq. (23).

$$1+\sum_{s=1}^{S}N^{s}=(N^{S+1}-1)/(N-1) \tag{22}$$

$$Y\left(1+\sum_{s=1}^{S}N^{s}\right)=Y\,(N^{S+1}-1)/(N-1) \tag{23}$$

It can be seen from Eq. (23) that the number of nodes in all MSDDs is determined by the number of state transitions, which is related to unit capacities in different modes, loads of subsystems, capacities and number of states of transmission lines.

Specifically, if the system MSDD construction is terminated because of no further state transition, i.e., all units have transited to complete failure states, this case is supposed to be the worst scenario with the largest number of nodes in system MSDD. In the worst scenario, the number of nodes in all system MSDDs is less than $Y\left(N^{1+\sum_{i=1}^{M}\sum_{j=1}^{n_i}(K_{ij}-1)}-1\right)/(N-1)$, which is associated with the numbers of system units and states of transmission lines, as well as the summation of state numbers for different units.

However, it should be noted that the number of nodes in the system MSDD is generally much smaller than Eq. (23) since the MSDD construction is usually stopped for the insufficient capacity to cover the system loads. Furthermore, the proposed technique can be further modified by constructing MSDD leading to an unreliable system if the number of reliable paths is much larger than that of unreliable paths. Therefore, in general, the complexity of the proposed MSDD algorithm can be reduced by avoiding enumerating all possible combinations.

For the computational burden of integration for calculating occurrence probabilities, it generally comes from multiple integrations where one or more units suffer from state transitions. Reference [36] elaborates details for the complexity analysis of multiple integrations for obtaining occurrence probabilities.

When the power systems are with large amounts of subsystems and system states, the number of state transitions might become relatively large resulting in complicated multiple integration for the reliability evaluation. It should be noted that the occurrence probabilities of some paths with larger number of state transitions are very small (close to zero). Therefore, the occurrence probabilities of paths with larger number of state transitions can be neglected during the calculation of system reliability.

Moreover, system state approximation based on fuzzy clustering can be employed to realize nodes with similar system states in the system MSDD for improving the algorithm performance without the decrease of result accuracy [45]. Based on the system MSDD, simulation methods can also be utilized to calculate the occurrence probabilities of the paths with a larger number of state transitions where the multiple integration could not be directly calculated by numerical integration.

4. Numerical studies

4.1. Case 1: exponential distributions

In this case, the power system illustrated in Fig. 2 is analyzed where the state transition time distributions for generating units in different states follow exponential distributions. The state transition rates of units in different modes are presented in Table 2 [36].

When the transmission line is in a state with the capacity 100 MW, there is always sufficient transmission capacity; if the transmission line is in a state with capacity 50 MW, sometimes the generation redistribution is restricted by the capacity of the transmission line; when the transmission line is in a complete failure state, no generation will be shared among different subsystems. There are 75, 69, 41 paths leading to a reliable system in the system MSDD, respectively, when the transmission line is in different states with 100, 50, 0 MW.

Table 2
The state transition rates of generating units in different modes.

Generating unit	Online mode (/day)		Warm standby mode (/day)	
State transition	1 → 2	2 → 3	1 → 2	2 → 3
$A_{1,1}$	1/120	1/100	–	–
$A_{1,2}$	1/150	1/120	–	–
$A_{2,1}$	1/150	1/100	–	–
$A_{2,2}$	1/150	1/100	1/300	1/200

Moreover, to validate the effectiveness of the proposed MSDD-based method, Monte Carlo simulation (MCS) with 100,000 samplings was conducted. Computer programs for both techniques were developed in Wolfram Mathematica 12 and implemented on a laptop with a 1.80 GHz processor. To make a clear illustration of the impacts of different transmission capacities on system reliability, Fig. 8 presents the comparisons of system reliability with different transmission capacities and methods where the successful activation probability of the warm standby unit is 1. Table 3 lists the comparison of computation time for the proposed method and MCS technique.

It can be seen from Fig. 8 and Table 3 that the proposed MSDD algorithm provides correct results and has a great advantage in computation time compared with MCS technique. In addition, the system reliability is higher when the transmission line is in a state with a larger capacity. When the capacity of the transmission line exceeds the threshold with the maximal capacity that should be transmitted, even if the transmission line capacity is unlimited, the system reliability would not be improved anymore. In this case, the threshold for the maximal capacity that should be transmitted is 80 MW. That is to say, the system reliability remains the same when the transmission capacity is not less than 80 MW.

Furthermore, in Case 1, to explore the multi-state characteristics of the transmission line, it is modeled as a three-state model. Different scenarios of power systems with different state probabilities of the transmission line and successful activation probabilities of warm standby units are presented in Table 4.

Fig. 9 illustrates the time-dependent system reliability of different scenarios in Case 1. Compared Scenario A with Scenario B where the systems are with different state probabilities of transmission capacity, the system is more reliable with larger state probabilities for the rated transmission capacity. This is because with larger transmission capacity, power deficiency has more possibilities to be balanced. Moreover,

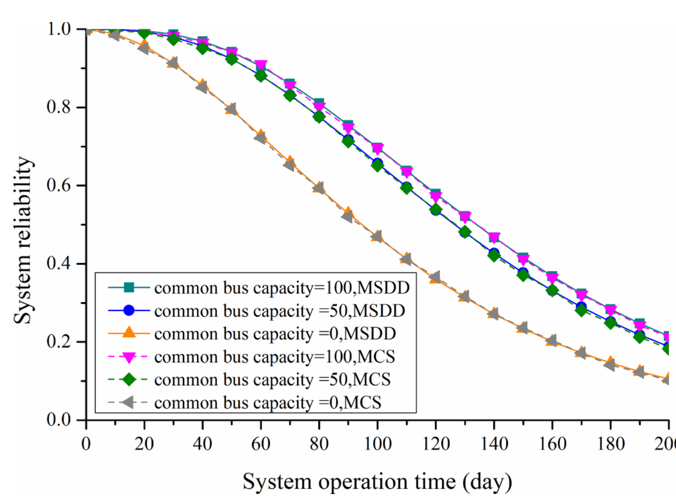

Fig. 8. System reliability of the power systems with different transmission line capacities and methods.

Table 3
Computation time of the power systems with different transmission line capacities and methods.

Capacity of the transmission line (MW)	100	50	0	100	50	0
Method	MSDD			MCS		
Computation time(s)	5.92	5.43	2.54	152.39	151.84	151.27

Table 4
Scenarios of power systems with different transmission state probabilities and successful activation probabilities.

Scenario		A	B	C	D
Transmission capacity (MW)	100	0.4	0.8	0.4	0.4
	50	0.3	0.1	0.3	0.3
	0	0.3	0.1	0.3	0.3
Successful start probability		1	1	0.9	0.8

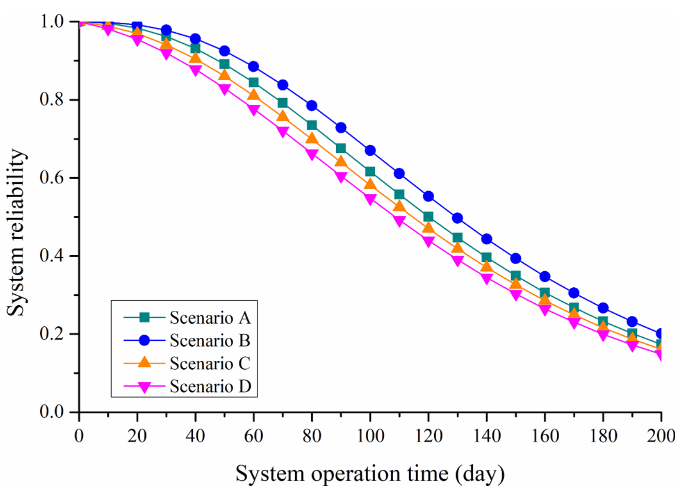

Fig. 9. System reliability of different scenarios in Case 1.

comparing Scenarios A, C and D, the reliability of a system with a smaller successful activation probability is lower than that of a system with a larger successful activation probability.

4.2. Case 2: Weibull distributions

To validate the applicability of the proposed MSDD method in power systems with non-exponential state transition time distributions, a power system with Weibull distributions is investigated considering the extensive application of Weibull distributions in reliability engineering. Table 5 presents parameters of the Weibull distributions for the state transition time distributions of generating units [36]. In Case

Table 5
The parameters of the Weibull distributions for state transition times in Case 2.

Unit	State transition	Online mode(/day)		Warm standby mode(/day)	
		Scale parameter	Shape parameter	Scale parameter	Shape parameter
$A_{1,1}$	1 → 2	300	1.5	–	–
	2 → 3	150	1.5	–	–
$A_{1,2}$	1 → 2	200	2	–	–
	2 → 3	200	2	–	–
$A_{2,1}$	1 → 2	300	1.5	–	–
	2 → 3	200	1.5	–	–
$A_{2,2}$	1 → 2	200	2	300	2
	2 → 3	150	2	200	2

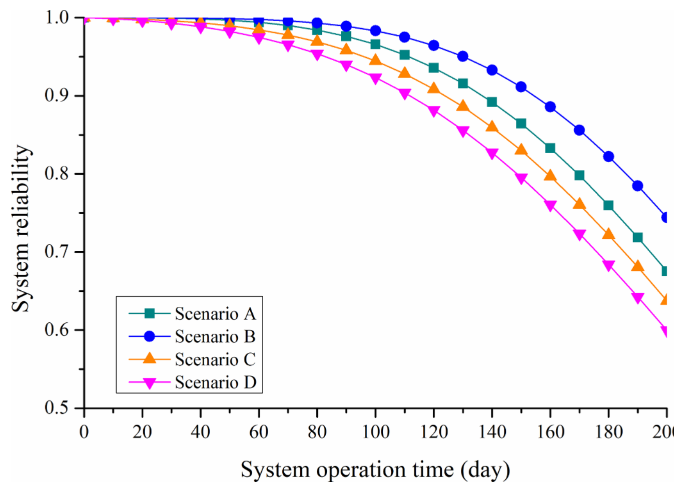

Fig. 10. System reliability of different scenarios in Case 2.

2, four scenarios with different transmission state probabilities and successful start probabilities are utilized as presented in Table 4.

The MSDD construction in Case 2 as well as the number of reliable paths leading to the system success is the same as that of Case 1. The differences lie in the occurrence probability calculation for different paths based on the multiple integration. The time-dependent reliability of the power system with Weibull distributions is illustrated in Fig. 10. The results keep consistency with those in Case 1 where distinct transmission state probabilities and successful activation probabilities are embedded.

Moreover, to compare the computation time of the proposed MSDD method applied to exponential distributions and Weibull distributions, the average computation times for a scenario in Case 1 and Case 2 are 13.89 s and 19.45 s, respectively. It can be seen that different multiple integrations do impact the computation time where the computation time for the case with Weibull distributions is greater than that of the case with exponential distributions because of the complicated integration for the system with Weibull distributions.

4.3. Case 3: A power system with 3 subsystems

A power system consisting of 3 subsystems is analyzed in this case, where the corresponding parameters of two subsystems are the same with those in Case 1. Three 50 MW binary-state generating units are installed in the third subsystem, where the state transition rates are 1/

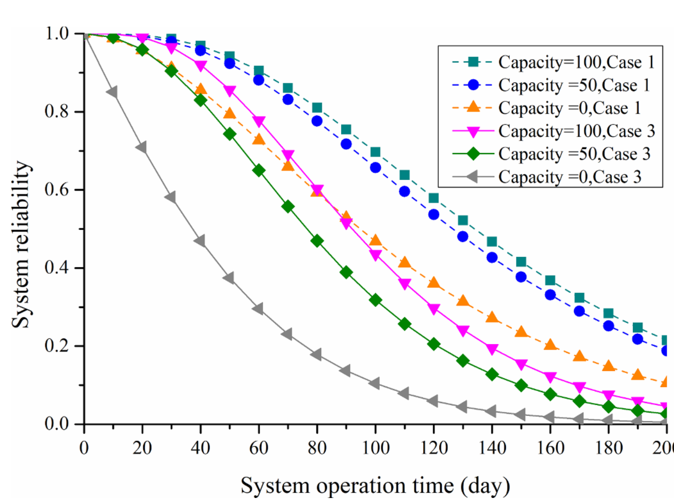

Fig. 11. System reliability when the transmission capacity is with different states in Case 1 and Case 3.

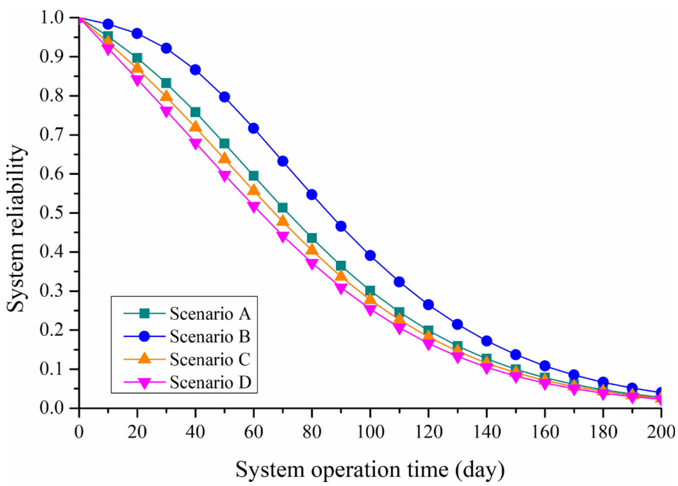

Fig. 12. System reliability of a power system with 3 subsystems in Case 3.

200 (/day) of the units. The maximal load of the third subsystem is 140 MW. Capacities and probabilities of transmission lines in this case is the same as Case 1, where three states are presented.

When the capacities of the transmission lines are with 100, 50 and 0 (MW), there are 336, 209, and 41 paths leading to a reliable system in the MSDDs, respectively. For comparison, assuming the warm standby units are perfectly activated, when the transmission lines are in different states, system reliability with 2 subsystems in Case 1 and 3 subsystems in Case 3 is respectively illustrated in Fig. 11. Comparing the system reliability of Case 1 and Case 3 with the same transmission capacity, it can be observed that with one more subsystem, the system reliability of Case 3 is lower than that of Case 1. Since generating units in the third subsystem are initially in online modes and only provide 10 MW surplus power, the failure of the third subsystem aggravates the failure of the entire power system.

Moreover, in this case, the same scenarios in Case 1 with different transmission state probabilities and successful start probabilities are utilized as presented in Table 4. The time-varying reliability of the power system with 3 subsystems is illustrated in Fig. 12. The results in Case 3 are consistent with those in Case 1 and Case 2, where a greater probability of a transmission state with a larger capacity and less start failure probability of warm standby units lead to a more reliable power system.

5. Conclusions

This paper considers the modeling of power systems where subsystems are configured with multi-state warm standby components and the performance can be sharing among different subsystems through multi-state transmission lines. The successful activation probabilities of warm standby units are embedded in the proposed model. Furthermore, the MSDD-based method is developed to evaluate the time-dependent reliability of the proposed systems, which allows systems with arbitrary state transition time distributions including the commonly used exponential distributions. Illustrative examples are provided to validate the effectiveness of proposed model and technique. The proposed MSDD method provides correct results and has great advantage in computation time compared with MCS technique.

In future research, systems with repairable generating units, optimal order for activating warm standby units, or phased-mission systems can be studied. Moreover, further research can also be devoted to improve the performance of the MSDD algorithm, for example, system state approximation using fuzzy clustering without reducing accuracy, hierarchical modeling, or simulation for the multiple integration where numerical integration cannot be solved.

CRediT authorship contribution statement

Heping Jia: Software, Validation, Writing - original draft. **Dunnan Liu:** Visualization, Investigation. **Yanbin Li:** Supervision. **Yi Ding:** Writing - review & editing. **Mingguang Liu:** Formal analysis. **Rui Peng:** Methodology, Writing - review & editing.

Declaration of Competing Interest

None.

Acknowledgements

This work was partly supported by China Postdoctoral Science Foundation (2020M670250, 2019TQ0090), the Fundamental Research Funds for the Central Universities (2020MS049), and the National Social Science Foundation of China (19ZDA081).

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