300 Solved Problems

Soil / Rock Mechanics
and
Foundations Engineering

These notes are provided to you by Professor Prieto-Portar, and in exchange, he will be grateful for your comments on improvements.

All problems are graded according to difficulty as follows:

*   Easy; defines general principles; typical of the PE examination;
**   Slightly more difficult; typical of Master's level problems;
*** Professional level ("real-life") problems.

by

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Conversion of Units

Multiplication Factor	Prefix	SI Symbol
1 000 000 000	giga	G
1 000 000	mega	M
1 000	kilo	k
0.001	milli	m
0.000 001	micro	μ
0.000 000 001	Nano	n

Base SI Units

Quantity	Unit	Symbol
length	meter	m
mass	kilograms (mass)	kg_m
force	Newton	N
time	second	s

Derived SI Units

Quantity	Derived SI Unit	Name	Symbol
area	square meter		m^2
volume	cubic meter		m^3
density	kilogram per cubic meter		kg_m/m^3
force	kilogram-meter per square second	Newtons	N
moment of force	Newton-meter		N-m
pressure	Newton per square meter	Pascal	Pa
stress	Newton per square meter	Pascal	Pa or N/m^2
work, energy	Newton-meter	joule	J
power	joule per second	watt	W

Conversion of SI Units to English Units

Lengths

Multiply by   *To* ⟶

From	inches	feet	yards	miles
mm	3.94×10^{-2}	3.28×10^{-3}	1.09×10^{-3}	6.22×10^{-7}
cm	3.94×10^{-1}	3.28×10^{-2}	1.09×10^{-2}	6.22×10^{-6}
m	3.94×10^{1}	3.28	1.09	6.22×10^{-4}
km	3.94×10^{4}	3.28×10^{3}	1.09×10^{3}	6.22×10^{-1}

$1\ \mu m = 1 \times 10^{-6}\ m$

$1\ Å = 1 \times 10^{-10}\ m = 3.28 \times 10^{-10}\ feet$

Area

Multiply by   *To* ⟶

From	square inches	square feet	square yards	square miles
mm²	1.55×10^{-3}	1.08×10^{-5}	1.20×10^{-6}	3.86×10^{-13}
cm²	1.55×10^{-1}	1.08×10^{-3}	1.20×10^{-4}	3.86×10^{-11}
m²	1.55×10^{3}	1.08×10^{1}	1.20	3.86×10^{-7}
km²	1.55×10^{9}	1.08×10^{7}	1.20×10^{6}	3.86×10^{-1}

$1\ acre = 43{,}450\ ft^2 = 4{,}047\ m^2 = 0.4047\ hectares$

Volume

Multiply by   *To* ⟶

From	cubic inches	cubic feet	cubic yards	quarts	gallons
cm³	6.10×10^{-2}	3.53×10^{-5}	1.31×10^{-6}	1.06×10^{-3}	2.64×10^{-4}
liter	6.10×10^{1}	3.53×10^{-2}	1.31×10^{-3}	1.06	2.64×10^{-1}
m³	6.10×10^{4}	3.53×10^{1}	1.31	1.06×10^{3}	2.64×10^{2}

Conversion of SI Units to English Units

Force

Multiply by / To → / From

From	ounces	pounds	kips	tons (short)
dynes	1.405×10^{-7}	2.248×10^{-6}	2.248×10^{-9}	1.124×10^{-9}
grams	3.527×10^{-2}	2.205×10^{-3}	2.205×10^{-6}	1.102×10^{-6}
kilograms	3.527×10^{1}	2.205	2.205×10^{-3}	1.102×10^{-3}
Newtons	3.597	2.248×10^{-1}	2.248×10^{-4}	1.124×10^{-4}
kilo-Newtons	3.597×10^{3}	2.248×10^{2}	2.248×10^{-1}	1.124×10^{-1}
tons (metric)	3.527×10^{4}	2.205×10^{3}	2.205	1.102

Pressure (or stress) σ

Multiply by / To → / From

From	lb/in^2	lb/ft^2	$kips/ft^2$	tons (short)/ft^2	feet of water	atmosphere
gm/cm^2	1.422×10^{-2}	2.048	2.048×10^{-3}	1.024×10^{-3}	3.281×10^{-2}	9.678×10^{-4}
kg/cm^2	1.422×10^{1}	2.048×10^{3}	2.048	1.024	3.281×10^{1}	9.678×10^{-1}
kN/m^2	1.450×10^{-1}	2.090×10^{1}	2.088×10^{-2}	1.044×10^{-2}	3.346×10^{-1}	9.869×10^{-3}
ton (metric)/m^2	1.422	2.048×10^{2}	2.048×10^{-1}	1.024×10^{-1}	3.281	9.678×10^{-2}

Torque (or moment) T or M

Multiply by / To → / From

From	lb-in	lb-ft	kips-ft
gm-cm	8.677×10^{-4}	7.233×10^{-5}	7.233×10^{-8}
kg-m	8.677	7.233	7.233×10^{-3}
kN-m	9.195×10^{3}	7.663×10^{2}	7.663×10^{-1}

Conversion of SI Units to English Units

Velocity v

Multiply by To → From	ft / s	ft / min	mi / h
cm / s	3.281×10^{-2}	1.9685	2.236×10^{-2}
km / min	5.467×10^{1}	3.281×10^{3}	3.728×10^{1}
km / h	9.116×10^{-1}	5.467×10^{1}	6.214×10^{-1}

1 mile = 1,610 meters = 5,282.152 feet

Unit weight γ

Multiply by To → From	lb / in³	lb / ft³
gm / cm³	3.613×10^{-2}	6.248×10^{1}
kg / m³	3.613×10^{-5}	6.248×10^{-2}
kN / m³	3.685×10^{-3}	6.368×10^{1}
tons (metric) / m³	3.613×10^{-2}	6.428×10^{1}

Power P

1 W = 1 J/sec = 1.1622 cal/hr = 3.41 Btu/hr = 0.0013 hp

1 hp = 745.7 W = 0.7457 kW

1 kW = 1.34 hp

Chapter 1
Soil Exploration

$C_B \rightarrow STP$ correction factor for the boreholes diameter.

$C_R \rightarrow STP$ correction factor for the rod length.

$C_S \rightarrow STP$ correction factor for the sampler type used.

$c_u \rightarrow$ Soil's un-drained cohesion.

$D_f \rightarrow$ Depth of the foundation's invert.

$E_m \rightarrow$ The efficiency of the STP hammer.

$N \rightarrow$ The "raw" value of the STP (as obtained in the field).

$p_o \rightarrow$ The original vertical stress at a point of interest in the soil mass.

$S \rightarrow$ The number of stories of a building.

$SPT \rightarrow$ Stands for *"Standard Penetration Test"*.

$N_{60} \rightarrow$ Corrected STP assuming 60% efficiency in the field.

$N_{70} \rightarrow$ Corrected STP assuming 70% efficiency in the field.

$\mu \rightarrow$ Correction factor for the shear vane test using the clay's Plasticity Index *PI*.

Find the required number of borings and their depth.

(Revised: Sept. 08)

A four story reinforced concrete frame office building will be built on a site where the soils are expected to be of average quality and uniformity. The building will have a 30 m x 40 m footprint and is expected to be supported on spread footing foundations located about 1 m below the ground surface. The site appears to be in its natural condition, with no evidence of previous grading. Bedrock is 30-m below the ground surface. Determine the required number and depth of the borings.

Solution:

A reinforced concrete building is heavier than a steel framed building of the same size. Hence, the design engineer will want soil conditions that are at least average or better. From Table-1 below, one boring will be needed for every 300 m^2 of footprint area. Since the total footprint area is 30 m x 40 m =1,200 m^2, use *four borings*.

Table-2 provides the minimum depth required for the borings, $5 \ S^{0.7} + D = 5(4)^{0.7} + 1 = $ *14 m*. Most design engineers want one boring to go to a slightly greater depth to check the next lower stratum's strength.

In summary, the exploration plan will be 4 borings to a depth of 14 m.

Table-1 - Spacing of the exploratory borings for buildings on shallow foundations.

Subsurface Conditions	Structural footprint Area for Each Boring	
	(m^2)	(ft^2)
Poor quality and / or erratic	200	2,000
Average	300	3,000
High quality and uniform	600	6,000

Table-2 - Depths of exploratory borings for buildings on shallow foundations.

Subsurface Conditions	Minimum Depth of Borings	
	(S = number of stories and D = the anticipated depth of the foundation)	
	(m)	(ft)
Poor and / or erratic	$6S^{0.7} + D$	$20S^{0.7} + D$
Average	$5S^{0.7} + D$	$15S^{0.7} + D$
High quality and uniform	$3S^{0.7} + D$	$10S^{0.7} + D$

(Revised: Sept. 08)

The most common soil and soft rock sampling tool in the US is the Standard Split Spoon.

Split spoon tubes split longitudinally into halves and permit taking a soil or soft rock sample. The tube size is designated as an NX. The NX outside diameter is Do = 50.8 mm (2 inches) and its inside diameter is Di = 34.9 mm (1-3/8 inches). This small size has the advantage of cheapness, because it is relatively easy to drive into the ground. However, it has the disadvantage of disturbing the natural texture of the soil. In soft rocks, such as young limestone, it will destroy the rock to such a degree that it may be classified as a "sand".

A better sampler is the Shelby (or thin-tube sampler). It has the same outside diameter of 2 inches (although the trend it to use 3 inches).

Compare the degree of sample disturbance of a US standard split-spoon sampler, versus the two Shelby thin-tube samplers (2" and 3" outside diameters) via their <u>*area ratio A_r*</u> (a measure of sample disturbance).

Solution:

The area ratio for a 2"-standard split-spoon sampler is,

$$A_r(\%) = \frac{D_o^2 - D_i^2}{D_i^2}(100) = \frac{(2.0)^2 - (1.38)^2}{(1.38)^2}(100) = 110\%$$

The area ratio for a 2"-Shelby-tube sampler is,

$$A_r(\%) = \frac{D_o^2 - D_i^2}{D_i^2}(100) = \frac{(2.0)^2 - (1.875)^2}{(1.875)^2}(100) = 13.8\%$$

The area ratio for a 3"-Shelby-tube sampler is,

$$A_r(\%) = \frac{D_o^2 - D_i^2}{D_i^2}(100) = \frac{(3.0)^2 - (2.875)^2}{(2.875)^2}(100) = 8.9\%$$

Clearly, the 3" O-D Shelby-tube sampler is the best tool to use.

(Revision Sept-08)

A standard penetration test (SPT) has been conducted in a loose coarse sand stratum to a depth of 16 ft below the ground surface. The blow counts obtained in the field were as follows: 0 – 6 in = 4 blows; 6 -12 in = 6 blows; 12 -18 in = 8 blows. The tests were conducted using a US-style donut hammer in a 6 inch diameter boring with a standard sampler and liner. The effective unit weight of the loose sand stratum is about 93.8 pcf.

Determine the corrected SPT if the testing procedure is assumed to only be 60% efficient.

Solution:

The raw SPT value is $N = 6 + 8 = 14$ (that is, only the last two sets of 6" penetrations).

The US-style donut hammer efficiency is $E_m = 0.45$, and the other parameters are obtained from the Tables provided on the next page: $C_B = 1.05$, $C_S = 1.00$, $C_R = 0.85$.

With these values, the SPT corrected to 60% efficiency can use Skempton's relation,

$$N_{60} = \frac{E_m C_B C_S C_R N}{0.60} = \frac{(0.45)(1.05)(1.00)(0.85)(14)}{0.60} = 9$$

Notice that the SPT value is always given as a whole number.

That corrected SPT N_{60} is then corrected for depth. For example, using the Liao and Whitman method (1986),

$$(N)_{60} = N_{60}\sqrt{\frac{2,000\ lb/ft^2}{(depth)(effective\ unit\ weight)}} = (9)\sqrt{\frac{2,000\ lb/ft^2}{(16\ ft)(93.8\ pcf)}} = 10$$

Other methods for corrections are discussed in Exploration-04.

SPT Hammer Efficiencies (adapted from Clayton, 1990).			
Country	Hammer Type	Release Mechanism	Hammer Efficiency
Argentina	donut	cathead	0.45
Brazil	pin weight	hand dropped	0.72
China	automatic	trip	0.60
	donut	hand dropped	0.55
	donut	cat-head	0.50
Colombia	donut	cat-head	0.50
Japan	donut	Tombi trigger	0.78 - 0.85
	donut	cat-head + sp. release	0.65 - 0.67
UK	automatic	trip	0.73
US	safety	2-turns on cat-head	0.55 - 0.60
	donut	2-turns on cat-head	0.45
Venezuela	donut	cat-head	0.43

Correction Factors for the Boring Diameter, Sampling Method and Boring Rod Length (adapted from Skempton, 1986).		
Correction Factor	Equipment Variables	Value
Borehole diameter factor C_B	65 – 115 mm (2.5 – 4.5 in)	1.00
	150 mm (6 in)	1.05
	200 mm (8 in)	1.15
Sampling method factor C_S	Standard sampler	1.00
	Sampler without liner (not recommended)	1.20
Rod length factor, C_R	3 – 4 m (10 – 13 ft)	0.75
	4 – 6 m (13 – 20 ft)	0.85
	6 – 10 (20 – 30 ft)	0.95
	>10 m (>30 ft)	1.00

(Revision Sept.-08)

A raw value of $N = 40$ was obtained from an *SPT* at a depth of 20 feet in a sand stratum that has a unit weight of 135 lb/ft^3. Correct it only for depth.

Solution:

Any of these three methods will provide acceptable answers. Notice how similar their results are from each other:

1. Using the Bazaraa Method (1967):

$$N_{corrected} = \frac{4N'}{1 + 2p_o} \quad if \ p_0 \le 1.5 \ kips / ft^2 \quad and$$

$$N_{corrected} = \frac{4N'}{3.25 + 0.5p_o} \quad if \ p_0 \ge 1.5 \ kips / ft^2$$

$$but \quad p_0 = \frac{(20 \ ft)(135 \ lb / ft^3)}{1000 \ lb / kip} = 2.70 \ kips / ft^2 > 1.5 kips / ft^2$$

$$therefore \ N_{corrected} = \frac{4N'}{3.25 + 0.5p_o} = \frac{4(40)}{3.25 + 0.5(2.70 \ kips / ft^2)} = 35$$

2. Using the Peck Method (1974):

$$N_{corrected} = N'C_N \quad where \ C_N = 0.77 \log_{10} \frac{20}{p_0} \ if \ p_0 \ is \ in \ tons / ft^2$$

$$or \quad C_N = 0.77 \log_{10} \frac{1915}{p_0} \ if \ p_0 \ is \ in \ kN / m^2$$

$$but \quad p_0 = \frac{(20 \ ft)(135 \ lb / ft^3)}{2000 \ lb / ton} = 1.35 \ tons / ft^2 = 2.70 \ kips / ft^2$$

$$\therefore \ C_N = 0.77 \log_{10} \frac{20}{1.35 \ tons / ft^2} = 0.90 \quad \therefore N_{corrected} = (40)(0.90) = 36$$

3. The Liao-Whitman Method (1986), as used in *Exploration-03*,

$$N_{corrected} = N' \sqrt{\frac{100}{p_o}} \ with \ p_o \ in \ kN / m^2 \quad or \quad = N' \sqrt{\frac{2,000 \ psf}{p_o}} \ with \ p_o \ in \ psf$$

$$but \quad p_o = (1.35 \ tons / ft^2) \left(\frac{96.1 \ kN / m^2}{1 \ ton / ft^2} \right) = 129.7 \ kN / m^2$$
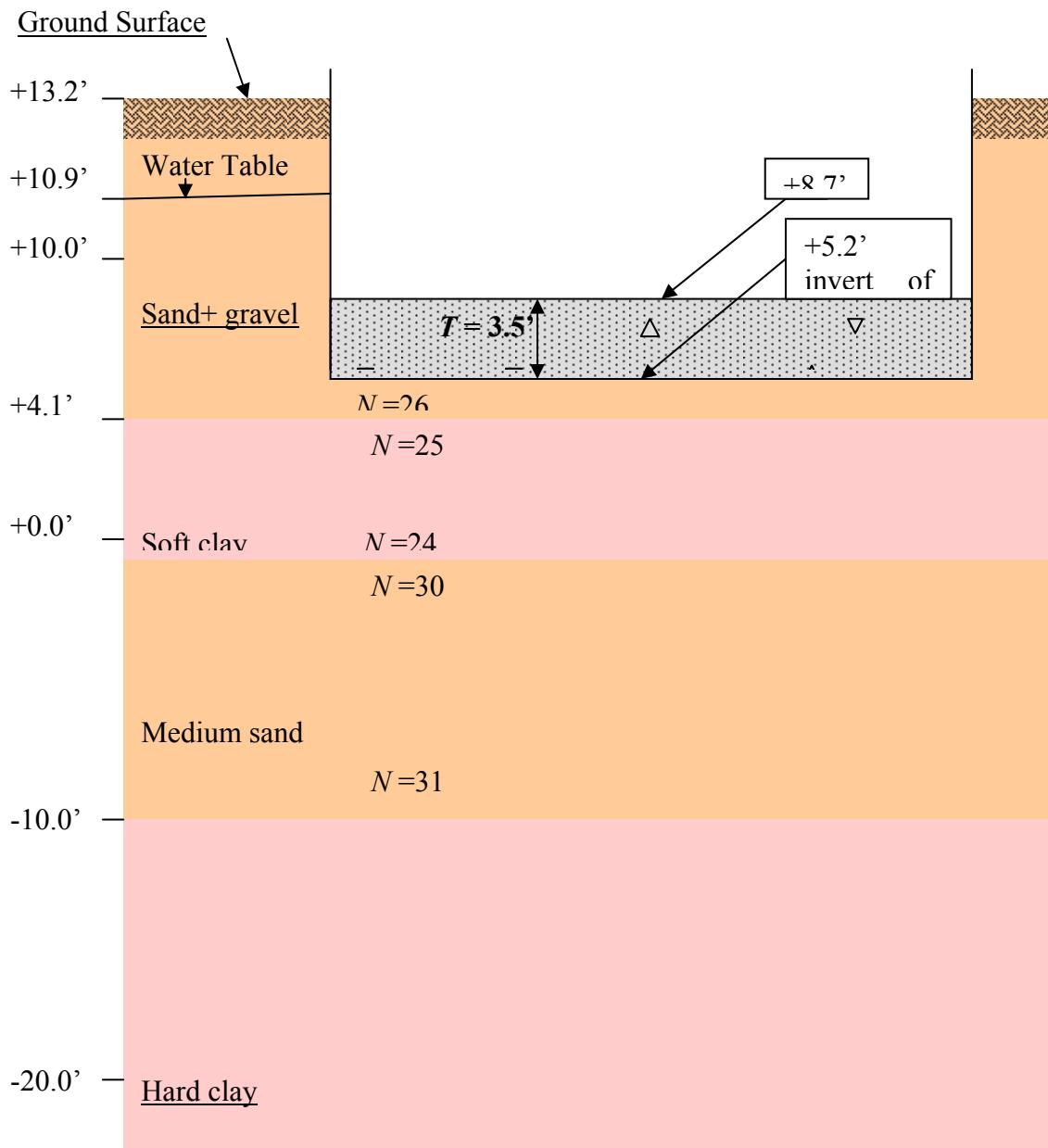
$$\therefore \ N_{corrected} = 40 \sqrt{\frac{100 \ kN / m^2}{129.7 \ kN / m^2}} = 35$$

Exploration–05. SPT corrections under a mat foundation.

(Revision Sept.-08)

Correct the *SPT* values shown below for an energy ratio of 60% using a high-efficient US-type donut hammer in a 2"-diameter boring. The invert (bottom) of the mat foundation is at elevation +5.2 feet.

+20'

Ground Surface

+13.2'

Water Table

+10.9'

+10.0'

Sand+ gravel

+8.7'

+5.2'
invert of

T = 3.5'

△   ▽

+4.1'

N =26

N =25

+0.0'

Soft clay   N =24

N =30

Medium sand

N =31

-10.0'

-20.0'   Hard clay

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Skempton proposed in 1986 the following correction for the sampling methods to the raw *SPT* value, assuming that only 60% of the energy of the hammer drives the sampler, $N_{60} = \dfrac{E_m C_B C_S C_R N}{0.60}$

where: N_{60} = SPT *N*-value corrected for field procedures assuming 60% efficiency

$E_m = 0.60 \rightarrow$ efficiency for a high-efficiency US-style safety hammer

$C_B = 1.00 \rightarrow$ borehole diameter correction

$C_S = 1.00 \rightarrow$ sampler correction, $= 0.75$ (10'-13')

$C_R = 0.85$ (13'-20') $\rightarrow$ rod length correction, $= 0.95$ (20'-30'), $= 1.0$ (>30')

$N = $ ***SPT***-value recorded in the field by the driller (known as the "raw" ***SPT***).

The depth correction is,

$$(N_1)_{60} = N_{60} \sqrt{\frac{2,000 \; lb/ft^2}{(depth)(effective\,unit\,weight)}}$$

At depth of +5.2 feet:

$$N_{60} = \frac{(0.60)(1)(1)(0.75)(26)}{0.60} = 20 \;\; and \;\; (N)_{60} = (20)\sqrt{\frac{2,000\,lb/ft^2}{(8\,ft)(127-62.4\,pcf)}} = 39$$

At +4.1'
$$N_{60} = \frac{(0.60)(1)(1)(0.75)(25)}{0.60} = 19 \; and \; (N)_{60} = (19)\sqrt{\frac{2,000\,lb/ft^2}{(9\,ft)(127-62.4\,pcf)}} = 35$$

At +2.0'
$$N_{60} = \frac{(0.60)(1)(1)(0.75)(24)}{0.60} = 18 \; and \; (N)_{60} = (18)\sqrt{\frac{2,000\,lb/ft^2}{(11\,ft)(125-62.4\,pcf)}} = 31$$

At -1.0'
$$N_{60} = \frac{(0.60)(1)(1)(0.85)(30)}{0.60} = 26 \; and \; (N)_{60} = (26)\sqrt{\frac{2,000\,lb/ft^2}{(14\,ft)(126-62.4\,pcf)}} = 39$$

At -5.0'
$$N_{60} = \frac{(0.60)(1)(1)(0.85)(31)}{0.60} = 26 \; and \; (N)_{60} = (26)\sqrt{\frac{2,000\,lb/ft^2}{(18\,ft)(126-62.4\,pcf)}} = 34$$

At -10'
$$N_{60} = \frac{(0.60)(1)(1)(0.95)(30)}{0.60} = 29 \; and \; (N)_{60} = (29)\sqrt{\frac{2,000\,lb/ft^2}{(23\,ft)(126-62.4\,pcf)}} = 34$$
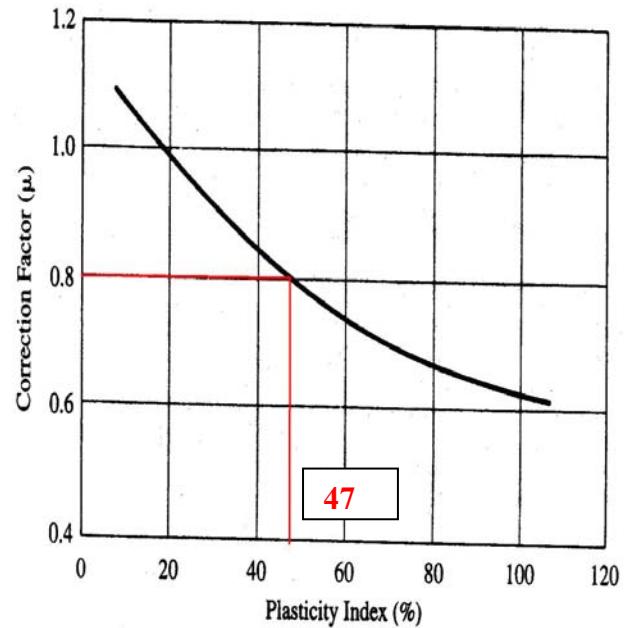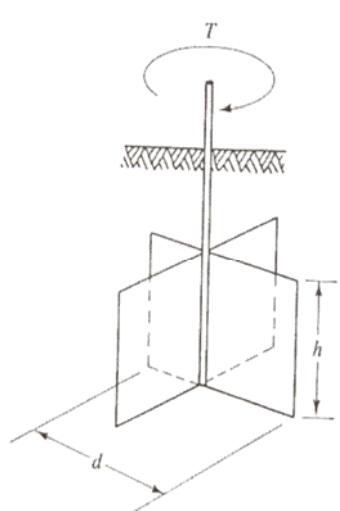
At -21'
$$N_{60} = \frac{(0.60)(1)(1)(1)(43)}{0.60} = 43 \; and \; (N)_{60} = (43)\sqrt{\frac{2,000\,lb/ft^2}{(33\,ft)(130-62.4\,pcf)}} = 41$$

Notice that the depth correction does not affect the deeper layers.

Exploration–06. **The Shear Vane Test determines the in-situ cohesion.**

(Revision Sept.-08)

A shear vane tester is used to determine an approximate value of the shear strength of clay. The tester has a blade diameter d = 3.625 inches and a blade height h = 7.25 inches. In a field test, the vane required a torque of 17.0 *ft-lb* to shear the clay sample, which has a plasticity index of 47% ($PI = LL - PL$). Determine the un-drained cohesion c_u corrected for its plasticity.

$$c_u = \frac{T}{\pi\left[(d^2 h/2)+(d^3/6)\right]} = \frac{17.0\ ft\cdot lb}{\pi\left[\dfrac{(0.3021\ ft)^2(0.6042\ ft)}{2}+\dfrac{(0.3021\ ft)^3}{6}\right]} = 168\ psf$$

The plasticity index helps correct the raw shear vane test value (Bjerrum, 1974) through the graph shown above. For a plasticity index of 47% read a correction factor $\mu = 0.80$. Therefore,

$$c_{u-corrected} = \mu c_u = (0.80)(168\ psf) = 134\ psf$$

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(Revision Sept.-08)

Read the boring log shown below and determine, (1) the location of the phreatic surface, (2) the depth of the boring and (3) the number of samples taken.

Solution:

(1) The phreatic surface (the water table) was *not* encountered in this boring and is noted at the bottom of the report;

(2) The boring was terminated at 21 feet in depth; and

(3) Five samples were taken. Only one sample (#2) was used for laboratory tests (dry density and moisture content). Samples #1 and #3 were complete split-spoon samples. Samples #4 and #5 were incomplete split-spoon samples.

Depth (feet)	Sample Type	Sample Number	Blow Counts (blows/foot)	Graphic Log	SOIL DESCRIPTION AND CLASSIFICATION	Dry Density (pcf)	Moisture Content (%)
					SILTY SAND (SM): brown, moist, fine sand, trace coarse sand and fine gravel		
-780-		1	10		medium dense		
5		2	21		SAND (SP): brown, moist, medium dense, fine to medium sand	110	5.2
-775-							
10		3	42		olive-brown, dense, fine to coarse sand, some fine gravel		
-770-							
15		4	50/5"		SANDY GRAVEL (GP): gray, moist, very dense, fine to coarse sand, fine to coarse gravel to 3 inches		
-765-							
20		5	90		SAND with GRAVEL (SP): olive-brown, moist, very dense, fine to coarse sand, fine to coarse gravel to 2 inches		
					Boring terminated at 21.0 feet Groundwater not encountered Hole backfilled and tamped using soil from cuttings		

(Revision Sept.-08)

Using the boring log and the *SPT* versus *Soil Engineering Parameters Table* shown on the next two pages, answer these four questions:

(1) Correct the values of the *SPT* of Sample S-4 to a 70% sampling efficiency with a standard sampling method and a US-donut hammer at elevation – 17 feet;

(2) Correct the same sample S-4 for depth assuming the unit weight is γ = 126 pcf;

(3) What are your estimates for the angle of internal friction and unit weight γ?

(4) What is the elevation (above sea level) of the groundwater and the elevation of the bottom of the boring?

Solution:

(1) The log shows a value of $N = 15$ (Sample S-4) at elevation -16.5'; at elevation -17' it has dropped a small amount to $N = 14$. Notice that the "Legend" portion denotes that the sampler was a 2" O.D. split spoon. Therefore, the sampling correction is,

$$N_{70} = \frac{EC_B C_S C_R N}{0.70} = \frac{(0.45)(1.0)(1.0)(0.85)(14)}{0.70} \approx 8$$

(2) Correct the same sample S-4 for depth.

$$(N_{70}) = N_{70}\sqrt{\frac{2000\ psf}{(\gamma h)}} = (8)\sqrt{\frac{2000\ psf}{(126)(17)\ psf}} \approx 8$$
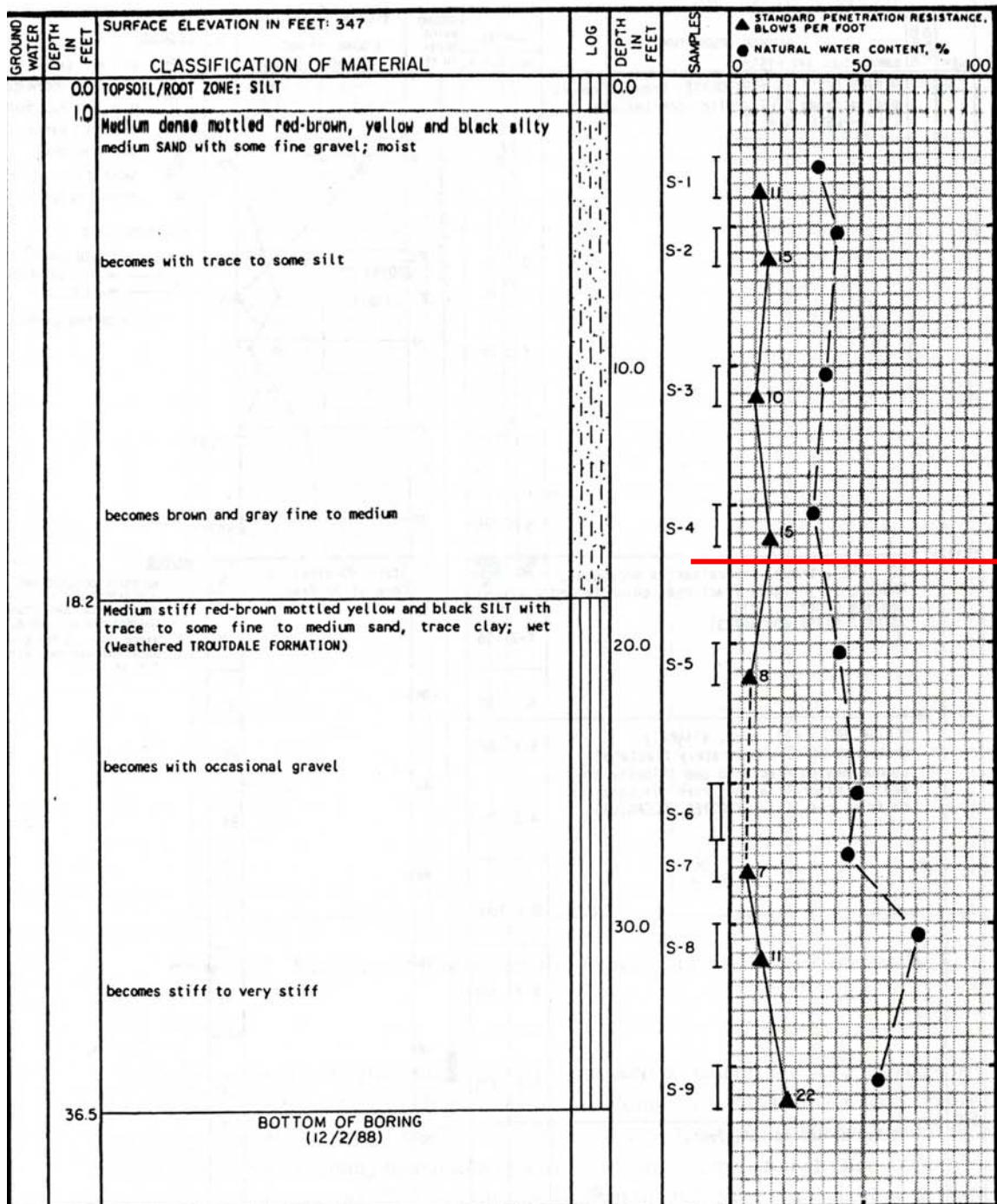
(3) What are your estimates for the angle of internal friction and unit weight γ?

The log identifies this level at -17' as a "brown and grey fine to medium SAND". Use the Table provided on page 23 to obtain an estimate of some of the engineering parameters for granular soils. Read the *SPT* for medium sands; then go to the Medium column and read the value of "$N = 8$" to obtain the values:

$\varphi = 32°$ and $\gamma_{wet} = 17\ kN/m^2$.

(4) What are the elevations (above sea level) of the groundwater and of the bottom of the boring?

- The boring did not report finding a ground water table.

- The bottom of the boring was at -36.5' from the surface, or 347.0' – 36.5' = +310.5'.

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GROUND WATER	DEPTH IN FEET	SURFACE ELEVATION IN FEET: 347 CLASSIFICATION OF MATERIAL	LOG	DEPTH IN FEET	SAMPLES
	0.0	TOPSOIL/ROOT ZONE: SILT		0.0	
	1.0	Medium dense mottled red-brown, yellow and black silty medium SAND with some fine gravel; moist			
					S-1
		becomes with trace to some silt			S-2
				10.0	S-3
		becomes brown and gray fine to medium			S-4
	18.2	Medium stiff red-brown mottled yellow and black SILT with trace to some fine to medium sand, trace clay; wet (Weathered TROUTDALE FORMATION)		20.0	S-5
		becomes with occasional gravel			S-6
					S-7
		becomes stiff to very stiff		30.0	S-8
					S-9
	36.5	BOTTOM OF BORING (12/2/88)			

STANDARD PENETRATION RESISTANCE, BLOWS PER FOOT ▲
NATURAL WATER CONTENT, % ●
0   50   100

S-1 ▲11
S-2 ▲15
S-3 ▲10
S-4 ▲15
S-5 ▲8
S-7 ▲7
S-8 ▲11
S-9 ▲22

0.0   0.5   1.0
SHEAR STRENGTH, T.S.F. (APPROX.)
■ TORVANE
◧ POCKET PENETROMETER

LOG
BORING B-3

DECEMBER 1988   88278
L.R. SQUIER ASSOCIATES, INC.
GEOTECHNICAL CONSULTANTS

Correlation between SPT values and some Engineering Parameters of Granular Soils						
Description		**Very loose**	**Loose**	**Medium**	**Dense**	**Very dense**
D_r	Relative density	0	0.15	0.35	0.65	0.85
SPT	fine	1 - 2	3 - 6	7 - 15	16 - 30	
(N'_{70})	medium	2 - 3	4 - 7	8 - 20	21 - 40	> 40
	coarse	3 - 6	5 - 9	10 - 25	26 - 45	
ϕ	fine	26 - 28	28 - 30	30 - 34	33 - 38	
	medium	27 - 28	30 - 32	32 - 36	36 - 42	< 50
	coarse	28 - 30	30 - 34	33 - 40	40 - 50	
γ_{wet}	pcf	70 - 102	89 - 115	108 - 128	108 -140	128 - 147
	kN/m^3	11 - 16	14 - 18	17 - 20	17 - 22	20 - 23

Note #1: These values are based on tests conducted at depths of about 6 m;
Note #2: Typical values of relative densities are about 0.3 to 0.7; values of 0 or 1.0 do not exist in nature;
Note #3: The value of the angle of internal friction is based on $\Phi = 28^o + 15^o D_r$;
Note #4: The typical value of an excavated soil ranges from 11 to 14 kN/m^3;
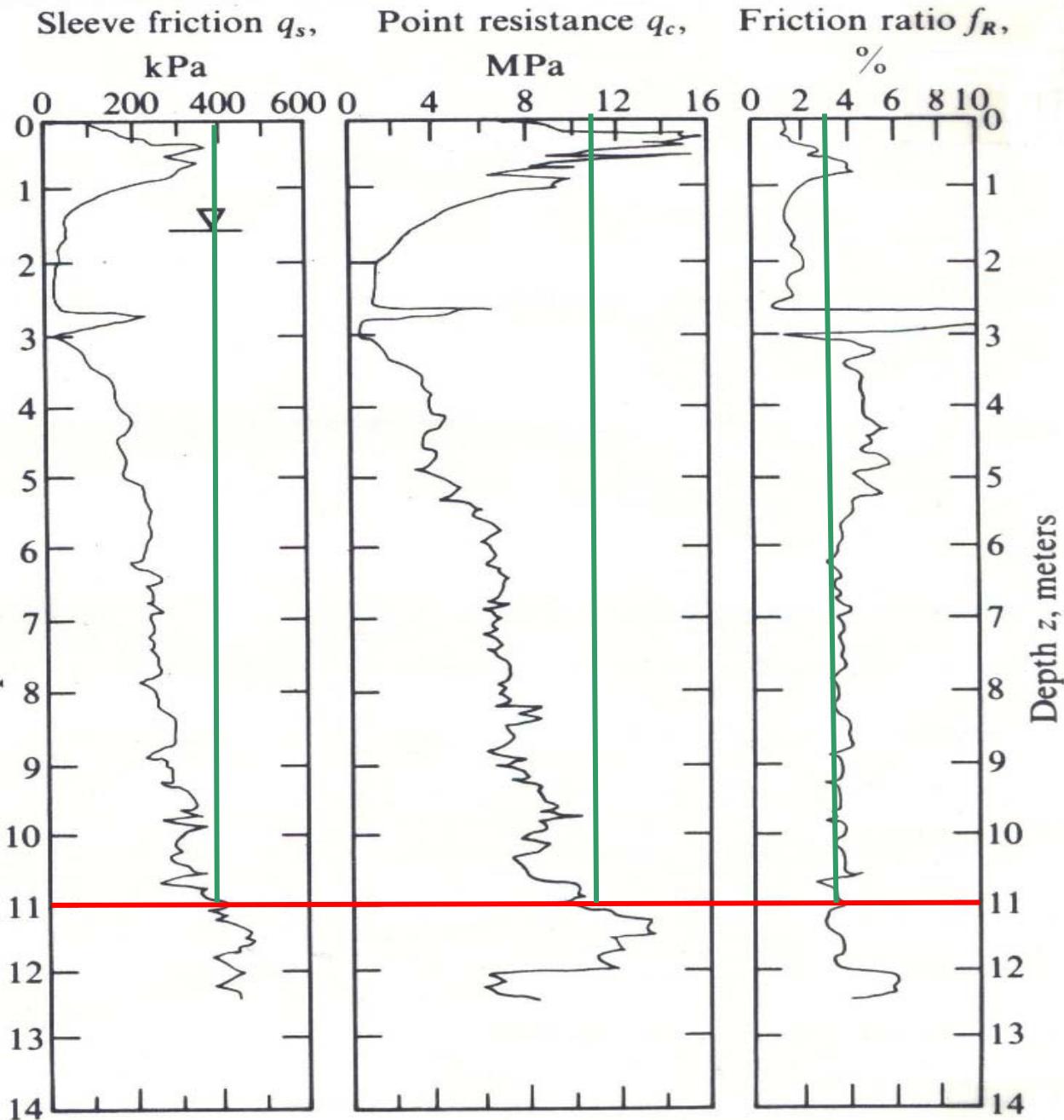
Correlation between SPT values and some Engineering Parameters of Cohesive Soils		
SPT - N_{70}	**Compressive Strength q_u**	**Description**
0 - 2	< 25 kPa	Very soft – squeezes between fingers Very young NC clay
3 - 5	25 - 50 kPa	Soft – easily deformed by fingers Young NC clay
6 - 9	50 - 100 kPa	Medium
10 - 16	100 - 200 kPa	Stiff – Hard to deform w/fingers Small OCR – aged clay
17 - 30	200 - 400 kPa	Very Stiff – Very hard w/fingers Increasing OCR – older clays
> 30	> 400 kPa	Hard – Does not deform w/fingers Higher OCR – cemented clays

(Revision: Sept.-08)
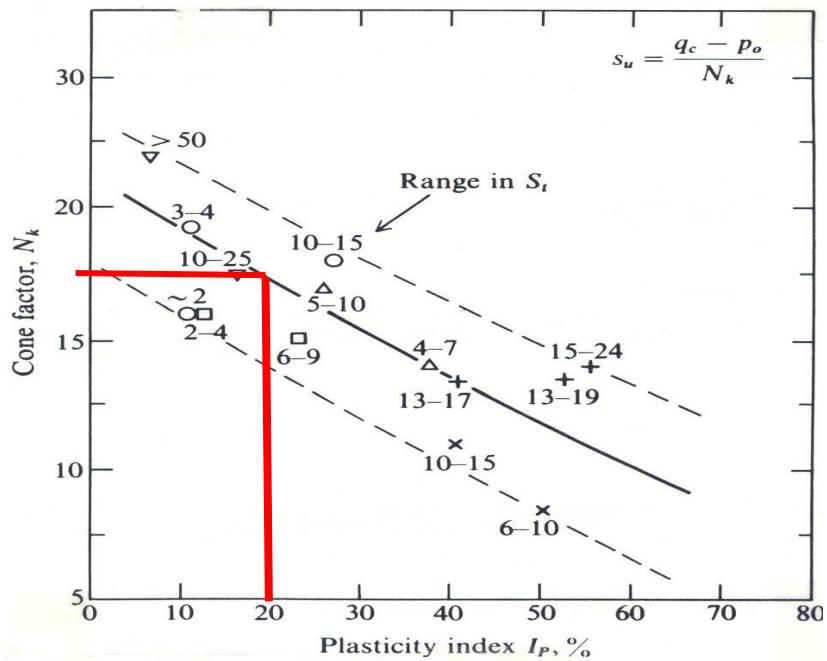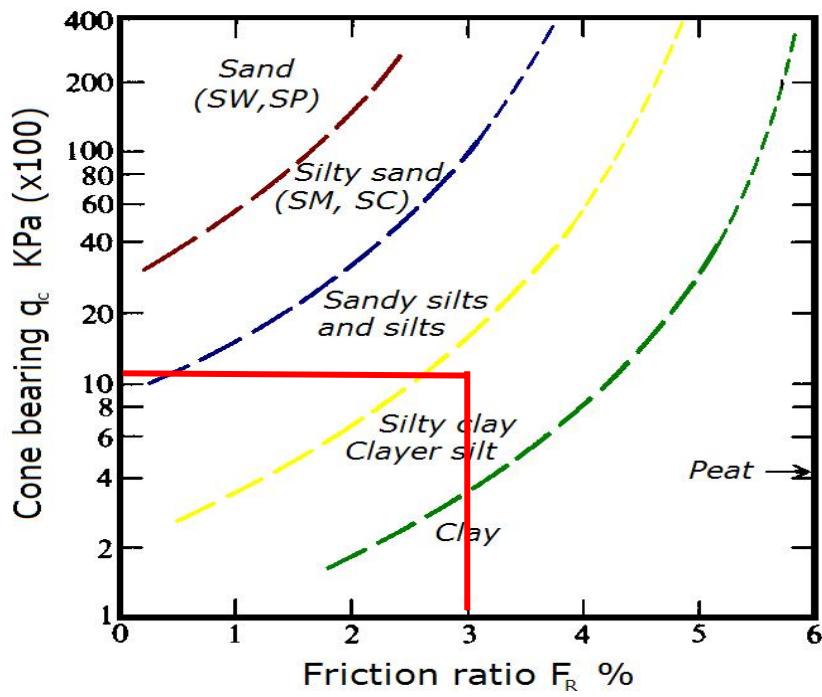
Classify a soil from the data provided by the Cone Penetration Test (CPT) shown below at a depth of 11 m. The clay samples recovered from that depth had $\gamma = 20$ kN/m^3 and $PI = I_p = 20$. Compare your estimate of the shear strength versus the lab test value of 550 kPa.

Solution.

Reading the data, $q_s \sim 400$ kPa and $q_c \sim 11$ MPa which results in a $f_R \sim 3\%$.

From the next chart, the soil appears to be a *silty clay*.

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At a depth of 11 m, the in-situ pressure p_o for a NC clay is,

$p_o = \gamma z = (20 \; kN/m^3)(11 \; m) = \; 220 \; kPa$

From the N_k versus I_p graph, for $I_p = 20$ yields an $N_k \sim 17.5$.

The un-drained shear strength s_u is,

$s_u = \dfrac{q_c - p_o}{N_k} = \dfrac{11,000 \; kPa - 220 \; kPa}{17.5} = 616 \; kPa \; versus \; lab = 550 \; kPa$ (a 12% error).

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Chapter 2
Phase Relations of Soil

$e \rightarrow$ *Voids ratio.*

$G_S \rightarrow$ *Specific gravity of the solids of a soil.*

$n \rightarrow$ *Porosity.*

$S \rightarrow$ *Degree of saturation.*

$V \rightarrow$ *Total volume (solids + water + air).*

$V_a \rightarrow$ *Volume of air.*

$V_V \rightarrow$ *Volume of voids (water + air).*

$V_S \rightarrow$ *Volume of solids.*

$V_W \rightarrow$ *Volume of water.*

$w \rightarrow$ *Water content (also known as the moisture content).*

$W_S \rightarrow$ *Weight of solids.*

$W_W \rightarrow$ *Weight of water.*

$g \rightarrow$ *Unit weight of the soil.*

$g_d \rightarrow$ *Dry unit weight of the soil.*
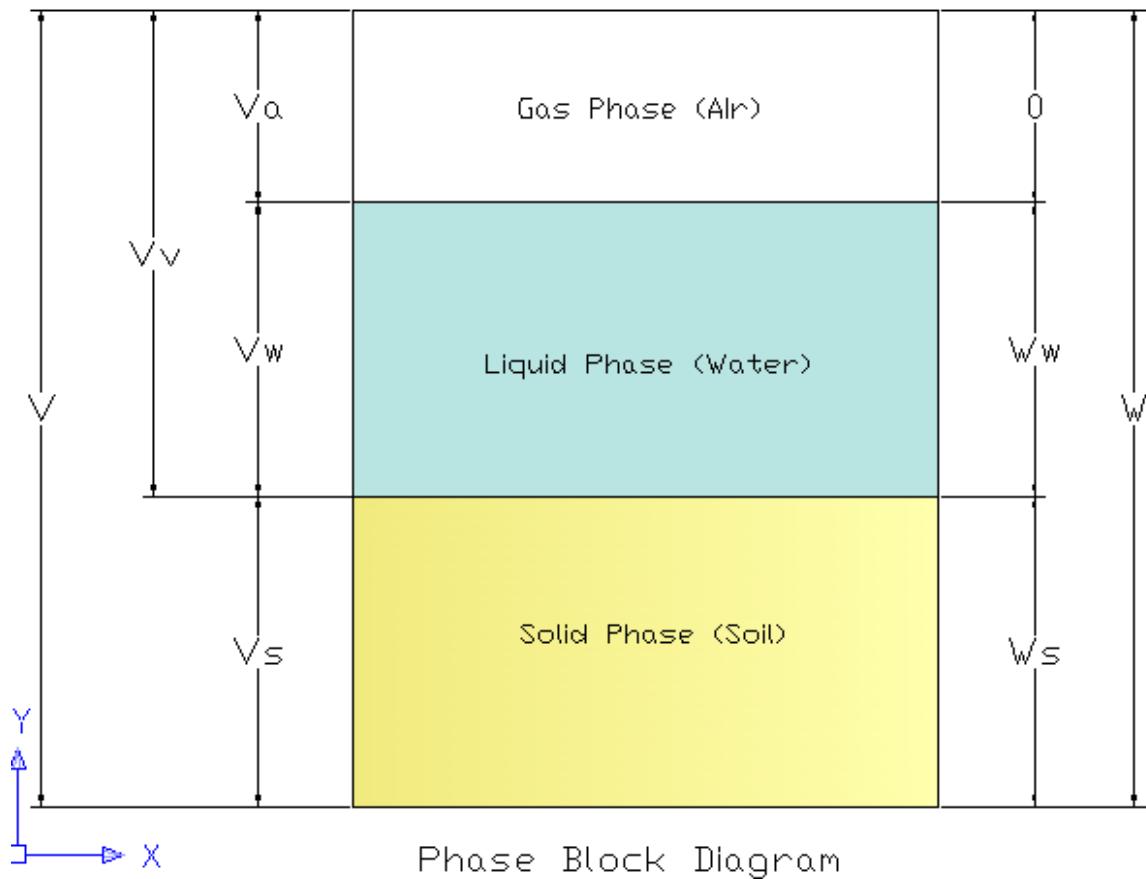
$g_b \rightarrow$ *Buoyant unit weight of the soil (same as g').*

$g_{SAT} \rightarrow$ *Unit weight of a saturated soil.*

$g_W \rightarrow$ *Unit weight of water*

Basic Concepts and Formulas for the Phases of Soils.

(A) Volumetric Relationships:

```
     Va          Gas Phase (Air)           0

Vv

     Vw          Liquid Phase (Water)      Ww
V                                                W


     Vs          Solid Phase (Soil)        Ws
Y

   ──► X          Phase Block Diagram
```

1. - Voids ratio e

$$e = \frac{V_V}{V_S}$$
2-1

ranges from 0 to infinity.

Typical values of sands are: very dense 0.4 to very loose 1.0

Typical values for clays are: firm 0.3 to very soft 1.5.

2. - Porosity n

$$n = \frac{V_V}{V}(100\%)$$
2.2

ranges from 0% to 100%.

The porosity provides a measure of the ***permeability*** of a soil.

The interrelationship of the voids ratio and porosity are given by,

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$$e = \frac{n}{1-n} \quad and \quad n = \frac{e}{1+e}$$ 2-3

3. - Saturation S

$$S = \frac{V_W}{V_V} x100\%$$ 2-4

ranges from 0% to 100%.

(B) Weight Relationships:

4. - Water content w

$$w = \frac{W_W}{W_S} x100\%$$ 2-5

Values range from 0% to over 500%; also known as ***moisture content***.

5. – Unit weight of a soil γ

$$\gamma = \frac{W}{V} = \frac{W_S + W_W}{V_S + V_W + V_A}$$ 2-6

The unit weight may range from being dry to being saturated.

Some engineers use "***bulk density ρ***" to refer to the ratio of mass of the solids and water contained in a unit volume (in Mg/m^3). Note that,

$$\gamma = \frac{W}{V} = \rho g = \frac{m}{V} g \quad which\ is\ the\ equivalent\ of\ F = ma.$$ 2-6

6. - Dry unit weight γ_d

$$\gamma_d = \frac{W_S}{V} = \frac{\gamma}{1+w}$$ 2-7

The soil is perfectly dry (its moisture is zero).

7. - The unit weight of water γ_w

$$\gamma_w = \frac{W_W}{V_W} \quad where\ \gamma = \rho g\ (F = ma)$$

$$\gamma_w = 62.4\ pcf = 1\ g/ml = 1\ kg/liter = 9.81\ kN/m^3$$

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Note that the above is for fresh water. Salt water is 64 pcf, etc.

8. - Saturated unit weight of a soil γ_{sat}

$$\gamma_{SAT} = \frac{W_S + W_W}{V_S + V_W + 0}$$

2-8

9. - Buoyant unit weight of a soil γ_b

$$\gamma_b = \gamma' = \gamma_{SAT} - \gamma_w$$

2-9

10. - Specific gravity of the solids of a soil G

$$G_S = \frac{\gamma_S}{\gamma_w}$$

2-10

Typical Values for the Specific Gravity of Minerals in Soils and Rocks		
Mineral	*Composition*	*Absolute specific gravity G_s*
Anhydrite	$CaSO_4$	2.90
Barites	$BaSO_4$	4.50
Calcite, chalk	$CaCO_3$	2.71
Feldspar	$KALSi_3O_8$	2.60 to 2.70
Gypsum	$CaSO_4\ 2H_2O$	2.30
Hematite	Fe_2O_3	5.20
Kaolinite	$Al_4Si_4O_{10}(OH)_8$	2.60
Magnetite	Fe_3O_4	5.20
Lead	Pb	11.34
Quartz (silica)	SiO_2	2.65
Peat	Organic	1.0 or less
Diatomaceous earth	Skeletons of plants	2.00

Other useful formulas dealing with phase relationships:

$Se = wG_S$

$e = \dfrac{\gamma_s}{\gamma_{dry}} - 1$

Unit weight relationships :

$$\gamma = \frac{(1+w)G_S\gamma_w}{1+e} = \frac{(G_S + Se)\gamma_w}{1+e} = \frac{(1+w)G_S\gamma_w}{1+\dfrac{wG_S}{S}} = G_S\gamma_w(1-n)(1+w)$$

Saturated unit weights :

$$\gamma_{SAT} = \frac{(G_S + e)\gamma_w}{1+e} = \left(\frac{e}{w}\right)\left(\frac{1+w}{1+e}\right)\gamma_w$$

$$\gamma_{SAT} = \gamma_d + n\gamma_w = \left[(1-n)G_s + n\right]\gamma_w = \left(\frac{1+w}{1+wG_s}\right)G_s\gamma_w$$

$$\gamma_{SAT} = \gamma' + \gamma_w$$

Dry unit weights :

$$\gamma_d = \frac{\gamma}{1+w} = G_s\gamma_w(1-n) = \frac{G_S\gamma_w}{1+e} = \frac{eS\gamma_w}{(1+e)w} = \frac{eG_s\gamma_w}{(S+wG_s)}$$

$$\gamma_d = \gamma_{SAT} - n\gamma_w = \gamma_{SAT} - \left(\frac{e}{1+e}\right)\gamma_w$$

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(Revision: Oct.-08)

A cohesive soil sample was taken from an *SPT* and returned to the laboratory in a glass jar. It was found to weigh 140.5 grams. The sample was then placed in a container of V = 500 cm^3 and 423 cm^3 of water were added to fill the container. From these data, what was the unit weight of the soil in kN/m^3 and pcf?

Solution.

Notice that the 140.5 grams is a mass. Therefore, the ratio of mass to volume is a density ρ,

$$\rho = \frac{m}{V} = \frac{140.5\, g_f}{(500-423)cm^3} = 1.82\, \frac{g_f}{cm^3}$$

$$\gamma = \rho g = \left(1.82\, \frac{g_f}{cm^3}\right)\left(\frac{1\, kg_f}{10^3\, g_f}\right)\left(9.806\, \frac{m}{sec^2}\right)\left(\frac{1\, kN}{10^3\, N}\right)\left(\frac{10^2\, cm}{1\, m}\right)^3 = 17.9\, \frac{kN}{m^3}\ (SI\ units)$$

$$\gamma = \left(17.9\, \frac{kN}{m^3}\right)\left(\frac{1000\, N}{1\, kN}\right)\left(\frac{0.2248\, lbs_f}{1\, N}\right)\left(\frac{1\, m^3}{35.3\, ft^3}\right) = 114\, pcf\ (US\ units)$$

(Revision: Sept.- 08)

A contractor has compacted the base course for a new road and found that the mean value of the test samples shows $w = 14.6\%$, $G_S = 2.81$, and $\gamma = 18.2\ kN/m^3$. The specifications require that $e \le 0.80$. Has the contractor complied with the specifications?

Solution:

$$\gamma = \frac{G_S \gamma_w (1+w)}{1+e} \quad \therefore \quad 1+e = \frac{G_S \gamma_w (1+w)}{\gamma}$$

$$1+e = \frac{2.81\left(9.81\dfrac{kN}{m^3}\right)(1+0.146)}{18.2\dfrac{kN}{m^3}} = 1.74$$

$$e = 1.74 - 1 = 0.74$$

$$\therefore \quad e = 0.74 < 0.8 \quad Yes, the\ contractor\ has\ complied.$$

(Revision: Oct.-08)

(1) Show that at saturation the moisture (water) content is $w_{sat} = \dfrac{(n\gamma_W)}{(\gamma_{sat} - n\gamma_W)}$.

(2) Show that at saturation the moisture (water) content is $w_{sat} = \gamma_w \left(\dfrac{1}{\gamma_d} - \dfrac{1}{\gamma_S} \right)$

Solution:

(1) In a fully saturated soil the relation, $Se = wG_S$ becomes simply $e = wG_s$

because $S = 1$ *or* $G_S = \dfrac{e}{w_{sat}} = \dfrac{n}{w_{sat}(1-n)}$

but $\gamma_{sat} = \gamma_w \left[(1-n)G_S + n \right]$

rearranging $\dfrac{\gamma_{sat}}{\gamma_w} = \left[(1-n)G_S + n \right] = \left[(1-n)\dfrac{n}{w_{sat}(1-n)} + n \right] = \dfrac{n}{w_{sat}} + n$

or $\dfrac{\gamma_{sat}}{\gamma_w} - n = \dfrac{n}{w_{sat}}$ *therefore* $w_{sat} = \dfrac{n\gamma_w}{\gamma_{sat} - n\gamma_w}$

(2) Again, in a fully saturated soil, $w_{sat} = \dfrac{e}{G_S} = \dfrac{V_V}{V_S}\dfrac{\gamma_w}{\gamma_S} = \dfrac{V_V}{V_S}\dfrac{\gamma_w}{1}\dfrac{V_S}{W_S} = \dfrac{\gamma_w V_V}{W_S}$

$\therefore$ $w_{sat} = \dfrac{\gamma_w V_V}{W_S} = \gamma_w \left(\dfrac{V_V}{W_S} \right) = \gamma_w \left(\dfrac{V_V + V_S - V_S}{W_S} \right) = \gamma_w \left(\dfrac{V_V + V_S}{W_S} - \dfrac{V_S}{W_S} \right)$

or $w_{sat} = \gamma_w \left(\dfrac{1}{\gamma_d} - \dfrac{1}{\gamma_S} \right)$

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(Revision: Oct.-08)

A geotechnical laboratory reported these results of five samples taken from a single boring. Determine which are not correctly reported, if any.

Sample #1: $w = 30\%$, $\gamma_d = 14.9$ kN/m³, $\gamma_s = 27$ kN/m³; clay.

Sample #2: $w = 20\%$, $\gamma_d = 18$ kN/m³, $\gamma_s = 27$ kN/m³; silt.

Sample #3: $w = 10\%$, $\gamma_d = 16$ kN/m³, $\gamma_s = 26$ kN/m³; sand.

Sample #4: $w = 22\%$, $\gamma_d = 17.3$ kN/m³, $\gamma_s = 28$ kN/m³; silt.

Sample #5: $w = 22\%$, $\gamma_d = 18$ kN/m³, $\gamma_s = 27$ kN/m³; silt.

Solution:

$$w_{sat} = \frac{e}{G_S} = \frac{V_V}{V_S}\frac{\gamma_w}{\gamma_S} = \frac{V_V}{V_S}\frac{\gamma_w}{1}\frac{V_S}{W_S} = \frac{\gamma_w V_V}{W_S} = \gamma_w \left(\frac{V_V}{W_S}\right) = \gamma_w \left(\frac{V_V + V_S - V_S}{W_S}\right)$$

$$w_{sat} = \gamma_w \left(\frac{V_V + V_S}{W_S} - \frac{V_S}{W_S}\right) = \gamma_w \left(\frac{1}{\gamma_d} - \frac{1}{\gamma_S}\right)$$

The water content is in error if it is greater than the saturated moisture, that is,

$$\therefore \quad w \le w_{SAT} = \gamma_w \left(\frac{1}{\gamma_d} - \frac{1}{\gamma_S}\right)$$

1) $w_{SAT} = \left(9.81\,kN/m^3\right)\left(\frac{1}{14.9} - \frac{1}{27}\right) = 30\% = w = 30\%\ GOOD$

2) $w_{SAT} = \left(9.81\,kN/m^3\right)\left(\frac{1}{18} - \frac{1}{27}\right) = 18.5\%\ v < w = 20\%\ WRONG$

3) $w_{SAT} = \left(9.81\,kN/m^3\right)\left(\frac{1}{16} - \frac{1}{26}\right) = 24\% > w = 10\%\ GOOD$

4) $w_{SAT} = \left(9.81\,kN/m^3\right)\left(\frac{1}{17.3} - \frac{1}{28}\right) = 22.1\% > w = 22\%\ GOOD$

5) $w_{SAT} = \left(9.81\,kN/m^3\right)\left(\frac{1}{18} - \frac{1}{27}\right) = 18.5\% < w = 22\%\ WRONG$

(Revision: Sept.-08)

A soil sample has a unit weight of 105.7 pcf and a saturation of 50%. When its saturation is increased to 75%, its unit weight raises to 112.7 pcf.

Determine the voids ratio e and the specific gravity G_s of this soil.

Solution:

$$\gamma = \frac{\gamma_W (G_S + Se)}{1+e}$$

$$\therefore \qquad 105.7 \; pcf = \frac{62.4(G_S + 0.50e)}{1+e} \qquad (1)$$

$$and \quad 112.7 \; pcf = \frac{62.4(G_S + 0.75e)}{1+e} \qquad (2)$$

Solving explicitly for G_s in equation (1),

$$G_s = \frac{(105.7)(1+e)}{62.4} - 0.50e$$

Replace G_s in equation (2) with the above relation from (1),

$$\therefore \quad (112.7)(1+e) = (105.7)(1+e) + (62.4)(0.25e)$$

$$\therefore \quad \underline{e = 0.814 \quad and \quad G_S = 2.67}$$

(Revision: Sept.-08)

The moist unit weight of a soil is 16.5 kN/m^3. Given that the w = 15% and G_s = 2.70, find:

(1) Dry unit weight γ_d,

(2) The porosity n,

(3) The degree of saturation S, and

(4) The mass of water in kg_m/m^3 that must be added to reach full saturation.

Solution:

a) $\quad \gamma_d = \dfrac{\gamma}{(1+w)} = \dfrac{16.5}{(1+0.15)} = 14.3 \dfrac{kN}{m^3}$

b) $\quad$ From the table of useful relationships,

$\gamma_d = \dfrac{G_s \gamma_w}{1+e} \quad \therefore \quad 1+e = \dfrac{G_s \gamma_w}{\gamma_d} = \dfrac{(2.70)(9.81)}{(14.3)} = 1.85 \quad \therefore \quad e = 0.85$

$n = \dfrac{e}{1+e} = \dfrac{0.85}{1+0.85}(100\%) = 46\%$

c) $\quad$ Since $Se = wG_s \quad \therefore S = \dfrac{wG_s}{e} = \dfrac{(0.15)(2.70)}{(0.85)}(100) = 48\%$

d) $\quad \gamma_{sat} = \dfrac{(G_s + e)\gamma_w}{1+e} = \dfrac{(2.70+0.85)(9.81)}{1+0.85} = 18.8 \dfrac{kN}{m^3}$

The water to be added can be found from the relation $\gamma = \rho g$

$\therefore \rho(\text{mass of water}) = \dfrac{\gamma}{g} = \left[\dfrac{(18.8 - 16.5)\,kN/m^3}{9.81\,kg\text{-}m/s^2}\right]\left(\dfrac{1,000\,N}{1\,kN}\right)\left(\dfrac{9.81\,kg\text{-}m/s^2}{N}\right) = 2,340 \dfrac{kg_m}{m^3}$

(Revision: Sept.-08)

A soil has an "in-situ" (in-place) voids ratio $e_o = 1.87$, $w_N = 60\%$, and $G_S = 2.75$. What are the γ_{moist} and S? (Note: All soils are really "moist" except when dry, that is when $w = 0\%$).

Solution: Set $V_S = 1$ m^3 (Note: this problem could also be solved by setting $V = 1.0$ m^3).

$$\therefore \quad e_o = \frac{V_V}{V_S} = \frac{1.87}{1} = 1.87 \quad \therefore \quad V = V_S + V_V = 1 + 1.87 = 2.87 \ m^3$$

The "natural" water content is $\quad w_N = \dfrac{W_w}{W_s} = 0.60 \quad \therefore \quad W_w = 0.60 W_s$

$$G_s = \frac{\gamma_s}{\gamma_w} = \frac{\frac{W_s}{V_s}}{\gamma_w} \quad \therefore \quad W_s = V_s \left(G_S \gamma_w\right) = \left(1 \ m^3\right)\left(2.75\right)\left(9.81 \ kN / m^3\right) = \quad 26.98 \ kN$$

$$W_w = 0.60\left(W_s\right) = \left(0.60\right)\left(26.98\right) \qquad\qquad = \underline{\quad 16.19 \ kN \quad}$$

$$W = W_s + W_w = 26.98 + 16.19 \qquad\qquad = \quad 43.17 \ kN$$

$$\therefore \quad \gamma_{moist} = \frac{W}{V} = \frac{43.17 \ kN}{2.87 \ m^3} = 15.0 \ \frac{kN}{m^3}$$

$$\therefore \quad S = \frac{V_w}{V_V} = \frac{\frac{W_w}{\gamma_w}}{V_V} = \frac{\left(\frac{16.19}{9.81}\right)}{1.87} = 88.2\%$$

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Phases of soils–08: Same as Prob-07 but setting the total volume V=1 m³.

(Revision: Oct.-08)

A soil has an "in-situ" (in-place) voids ratio $e_o = 1.87$, $w_N = 60\%$, and $G_S = 2.75$. What are the γ_{moist} and S? (Note: All soils are really "moist" except when dry, that is when w = 0%).

_____

Solution: Set $V = 1\ m^3$ (instead of $V_S = 1\ m^3$ used in Phases-07).

$but\ e_o = \dfrac{V_V}{V_S} = 1.87 \therefore\ but\ V = 1\ m^3 = V_S + V_V = V_S + 1.87V_S = 2.87V_S\ \therefore V_S = 0.348\ and\ V_V = 0.652$

The "natural" water content is $w_N = \dfrac{W_w}{W_s} = 0.60 \therefore\ W_w = 0.60W_s$

$G_s = \dfrac{\gamma_s}{\gamma_w} = \dfrac{\dfrac{W_s}{V_s}}{\gamma_w} \therefore\ W_s = V_s(G_s\gamma_w) = (0.348\ m^3)(2.75)(9.81\ kN/m^3) = 9.39\ kN$

$W_w = 0.60(W_s) = (0.60)(9.39)$ $= \underline{5.63\ kN}$

$W = W_s + W_w = 9.39 + 5.63$ $= 15.02\ kN$

$\therefore\ \gamma_{moist} = \dfrac{W}{V} = \dfrac{15.0\ kN}{1\ m^3} = 15.0\ \dfrac{kN}{m^3}$

$\therefore\ S = \dfrac{V_w}{V_V} = \dfrac{\dfrac{W_w}{\gamma_w}}{V_V} = \dfrac{\left(\dfrac{5.63}{9.81}\right)}{0.652} = 88.0\%$

(Revision: Sept.-08)

A soil sample has a unit weight of 105.7 pcf and a water content of 50%. When its saturation is increased to 75 %, its unit weight raises to 112.7 pcf. Determine the voids ratio e and the specific gravity G_s of the soil. (NB: This is the same problem as Phase–06, but solved with a block diagram).

Solution:

Set $V = 1\,ft^3$

$\gamma_2 - \gamma_1 = 112.7 - 105.7 = 7.0 \quad lbs \quad are \quad 25\% \quad of \quad water$

$\therefore \qquad 21.0 \quad lbs \quad are \quad 75\% \quad of \quad water$

$\therefore \qquad W_S = 112.7 - 20.8 \quad = \quad 91.9 \quad lb$

$$V_w = \frac{W_w}{\gamma_w} = \frac{20.8\ lb}{62.4\ pcf} = 0.333\ ft^3$$

$$V_a = \frac{1}{3}V_w = 0.111\ ft^3 \quad \therefore \quad V_v = V_a + V_w = 0.111 + 0.333 = 0.444$$

$$\therefore \qquad V_s = 1 - V_v = 1 - 0.444 = 0.556$$

$$e = \frac{V_V}{V_S} = \frac{0.444}{0.556} = 0.80$$

$$and \quad G_S = \frac{\gamma_S}{\gamma_w} = \frac{W_S}{V_S\gamma_w} = \frac{91.9\ lb}{(0.556)(62.4)} = 2.65$$

(Revision: Sept.-08)

A saturated soil sample has a unit weight of 122.5 pcf and G_s = 2.70. Find γ_{dry} , *e, n,* and *w*.

Solution:

$$V = V_S + V_w = \frac{1}{\gamma_w}\left(\frac{W_S}{G_S} + W_w\right) \qquad (1)$$

$$W = W_S + W_w = 122.5 \, lb \qquad (2)$$

Combining equations (1) and (2) yields $\quad 1 = \frac{1}{(62.4 \, pcf)}\left(\frac{122.5 - W_w}{2.70} + W_w\right)$

$\therefore W_w = 27.0 \, lb \quad \therefore \quad V_w = \frac{W_w}{\gamma_w} = \frac{27.0 \, lb}{62.4 \, pcf} = 0.433 \, ft^3$

$\therefore W_S = 95.5 \, lb \quad \therefore \quad V_S = \frac{W_S}{G_S \gamma_w} = \frac{95.5 \, lb}{(2.70)(62.4 \, pcf)} = 0.567 \, ft^3$

$\therefore \qquad \gamma_{dry} = \frac{W_S}{V} = \frac{95.5 \, lb}{1 \, ft^3} = 95.5 \, pcf$

$\therefore \qquad e = \frac{V_V}{V_S} = \frac{0.433}{0.567} = 0.764$

$\therefore \qquad n = \frac{V_V}{V} = \frac{0.433}{1} = 0.433 \qquad n = 43.3\%$

$\therefore \qquad w = \frac{W_w}{W_S} = \frac{27}{95.5} = 0.283 \qquad w = 28.3\%$

*Phases of soils–11: *Find the weight of water needed for saturation.*

(Revision: Sept.-08)

Determine the weight of water (in *kN*) that must be added to a cubic meter of soil to attain a 95 % degree of saturation, if the dry unit weight is 17.5 *kN/m³*, its moisture is 4%, the specific gravity of solids is 2.65 and the soil is entirely made up of a clean quartz sand.

Solution:

$$\gamma_d = 17.5 \frac{kN}{m^3} = \frac{\gamma}{1+w} = \frac{\gamma}{1+0.04} \quad \therefore \quad \gamma = 18.2 \frac{kN}{m^3}$$

$$W = 18.2 = W_S + W_w = W_S + w W_S = (1.04) W_S$$

$$\therefore \quad W_S = 17.5 \ kN, \quad and \quad W_w = 0.70 \ kN$$

$$V_S = \frac{W_S}{\gamma_S} = \frac{17.5 \ kN}{G_S \gamma_w} = \frac{17.5 \ kN}{(2.65)(9.81 \ kN \ / \ m^3)} = 0.673 \ m^3$$

$$V_w = \frac{W_w}{\gamma_w} = \frac{0.70 \ kN}{(9.81 \ kN \ / \ m^3)} = 0.07 \ m^3 \quad \therefore \quad V_a = V - V_S - V_w = 0.257 \ m^3$$

$$e = \frac{V_V}{V_S} = \frac{0.07 + 0.257}{0.673} = 0.49$$

$$The \ existing \ S = \frac{w G_S}{e} = \frac{(0.04)(2.65)}{0.49}(100) = 21.6\%$$

$$We \ require \ a \ S = 95\%, \ therefore,$$

$$w = \frac{Se}{G_S} = \frac{(0.95)(0.49)}{2.65} = 0.17$$

$$W_w = w W_S = (0.17)(17.5 \ kN) = 2.98 \ kN$$

$$already \ have \ W_w = 0.70 \ kN$$

$$\therefore \quad must \ add \ water = 2.28 \ kN$$

Answer: *Add 2.28 kN of water per m³.*

(Revision: Sept.-08)

A project engineer receives a laboratory report with tests performed on marine marl calcareous silt). The engineer suspects that one of the measurements is in error. Are the engineer's suspicions correct? If so, which one of these values is wrong, and what should be its correct value?

$Given \quad \gamma = unit\ weight\ of\ sample = 18.4\ \dfrac{kN}{m^3}$

$\gamma_S = unit\ weight\ of\ solids = 26.1\ \dfrac{kN}{m^3}$

$w = water\ content = 40\%$

$e = voids\ ratio = 1.12$

$S = degree\ of\ saturation = 95\%$

Solution:

Check the accuracy of 4 out of 5 of the variables using,

$Se = wG_S \quad \therefore \quad Se = (0.95)(1.12) = 1.06$

$wG_S = (w)\dfrac{\gamma_S}{\gamma_w} = (0.4)\dfrac{26.1}{9.81} = 1.06 \quad \therefore \quad$ Therefore, these four are correct.

The only possibly incorrect value is γ. Assume that $V = 1\ m^3$.

$V = 1\ m^3 = V_a + V_w + V_S \qquad\qquad (1)$

$but \quad e = \dfrac{V_V}{V_S} = 1.12 \quad \therefore \quad 0 = -V_a - V_w + 1.12 V_S \quad (2)$

$\therefore \quad V_S = 0.472\ m^3,\ V_V = 0.528\ m^3 \quad but \quad V_w = 0.95 V_V = 0.502\ m^3$

$\therefore \quad V_a = 0.026\ m^3$

$\therefore W_S = \gamma_S V_S = \left(26.1\dfrac{kN}{m^3}\right)(0.472\ m^3) = 12.3\ kN$

$W_w = wW_S = (0.40)\left(12.3\dfrac{kN}{m^3}\right) \qquad = 4.9\ kN$

$W = 12.3\ kN + 4.9\ kN \qquad\qquad = 17.2\ kN$

Therefore, the actual unit weight of the soil is,

$\therefore \gamma = \dfrac{W}{V} = \dfrac{17.2\ kN}{1\ m^3} = 17.2\ \dfrac{kN}{m^3} \neq 18.4\ \dfrac{kN}{m^3}$

You are a Project Engineer on a large earth dam project that has a volume of 5x10^6 yd^3 of select fill, compacted such that the final voids ratio in the dam is 0.80. Your boss, the Project Manager delegates to you the important decision of buying the earth fill from one of three suppliers. Which one of the three suppliers is the most economical, and how much will you save?

            Supplier A   **Sells fill at \$ 5.28/ yd^3 with   e = 0.90**

            Supplier B   **Sells fill at \$ 3.91/ yd^3 with   e = 2.00**

            Supplier C   **Sells fill at \$ 5.19/ yd^3 with   e = 1.60**

Solution:

Without considering the voids ratio, it would appear that Supplier B is cheaper than Supplier A by \$1.37 per yd^3.

Therefore: To put 1yd^3 of solids in the dam you would need 1.8 yd^3 of soil.

                For 1yd^3 of solids from A you would need 1.9 yd^3 of fill.

                For 1yd^3 of solids from B you would need 3.0 yd^3 of fill.

                For 1yd^3 of solids from C you would need 2.6 yd^3 of fill.

The cost of the select fill from each supplier is (rounding off the numbers):

$$A = \frac{1.9}{1.8}(5)(10^6 \; yd^3)\left(\frac{5.28\$}{yd^3}\right) \approx \$ \; 27,900,000$$

$$B = \frac{3.0}{1.8}(5)(10^6 \; yd^3)\left(\frac{3.91\$}{yd^3}\right) \approx \$ \; 32,600,000$$

$$C = \frac{2.6}{1.8}(5)(10^6 \; yd^3)\left(\frac{5.19\$}{yd^3}\right) \approx \$ \; 37,500,000$$

Therefore Supplier *A* is the cheapest by about \$ 4.7 Million compared to Supplier B.

(Revision: Sept.-08)

Based on the previous problem (Phases–13), if the fill dumped into the truck has an e = 1.2, how many truck loads will you need to fill the dam? Assume each truck carries 10 yd³ of soil.

Solution:

$Set \ V_S = 1 \quad e = \dfrac{V_V}{V_S} = \dfrac{V_V}{1} = V_V = 1.2$ which means that there is 1 yd³ of solids per 1.2 yd³ of voids.

$\therefore 2.2 \ yd^3 \ of \ soil \ for \ each \ 1 \ yd^3 \ of \ solids.$

$10 \ yd^3 \ of \ soil \ for \ each \ x \ yd^3 \ in \ a \ truck \ load$

$\therefore \quad x = 4.54 \ yd^3 \ of \ solids \ per \ truck \ trip.$

The required volume of solids in the dam is,

$$V_{solids} = \frac{\left(5x10^6 \ yd^3 of \ soil\right)\left(1 \ yd^3 of \ solids\right)}{1.8 \ yd^3 of \ soil} = 2.8x10^6 \ yd^3 \ of \ solids$$

Therefore, (rounding off)

$$Number \ of \ Truck-trips = \frac{\left(2.8x10^6 \ yd^3 of \ solids\right)}{\left(4.54 \ yd^3 of \ solids \ / \ truck - trip\right)} = 616,800$$

(Revision: Sept.-08)

You have been hired as the Project Engineer for a development company in South Florida to build 610 housing units surrounding four lakes. Since the original ground is low, you will use the limestone excavated from the lake to fill the land in order to build roads and housing pads. Your estimated fill requirements are 700,000 m³, with a dry density equivalent to a voids ratio e = 0.46. The "in-situ" limestone extracted from the lakes has an e = 0.39, whereas the limestone dumped into the trucks has an e = 0.71. How many truckloads will you need, if each truck carries 10 m³?

Solution:

Assume: $V_S = 1\ m^3$ $\therefore$ $e = \dfrac{V_V}{V_S} = \dfrac{V_V}{1} = V_V = 0.46\ m^3$ in the compacted fill

The required 700,000 m³ of fill have 1.46 m³ of voids per each 1 m³ of solids

Therefore, the 700,000 m³ of fill have 479,400 m³ of solids

Each truck carries 1.71 m³ of fill per 1 m³ solids

In order for the trucks to carry 479,000 m³ of solids they must carry 820,000 m³ of fill

Since each truck carries 10 m³ of fill,

$\therefore$ The number of truck-loads $= \dfrac{820,000\ m^3}{10\ m^3} = $ 82,000 truck-loads.

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(Revised: Sept.-08)

A large housing development requires the purchase and placement of the fill estimated to be 200,000 cubic yards of lime-rock compacted at 95% Standard Proctor with an *OMC* of 10%. Two lime-rock suppliers offer to fill your order: Company A has a borrow material with an in-situ γ = 115 pcf, *w* = 25%, G_S = 2.70; Standard Proctor yields a maximum γ_d = 112 pcf; at a cost of $0.20/yd³ to excavate, and $ 0.30/yd³ to haul. Company B has a borrow material with an in-situ γ = 120 pcf, *w* = 20%, G_S = 2.70; Standard Proctor yields a maximum γ_d = 115 pcf; a cost of $0.22/yd³ to excavate, and $ 0.38/yd³ to haul.

(1) What volume would you need from company A?

(2) What volume would you need from company B?

(3) Which would be the cheaper supplier?

Solution:

(1) The key idea: 1 yd^3 of solids from the borrow pit supplies 1 yd^3 of solids in the fill.

(2) Pit *A*: W_S = 92 lb, W_W = 23 lb ⟶ V_W = 0.369 ft³, V_S = 0.546 ft³, V_a = 0.085 ft³

$$e = \frac{V_V}{V_S} = \frac{0.454}{0.546} = 0.83 \quad \therefore \quad 1.83 \; yd^3 of \; soil \; contains \; 1.0 \; yd^3 of \; solids.$$

Pit *B*: W_S = 100 lb, W_W = 20 lb, V_W = 0.321 ft³, V$_S$ = 0.594 ft³, V_a = 0.08 ft³

$$e = \frac{V_V}{V_S} = \frac{0.401}{0.594} = 0.68 \quad \therefore \quad 1.68 yd^3 of \; soil \; contains \; 1.0 yd^3 of \; solids.$$

(3) Material needed for fill from company A:

$$\gamma = 0.95\gamma_d (1+w) = 0.95(112)(1+0.10) = 117 \, pcf \; \therefore \; W_S = 106.4 \, lb, \quad W_w = 10.6 \, lb$$

$$e = \frac{V_V}{V_S} = \frac{0.37}{0.63} = 0.59 \quad \therefore \quad 1.59 yd^3 of \; soil \; contains \; 1.0 yd^3 \; of \; solids$$

$$\therefore Site \; A \; requires \; \frac{200,000 \; yd^3 \; of \; fill}{1.59} = 125,800 \; yd^3 \; of \; solids$$

Material needed for fill from company B:

$$\gamma = 0.95\gamma_d (1+w) = 0.95(115)(1+0.10) = 120 \, pcf \; \therefore \; W_S = 109.1 \, lb, \quad W_w = 10.9 \, lb$$

$$e = \frac{V_V}{V_S} = \frac{0.35}{0.65} = 0.54 \quad \therefore \quad 1.54 yd^3 of \; soil \; contains \; 1.0 \; yd^3 \; of \; solids$$

$$\therefore Site \; B \; requires \; \frac{200,000 \; yd^3 \; of \; fill}{1.54} = 130,000 yd^3 \; of \; solids$$

(4) a) Cost of using Company A:

$$Cost_A = \left(125,800 \ yd^3\right)\left(1.83\right)\left(\frac{\$0.50}{yd^3}\right) = \$115,100$$

Cost of using Company B:

$$Cost_B = \left(130,000 \ yd^3\right)\left(1.68\right)\left(\frac{\$0.60}{yd^3}\right) = \$131,100$$

Using Company A will save about $ 16,000.

Use a matrix to the find the missing data.

(Revision: Sept.-08)

A contractor obtains prices for 34,000 yd^3 of compacted "borrow" material from three pits: Pit #3 is $11,000 cheaper than Pit #2 and $39,000 cheaper than Pit #1. The fill must be compacted down to a voids ratio of 0.7. Pit #1 costs $ 6.00/yd^3 and Pit #3 costs $ 5.50/yd^3. Pits #2 and #3 reported their voids ratios as 0.88 and 0.95 respectively. Use a matrix to find,

   a) The missing unit cost C_2 for Pit #2;

   b) The missing voids ratio e for Pit #1;

   c) The missing volume of fill V required from each pit; and

   d) The amount paid by the contractor for each pit.

Solution:

A summary of the data provided is herein shown in matrix form,

Item	Borrow Sites			Project Site
	Pit-1	Pit-2	Pit-3	
Unit Cost ($/$yd^3$)	6.00	C_2	5.50	
Voids ratio e	e_1	0.88	0.95	0.70
Volume (yd^3)	V_1	V_2	V_3	34,000
Total Cost ($)	$TC_1 = TC_2 + 28,000$	TC_2	$TC_3 = TC_2 - \$11,000$	

The volume of solids V_s contained in the total volume of fill $V = 34,000 \ yd^3$ can be found from,

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$$V = V_V + V_S = 0.7V_S + V_S = V_S\left(0.7+1\right) = 34,000 \ yd^3 \quad \therefore \ V_S = \frac{34,000}{1.7} = 20,000 \ yd^3 \ of \ solids$$

At Pit #3, $\dfrac{V_3}{V_S} = 1 + e_3 \quad \therefore \ V_3 = V_S\left(1+e_3\right) = \left(20,000 \, yd^3\right)\left(1+0.95\right) = 39,000 \, yd^3 \ of \ soil$

The total cost of Pit #3 is $TC_3 = \left(39,000 \, yd^3\right)\left(\$ \, 5.50 / \, yd^3\right) = \$ \, 214,500$

At Pit #2: $\dfrac{V_2}{V_S} = 1 + e_2 \quad \therefore V_2 = V_S\left(1+e_2\right) = \left(20,000 \, yd^3\right)\left(1+0.88\right) = 37,600 \, yd^3 \ of \ soil$

But, the total cost of Pit #2 is $TC_2 - \$ \, 11,000 = TC_3 = \$ \, 214,500 \quad \therefore \ TC_2 = \$ \, 225,500$

$\qquad$ The unit cost of Pit #2 $C_2 = \dfrac{TC_2}{V_2} = \dfrac{\$ \, 225,500}{37,600 \, yd^3} = \$ \, 6.00 / \, yd^3$

At Pit #1: $V_1 = \dfrac{TC_1}{\$ \, 6.00 / \, yd^3} = \dfrac{TC_2 + 28,000}{\$ \, 6.00 / \, yd^3} = \dfrac{225,500 + 28,000}{\$ \, 6.00 / \, yd^3} = 42,250 \, yd^3 \ of \ soil$

But, $V_1 = V_S\left(1+e_1\right) = \left(20,000 \, yd^3\right)\left(1+e_1\right) = 42,250 \, yd^3 \quad \therefore \quad e_1 = 1.11$

(Revision: Sept.-08)

You have been retained by a local municipality to prepare a study of their "muck" soils. Assume that you know the dry unit weight of the material (solids) γ_{sm} and the dry unit weight of the organic solids γ_{so}. What is the unit weight γ_s of the combined dry organic mineral soil whose organic content is M_0? (The organic content is the percentage by weight of the dry organic constituent of the total dry weight of the sample for a given volume.) What is the voids ratio e of this soil if it is known that its water content is w and its degree of saturation is S?

Solution:

Set $W_s = 1$ unit   and $\gamma_s = \dfrac{W_s}{V_s} = \dfrac{1}{(V_{so} + V_{sm})}$

(a) Assume $M_o = W_o$ for a unit weight of the *dry soil*

　　Therefore $1 - M_o = W_m$

$\dfrac{M_o}{\gamma_{so}}$ = volume of organic V_{so} solids

$\dfrac{(1 - M_o)}{\gamma_{sm}}$ = volume of mineral V_{sm} solids

The total unit weight is the weight of a unit volume.

Therefore $\gamma_s = \dfrac{1}{\left(\dfrac{M_o}{\gamma_{so}} + \dfrac{(1 - M_o)}{\gamma_{sm}} \right)} = \gamma_{so} \left[\dfrac{\gamma_{sm}}{M_o(\gamma_{sm} - \gamma_{so}) + \gamma_{so}} \right]$

(b) $e = \dfrac{V_v}{V_s} = \dfrac{\left(\dfrac{\text{volume of water}}{S} \right)}{V_s} = \dfrac{\left(\dfrac{\text{weight of water}}{\gamma_w S} \right)}{V_s} = \dfrac{\left(\dfrac{w(\text{weight of solids})}{\gamma_w S} \right)}{V_s}$

Therefore $e = \dfrac{\left(\dfrac{w}{\gamma_w S(1)} \right)}{\left(\dfrac{M_o}{\gamma_{so}} + \dfrac{(1 - M_o)}{\gamma_{sm}} \right)} = \dfrac{w\gamma_{sm}\gamma_{so}}{\gamma_w S\left[M_o(\gamma_{sm} - \gamma_{so}) + \gamma_{so} \right]}$

Chapter 3
Classification of Soils and Rocks

Symbols for Classification of soils

$C_c \rightarrow$ Coefficient of gradation (also coefficient of curvature).

$C_u \rightarrow$ Coefficient of uniformity.

$RC \rightarrow$ Relative compaction.

$D_x \rightarrow$ Diameter of the grains (at % finer by weight).

$D_r \rightarrow$ Relative density of a granular soil.

$e \rightarrow$ Voids ratio.

$e_{min} \rightarrow$ Minimum voids ratio.

$e_{max} \rightarrow$ Maximum voids ratio.

$I_P \rightarrow$ Index of plasticity (also referred to as *PI*).

$K \rightarrow$ Constant of the yield value.

$LL \rightarrow$ Liquid limit.

$PL \rightarrow$ Plastic limit.

$SL \rightarrow$ Shrinkage limit.

$V \rightarrow$ Volume of the soil sample.

$W \rightarrow$ Weight of the soil sample.

$\gamma_{d(min)} \rightarrow$ Dry unit weight in loosest condition (voids ratio e_{max}).
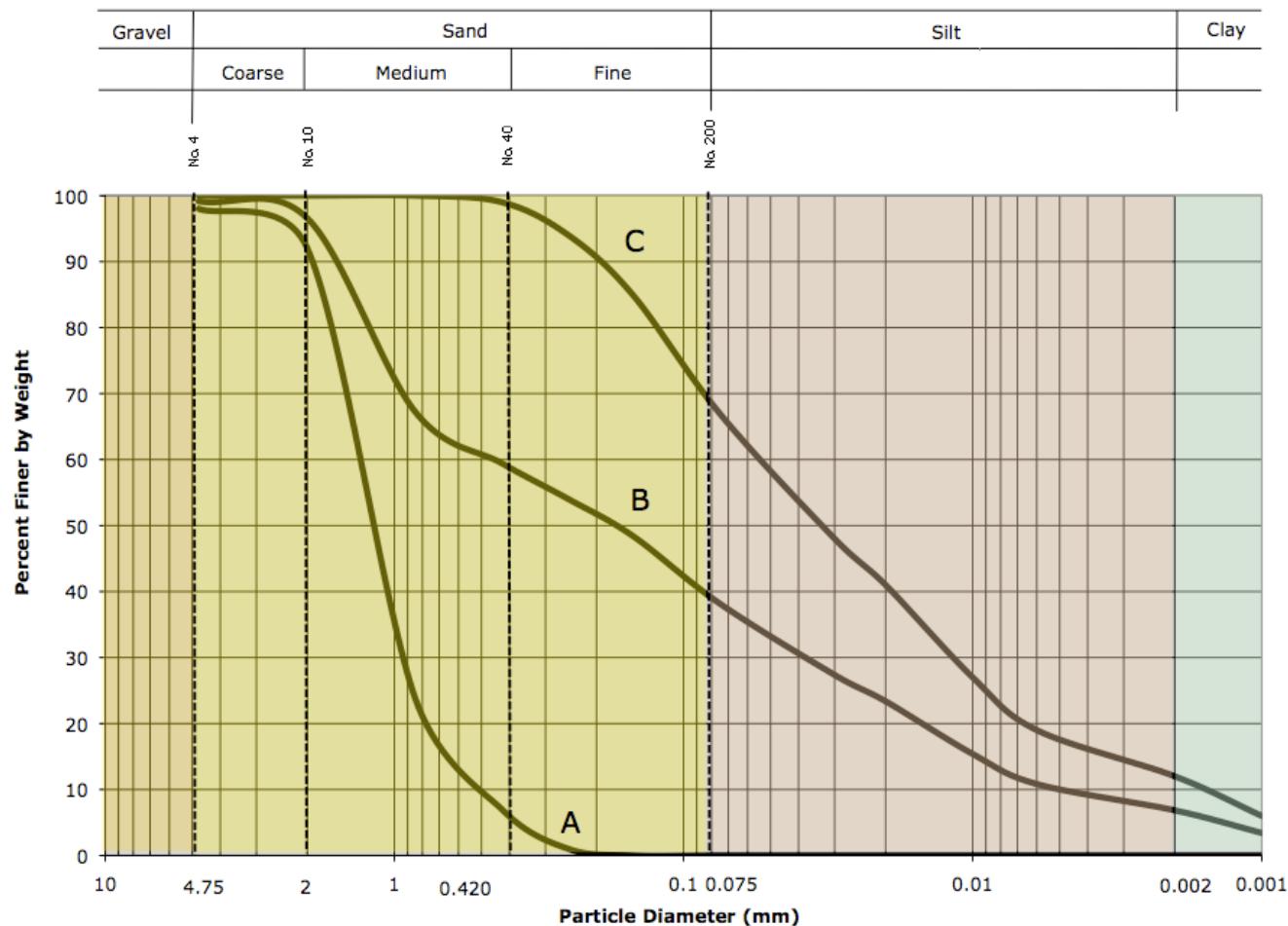
$\gamma_d \rightarrow$ In-situ dry unit weight (voids ratio e).

$\gamma_{d(max)} \rightarrow$ Dry unit in densest condition (voids ratio e_{min})

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*Classify–01: Percentage of each of the four grain sizes (G, S, M & C).

(Revision: Sept.-08)

Determine the percentage of gravels (G), sands (S), silts (M) and clays (C) of soils A, B and C

shown below.

Solution:

Notice that the separation between gravels (G) and sands (S) is the #4 sieve which corresponds to a particle size of 4.75 mm. The separation between sands (S) and silts (M) is the #200 sieve which corresponds to a particle size of 0.075 mm. Finally, the separation between silts (M) and clays (C) is the 0.002 mm (or 2 micro-meters = 2 μm). These divisions are shown above through color differentiation. Each soil A, B and C is now separated into the percentage of each:

Soil A: 2% G; 98% S; 0% M; 0%C. This soil is a _uniform or poorly-graded sand (SP)_.
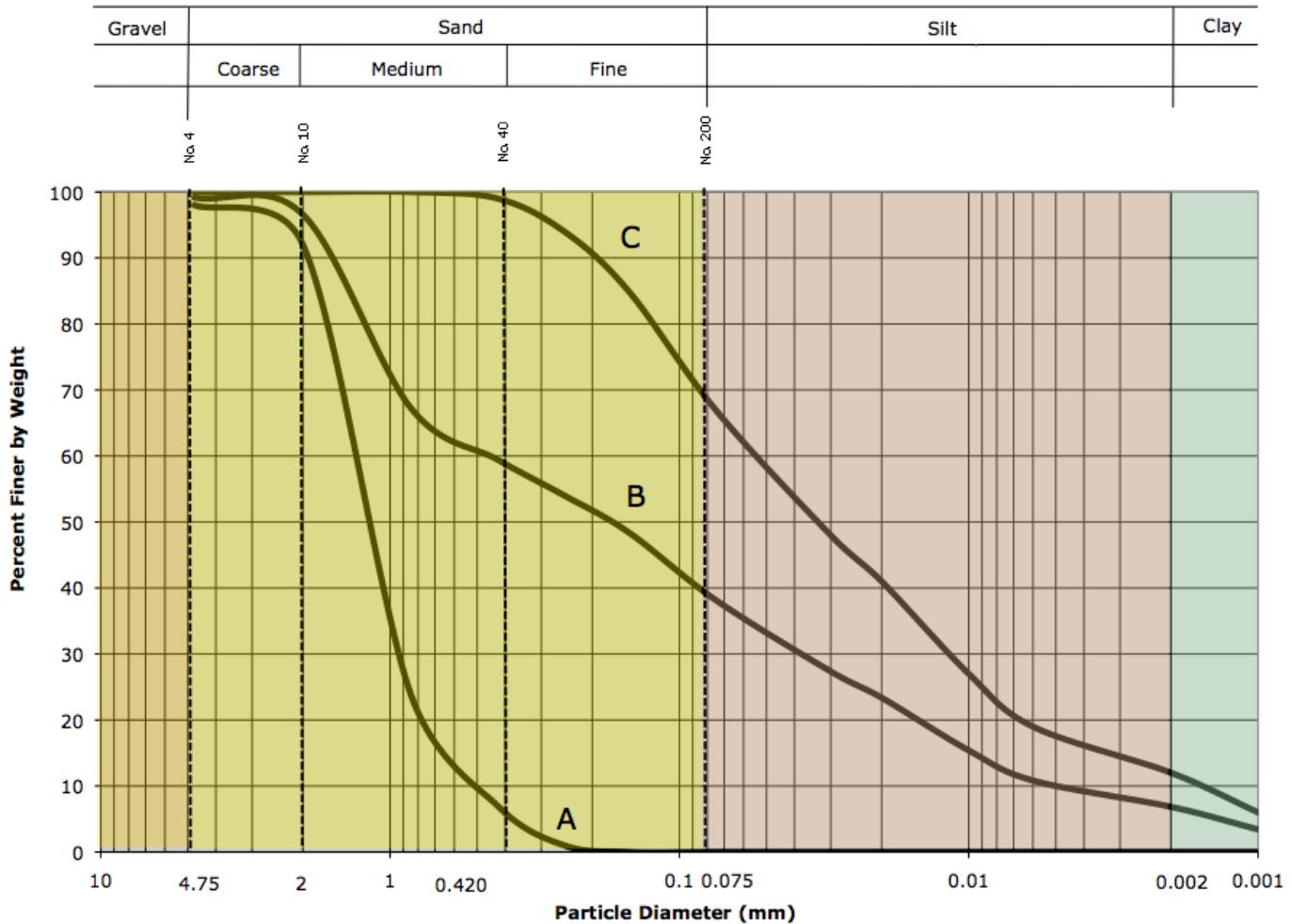
Soil B: 1% G; 61% S; 31% M; 7%C. This soil is a _well-graded silty sand (SM)_.

Soil C: 0% G; 31% S; 57% M; 12%C. This soil is a _well-graded sandy silt (M)_.

(Revision: Sept.-08)

Determine the uniformity coefficient C_u and the coefficient of gradation C_c for soil A.

Solution:

From the grain distribution curve, D_{60} = 1.4 mm, D_{30} = 0.95 mm and D_{10} = 0.50 mm, therefore the coefficients are,

$$C_U = \frac{D_{60}}{D_{10}} = \frac{1.40\ mm}{0.50\ mm} = 2.8 \quad and \quad C_C = \frac{D_{30}^2}{D_{60}D_{10}} = \frac{(0.95)^2}{(1.40)(0.50)} = 1.29$$

A **_uniform_** soil has a coefficient of uniformity C_u less than 4, whereas a **_well-graded_** soil has a uniformity coefficient greater than 4 for gravels and greater than 6 for sands. Since soil A has a low value of 2.8, and it is sand, this corresponds to a **_poorly-graded sand (SP)_**. Steep curves are uniform soils (low C_u) whereas diagonal curves are well-graded soils (high C_u).
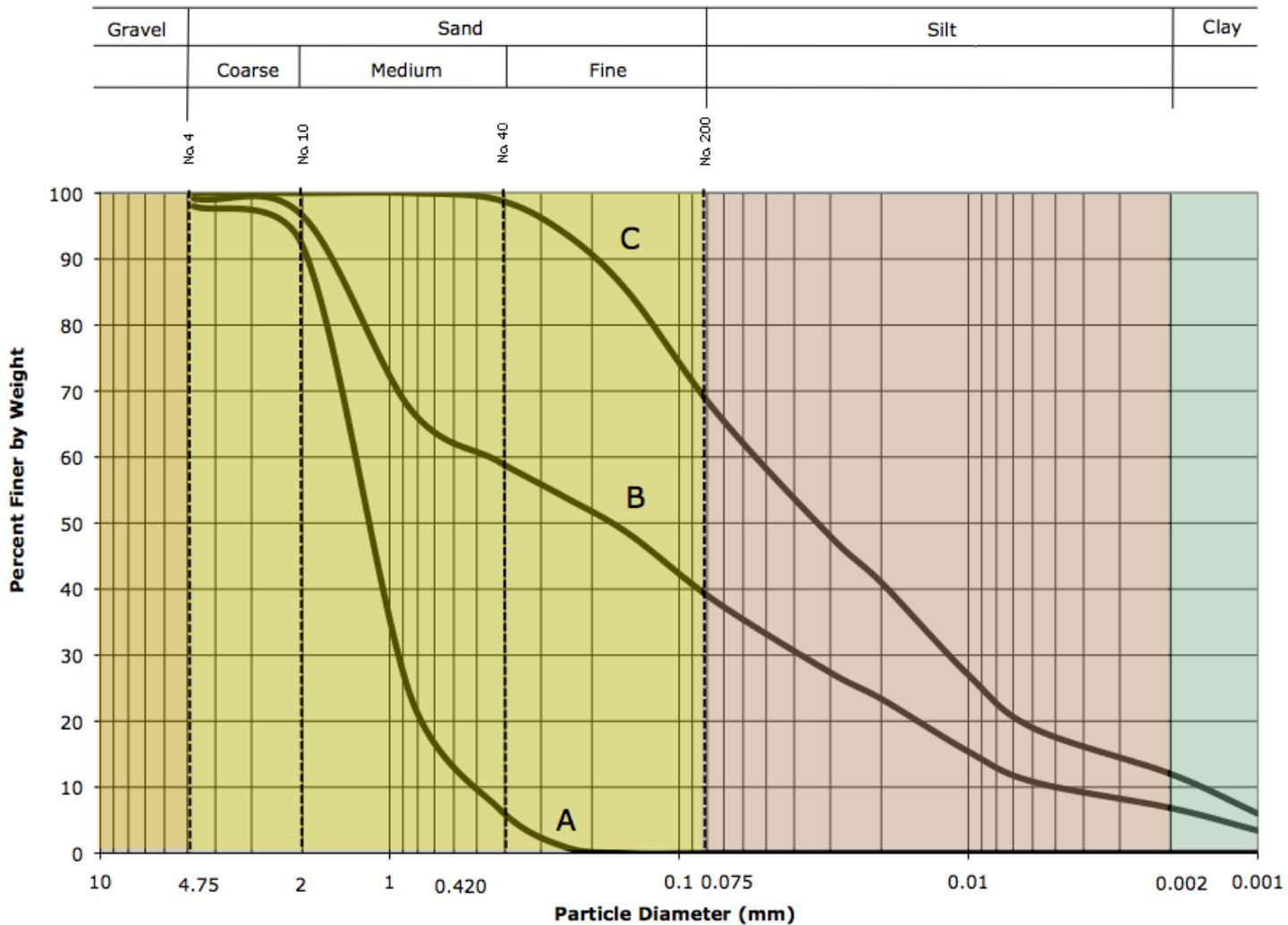
Smooth curved soils have coefficients of curvature C_c between 1 and 3, whereas irregular curves have higher or lower values. Soils that are missing a type of soil (a gap) are called **_gap-graded_** (C_c will be less than 1 or greater than 3 for gap-graded soils).

Therefore, this soil is classified as **_poorly-graded sand (or SP)._**

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(Revision: Sept.-08)

Use the grain-size distribution curve shown below to classify soils *A* and *B* using the USCS. Soil *B*'s Atterberg limits are *LL* = 49% and *PL* = 45%?

Solution:

Classify Soil A:

For soil A, the distribution is $G = 2\%$, $S = 98\%$, $M = 0\%$ and $C = 0\%$.

$$C_U = \frac{D_{60}}{D_{10}} = \frac{1.40 \ mm}{0.50 \ mm} = 2.8, \text{ therefore, soil } A \text{ is a } \textit{\textbf{poorly graded sand (SP)}}.$$

Classify Soil B:

For soil B, the distribution is $G = 0\%$, $S = 61\%$, $M = 35\%$ and $C = 4\%$.

$$C_U = \frac{D_{60}}{D_{10}} = \frac{0.45 \ mm}{0.005 \ mm} = 90, \text{ therefore, soil } A \text{ is } \textit{\textbf{very well graded silty sand (SM)}}.$$

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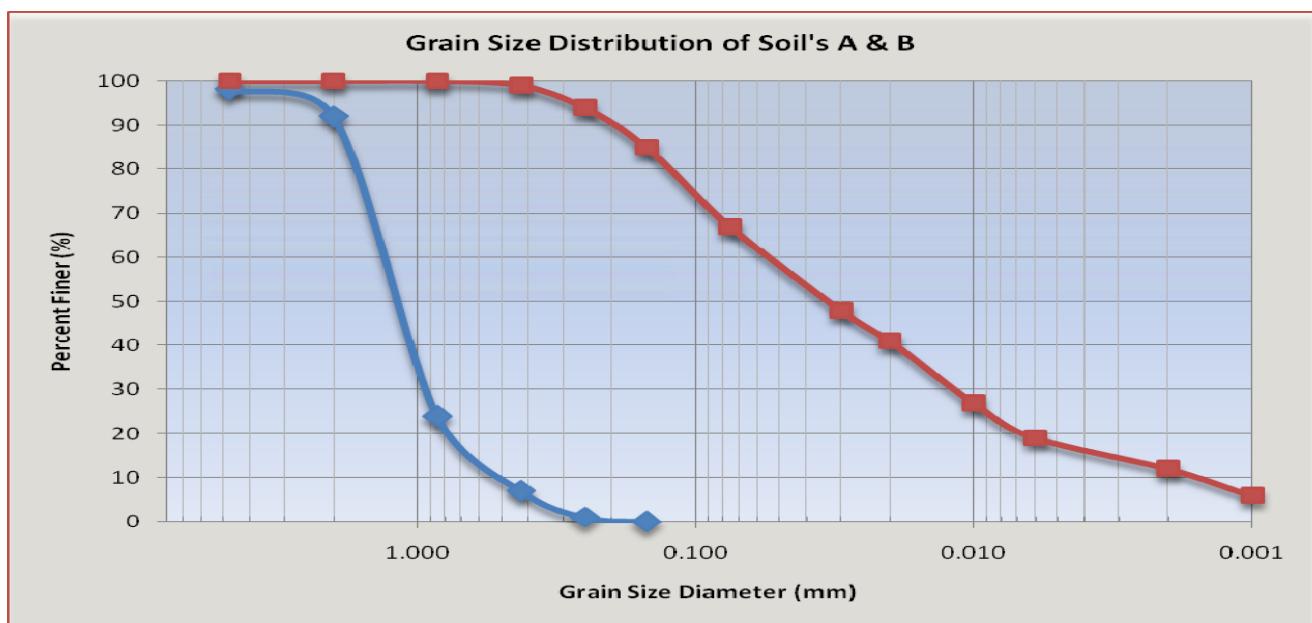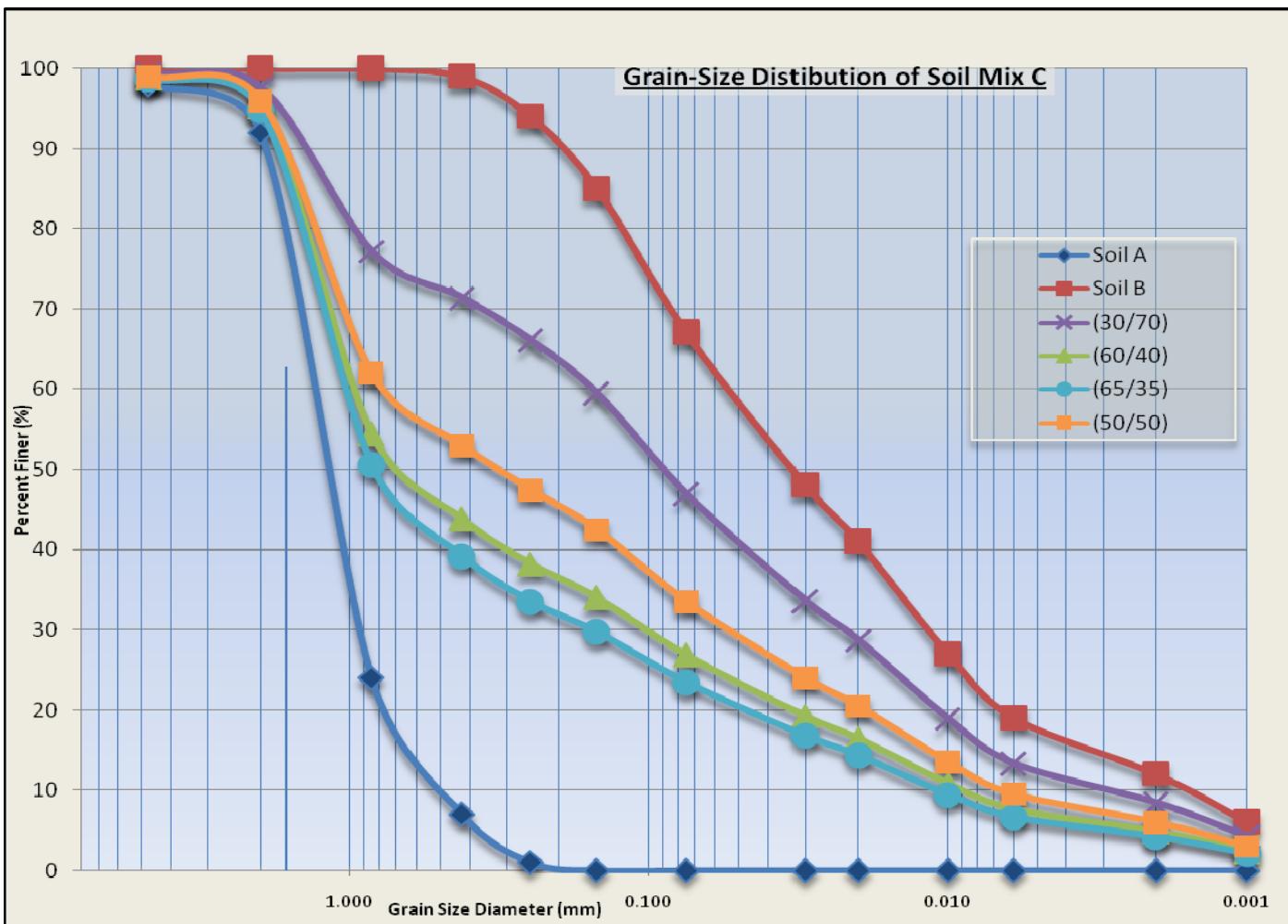
(Revision: Sept.-08)

A site has an unsuitable in-situ soil A that does not compact properly. In lieu of removing that soil A, you have decided to improve it by mixing it with a borrow pit soil B to produce an improved new soil C that will compact better.

You desire a coefficient of uniformity C_u of about 100 for the new soil C. Determine the relative percentages of these two uniform soils A and B so that they will result in better graded soil C. Plot your results.

The plots of soils A and B are as shown below,

Soil A is composed of 2% *G*, and 98% *S*: (6% coarse sand, 85% medium sand and 7% fine sand). It is obviously a *poorly graded sand (SP).*

Soil B is composed of approximately 33% *S*, 55% *M* and 12% *C*. It is a *well-graded sandy silt.*

Consider several solutions as shown below with A/B ratios of 30/70, 35/65, 40/60 and 50/50. The best is the 50/50 solution via *D₁₀* = 0.006 mm,

$$C_U = \frac{D_{60}}{D_{10}} = 100 = \frac{D_{60}}{0.006 mm} \quad \therefore D_{60} = 0.6 mm$$

The best fit is a 50% of A plus 50% of B mix.

Grain-Size Distribution of Soil Mix C

(Revision: Sept.-09)

A sample of soil weights 1.5 N. Its clay fraction weighs 0.34 N. If its liquid limit is 60% and its plastic limit is 26%, classify the clay.

Solution:

W = 1.5 N

W_{clay} = 0.34 N (or 23% of W)

I_p = PI = LL – PL = 60% – 26% = 34 %

$$A = \frac{I_P}{\%\ of\ clay\ fraction} = \frac{34\%}{23\%} \approx 1.5$$

The activity number 1.5 falls above the U-line in Skempton's diagram (see Classify-03). Therefore, this is a CH clay, and is probably a member of the Montmorillonite family.

Classify – 06

During a hydrometer analysis a soil with a G_s = 2.60 is immersed in a water suspension with a temperature of 24°C. An R = 43 cm is obtained after 60 minutes of sedimentation. What is the diameter D of the smallest-size particles that have settled during that time?

Solution:

Using the table below, for G_s = 2.60 and T= 24°C, K= 0.01321.

$$\therefore \quad L = 16.29 - (0.164R) = 16.29 - [0.164(43)] = 9.2 \quad cm$$

$$D = K\sqrt{\frac{L}{t}} = 0.01321\sqrt{\frac{9.2\ cm}{60\ min}} = 0.00517\ mm = 5.2 \times 10^{-3}\ mm\ (\text{a } \textit{silt})$$

Table of constant K versus Temperature T (°C)

Temparature (°C)	G_s							
	2.45	2.50	2.55	2.60	2.65	2.70	2.75	2.80
16	0.01510	0.01505	0.01481	0.01457	0.01435	0.01414	0.01394	0.01374
17	0.01511	0.01486	0.01462	0.01439	0.01417	0.01396	0.01376	0.01356
18	0.01492	0.01467	0.01443	0.01421	0.01399	0.01378	0.01359	0.01339
19	0.01474	0.01449	0.01425	0.01403	0.01382	0.01361	0.01342	0.01323
20	0.01456	0.01431	0.01408	0.01386	0.01365	0.01344	0.01325	0.01307
21	0.01438	0.01414	0.01391	0.01369	0.01348	0.01328	0.01309	0.01291
22	0.01421	0.01397	0.01374	0.01353	0.01332	0.01312	0.01294	0.01276
23	0.01404	0.01391	0.01358	0.01337	0.01317	0.01297	0.01279	0.01261
24	0.01388	0.01365	0.01342	0.01321	0.01301	0.01282	0.01264	0.01246
25	0.01372	0.01349	0.01327	0.01306	0.01286	0.01267	0.01249	0.01232
26	0.01357	0.01334	0.01312	0.01291	0.01272	0.01253	0.01235	0.01218
27	0.01342	0.01319	0.01397	0.01277	0.01258	0.01239	0.01221	0.01204
28	0.01327	0.01304	0.01283	0.01264	0.01244	0.01225	0.01208	0.01191
29	0.01312	0.01290	0.01269	0.01249	0.01230	0.01212	0.01195	0.01178
30	0.01298	0.01276	0.01256	0.01236	0.01217	0.01199	0.01182	0.01169

The fines fraction of a soil to be used for a highway fill was subjected to a hydrometer analysis by placing 20 grams of dry fines in a 1 liter solution of water (dynamic viscosity 0.01 Poise at 20 degrees centigrade). The specific gravity of the solids was 2.65.

a) **Estimate the maximum diameter D of the particles found at a depth of 5 cm after a sedimentation time of 4 hours has elapsed, if the solution's concentration has reduced to 2 grams/ liter at the level.**

At that moment,

b) **What percentage of the sample would have a diameter smaller than D?**

c) **What type of soil is this?**

Solution:

a) Using Stoke's relation: $\quad v = \dfrac{\gamma_s - \gamma_w}{18\eta} d^2 \quad$ or $\quad d\ (mm) = \sqrt{\dfrac{18\eta}{\gamma_s - \gamma_w}\left(\dfrac{L(cm)}{t(\min)}\right)}$

where $\quad t = 4$ hours $= 14,400$ sec, $\quad L = 5$ cm, $\eta = 10^{-2}\,Poise\ \left(i.e.,\ \dfrac{dyne \cdot s}{cm^2}\right)$

$\quad$ but $\ G_s = \dfrac{\gamma_s}{\gamma_w} \ \therefore \gamma_s = G_s \gamma_w = (2.65)(9.81\ \ dynes/cm^3) = 26\ \ dynes/cm^3$

$$d = \sqrt{\dfrac{18x10^{-2}\ \left(\dfrac{dynes \cdot \sec}{cm^2}\right)(5\ cm)}{9.81\left(\dfrac{dynes}{cm^3}\right)(2.65 - 1.00)(14,400\sec)}} = \underline{0.020\ \ mm}$$

b) The unit weight γ of the solution after 4 hours is,

$$\gamma = \dfrac{weight\ of\ soil\ in\ solution}{volume\ of\ solution} = \dfrac{2g + [1000cm^3 - 2g/2.65]\ x\ 1\ g/cm^3}{1000\ cm^3} = 1.001\ g/cm^3$$

The portion of soil having a diameter smaller than D is,

$$Portion\ \ smaller = \dfrac{V}{W} \bullet \dfrac{\gamma_s \gamma_w}{\gamma_s - \gamma_w}\ (\gamma - 1) = \dfrac{1000\ cm^3}{20\ g}\left(\dfrac{2.65\ \ x\ \ 1}{2.65\ \ -\ \ 1}\right)(1.001 - 1) = 0.08$$

$\therefore$ *The remaining soil is only 8% of the original sample.*

c) The diameter $d = 0.020$ mm corresponds to a silt.

(Revision: Sept.-09)

The formula for the relative compaction D_r is, $\qquad D_r = \dfrac{e_{max} - e}{e_{max} - e_{min}}$

Derive an equivalent equation as a function of dry unit weights, such that

$$D_r = \frac{\left(\gamma_{d(field)} - \gamma_{d\,min}\right)}{\left(\gamma_{d\,max} - \gamma_{d\,min}\right)} \frac{\left(\gamma_{d\,max}\right)}{\left(\gamma_{d(field)}\right)}$$

Solution:

$\gamma_{d\,(min)}$ = dry unit weight in loosest condition (voids ratio e_{max})

γ_d = in-situ dry unit weight (voids ratio e)

$\gamma_{d\,(max)}$ = dry unit in densest condition (voids ratio e_{min})

where

$$\gamma_d = \frac{W_S}{V} = \frac{G_S \gamma_W}{1+e} \qquad \text{and} \quad \boldsymbol{D_r} = 0 = \text{loose, to } 1 = \text{very dense}$$

$$D_r = \frac{\dfrac{1}{\gamma_{d\,min}} - \dfrac{1}{\gamma_d}}{\dfrac{1}{\gamma_{d\,min}} - \dfrac{1}{\gamma_{d\,max}}} = \left[\frac{\gamma_d - \gamma_{d\,min}}{\gamma_{d\,max} - \gamma_{d\,min}}\right]\left[\frac{\gamma_{d\,max}}{\gamma_d}\right] = \left[\frac{\gamma_{d(field)} - \gamma_{d\,min}}{\gamma_{d\,max} - \gamma_{d\,min}}\right]\left[\frac{\gamma_{d\,max}}{\gamma_{d(field)}}\right]$$

For example, what is the RC (relative density) of a sand in the field if it was tested to be at 98% Standard Proctor, its maximum unit weight was 18.8 kN/m^3 and its minimum unit weight was 14.0 kN/m^3?

$$RC = 98\% = \frac{\gamma_{d(field)}}{\gamma_{d(Std.Proct.)}} = \frac{\gamma_{d(field)}}{18.8} \quad \therefore \quad \gamma_{d(field)} = 18.4\,kN/m^3$$

$$D_r = \left[\frac{\gamma_d - \gamma_{d\,min}}{\gamma_{d\,max} - \gamma_{d\,min}}\right]\left[\frac{\gamma_{d\,max}}{\gamma_d}\right] = \frac{(18.4-14.0)(18.8)}{(18.8-14.0)(18.4)} = 94\%$$

Classify – 09

The data obtained from relative density tests is shown below. Calculate the range of relative densities.

	Limiting γ	**Average γ in kN/m³**
γ_{max}	**18.07**	**17.52**
γ_{min}	**14.77**	**15.56**
γ_{field}		**16.97**

Solution:

$$D_r = \left(\frac{\gamma_n - \gamma_{min}}{\gamma_{max} - \gamma_{min}} \right)\left(\frac{\gamma_{max}}{\gamma_n} \right)$$

$$range\ 1\ (low\ \gamma_{min})(high\ \gamma_{max}) \quad D_r = \left(\frac{16.97-14.77}{18.07-14.77} \right)\left(\frac{18.07}{16.97} \right) = 0.71$$

$$range\ 2\ (avg\ \gamma_{min})(high\ \gamma_{max}) \quad D_r = \left(\frac{16.97-15.56}{18.07-15.56} \right)\left(\frac{18.07}{16.97} \right) = 0.60$$

$$range\ 3\ (low\ \gamma_{min})(avg\ \gamma_{max}) \quad D_r = \left(\frac{16.97-14.77}{17.52-14.77} \right)\left(\frac{17.52}{16.97} \right) = 0.83$$

$$range\ 4\ (avg\ \gamma_{min})(avg\ \gamma_{max}) \quad D_r = \left(\frac{16.97-15.56}{17.52-15.56} \right)\left(\frac{17.52}{16.97} \right) = 0.74$$

$$\therefore \quad \underline{60\% \quad \leq \quad D_r \quad \leq \quad 83\%}$$

(Revision: Sept.-09)

South Florida has two types of sand, a quartzitic sand (γ_{s2} = 165.5 pcf) and calcareous sand (γ_{s1} = 146.3 pcf). At a particular site, their voids ratios were found to be:

for the quartzitic sand, e_{max} = 0.98 and e_{min} = 0.53

for the calcareous sand, e_{max} = 0.89 and e_{min} = 0.62

These voids ratios were measured by using a mold with a diameter of 4 inches and a height of 4.59 inches. The dry quartzitic sand weight was 3.254 lbs, and the dry calcareous sand was 2.868 lbs. Find their relative densities and dry unit weights. Comment on these results.

Solution:

By definition $D_r = \dfrac{e_{max} - e}{e_{max} - e_{min}}$

For the calcareous sand, $e_1 = \dfrac{\dfrac{\pi d^2 h}{4} - \dfrac{P_1}{\gamma_{S1}}}{\dfrac{P_1}{\gamma_{S1}}} = \dfrac{\dfrac{\pi(4)^2(4.59)}{4} - (\dfrac{2.868}{146.3}) \times 12^3}{\dfrac{2.868 \times 12^3}{146.3}} = 0.70$

For the quartzitic sand, $e_2 = \dfrac{\dfrac{\pi d^2 h}{4} - \dfrac{P_2}{\gamma_{S2}}}{\dfrac{P_2}{\gamma_{S2}}} = \dfrac{\dfrac{\pi(4)^2(4.59)}{4} - (\dfrac{3.254}{165.6}) \times 12^3}{\dfrac{3.254 \times 12^3}{165.6}} = 0.70$

Notice that $e_1 = e_2$

For the calcareous sand, $D_{r_1} = \dfrac{0.89 - 0.70}{0.89 - 0.62} = 0.70$ and

For the quartzitic sand, $D_{r_2} = \dfrac{0.98 - 0.70}{0.98 - 0.53} = 0.62$

The two types of sand have different relative densities because the calcareous sand grains are more tightly packed than the quartzitic sand grains.

For the calcareous sand, $\gamma_{d_1} = \dfrac{\gamma_{s_1}}{1+e_1} = \dfrac{146.3}{1+0.7} = 86.1\,\text{pcf}$ (but $D_{r_1} = 0.70$)

For the quartzitic sand, $\gamma_{d_2} = \dfrac{\gamma_{s_2}}{1+e_2} = \dfrac{165.6}{1+0.7} = 97.4\,\text{pcf}$ (but $D_{r_{21}} = 0.62$)

As a result, the dry unit weight is greater for the soil with the lower relative density.
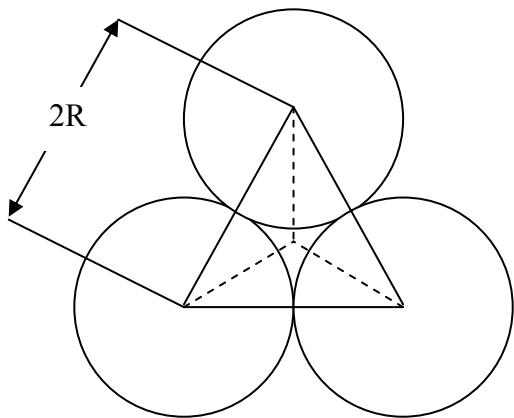
(Revision: Sept.-09)

Prove that $e_{min} = 0.35$.

$$V_{tetr} = 0.1179a^3 = 0.1179(2R)^3 = 0.943R^3$$

$$V_{sphere} = \frac{4}{3}\pi R^3$$
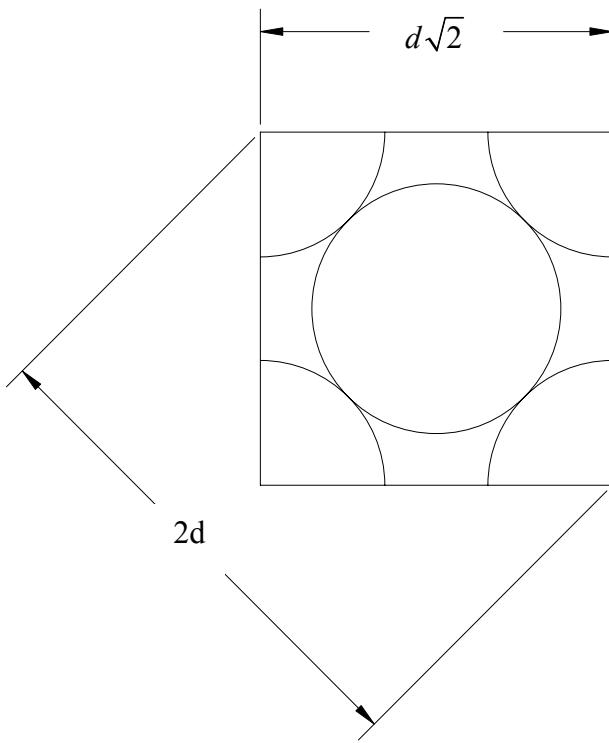
The volume of the sphere occupied by the tetrahedron is:

$$\therefore \quad V_{sphere(tetr)} \equiv \frac{60}{360} = 0.167 = 16.7\%$$

$$\therefore \quad e = \frac{V_V}{V_S} = \frac{V - V_{sphere}}{V_{sphere}} = \frac{0.943R^3 - 0.167(4/3\pi R^3)}{0.167(4/3\pi R^3)} = \underline{0.35}$$

ALTERNATE METHOD:

$$Volume\ of\ cube = \left(d\sqrt{2}\right)^3 = d^3 2\sqrt{2}$$

$$Volume\ of\ sphere = 4\left(\frac{\pi d^3}{6}\right) = \frac{2}{3}\pi d^3$$

$$e_{min} = \frac{V_{cube} - V_{sphere}}{V_{sphere}} = \frac{2d\sqrt{2} - \left(\frac{2}{3}\pi d^3\right)}{\frac{2}{3}\pi d^3} = 0.35$$

Chapter 4
Compaction and Soil Improvement

Symbols for Compaction

$e \rightarrow$ Voids ratio.

$G_S \rightarrow$ Specific gravity of the solids of a soil.

$n \rightarrow$ Porosity of the soil.

$OMC \rightarrow$ Optimum moisture content.

$S \rightarrow$ Degree of saturation.

$V \rightarrow (\frac{1}{30} ft^3 \equiv 9.44*10^{-4} m^3$ Standard Proctor mold, ASTM D-698).

$V_a \rightarrow$ Volume of air.

$V_S \rightarrow$ Volume of solids.

$V_V \rightarrow$ Volume of voids (water + air).

$V_W \rightarrow$ Volume of water.

$w \rightarrow$ Water content.

$V_S \rightarrow$ Volume of soil sample.

$\gamma \rightarrow$ Unit weight of the soil.

$\gamma_d \rightarrow$ Dry unit weight.

$\gamma_b \rightarrow$ Buoyant unit weight of the soil.

$\gamma_{SAT} \rightarrow$ Saturated unit weight of the soil.

$\gamma_S \rightarrow$ Unit weight of the solid.

$\gamma_W \rightarrow$ Unit weight of water.

$\gamma_{d.field} \rightarrow$ Dry unit weight in the field.

(Revision: Aug-08)

A Standard Proctor test has yielded the values shown below. Determine:
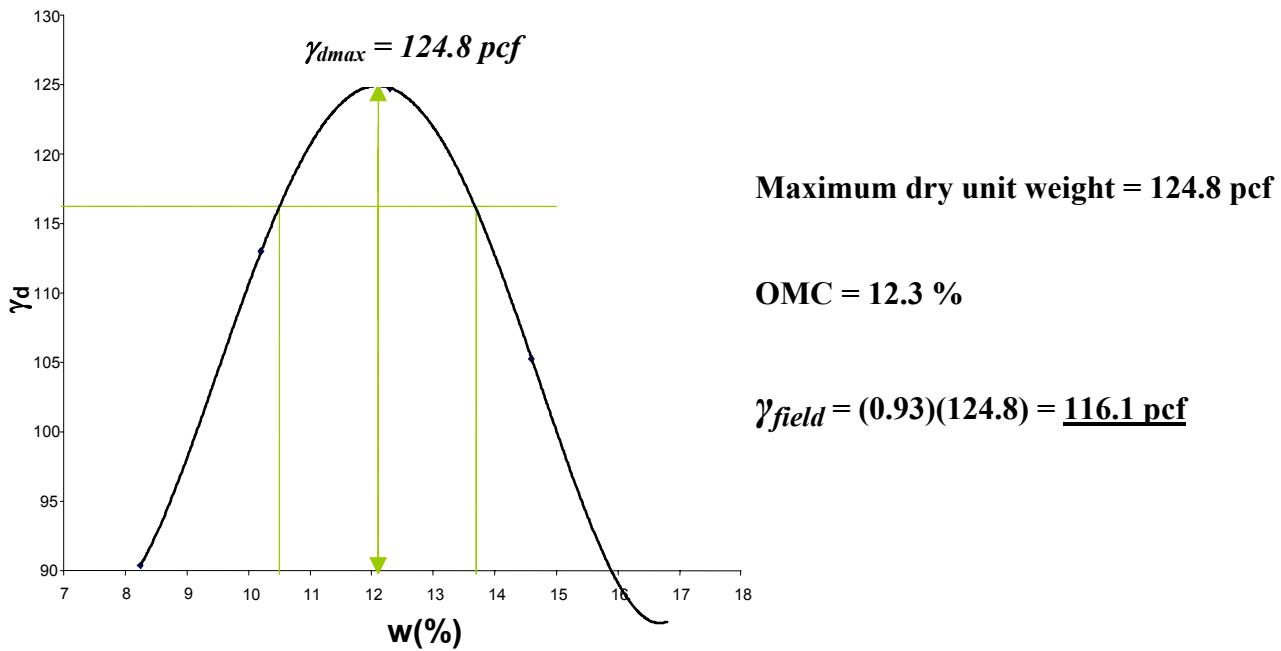
(5) The maximum dry unit weight and its OMC; remember $V = 1/30$ ft^3.

(6) The moisture range for 93% of maximum dry unit weight.

No	Weight of wet soil (lb)	Moisture %
1	3.26	8.24
2	4.15	10.20
3	4.67	12.30
4	4.02	14.60
5	3.36	16.80

Solution:

Formulas used for the calculations: $\gamma = \dfrac{W}{V}$ and $\gamma_d = \dfrac{\gamma}{1+w}$

W (lb)	w(%)	γ (lb/ft^3)	γ_d (lb/ft^3)
3.26	8.24	97.8	90.35
4.15	10.20	124.5	113.0
4.67	12.30	140.1	124.8
4.02	14.60	120.6	105.2
3.36	16.80	100.8	86.30

$\gamma_{dmax} = 124.8\ pcf$

Maximum dry unit weight = 124.8 pcf

OMC = 12.3 %

$\gamma_{field} = (0.93)(124.8) = \underline{116.1\ pcf}$

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(Revision: Aug-08)

Using the table shown below:

(7) Estimate the maximum dry weight of a sample of road base material, tested under Standard Proctor ASTM D-698 (all weights shown are in Newton).

(8) Note that the volume $V = \dfrac{1}{30} ft^3 \left(\dfrac{1\, m^3}{35.32\, ft^3} \right) = 9.44 \times 10^{-4}\, m^3$

(9) Find the OMC.

(10) What is the appropriate moisture range when attaining 95% of Standard Proctor?

Trial No.	1	2	3	4	5
W(Newton)	14.5	15.6	16.3	16.4	16.1
ω (%)	20	24	28	33	37

Solution:

Trial No.	1	2	3	4	5
$\gamma = \dfrac{W}{V} \left(kN/m^3 \right)$	15.4	16.5	17.3	17.4	17.1
$\gamma_d = \dfrac{\gamma}{1+\omega} \left(kN/m^3 \right)$	12.8	13.3	13.5	13.1	12.5
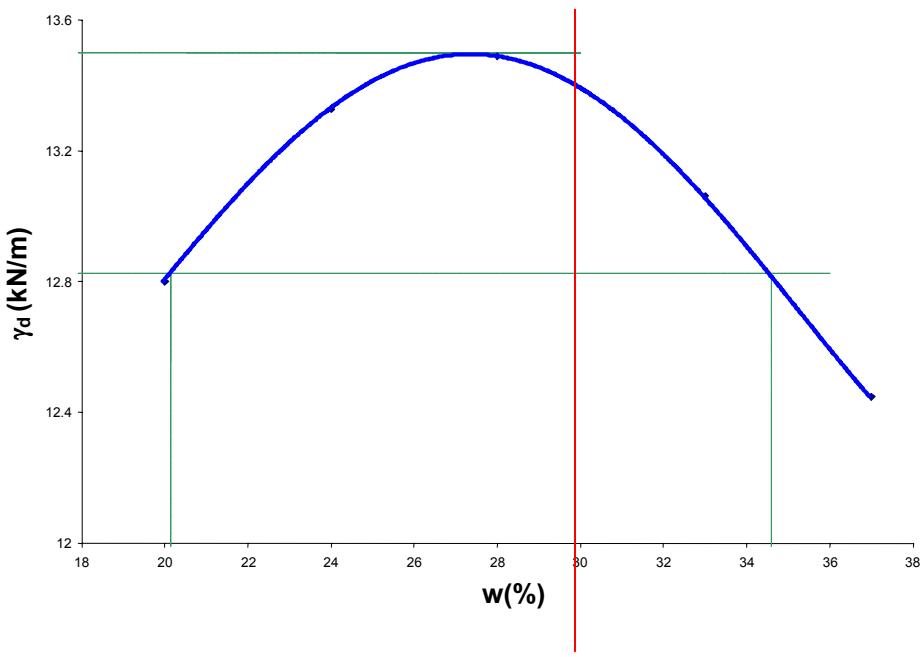
$$\gamma_{d\,max} = 13.5\ kN/m^3$$
$$OMC = 28\ \%$$

$$\gamma_{d\,max} = 13.5\ kN/m^3$$
$$OMC = 28\%$$

$$\gamma_{d\text{-field}} = 0.95(13.5) = 12.8\ kN/m^3$$

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(Revision: Sept.-08)

The results of a Standard Compaction test are shown in the table below:
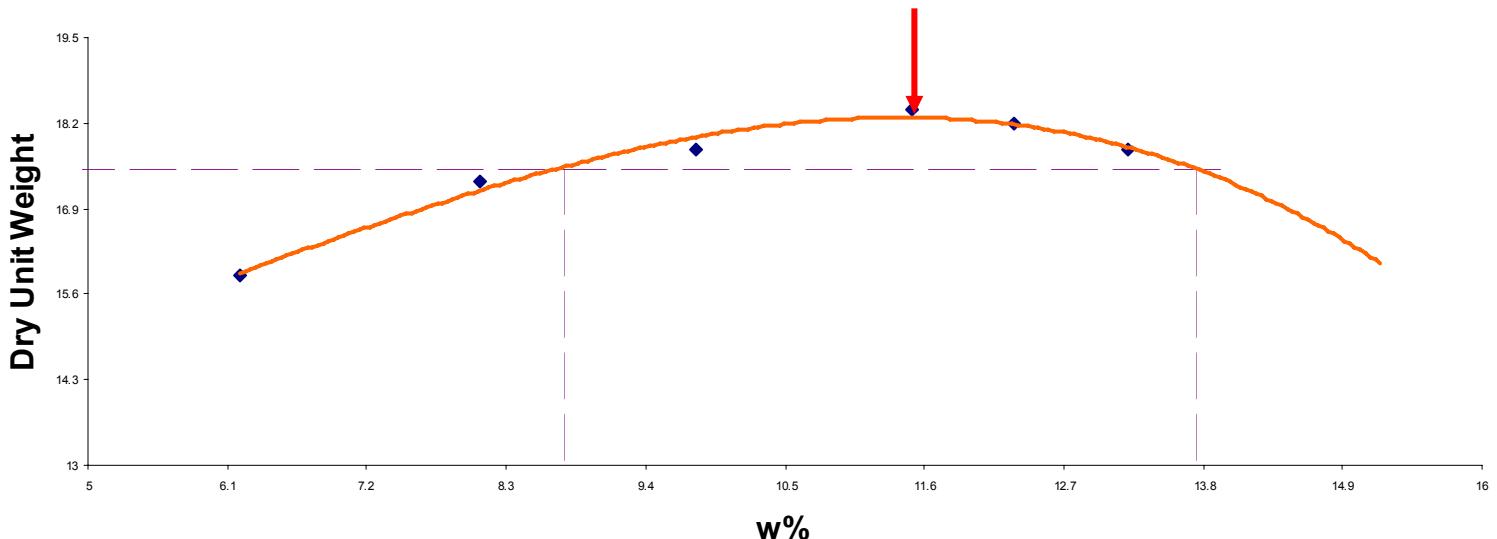
ω (%)	6.2	8.1	9.8	11.5	12.3	13.2
γ (kN/m^3)	16.9	18.7	19.5	20.5	20.4	20.1
$\gamma_d = \dfrac{\gamma}{1+\omega}$	15.9	17.3	17.8	18.4	18.2	17.8

(11)   a) Determine the maximum dry unit weight and the OMC.

(12)   b) What is the dry unit weight and moisture range at 95% RC (Relative Compaction)?

(13)   c) Determine the degree of saturation at the maximum dry density if G_s = 2.70.

Solution:

a) $\gamma_{d\,max}$ = 18.4 kN/m³, OMC = 11.5%
b) $\gamma_{d\,at\,95\%}$ = (0.95)(18.4) = 17.5 kN/m^3

The moisture range w for 95% RC is from 8.75% to 13.75%.

$\gamma_{d\,max}$ = *13.5 kN/m³ OMC = 11.5 %*

c)

$$S = \frac{\dfrac{wG_s\gamma_{d\max}}{\gamma_w}}{G_s - \dfrac{\gamma_{d\max}}{\gamma_w}} = \frac{\dfrac{(0.115)(2.70)(18.4)}{(9.8)}}{2.7 - \left(\dfrac{18.4}{9.8}\right)} = 0.71$$ ***Saturation S = 71%***

*Compaction-04: Number of truck loads required.

(Revision: Sept.-08)

The in-situ moisture content of a soil is 18% and its moist unit weight is 105 pcf. The specific gravity of the soil solids is 2.75. This soil is to be excavated and transported to a construction site, and then compacted to a minimum dry weight of 103.5 pcf at a moisture content of 20%.

a) How many cubic yards of excavated soil are needed to produce 10,000 yd³ of compacted fill?

b) How many truckloads are needed to transport the excavated soil, if each truck can carry 20 tons?

Solution:

$$W_{borrow\ site} = W_{construction\ site}$$

$$V_{borrow\ site}\gamma_{borrow\ site} = V_{construction\ site}\gamma_{construction\ site}$$

$$\gamma_{d(borrow\ site)} = \frac{\gamma}{1+w} = \frac{105\,pcf}{1+0.18} = 89\,pcf \ versus \ \gamma_{d(construction\ site)} = 103.5\,pcf$$

$a)$ Volume to be excavated $= 10,000\ yd^3\left(\dfrac{103.5\,pcf}{89\,pcf}\right) = 11,630\ yd^3$

$b)$ Number of truck loads $= \dfrac{\left(11,630\ yd^3\right)\left(\dfrac{27\,feet^3}{yd^3}\right)\left(\dfrac{105\,lb}{feet^3}\right)}{\left(\dfrac{20\,ton}{truck}\right)\left(\dfrac{2,000\,lb}{ton}\right)} = 824\ truck-loads$

*Compaction-05: *What is the saturation S at the OMC?*

(Revision: Sept.-08)

A Standard Proctor test was performed on a clayey gravel soil; the test results are shown below. Find the degree of saturation at the optimum condition; assume that $G_s = 2.60$.

Test	1	2	3	4	5	6	7
$w\%$	3.00	4.45	5.85	6.95	8.05	9.46	9.90
γ_d / γ_w	1.94	2.01	2.06	2.09	2.08	2.06	2.05
γ_d kN/m³	19.4	20.1	20.6	20.9	20.8	20.6	20.5

Use $\gamma_w = 10$ kN/m³ for simplicity.
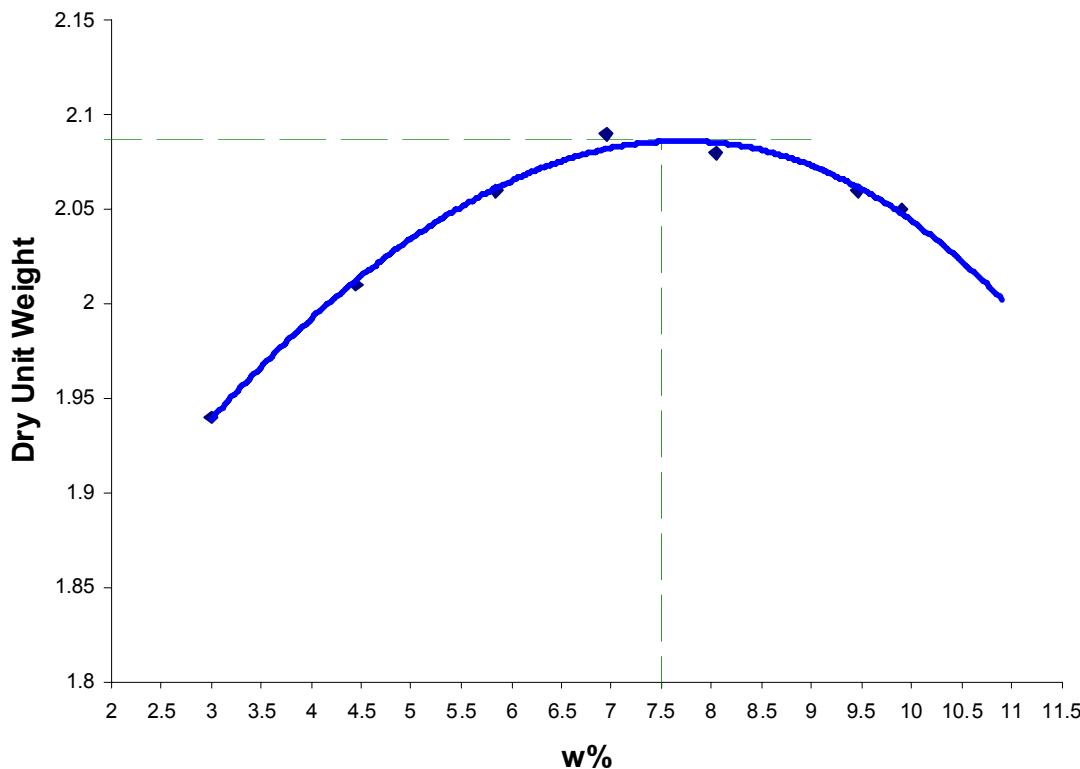
Solution:

It is known that $Se = wG_s$ $\therefore$ $S = \dfrac{wG_s}{e}$ but $e = \left(\dfrac{\gamma_s}{\gamma_d} - 1 \right) = \dfrac{\gamma_s - \gamma_d}{\gamma_d}$

therefore, at the OMC the saturation is,

$$S_{OMC} = w_{OMC}G_s \left(\frac{\gamma_{d-OMC}}{\gamma_s - \gamma_d} \right) = (0.0695)(2.60)\left(\frac{20.9}{(2.60)(10) - 20.9} \right) = 0.74$$

The degree of saturation at the OMC is 74%.

*Compaction-06: Definition of the relative compaction (RC).

(Revision: Sept.-08)

The relative compaction (*RC*) of a sandy road base in the field is 90%. The maximum and minimum dry unit weights of the sand are $\gamma_{d(max)}$ = 20.4 kN/m³ and $\gamma_{d(min)}$ = 13.9 kN/m³. Determine the field values of:

a) The dry unit weight in the field;
b) Relative density (of compaction) D_r;
c) The moist unit weight γ when its moisture content is 15%.

Solution:

The relative compaction *RC* is the dry unit weight obtained in the field, as compared to the Standard Proctor obtained in the laboratory.

a) The relative compaction *RC* is,

$$RC = 0.90 = \frac{\gamma_{d(field)}}{\gamma_{d(max)}} = \frac{\gamma_{d(field)}}{20.4} \quad \therefore \gamma_{d(field)} = (0.90)(20.4) = 18.4\ kN/m^3$$

b) The relative density D_r is,

$$D_r = \frac{\gamma_{d(field)} - \gamma_{d(min)}}{\gamma_{d(max)} - \gamma_{d(min)}} \times \frac{\gamma_{d(max)}}{\gamma_{d(field)}} = \frac{(18.4 - 13.9)}{(20.4 - 13.9)}\left(\frac{20.4}{18.4}\right) = 0.768 = 76.8\%$$

c) The moist unit weight γ is,

$$\gamma = \gamma_d(1 + w) = 18.4(1 + 0.15) = 21.2\ kN/m^3$$

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(Revision: Aug-08)

A Standard Proctor compaction test performed on a sample of crushed limestone (G_s = 2.70) obtained a maximum dry unit weight of 90 pcf at OMC. A field compacted sample showed a moisture of 28% and a unit weight of 103.7 pcf.

Find the relative compaction (*RC*).

Find the degree of saturation *S* of the field soil sample.

Solution:

$$\gamma_{d(field)} = \frac{\gamma_{moist}}{1+w} = \frac{103.7}{1+0.28} = 81.0\,pcf$$

a) $$RC = \frac{\gamma_{d(field)}}{\gamma_{d\max}} = \left(\frac{81.0\,pcf}{90.0\,pcf}\right) = 0.90$$

The Relative Compaction = 90%

b) $$\gamma_d = \frac{G_s\gamma_w}{1+e} \quad \therefore \quad 1+e = \frac{G_s\gamma_w}{\gamma_d}$$

$$\therefore \quad e = \left(\frac{G_s\gamma_w}{\gamma_d} - 1\right) = \frac{(2.70)(62.4)}{(81.0)} - 1 = 1.08$$

$$Se = wG_s$$

$$\therefore \quad S = \frac{wG_s}{e} = \frac{(0.28)(2.70)}{(1.08)} = 0.70 \quad \textit{Saturation S = 70\%}$$

*Compaction-08: Converting volumes from borrow pits and truck loads.

(Revision: Oct.-08)

An embankment for a highway 30 m wide and 1.5 m thick is to be constructed from a sandy soil, trucked in from a borrow pit. The water content of the sandy soil in the borrow pit is 15% and its voids ratio is 0.69. Specifications require the embankment to be compacted to a dry unit weight of 18 kN/m^3. Determine, for 1 km length of embankment, the following:

a) The dry unit weight of sandy soil from the borrow pit required to construct the embankment, assuming that $G_S = 2.70$;

b) The number of 10 m^3 truckloads of sandy soil required to construct the embankment;

c) The weight of water per truck load of sandy soil; and

d) The degree of saturation of the in-situ sandy soil.

Solution:

a) The borrow pit's dry unit weight $\gamma_d = \dfrac{G_S \gamma_w}{1 + e} = \dfrac{2.7(9.8)}{1 + 0.69} = 15.7 \dfrac{kN}{m^3}$

b) The volume of the finished embankment $V = 30m (1.5m)(1km \ long) = 45 \times 10^3 \ m^3$

Volume of borrow pit soil required $= \dfrac{\gamma_{d(reqd)}}{\gamma_{d(borrow \ pit)}} (V) = \left(\dfrac{18}{15.7} \right)(45 \times 10^3) m^3$

Number of truck trips $= \left(\dfrac{18}{15.7} \right) \left(\dfrac{45 \times 10^3 \ m^3}{10 \ m^3} \right) = 5,160$ truck-loads

c) Weight of dry soil in 1 truck-load $W_d = 10 m^3 \left(15.7 \dfrac{kN}{m^3} \right) = 157 \ kN$

Weight of water $= w W_d = (0.15)(157 \ kN) = 23.6 \ kN$ per truck load

d) Degree of saturation $S = \dfrac{w G_S}{e} = \dfrac{(0.15)(2.70)}{0.69} = 0.59 = 59\%$

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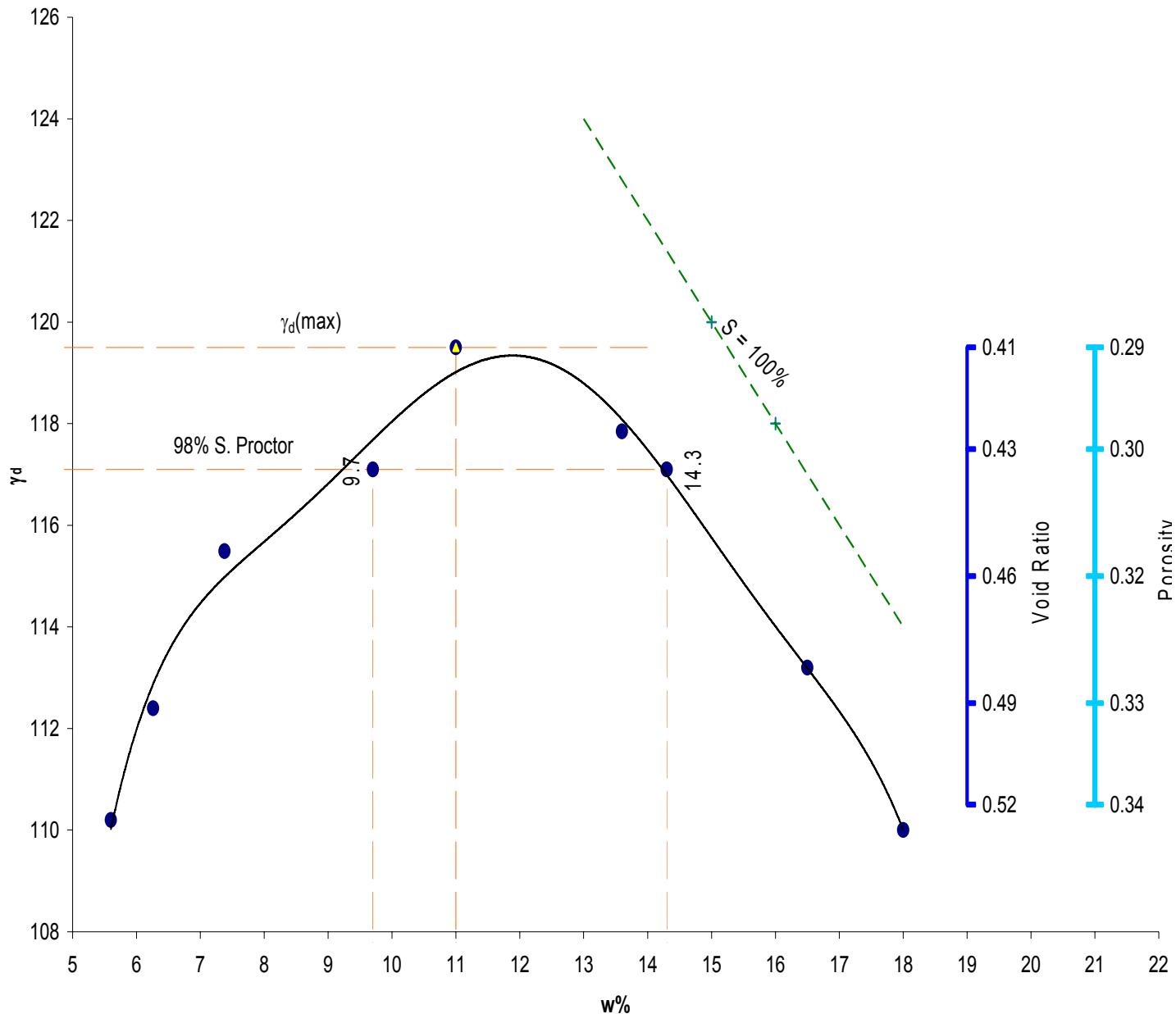
(Revision: Octt.-08)

From the Standard Proctor compaction curve shown below:

Give two possible reasons that may cause a Proctor test to cross the ZAV curve?

What is the water content range (in gallons) needed to build a street 1,000 feet long of compacted 16" base at 98% Standard Proctor for two lanes, each 12 ft wide?

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Solution:

a) Since $\gamma_{zav} = \dfrac{G_s \gamma_w}{1 + \left(wG_s \big/ S \right)}$ and $\gamma_{zav} = \dfrac{\gamma_{wet}}{1+w}$

The crossing of the ZAV (zero air voids) curve is due to an incorrect assumption of G_s, and/or a miscalculation of the water content w and γ_{wet}.

b) The soil volume V required for the road is,

$$V = \left(16\, \text{in thick} \right) \left(\frac{1\,ft}{12\,in} \right) \left(24\,ft\ \text{wide} \right) \left(1,000\,ft\ \text{long} \right) = 32,000\,ft^3$$

The peak dry density $\gamma_{d\,max} = 119.5\,pcf$ at an OMC=12%, and the range of the water content is from 9.7% to 14.3% for 98% of Standard Proctor or $\left(98\% \right) \gamma_{d\,max} = \left(0.98 \right) \left(119.5 \right) = 117.1\,pcf$

The total weight of soil in the pavement is $W = V\gamma = \left(32,000\,ft^3 \right) \left(117.1\,pcf \right) = 3.75x10^6\,lbs$

The weight of water W_w at the low end (9.7%) is,

$$V_w = \frac{W_w}{\gamma_w} = \frac{\left(0.097 \right) \left(\dfrac{3.75x10^6\,lbs}{1.097} \right)}{62.4\,pcf} \left(\frac{7.45\,gal}{ft^3} \right) \square\, 40,000\,gallons$$

The weight of water W_w at the high end (14.3%) is,

$$V_w = \frac{W_w}{\gamma_w} = \frac{\left(0.143 \right) \left(\dfrac{3.75x10^6\,lbs}{1.143} \right)}{62.4\,pcf} \left(\frac{7.45\,gal}{ft^3} \right) \square\, 56,000\,gallons$$

Therefore, the average volume of water is $\left(1/2 \right) \left(56,000 + 40,000 \right) = 48,000\,gallons$

Therefore, the volume of water required $= 48,000 \pm 8,000\ gallons.$

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Compaction-10: *Find the family of saturation curves for compaction.*

(Revision: Oct.-08)
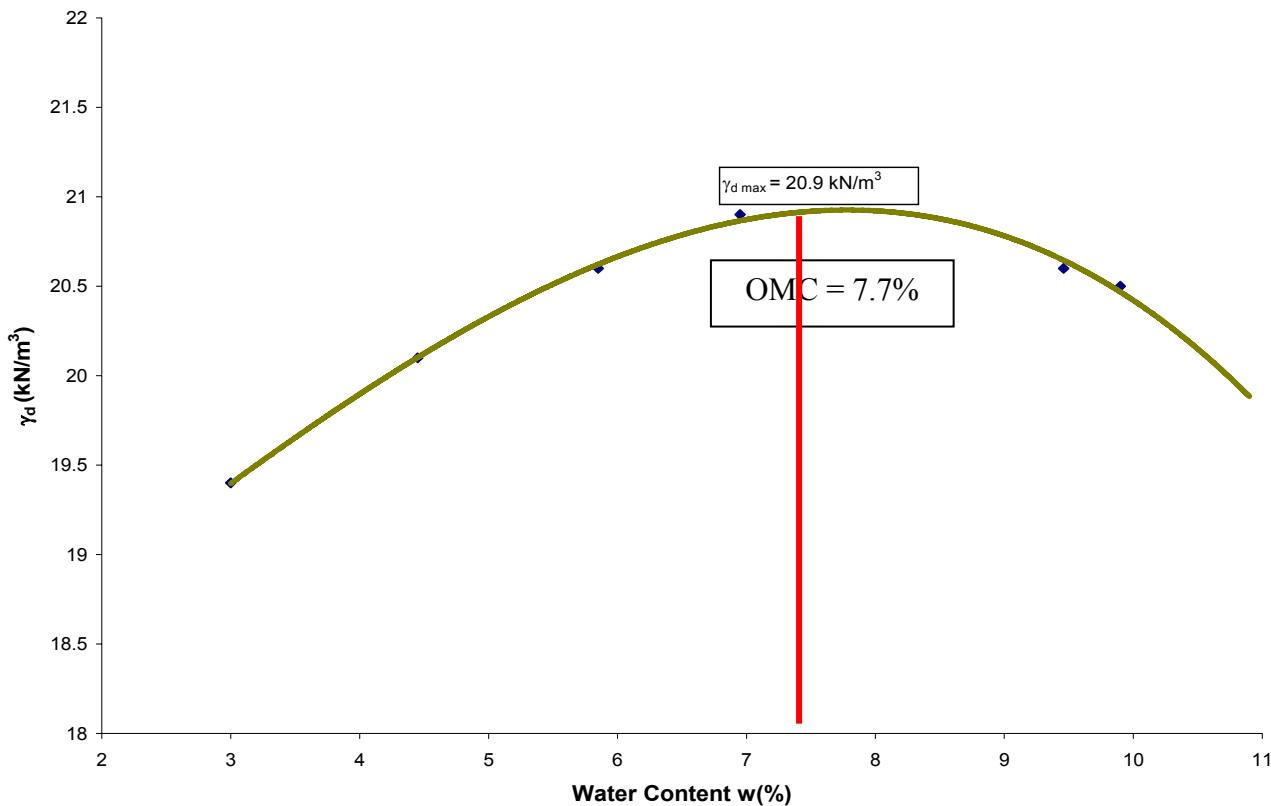
This problem expands Compaction-05: A Modified Proctor compaction test is performed on a clayey gravel road base. The solids have a specific gravity of 2.65. The compaction data yielded the following binomial values for γ_d/γ_w versus w %:

w(%)	3.00	4.45	5.85	6.95	8.05	9.46	9.90
γ_d / γ_w	1.94	2.01	2.06	2.09	2.08	2.06	2.05

(14) Find the $\gamma_{d\,max}$ and the OMC from the compaction curve;

(15) Find the degree of saturation at the above conditions.

(16) Calculate the percentage of air for a given porosity n and the saturation S.

(17) Find the equation that describes the points of equal saturation.

(18) Determine the equation for $S = 100\%$

(19) Discuss the characteristics of this last curve and equation.

Solution:

a)

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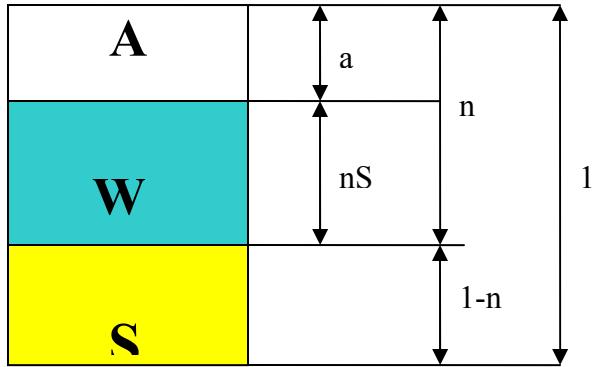
b) $Se = wG_s$ $\therefore$ $S = \dfrac{wG_s}{e}\left(\dfrac{\gamma_{d\max}}{\gamma_w - \gamma_{d\max}}\right) = (7.7\%)(2.65)\left(\dfrac{2.09}{2.65 - 2.09}\right) = 76\%$

c)

$$ns = \frac{V_V}{V}\frac{V_W}{V_V} = \frac{V_W}{V} = \frac{V_W}{1} = V_W$$

$$a = 1 - nS - (1 - n) = 1 - nS - 1 + n$$

$$a = n(1 - S) = V_a$$
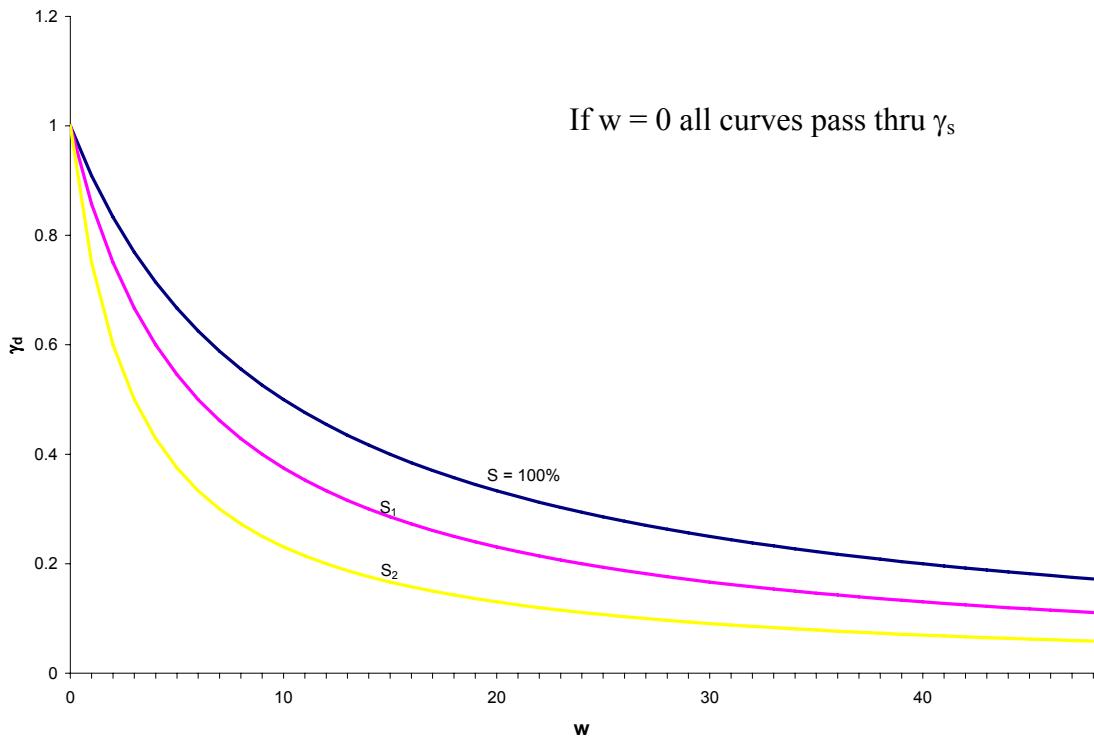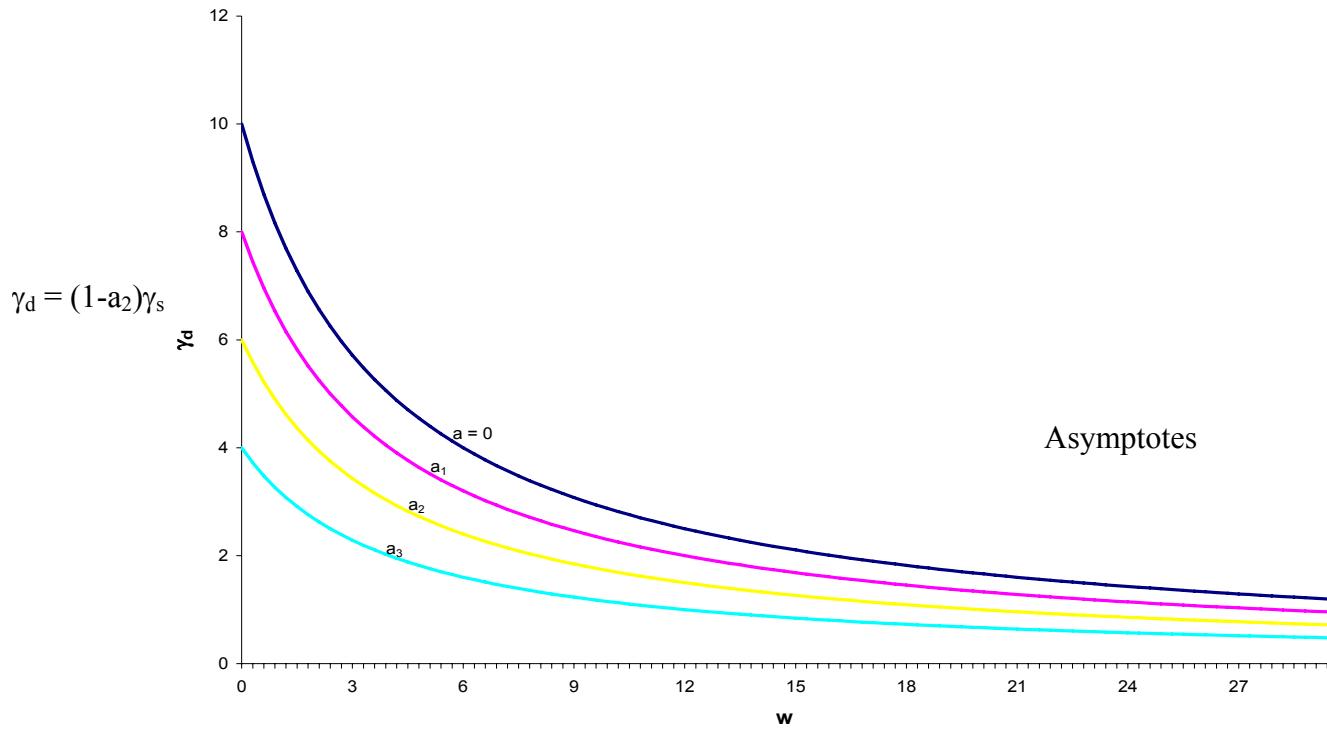
That is, the percentage of air a is equal to the porosity n times the factor (1 - S), where S is the degree of saturation.

d) Consider a family of curves of equal saturation,

$$S = \frac{\text{volume of water}}{\text{volume of air} + \text{volume of water}} = \frac{\text{volume of water}}{1 - \text{volume of soil grains}} =$$

$$S = \frac{w\left(\dfrac{\gamma_d}{\gamma_w}\right)}{1 - \left(\dfrac{\gamma_d}{\gamma_s}\right)} \quad \therefore \gamma_d\left(\dfrac{w}{\gamma_w} + \dfrac{S}{\gamma_s}\right) - S \quad \therefore \dfrac{\gamma_d}{\gamma_w} = \dfrac{S\gamma_s}{w\gamma_s + S\gamma_w}$$

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Curves can be plotted for varying values of *a* and saturation *S*.

$$\gamma_d = (1-a_2)\gamma_s$$

Asymptotes

If w = 0 all curves pass thru γ_s

These are hyperbolas with the w-axis as an asymptote.

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Compaction-11: Water needed to reach maximum density in the field.
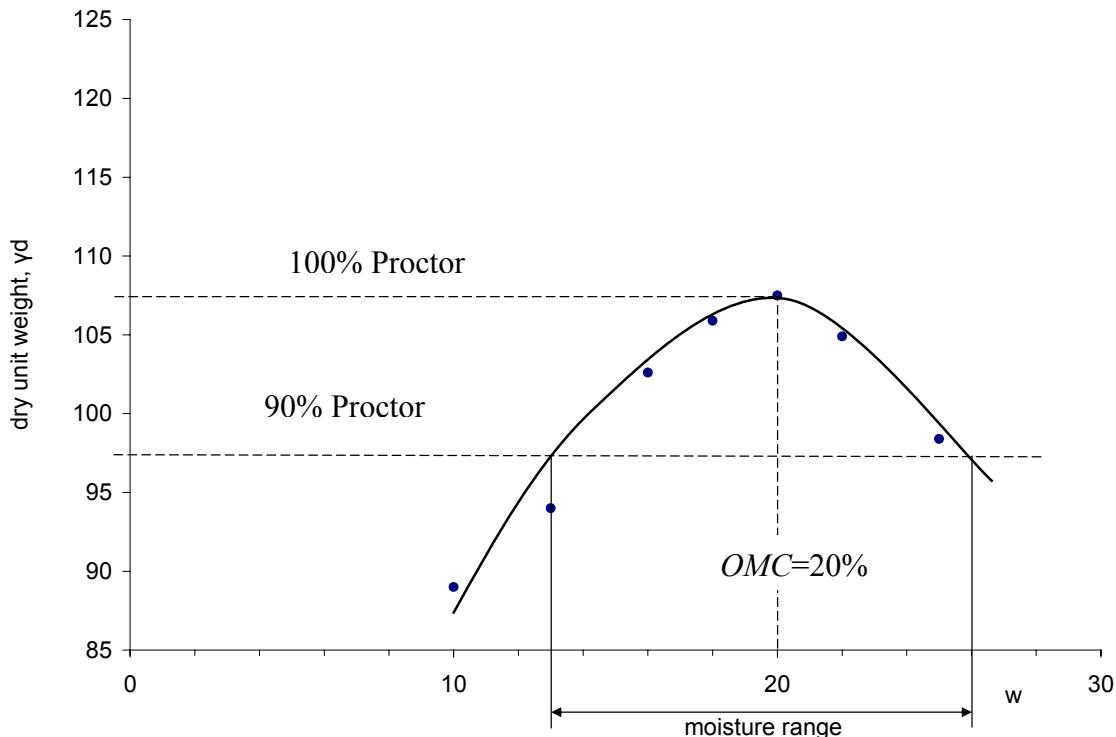
A Standard Proctor test yields the values listed below for a soil with $G_s = 2.71$. Find:

(20)   The plot of the dry unit weight versus the water content;

(21)   The maximum dry unit weight;

(22)   The optimum moisture content;

(23)   The dry unit weight at 90% of Standard Proctor;

(24)   The moisture range for the 90% value;

(25)   The volume of water (in gallons) that must be added to obtain 1 cubic yard of soil at the maximum density, if the soil was originally at 10% water content.

w (%)	10	13	16	18	20	22	25
γ (pcf)	98	106	119	125	129	128	123
γ_d (pcf)	89	94	102.6	105.9	107.5	104.9	98.4

Solution:
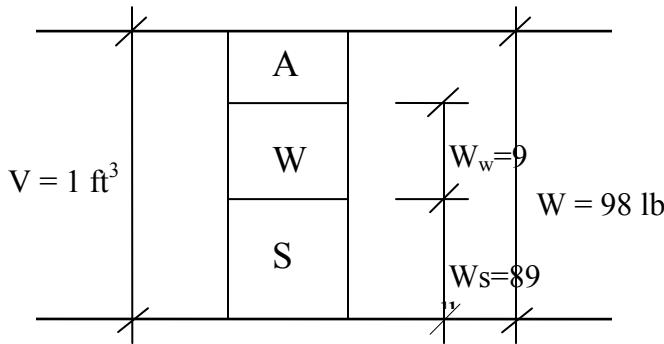
a) The plot of γ_d (dry unit weight) versus w (water content):

(b) From the plot, the maximum dry unit weight is $\gamma_{dmax} =$ **107.5 pcf.**

(c) From the plot, the optimum moisture content is $OMC =$ **20%.**

(d) The 90% value of the maximum dry unit weight $\gamma_{dmax} = (0.9)(107.5) =$ **96.8 pcf**

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(e) The moisture range for the 90% value is approximately from **13% to 26%.**
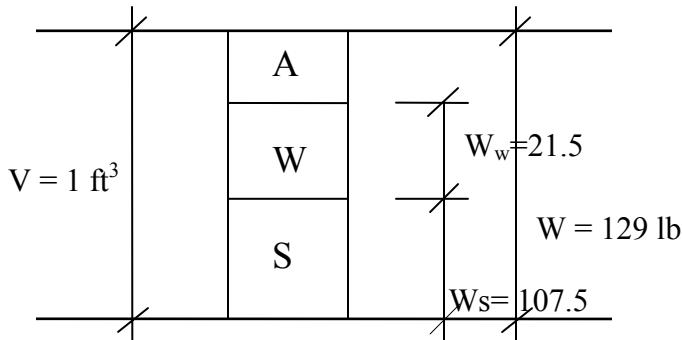
(f) Soil at 10% moisture,

$$w = 0.10 = \frac{W_w}{W_s} \quad \therefore \quad W_w = 0.10W_s$$

but $W_s = 89$ *lb and* $W_w = 0.10W_s = (0.1)(89) = 9\ lb \therefore\ W = W_s + W_w = 89 + 9 = 98\ lb$

Soil at 20% moisture,

$$w = 0.20 = \frac{W_w}{W_s} \quad \therefore \quad W_w = 0.20W_s$$

but $W_s = 107.5$ *lb and* $W_w = 0.20W_s = (0.2)(107.5) = 21.5\ lb \therefore W = W_s + W_w = 129\ lb$

Therefore, need to add the following water: (21.5 lb) – (9 lb) = 12.5 lb/ft^3

$$\therefore \quad Added\ water = \frac{12.5lb}{ft^3} \frac{ft^3}{62.4\,lb} \frac{7.48\,gallons}{ft^3} \frac{27\ ft^3}{1\,yd^3} = 40\ gallons/yd^3$$
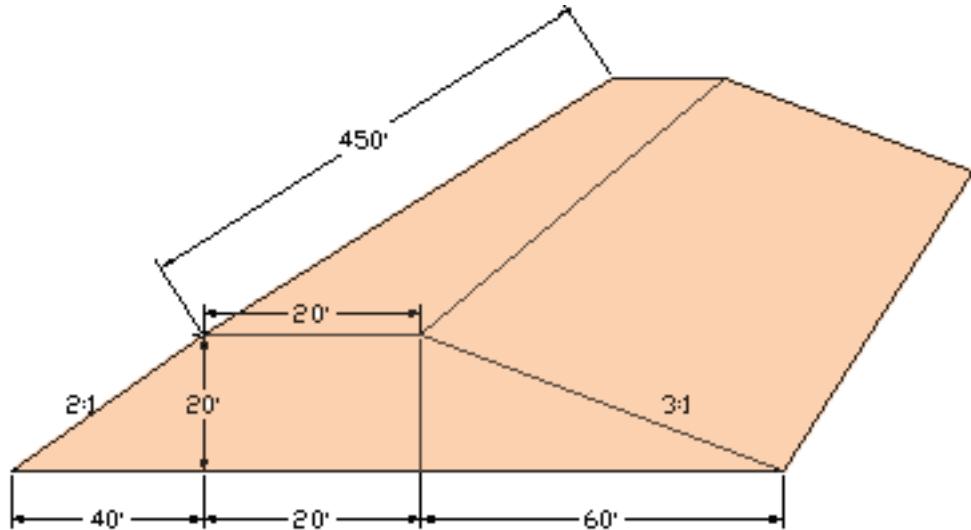
Answer: Add 40 gallons of water per cubic yard of compacted soil.

**Compaction-12: *Fill volumes and truck load requirements for a levee.*

(Revision: Aug-08)

Your company has won a contract to provide and compact the fill material for an earth levee, with the dimensions shown below. The levee fill is a silty clay soil to be compacted to at least 95% of maximum Standard Proctor of γ_d = 106 pcf at an *OMC* of 18%. Your borrow pit has a silty clay with an in-situ moist density of 112.1 pcf at 18%, and a G_s = 2.68. When the soil is excavated and loaded on to your trucks, the voids ratio of the material is e = 1.47. Your trucks can haul 15 cubic yards of material per trip.

(26) Determine the volume of fill required for the levee;

(27) Determine the volume required from the borrow pit;

(28) Determine the required number of truckloads.

Solution:

a) Volume of levee $= \left[\left(\dfrac{1}{2}\right)(20')(40') + (20')(20') + \left(\dfrac{1}{2}\right)(20')(60')\right](ft^2)\left(\dfrac{450\ ft\ long}{27\ ft^3\ /\ cy}\right) = 23,300\ cy$

b) To find the volume required from the borrow pit consider that the weight of solids is the same in both, $W_{s(borrow)} = W_{s(levee)}$

But $\gamma_d = \dfrac{W_s}{V}$ or $W_s = \gamma_d V$ or $\gamma_{d(borrow)}V_{borrow} = \gamma_{d(levee)}V_{levee}$

where $\gamma_{d(borrow)} = \dfrac{\gamma}{1+w} = \dfrac{112.1}{1+0.18} = 95\,pcf$ *and* $\gamma_{d(levee)} = (0.95)(106) = 100.7\,pcf$

$\therefore\ V_{borrow} = V_{levee}\dfrac{\gamma_{d(levee)}}{\gamma_{d(borrow)}} = (23,300\ cy)\left[\dfrac{100.7\,pcf}{95\,pcf}\right] = 24,700cy$

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Number of truck-loads required is based on,

$$W_{s(hauled)} = W_{s(levee)}$$

$$\gamma_{d(hauled)} V_{hauled} = \gamma_{d(levee)} V_{levee}$$

$$\therefore \quad V_{hauled} = V_{levee} \left(\frac{\gamma_{d(levee)}}{\gamma_{d(hauled)}} \right)$$

$$but \quad \gamma_{d(levee)} = 100.7\,pcf \quad and \quad \gamma_{d(hauled)} = \frac{G_s \gamma_w}{1+e} = \frac{(2.68)(62.4)}{1+1.47} = 67.7\,pcf$$

$$V_{hauled} = V_{levee} \left(\frac{\gamma_{d(levee)}}{\gamma_{d(hauled)}} \right) = \left(23,300\ yd^3 \right) \left(\frac{100.7\,pcf}{67.7\,pcf} \right) = 34,700\ yd^3$$

$$\text{Number of truck-loads} = \left(\frac{V_{hauled}}{truck\ capacity} \right) = \left(\frac{34,700\ yd^3}{15\ yd^3 / truck - load} \right) = 2,314$$

(Revision: Aug-08)

A town's new reservoir will impound its fresh water with a small earth dam, rectangularly shaped in plan. The perimeter of the dam will be 2,200 ft long by 1,750 ft wide, and its cross-section is shown below in figure B. The dam is to be built with a silty clay soil, with a specific gravity of 2.70, available from three different local sources. Specifications call for a compacted soil at the dam with a dry unit weight of 97 pcf at an OMC of 31 percent. Assume all voids are totally devoid of any gas (or air). The borrow suppliers quoted the following:

Pit	Price ($/yd^3)	G_s	S (%)	w (%)
A	1.05	2.69	65	22
B	0.91	2.71	49	22
C	0.78	2.66	41	18
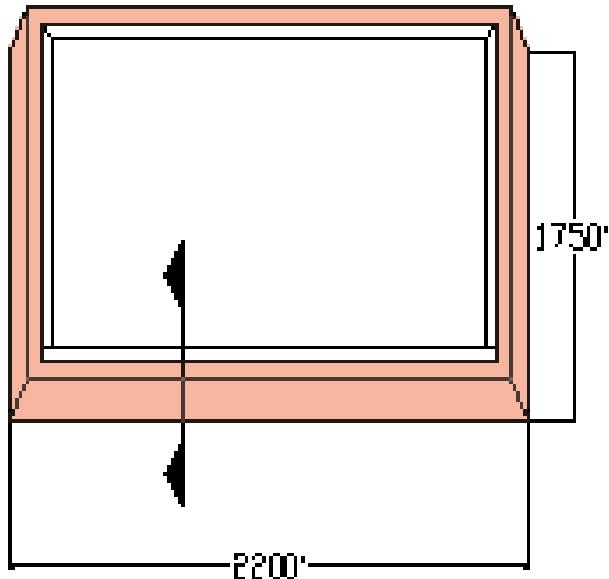
FIGURE A
WATER SUPPLY RESERVOIR
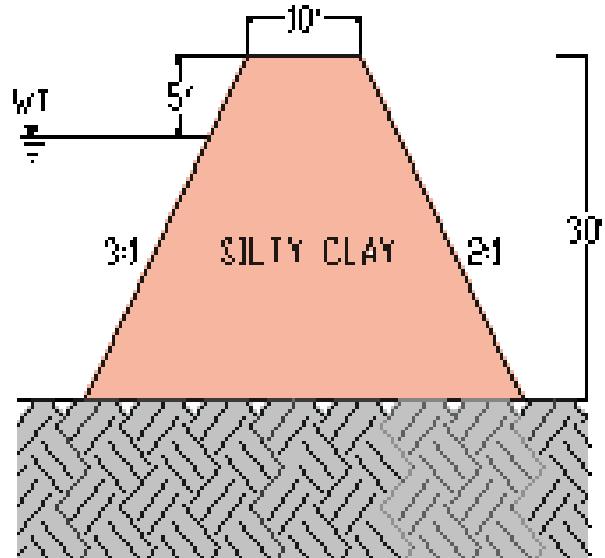
FIGURE B
DAM CROSS-SECTION

Questions:

1. What is the dam's cross-sectional area?
   a) 675 ft^2
   b) 2,100 ft^2
   c) 2,350 ft^2
   d) 2,700 ft^2
   e) 2,550 ft^2

2. What is the approximate total volume V of soil required for the dam?
   a) 2,550 ft^3
   b) 300,000 yd^3
   c) 900,000 yd^3
   d) 1.22 Myd^3
   e) 0.75 Myd^3

3. What is the approximate volume of water impounded (stored) in the dam?
   a) 6,300,000 ft^3
   b) 134 acre-ft
   c) 7 Mft^3
   d) 154 acre-ft
   e) 45 · 10^6 gallons

4. What is the unit weight γ of soil of the compacted earth dam?
   a) 127 pcf
   b) 97 pcf
   c) 86 pcf
   d) 98 pcf
   e) 128 pcf

5. What is the designed voids ratio e of the compacted soil in the dam?
   a) 1.1
   b) 0.92
   c) 0.84
   d) 0.79
   e) 1.2

6. What is the voids ratio e of the material in pit A?
   a) 0.98
   b) 0.91
   c) 1.02
   d) 1.1
   e) 0.72

7. What is the voids ratio e of the material in pit B?
   a) 0.93
   b) 1.22
   c) 0.81
   d) 1.01
   e) 1.00

8. What is the voids ratio e of the material in pit C?
   a) 1.10
   b) 1.12
   c) 1.08
   d) 1.05
   e) 1.17

9. Assume that the voids ratio e of pit A is 0.98. What is the equivalent volume required of pit A to place 1.2 million cubic yards of compacted soil in the dam?
   a) 1.29 Myd^3
   b) 1.20 Myd^3
   c) 0.95 Myd^3
   d) 0.97 Myd^3
   e) 0.96 Myd^3

10. Assume that the voids ratio e of pit B is 0.81, what is the equivalent volume required of pit B to place 1.2 million cubic yards of compacted soil in the dam?
   a) 1.00 Myd^3
   b) 1.02 Myd^3
   c) 1.18 Myd^3
   d) 1.05 Myd^3
   e) 1.07 Myd^3

11. Assume that the voids ratio e of pit C is 1.10, what is the equivalent volume required of pit C to place 1.2 million cubic yards of compacted soil in the dam?
   a) 1.34 Myd^3
   b) 1.37 Myd^3
   c) 1.25 Myd^3
   d) 1.23 Myd^3
   e) 1.21 Myd^3

12. Which pit offers the cheapest fill?
   a) Pit A
   b) Pit B
   c) Pit C
   d) Both A and C
   e) Both B and C

Chapter 5
Permeability of Soils

Symbols for Permeability

$A\rightarrow$ *area of a seepage surface.*

$C\rightarrow$ *Hazen's coefficient.*

$d\rightarrow$ *Diameter of a capillary tube*

$D_x\rightarrow$ *Diameter of a soil % finer (represents % finer by weight)*

$e\rightarrow$ *The voids ratio.*

$G_S\rightarrow$ *Specific gravity of the solids of a soil.*

$h\rightarrow$ *Thickness of the aquifer.*

$H\rightarrow$ *Thickness of the soil layer.*

$h_C\rightarrow$ *Height of the rising capillary*

$i\rightarrow$ *Hydraulic gradient.*

$k\rightarrow$ *coefficient of permeability in D'Arcy's equation .*

$k_H\rightarrow$ *Coefficient of horizontal permeability..*

$k_V\rightarrow$ *Coefficient of vertical permeability*

$L\rightarrow$ *Distance of the hydraulic head loss.*

Ludgeon$\rightarrow$ Standard unit for permeability (10^{-4} mm/sec).

$N_f\rightarrow$ *The number of flow channels in Forheimer's equation.*

$N_{eq}\rightarrow$ *The number of equipotentials drops in Forheimer's equation.*

$p_o\rightarrow$ *In-situ vertical pressure at any depth .*

$q\rightarrow$ *Flow rate (ft^3 per second per foot of width).*

$Q\rightarrow$ *Total seepage (total flow).*

$S_c\rightarrow$ *Seepage capacity.*

$T_S\rightarrow$ *Surface tension (typically given as 0.073 N per meter).*

$u\rightarrow$ *pore water pressure.*

$u_{max}\rightarrow$ *Maximum pore water pressure.*

$\gamma_S\rightarrow$ *Unit weight of solids.*

$\gamma_W\rightarrow$ *Unit weight of water .*

$\sigma_V\rightarrow$ *Vertical effective stress.*

(Revision: Aug-08)

(a) When is it appropriate to use a constant-head permeability test versus a falling-head permeability test?

(b) What are the "standard" units of permeability versus the "common" unit?

<u>**Solution:**</u>

(a) The constant head test is performed for granular soils (gravels **G**, and sands **S**), whereas the falling head test is used for fine-grained (cohesive) soils (silts **M**, and clays **C**).

(b) In Europe, the "standard" unit is the <u>***Ludgeon***</u> (10^{-4} mm/sec), and in the USA the standard unit is the <u>***Meinzer***</u>, which is the rate of flow in gallons per day through an area of 1 square foot under a hydraulic gradient of unity (1 foot/foot).

The "common" unit of permeability is ***cm / sec***.

Permeability-02: *Use of Hazen's formula to estimate the k of an aquifer.*

(Revision: Aug-08)

A test boring was performed at an elevation 955 feet MSL, and it found the phreatic surface (water table) 5 feet below the ground surface. An aquifer stratum was identified, and a sample of its soil showed the grain size distribution below. Estimate the permeability using Hazen's formula with the coefficient $C = 12$. A piezometer (measures the location of the WT) was installed 2500 feet downstream from the boring, and showed its phreatic surface at elevation 942 feet MSL. If the thickness of the aquifer was a uniform 12 feet between both points, estimate the quantity of flow per foot of width in gallons/hour $(1 ft^3 \approx 7.45 gallons)$.
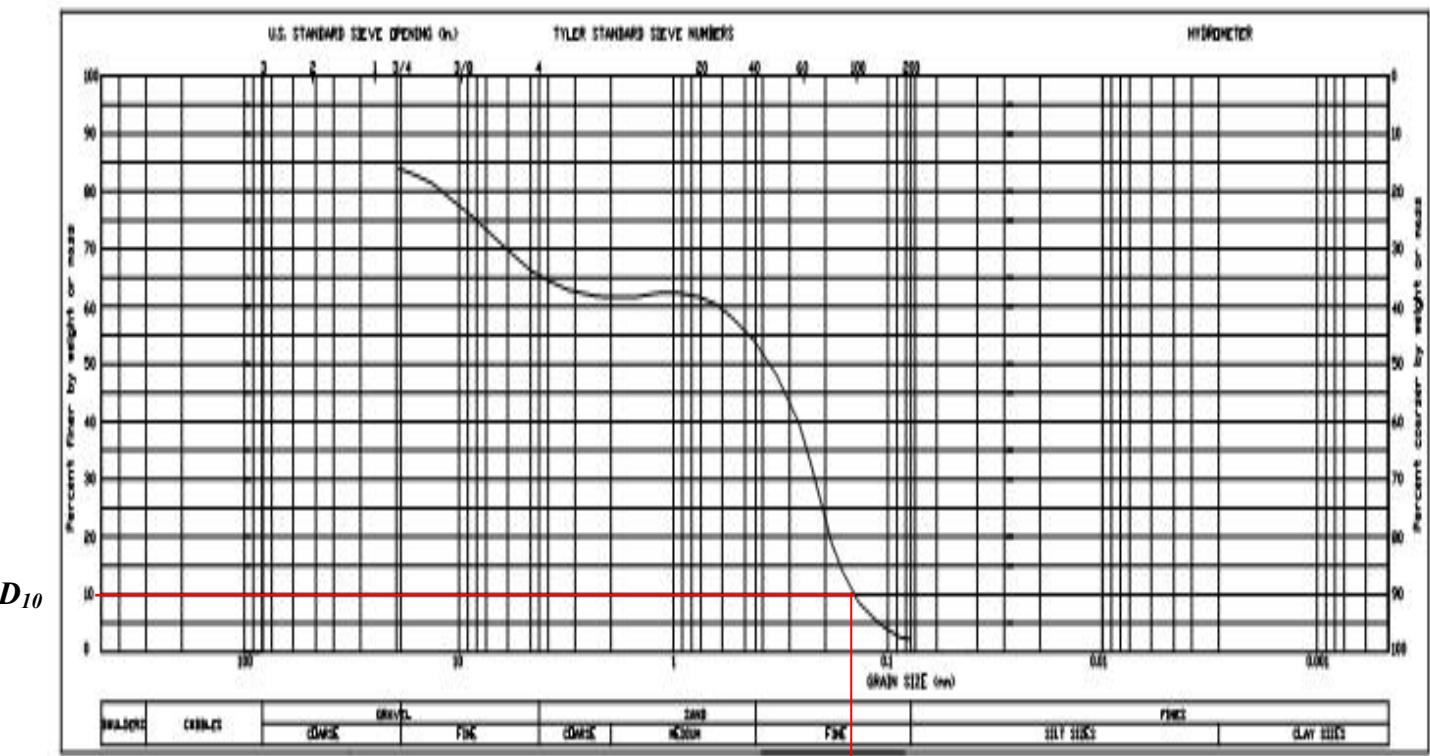
Solution:

Allen Hazen's (1893) formula for the permeability $k = C(D_{10})^2 = (12)(0.16mm)^2 = 0.31 \dfrac{mm}{s}$

The hydraulic head drop is (955 ft – 5 ft) – (942 ft) = 8 feet

Applying D'Arcy formula,

$$q = k\frac{\Delta h}{L}A = \left(0.31\frac{mm}{sec}\right)\left(\frac{3600s}{hour}\right)\left(\frac{1in}{25.4mm}\right)\left(\frac{1ft}{12in}\right)\left(\frac{7.45gal}{1ft^3}\right)\left(\frac{8ft}{2,500ft}\right)(12ft)(1ft) = 1.05\frac{gallons}{hour}\ per\ ft\ width$$
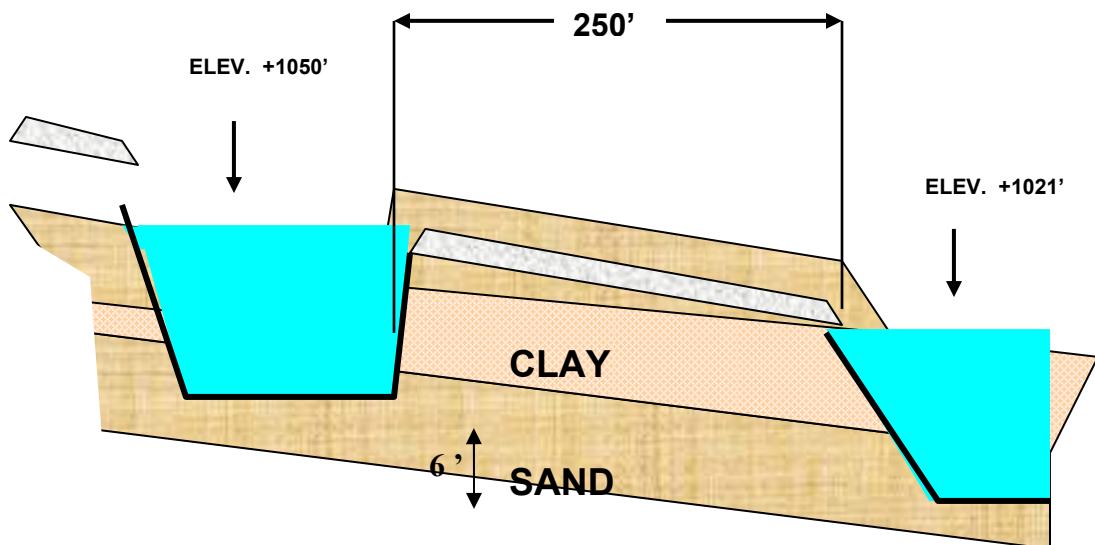
$D_{10} = 0.16\ mm$

GRAIN SIZE DISTRIBUTION DIAGRAM

(Revision: Aug-08)

A canal and a river run parallel an average of 250 feet apart. The elevation of the water surface in the canal is at +1050 feet and in the river at +1021 feet. A stratum of sand intersects both the river and the canal below their water levels. The sand is 6 feet thick, and is sandwiched between strata of impervious clay. Compute the seepage loss q from the canal in $\dfrac{ft^3}{day - mile}$ if the permeability of the sand is 2 x10^{-3} $\dfrac{ft}{\sec}$.

Solution:

D'Arcy's formula for q yields,
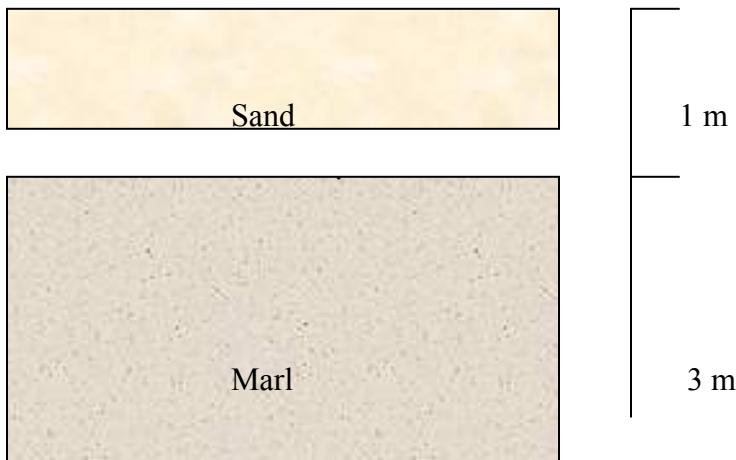
$$q = k\frac{\Delta h}{L} A = (2)(10)^{-3} \frac{ft}{s}\left(\frac{86,400 s}{day}\right)\left[\frac{(1050-1021)ft}{250 ft}\right](6.0 ft)\left(\frac{5,280 ft}{mile}\right)$$

$$q = 635,000 \frac{ft^3}{day - mile}$$

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(Revision: Aug-08)

The topmost layer is loose, clean sand, 1 meter thick. Its vertical permeability k_V can be estimated using Hazen's formula with $C = 1.5$ (to over-estimate) and the sieve analysis shown here. Its k_H is known to be approximately 500% of the k_V. Below the sand stratum is a marine marl, 3 meters thick, with a $k_V = k_H = 10^{-6}$ m/s. What is the combined k_{Hcomb} for the upper 4 m in cm/sec?

Sand	1 m
Marl	3 m

───

Solution:

Use Hazen's formula to find the permeability, where k is in cm/s if C ranges from 0.8-1.5. D_{10} is in *mm*. Use the grain-size distribution curve of Permeability-02.

$$k_{H_1} = 5k_{V_1} = 5(CD_{10}^2) = (5)(1.5)(0.16 \text{ mm})^2 = 1.92 \text{ cm}/s = 1,920(10^{-6}) \text{ m}/s$$

The formula for combining several horizontal layers is,

$$k_{Hcomb} = \frac{k_1 H_1 + k_2 H_2}{H_1 + H_2} = \frac{(1,920x10^{-6} m/s)(1m) + (10^{-6} m/s)(3m)}{1m + 3m} = 4.8 \times 10^{-4} cm/s$$

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(Revision: Aug-08)

The soil profile shown below is typical of Miami-Dade County. Estimate the equivalent permeabilities $k_{V(eq)}$ and $k_{H(eq)}$ in cm/sec, and the ratio of $k_{H(eq)}$ / $k_{V(eq)}$.

1 m	Fine Sand	$k = 1 \times 10^{-1}$ cm/sec	Pamlico Formation
5 m	Porous Limestone	$k = 2 \times 10^{-3}$ cm/sec	Miami Formation
3 m	Fine Sand	$k = 1 \times 10^{-3}$ cm/sec	Fort Thompson Formation
12 m	Upper sandy Limestone	$k = 2 \times 10^{-4}$ cm/sec	Fort Thompson Formation

Solution:

$$k_{V(EQ)} = \frac{H}{\dfrac{H_1}{k_1} + \dfrac{H_2}{k_2} + \dfrac{H_3}{k_3} + \dfrac{H_4}{k_4}} = \frac{(1+5+3+12)\,m}{\dfrac{1\,m}{10^{-1}\,cm/s} + \dfrac{5\,m}{2\times10^{-3}\,cm/s} + \dfrac{3\,m}{10^{-3}\,cm/s} + \dfrac{12\,m}{2\times10^{-4}\,cm/s}}$$

$$= \frac{(2,100\,cm)}{(6,551x10^3)s} = 0.32\times10^{-3}\,cm/s$$

$$k_{H(EQ)} = \frac{1}{H}\left[H_1k_1 + H_2k_2 + H_3k_3 + H_4k_4\right] = \frac{1}{(21\,m)}\left[(1)(10^{-1}) + (5)(2\times10^{-3}) + (3)(10^{-3}) + (12)(2\times10^{-4})\right]$$

$$k_{H(EQ)} = 4.8\times10^{-3}\,cm/s$$
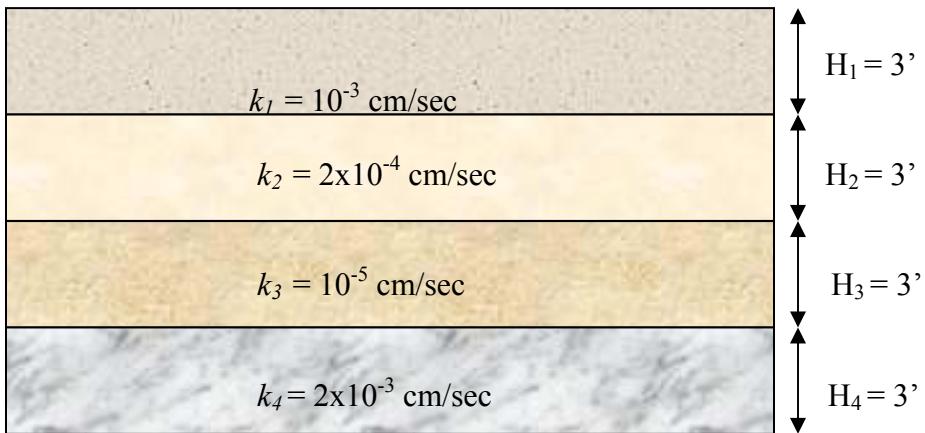
Therefore $\quad \dfrac{k_H}{k_V} = \dfrac{4.8\times10^{-3}\,cm/s}{0.32\times10^{-3}\,cm/s} = 15$

The horizontal permeability is 15 times larger than the vertical permeability.

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(Revision: Aug-08)

Estimate the ratio of the horizontal to the vertical permeability of these four strata.

$k_1 = 10^{-3}$ cm/sec $H_1 = 3'$

$k_2 = 2 \times 10^{-4}$ cm/sec $H_2 = 3'$

$k_3 = 10^{-5}$ cm/sec $H_3 = 3'$

$k_4 = 2 \times 10^{-3}$ cm/sec $H_4 = 3'$

Solution:

The equivalent horizontal permeability of all four layers is:

$$k_{H(eq)} = \frac{1}{H}(k_1 H_1 + k_2 H_2 + k_3 H_3 + k_4 H_4) = \frac{(ft)(cm/s)}{(12ft)}\left[(3)(10^{-3}) + (3)(2)(10^{-4}) + (3)(10^{-5}) + (3)(2)(10^{-3})\right]$$

$$k_{H(eq)} = 8 \times 10^{-4} cm/\sec$$

The equivalent vertical permeability of all four layers is:

$$k_{Veq} = \frac{H}{\dfrac{H_1}{k_{V_1}} + \dfrac{H_2}{k_{V_2}} + \dfrac{H_3}{k_{V_3}} + \dfrac{H_4}{k_{V_4}}} = \frac{12\ ft}{\dfrac{3'}{10^{-3}} + \dfrac{3'}{2 \times 10^{-4}} + \dfrac{3'}{10^{-5}} + \dfrac{3'}{2 \times 10^{-3}}} = 0.37 \times 10^{-4} cm/\sec$$
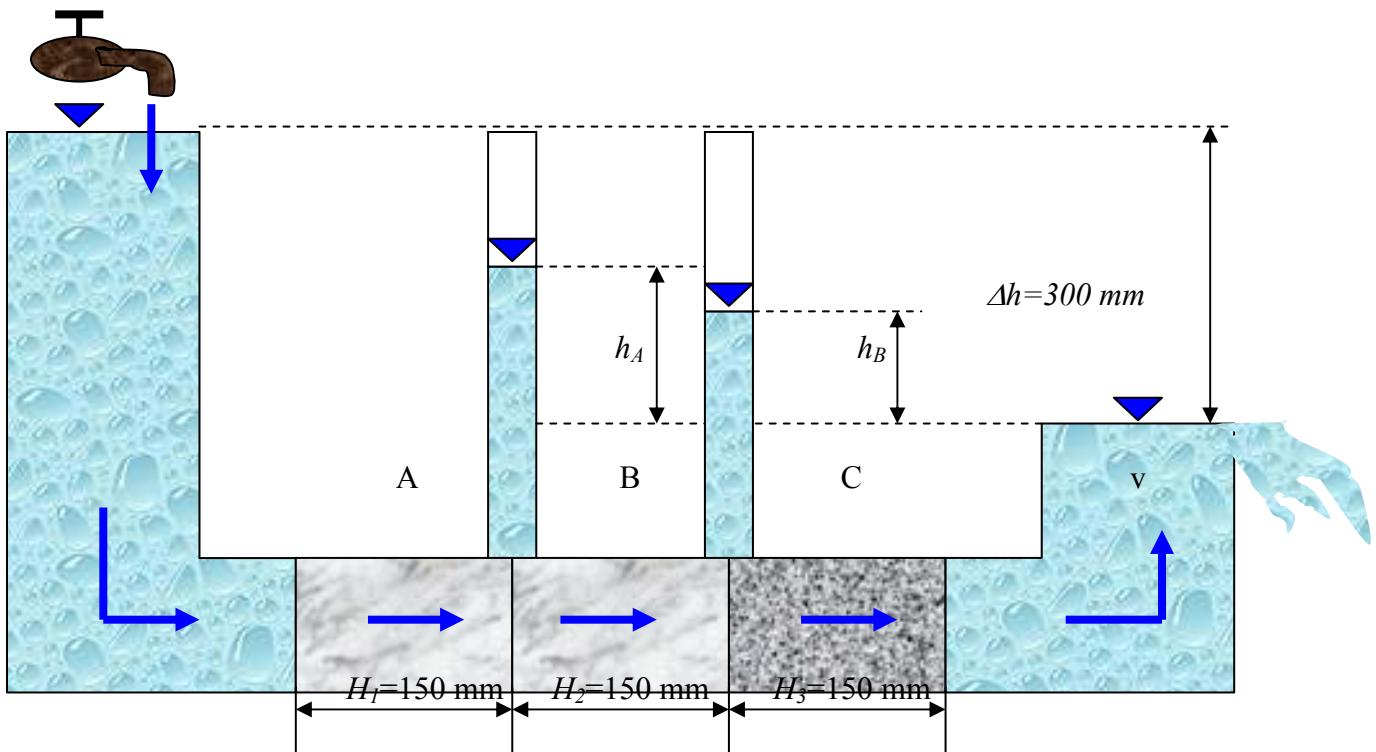
Therefore the ratio of the horizontal to the vertical permeability is:

$$\frac{k_{Heq}}{k_{Veq}} = \frac{8 \times 10^{-4}\ cm/s}{0.37 \times 10^{-4}\ cm/s} = 22$$

*Permeability–07: Do not confuse a horizontal with a vertical permeability.

(Revision: Aug-08)

The soil layers below have a cross section of 100 mm x 100 mm each. The permeability of each soil is: $k_A = 10^{-2}$ cm/sec.; $k_B = 3 \times 10^{-3}$ cm/sec; $k_C = 4.9 \times 10^{-4}$ cm/sec. Find the rate of water supply in cm^3/hr.

Solution:

This is a trick drawing: it "looks" like a horizontal flow, but in reality it is a vertical flow because the flow has to cross through every layer; it can not "bypass" any layer. Therefore, every soil layer has the same flow $v = v_1 = v_2 = v_3$ and the total head $\Delta h = \Delta h_1 + \Delta h_2 + \Delta h_3$.

$$k_{V(eq)} = \frac{H}{\dfrac{H_1}{k_1} + \dfrac{H_2}{k_2} + \dfrac{H_3}{k_3}} = \frac{450mm}{\dfrac{150mm}{1 \times 10^{-2} cm/s} + \dfrac{150mm}{3 \times 10^{-3} cm/s} + \dfrac{150mm}{4.9 \times 10^{-4} cm/s}} = 1.2(10^{-3}) cm/\sec$$

$$\therefore q = k_{eq} iA = k_{eq} \frac{\Delta h}{H} A = (0.0012\, cm/s)\left(\frac{300\, mm}{450\, mm}\right)(10\, cm)(10\, cm)\left(\frac{3,600\, sec}{1\, hr}\right) = 291\, cm^3/hour$$

Permeability-08: Permeability as a function of the voids ratio e.

The coefficient of permeability of fine sand is 0.012 cm/sec at a voids ratio of 0.57. Estimate the increased permeability using the Kozeny-Carman formula of this same sand when its voids ratio has increased to 0.72.

Solution:

Using the Kozeny-Carman formula,
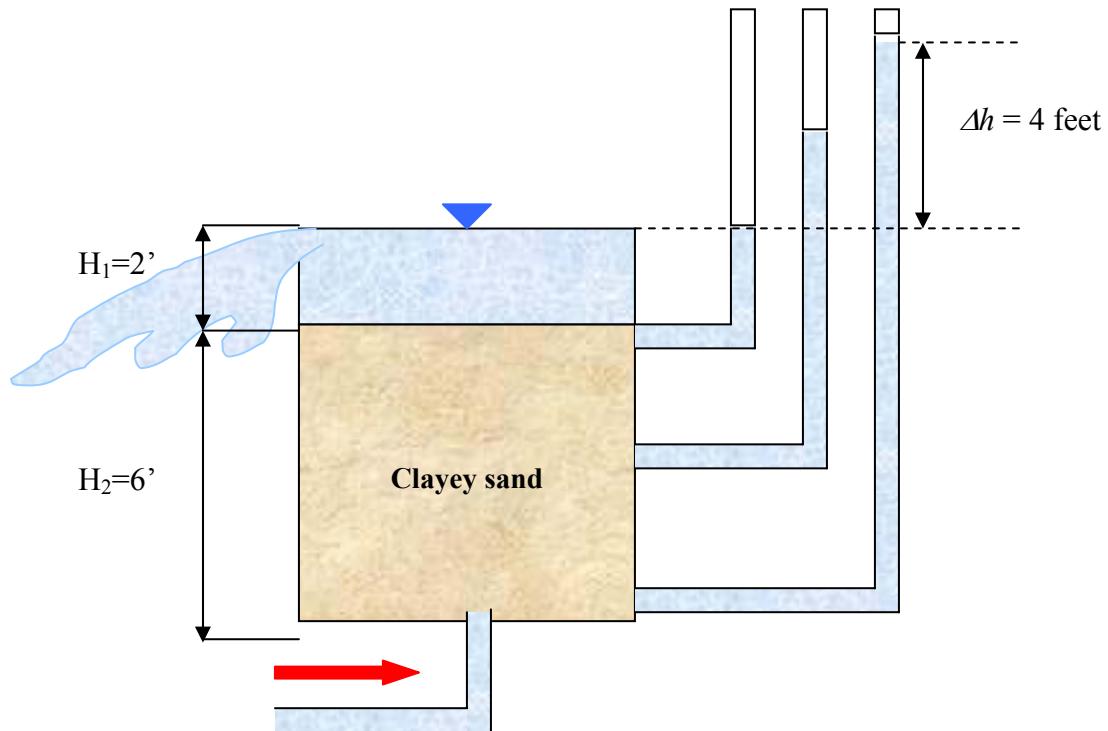
$$k = \frac{e^3}{(1+e)}$$

$$\therefore \frac{k_1}{k_2} = \frac{0.012}{k_2} = \frac{\dfrac{e_1{}^3}{1+e_1}}{\dfrac{e_2{}^3}{1+e_2}} = \frac{\dfrac{(0.57)^3}{1+0.57}}{\dfrac{(0.72)^3}{1+0.72}} = 0.544$$

$$\therefore \quad k_2 = \frac{0.012}{0.544} = 0.022 \; cm/s$$

Notice that since $k_1 = 0.012$ cm/s the permeability has almost doubled. A 26% increase of the voids ratio has effected a doubling the permeability.

*Permeability–09: Uplift pressures from vertical flows.

(Revision: Aug-08)

The soil below is a dense well-graded clayey sand with γ_d = 112 pcf and G_s = 2.63, a permeability k = 240 mm/min at a voids ratio of e = 0.85; the cross-sectional area of the tank is 36 ft². Find (a) the seepage rate q in ft³/min., and (b) the direction of the flow.

Δh = 4 feet

H_1=2'

H_2=6'

Clayey sand

Solution:

a) The hydraulic gradient i and the voids ratio e are:

$$i = \frac{\Delta h}{H_2} = \frac{4\,ft}{6\,ft} = 0.667 \quad and \quad e = \frac{G_s \gamma_w}{\gamma_d} - 1 = \frac{(2.63)(62.4\,pcf)}{112\,pcf} - 1 = 0.465$$

The Casagrande formula relates the known permeability $k_{0.85}$ at e = 0.85 to an unknown permeability k at any voids ratio e,

$$k = 1.4e^2 k_{0.85} = 1.4(0.465)^2 \left(240\frac{mm}{min}\right)\left(\frac{1\,in}{25.4\,mm}\right)\left(\frac{1ft}{12\,in}\right) = 0.239\,ft\,/\min$$

Therefore, the seepage rate q is:

$$q = kiA = \left(0.238\frac{ft}{min}\right)(-0.667)(36\,ft^2) = -5.74\frac{ft^3}{min}$$

b) The flow direction is **UP.**

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(Revision: Sept.-2008)
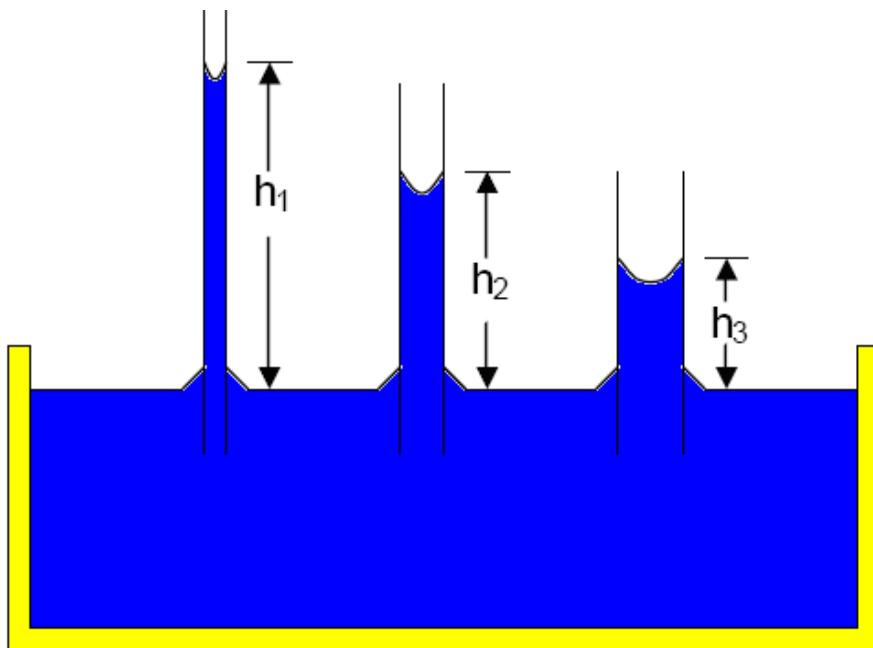
Determine the different heights h_c that water will raise in three different capillary tubes, with diameters:

d_1 = 0.00075 mm (corresponding to a fine clay sized particle),

d_2 = 0.075 mm (corresponding to the smallest sand sized particle) and

d_3 = 0.75 mm (corresponding to a medium sand sized particle).

Assume that the surface tension is 0.075 *N/m* with an angle $\alpha = 3^{o}$.

h_1

h_2

h_3

Solution:

The surface tension of water T_s ranges from about 0.064 to 0.075 *N/m* (0.0044 to 0.0051 lb/ft). In this problem we have chosen the largest value. Notice that the negative sign indicates that the water has risen due to the capillary tension.

$$h_c = \frac{-4\,(\mathrm{T_s})(\cos\alpha)}{d\,\gamma_w} = \frac{-4\,(0.075\,N/m)(\cos 3°)}{d\,(9.81\,kN/m^3)}$$

$$h_1 = \frac{-4\,(7.5\,x10^{-2}\,N/m)(\cos 3°)}{(7.5\,x10^{-7}\,m)(9.81\,kN/m^3)} = -41\ m$$

$$h_2 = \frac{-4\,(7.5\,x10^{-2}\,N/m)(\cos 3°)}{(7.5\,x10^{-5}\,m)(9.81\,kN/m^3)} = -0.41\ m = 410\,mm$$

$$h_3 = \frac{-4\,(7.5\,x10^{-2}\,N/m)(\cos 3°)}{(7.5\,x10^{-4}\,m)(9.81\,kN/m^3)} = -0.041\ m = 41\ mm$$
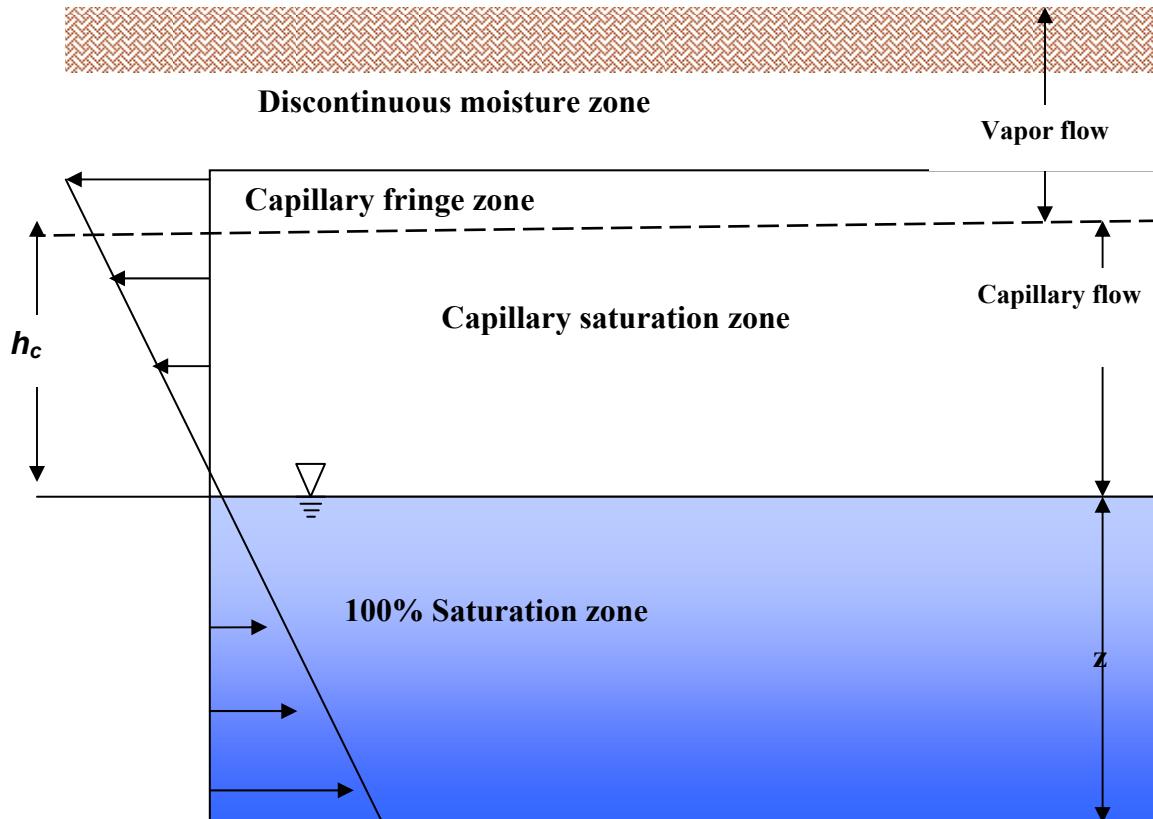
*Permeability-11: Rise of the water table due to capillarity saturation.

How much does the capillary water rise above the water table in a very fine sand (d = 0.1 mm) if the surface tension force is T_o = 0.064 N/m with an α = 3º?

Solution:
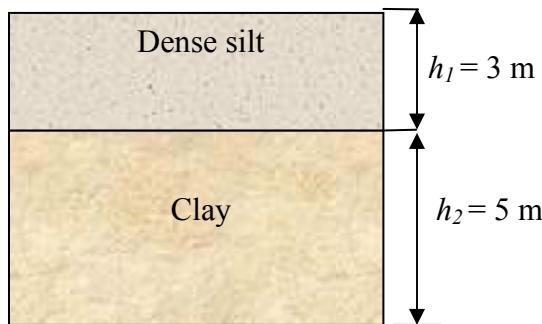
$$h_c = \frac{-4T_o \cos\alpha}{d\gamma_w} = \frac{-4(0.064 N/m)\cos 3°}{(10^{-4}\ m)(9.81\ kN/m^3)} = -0.26\ m$$

Discontinuous moisture zone

Vapor flow

Capillary fringe zone

Capillary flow

Capillary saturation zone

h_c

100% Saturation zone

z

*Permeability-12: Find the capillary rise h_c in a silt stratum using Hazen.

(Revision: Aug-08)

Another method of determining the capillary rise in a soil is to use **Hazen's capillary formula**. The 3 m thick dense silt layer shown below is the top stratum of a construction site, has an effective diameter of 0.01 mm. What is the approximate height of the capillary rise in that silt stratum? What are the vertical effective stresses at depths of 3 m and 8 m below the surface? The "free ground water" level is 8 meters below the ground surface, the $\gamma_S = 26.5\ kN/m^3$, and the soil between the ground surface and the capillary level is partially saturated to 50%.

Solution:

1. The Hazen empirical formula for capillary rise is $h_C = \dfrac{0.0306}{0.2D_{10}} = \dfrac{0.0306}{(0.2)(0.01\ mm)} = 15.3\ m$

In essence, the entire silt stratum is saturated through capillarity.

2. For full saturation, $S = 100\%$,

$$Se = wG_S \quad \therefore \quad w = \frac{Se}{G_S} = \frac{Se\gamma_\omega}{\gamma_S} = \frac{(1)(0.40)(9.81)}{26.5} = 0.148$$

$$\therefore \gamma_{SAT} = \gamma_S \left(\frac{1+w}{1+e}\right) = \frac{(26.5)(1.148)}{1.40} = 21.8\ \frac{kN}{m^3}$$

For 50% saturation,

$$w = \frac{Se\gamma_\omega}{\gamma_S} = \frac{(0.5)(0.40)(9.81)}{26.5} = 0.074$$

$$\therefore \gamma = \gamma_S \left(\frac{1+w}{1+e}\right) = \frac{(26.5)(1.074)}{1.40} = 20.3\ \frac{kN}{m^3}$$

Therefore,
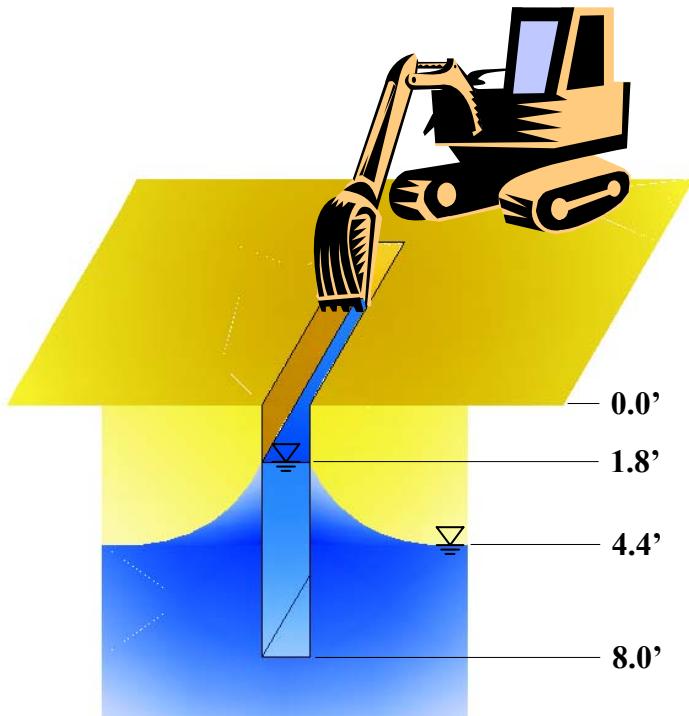
$$\therefore (\sigma_V')_3 = h_1\gamma = (3)(20.3) = \underline{61\ kPa}$$

$$\therefore (\sigma_V')_8 = h_1\gamma + (h_2)\gamma_{SAT} = (3)(20.3) + (5)(21.8) = \underline{170\ kPa}$$

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*Permeability-13: Back-hoe trench test to estimate the field permeability.

(Revision: Sept.-08)

A common method of determining a site's drainage capabilities is the constant-head trench percolation test shown below. The trench is dug by a backhoe to roughly the dimensions shown. The testing crew uses a water truck to fill the trench with water above the WT, and then they attempt to maintain the head constant for about 10 minutes. The amount of water that has flowed out during the test is Q (in gallons/minute). The seepage capacity $S_c = Q/CL'H$, where C is a units conversion factor, L' is the trench semi-perimeter (length plus width, in ft) and H is the head. The units of the seepage capacity are commonly given in *cfs/ft/ft*. Based on the reported geometric conditions shown below, and that the crew used 1,540 gallons during a 10 minutes test, what is the surface seepage capacity of that site?

— 0.0'

— 1.8'

— 4.4'

— 8.0'

Solution:

$$Q = \frac{1,540 \, gal}{10 \, min} = 154 \; gallons / minute$$

The perimeter of the trench is $L' = 2(6\,ft + 1.5\,ft) = 15\,ft$ and $H = (4.4\,ft - 1.8\,ft) = 2.6\,ft$

The seepage capacity of the surface stratum is S_c,
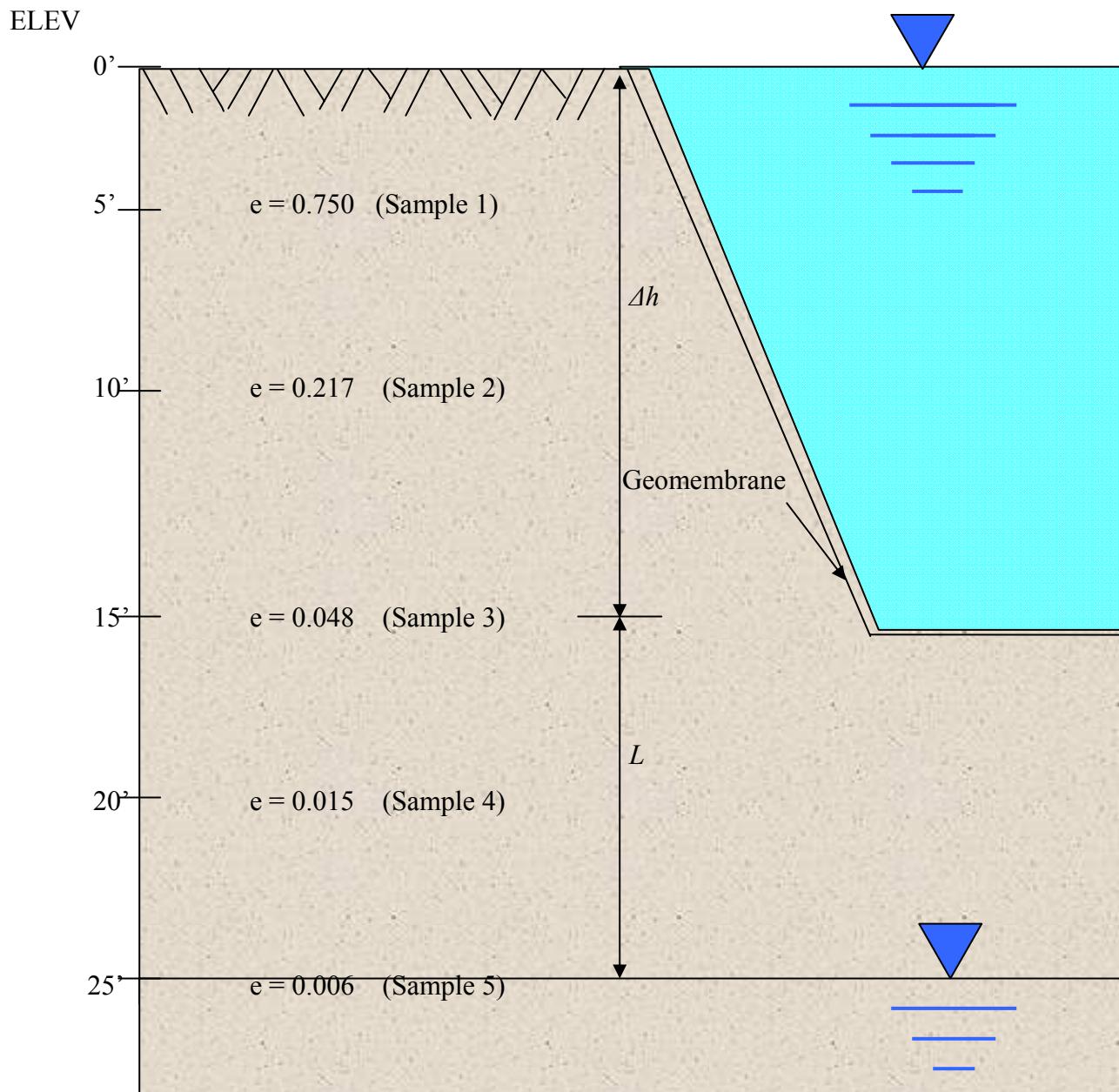
$$S_c = \frac{Q}{C \cdot L' \cdot H} = \frac{154 \; gallons/min \; (ft^3)(min)}{(15\,ft)(2.6\,ft)(7.45 \, gallons)(60 \, sec)} = 8.8x10^{-3} \; cfs / ft / ft$$

Borings were taken at the site of an intended impoundment pond and the in-situ voids ratios at various depths are shown in the figure below. A constant-head permeability test was performed on sample #1 (which was 6" high and 2" in diameter) subjected to a pressure head of 27": after 5 seconds, 50 grams of water were collected through the sample.

Impoundment pond area = 4.4 miles^2

ELEV

(29) Determine the permeability of sample #1;

A trial test of a vibroflotation (densification) probe was taken to a depth of 5 feet and showed a densified voids ratio $e = 0.55$. If this densification ratio is attained to the full depth of 25 feet, at

what depth (to the nearest 5 feet) would you place the bottom of the pond in order to keep the total seepage _Q_ below 50,000 gal/min?

Assume seepage only through the bottom of the pond, and that the pond is kept filled.

Solution:

Step 1: Determine the permeability _k_ of sample # 1.

From Darcy;   Q= _k i_ A, therefore $k = \dfrac{q}{iA} = \dfrac{4q}{i\pi d^2}$

Since 50 grams of water is equivalent to 50 cm^3 (for γ_w = 1g/cm^3)

but , $q = \dfrac{50cm^3}{5\,sec} = 10\dfrac{cm^3}{sec}$,   and $i = \dfrac{\Delta h}{L} = \dfrac{27in}{6in} = 4.5$

$\therefore \quad k_a = \dfrac{4q}{i\pi d^2} = \dfrac{(4)\left(10\dfrac{cm}{in}\right)(10mm/cm)}{4.5\pi\,(6in)^2\left(2.54\dfrac{cm}{in}\right)^2} = 0.122\,\dfrac{mm}{sec}$

Step 2: Determine the new permeability of the sand due to vibroflotation densification.

Using Casagrande's relation $\qquad\qquad k = 1.4k_{0.85}e^2$

Therefore $\qquad\qquad k_b = k_a * \dfrac{e_a^2}{e_b^2} = 0.122\dfrac{(0.55)^2}{(0.75)^2} = 0.066\ mm/\sec$

Step 3: The ratio of densified permeability to in-situ permeability

$\dfrac{k_b}{k_a} = \dfrac{0.066}{0.122} = 0.54$ $\therefore$ Vibroflotation has reduced the permeability by half.

Step 4: Find the densified permeability at each sample depth,

$$k = 1.4k_{0.85}e^2 \quad \text{therefore} \quad k_{0.85} = \frac{k}{1.4e^2} = \frac{(0.122mm/\sec)}{1.4(0.75^2)} = 0.155 \ mm/\sec$$

The corresponding densified permeability

$$k = 1.4k_{0.85}e^2 \quad \text{(x ratio)}$$

Depth (feet)	Original e_0	Original k (mm/sec)	Densified k (mm/sec)
0.	0.	0.	0.
-5	0.750	0.122	0.066
-10	0.217	0.010	0.005
-15	0.048	0.005	2.76×10^{-4}
-20	0.015	5.08×10^{-5}	2.74×10^{-5}
-25	0.006	1.03×10^{-5}	0.56×10^{-5}

Step 5: Estimate the required depth h of the pond.

$Q = 50{,}000$ gallons/min $(1\text{ft}^3 / 7.48 \text{ gal}) = 6{,}680 \text{ ft}^3$/min.

Consider this rate to be constant.

From Darcy's equation $\quad Q = kiA \quad$ or $\quad ki = \dfrac{k\Delta h}{L} = \dfrac{Q}{A}$

$$\frac{Q}{A} = \frac{\left(6,680 \ ft^3 \middle/ \min\right)\left(\dfrac{1mm}{3.281 \times 10^{-3} \ ft}\right)}{\left(4.4mile^2\right)\left(5280 \ ft \middle/ mile\right)^2 (1\min)\left(60 \ \sec \middle/ \min\right)} = 2.79 \times 10^{-4} \ mm \middle/ \sec$$

Consider the depth -15 ft.

$$k\frac{\Delta h}{L} = \frac{15}{10}\left(2.76x10^{-4}\frac{mm}{\sec}\right) = 4.14x10^{-4}\,mm/\sec$$
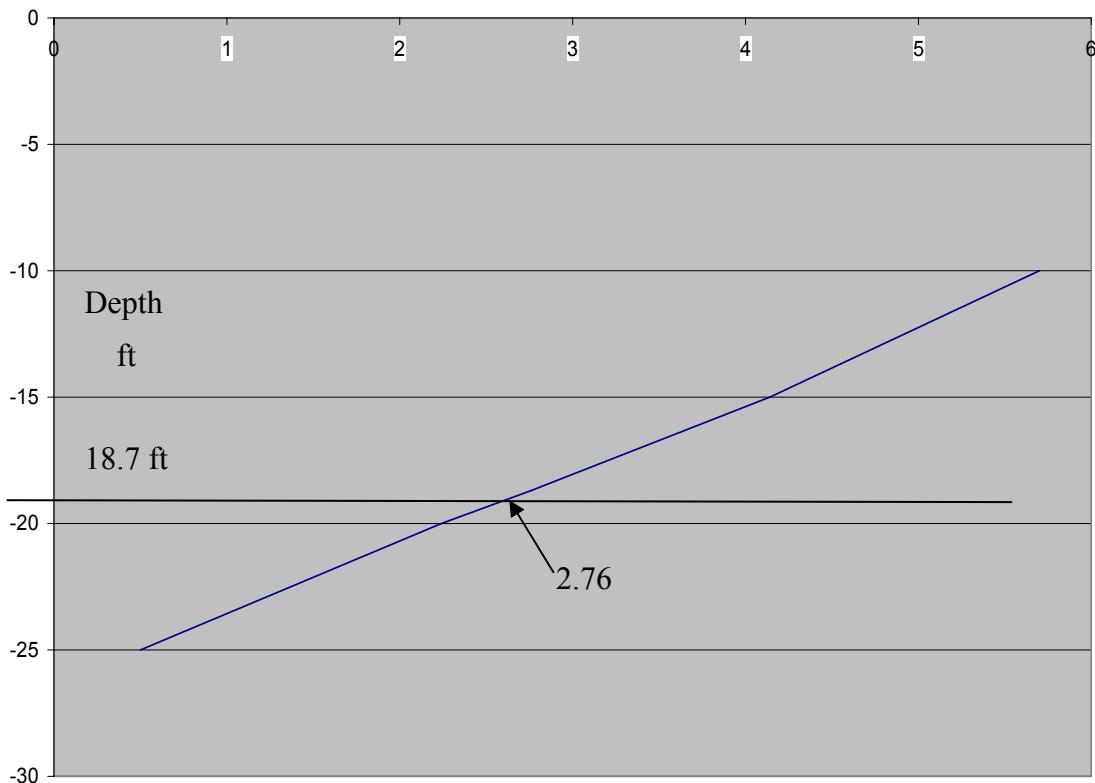
or at a depth of: -20 ft.

$$k\frac{\Delta h}{L} = \frac{20}{5}\left(0.56x10^{-5}\frac{mm}{\sec}\right) = 0.224x10^{-4}\frac{mm}{\sec}$$

Therefore, place the bottom of pond at -19 feet.

Chapter 6
Seepage and Flow-nets
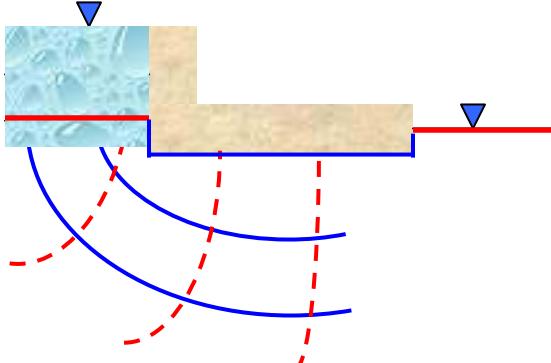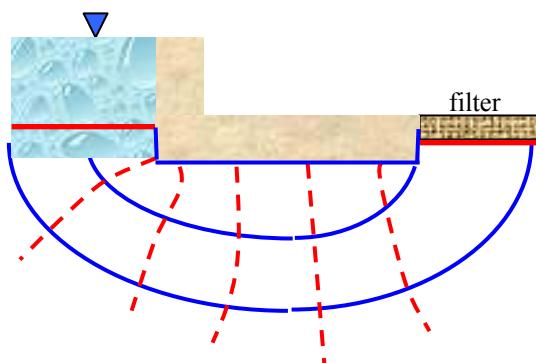
(Revision: Aug-08)

Do you recognize something wrong with each of the following flow-nets?

a)

b)

filter

c)

Well
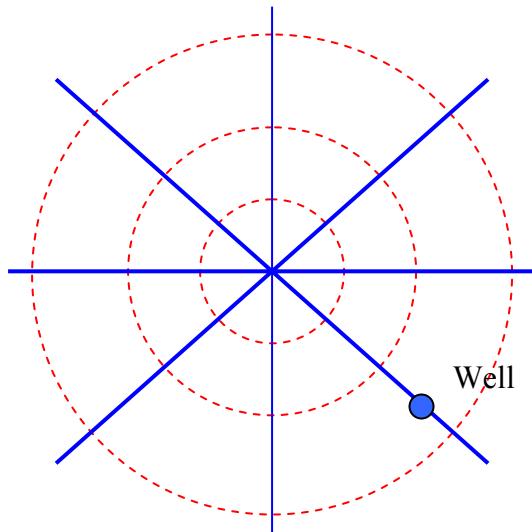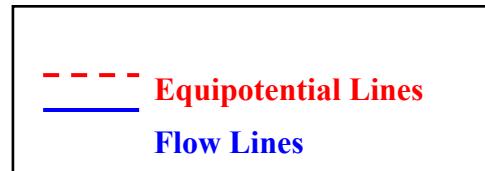
- - - -	**Equipotential Lines**
————	**Flow Lines**
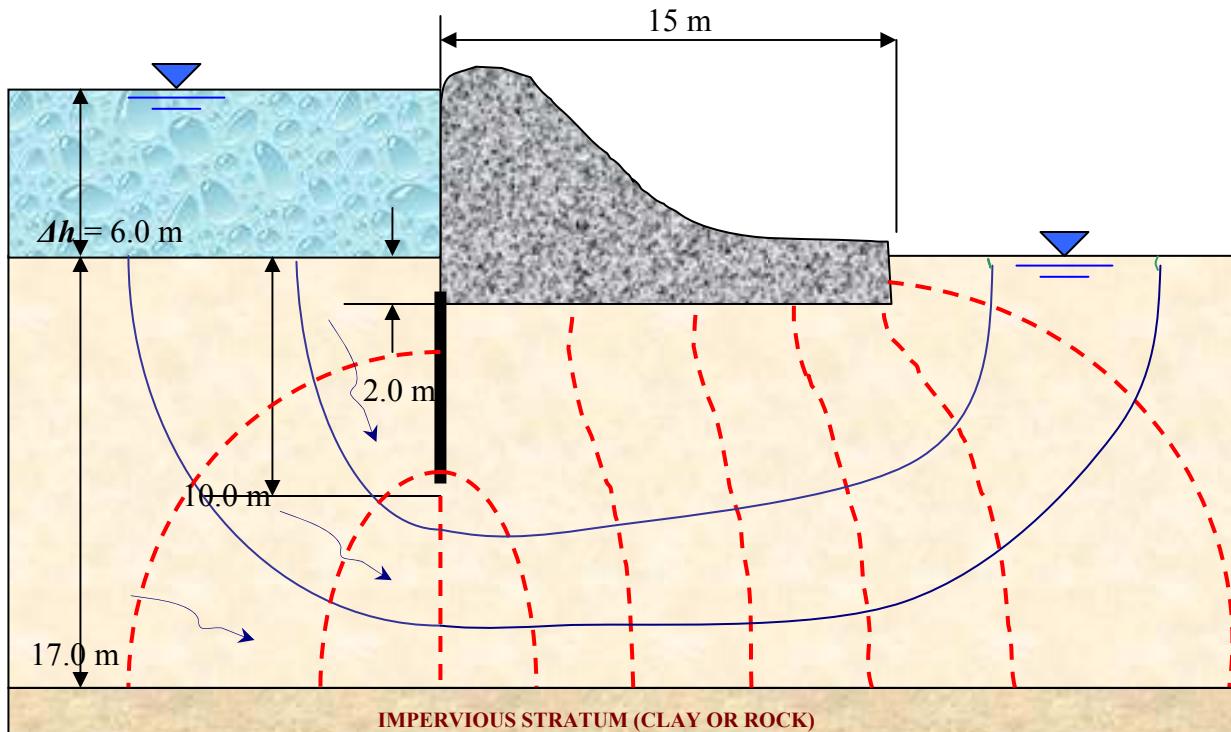
Solution:

a) Incorrectly drawn mesh, because two equipotential lines intersect each other (equipotential lines and flowlines must intersect orthogonally to each other).

b) Incorrectly drawn mesh, because two flow-lines intersect each other (same as above).

c) The well should be at the center of the net (a sink or a source point).

Flow-nets-02: *A flow-net beneath a dam with a partial cutoff wall.*

The completed flow net for the dam shown below includes a steel sheet-pile cutoff wall located at the head-water side of the dam in order to reduce the seepage loss. The dam is half a kilometer in width (shore to shore) and the permeability of the silty sand stratum is 3.5 x 10^{-4} cm/s. Find, (a) the total seepage loss under the dam in liters per year, and (b) would the dam be more stable if the cutoff wall was placed under its tail-water side?

Figure shows: 15 m (crest width), $\Delta h = 6.0$ m, 2.0 m, 10.0 m, 17.0 m, IMPERVIOUS STRATUM (CLAY OR ROCK)

Solution:

(a) Notice that $\Delta h = 6.0$ m, the number of flow channels $N_f = 3$ and the equipotentials $N_{eq} = 10$.

Using *Forcheimer's* equation,

$$q = k\Delta h \frac{N_f}{N_{eq}} = \left(3.5 \times 10^{-4} \ \frac{cm}{\sec}\right)\left(\frac{m}{100 \ cm}\right)(6.0 \ m)\left(\frac{3}{10}\right) = 6.3 \times 10^{-6} \ m^2/\sec/ \ per \ m \ of \ dam \ width$$

Since the dam is 500 meters wide (shore-to-shore) the total flow Q under the dam is,
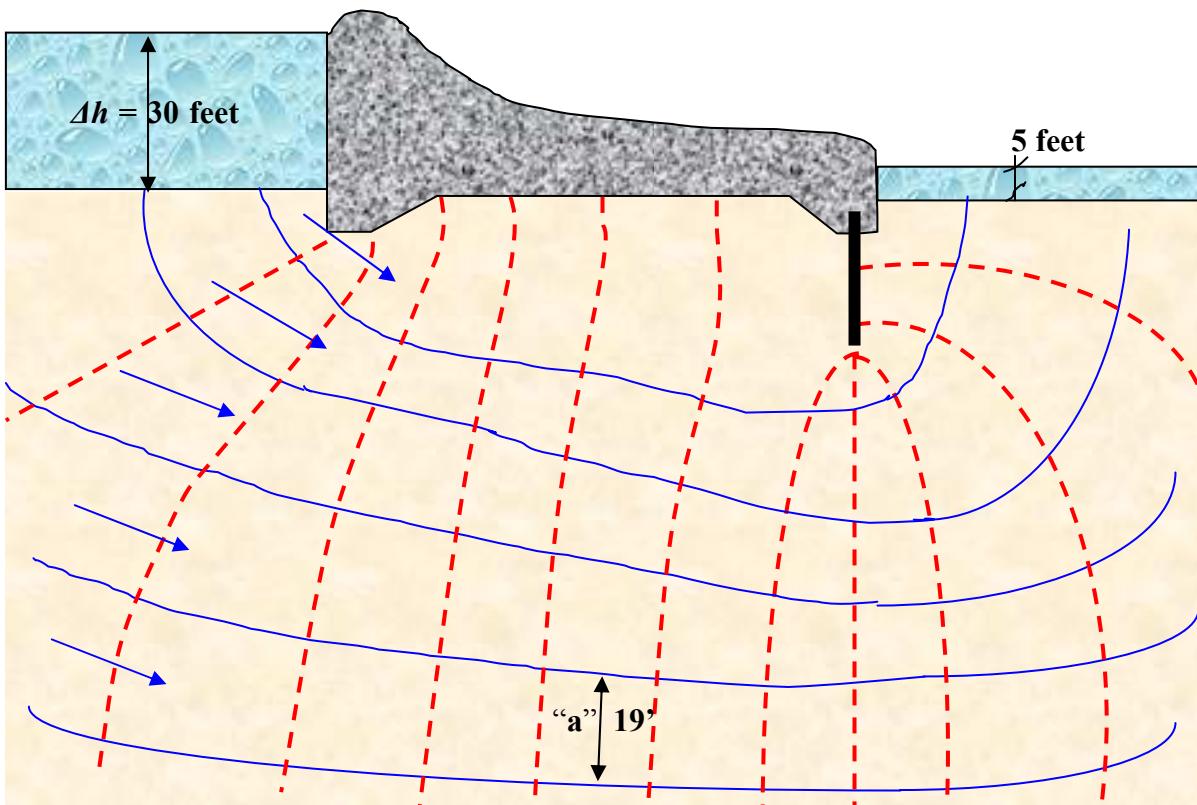
$$Q = Lq = 500 \ m \left[6.3 \times 10^{-6} \ m^3/\sec\right]\left(\frac{10^3 \ liters}{1 \ m^3}\right)\left(31.5 \times 10^6 \ \frac{\sec}{year}\right) = 100 \ \frac{million \ liters}{year}$$

b) No. Placing the cutoff wall at the toe would allow higher uplift hydrostatic pressures to develop beneath the dam, thereby decreasing the dam's stability against sliding toward the right (down-stream).

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Using the flow net shown below, (1) determine the seepage underneath the 1,000 foot wide concrete dam, and (2) the velocity at point "*a*" in feet/hour, where the height of the net's square is 19 feet. The soil has a $G_S = 2.67$, $D_{10} = 0.01$ mm. Overestimate the flow by using Hazen's coefficient $C = 15$ to determine the permeability k.

Solution:

Find the permeability k using Hazen's formula:

$$k = C(D_{10})^2 = 15(10^{-2} mm)^2 = 0.0015 \frac{mm}{\sec}$$

Using Forheimer's equation with flow lines $N_f = 5$ and equipotentials $N_{eq} = 12$,

$$q = k\Delta h \frac{N_f}{N_{eq}} = \left(0.0015 \frac{mm}{\sec}\right)\left(\frac{1 \ in}{25.4 \ mm}\right)\left[\frac{1 \ ft}{12 \ in}\right]\left(\frac{3,600 \ \sec}{1 \ hour}\right)(30'-5')\left(\frac{5}{12}\right) = 0.185 \frac{ft^3}{hr - ft \ of \ dam}$$

$$\therefore Q = Lq = (1,000 \ ft)\left(0.185 \frac{ft^2}{hr}\right) = 185 \frac{ft^3}{hr}$$
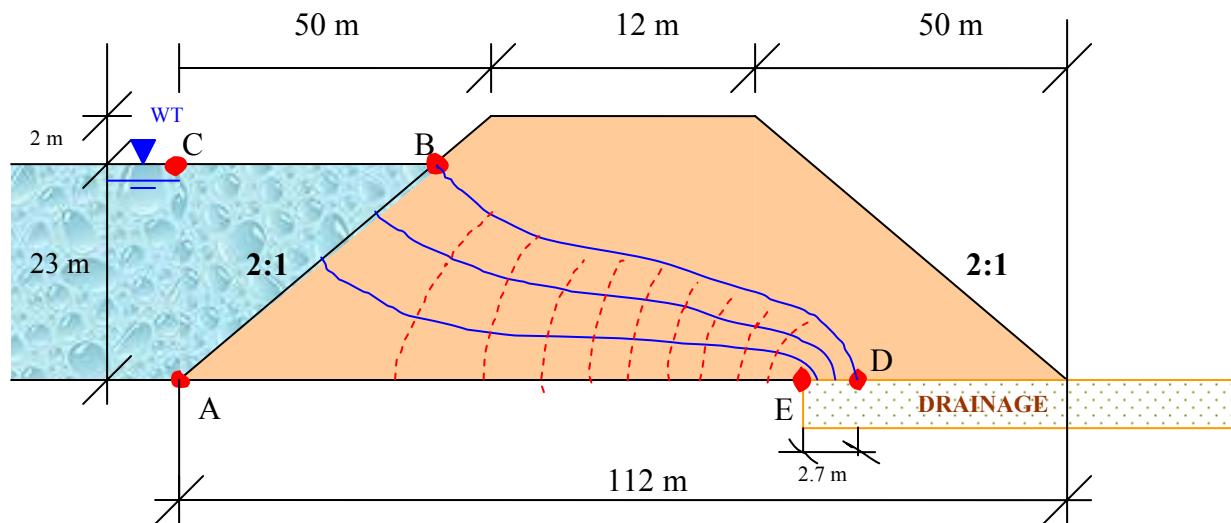
The velocity at "*a*" has a flow q in only that channel, or $q/5$,

$$v = \frac{q}{A} = \left(\frac{\dfrac{0.185 \ ft^3}{5 \ hr}}{(19 \ ft \ high)(1 ft \ wide)}\right) \approx 0.002 \frac{ft}{hr}$$

*Flow-nets-04: *Flow through an earth levee.*

(Revision: Aug-08)

In western Miami-Dade County, the Everglades are contained with levees. Levee #111 runs North-South about 2 kilometers west of Krome Avenue and its cross section is show below. Laboratory tests indicate that the permeability of the 80-year old levee is 0.30 m/day. What is the volume of water lost through the levee along each kilometer in m^3/day?

Cross-section of levee looking north.

Solution:

Using Forheimer's equation,

$$Q = Lq = L\left(k\Delta h \frac{N_f}{N_{eq}} \right) = (1,000\,m)\left(0.3\frac{m}{day} \right)(23\,m)\left(\frac{3}{10} \right) = 2,070\frac{m^3}{day}$$

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(Revision: Aug-08)
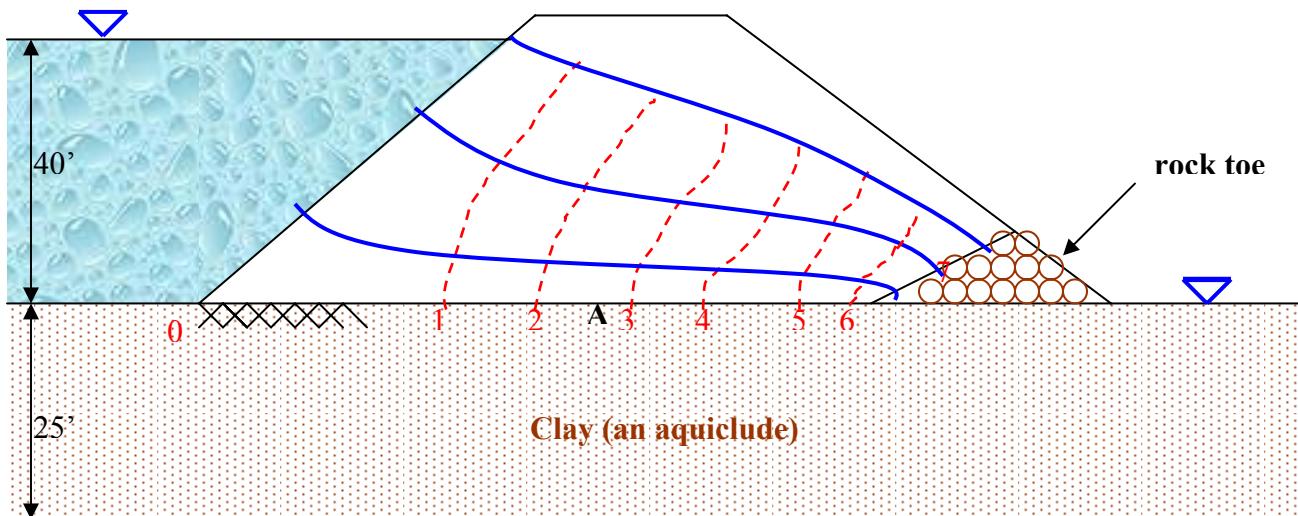
Find the seepage through the earth dam shown below in gallons/day if the sieve analysis shows the D_{10} to be 0.17 mm, and the dam is 1,200 feet wide. What is the pressure head at the top of the aquiclude and at mid-dam (point A)?

Number of flow channels $N_f = 3$

Number of equipotential drops $N_{eq} = 7$

Using Hazen's formula $k = CD_{10}^2 = 15D_{10}^2 = 15(0.17mm)^2 = 0.43\dfrac{mm}{sec}$

Note: $8 \leq C \leq 15$ for D_{10} mm; to overestimate flows, use $C=15$.

Solution:

$$Q = Lq = L\left(k\Delta h\frac{N_f}{N_{eq}}\right) = (1,200\,ft)\left(0.43\frac{mm}{sec}\right)\left(\frac{1\,inch}{25.4mm}\right)\left(\frac{1\,ft}{12\,inches}\right)\left(\frac{7.5\,gallons}{1\,ft^3}\right)(40\,ft)\left(\frac{3}{7}\right)\left(\frac{86,400\,sec}{1\,day}\right)$$

$$Q = 18.8 \times 10^6 \frac{gallons}{day}$$

At point "A" the dynamic pressure head is $\left(\dfrac{4.4}{7}\right)(40\,ft) = 25.1\,feet$
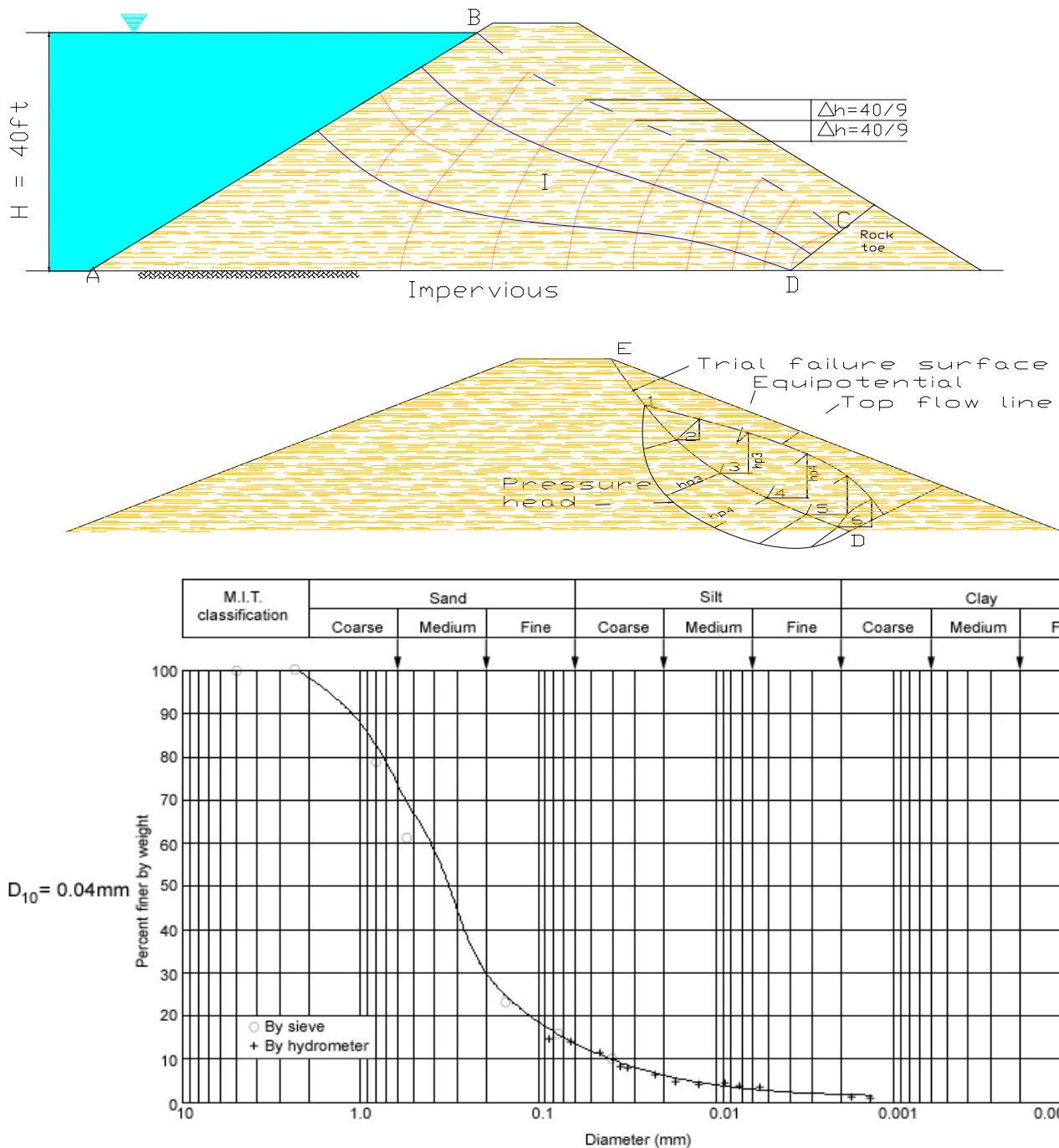
The static head at "A" is approximately $\dfrac{2}{3}(40\,ft) = 26.7\,feet$

Therefore, the total head = static + dynamic = $25.1\,ft + 26.7\,ft = 51.8\,feet$

(Revision: Aug-08)

The cross-section of an earth dam 5,000 feet wide is shown below. Determine (a) the seepage flow through the dam, in ft^3 / minute, (b) the hydraulic gradient in square I, and (c) the pore pressures along a trial failure surface along the line *ED.*

D_{10}= 0.04mm

Solution:

(a) From graph D_{10} = 0.04 mm. Using Hazen's relation, with C = 15 to overestimate the permeability of the dam,

$$k = C(D_{10})^2 = (15)(0.04 \, mm)^2 = 0.024 \, mm/\sec$$

$$Q = Lq = L\left(k\Delta h \frac{N_f}{N_{eq}}\right) = (5{,}000 \, ft)\left(0.024\frac{mm}{\sec}\right)\left(\frac{1 \, inch}{25.4 \, mm}\right)\left(\frac{1 \, ft}{12 \, inches}\right)\left(\frac{86{,}400 \, \sec}{day}\right)(40 \, ft)\left(\frac{3}{9}\right)$$

$$Q = 454{,}000 \frac{ft^3}{day}$$

(b) The gradient in square I is,

$$i_I = \frac{\Delta h}{l_I} = \frac{40/9}{11.2} = \underline{0.40}$$

(c) The pore pressures along ED are approximately,

at	pore pressure		
E	0		0
1	$u = \left[40' - 2.5 \left(\frac{40}{9}\right)\right] 62.4 =$	1803 psf =	1.80 ksf
2	$u = \left[40' - 3 \left(\frac{40}{9}\right)\right] 62.4 =$	1664 psf =	1.66 ksf
3	$u = \left[40' - 4 \left(\frac{40}{9}\right)\right] 62.4 =$	1387 psf =	1.39 ksf
4	$u = \left[40' - 5 \left(\frac{40}{9}\right)\right] 62.4 =$	1109 psf =	1.11 ksf
5	$u = \left[40' - 6 \left(\frac{40}{9}\right)\right] 62.4 =$	832 psf =	0.83 ksf
6	$u = \left[40' - 7 \left(\frac{40}{9}\right)\right] 62.4 =$	555 psf =	0.55 ksf
D	$u = \left[40' - 8 \left(\frac{40}{9}\right)\right] 62.4 =$	277 psf =	0.28 ksf
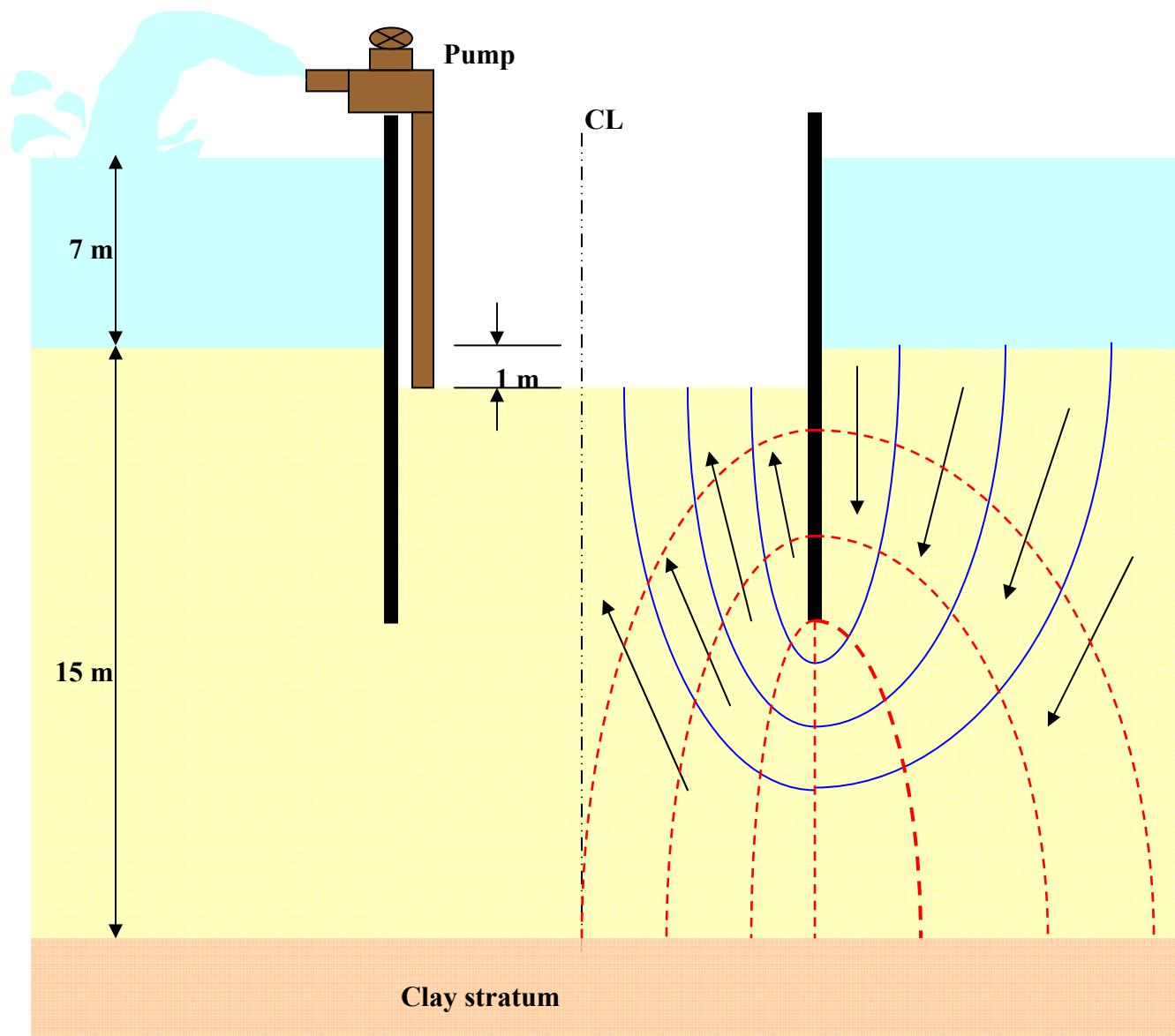
(Revision: Aug-08)

A cofferdam is to be built in the middle of a bay to place the foundations of a tall television tower. A plan area of the cofferdam is 30 m long by 10 m wide. A sample taken from the bay bottom was subjected to a hydrometer analysis: 20 grams of bay bottom dry fines were mixed with 1 liter of water. The specific gravity of the solids was found to be 2.65. The dynamic viscosity of water is 10^{-2} Poise (dynes-sec/cm^2) at 20ºC. After 1 hour of precipitation, the hydrometer dropped 16 cm. The soil is uniform in size, with 80 % passing the # 200 sieve.

(30)   What type of soil was the sample?

(31)   Will a large 3 m^3 per minute pump be adequate to maintain a 1 m draw down below the bay bottom? Use *FS* > 2.

Clay stratum

Solution:

a) Use Stoke's formula to find diameter of the bay bottom particles:

$$D_{10} = d = \sqrt{\frac{18\eta}{\gamma_S - \gamma_W}} \cdot \sqrt{\frac{L}{t}} = \left(\sqrt{\frac{18\eta}{\gamma_W (G_S - 1) t}}\right)\left(\sqrt{\frac{18x10^{-2} dynes - sec \ x \ cm^3 \ x \ 16cm}{cm^2 (2.65 - 1)(9.81 dynes \ x \ 3600 sec)}}\right)$$

$d = 0.070 \ mm$

Therefore the soil is <u>silt</u> (0.075 mm to 0.002 mm).

b) Hazen's formula permits us to estimate the permeability k of the soil:

$$k = CD_{10}^2 \frac{mm}{sec} = 15(0.07mm)^2 = 0.074 \frac{mm}{sec}$$

Use Forheimer's formula to estimate the total flow Q into the cofferdam:

$$q = k\Delta h \frac{N_f}{N_{eq}} = \left(7.4x10^{-2} \frac{mm}{sec}\right)(8m)\left(\frac{4}{8}\right)\left(\frac{m}{10^3 mm}\right) = 29.6x10^{-5} \frac{m^3}{m-s}$$

$$Q = (perimeter)(q) = 80m\left(29.6x10^{-5} \frac{m^3}{m-s} \cdot \frac{60s}{min}\right) = 1.42 \frac{m^3}{min}$$

$$Pump \ \ FS = \frac{3 \ m^3 / min}{1.42 \ m^3 / min} = 2.11 > 2 \quad OKAY \ \ Pump \ is \ adequate!$$

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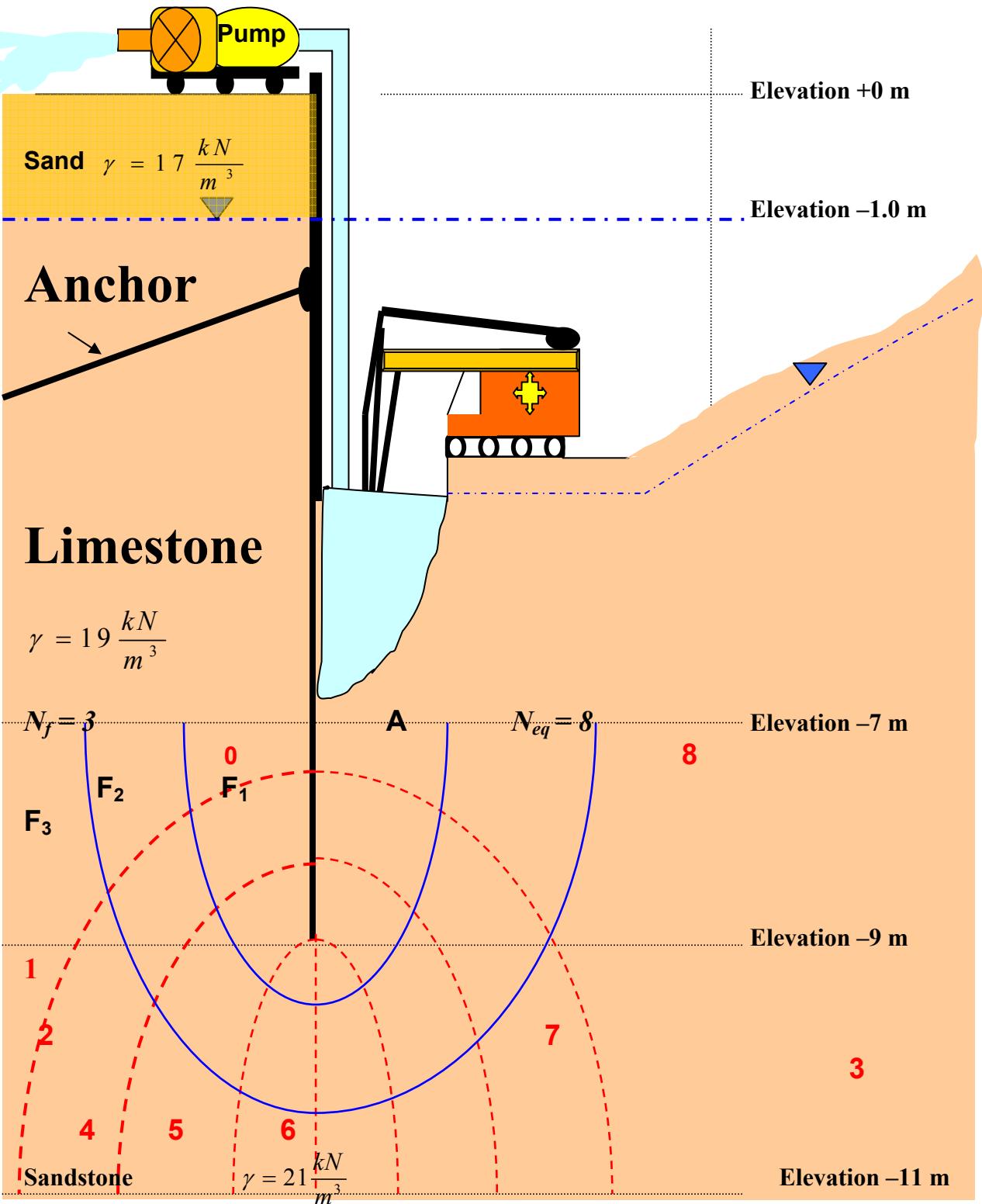
*Flow-nets-08: Drainage of deep excavations for buildings.

(Revision: Aug-08)

A new office building will require a two-level underground parking garage. The plan size of the site is 100 x 80 meters. Some of the soil properties are shown below.

a) At what depth of the excavation will the limestone (shear strength = 0.1 MN/m^2) have a punching shear failure? Suggest using a 1m x 1m plug as a model.

b) What size pump do you need (m^3/minute) with a factor of safety of 3?

Pump

Elevation +0 m

Sand $\gamma = 17 \dfrac{kN}{m^3}$

Elevation –1.0 m

Anchor

Limestone

$\gamma = 19 \dfrac{kN}{m^3}$

$N_f = 3$ **A** $N_{eq} = 8$ Elevation –7 m

0   8

F$_2$ F$_1$

F$_3$

1

2   7

3

4   5   6

Sandstone $\gamma = 21 \dfrac{kN}{m^3}$ Elevation –11 m

Solution:

a) The uplift force at point A is found by,

$$\sum F_y = 0 \quad F_{uplift} - F_{shear\ resistance} = 0$$

$$F_{uplift} = uA = (\Delta h \gamma_w) A = (6\,m)\left(9.81\frac{kN}{m^3}\right)(1\,m^2) = 59\,kN$$

$$F_{shear} = \tau A = 0.1\frac{MN}{m^2}(4x)m^2 = 400x\ \frac{kN}{m}$$

$$\therefore \quad x = \frac{59}{400} = 0.147\,m \quad with \quad FS = 7 \quad \therefore \quad x = 1\,m$$

This corresponds to elevation - 6 m.

b) Determine the flow quantity with Forheimer's formula,

$$Q = qL = Lk\left(\frac{a}{b}\right)\left(\frac{N_f}{N_q}\right)\Delta h$$

$$where \quad k = CD_{10}^2 = (12.5)(0.095\,mm)^2 = 0.113\ \frac{mm}{sec}$$

$$\Delta h = 5\,m$$
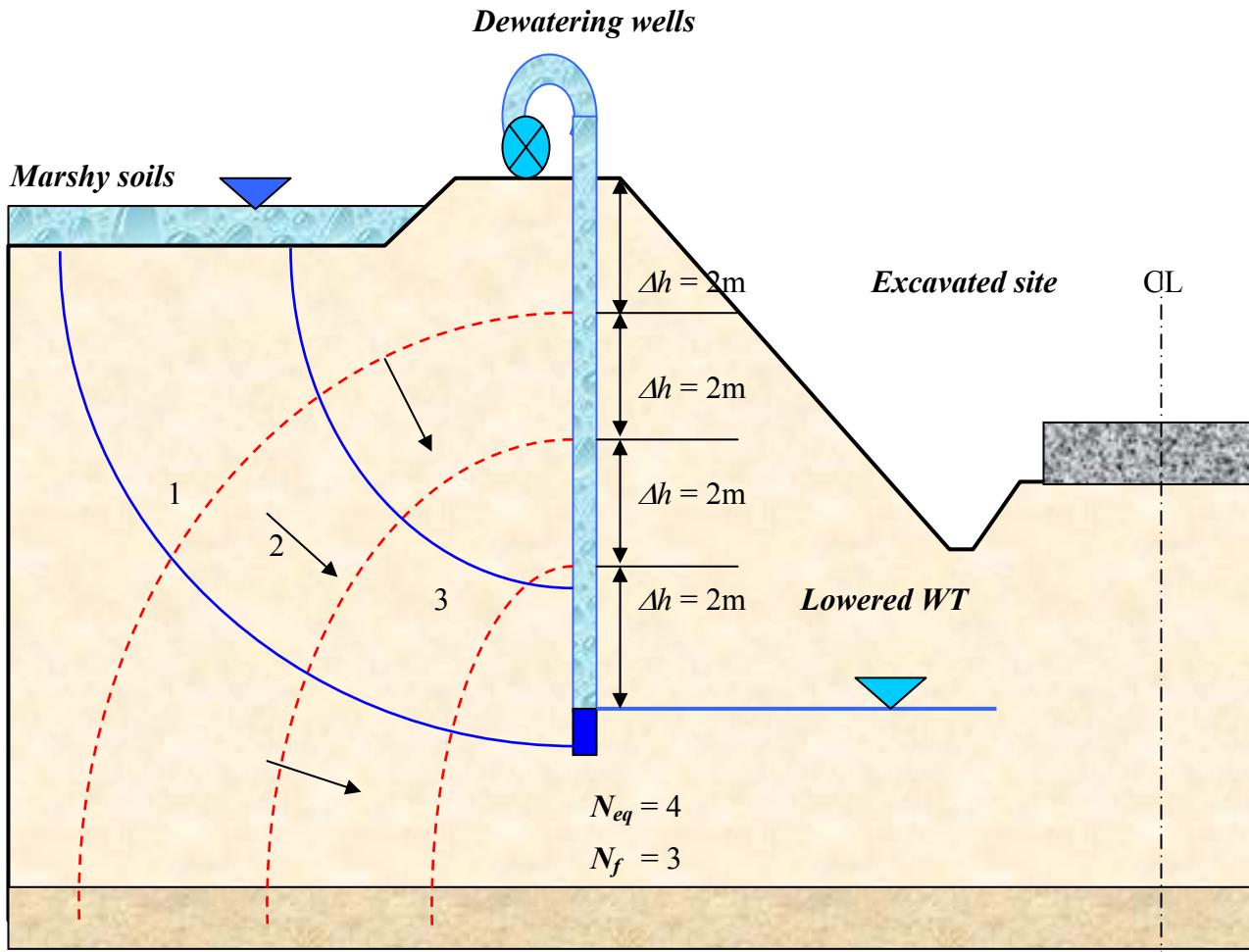
$$\therefore \quad Q = (360\,m)\left(0.113\frac{mm}{sec}\right)\left(\frac{3}{8}\right)(5\,m)\left(\frac{60}{m}\right)\left(10^{-3}\frac{m}{mm}\right) = 4.6\ m^3/min \approx 5\,m^3/min$$

For a FS = 3 use a 15 $\frac{m^3}{min}$ pump.

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*Flow-nets-09: Dewatering a construction site.

(Revision: Aug-08)

The figure below shows a dewatering plan to build the foundations of an office building below the water table and without sheet-piling. The plan area of the excavation is 400 m long by 100 m wide. The soil has a D_{10} of 0.02 mm. What size pump do you need (*gpm*) with a *Factor of Safety* = 2?

Dewatering wells

Marshy soils

$\Delta h = 2m$ *Excavated site*   CL

$\Delta h = 2m$

1     $\Delta h = 2m$

2

3     $\Delta h = 2m$ *Lowered WT*

$N_{eq} = 4$
$N_f = 3$

Solution:

Notice that $\Delta h = 8\ m$, $N_f = 3$, $N_{eq} = 4$ and $L = perimeter = 1,000\ m$.

The permeability $k = C\left(D_{10}\right)^2 = \left(15\right)\left(0.02mm\right)^2 = 0.006\ mm\,/\,s$

$$Q = L\ q = L\ k\ \Delta h\left(\frac{N_f}{N_{eq}}\right) = \left(200 + 800\right)m\left(0.006\frac{mm}{s}\right)\left(8m\right)\left(\frac{3}{4}\right)\left(\frac{m}{10^3\,mm}\right)\left(\frac{60s}{1\min}\right) = 2.16\frac{m^3}{\min}$$
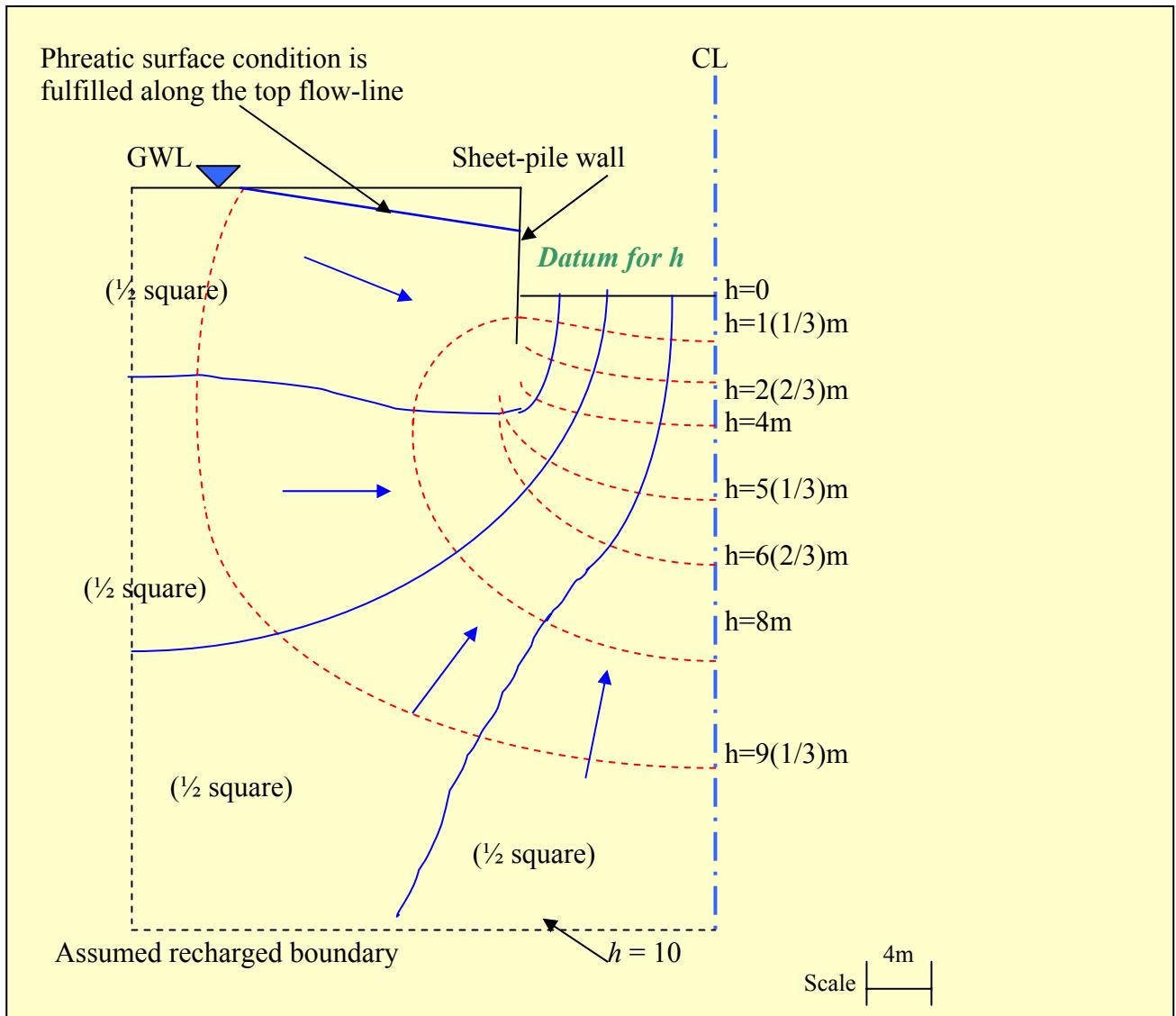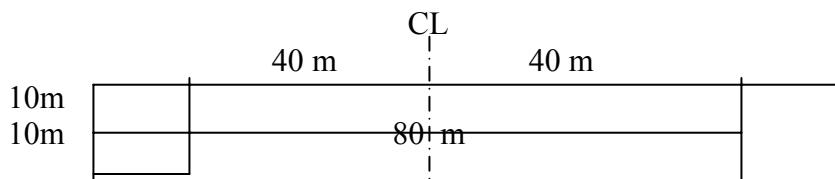
$$Q = \left(2.16\frac{m^3}{\min}\right)\left(\frac{ft}{0.30\ m}\right)^3\left(\frac{7.45\,gal}{ft^3}\right) = 600\frac{gallons}{\min}$$

Therefore, for a factor of safety of 2 use at least a 1,200 gallons per minute pump or two 600 gallons per minute pumps.

*Flow-net-10: Dewatering in layered strata.

(Revision: Aug-08)

The figure below shows the profile of a square excavation (in plan view) in a layered soil, where the vertical permeability is 5×10^{-5} m/s and the horizontal permeability is roughly ten times higher than the vertical. Estimate the dewatering capacity requirements, in m^3/hour, to prevent the excavation from flooding. The value of Δh is to scale, but you may use 10 m.

CL

40 m   40 m

10m

10m   80 m

Phreatic surface condition is
fulfilled along the top flow-line

CL

GWL   Sheet-pile wall

Datum for h

(½ square)

h=0
h=1(1/3)m

h=2(2/3)m
h=4m

h=5(1/3)m

h=6(2/3)m

(½ square)

h=8m

(½ square)

h=9(1/3)m

(½ square)

Assumed recharged boundary   h = 10

Scale |—4m—|

Solution:

$\Delta h = 10m$, $N_f = 4$, $N_{eq} = 8$

$k = \sqrt{k_x k_y} = \sqrt{(5x10^{-4} \ m \ / \ s)(5x10^{-5} \ m \ / \ s)} = 1.6 \left(10^{-4} m \ / \ s \right)$

The perimeter of the cofferdam $p = (4)(80m) = 320m$

$Q = qp = \left[k \Delta h \left(\dfrac{N_f}{N_{eq}} \right) \right] p = \left[(1.6)\left(10^{-4} \ \dfrac{m}{s} \right)(10m)\left(\dfrac{4}{8} \right) \right](320m)$
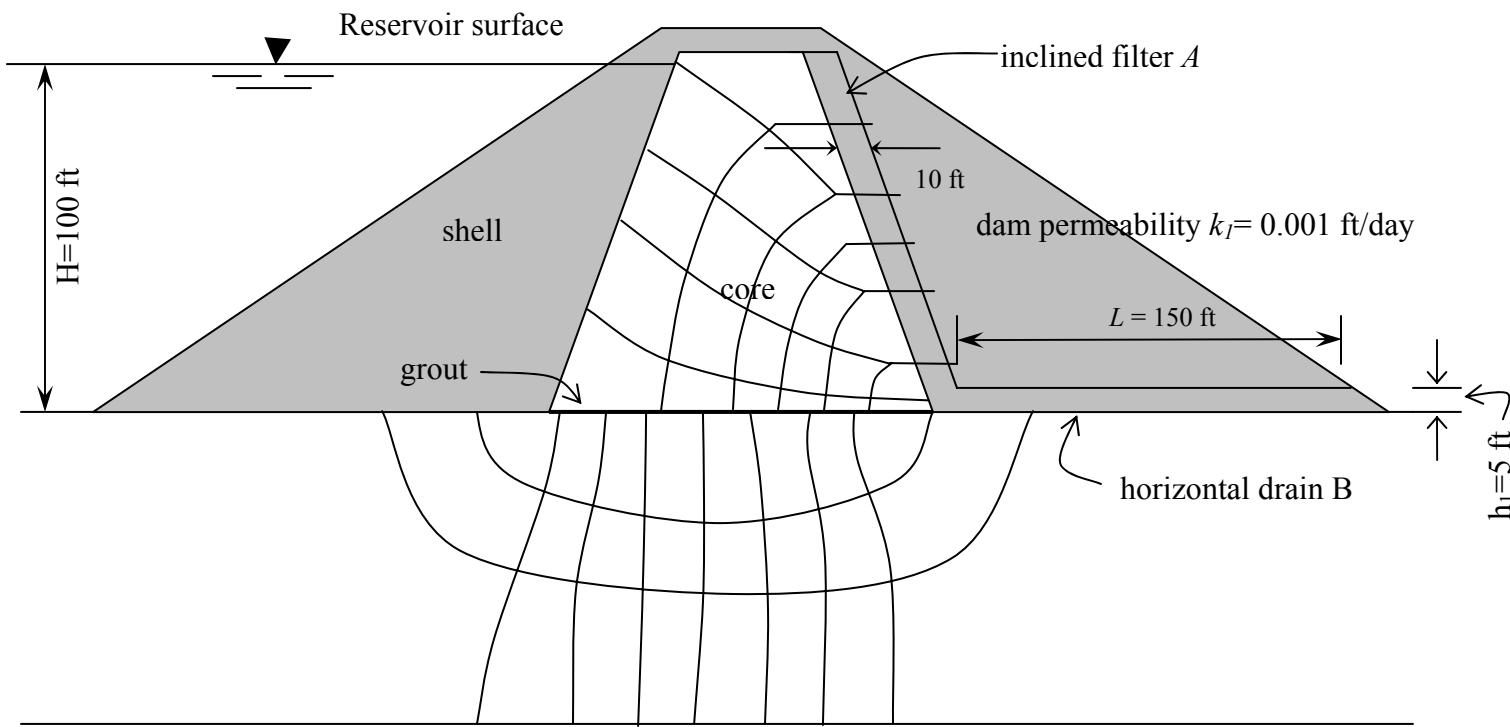
$Q = 0.253 \left(\dfrac{m^3}{s} \right)\left(\dfrac{3,600 s}{hr} \right) = 911 \dfrac{m^3}{hr}$

(Revision: Aug-08)

An earth dam on a pervious but strong earth foundation has the cross-section shown in the figure below. The core of the dam is sealed from the jointed rock foundation with a thin layer of grout.

(32)   State the function and properties of the core shell and drains;

(33)   What is the function of the grout between the core and foundation? Under what conditions is it most important?

(34)   Calculate the seepage quantity per foot of length of the dam through the dam, through the foundation, and the total seepage quantity.

(35)   What grading requirements should be specified for the inclined filter A?

(36)   What minimum permeability k is required in the horizontal drain B to prevent saturation from rising into the random fill zone? Give the results of k in ft/day.

Note: The grain size of core = 100% passes 1", 15% size = 1/8", and 85% size = 0.001 in.

Foundation layer permeability $k_2 = 0.1$ ft/day

Solution:

(a)   The *core* is used to retain water within the dam, that is, to resist seepage. The material should be relatively impermeable (clay) and should not shrink or swell excessively.

The *shell* provides the structural strength to support and protect the core. The material must be more permeable than the core material, strong and durable.

The ***drains*** are provided to reduce the pore water pressures in the foundation and in the embankment to increase stability. The drains also remove seepage water to reduce soil erosion. The drain material must be permeable enough to permit drainage with a low head loss and yet fine enough to keep the adjacent soil in place.

(b)   The primary function of the grout between the core and foundation is to form an impervious layer which prevents seepage along the contact surface. This becomes most important when the ratio k_2/k_1 becomes large.

(c)   Calculate the seepage Q by using the flow net shown in the figure.

    1)   Through the dam: $Q = k_1(\Delta h/L)b$ in ft³/day/ft where b is the normal distance between streamlines. The flow net divides the core into 4 zones (#1 at the bottom, #4 at the top).

$$Q = k_1 \sum_{j=1}^{4} i_j\, b_j = k_1 \sum_{j=1}^{4} (\Delta h)_j\, (\underline{b})_j$$

In zone #1, the flow net is nearly rectangular, so $(\underline{b})_1 = 2$; for zone # 4, $(\underline{b})_j = 1$

$$\qquad\qquad\qquad\qquad\qquad\qquad \ell \qquad\qquad\qquad\qquad \ell$$

The average head loss h_l across the core in each zone

$$\frac{(\underline{p})}{\gamma} + Z_u = \text{constant} = 100' \text{ on the upstream face}$$

On down stream face of core $(\underline{p}) = 0$ is assumed in the drain,

$$\frac{}{\gamma}$$

so that $Z_L + h_L = \text{constant} = 100'$ on the downstream face

Using an average Z_L for each zone by scaling,

Zone#1 $Z_L = 2'$ $h_L = 98'$ $\dfrac{\Delta h_L}{6} = \dfrac{98}{6}$

Zone#2 $Z_L = 10'$ $h_L = 90'$ $\dfrac{\Delta h_L}{5.5} = \dfrac{90}{5.5}$

Zone#3 $Z_L = 25'$ $h_L = 75'$ $\dfrac{\Delta h_L}{4.5} = \dfrac{75}{4.5}$

Zone#4 $Z_L = 55'$ $h_L = 45'$ $\dfrac{\Delta h_L}{3} = \dfrac{45}{3}$

$$Q_1 = k_1 \sum_{j=1}^{4} \Delta h_j\, (b/L)_j = 0.001 \left[\frac{98'(2)}{6} + \frac{90'}{5.5} + \frac{75'}{4.5} + \frac{45'}{3} \right] = \underline{\textbf{0.081 ft}^3\textbf{/day/ft of dam}}$$

    2) Through the foundation $Q_2 = \underline{N}_F\, kh$

where N_F = number of flow paths, N_d = number of equipotential drops and

h = total head dissipated.

$$Q_2 = \frac{3}{8} (0.1)(100) = \textbf{3.75 ft}^3 \textbf{/day/ft of dam}$$

3) The total seepage **Q** is therefore,

$$\textbf{Q} = Q_1 + Q_2 = 0.081 + 3.75 = \textbf{3.83 ft}^3\textbf{/day /ft of dam}$$

(d) The grading requirements for the inclined filter A,

(1) Free drainage, require D_{15} (filter) ≥ 4 D_{15} (soil)

$$D_{15} \text{ (filter)} \geq 4 \text{ (0.001 in)}$$

$$\geq 0.004 \text{ in}$$

(2) To prevent erosion of the core material requires D_{15} (filter) ≥ 4 D_{85} (soil)

$$\geq 4 \text{ (0.001 in)}$$

$$\geq 0.004 \text{ in}$$

So 85% of the filter material must be coarser than 0.01" to 0.2". The filter grain size grading curve should be parallel to or flatter than the core material grading curve. See the graph on the next page for one possible grading curve, which gives

100% passes = 10 inch

15% passes = 2 inch

85% passes = 0.01 inch
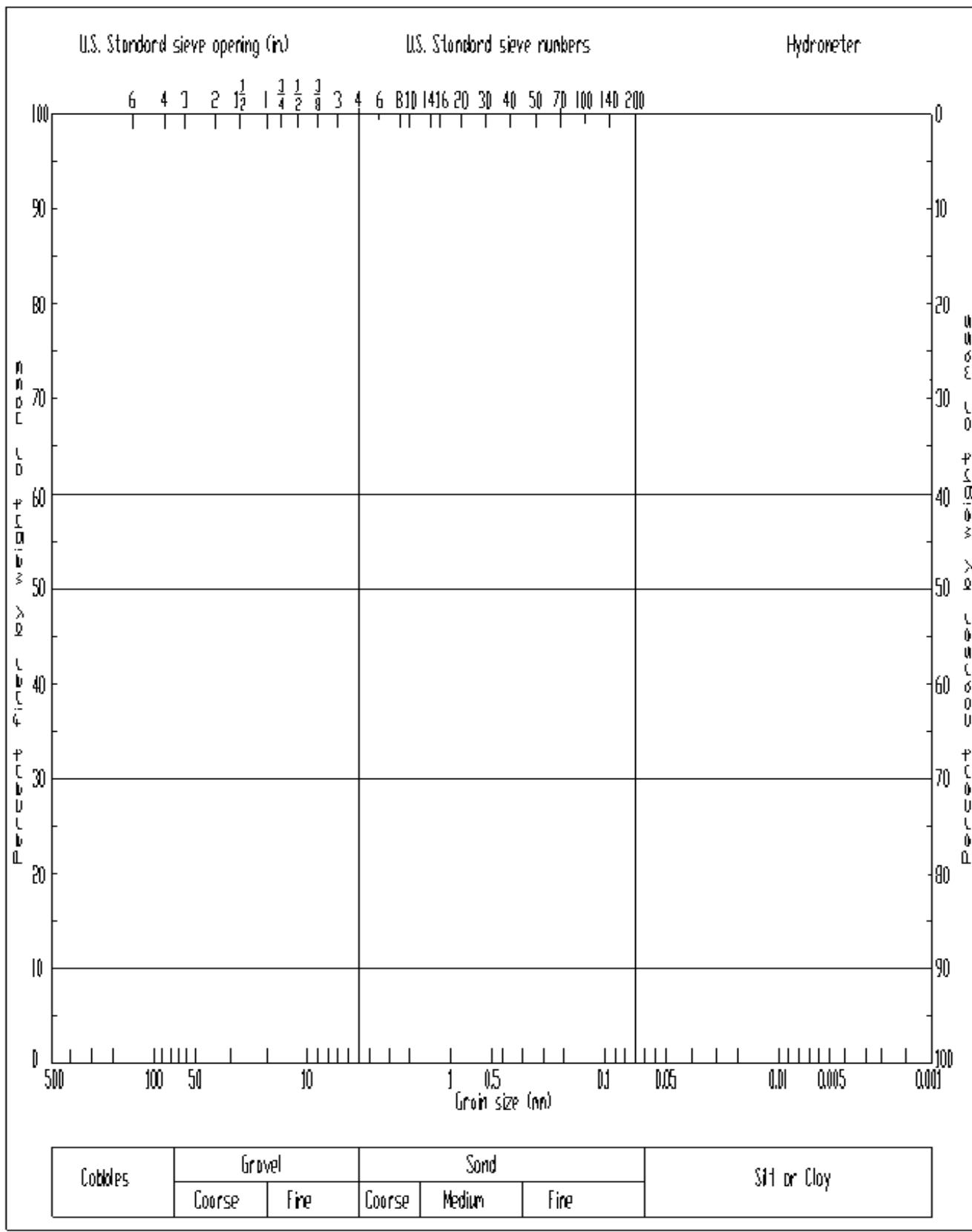
(e) The drain B must carry the total seepage flow Q = 3.83 ft³/day/ft of dam calculated above. The Dupuit formula for two-dimensional flow on a horizontal impervious boundary is

$$Q = \frac{k}{2L} (h_1^2 - h_2^2)$$

where Q = 3.83ft³/day/ft, L = 150 ft, h_1 = 5 ft and h_2 < 5 ft.

At what value of h_2 will it minimize k? Clearly it is when $h_2 = 0$, although this does seem unrealistic since we are saying that the flow at the lower end of the drain has zero depth. Nevertheless, it gives us a minimum value, which is:

$$k_{min} = \frac{2LQ}{h_1{}^2} = \frac{2(150)(3.83)}{5^2} = \textbf{46 ft/day}$$

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Chapter 7
Effective Stresses and Pore Water Pressure

Symbols for Effective Stresses and Pore Water Pressure

*Effective Stress–01: *The concept of buoyancy.*

What force is required to hold an empty box that has a volume of 1 cubic foot, just below the water surface?

Solution:

The volume of the displaced water is 1 ft^3.
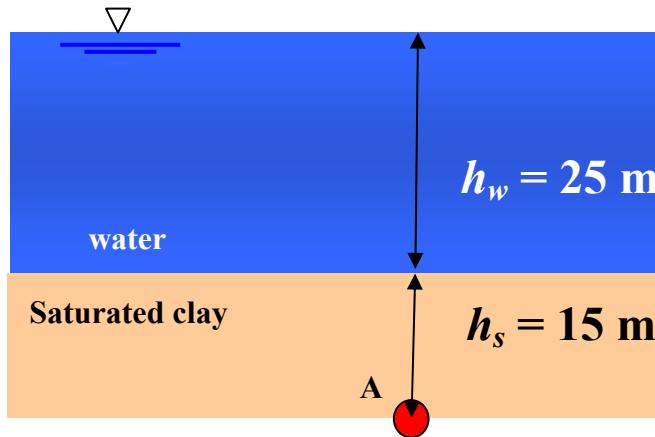
Therefore, the force is the weight of 1 ft^3 of water = *62.4 lbs / ft^3*.

What is the force required to hold the same box 10 feet below the surface?

(Revision: Aug-08)

A sample was obtained from point A in the submerged clay layer shown below. It was determined that it had a $w = 54\%$, and a $G_s = 2.78$. What is the effective vertical stress at A?

Solution:

The effective stress σ' at the point A consists solely of the depth of the soil (not of the water) multiplied by the soil buoyant unit weight.

$$\sigma' = \gamma' h_{soil} \quad where \quad \gamma' = \gamma_b = \gamma_{SAT} - \gamma_W$$

In order to find γ' there are a number of derivations, such as this one,

$$\gamma' = \left[\frac{(G_S + e)\gamma_W}{1+e} \right] - \gamma_W \quad \text{where the voids ratio } e \text{ can be found through } Se = wG_S$$

and noticing that $S = 1$ because the soil is 100% saturated, $e = wG_S = (0.54)(2.78)$

$$\sigma' = \gamma' h_{soil} = \left\{ \left[\frac{(G_S + e)\gamma_W}{1+e} \right] - \gamma_W \right\} h_{soil} = \left[\frac{[2.78 + (0.54)(2.78)](9.81)}{1+(0.54)(2.78)} - 9.81 \right] (15\,m)$$

$$\sigma' = 105\ kPa$$

(Revision: Aug-08)

The City of Houston, Texas has been experiencing a rapid lowering of its phreatic surface (draw-downs) during the past 49 years due to large volumes of water pumped out of the ground by industrial users.

- a) **What was the effective vertical stress at a depth of 15 m in 1960?**
- b) **What is the effective stress at the same depth in 2009?**
- c) **What happens to the ground surface as a result of the draw-downs?**

Solution:

a)
$$\sigma_V' = \left[\gamma h + \gamma'h'\right]_{SAND} + \left[\gamma'h'\right]_{SILT} + \left[\gamma'h'\right]_{CLAY} \quad where \quad \gamma' = \gamma_{SAT} - \gamma_W$$
$$\sigma_V' = \left[(20.4)(3) + (18.8 - 9.81)(3)\right] + \left[(14.9 - 9.81)(6)\right] + \left[(12.6 - 9.81)(3)\right]$$
$$\sigma_V' = 128\ kPa$$

b) $\sigma_V' = \left[(20.4)(6) + (16.5)(6)\right] + \left[(12.6 - 9.81)(3)\right] = 230\ kPa$
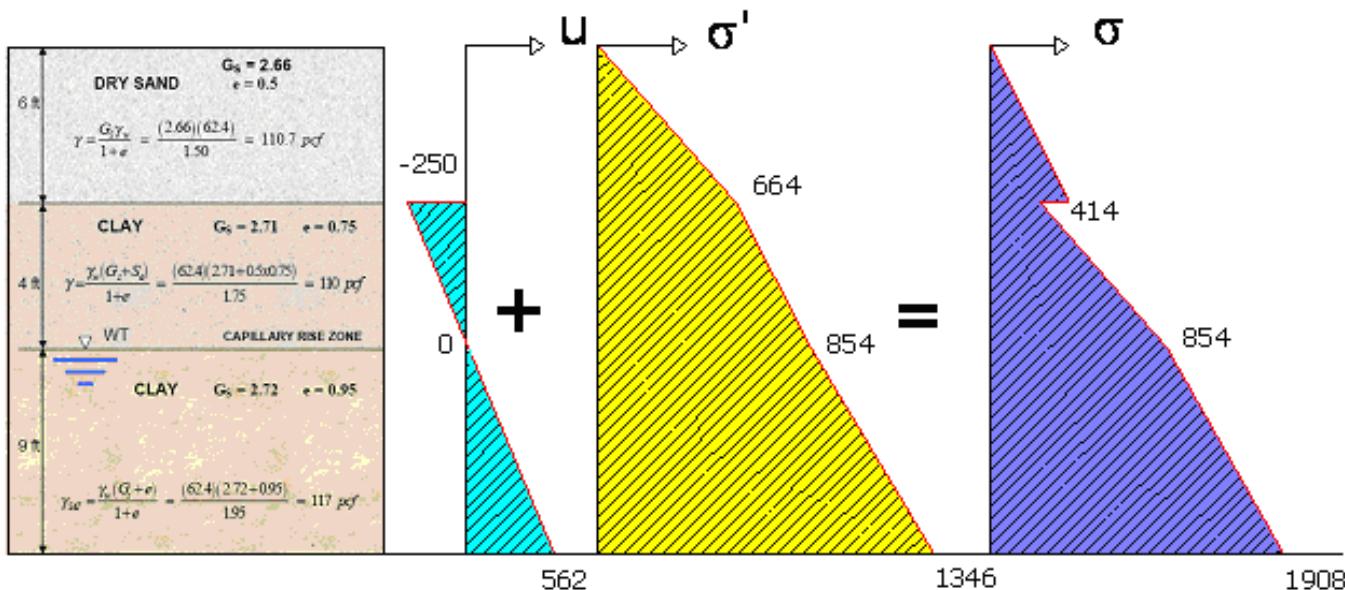
This is an 80% increase in stress due solely to a dropping water table.

c) The ground surface has also been lowered, due to the decreasing thickness of the sand and the silt strata due to their loss of the volume previously occupied by the water.

Revision

In the soil profile shown below, show a plot of the pore water pressure and the effective stress along the right margin of the figure, with numerical values at each interface. Pay heed to the capillarity in the upper clay. Assume S = 50%.

Solution:

$\Delta h = (62.4) \times (4) = -250$

$\Delta h = (62.4) \times (6) = 562$

$\Delta h = (110.7) \times (6) = 664$

$\Delta h = \{[(110-62.4) \times (4)] + 664\} = 854$

$\Delta h = \{[(117-62.4) \times (9)] + 854\} = 854$

$\Delta h = 664 - 250 = 414$

$\Delta h = 0 + 854 = 854$

$\Delta h = 562 + 1346 = 1908$

Depth (ft)	u + σ'		σ
0	0	0	0
6	-250	664	414
10	0	854	854
19	562	1346	1908
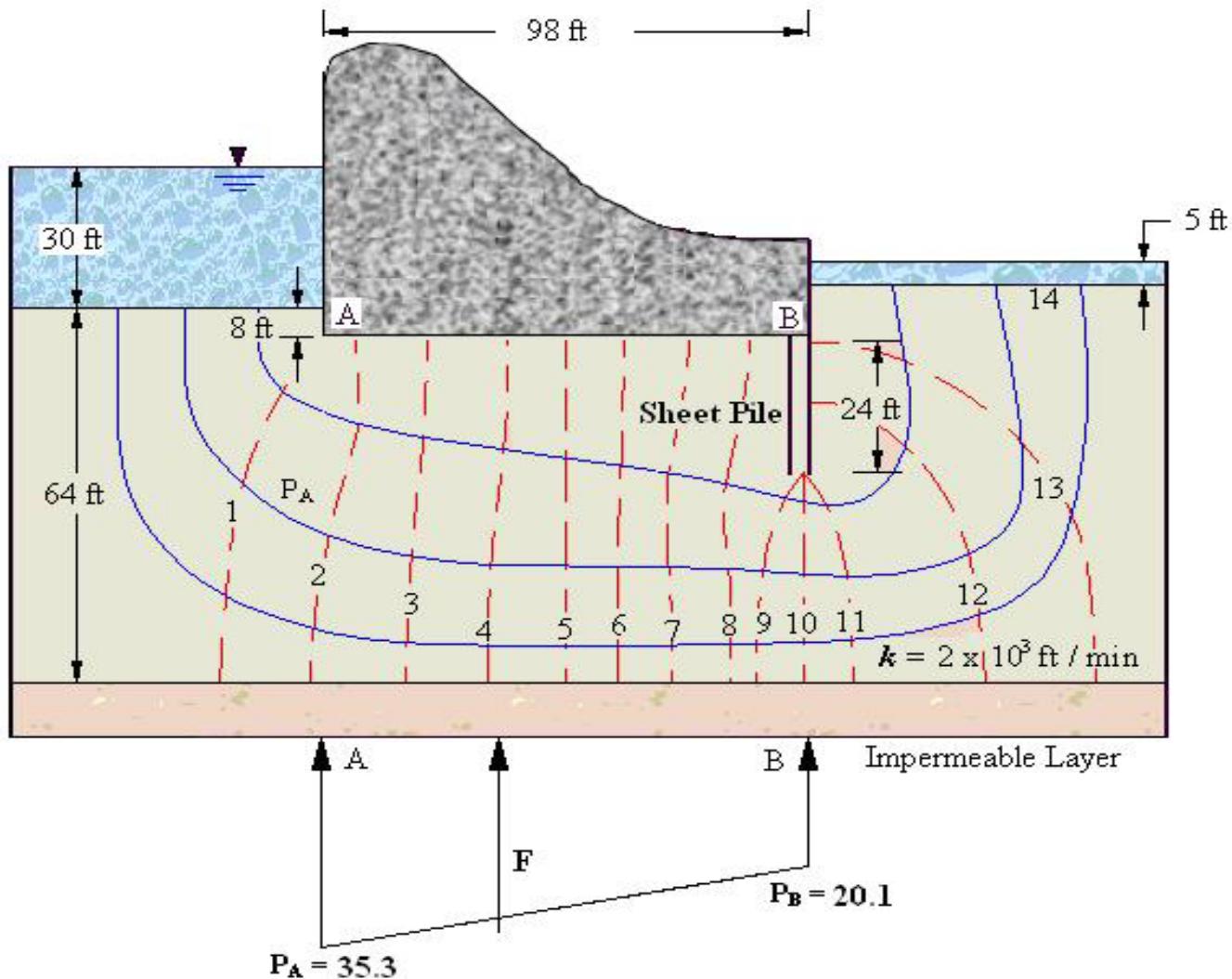
Chapter 8
Dams and Levees

Symbols for Dams and Levees

*Dams-01: Find the uplift pressure under a small concrete levee.

(Revision: Sept.-08)

Calculate the uplift force at the base of the weir, per foot of width. Points A and B are at the corners of the concrete levee.

Solution:

The dynamic head drop per equipotential is,

$$\Delta(\Delta h) = \frac{H_A - H_B}{N_{eq}} = \frac{30\,ft - 5\,ft}{14\,drops} = 1.8\ ft\,/\,drop$$

Pressure head at A = 30 ft + 8 ft - Δh (1.5) = 35.3 ft

Pressure head at B = 30 ft + 8 ft - Δh (10) = 20.1 ft

The uplifting force F is,
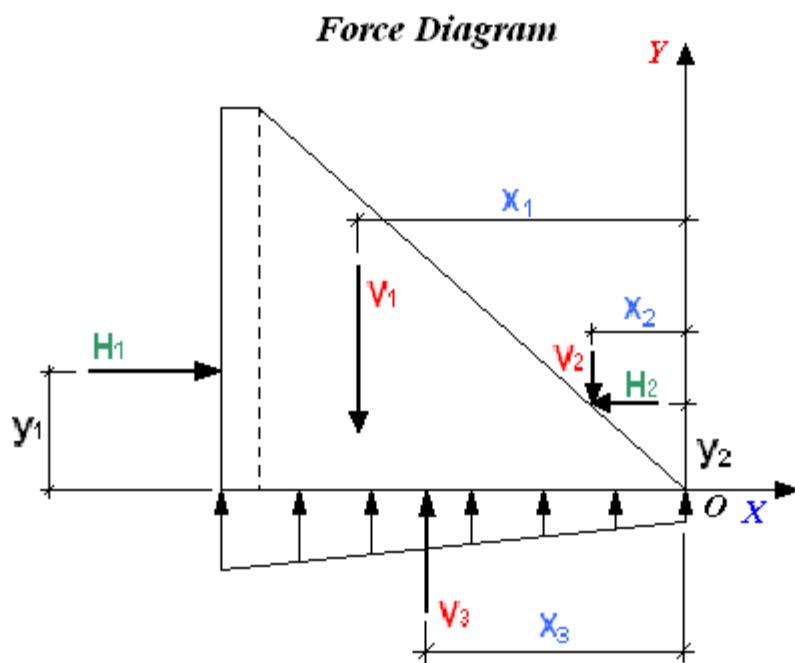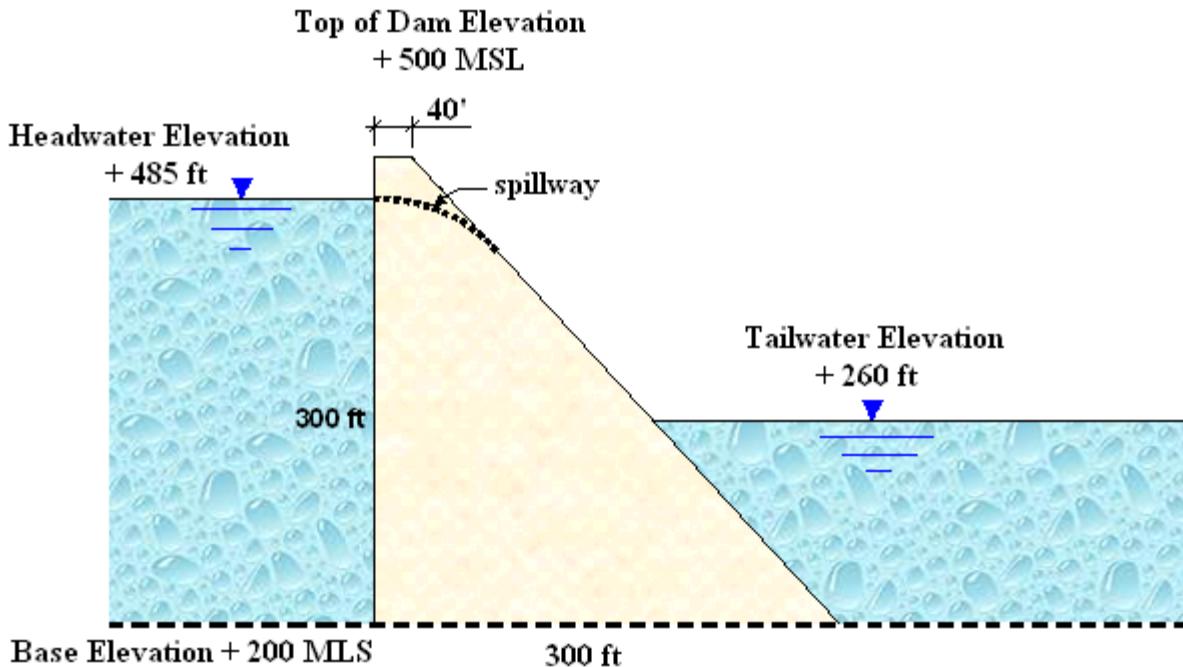
$$F = L\left(\frac{p_A + p_B}{2}\gamma_w\right) = 98'\left(\frac{35.3 + 20.1}{2}\right)(62.4)\frac{lb}{ft^3} = 169,000\frac{lb}{ft} = 169\ \frac{kip}{ft}$$

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*Dams-02: Determine the uplift forces acting upon a concrete dam.

(Revision: Aug-08)

The uplift (hydrostatic) force under the concrete gravity dam shown below varies as a straight line from **67% of the headwater pressure at the heel, to 100% of the tail-water at the toe.** Assume $\gamma_{Concrete} = 145 \ pcf$

a) Determine the Factor of Safety against overturning; and

b) Determine the FS against sliding, if the sand that underlay the dam has $\varphi = 37°$.

Top of Dam Elevation
+ 500 MSL

40'

Headwater Elevation
+ 485 ft

spillway

Tailwater Elevation
+ 260 ft

300 ft

Base Elevation + 200 MLS

300 ft

Force Diagram

Y

X_1

V_1

X_2

H_1

V_2

H_2

y_1

y_2

O X

V_3

X_3

Solution:

Step 1: Determine all forces on the dam

$$\Sigma F_V = 0$$

$$V_1 = weight\ of\ dam = \left(0.145\frac{k}{ft^3}\right)\left[(300\,ft)(40\,ft) + \left(\frac{1}{2}\right)(260\,ft)(300\,ft)\right] = 7{,}395\ kips\,/\,ft$$

$$\Sigma M_0 = 0$$

$$x_1,\ from\ toe\ O = \frac{(1740\,k)(280\,ft) + (5655\,k)\left(\frac{2}{3}(260\,ft)\right)}{(1740\,k + 5655\,k)} = 198\ ft\ left\ of\ toe$$

$$V_2 : vertical\ weight\ of\ water\ upon\ toe\ section = 0.0624\frac{k}{ft^3}\left(\frac{1}{2}(60\,ft)(52\,ft)\right) = 97\ kips\,/\,ft$$

$$x_2,\ from\ toe\ O = \frac{1}{3}(52\,ft) = 17.3\ ft\ left\ of\ toe$$

$$H_1 : lateral\ force\ from\ headwater = \left(\frac{1}{2}0.0624\frac{k}{ft^3}\right)(285\,ft)^2 = 2{,}534\ kips\,/\,ft$$

$$y_1,\ from\ toe\ O = \frac{1}{3}(285\,ft) = 95\ ft\ above\ the\ toe.$$

$$H_2 : lateral\ force\ from\ tailwater = \left(\frac{1}{2}0.0624\frac{k}{ft^3}\right)(60\,ft)^2 = 112\ kips\,/\,ft$$

$$y_2,\ from\ toe\ O = \frac{1}{3}(60\,ft) = 20\ ft\ above\ the\ toe.$$

Step 2: The hydrostatic uplift at the base of the dam.

$$V_3 : uplift\ force = \frac{1}{2}(p_{LEFT} + p_{RIGHT})(300\,ft) = 2{,}534\ kips\,/\,ft$$

$$where\ the\ pressure\ p_{LEFT} = (0.67)\gamma_w h = 0.67(0.0624)(285\,ft) = 11.9ksf$$

$$and\ the\ pressure\ p_{RIGHT} = (1.00)\gamma_w h = 1.0(0.0624)(60\,ft) = 3.7\ ksf$$

$$\therefore V_3 = \frac{1}{2}(11.9 + 3.7)\frac{k}{ft^2}(300\,ft) = 2{,}340\ kips\,/\,ft$$

$$x_3,\ from\ toe\ O = \frac{\left(3.7\frac{k}{ft^2}\right)(300\,ft)\left(\frac{300\,ft}{2}\right) + \frac{1}{2}\left(11.9\frac{k}{ft^2} - 3.7\frac{k}{ft^2}\right)(300\,ft)\left(\frac{2}{3}(300\,ft)\right)}{\left(3.7\frac{k}{ft^2}\right)(300\,ft) + \frac{1}{2}\left(11.9\frac{k}{ft^2} - 3.7\frac{k}{ft^2}\right)(300\,ft)} = 176.3\ ft\ left\ of\ toe$$

Step 3: The factor of safety (FS) against overturning (taken about the toe),

$$FS = \frac{resisting\ moments}{overturning\ moments} = \frac{V_1 x_1 + V_2 x_2 + H_2 y_2}{V_3 x_3 + H_1 y_1}$$

$$FS_{overturning} = \frac{(7395)(198) + (97)(17.3) + (112)(20)}{(2534)(95) + (2340)(176.3)} = 2.2 > 2 \ \ GOOD$$

Step 4: The F.S. against sliding,

$$FS_{sliding} = \frac{resisting\ forces}{driving\ forces} = \frac{H_2 + (V_1 + V_2 - V_3)\tan\phi}{H_1}$$

$$FS_{sliding} = \frac{(112) + (7395 + 97 - 2340)\tan 37°}{2534} = 1.58 < 2 \ NOT\ GOOD\ ENOUGH$$

Chapter 9
Stresses in Soil Masses

Symbols for Stresses in Soil Masses
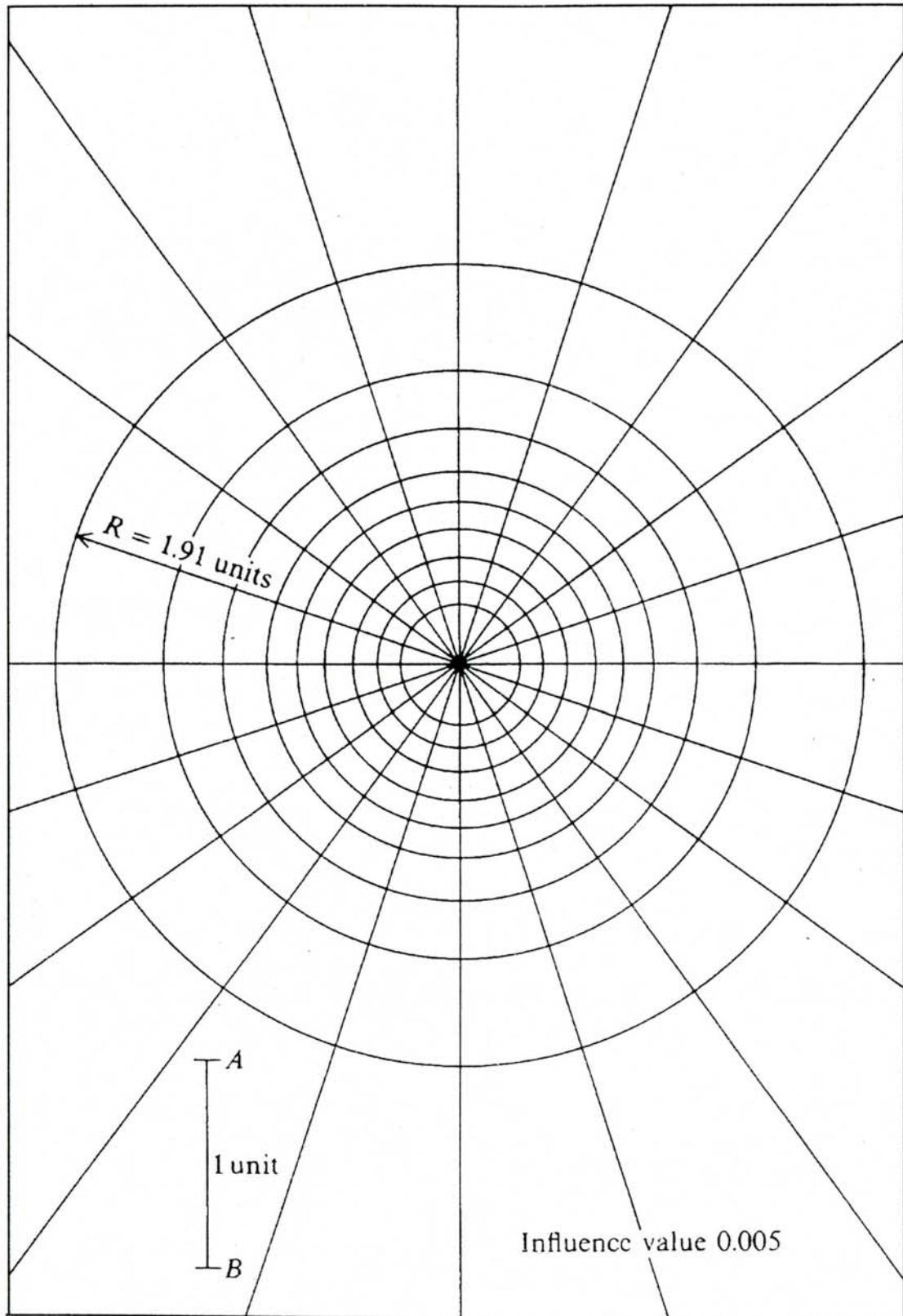
$\theta \rightarrow$ The angle of the plane of interest angle with respect to the major principal stress (σ_1).

$\sigma_{max} \rightarrow$ Maximum normal axial stress.

$\sigma_{min} \rightarrow$ Minimum normal axial stress.

$\sigma_\theta \rightarrow$ The normal stress on a plane with an angle θ with respect to the major principal stress plane (σ_1).

$\tau_{max} \rightarrow$ maximum shear stress.

$\tau_{minx} \rightarrow$ Minimum shear stress.

$\tau_\theta \rightarrow$ The shear stress on aplane with an angle θ respect to the major principal stress plane (σ_1).

$\phi \rightarrow$ The angle of internal friction of the soil.

$\gamma \rightarrow$ Unit weight of the soil.

$\tau_n \rightarrow$ Shear stress.

$\sigma_n \rightarrow$ Normal stress.

$q_u \rightarrow$ Ultimate shear strength of a soil.

Symbols for Boussinesq Stresses

$B \rightarrow$ Width of the loaded selected region.

$G_S \rightarrow$ Specific gravity of the solids of a soil.

$L \rightarrow$ Length of the loaded selected region.

$m \rightarrow$ The ratio *(B/Z)*.

$n \rightarrow$ The ratio *(L/Z)*.

$N \rightarrow$ Normal load carried by a foundation.

$Dp \rightarrow$ Increased stress on the soil from a surface loaded area.

$p \rightarrow$ Stress of the loaded area.

$z \rightarrow$ Depth of the soil at the point of interest.

$\gamma \rightarrow$ Unit weight of the soil.

Symbols for Newmark

$IV \rightarrow$ The influence value in the Newmark' chart (for example, a chart divided into 100 areas, each is IV=0.01.

$AB \rightarrow$ Scale to the depth of interest to determine the size of the surface structure graph the Newmark's graph.

$C \rightarrow$ consolidation.

$M \rightarrow$ Nomber of squares (enclosed in the Newmark' chart).

$p_o \rightarrow$ The effective stress at the point of interest.

$q \rightarrow$ The load of the footing.

$q_u \rightarrow$ Ultimate shear strength of a soil.

$\sigma_x \rightarrow$ stress at an specific point (x).

$z \rightarrow$ Depth of the stratum stat of the soil.
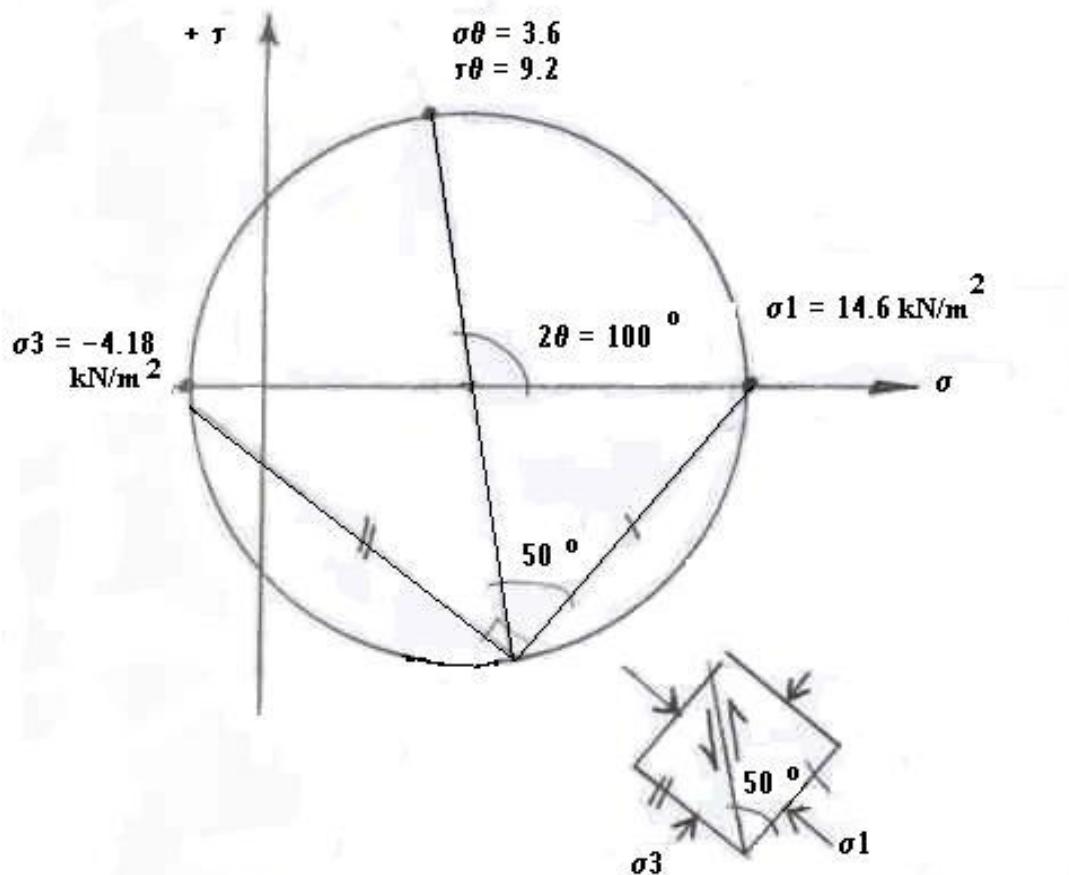
Influence chart for vertical pressure. *[After Newmark (1942).]*

(Revision: Jan-09)

A soil particle is found to be subjected to a maximum stress of 14.6 kN/m², and a minimum stress of − 4.18 kN/m². Find the σ and τ on the plane of $\theta = 50°$ with respect to the major principal stresses, and also find τ_{max}.

(a) The graphical solution,

(b) The calculated solution,

$$\sigma_\theta = \left(\frac{\sigma_1 + \sigma_3}{2}\right) + \left(\frac{\sigma_1 - \sigma_3}{2}\right)\cos 2\theta = \left(\frac{14.6 - 4.18}{2}\right) + \left(\frac{14.6 + 4.18}{2}\right)\cos 2(50°) = 3.6\frac{kN}{m^2}$$
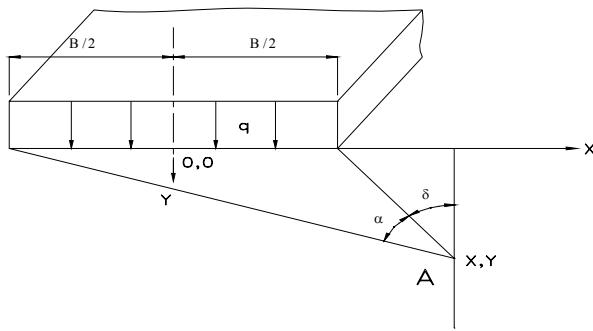
$$\tau_\theta = \left(\frac{\sigma_1 - \sigma_3}{2}\right)\sin 2\theta = \left(\frac{14.6 + 4.18}{2}\right)\sin 2(50°) = 9.2\frac{kN}{m^2}$$

$$\tau_{maximum} = \left(\frac{\sigma_1 - \sigma_3}{2}\right) = 9.4\frac{kN}{m^2}$$

*Mohr – 02: Find the principal stresses and their orientation.
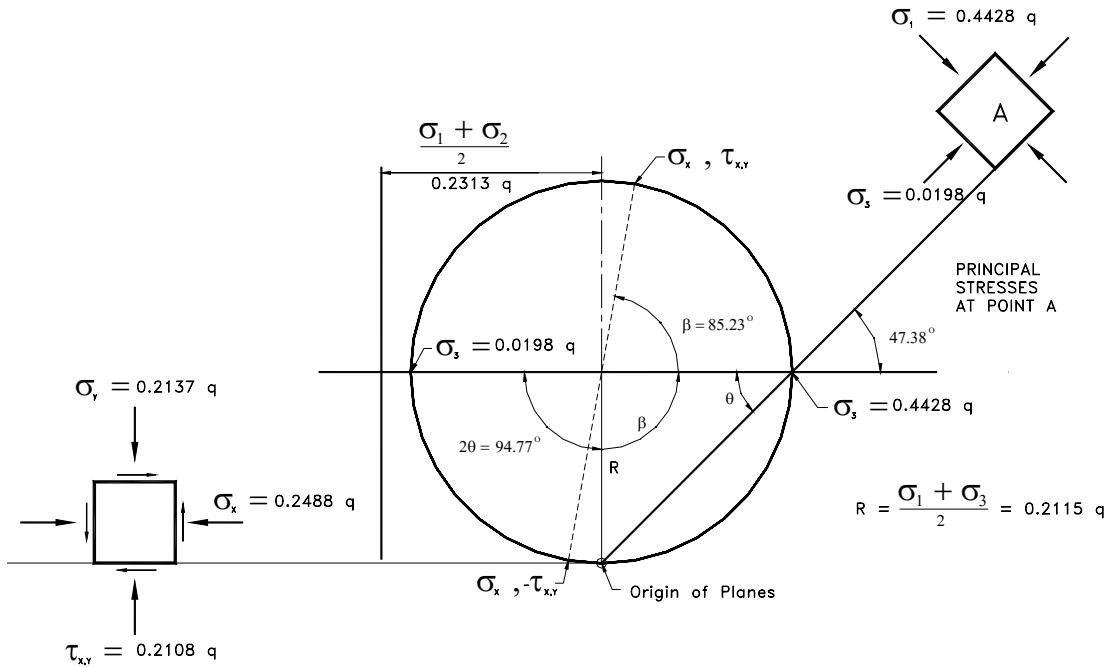
(Revision: Sept.-08)

Equations for the principal stresses in the elastic half-space shown below for a uniformly loaded strip footing are as follows: $\sigma_1 = q/\pi(\alpha + \sin \alpha)$ **and** $\sigma_3 = q/\pi(\alpha - \sin \alpha)$.

The direction of the major principal stress bisects the angle α. Calculate the vertical stress σ_y, the horizontal stress σ_x, and τ_{xy} at point A if x = 0.75B and y = 0.5B using Mohr's diagram.

Solution:

$\alpha + \delta = \arctan 2.5 = 68.20°$ $2\theta + \beta = 180°$

$\delta = \arctan 0.5 = 26.57°$ $\beta = 85°$

$68.20° - 26.57° = 41.63°$
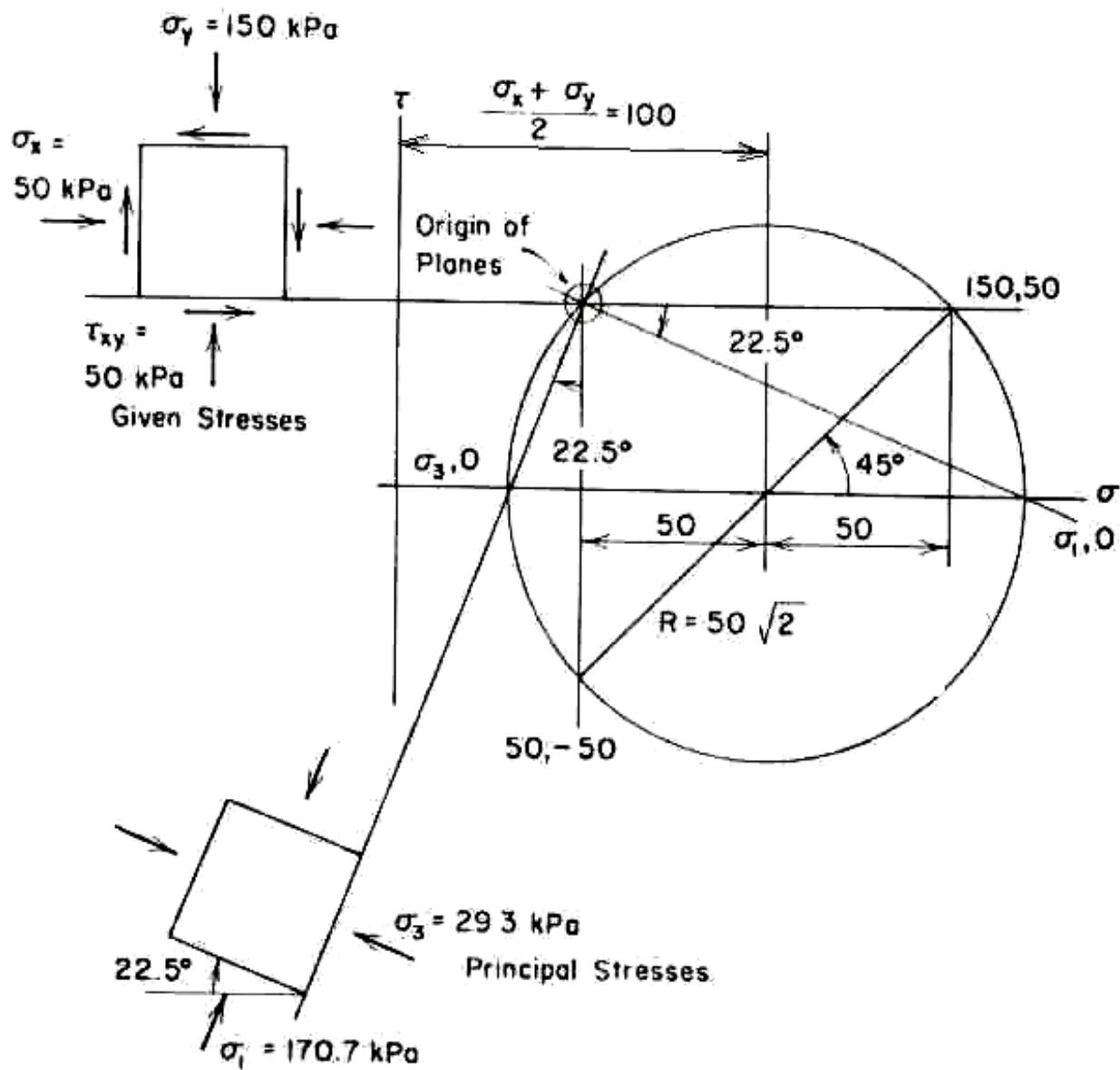
if q = 432.5 kPa then $\sigma_1 = (\sigma_x + \sigma_y)/2 + R = 100 + 70.7 = 170.7$ kPa

$\sigma_3 = (\sigma_x + \sigma_y)/2 - R = 100 - 70.7 = 29.3$ kPa

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(Revision: Sept-09)

Given the general stresses at a point in a soil, determine the principal stresses and show them on a properly oriented element.
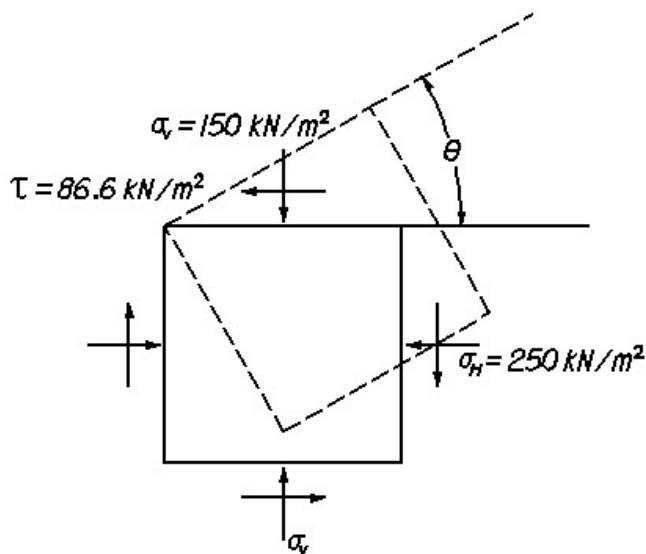
(Revision: Sept-09)

A sample of clean sand was retrieved from 7 m below the surface. The sample had been under a vertical load of 150 kN/m², a horizontal load of 250 kN/m², and a shear stress of 86.6 kN/m². If the angle θ between the vertical stress and the principal stress is 60°, what is the angle of internal friction ϕ of this sample?

Solution:
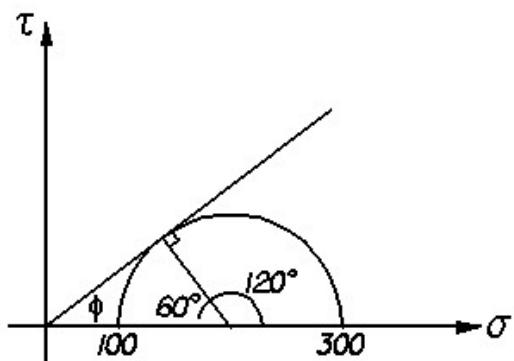
$$\sigma_1 = \frac{\sigma_v + \sigma_h}{2} \pm \sqrt{\left(\frac{\sigma_v - \sigma_h}{2}\right)^2 + (\tau)^2}$$

$$\sigma_1 = \frac{150+250}{2} \pm \sqrt{\left(\frac{150-250}{2}\right)^2 + (86.6)^2}$$

$$\sigma_1 = 200 + 100 = \boxed{300 \ kN/m^2}$$

$$\therefore \sigma_3 = 200 - 100 = \boxed{100 \ kN/m^2}$$

$$\sin\phi = \frac{\tfrac{1}{2}(\sigma_1 - \sigma_3)}{\tfrac{1}{2}(\sigma_1 + \sigma_3)} = \frac{300 - 100}{300 + 100}$$

$$\sin\phi = 0.5$$

$$\phi = \sin^{-1}(0.5)$$

$$\boxed{\phi = 30°}$$
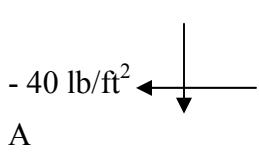
(Revision: Sept-09)

Using a Mohr circle, determine the normal and shear stresses on the plane AB.

Solution:

$\sigma_y = 90 \text{ lb/ft}^2$

- 40 lb/ft^2

A

τ_n θ 30°

B

σ_x

σ_n

125 lb/ft^2

τ_{xy} - 40 lb/ft^2
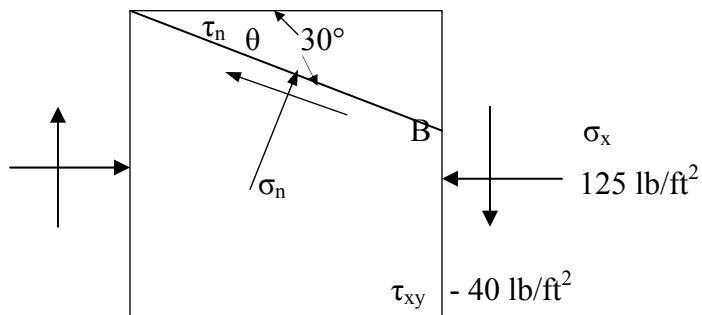
(a) $\sigma_1 = \dfrac{\sigma_y + \sigma_x}{2} + \sqrt{\left(\dfrac{\sigma_y - \sigma_x}{2}\right)^2 + \tau_{xy}^2}$

$\sigma_3 = \dfrac{90 + 150}{2} - \sqrt{\left(\dfrac{90 - 125}{2}\right)^2 + (-40)^2}$

σ_1	=	151	psf
σ_3 =	64 psf		

(b) $\sigma_n = \dfrac{\sigma_1 + \sigma_3}{2} + \dfrac{\sigma_1 - \sigma_3}{2} \cos 2\theta$

$= \dfrac{151 + 64}{2} + \dfrac{151 - 64}{2} \cos 60°$

$\sigma_n = 129 \text{ psf}$

$\tau_n = \dfrac{\sigma_1 - \sigma_3}{2} \sin 2\theta$

$= \dfrac{151 - 64}{2} \sin 60°$

$\tau_n = 38 \text{ psf}$

133
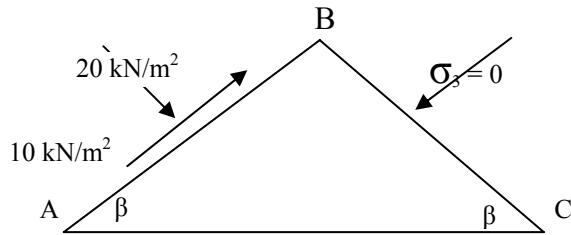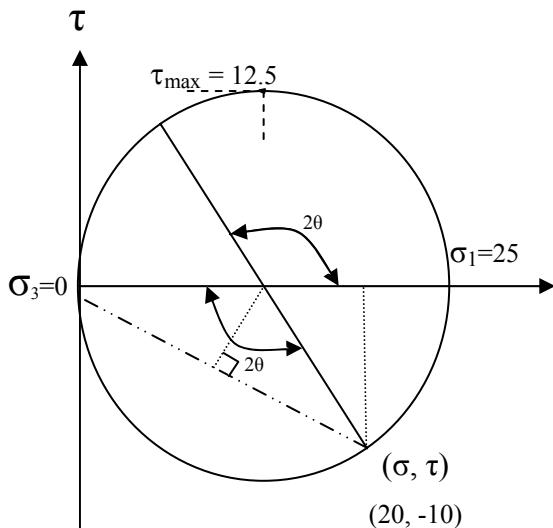
(Revised: Sept-09)

From the stress triangle shown below, find (a) the maximum and minimum principle stresses, (b) the angle alpha, as shown, (c) the angle theta, and (d) the value for the maximum shear stress.

(A) Graphical Solution: OR   **(B)** Analytical Solution:

$$\textcircled{1} \quad \sigma_1 = \frac{\sigma_1 + \sigma_1}{2} + \frac{\sigma_1 + \sigma_1}{2} COS\ 2\theta$$

$$20 = \frac{\sigma_1}{2} + \frac{\sigma_1}{2} COS\ 2\theta \quad \therefore \quad \sigma_1 = \frac{40}{1 + cos\ 2\theta}$$

$$1 + cos2\theta$$

$$\textcircled{2} \quad \tau_n = \frac{\sigma_1 - \sigma_3}{2} SIN\ 2\theta$$

$$2$$

$$10 = \frac{\sigma_1}{2} SIN\ 2\theta \qquad \therefore \quad \vartheta_1 = \frac{20}{SIN\ 2\theta}$$

$$\therefore \quad 1 - 2\ sin\ 2\theta + cos\ 2\theta = 0$$

(a) $\boxed{\sigma_1 = 25\ kN/m^2 \ \text{ and } \ \sigma_3 = 0}$

(b) α in the figure is in the angle 2θ between (σ, τ) and σ_3.

...... $\boxed{\theta = 63.4°}$

...... $\boxed{\alpha = 2\theta = 126.9°}$

..... $\boxed{\sigma_1 = \dfrac{20}{sin\ 126.9^0} = 25\ KN/m^2, \qquad \sigma_3 = 0}$

(c)   ...... $\boxed{\theta = 63.4°}$

and $\boxed{\tau_{max} = 12.5\ kN/m^2}$

...... $\boxed{\tau_{max} = 12.5\ kN/m^2}$
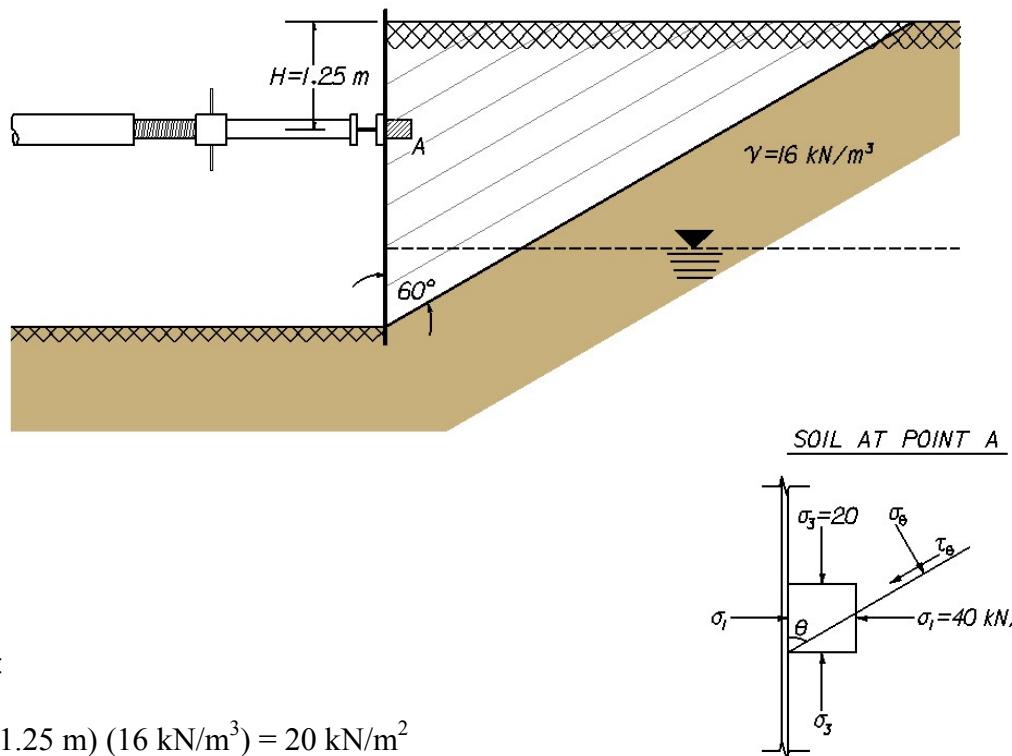
(Revised Sept-09)

The temporary excavation shown below is braced with a steel tube strut. Every morning, a misguided foreman tightens the screw mechanism on the strut "just to be safe". The stress on a soil particle at point A, just behind the wall, has been measured with a pressure sensor installed by the Engineer. It now measures 40 kN/m². If the potential failure planes in the soil behind the wall sustain 60° angles with respect to the vertical wall, estimate the normal and shear stresses at that point A along a potential failure plane.

Solution:

At point A:

$\sigma_v = h\gamma = (1.25 \text{ m}) (16 \text{ kN/m}^3) = 20 \text{ kN/m}^2$

∴ σ_v is the minor principal stress at A,

Since $\theta = 60°$ is with respect to the <u>major</u> principal stress (σ_1) plane, then $\sigma_v = \sigma_3$

∴ $\sigma_\theta = (\sigma_1 + \sigma_3)/2 + (\sigma_1 - \sigma_3)/2 \cos 2\theta = (40+20)/2 + (40-20)/2 \cos 120°$

∴ $\sigma_\theta = \boxed{25 \text{ kN/m}^2}$

and $\tau_\theta = (\sigma_1 - \sigma_3)/2 \sin 2\theta = (40-20)/2 \sin 120°$
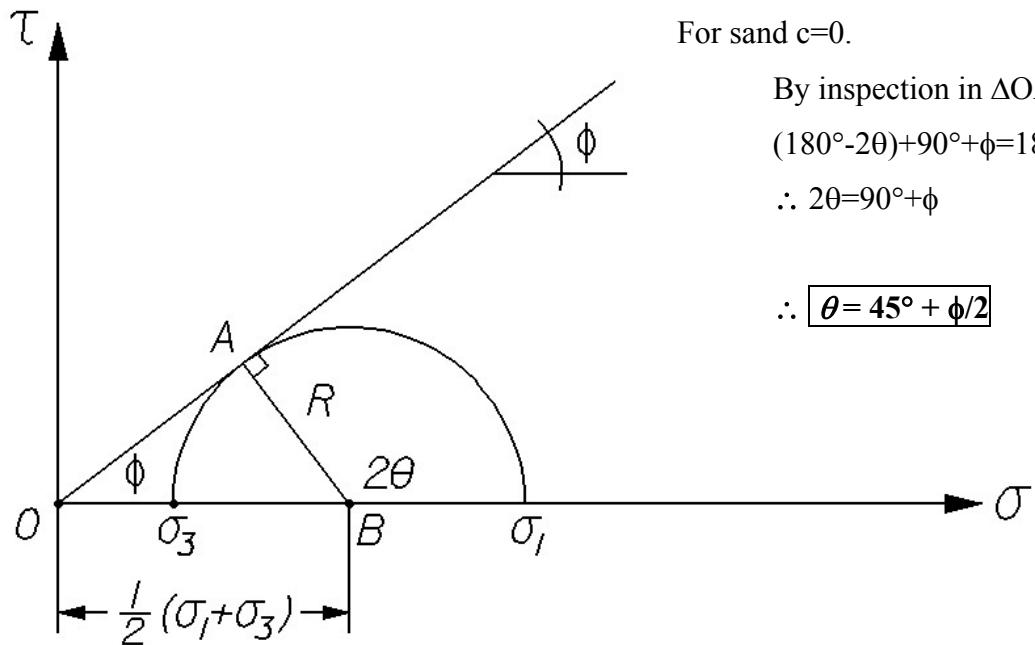
∴ $\tau_\theta = \boxed{8.7 \text{ kN/m}^2}$

(Revised Sept-09)

For a clean sand, prove that $\theta = 45° + \phi/2$ using Mohr's circle.

Solution:

For sand c=0.

By inspection in $\triangle OAB$

$(180°-2\theta)+90°+\phi=180°$

$\therefore 2\theta=90°+\phi$

$\therefore \boxed{\theta = 45° + \phi/2}$

A failure test on a clean sand (i.e. c=0) shows that σ_1=11.5 ksf and σ_3=3.2 ksf at failure. Find the angle ϕ for this sand.

In the $\triangle OAB$ above, $R = \frac{1}{2}\left(\sigma_1 - \sigma_3\right)$
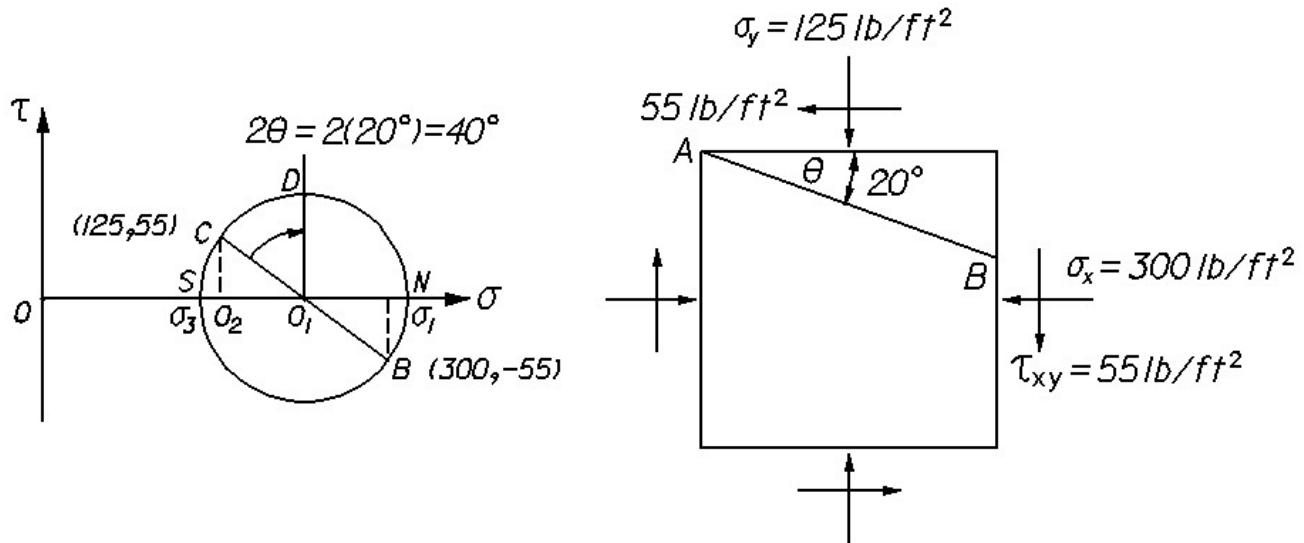
$\therefore \sin\phi = \dfrac{\left[\frac{1}{2}\left(\sigma_1 - \sigma_3\right)\right]}{\left[\frac{1}{2}\left(\sigma_1 + \sigma_3\right)\right]} = \dfrac{(11.5 - 3.2)}{(11.5 + 3.2)} = 0.565$

$\therefore \boxed{\phi = 34.5°}$

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(Revised Sept-09)

Determine the normal and shear stresses on the plane AB.

$00_1 = (300+125)/2 = 212.5$ *psf*

$0_1 0_2 = (300-125)/2 = 87.5$ *psf*

$0_1 B = \sqrt{87.5^2 + 55^2} = 103$ *psf*

$\sigma_1 = 212.5 + 103 = \boxed{315.5 \ psf}$

$\sigma_3 = 212.5 - 103 = \boxed{109.5 \ psf}$

$\pi C 0_1 0_2 = tan^{-1} \ (55/87.5) = 32°$

$\sigma_n = 212.5 - 103 \ cos \ (32+40) = \boxed{181 \ psf}$

$\tau_n = 103 \ sin \ (32+40) = \boxed{98 \ psf}$

137
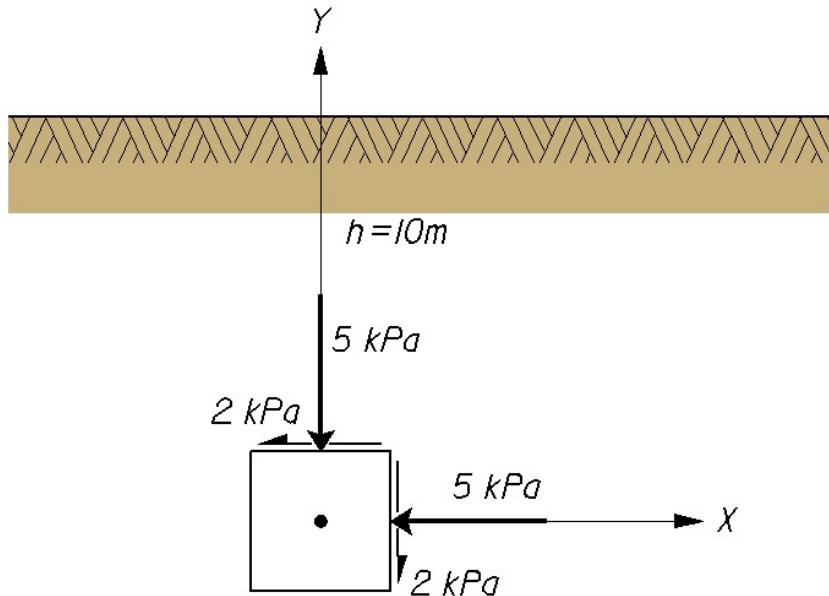
(Revised Sept-09)
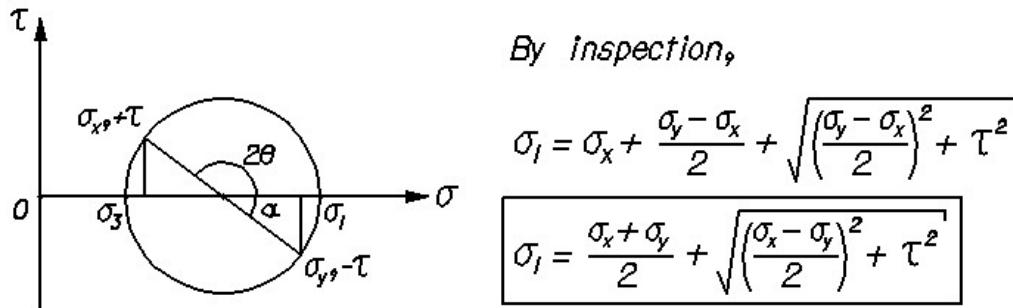
(37)   Derive the equation that transforms a general state of stress to the principal state of stress. (Hint: Use Mohr's circle for a graphical solution).

(38)   Determine the value of the major principal stress.

(39)   Determine the angle θ between the major principal stress and the state of stress shown in the figure above.

a)

By inspection,

$$\sigma_I = \sigma_x + \frac{\sigma_y - \sigma_x}{2} + \sqrt{\left(\frac{\sigma_y - \sigma_x}{2}\right)^2 + \tau^2}$$

$$\boxed{\sigma_I = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2}}$$

b) $\sigma_I = \frac{5+5}{2} + \sqrt{0 + 2^2} = \boxed{7\,kPa}$
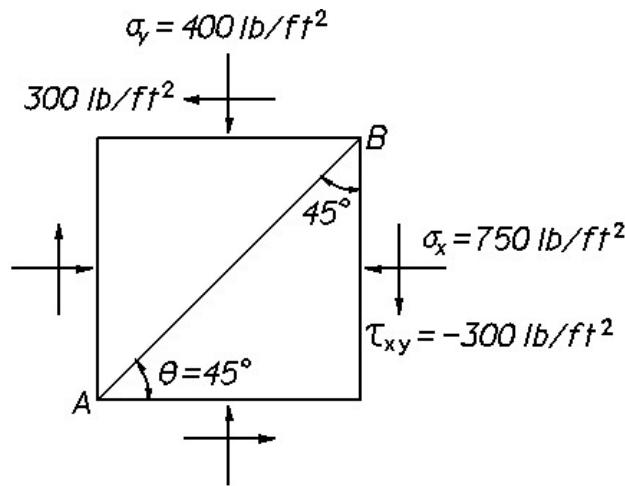
c) $\sin \alpha = \frac{\tau}{\sqrt{\left(\frac{\sigma_y - \sigma_x}{2}\right)^2 + 2^2}} = \frac{2}{\sqrt{0 + 2^2}} = 1$ $\therefore \alpha = 90°$

but $\alpha + 2\theta = 180°$

$\therefore 2\theta = 180°$

$\boxed{\theta = 45°}$

(Revised Sept-09)

Determine the maximum and minimum principal stresses, and the normal and shear stresses on plane AB.

Maximum and minimum principal stresses:

$$\begin{matrix} \sigma_1 \\ \sigma_3 \end{matrix} = \frac{\sigma_y + \sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_y - \sigma_x}{2}\right)^2 + (\tau_{xy})^2}$$

$$= \frac{400 + 750}{2} \pm \sqrt{\left(\frac{400 - 750}{2}\right)^2 + (-300)^2}$$

$$\therefore \boxed{\begin{array}{l} \sigma_1 = 922 \ psf \\ \sigma_2 = 228 \ psf \end{array}}$$

Normal and shear stresses:

$$\sigma_n = \frac{\sigma_1 + \sigma_3}{2} + \frac{\sigma_1 - \sigma_3}{2} \cos \ 2\theta + \tau_{xy} \sin \ 2\theta$$

$$= \frac{400 + 750}{2} + \frac{400 - 750}{2} \cos \ 90° + (-300) \sin \ 90°$$

$$\boxed{\sigma_n = 200 \ psf}$$
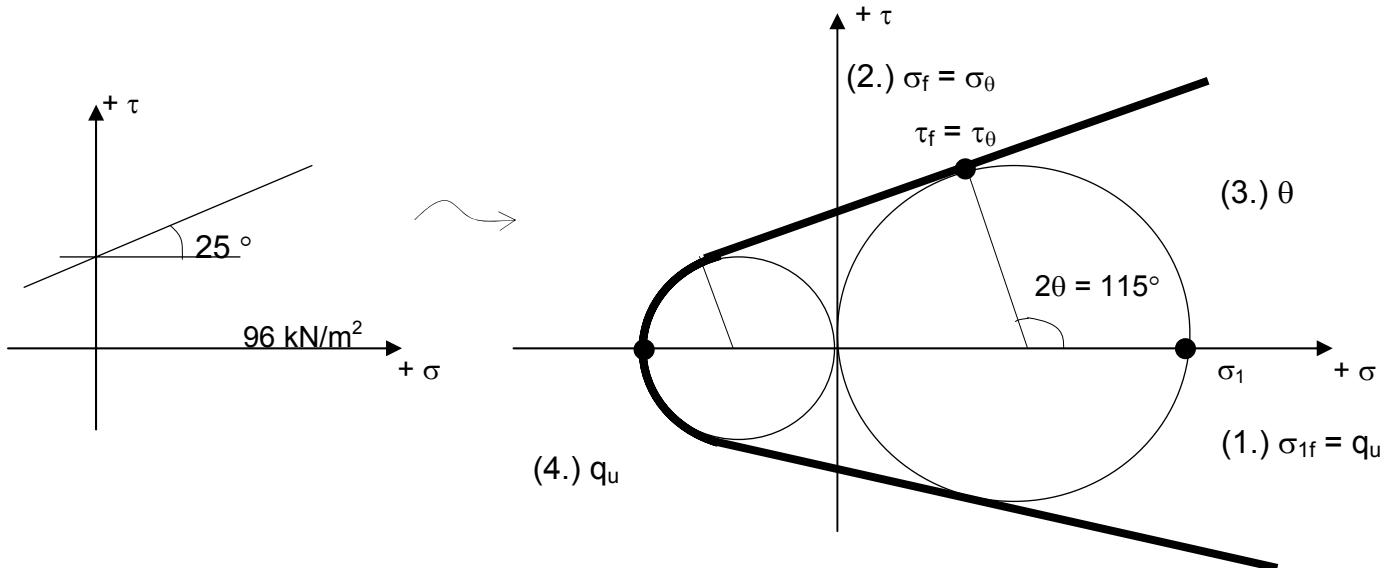
$$\tau_n = \frac{\sigma_1 - \sigma_3}{2} \sin \ 2\theta - \tau_{xy} \cos \ 2\theta$$

$$= \frac{400 - 750}{2} \sin \ 90° - (-300) \cos \ 90°$$

$$\boxed{\tau_n = -175 \ psf}$$

139

Data from Mohr-Coulomb failure envelope.

(Revised Sept-09)

A soil sample has been tested and when plotted developed the Mohr-Coulomb envelope of failure shown below. Find (1) axial stresses at failure, (2) the normal and shear stresses on the failure plane, (3) the angle of failure with respect to the principal axis, and (4) the soil tensile strength.

Solution:

$\sigma_{1f} = q_u = 300$ Pa

$\sigma_f = \sigma_\theta = 86$ Pa

$\tau_f = \tau_\theta = 136$ Pa

$\theta = 115°/2 = 57.5°$

$q_u = -123$ Pa

140
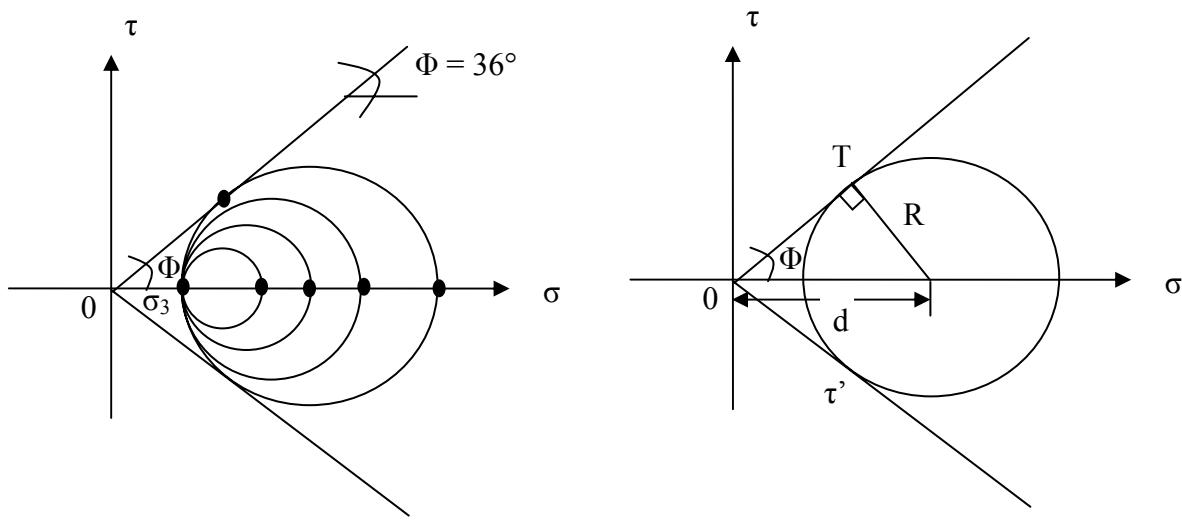
(Revised Sept-09)

A dry sample of sand was tested in a triaxial test. The angle of internal friction was found to be 36°. If the minor principal stress was 300 kPa, at what value of maximum principal stress will the sample fail? The same test was then performed on a clay sample that had the same φ, and cohesion of 12 kPa. What was the new maximum principal stress?

Solution:

a)   Failure will occur when the Mohr circle becomes tangent to the failure envelopes.

$$\frac{R}{d} = \sin\phi$$

$$\sigma_3 = d - R = \frac{R}{\sin\phi} - R = R\frac{(1 - \sin\phi)}{\sin\phi}$$

$$\sigma_1 = d + R = \frac{R}{\sin\phi} + R = R\frac{(1+\sin\phi)}{\sin\phi}$$

$$\therefore \frac{\sigma_1}{\sigma_3} = \frac{(1+\sin\phi)}{(1-\sin\phi)} = \tan^2(45° + \frac{\phi}{2})$$

$$\therefore \sigma_1 = (300\ kPa)\tan^2(45° + \frac{36°}{2}) = 1160\ kPa$$
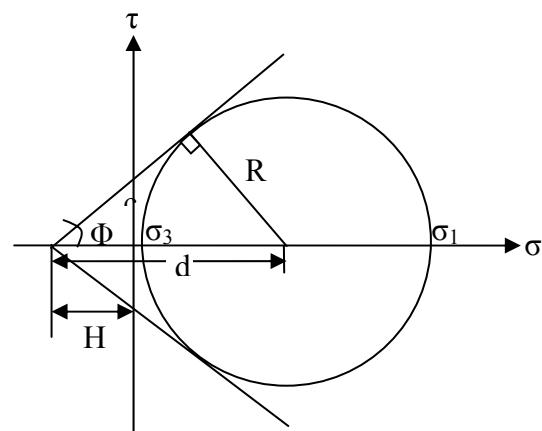
b)

$$H = c(\cot\phi)$$

$$\sigma_1 + H = (\sigma_3 + H)\tan^2(45° + \frac{\phi}{2})$$

$$H = (12\,kPa)(\cot 36°) \approx 17\,kPa$$

$$\sigma_1 = (300+17)\tan^2(45° + \frac{36°}{2}) - 17 = 1,200\,kPa$$
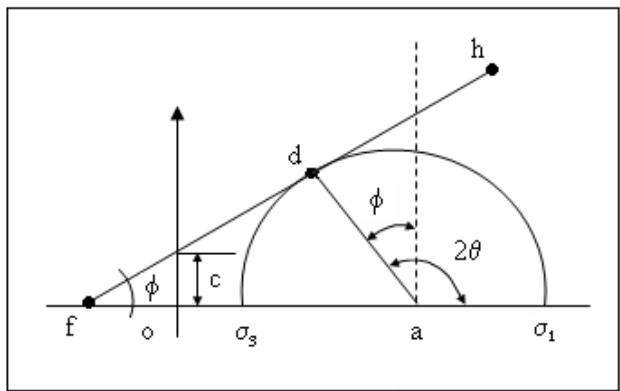
Derive the general formula for horizontal stress.
(Revised Sept-09)

Derive the general formula of the horizontal stress as a function of the vertical stress, cohesion and the angle of internal friction.

Solution:

$$\theta = 45° + \frac{\theta}{2} \quad \therefore \ 2\theta = 90° + \phi$$

$$\overline{ad} = \frac{\sigma_1 - \sigma_3}{2}$$

$$\sin\phi \approx \frac{\overline{ad}}{\overline{af}} \tag{1}$$

$$\overline{af} = \overline{af} + \overline{af} = (c)\cot\theta + \frac{\sigma_1 + \sigma_3}{2} \tag{2}$$

$$\tan\theta = \frac{c}{\overline{of}} \quad \therefore \ \frac{\overline{of}}{c} = \cot\theta \quad \therefore \ \overline{of} = (c)\cot\theta \tag{3}$$

According to the Mohr's circle properties:

$$\overline{oa} = \sigma_3 + \frac{\sigma_1 - \sigma_3}{2} \quad \therefore \ \overline{oa} = \frac{\sigma_1 - \sigma_3}{2} \quad \therefore \ \overline{af} = (c)\cot\phi + \frac{\sigma_1 - \sigma_3}{2} \tag{4}$$ **From (1) & (4):**

$$\sin\phi = \frac{\dfrac{\sigma_1 - \sigma_3}{2}}{(c)\cot\phi + \dfrac{\sigma_1 + \sigma_3}{2}} \quad \therefore \ \sin\phi\left((c)\cot\phi + \frac{\sigma_1 + \sigma_3}{2}\right) = \frac{\sigma_1 - \sigma_3}{2}$$

$$(c)\sin\phi\cot\phi + \frac{\sigma_1 + \sigma_3}{2}\sin\phi = \frac{\sigma_1 - \sigma_3}{2} \quad \therefore \ (c)\sin\phi\cot\phi = \frac{\sigma_1 - \sigma_3}{2} - \frac{\sigma_1 + \sigma_3}{2}\sin\phi$$

$$\therefore \ 2\left((c)\sin\phi\cot\phi\right) = \sigma_1 - (\sin\phi(\sigma_1) - (\sigma_3 + \sin\phi(\sigma_3)))$$

$$\therefore \ 2\left((c)\sin\phi\frac{\cot\phi}{\sin\phi}\right) = \sigma_1\ (1 - \sin\phi) - \sigma_3 + (1 + \sin\phi) \qquad \therefore \ 2\frac{(c)\cos\phi}{1 - \sin\phi} + \sigma_3\frac{1 + \sin\phi}{1 - \sin\phi} = \sigma_1$$

Since $\dfrac{\cos\phi}{1 - \sin\phi} \approx \tan\left(45^0 + \phi\right)$ **and** $\dfrac{1 + \sin\phi}{1 - \sin\phi} \approx \tan^2\left(45^0 + \dfrac{\phi}{2}\right)$

$$\boxed{\sigma_1 = \sigma_3\ \tan^2\left(45^0 + \frac{\phi}{2}\right) + 2c\ \tan\left(45^0 + \frac{\phi}{2}\right)}$$

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(Revision: Sept.-08)

A construction site has a surface layer of Aeolic sand 2 m thick, underlain by a 10 m thick clay stratum. The project involves placing a wastewater treatment tank, 10 m x 10 m in plan, with a contact pressure $p_o = 400\ kN/m^2$. Find the stress down the centerline of the tank at the top and the bottom of the clay stratum using Newmark's influence chart shown below.

Solution:

B = 10 m

SAND   2 m

①

CLAY   10 m

②

AB = 2 m

AB = 12 m

A   *B*

Influence Value *(IV)* = 1/200 = 0.005

B = 10 m for point #1

B = 10 m for point #2

x

y

The increase in the vertical stress is found from $\Delta p = p_o\ M\ (IV)$ where *M* is the number of "squares" enclosed in the Newmark chart and *(IV)* is the influence value.

For Δp_1, $AB = $ 2 m → $\Delta p_{1=}\ p_o\ M(IV) = $ (400 kN/m²) (190) (0.005) → $\underline{\Delta p_1 = \ 380\ kN/m^2}$

For Δp_2, $AB = 12$ m → $\Delta p_{2=}\ p_o\ M(IV) = $ (400 kN/m²) (42) (0.005) → $\underline{\Delta p_2 = \ \ \ 84\ kN/m^2}$____(which represents a 78% reduction in the vertical stress).

143
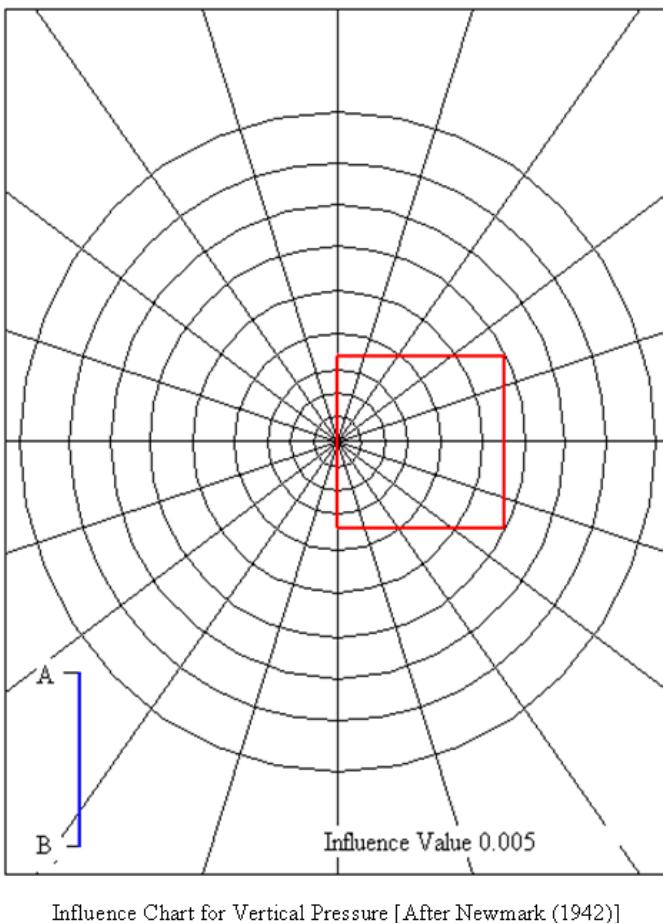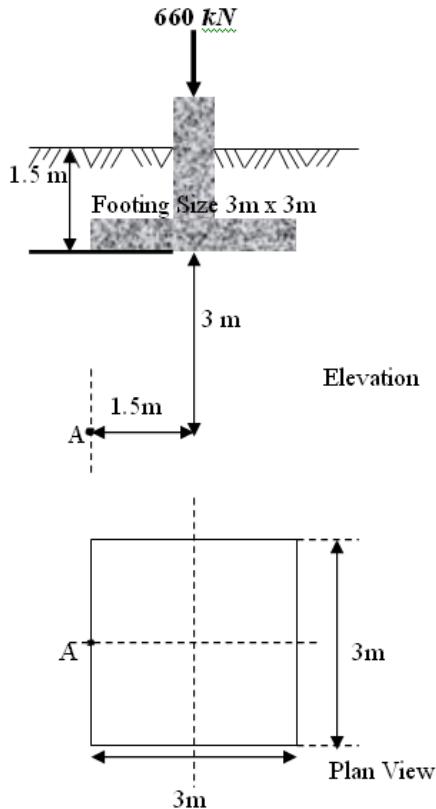
(Revision: Aug-08)

Find the stress at the point A shown below, at a depth of 3 m below the edge of the footing. The plan of the square footing has been plotted on top of the Newmark graph to a scale of $AB = 3m$ and placed in such a way that point A falls directly over the center of the chart.

Solution:

The distance AB is now $= 3m$

660 kN

1.5 m

Footing Size 3m x 3m

3 m

Elevation

1.5m

A

A

3m

3m

Plan View

A

B

Influence Value 0.005

Influence Chart for Vertical Pressure [After Newmark (1942)]

The number of elements inside the outline of the plan is about $M = 45$. Hence,

$$\Delta p = q \ M \ (IV) = \left[\frac{660 \ kN}{(3 \ m)(3 \ m)} \right] (45)(0.005) = 16.5 \ kN \ / \ m^2$$

The number of elements inside the outline of the plan is about $M = 45$. Hence,
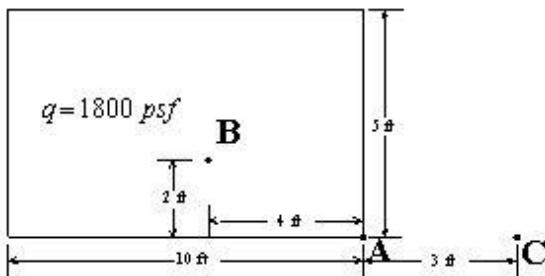
$$\Delta p = q \ M \ (IV) = \left[\frac{660 \ kN}{(3 \ m)(3 \ m)} \right] (45)(0.005) = 16.5 \ kN \ / \ m^2$$

144
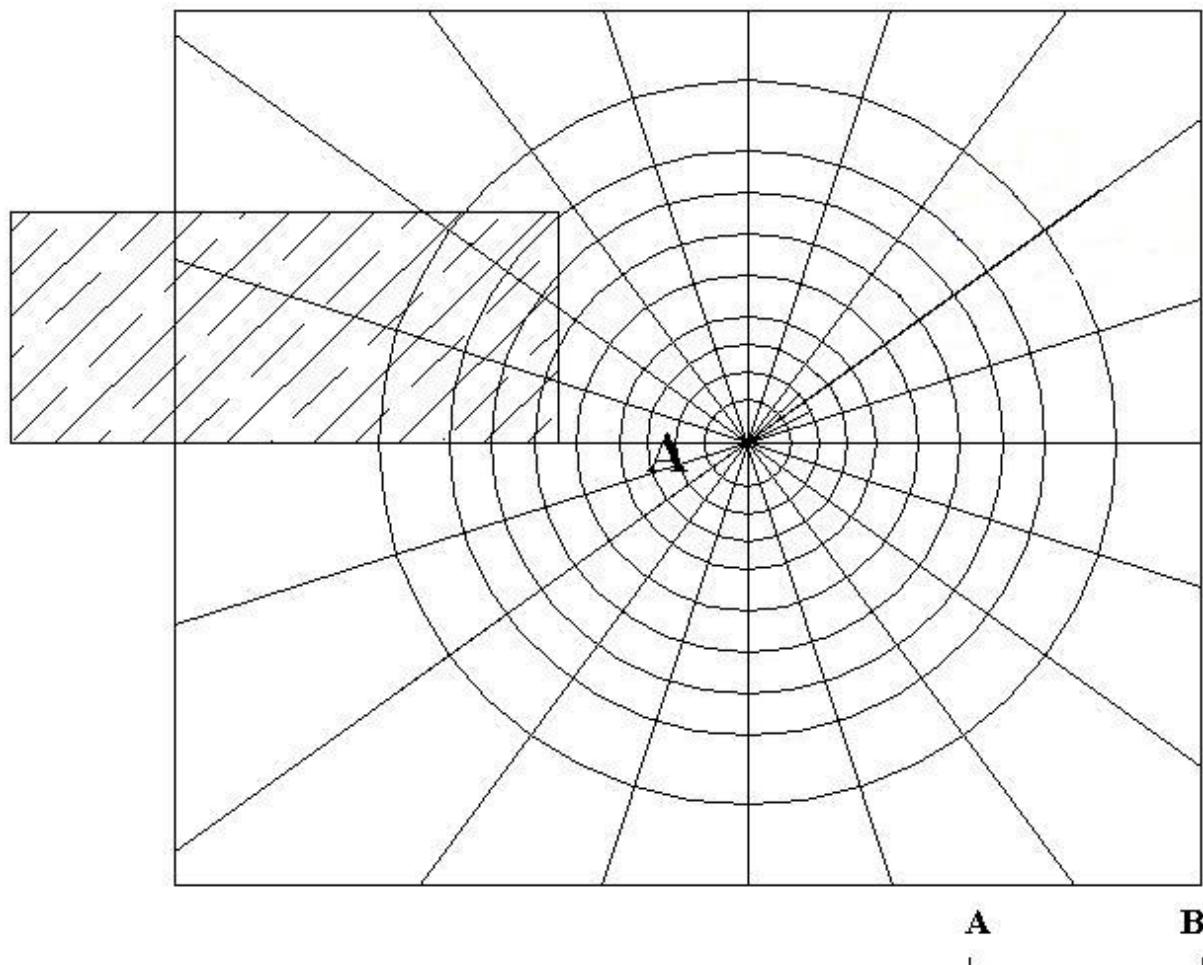
(Revision: Aug-08)

The footing shown below has a load q = 1.8 *ksf*. Find the stress at a depth of 5 feet below the footing invert, at the point C.

Influence Value (IV) = 0.005   Depth Point = Z

Solution:

A   B
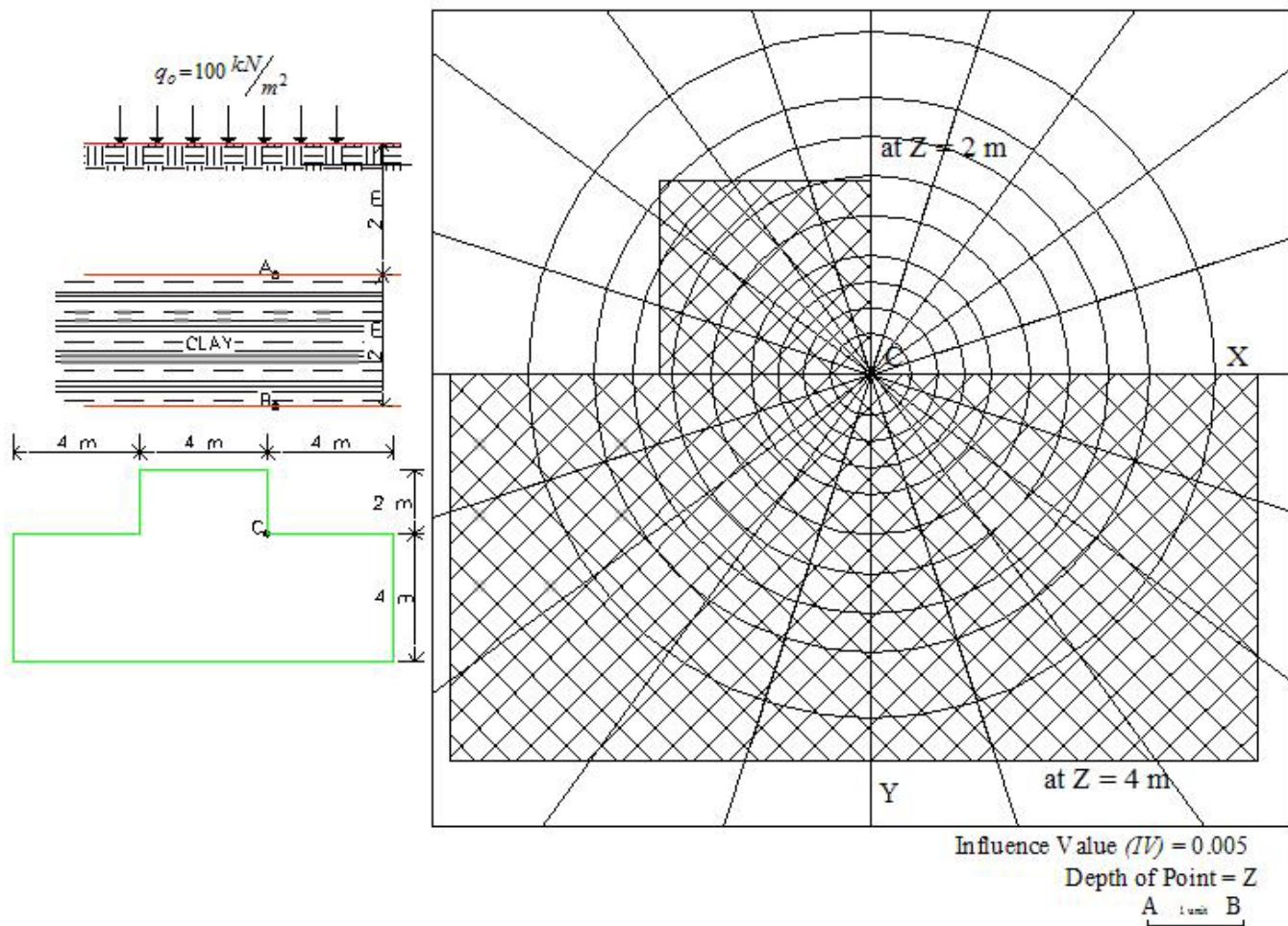
Set AB = 5 and draw the footing to that scale. The number of affected areas *M* =8, therefore

$\Delta p = q \, M \, (IV) = (1{,}800 \; psf)(8)(0.005) = $ *72 psf*

145

Newmark-04: *Stresses coming from complex shaped foundations.*

(Revision: Aug-08)

A small but heavy utility building will be placed over a 2 m thick sand stratum. Below the sand is a clay stratum 2 m thick. Find the stress at points A and B in the clay stratum directly below point C at the surface.

$q_o = 100 \, ^{kN}/_{m^2}$

at Z = 2 m

CLAY

4 m   4 m   4 m

2 m

4 m

X

Y

at Z = 4 m

Influence Value *(IV)* = 0.005
Depth of Point = Z

A ←1 unit→ B

Solution:

Point A (top of clay stratum) $q_A = q \, M \, (IV) = (100)(136)(0.005) = 68 \dfrac{kN}{m^2}$

Point B (bottom of clay) $\quad q_B = q \, M \, (IV) = (100)(100)(0.005) = 50 \dfrac{kN}{m^2}$

$q_{average} = \dfrac{q_A + q_B}{2} = \dfrac{(70 + 50)}{2} = 59 \dfrac{kN}{m^2}$

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(Revision: Aug-08)
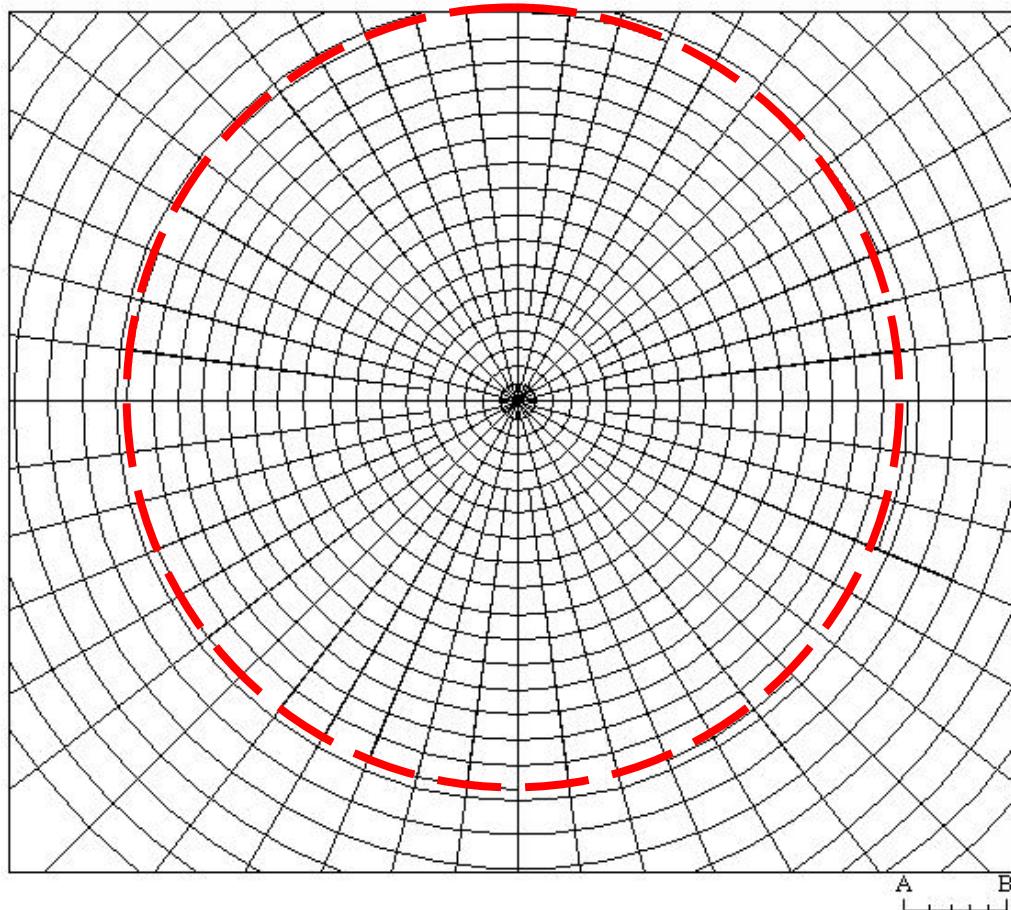
A circular oil storage tank will be built at the shore of Tampa Bay. It will be 20 m in diameter, and 15 m high. The tank sits upon a 2 m thick sand deposit that rests upon a clay stratum 16 m thick. The water table is at practically at the surface. Find the stress increase from a fully loaded tank, at mid-clay stratum, (a) directly under the center of the tank, and (b) at its outer edge, using the Newmark influence chart shown below.

<p style="text-align:center">Set AB 10 m   Influence Value (IV) = 0.001</p>

Solution:

The contact stress is $q_o = (0.95)(9.81 \text{ kN/m}^3)(15\text{m}) = 140 \ kN \ / \ m^2$

At mid-clay depth along the centerline of the tank (depth = 10 m) $\therefore$ OQ = 10 m
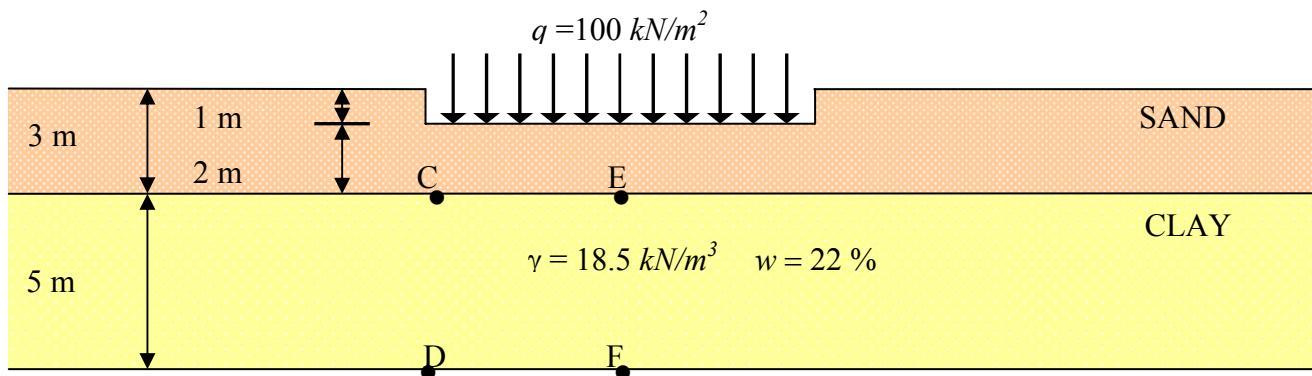
$$\sigma'_v = (q_o - \gamma_w h) M(IV) = [140 - (9.81)(10)](680)(0.001) = 28.5 \frac{kN}{m^2}$$

(Revision: Aug-08)

A small but heavily loaded utility building has dimensions of 20 m x 20 m. It applies a uniform load on its mat foundation of 100 kN/m^2. Its mat foundation sits 1 m below the surface. The soil profile consists of 3 m of a dry sand, with $\gamma = 17.5$ kN/m^3 under laid by a 5 m thick clay layer with a $\gamma = 18.5$ kN/m^3, a moisture content of 22%, $C_c = 0.30$ and a $G_s = 2.70$. The clay stratum is under laid by another sand stratum, and the phreatic surface coincides with the top of the clay stratum.

(40) Using the Newmark method, what are the new stresses at the top and bottom of the clay stratum due to the building's loading?

(41) What is the expected differential settlement between the building's center and one of its corners, in mm?

(42) If a laboratory sample 4" thick of the field clay attained 50% consolidation in 5 hours, what time will the clay layer in the field attain 60% consolidation?

$q = 100 \ kN/m^2$

3 m   1 m

2 m   C   E

SAND

CLAY

$\gamma = 18.5 \ kN/m^3$ $w = 22 \ \%$

5 m

D   F

Solution:

(1) Set AB = 2 m and observe that the building's foot-print covers the entire graph.

$\therefore \quad \Delta\sigma_E = 100 \ kN / m^2$

Set AB = 7 m and the number of units $M = 175$

$\therefore \quad \Delta\sigma_F = qM \ (IV) = (100)(175)(0.005) = 87.5 \ kN / m^2$

The stress at point C has dropped to 50% of the stress at E,

$\therefore \quad \Delta\sigma_C = 50 \ kN / m^2 \qquad and \quad \therefore \quad \Delta\sigma_D = (0.96)(50) = 48 \ kN / m^2$

Average stress in the clay stratum beneath the center $= \dfrac{(100 + 87.5)}{2} = 93.8 \ \dfrac{kN}{m^2}$

Average stress in the clay stratum beneath the corner $= \dfrac{(50 + 48)}{2} = 49 \ \dfrac{kN}{m^2}$

(2) The differential settlement between the center and a corner of the building is,

$C_c = 0.30 \ and \ G_S = 2.70$

$Se_o = wG_S \quad \therefore \quad e_o = \dfrac{wG_s}{S} = \dfrac{(0.22)(2.70)}{1} = 0.59$

The in-situ stress at mid-clay stratum before the building is built is,

$p_o = \gamma_{sand} h_{sand} + \gamma_{clay} h_{at \ mid-clay} = (3m)(17.5) + (2.5m)(18.5 - 9.8) = 74.3 \dfrac{kN}{m^2}$

Therefore,

the settlement at the center $\Delta H = \dfrac{C_C H}{1+e} \log\left(\dfrac{p_o + \Delta\sigma}{p_o}\right) = \dfrac{(0.30)(5m)}{1+(0.59)} \log\left(\dfrac{(74.3)+(93.8)}{(74.3)}\right) = 0.33 m$

the settlement at the corner $\Delta H = \dfrac{C_C H}{1+e} \log\left(\dfrac{p_o + \Delta\sigma}{p_o}\right) = \dfrac{(0.30)(5m)}{1+(0.59)} \log\left(\dfrac{(74.3)+(49)}{(74.3)}\right) = 0.21 m$

the differential settlement is $\Delta(\Delta H) = 0.33 \, m - 0.21 \, m = 0.12 \ m = 120 \ mm$

(3) The time required to attain 60% consolidation in the field is,

$c_v = \dfrac{T_\sigma H_{drained}^2}{t} = \dfrac{(0.26)(2,500 \, mm)^2}{t_F} = \dfrac{(0.18)(2 x 25.4 \, mm)^2}{(5 \ hours)}$
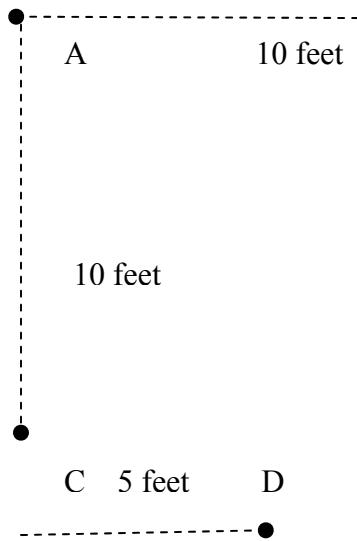
Solving for the time of settlement in the field t_f,

$t_F = \dfrac{(0.26)(2,500 \, mm)^2}{(0.18)(2 x 25.4 \, mm)^2} \dfrac{(5 \, hours)}{1} \left(\dfrac{1 \, day}{24 \, hours}\right)\left(\dfrac{1 \, year}{365 \, days}\right) \approx 2 \ years$

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(Revision: Aug-08)

Point loads of 2000, 4000, and 6000 lbs act at points A, B and C respectively, as shown below. Determine the increase in vertical stress at a depth of 10 feet below point D.

Solution.

Using the Boussinesq (1883) table on page 202 for vertical point loads, the vertical increase in stress contributed by each at a depth $z = 10$ feet is found by,

$$\Delta p_z = \frac{P}{z^2}\left\{ \frac{3}{2\pi}\frac{1}{\left[(r/z)^2+1\right]^{5/2}}\right\} = \frac{P}{z^2}I_1$$

Increase in the load at:	P (lbs)	r (ft)	z (ft)	r/z	I_1	Δp (psf)
Δp from A	2,000	$(10^2+5^2)^{1/2} = 11.18$	10	1.12	0.0626	1.25
Δp from B	4,000	$(10^2+5^2)^{1/2} = 11.18$	10	1.12	0.0626	2.50
Δp from C	6,000	5	10	0.50	0.2733	16.40
						Total = *20.2 psf*

Therefore, the vertical stress increase at D from the three loads A, B and C is 20.2 psf.
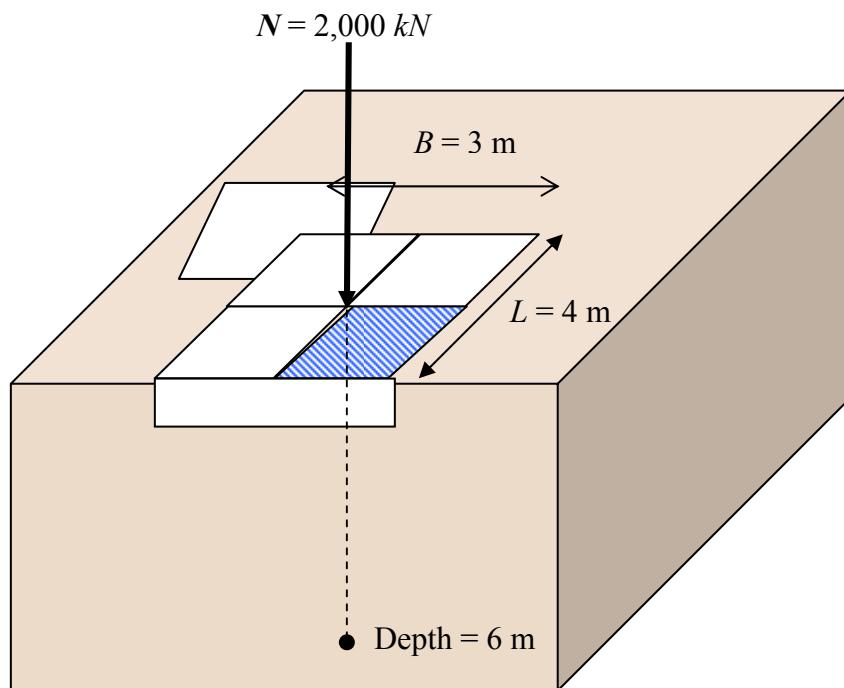
(Revision: Aug-08)

Determine the vertical stress increase in a point at a depth of 6 m below the center of the invert of a newly built spread footing, 3 m by 4 m in area, placed on the ground surface carrying a columnar axial load of N = 2,000 kN.

Solution:

The Boussinesq solution for a rectangular loaded area only admits finding stresses below a corner of the loaded area. Therefore, the footing must be cut so that the load is at a "corner" (shown as the quarter of the area), where the reduced footing dimensions for the shaded area are $B_1 = 1.5$ m and $L_1 = 2.0$ m.

$N = 2,000 \ kN$

$B = 3$ m

$L = 4$ m

Depth = 6 m

$$m = \frac{B_1}{z} = \frac{1.5m}{6.0m} = 0.25 \quad and \quad n = \frac{L_1}{z} = \frac{2.0m}{6.0m} = 0.33$$
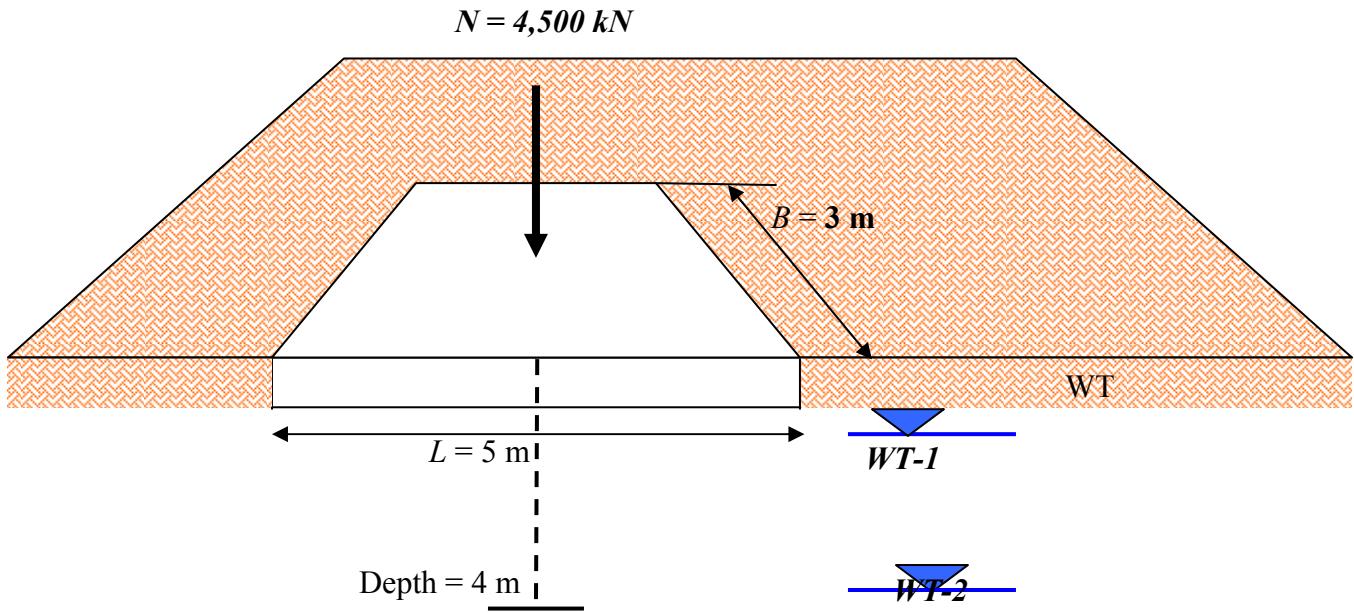
Use the table and extrapolate and find $I_4 = 0.0345$

$$\Delta q_z = q_o \left(4 \ I_4\right) = \left(\frac{N}{BL}\right)\left(4 \ I_4\right) = \left(\frac{2,000 \ kN}{(3m)(4m)}\right)(4)(0.0345) = 23 \ kN/m^2$$

(Revision: Aug-08)

Find the effective stress increase in the soil at a depth of 4 m below the footing, and then find the increase in the stress due to a drop of the *WT* from originally 1 m below the footing to 5 m below the footing.

N = 4,500 kN

B = 3 m

WT

WT-1

L = 5 m

Depth = 4 m

WT-2

<u>*Solution:*</u>

$$q = \frac{N}{A} = \frac{4,500 \, kN}{15 \, m^2} = 300 \frac{kN}{m^2} \quad m = \frac{B}{z} = \frac{1.5 \, m}{4 \, m} = 0.375 \; and \; n = \frac{L}{z} = \frac{2.5 \, m}{4 \, m} = 0.625$$

$$\therefore \quad I_4 = 0.076$$

a) The total stress increase from the footing is,

$$\Delta p_o = q(4I_4) = (300)(4)(0.076) = 91.2 \, kN / m^2$$

and the effective stress when the *WT* is 1 m below the footing is,

$$\Delta p_o' = \Delta p_o - u = (91.2) - (3m)(9.8) = 61.8 \, kN / m^2$$

b) When the WT drops from -1 m to -5 m below the footing, the effective stress is identical to the total stress. Therefore the effective stress increase is,

$$\Delta p_o = 91.2 \, kN / m^2 \text{ which is a 48\% increase in stress.}$$

(Revision: Aug-08)
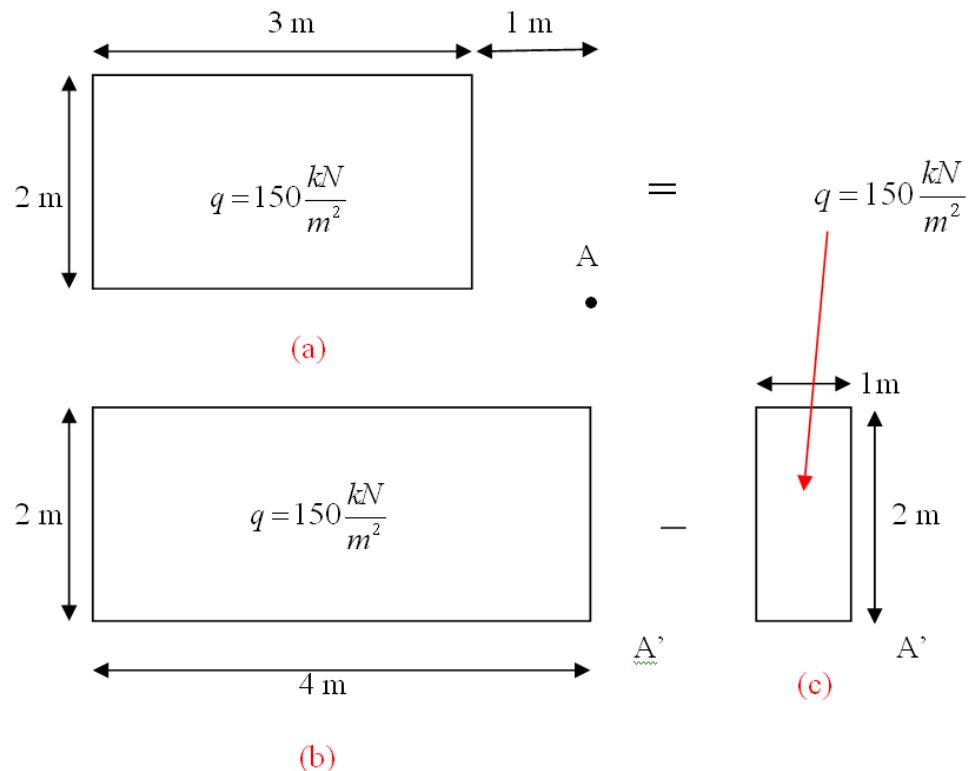
Find the vertical stress increase Δp below the point A at a depth $z = 4$ m.

Solution:

The stress increase, Δp, can be written as : $\Delta p = \Delta p_1 - \Delta p_2$

where Δp_1 = stress increase due to the loaded area shown in (b).

$\quad\quad \Delta p_2$ = stress increase due to the loaded area shown in (c).

(a)

(b)

(c)

For the loaded area shown in (b):

$$m = \frac{B}{Z} = \frac{2}{4} = 0.5 \quad and \quad n = \frac{L}{Z} = \frac{4}{4} = 1.0$$

$$\Delta p_1 = qI_4 = (150)(0.12) = 18 \; kN / m^2$$

Similarly, for the loaded area show in (c):

$$m = \frac{B}{Z} = \frac{1}{4} = 0.25 \quad and \quad n = \frac{L}{Z} = \frac{2}{4} = 0.5$$
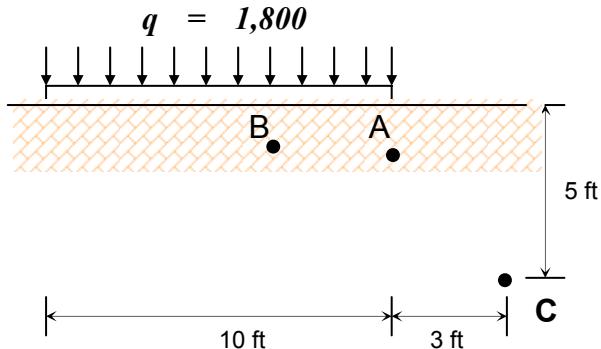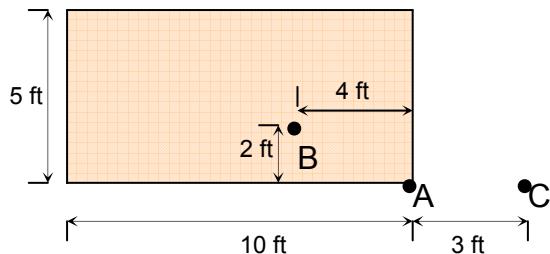
$$\Delta p_2 = (150)(0.048) = 7.2 \; kN / m^2$$

Therefore, $\Delta p = \Delta p_1 - \Delta p_2 = 18 - 7.2 = 10.8 \; kN / m^2$

(Revision: Sept.-08)

A clay sanitary pipe is located at a point C below the footing shown below. Determine the increase in the vertical stress Δp at the depth of the pipe, which is z = 5 feet below the footing invert, and 3 feet away from its edge. The footing has a uniformly distributed load q = 1,800 psf.

PLAN VIEW   SECTION

Solution:

For the expanded 5' x 13' area,

$$m = \frac{B}{Z} = \frac{5}{5} = 1 \quad and \quad n = \frac{L}{Z} = \frac{13}{5} = 2.6 \quad therefore, \ I_4 = 0.200$$

For virtual 3' x 5' area

$$m = \frac{B}{Z} = \frac{3}{5} = 0.6 \quad and \quad n = \frac{L}{Z} = \frac{5}{5} = 1 \quad therefore, \ I_4 = 0.136$$
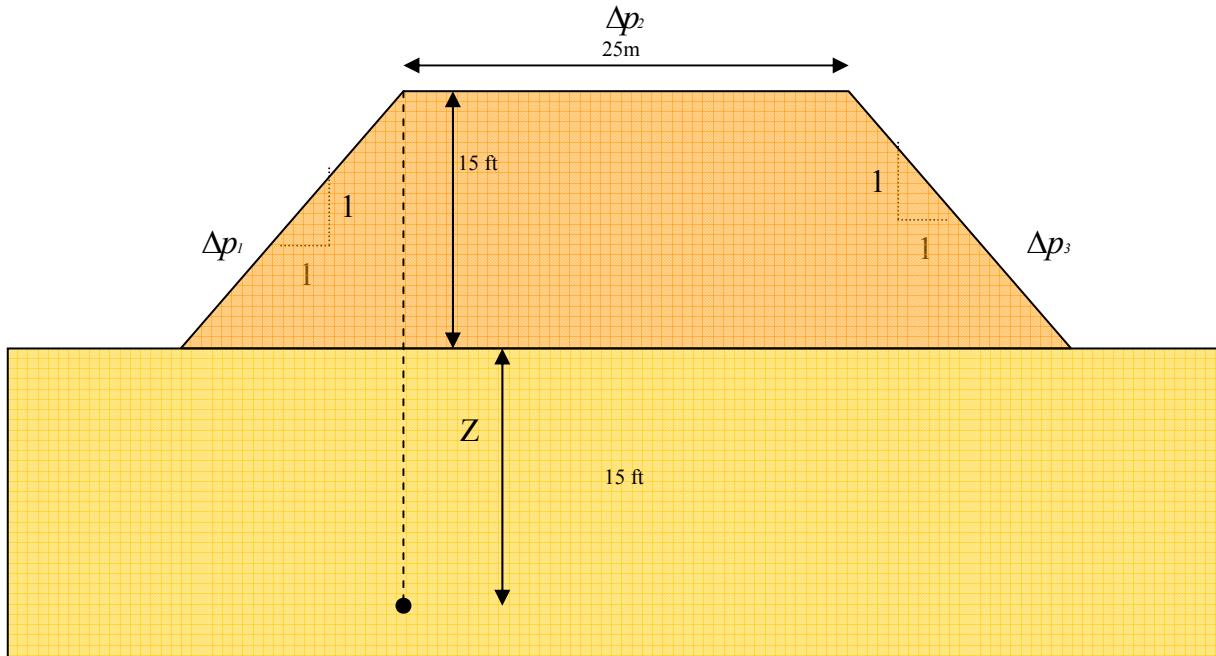
The increase in stress at point C from the footing is therefore,

$$\Delta p = q(I_4 - I_4^{'}) = \left(1,800 \ \frac{lb}{ft^2}\right)(0.200 - 0.136) = 115 \ psf$$

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(Revision: Aug-08)

Calculate the stress increase at the point A due to the new road embankment.

<u>*Solution:*</u>

The contribution from the central portion of the fill is Δp_2, whereas the contribution from the left and right hand slopes are Δp_1 and Δp_3 respectively. Using Boussinesq,

$$\Delta p_1 \Rightarrow \frac{2x_1}{B_1} = \frac{2(15')}{15'} = 2 \qquad \frac{2z}{B_1} = \frac{2(15')}{15'} = 2 \qquad \frac{\Delta p_1}{q} = 0.25$$

$$\therefore \Delta p_1 = (0.25)(15')\left(120\frac{lb}{ft^3}\right) = 450 \ psf$$

$$\Delta p_2 \Rightarrow \frac{2x_2}{B_2} = \frac{2(-12.5')}{25'} = -1 \qquad \frac{2z}{B_2} = \frac{2(15')}{25'} = 1.2 \qquad \frac{\Delta p_2}{q} = 0.47$$

$$\therefore \Delta p_2 = (0.47)(15')\left(120\frac{lb}{ft^3}\right) = 846 \ psf$$

$$\Delta p_3 \Rightarrow \frac{2x_3}{B_3} = \frac{2(40')}{15'} = 5.3 \qquad \frac{2z}{B_3} = \frac{2(15')}{15'} = 2 \qquad \frac{\Delta p_3}{q} = 0.02$$
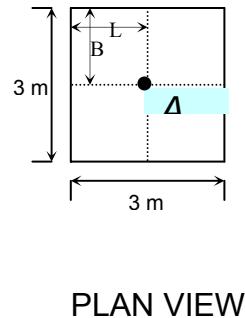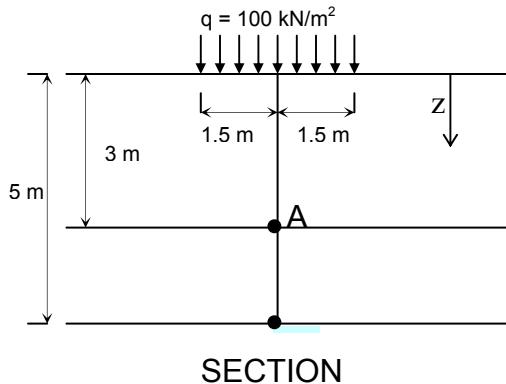
$$\therefore \Delta p_3 = (0.02)(15')\left(120\frac{lb}{ft^3}\right) = 306 \ psf$$

$$\Delta p = \Delta p_1 + \Delta p_2 + \Delta p_3 = 450 + 846 + 36 = 1,332 \ psf$$

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*Stress-07: Finding a stress increase from a surface load of limited width.

(Revision: Aug-08)

Determine the average stress increase below the center of the loaded area, between z = 3 m and z = 5 m.

SECTION   PLAN VIEW

Solution:

The stress increase between the required depths (below the <u>corner</u> of each rectangular area) can be given as:

$$\Delta p_{avg(H_2/H_1)} = q\left[\frac{(H_2)(I_{4(H_2)})-(H_1)(I_{4(H_1)})}{H_2-H_1}\right] = 100\left[\frac{(5)(I_{4(H_2)})-(3)(I_{4(H_1)})}{5-3}\right]$$

For $I_{4(H2)}$:   m′ = B / H$_2$ = 1.5 / 5 = 0.3

n′ = L / H$_2$ = 1.5 / 5 = 0.3

For m′ = n′ = 0.3, $I_{4(H2)}$ = 0.038

For $I_{4(H1)}$:   m′ = B / H$_1$ = 1.5 / 3 = 0.5

n′ = L / H$_1$ = 1.5 / 3 = 0.5

For m′ = n′ = 0.5, $I_{4(H1)}$ = 0.086

Therefore:

$$\Delta p_{av(H2/H1)} = 100 \times \frac{(5)(0.038)-(3)(0.086)}{5-3} = \textbf{3.4 kN/m}^2$$

The stress increase between z = 3 m and z = 5 m below the <u>center</u> of the load area is equal to:

$$4\Delta p_{avg(H_2/H_1)} = (4)(3.4) = 13.6\ kN/m^2$$

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(Revision: Aug-08)

The vertical stress σ_v in a soil at any depth below the surface can be estimated as a function of the soil unit weight γ by the equation,

$$\sigma_v = \int_0^Z \gamma(\sigma_v)\,dz = \int_0^{100} (95 + 0.0007\sigma_v)\,dz$$

If a particular stratum has a function $\gamma = 95 + 0.0007\,\sigma_v$, where γ is in pcf and σ_v is in psf, find the vertical stress at a depth of 100 feet below the surface.

Solution:

Rearranging, and integrating by parts,

$$\int_0^{\sigma_v} \frac{d\sigma_v}{(95 + 0.0007\sigma_V)} = \int_0^{100} dz$$

$$\frac{1}{0.0007} \ln(95 + 0.0007\sigma_V)\Big|_0^{\sigma_V} = z\Big|_0^{100}$$

$$\sigma_v = 135,800\left(e^{0.0007z} - 1\right)\Big|_0^{100} = 9,840 \; psf$$

At Z = 100 $\sigma_v = 135,800(1.0725 - 1)$

$$\therefore \quad \sigma_v = 9,840 \; psf$$

Chapter 10
Elastic Settlements

Symbols for Elastic Settlements

$N \rightarrow$ Raw value of the STP (obtained in the field).

$q_o \rightarrow$ Contact pressure.

$C_1 \rightarrow$ Embedment coefficient.

$C_2 \rightarrow$ Creep correction factor.

$E_S \rightarrow$ Soil elastic modulus.

$E_{eq} \rightarrow$ Equivalent modulus.

$\Delta(\Delta_e) \rightarrow$ Differential settlement between adjacent foundation.

$\Delta H_i \rightarrow$ Elastic settlement.

$I \rightarrow$ Influence factor; essentially equivalent to the strain ε in the soil.

$I_Z \rightarrow$ Simplifying influence factor.

$\varepsilon \rightarrow$ Strain at mid stratum.

$\gamma \rightarrow$ Unit weight of the soil.

$v \rightarrow$ Poisson's ratio.

$M_T \rightarrow$ Transverse moment.

*Elastic Settlement-01: Settlement (rutting) of a truck tire.

(Revision: Jan-08)

You are required to move a 60-ton truck-mounted crane onto your construction site. The front wheels carry 20% of the load on tires inflated with 80 *psi* air pressure. Calculate the possible rutting depth to your temporary jobsite road built from in-situ compacted medium sand. A surface SPT test shows an N = 12 and the tire's bearing area is roughly square. Use the Schmertmann method to estimate the rutting.

Solution:

Each front tire has a square bearing area of *BxB* such that:

$$B^2 = \frac{\text{tire's load}}{\text{tire pressure } q_o} = \frac{(0.5)(20\%)(120,000 \ lb)}{(80 \ lb/in^2)} = 150 \ in^2 \quad \therefore \quad B = 12.2 \ inches$$

A rough estimate of the soil's elastic modulus is $E_s = 14N = 14(12) = 168$ ksf.

Since the sand is compacted, it is a dense sand, and the influence factor I_z is equivalent to the strain ε. The strain reaches a maximum value of 0.6. Therefore, the average value of the strain is about 0.3 throughout its depth to $2B = 2(12.2 \ inches) = 24.4$ inches. Since the crane loads are on the surface and only for a few days, it is permissible to assume that there is no creep and therefore $C_1 = C_2 = 1$. Therefore, for the single layer of soil, the rutting is,

$$\Delta = C_1 C_2 q_o \left(\frac{\varepsilon}{E_s}\right) dz = (1)(1)(80 \frac{lb}{in^2})(\frac{0.30}{168 \ k/ft^2})(24.4 \ in) \times \left(\frac{144 \ in^2}{ft^2}\right)\left(\frac{k}{1,000 \ lb}\right)$$

$\therefore \Delta \approx 0.5 \ inches \ of \ rutting.$

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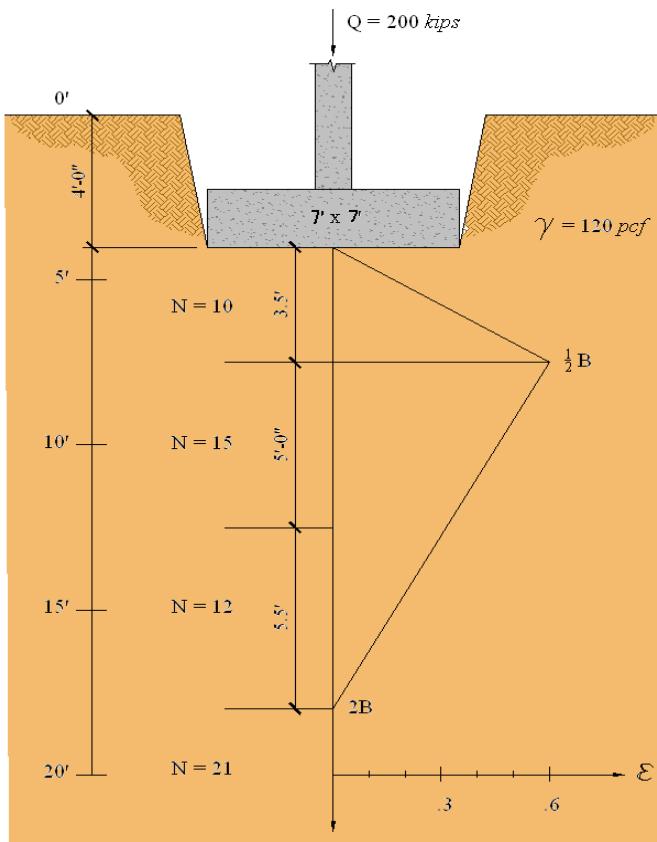
*Elastic Settlement-02: Schmertmann method used for granular soils.

(Revision: Aug-08)

Estimate the settlement of a square footing placed on a fine, medium dense sand, embedded 4 _ft_ below the ground surface, for long-term use. Use the Schmertmann method.

Assume $\dfrac{E_s}{N} \cong 14$ where E_s is in _ksf_; used for fine medium sands.

Solution:

Layer	Δz	E_s	$I_z = \varepsilon$	$\dfrac{I_z \Delta z}{E_s}$
	(in)	(ksf)		
1	42	140	0.30	0.090
2	60	210	0.46	0.130
3	66	168	0.16	0.061
			$\sum=$	0.281

The contact pressure on the soil is, $q_o = \dfrac{Q}{B^2} - D_f \gamma = \dfrac{200\,k}{49\,ft^2} - (4\,ft)(0.120\,kcf) = 3.60\,ksf$

The coefficients for the Schmertmann method are C_1 and C_2 :

Depth factor $C_1 = 1.0 - 0.5\left(\dfrac{D_f \gamma}{q_o}\right) = 1.0 - 0.5\left(\dfrac{0.48}{3.60}\right) = 0.93$

Creep factor $C_2 = 1.35$ _for a five year period_.
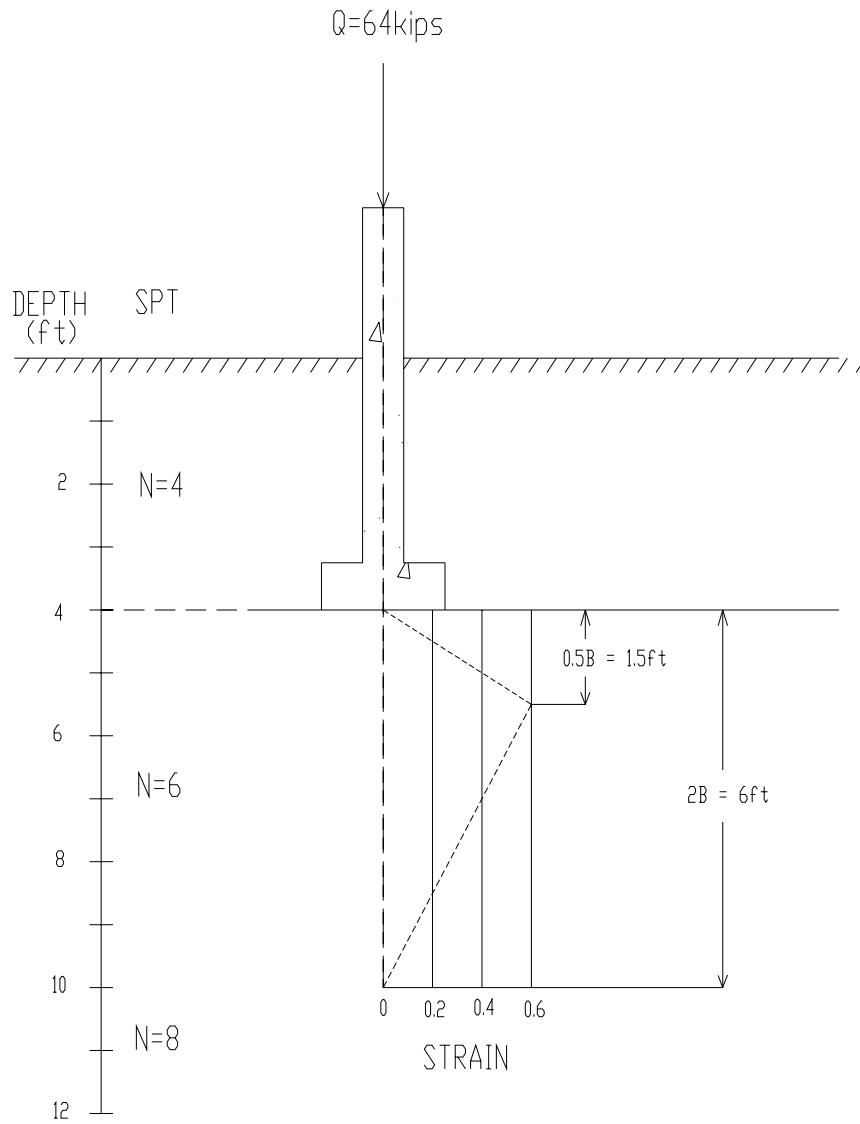
The Schmertmann formula for the elastic settlement Δ is,

$$\therefore \Delta = C_1 C_2 q_o \sum_0^{2B} \dfrac{\varepsilon}{E}\Delta z = (0.93)(1.35)(3.60)(0.281) = 1.27 \ inches$$

(Revision: Aug-08)

Determine the elastic settlement of a deep spread footing after five years of the 3 ft. x 3 ft. footing when it is placed on a uniform clean sand with γ = 110 pcf.

Solution:

The contact pressure on the soil is, $q_0 = \dfrac{Q}{B^2} = \dfrac{64\,kips}{(3\,ft)^2} = 7.11\,ksf$

The SPT value indicates that the soil is a loose sand. The modulus E for loose sand can be calculated using the following formula:

$$E_S \approx 10(N+15)\ ksf$$

The following table summarizes the data and calculations:

Layer number	Layer's thickness ΔZ (feet)	SPT (average) N	Soil's elastic modulus E (ksf)	Average strain at mid stratum ε	$\varepsilon_z \Delta Z/E$ (ft³/kip)
1	1.5	6	210	0.35	0.0025
2	4.5	7	220	0.30	0.0061

$\Sigma = 0.0086$

The correction factors are as follows:

1. Depth factor, $C_1 = 1 - \left(\dfrac{0.5\gamma D_f}{q_0 - \gamma D_f} \right) = 1 - \left(\dfrac{(0.5)(0.110)(4)}{7.11 - (0.110 \times 4)} \right) = 0.97$

   Since $C_1 > 0.5$, this is **FINE**.

2. Creep factor, $C_2 = 1 + 0.2\log\left(\dfrac{t_{yr}}{0.1} \right) = 1 + (0.2)\log\left(\dfrac{5}{0.1} \right) = 1.34$
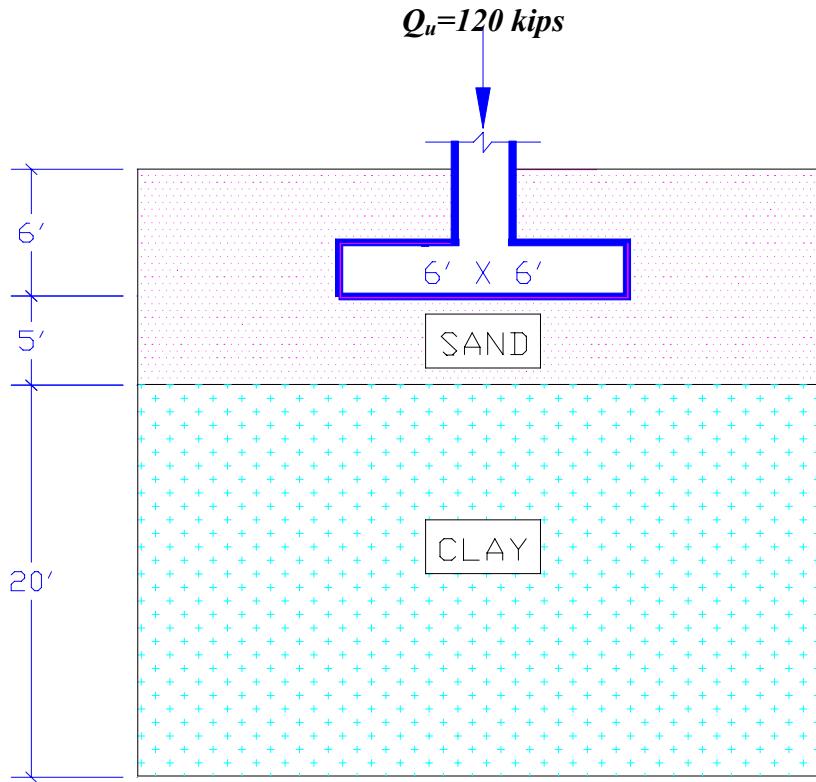
The total elastic settlement is,

$$\Delta = C_1 C_2 (q_o - \gamma D_f) \sum_0^{2B} \frac{\varepsilon}{E_s} dz = (0.97)(1.34)\left[7.11 - (0.110)(4) \right](0.0086) = 0.07\,ft = 0.84\,in$$

(Revision: Aug-08)

Use the 2:1 method to find the average stress increase (Δq) due to the applied load Q_u in the 5-foot sand stratum directly beneath the footing. If E_S = 400 ksf and v = 0.3, what is the expected immediate settlement ΔH_i?

Qu=120 kips

6'

5'

6' X 6'

SAND

CLAY

20'

Solution:

The settlement ΔH of an elastic media (the 5 foot thick sand stratum in this case) can be found from the theory of elasticity as,

$$\Delta H = q_o B \frac{1-v^2}{E_s} I_w$$

For square and flexible footings the influence factor is about I_W = 0.95. The 2:1 method essentially assumes that the stress reduces vertically by a vertical slope of 2 units vertically to 1 unit horizontally. The stress increase can be found by integrating the above equation,

$$\Delta q = \frac{1}{H} \int_{H_1}^{H_2} \frac{Q_u}{(B+Z)^2} dz$$

where H_1 = 0 feet (the footing's invert) to H_2 = 5 fee (bottom of the sand stratum).

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$$\Delta q = \frac{1}{H} \left[\frac{Q}{B+z} \right]_0^5 = \frac{1}{5\,ft} \left(\frac{120kip}{6\,ft} - \frac{120kip}{11\,ft} \right) = 1.82ksf$$

but

$$\Delta H = q_o B \frac{1-v^2}{E_s} I_w = \left(\frac{120}{36} \frac{kip}{ft^2} \right) (6\,ft) \left(\frac{1-0.3^2}{400\ \ ksf} \right) (0.95) \left(\frac{12}{ft} \frac{in}{ft} \right) = 0.52\ inches$$

(Revision: Aug-08)

The allowable bearing capacity of a 30-ft thick, medium dense sand stratum (with $\varphi = 36^o$ and $\gamma = 112$ pcf) is 3 ksf. Column A has a design load of 430 kips and column B has a design load of 190 kips. Select footing sizes and determine the differential settlement $\Delta(\Delta H)$ between them. Is this $\Delta(\Delta H)$ acceptable for columns spaced 30 ft apart?

Solution:

Footing size for column A:

$$B_A = \sqrt{\frac{Q_A}{q_{all}}} = \sqrt{\frac{430 \ kips}{3 \ ksf}} = 12 \ feet$$

Footing size for column B:

$$B_B = \sqrt{\frac{Q_B}{q_{all}}} = \sqrt{\frac{190 \ kips}{3 \ ksf}} = 8 \ feet$$

A quick estimate of the ratio of settlement to the proportionality is $\dfrac{\Delta H_A}{\Delta H_B} = \dfrac{12 \ ft}{8 \ ft} = 1.5$

Therefore if the settlement at column B is $\Delta H_B = 1$ in., then the settlement at column A will be $\Delta H_A = 1.5$ in. Then,

$$\Delta(\Delta H) = 1.5 \ in - 1.0 \ in = 0.5 \ inches$$

and the rotation between the two columns is

$$\theta = \frac{\Delta(\Delta H)}{L} = \frac{(0.5 \ in)}{(30 ft)\left(\dfrac{12 in}{ft}\right)} = \underline{\underline{0.0014}}$$

Both of these values [$\Delta(\Delta H)$ and θ] are acceptable, since $\Delta(\Delta H)$ should be < 1" and θ < 0.0033.

(Revision: Aug-08)

Compute the average stress *Δq* at mid-clay stratum, for the values shown below, using: (a) Boussinesq's method, (b) Westergaard's method, and (c) the 2:1 method.
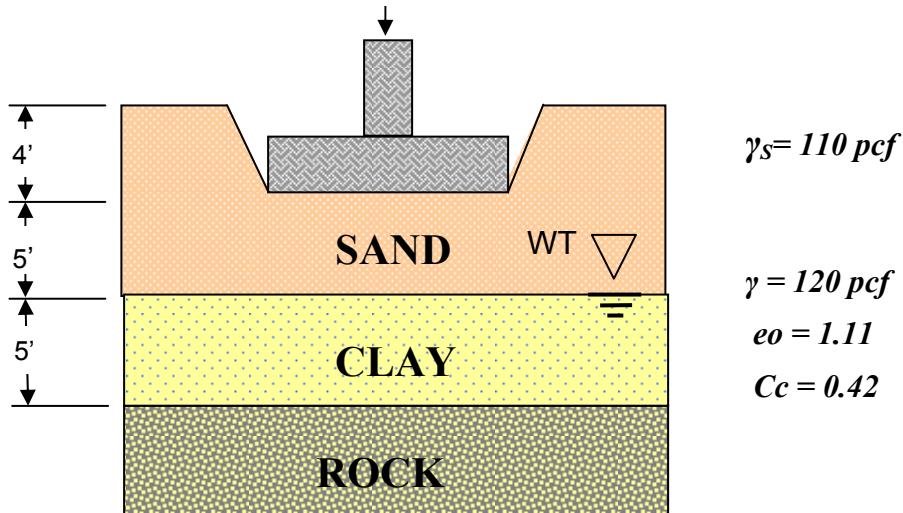
Also, determine the size of a square spread footing, in order to limit total settlement $\Delta H = \Delta H_i + \Delta H_c + \Delta H_S$ to only 1.5 inches.

Estimate the initial settlement $\Delta H_i = 0.05\sqrt{Z_s}$ where Z_s is the thickness of the granular stratum beneath the footing in feet, to give *ΔH$_i$* in inches.

Solution:

Assume an initial value of B = 10 feet.

$Q = 240$ kips

4'

5'

SAND   WT

5'

CLAY

ROCK

$\gamma_s = 110\ pcf$

$\gamma = 120\ pcf$

$e_o = 1.11$

$Cc = 0.42$

The contact pressure q$_o$ of the footing is:

$$q_o = \frac{Q}{A_B} = \frac{240\ kips}{100\ ft^2} = 2.4\ ksf$$

(a) Stress at mid-clay stratum using **_Boussinesq's method_** (use the charts on page 205) for a square footing:

$$\frac{q}{q_o} = 0.52 \quad \therefore \quad q = (0.52)(2.4ksf) = 1.25\,ksf$$
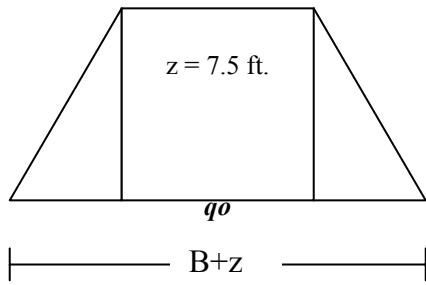
(b) Stress at mid-clay stratum using ***Westergaard's method*** for a square footing:

$$\frac{q}{q_o} = 0.33 \quad \therefore \quad q = (0.33)(2.4 \ \ ksf) = 0.79 \ ksf$$

(c) Stress at mid-clay stratum using the ***2:1 method*** for a square footing:

The depth (Z) from the footing invert to mid-clay is 7.5 feet:

B = 10 ft.

z = 7.5 ft.

qo

B+z

$$q = \frac{Q}{A_{B+Z}} = \frac{240 \ \ kips}{(10 ft + 7.5 ft)^2} = 0.78 \, ksf$$

Note that the Boussinesq method provides the highest predicted stress. Since this would predict faster consolidation rates, it is the least conservative method. Therefore, for this problem, use the 2:1 method's stress of 0.78 ksf. The instantaneous settlement (ΔH_i):

$$\Delta H_i = 0.05\sqrt{Z_s} = 0.05\sqrt{5 ft} = 0.11 \ \ in$$

The in-situ effective stress q_o' at mid-clay layer, <u>before</u> placing the footing is:

$$q_o' = \gamma_s h_s + \gamma_c h_c = \left(0.11 \ \frac{kip}{ft^3} \times 9 ft \right) + \left[\left(0.120 \frac{kip}{ft^3} - 0.0624 \frac{kip}{ft^3} \right) \times 2.5 ft \right] = 1.13 \ \ ksf$$

Using *q* = 0.78 ksf from the 2:1 method, the total settlement ΔH is equal to the immediate ΔH_i, plus the consolidation ΔH_c, and the secondary settlement ΔH_s, but limited to no more than 1.50 inches.

$$\Delta H = \Delta H_i + \Delta H_C + \Delta H_s = 1.50 \ \ inches$$

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But ΔH_i = 0.11 in, and ΔH_s is negligible for this problem. Therefore the maximum permissible consolidation settlement ΔH_c is limited to:

$$\Delta H_c = 1.50 \ \ in - \Delta H_i = 1.50 \ \ in - 0.11 \ \ in = 1.39 \ \ in$$

or

$$\Delta H_c = \frac{C_c H}{1+e_o} \log \frac{q_o' + \Delta q}{q_o'} = 1.39 \ \ in = \frac{(0.42)\left(5 \, ft \times \dfrac{12 \ in}{ft}\right)}{1+1.11} \log\left(\frac{1.13 \ \ ksf + \Delta q}{1.13 \ \ ksf}\right) = 1.39 \ \ in$$

Therefore $\Delta q = 0.39$ ksf

Using the 2:1 method:

$$\Delta q = \frac{Q}{A_{B+Z}} \qquad B + Z = \sqrt{\frac{Q}{\Delta q}} = \sqrt{\frac{240 \ \ kips}{0.34 \ \ ksf}} = 26.6 \ \ ft$$

Therefore $B = 19$ feet
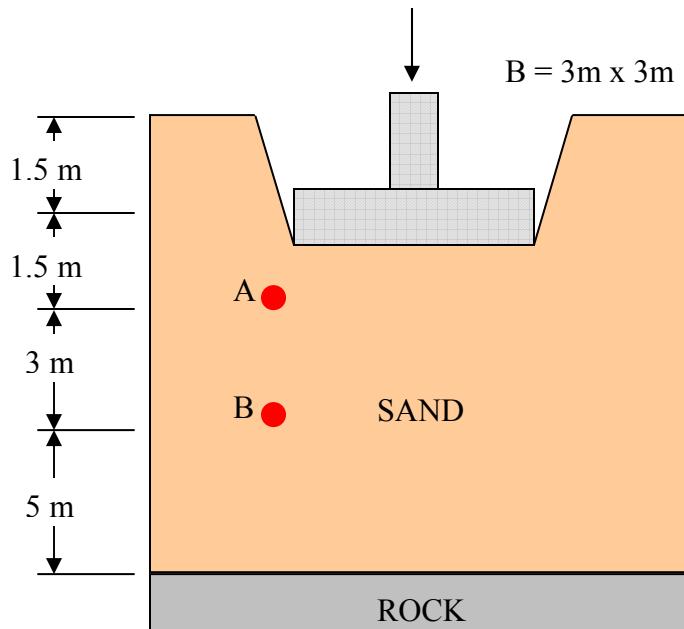
Since the initial B = 10 feet, the new value of 19 feet should be used to re-iterate towards a better solution that converges.

*Elastic Settlement-07: Schmertmann versus the strain methods.

(Revision: Aug-08)

Compute the immediate settlement ΔH_i using the Schmertmann formula using an average Δq value (q_{v1} = 233.3 kPa, q_{v2} = 163.3 kPa, q_{v3} = 77.0 kPa, q_{v4} =44.0 kPa and q_{v5} = 28.0). E_s at point A is 20,400 kPa, $\dfrac{D_f}{B} = 0.5$ and $C_1 = C_2 = 1$. Compare the results with an alternate method using $\Delta H = q_o B \left(\dfrac{1-v^2}{E_s} \right) I_w$, where $v = 0.3$ and $I_w = 0.95$.

$$Q = 2{,}100 \ kN$$

$$B = 3m \ x \ 3m$$

1.5 m

1.5 m

A

3 m

B   SAND

5 m

ROCK

Solution:

The average stress from multiple layers is solved via this formula,

$$\Delta q_v = \frac{\Delta H}{H}\left[\left(\frac{q_1 + q_n}{2}\right) + q_2 + q_3 + \ldots + q_{n-1}\right] = \frac{1.5m}{6m}\left[\left(\frac{233.3 kPa + 28.0 kPa}{2}\right) + 163.3 kPa + 77.0 kPa + 44.0 kPa \right]$$

$$\Delta q_v = 104 \ kPa$$

The elastic settlement via Schmertmann is,

$$\Delta H = C_1 C_2 (0.6B)\frac{\Delta q}{E_s} = (1)(1)(0.6)(3 \ m)\left(\frac{1000 \ mm}{m} \right)\left(\frac{104 \ kPa}{20,400 \ kPa} \right) = 9.2 \ mm$$
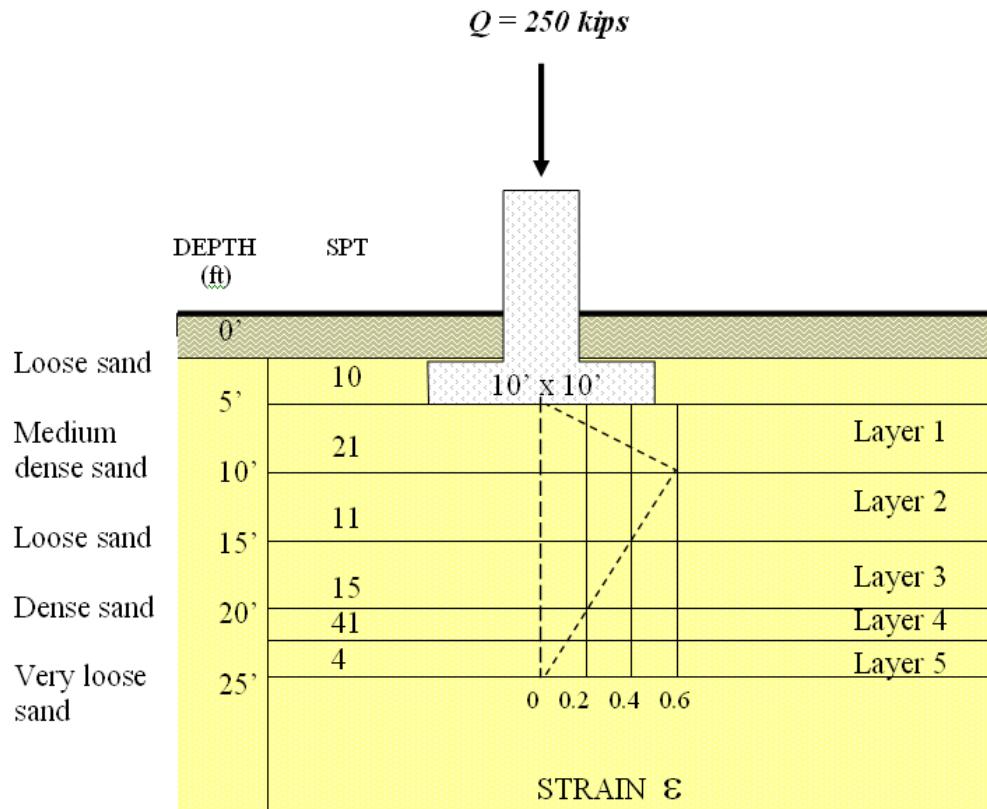
The alternative method from the theory of elasticity would yield,

$$\Delta H = q_{v1} B \left(\frac{1-v^2}{E_s} \right) I_w = (233.3 \ kPa)(3 \ m)\left(\frac{1-0.3^2}{20,400 \ kPa} \right)(0.95)\left(\frac{1000 \ mm}{m} \right) = 10 \ mm$$

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(Revision: Aug-08)

Determine the elastic settlement using the Schmertmann method of the 10'x 10' footing as shown below. Estimate the elastic modulus using $E_S = 10(N + 15)$, where E_S is in ksf and N is the corrected SPT value.

$Q = 250\ kips$

Solution:

The data from these strata are placed into a table below.

Layer No.	Layer Thickness ΔZ, (feet)	Soil Modulus E_S, (ksf)	$I_Z = \varepsilon$ (average strain)	$\dfrac{I_Z \Delta Z}{E}$ (ft/kp)
1	5.0	360	0.35	0.00486
2	5.0	260	0.50	0.00962
3	2.5	300	0.35	0.00292
4	2.5	560	0.25	0.00112
5	5.0	190	0.10	0.00263
	$\Sigma = 2B = 20$ ft		$\Sigma =$	0.02115

The Schmertmann coefficients are,

The depth coefficient $C_1 = 1 - [0.5(\dfrac{\gamma D_f}{q - \gamma D_f})] = 1 - \left[\dfrac{0.5(0.1)(5)}{1.5 - (0.1)(5)}\right] = 0.75$

The time coefficient $C_2 = 1.35$

$\Delta H_i = C_1 C_2 (q - \gamma D_f) \sum\limits_{0}^{2B} \dfrac{\varepsilon}{E} dz = (0.88)(1.35)\left[2.5 - (0.1)(5)\right](0.02115) = 0.042\, ft = 0.05\, in$

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Elastic Settlement-09: *Settlement of a mat foundation.*

(Revision: Aug-08)

A mat foundation located 8 feet below grade supports a ten story building upon an area of 50 ft by 150 ft, and carries a uniform load of 6 ksf. For the soil profile conditions shown below, determine the total settlement at the center and a corner of the foundation. The structure is of reinforced concrete with column spacing at 25 ft. Is the calculated differential settlement acceptable?

Solution:

a) Using the Schmertmann method,

Layer Δz *(in)* E_s *(ksi)* $I_z = \varepsilon$ $(I_z/E_s)\Delta z$ *(in³/kip)*

Layer	Δz (in)	Es (ksi)	Iz =ε	(Iz/Es)Δz (in³/kip)
1	300	3.47	0.35	30.26
2	300	3.47	0.50	43.23
3	300	8.33	0.30	10.80
				$\Sigma = 84.29$

$$The\,depth\,factor\,C_1 = 1.0 - 0.5\frac{\gamma D_f}{\left(q - \gamma D_f\right)} = 1.0 - 0.5\left[\frac{(0.100)(8)}{6-(0.100)(8)}\right] = 0.92$$

$The\,time\,factor\,(creep)\,C_2 = 1.0$

$$\Delta = C_1 C_2 \left(q_o - \gamma D_f\right)\sum_0^{2B}\frac{\varepsilon}{E_s}dz = (0.92)(1.0)\left[6-(0.100)(8)\right](84.29) = 3.78\,in$$

b) Consolidation settlement **Method A** (take each layer at a time),

Set $e_{o1} = 1.00$, and from e = 125/γ, $e_{o2} = 0.96$, $e_{o3} = 0.89$.

For clay 1: $q_o = (0.100)(50) + (0.110 - 0.0624)25 + (0.125 - 0.0624)12.5 = \underline{5.95\ ksf}$

At the mid clay stratum is at 87.5 ft below the surface,

(ie. 87.5/50=1.75 B = 3.5 B/2) $\Delta q = 0.12q = 0.12(6) = 0.72$ ksf

$$\Delta_1 = \frac{C_c H_1}{1 + e_o}\log_{10}\frac{q_o + \Delta q}{q_o} = \frac{0.20(25x12)}{1+1}\log_{10}\frac{5.95 + 0.72}{5.95} = 1.48\,in$$

At corner $\Delta q = 0.09q = 0.09(6) = 0.54$

$$\Delta_1 = \frac{0.20(25x12)}{2}\log_{10}\frac{5.95 + 0.54}{5.95} = 1.13\,in$$

For clay 2: $q_o = 5.95 + (0.125 + 0.0624)12.5 + (0.130 - 0.0624)12.5 = \underline{7.58\ ksf}$

midlayer at 112 ft = 4.5 B/2, $\Delta q \cdot 0.07q = 0.42$

$$\Delta_2 = \frac{0.35(25x12)}{1+0.96} \log_{10} \frac{7.58+0.42}{7.58} = 1.25 \, in \, at \, the \, centerline$$

$\Delta q \cdot 0.05q = 0.30$

$$\Delta_2 = \frac{0.35(25x12)}{1+0.96} \log_{10} \frac{7.58+0.30}{7.58} = 0.96 in$$

For clay 3: $q_o = 7.58 + (0.130 + 0.0624)12.5 + (0.140 - 0.624)12.5 = \underline{8.65 \, ksf}$

at midlayer (137.5' / 5.5 B/2) $\Delta q = 0.04q = 0.24$ ksf

$$\Delta_3 = \frac{0.15(25x12)}{1+0.89} \log_{10} \frac{8.65+0.24}{8.65} = 0.28 in$$

at corner $\Delta q = 0.03q = 0.18$ ksf

$$\Delta_3 = \frac{0.15(25x12)}{1+0.89} \log_{10} \frac{8.65+0.18}{8.65} = 0.21 in$$

Method B: the equivalent layer equation

$$\frac{\Delta}{H} = \frac{\Delta e}{1+e_o} \quad where \quad \Delta e = C_c \log \frac{q_o + \Delta q}{q_o}$$

The total settlement on the centerline is,

$\Delta_6 = 3.78 + 1.48 + 1.25 + 0.28 = 6.78$ inches

and along the foundation edge,

$\Delta_{edge} = 3.78 + 1.13 + 0.96 + 0.21 = \underline{6.08 \, inches}$

The differential settlement is $\Delta(\Delta) = \underline{0.70 \, inches}$

The allowable for reinforced concrete buildings $\Delta(\Delta) < 0.003(span) = 0.003(25x12 \, inches)$

$= 0.90$ inches

Therefore, the design is acceptable.

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Chapter 11
Plastic Settlements

$e \rightarrow$ *Voids ratio.*

$E_S \rightarrow$ *Soil elastic modulus.*

$C_C \rightarrow$ *Compression index.*

$G_S \rightarrow$ *Specific gravity of the solids of a soil.*

$OCR \rightarrow$ *Over-consolidation ratio (ratio of in-situ stress divided by the overburden stress).*

$H \rightarrow$ *Depth of zone influence.*

$DH_c \rightarrow$ *Plastic settlement (also called primary consolidation).*

$DH_{total} \rightarrow$ *Total settlement of a structure.*

$DH_s \rightarrow$ *Second consolidation settlement.*

$Dp \rightarrow$ *Increasing pressure on the surface.*

$u \rightarrow$ *Pore water pressure.*

$P_e \rightarrow$ *Pre-consolidation pressure of a specimen.*

$\gamma_{SAT} \rightarrow$ *Saturated unit weight of the soil.*
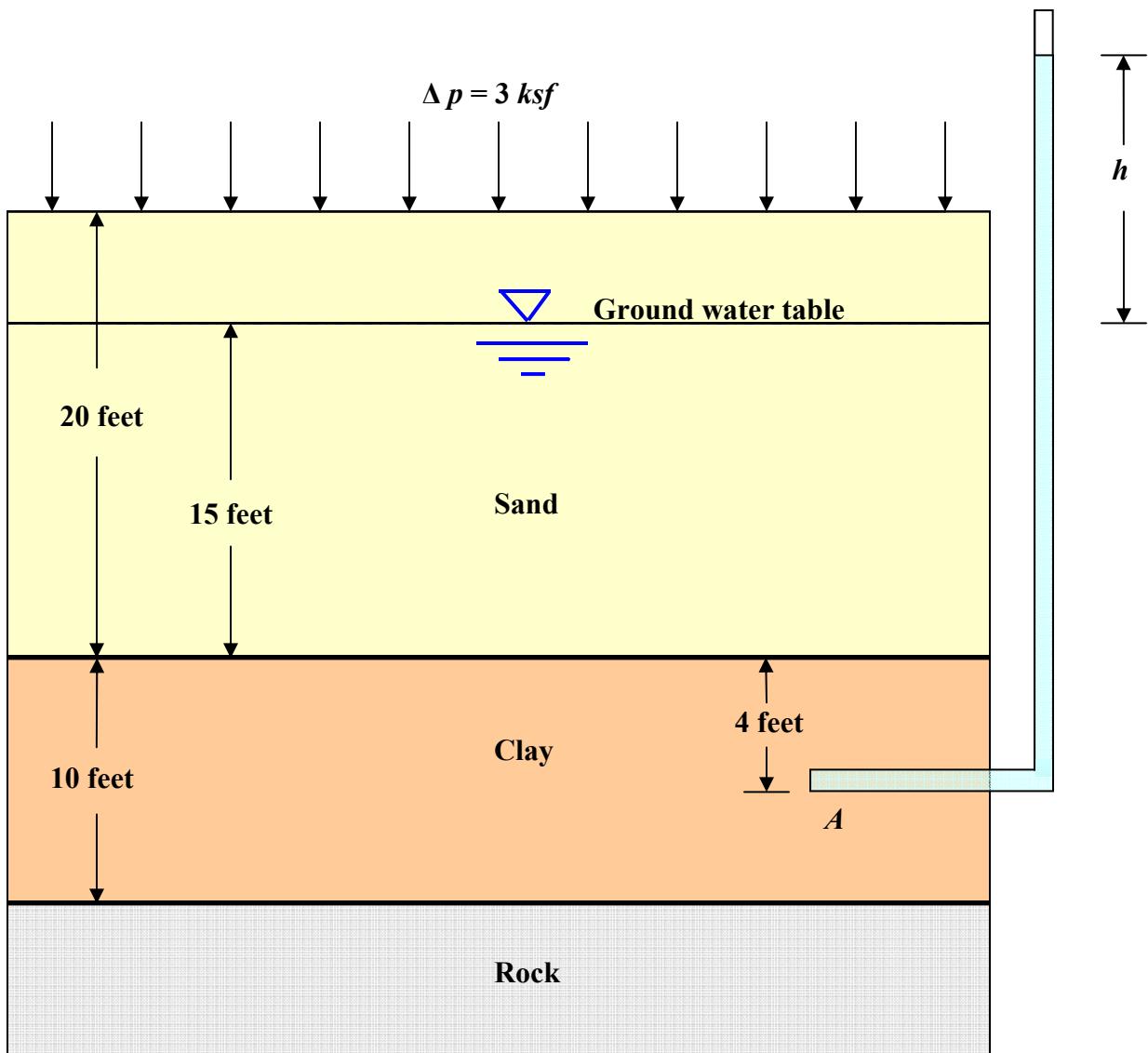
$\gamma_W \rightarrow$ *Unit weight of water.*

$U \rightarrow$ *Degree of consolidation.*

*Plastic Settlement–01: Porewater pressure in a compressible soil.

(Revision: Oct.-08)

a. How high will the water rise in the piezometer immediately after the application of the surface load of 3 ksf?
b. What is the degree of consolidation from the 3 ksf at point A, when h =15 ft.?
c. Find *h* when the degree of consolidation at A is 60%.

Solution:

a) Assume a uniform increase of the initial excess pore water pressure throughout the 10-foot thickness of the clay layer:

The pore water pressure is $u_0 = \Delta p = \gamma_w h = 3{,}000 \, lb / ft^2$ $\therefore \ h = \dfrac{\Delta p}{\gamma_w} = \dfrac{3{,}000}{62.4} = 48.1 \ feet$

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b) The degree of consolidation at A is U_A (%) when h = 15 feet:

$$U_A\% = \left(1 - \frac{u_A}{u_0}\right)100 = \left(1 - \frac{(15\ ft)(62.4\ pcf)}{(48.1\ ft)(62.4\ pcf)}\right)100 = 69\%$$

c) When U_A = 60 %, what is the value of h?

$$U_A = 0.6 = \left(1 - \frac{u_A}{u_0}\right) = \left(1 - \frac{u_A}{3,000\ psf}\right)$$
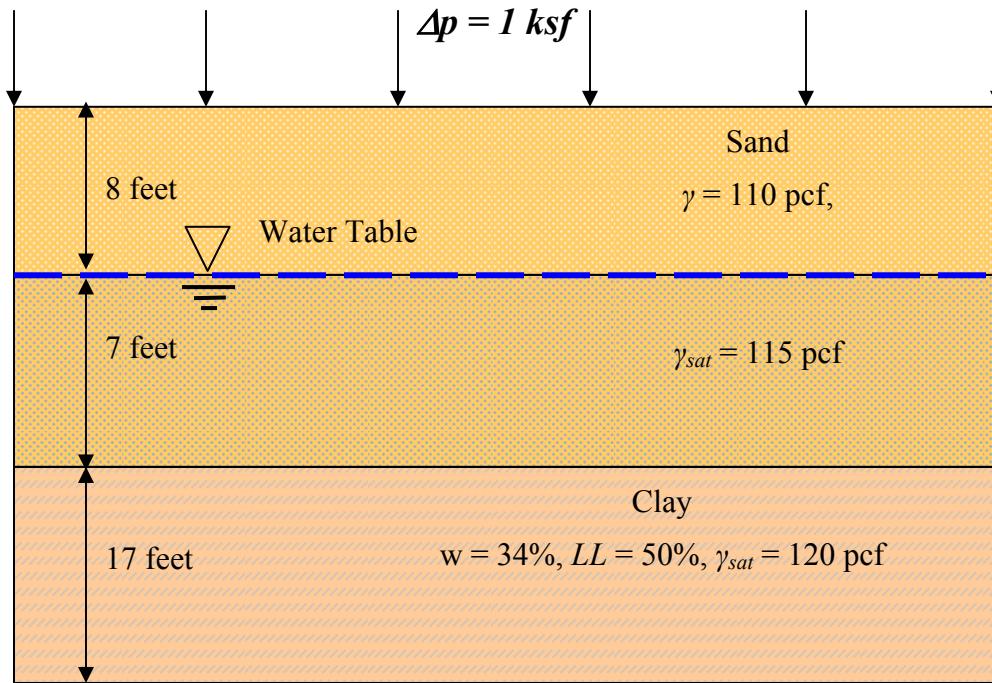
$$\therefore \quad u_A = (1 - 0.6)(3,000\ psf) = 1,200\ psf$$

$$h = \frac{u_A}{\gamma_w} = \frac{(1,200\ psf)}{(62.4\ pcf)} = 19.2\ feet$$

(Revision: Aug-08)

A new building is planned upon the site shown below. Assume that the clay solids have a specific gravity of 2.67. Find the primary consolidation settlement if the clay is normally consolidated.

Solution:

$Skempton\ formula\ for\ the\ C_c = 0.009(LL-10) = 0.009(50-10) = 0.36$

$Se = wG_s \quad but\ S = 1 \quad \therefore e = wG_s = (0.34)(2.67) = 0.91$

$The\ stress\ of\ the\ clay\ at\ its\ mid-stratum\ before\ the\ building\ was\ built\ is,$

$p_o = \gamma_{d-sand} H_{dry-sand} + (\gamma_{sat} - \gamma_w)H_{sat-sand} + (\gamma_{sat} - \gamma_w)H_{mid-clay}$

$\quad = (0.11 kcf)(8\ ft) + (0.115 - 0.0624)kcf(7\ ft) + (0.120 - 0.0624)kcf(8.5\ ft)$

$p_o = 1.74\ ksf$
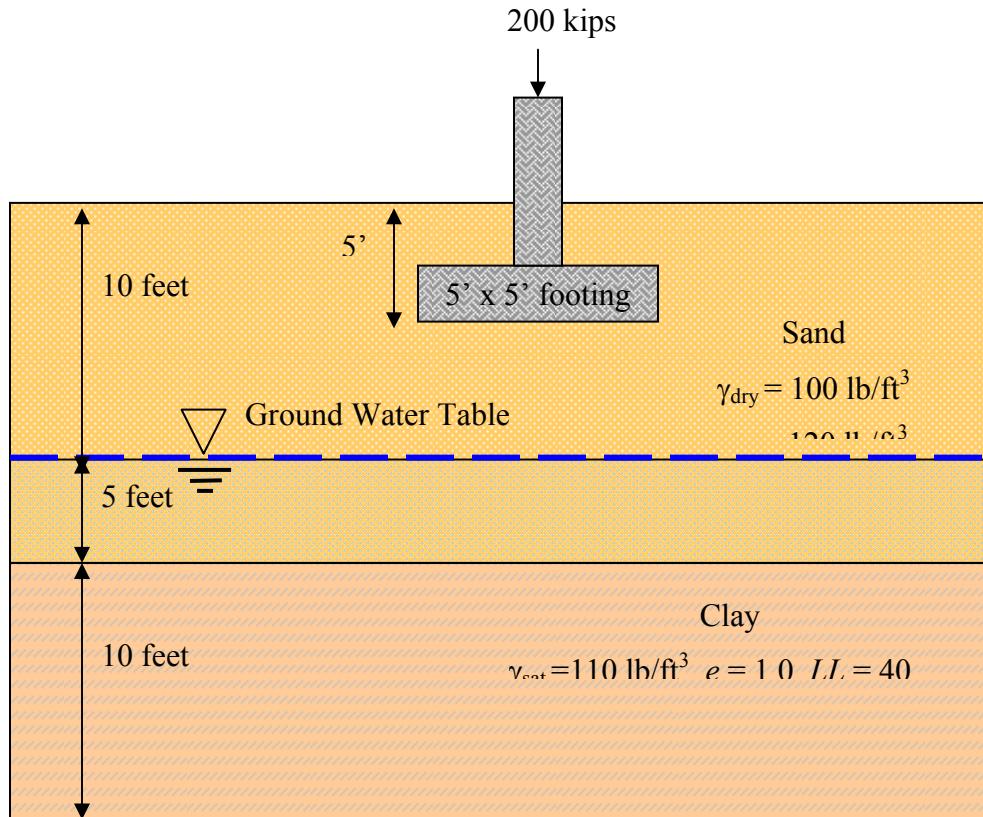
$The\ consolidation\ settlement\ is,$

$$\Delta H = \frac{HC_c}{1+e_o}\log_{10}\frac{p_o + \Delta p}{p_o} = \frac{(17)(12)(0.36)}{1+0.91}\log_{10}\frac{1.74+1}{1.74} = 7.6\ inches$$

Calculate the settlement of the 10-foot thick clay layer shown below that will result from the column's load carried by a 5-foot square footing. The clay is normally consolidated. Apply Boussinesq's formula to find the reduction of the vertical stress with depth.

200 kips

5'

10 feet

5' x 5' footing

Sand

$\gamma_{dry} = 100\ lb/ft^3$

Ground Water Table

5 feet

Clay

$\gamma_{sat} = 110\ lb/ft^3 \quad e = 1.0 \quad LL = 40$

10 feet

Solution:

Skempton formula $C_c = 0.009(LL - 10) = 0.009(40 - 10) = 0.27$

The stress of the clay at its mid − stratum before the building was built is,

$p_o = \gamma_{d-sand} H_{dry-sand} + (\gamma_{sat} - \gamma_w) H_{sat-sand} + (\gamma_{sat} - \gamma_w) H_{mid-clay}$

$p_o = (0.10 kcf)(10 ft) + (0.120 - 0.0624) kcf (5 ft) + (0.110 - 0.0624) kcf (5 ft)$

$p_o = 1.53\ ksf$

The Δp_{avg} below the center of the footing between $z_1 = 15\ feet$ to $z_2 = 25\ feet$ is given as,

$$\Delta p_{avg} = 4q \left[\frac{H_2 I_{4(H_2)} - H_1 I_{4(H_1)}}{H_2 - H_1} \right] \quad where \quad I_{4(H_2)} = f \left[m' = \frac{B}{H_2}, n' = \frac{L}{H_2} \right]$$

$B = L = \frac{5}{2} = 2.5\ feet$

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$$m' = \frac{B}{H_1} = \frac{2.5}{15} = 0.167 \quad and \quad n' = \frac{L}{H_1} = \frac{2.5}{15} = 0.167$$

From Boussinesq's Table for I_4

$$I_{4(H_2)} = 0.05 \quad and \quad I_{4(H_1)} = 0.075$$

The average pressure at mid-clay layer is thus given by,

$$\Delta p_{avg} = 4q \left[\frac{H_2 I_{4(H_2)} - H_1 I_{4(H_1)}}{H_2 - H_1} \right] = 4 \left(\frac{200\,kips}{(5')(5')} \right) \left[\frac{(25)(0.05) - (15)(0.075)}{25' - 15'} \right] = 0.4\,ksf = 400\,psf$$
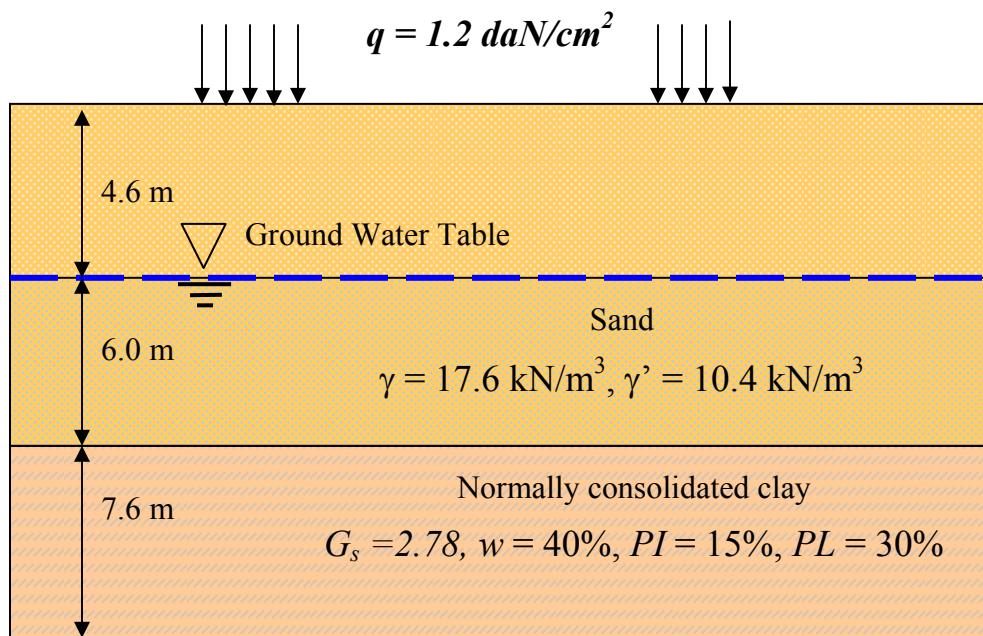
The consolidation settlement is,

$$\Delta H = \frac{HC_c}{1+e_o} \log_{10} \frac{p_o + \Delta p_{avg}}{p_o} = \frac{(10)(12)(0.27)}{1+1} \log_{10} \frac{1.53 + 0.40}{1.53} = 1.6\,inches$$

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(Revision: Aug-08)

Find the total settlement under a building that applies the load shown below.

Solution:

Notice that the data provided does not include a unit weight for the clay stratum. Therefore, this value must be determined through the other information provided.

$$\gamma_s = \gamma_w G_s = 27.8\,kN/m^3 \qquad Se = (1)e = wG_s = 0.40(2.78) = 1.11$$

Therefore, the clay unit weight is $\gamma_{sat} = \dfrac{(G_s + e_o)\gamma_w}{1 + e_o} = \dfrac{[(2.78) + (1.11)](9.81)}{1 + (1.11)} = 18.1\,kN/m^3$

The effective unit weight for the clay is,

$$\gamma' = \gamma_{sat} - \gamma_w = 18.1 - 9.81 = 8.3\,kN/m^3$$

Also, the Skempton relation is, $C_c = 0.009(w_L - 10) = 0.009(45 - 10) = 0.32$

The stress at the mid-clay stratum,

$$\sigma' = (4.6m)(17.6\,kN/m^3) + (6.0m)(10.4\,kN/m^3) + (7.6m/2)(8.3\,kN/m^3) = 175kN/m^2$$

The consolidation settlement is,

$$\Delta H = \frac{HC_c}{1 + e_o}\log_{10}\frac{\sigma' + \Delta\sigma}{\sigma'_o} = \frac{(7.6m)(0.32)}{1 + 1.11}\log_{10}\frac{175 + 120}{175} = 0.26\,m = 260\,mm$$

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Plastic Settlement-05: Pre-consolidation pressure p$_c$ and index C$_c$.
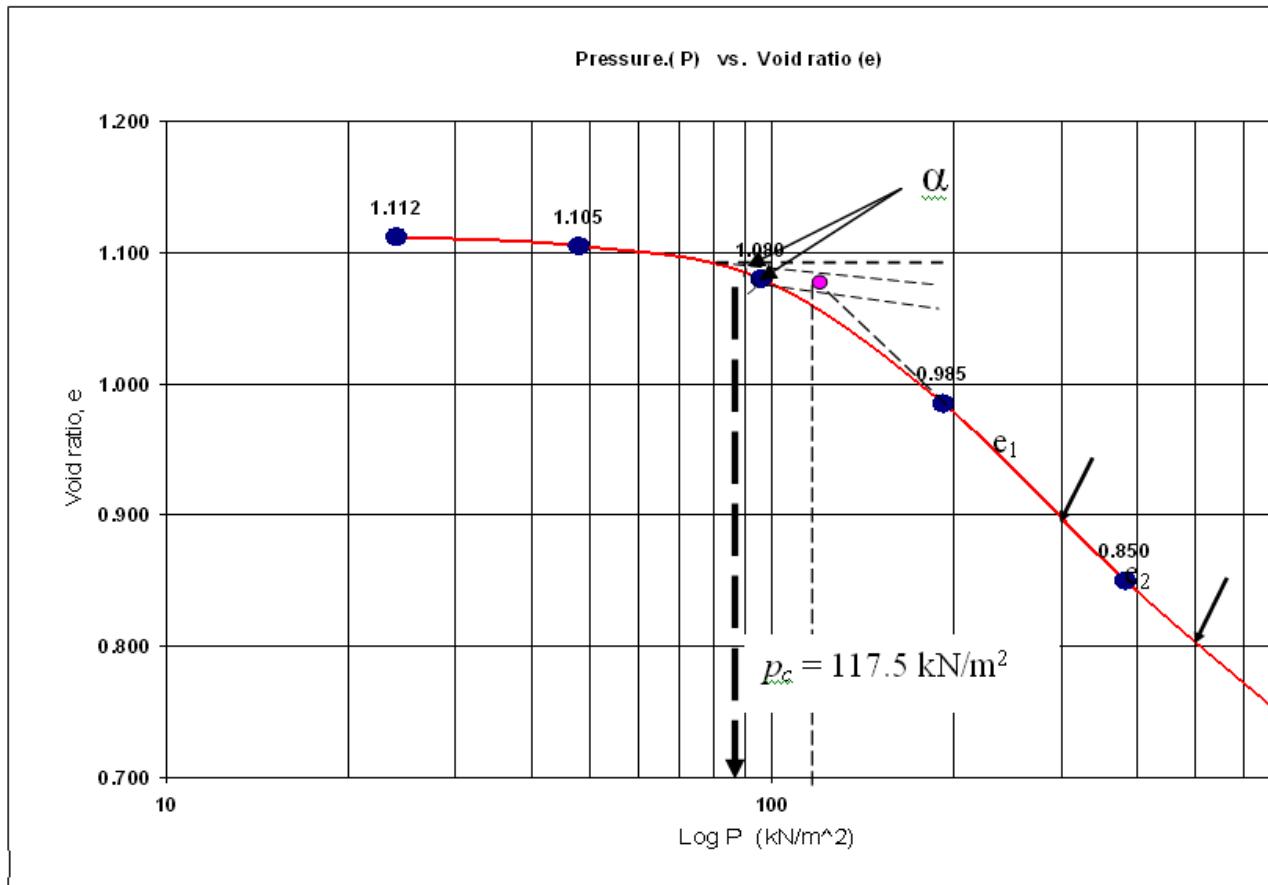
(Revision: Aug-08)

The results of a laboratory consolidation test on a clay sample are given below:

Pressure, p (kN/m^2)	Void ratio, e
23.94	1.112
47.88	1.105
95.76	1.080
191.52	0.985
383.04	0.850
766.08	0.731

(43) **a) Draw an *e*-log *p* plot.**

(44) **b) Determine the pre-consolidation pressure, p$_c$.**

(45) **c) Find the compression index, C$_c$.**

Solution:

Pressure.(P) vs. Void ratio (e)

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b) Determine the pre-consolidation pressure, p_c. From the e-log p plot,

$$p_2 = 500 \, kN/m^2 \ and \ e_2 = 0.8 \qquad p_1 = 300 \, kN/m^2 \ and \ e_1 = 0.9 \quad \therefore p_c = 117.5 \, kN/m^2$$
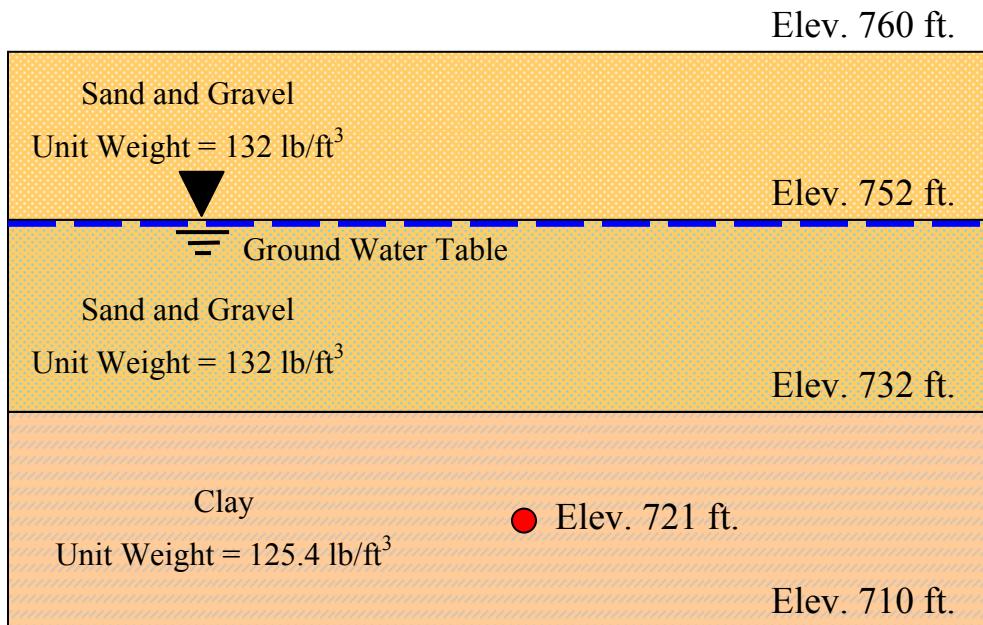
c) Find the compression index, C_c. From the slope of the graph,

$$C_c = \frac{e_1 - e_2}{\log\left(\dfrac{p_2}{p_1}\right)} = \frac{0.9 - 0.8}{\log\left(\dfrac{500}{300}\right)} = 0.451$$

(Revision: Aug-08)

The clay stratum shown in the profile below has a total vertical stress of 200 kN/m^2 at its mid-height with a voids ratio of 0.98. When the vertical stress increases to 500 kN/m^2 the voids ratio decreases to 0.81. Find (a) the effective overburden pressure at mid-height of the compressible clay layer, and (b) the voids ratio of the clay if the total pressure at its mid-height is 1000 kN/m^2.

Elev. 760 ft.

Sand and Gravel

Unit Weight = 132 lb/ft^3

Elev. 752 ft.

Ground Water Table

Sand and Gravel

Unit Weight = 132 lb/ft^3

Elev. 732 ft.

Clay

Elev. 721 ft.

Unit Weight = 125.4 lb/ft^3

Elev. 710 ft.

Solution:

$$a)\ p_o = \left[\gamma h\right]_{dry\ sand-gravel} + \left[\gamma' h\right]_{saturated\ sand-gravel} + \left[\gamma' h\right]_{mid-clay\ stratum}$$

$$p_o = (132\,lb/ft^3)(8\,ft) + (132 - 62.4)\,lb/ft^3\,(20\,ft) + (125.4 - 62.4)\,lb/ft^3\,(11\,ft)$$

$$p_o = 3.14\,ksf$$
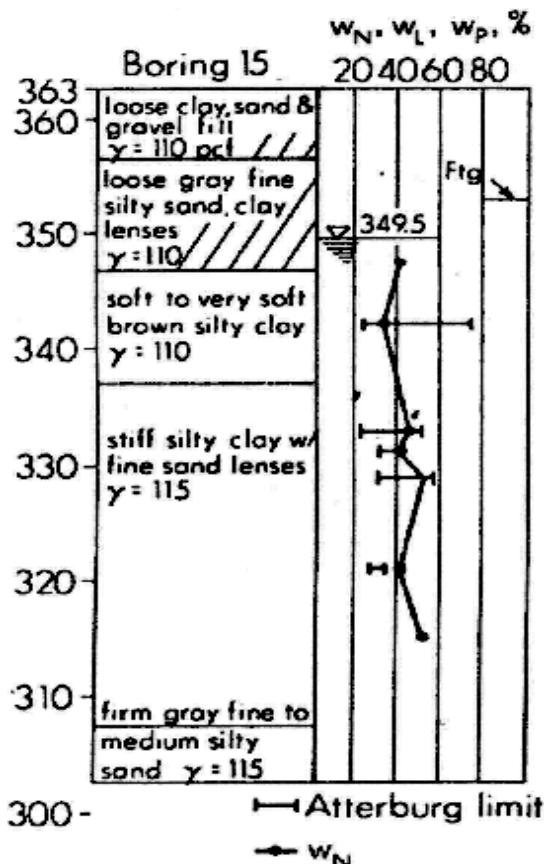
$$b)\quad C_c = \frac{e_1 - e_2}{\log(p_2/p_1)} = \frac{0.98 - 0.81}{\log(500/200)} = 0.427$$

$$0.427 = \frac{0.98 - e_2}{\log(1000/200)} \quad \therefore e_2 = 0.68$$

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Plastic Settlement-07: Settlement due to a lowered WT.

(Revision: Aug-08)

Find the settlement due to lowering of the phreatic surface from elevation 349.5' to 344.0' using the boring report shown below.

Solution:

$$\bar{p}_o = (0.110\,pcf)(7\,ft)+(0.110\,pcf)(6\,ft)+$$
$$(0.110-0.0624\,pcf)(3\,ft)+(0.110-0.0624\,pcf)(3\,ft)$$
$$\bar{p}_o = 1.71\,ksf$$

$$\Delta p = \frac{\Delta q}{H} + \Delta p \text{ due to the lowering } WT$$

$$\Delta q_2 = \Delta p - \Delta q_1 = 1.853 - 1.16 = 0.693$$

$$\Delta H_1 = \frac{C_c H}{1+e_0}\log\frac{p_o+\Delta q_1}{p_o}$$

$$\Delta H_1 = \frac{0.030(10)}{1+0.96}\log\frac{3+1.16}{3} = 0.032\,ft$$

$$\Delta H_2 = \frac{0.034(10)}{1+0.96}\log\frac{3+0.693}{3} = 0.157\,ft$$

versus 0.083ft the additional settlement.
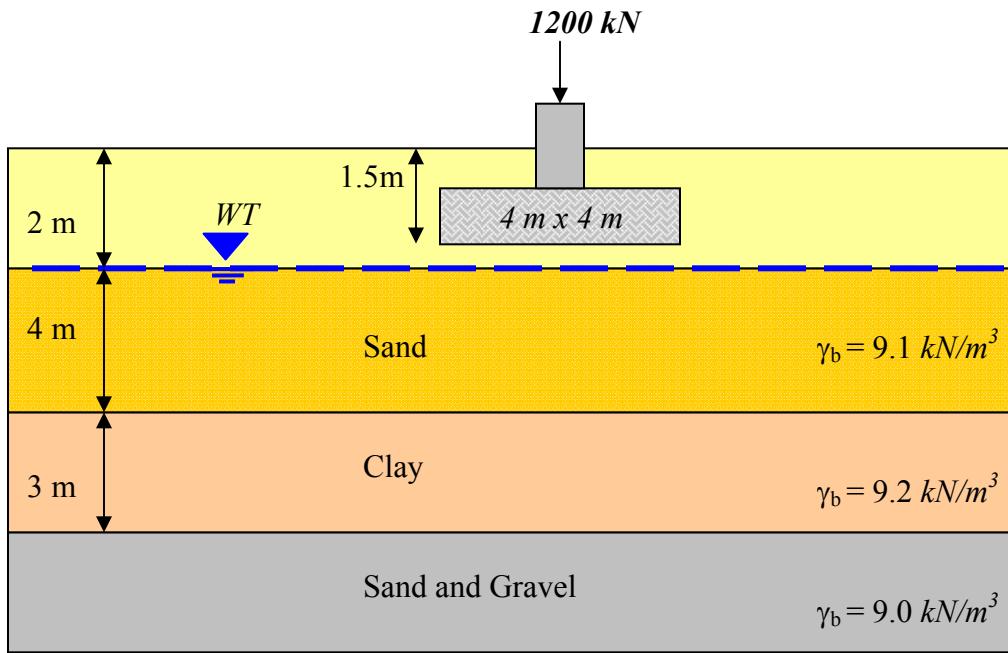
The rising *WT* may reduce settlement.

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(Revision: Sept-08)

Oedometer (consolidation) tests of several samples from the clay stratum yields the consolidation curve shown below. Given that Gs = 2.65, find (a) the value of p_o, (b) The value of p_c and (c) the OCR of the clay.

<u>Solution:</u>

1200 kN

1.5m

2 m

WT

4 m x 4 m

4 m

Sand

$\gamma_b = 9.1 \ kN/m^3$

Clay

3 m

$\gamma_b = 9.2 \ kN/m^3$

Sand and Gravel

$\gamma_b = 9.0 \ kN/m^3$

a) In order to find the in-situ stress p_o before the footing was built, we need to find the unit weight of the sand stratum,

$$\gamma_d = \frac{\gamma_b G_s}{G_s - 1} = \frac{(9.1 kN/m^3)(2.65)}{1.65} = 14.6 \ kN/m^3$$

The stress p_o is found at mid-clay stratum,

$$p_o = [h(\gamma_d)]_{dry(sand)} + [h(\gamma')]_{sand} + [h(\gamma')]_{clay}$$

$$p_o = [(2m)(14.6kN/m^3)] + [(4m)(9.1kN/m^3)] + [(1.5m)(9.2kN/m^3)] = 79.4 kPa$$

(Revision: Aug-08)

An odometer test was performed on a peat soil sample from an FDOT project in the Homestead area. The results are shown below. The initial sample thickness is 20 mm, with two-way drainage through porous stones, simulating field conditions. The vertical stress increment is 10 kPa. Estimate of the coefficient of consolidation c_v as,
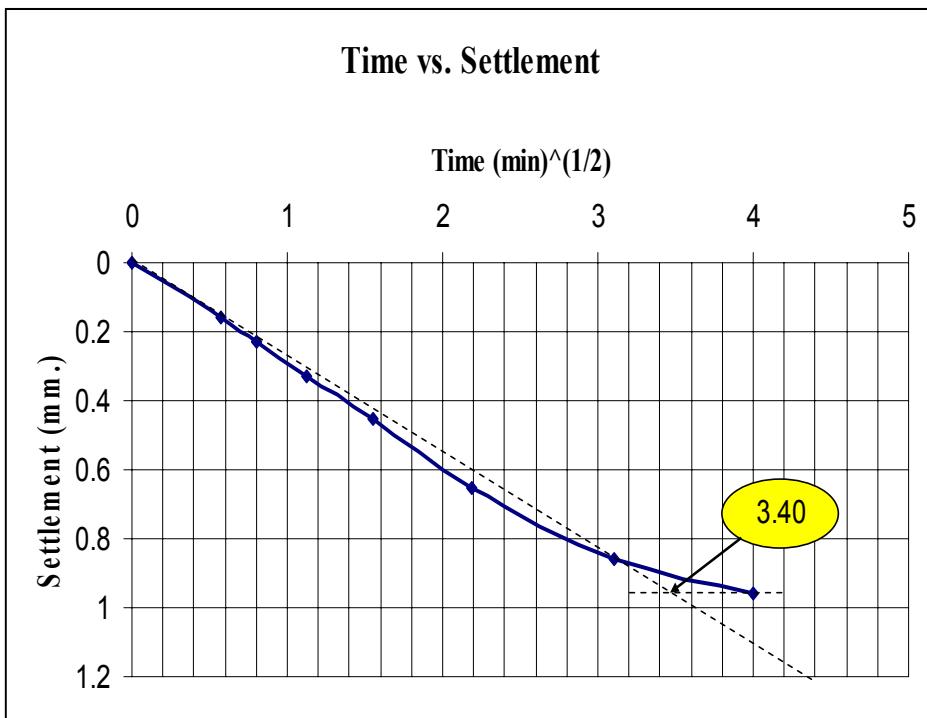
$$Coefficient\,of\,consolidation(C_v) = \frac{3H_{dr}{}^2}{4t}$$

Time (minutes)	0	0.32	0.64	1.28	2.40	4.80	9.60	16.0
Settlement (mm.)	0	0.16	0.23	0.33	0.45	0.65	0.86	0.96
$\sqrt{time} = (min)^{(\frac{1}{2})}$	0	0.57	0.8	1.13	1.55	2.19	3.10	4.0

(1) Plot a graph of settlement against the square root of time.

(2) Determine the value of the coefficient of consolidation cv (in m2/s).

(3) Estimate what could be a good estimate of the elastic modulus of this soil Eo (kPa)

(4) What sort of permeability k (in m/s) could you estimate for this soil (from Eo)?

Solution:

(1)

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(2)

$$vt_x = 3.40\,\text{min}^{1/2} \quad and \quad time(t_x) = 11.56\,\text{min}$$

$$c_v = \frac{3d^2}{4t_x} = \frac{3(10mm)^2(1\,\text{min})(1m)^2}{4(11.56\,\text{min})(60\,\text{sec})(1000mm)^2} = 1.08 \times 10^{-7}\,\frac{m^2}{\text{sec}}$$

(3)

$$Voids - ratio\ (e_v) = \frac{1\ mm}{20\ mm} = 0.05 \quad \therefore e_v = 5\%$$

$$E_o = \frac{\sigma_v}{e_v} = \frac{10 \times 10^3\,N}{0.05\ m^2} = 200 \times 10^3\,Pa = 200\ kPa$$

(4)

$$e \approx \frac{C_c}{1+e_o} \quad \therefore \quad c_v = \left[\frac{2.30\sigma_z{}'k}{\gamma_w}\right]\left[\frac{1+e}{C_c}\right] \quad \therefore c_v = \left(\frac{\sigma_z}{e_z}\right)\left(\frac{k}{\gamma_w}\right) = E_o\left(\frac{k}{\gamma_w}\right)$$

$$k = \frac{c_v\gamma_w}{E_o} = \frac{\left(1.08 \times 10^{-7}\,m^2\middle/s\right)\left(9.81\,kN\middle/m^3\right)}{\left(200\,kN\middle/m^2\right)} = 5.3 \times 10^{-9}\,\frac{m}{\text{sec}}$$

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(Revision: Aug-08)

An oedometer (consolidation) test is performed on a normally consolidated clay stratum that is 8.5 feet thick, and it found that the clay's initial voids ratio was e_o = 0.8 and its primary compression index is C_c = 0.28. The in-situ stress at mid-clay layer is p_o = 2,650 psf, and the building exerts a pressure through its mat foundation of 970 psf. The secondary compression index C_α = 0.02.

The time of completion of the primary settlement is approximately 18 months. What is the total consolidation of the 8.5 foot clay stratum 5 years after the primary consolidation?

Solution:

The primary consolidation ΔH_p is,

$$\Delta H_p = \frac{C_c H}{1+e_o} \log\left(\frac{p_o' + \Delta p'}{p_o'}\right) = \frac{(0.28)(8.5\,ft)(12\,in/ft)}{1+(0.8)} \log\left(\frac{2,650+970}{2,650}\right) = 2.15\,inches$$

The secondary consolidation ΔH_s is,

$$\Delta H_s = \frac{C_\alpha H}{1+e_p} \log\left(\frac{t_2}{t_1}\right)$$

We must find e_p first, by finding the change in the voids ratio during primary consolidation,

$$e_p = e_o - \Delta e = e_o - C_c \log\left(\frac{p_o' + \Delta p'}{p_o'}\right) = 0.8 - (0.28)\log\left(\frac{2,650+970}{2,650}\right) = 0.76$$

$$\therefore \quad \Delta H_s = \frac{C_\alpha H}{1+e_p} \log\left(\frac{t_2}{t_1}\right) = \frac{(0.02)(8.5-0.18)(12\,in/ft)}{1+(0.76)} \log\left(\frac{5}{1.5}\right) = 0.59\,inches$$

The total consolidation settlement is thus $= \Delta H_p + \Delta H_s = 2.15 + 0.59 = 2.74\,inches$
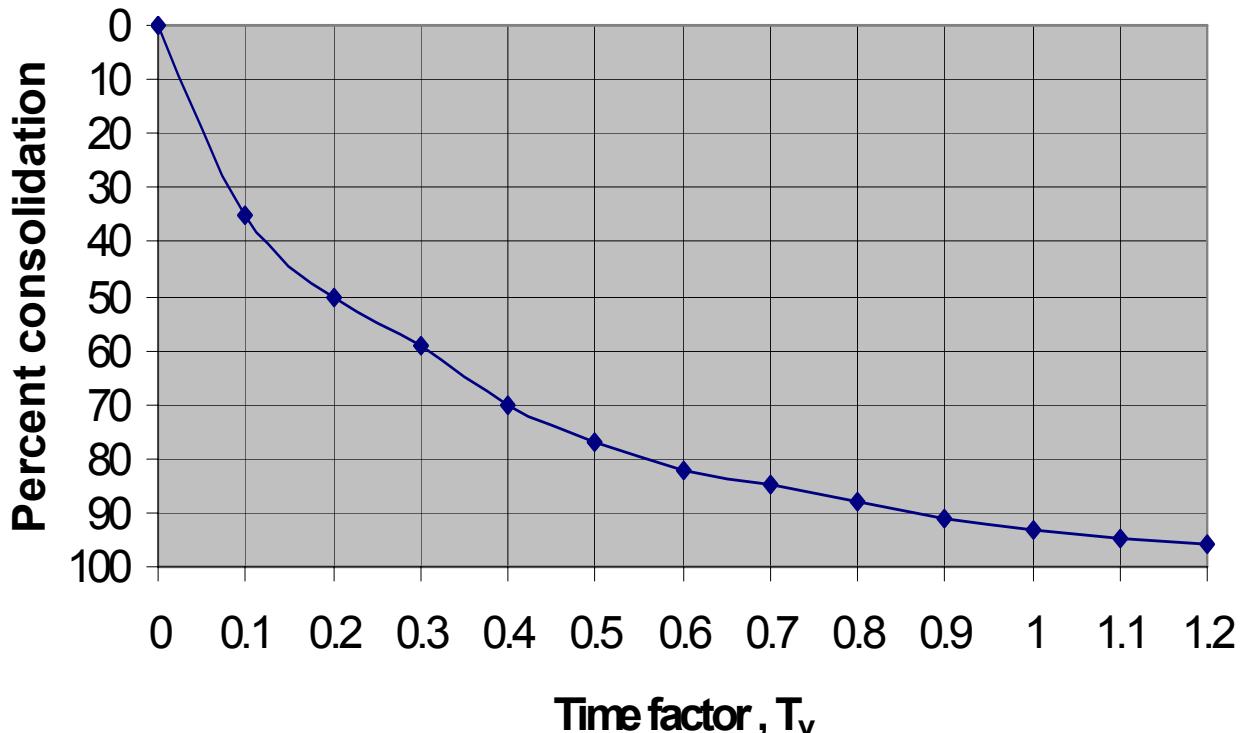
(Revision: Aug-08)

A 3-m thick, doubly-drained saturated stratum of clay is under a surcharge loading that underwent 90% primary consolidation in 75 days. Find the coefficient of consolidation c_v of this clay in cm²/sec.

Solution:

The clay layer has two-way drainage, and $T_v = 0.848$ for 90% consolidation.
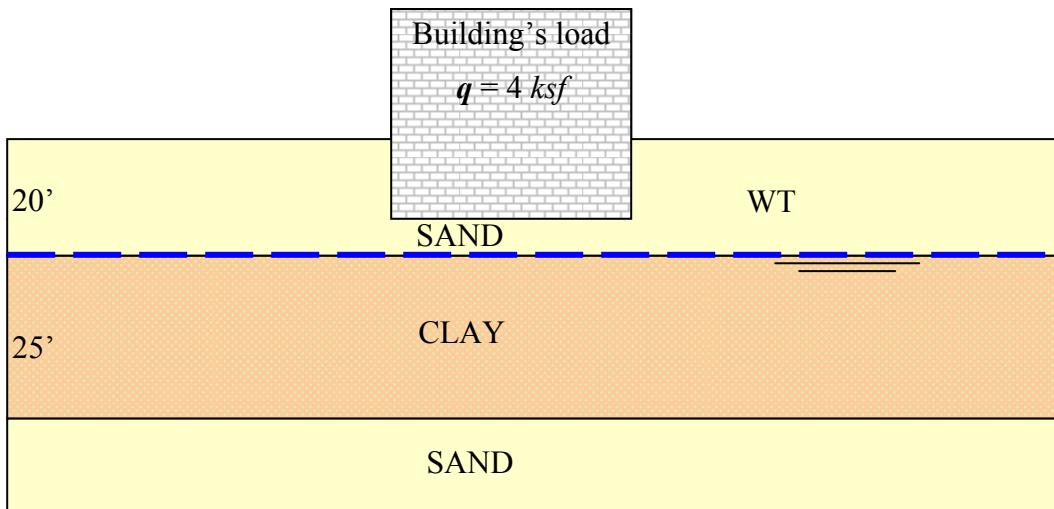
$$c_v = \frac{T_v H_{dr}^2}{t} = \frac{(0.848)(150\ cm)^2}{(75\ days \times 24 \times 60 \times 60)} = \mathbf{0.00294\ cm^2/sec}$$

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*Plastic Settlement-12: *The time rate of consolidation.*

(Revision: Aug-08)

An oedometer (consolidation) test is performed on a 4" thick specimen, drained on top and bottom. It was observed that 45 percent consolidation (T_v = 0.15) was attained in 78 hours. Determine the time required to attain 70 percent consolidation (T_v = 0.40) in a job site where the clay stratum is shown below.

Solution:

The coefficient of consolidation $c_v = \dfrac{T_v H_{dr}^2}{t}$ is the same for the lab and field samples.

$$c_v = \frac{T_v H_{dr}^2}{t} = \left(\frac{T_v H_{dr}^2}{t}\right)_{laboratory} = \left(\frac{T_v H_{dr}^2}{t}\right)_{field}$$

$$\therefore \quad \frac{(0.15)(2\,in)^2}{(78\,hours)} = \frac{(0.40)(12.5\,ft \times 12\,in/ft)^2}{t_{field}}$$

$$\therefore \quad t_{field} = (78\,hours)\left(\frac{1\,day}{24\,hours}\right)\left(\frac{1\,year}{365\,days}\right)\left(\frac{0.40}{0.15}\right)\left(\frac{(12.5\,ft \times 12\,in/ft)^2}{(2\,in)^2}\right)$$

$$\therefore \quad t_{field} = 134 \; years \; to \; attain \; 70\% \; consolidation$$

(Revision: Oct.-08)

Using the information derived from Problem 11, how long will it take a 30-mm thick undisturbed clay sample obtained from the field to undergo 90% consolidation in the laboratory?

Solution:

The Time Factor T_v is the same 90% in the field as in the laboratory, therefore,

$$T_{90} = \frac{c_v t_{90(field)}}{H_{dr(field)}^2} = \left[\frac{c_v (75 days \times 24 \times 60 \times 60)}{(\dfrac{3,000mm}{2})^2} \right]_{field} = \left[\frac{c_v t_{90}}{(\dfrac{30mm}{2})^2} \right]_{laboratory}$$

$$\therefore \quad t_{90(lab)} = \frac{(75 days \times 24 \times 60 \times 60)(15mm)^2}{(1,500mm)^2} = 648 \ \text{seconds} = 10 \ \text{minutes}$$

*Plastic Settlement-14: *Laboratory versus field time rates of settlement.*

(Revision: Aug-08)

Laboratory tests on a 25mm thick clay specimen drained at both the top and bottom show that 50% consolidation takes place in 8.5 minutes.

(1) How long will it take for a similar clay layer in the field, 3.2 m thick, but drained at the top only, to undergo 50% consolidation?

(2) Find the time required for the clay layer in the field as described in part (a) above, to reach a 65% consolidation.

Solution:

(1)

$$\frac{t_{lab}}{H_{dr(lab)}^2} = \frac{t_{(field)}}{H_{dr(field)}^2}$$

$$\therefore t_{(field)} = \frac{(3.2m)^2(8.5\min)}{\left(\dfrac{25mm}{2\times1000mm}\right)^2} = 557,000\min = 387\,days$$

(2)

$$c_v = \frac{T_v H_{dr}^2}{t_{50}} = \frac{(0.197)\left(\dfrac{25}{2\times1000}\right)^2}{(8.5\min)} = 0.36\times10^{-5}$$

$$\therefore t_{(field\,65)} = \frac{T_{65} H_{dr}^2}{c_v} = \frac{(0.34)(3.2m)^2}{0.36\times10^{-5}m^2/\min} = 961,400\min = 668\,days$$

(Revision: Aug-08)

A clay layer 20 feet thick sitting on top of granite bedrock, experiences a primary consolidation of 8.9 inches. Find:
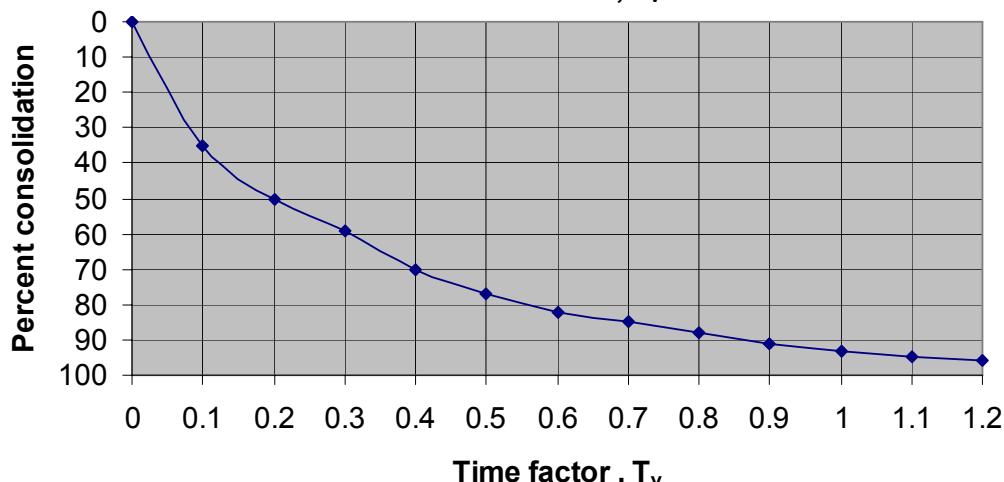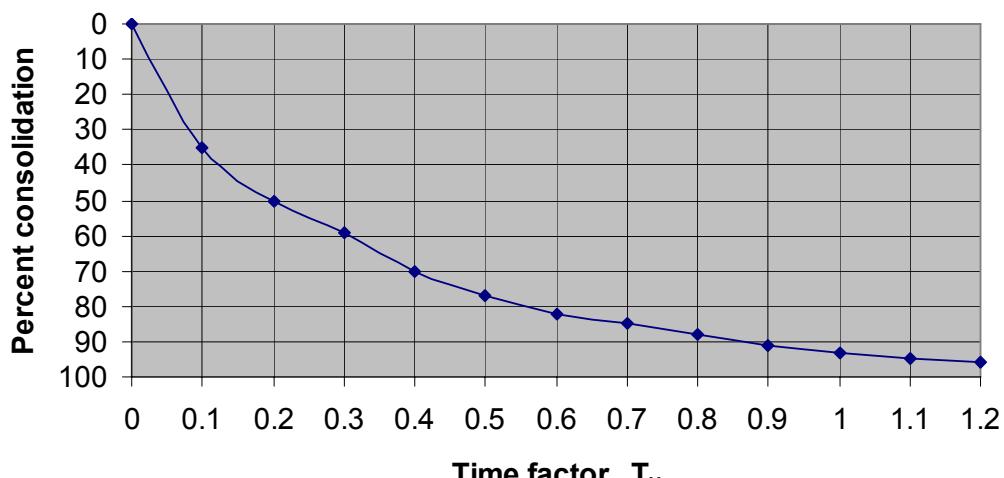
(a) The degree of consolidation when the settlement reaches 2 inches.

(b) The time to reach 50% settlement if cv is 0.002 cm2/sec.

(c) The time for 50% consolidation if the clay stratum is doubly-drained?

Solution:

(a) $U\% = \left(\dfrac{2}{8.9}\right) 100 = 22.5\%$

(b) $T_{50} = 0.197 \quad and \quad t = \dfrac{T_v}{c_v} H_{dr}^2 = \dfrac{(0.197)\left[(20\,ft)(30.48cm/\,ft)\right]^2}{\left(0.002\ cm^2/\sec\right)\left(\dfrac{1hr}{360\sec}\right)\left(\dfrac{days}{24h}\right)} = 424 \quad days$

(c) $t = \dfrac{T_v}{c_v} H_{dr}^2 = \dfrac{(0.197)\left[(10\,ft)(30.48cm/\,ft)\right]^2}{\left(0.002\ cm^2/\sec\right)\left(\dfrac{1hr}{360\sec}\right)\left(\dfrac{days}{24h}\right)} = 106 \quad days$

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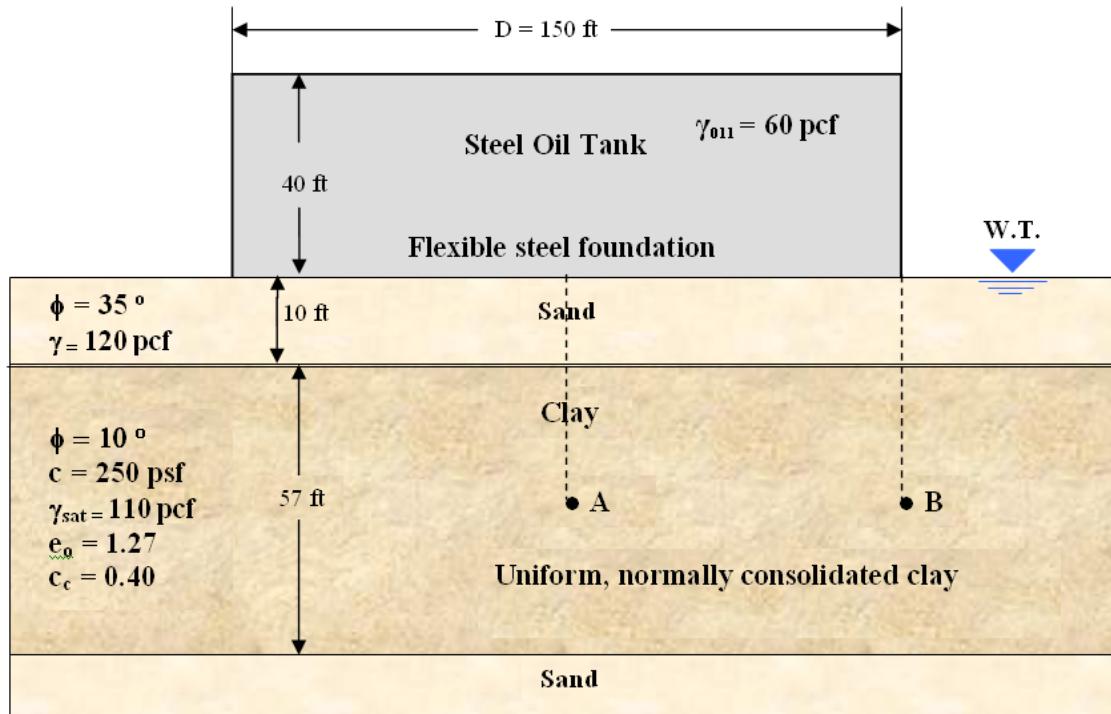
(Revision: Oct.-08)

An oedometer test on a 1" thick, doubly-drained sample from the clay stratum (shown below) attained 50% consolidation in 6.5 minutes. Find:

(a) The total differential settlement of the fully loaded tank.

(b) The time required for 75% consolidation in the field.

(c) The depth of excavation for minimal settlement.

Solution:

a) The surface load from the oil tank (neglecting the weight of the tank) is Δp:

$$\Delta p = h\gamma_{oil} = (40\,ft)(60\,pcf) = 2.4\ ksf$$

Using the Boussinesq pressure diagram (next two pages) for $B = 75'$, provides the stress levels at any point in the soil mass, thus,

The stress at point A $\Rightarrow 0.91\Delta p = 0.90(2.4) = 2.2$ ksf

point B $\Rightarrow 0.43\Delta p = 0.43(2.4) = 1.0$ ksf

The in-situ effective stress at point A was p_o, before the tank was built:

$$p_o = (10')(0.120 - 0.062) + (28.5')(0.110 - 0.062) = 1.95\ ksf$$

The settlements at point A (below the center of the tank), and point B (at the edge of the tank) are,

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$$\Delta H_A = \frac{C_c H}{1+e_o} \log\left(\frac{p_o'+\Delta p}{p_o'}\right) = \frac{0.4(57')}{(1+1.27)} \log\left(\frac{1.95+2.2}{1.95}\right) = 3.34 \ ft$$

$$\Delta H_B = \frac{C_c H}{1+e_o} \log\left(\frac{p_o'+\Delta p}{p_o'}\right) = \frac{0.4(57')}{(1+1.27)} \log\left(\frac{1.95+1.0}{1.95}\right) = 1.85 \ ft$$

$\therefore$ *The differential settlement between A and B is* $= 3.34 ft - 1.85 ft = 1.49 ft = 18 inches$

b) Since the time required for consolidation is,

T$_v$ for 50% = 0.197, and

T$_v$ for 75% = 0.480, using the relationship between field and lab conditions, through the coefficient of consolidation,

$$\frac{Tv_{field}H^2_{field}}{t_{75}} = \frac{Tv_{lab}H^2_{field}}{t_{lab}} \quad \therefore \quad c_v = \frac{(0.2)\left[0.5in\left(\frac{ft}{12in}\right)\right]^2}{6.5\,min} = \frac{(0.480)(28.5ft)^2}{t_{75}}$$

$\therefore \quad t_{75} = 13.8 \ years \ for \ 75\% \ consolidation.$

c) The settlement can be reduced by excavating weight of soil equal to the weight of the structure (note: total, not effective, since the water is also removed). Assume need to excavate "x" feet from the clay layer.

$$\sigma_{total} = (10')(0.120) + x(0.110) = 2.4 \ ksf$$

Solving for x: $\therefore x = 10.9 ft \cong 11$ feet of clay

Add the 11 feet of the clay to the 10 feet of the sand on top of the clay for a

$\therefore$ *Total excavation depth* $= 11 ft + 10 ft = 21 \ feet$

(Revision: Aug-08)

A common method used to accelerate the consolidation of a clay stratum is a *sand surcharge*, as shown below. The surcharge load will force the clay to attain a large part of its settlement before the structure with the same load is built. This method minimizes the settlement of the structure.

An office building is planned to be built on the site shown below. The total weight of the building is 136,000 kips, spread over a square foundation 200 ft by 200 ft. Field tests showed that the clay stratum has a liquid limit of 28 percent, an initial void ratio of 0.95, a γ = 130 pcf and a consolidation coefficient of 10^{-3} in^2/second. The sand stratum has a C_C = 0.01, a γ = 125 pcf and an initial void ratio of 0.70. The sand surcharge has a γ = 115 pcf.

(d) Determine the total settlement at mid-clay under the center of the surcharge.

(e) The time required to attain 60% consolidation of the clay stratum (i.e. T_V = 0.30). This is the *lead time* required to place the surcharge before construction.

(f) The *SPT* in the sand stratum is N = 15.

Solution:

(a) The weight of the new building is estimated at Q = 136,000 kips. The surcharge will have to weigh the same, spread over an area = 200' x 200' = 40,000 ft^2, using a fill with a unit weight of γ = 115 pcf. The unit pressure of the surcharge $\gamma_{surcharge}$ is,

$$q_{sur} = \frac{Q}{A} = \frac{136,000 kips}{40,000 \, ft^2} = 3.4 \, ksf$$

$$but, \; q_{sur} = \gamma_{sur}h = 115\frac{lb}{ft^3}(h) = 3.4 \; ksf \quad \therefore h = 30 \; feet \; high$$

Both the sand and clay strata contribute to the settlement. However, the settlement from the clay stratum is a consolidation settlement, such that,

Using Skempton's formula $C_c = 0.009(LL-10) = 0.009(28-10) = 0.162$

Also $e_o = 0.95$ *and* $H_{clay\,stratum} = 20\,ft = 240\,in.$

The initial stress at mid-clay stratum is,

$p_0 = \gamma_s h + \gamma_s^b h = \gamma_c^b h = (0.125kcf)(15\,ft) + (0.125 - 0.0624)(15) + (0.130 - 0.0624)(10) = 3.5\ ksf$

The increased load from the surcharge Δp is,

$\Delta p = \gamma h = (0.115kcf)(30\,ft) = 3.45\ ksf$

The settlement created by the surcharge is,

$$\Delta H_c = \frac{HC_c}{(1+e_0)}\log_{10}\left(\frac{p_0 + \Delta p}{p_0}\right) = \frac{(240in)(0.162)}{(1+0.95)}\log_{10}\left(\frac{3.5 + 3.45}{3.5}\right) = 5.94\ inches$$

The settlement from the sand stratum is an elastic settlement,

$H_{sand} = 30\,ft = 360in \quad E_S \approx 14N = (14)(15) = 210k/ft^2;\ depth\ factor\ C_1 = 1 \quad and \quad creep\ factor\ C_2 = 1$

$$\Delta = C_1 C_2 q_o\left(\frac{\varepsilon}{E}\right)dz = (1)(1)(3.4ksf)(\frac{0.30}{210\ ksf})(360\ in) = 1.75in$$

The total settlement $= 5.94in + 1.75in = 7.69in$

b) The lead time t required for the surcharge to accomplish its task is,

$$t = \frac{T_v H_{dr}^2}{c_v} = \frac{(0.30)\left[(20\,ft)(12in/1ft)\right]^2}{(10^{-3}in^2/\sec)}\left(\frac{1\min}{60\sec}\right)\left(\frac{1hr}{60\min}\right)\left(\frac{1day}{24hr}\right) = 200\ days\ for\ 60\%\ consolidation$$
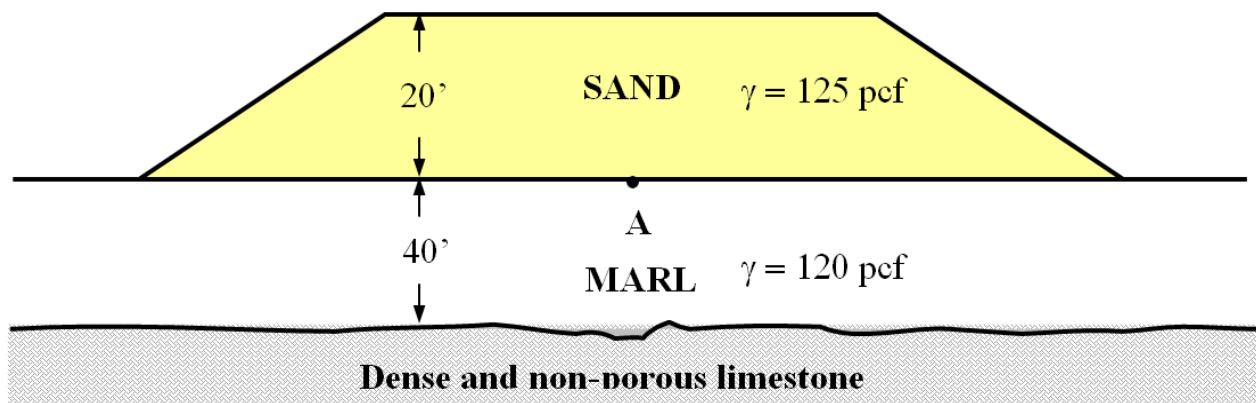
The total settlement that has taken place at 200 days (60% consolidation) is,

$Total\ Time = \Delta H_S + (60\%)\Delta H_C = 1.73in + (0.6)(5.94in) = 5.3\ inches$

(Revision: Aug-08)

A uniform surcharge of sand 20 feet in height will be placed over the marl stratum as shown below, in order to preconsolidate that layer for a future building. The in-situ voids ratio of the marl is 0.59, and its index of compression can be found from a relation proposed by Sowers as $C_c = 0.75 (e_o – 0.30)$. Find the total settlement of the surcharge at its point A.
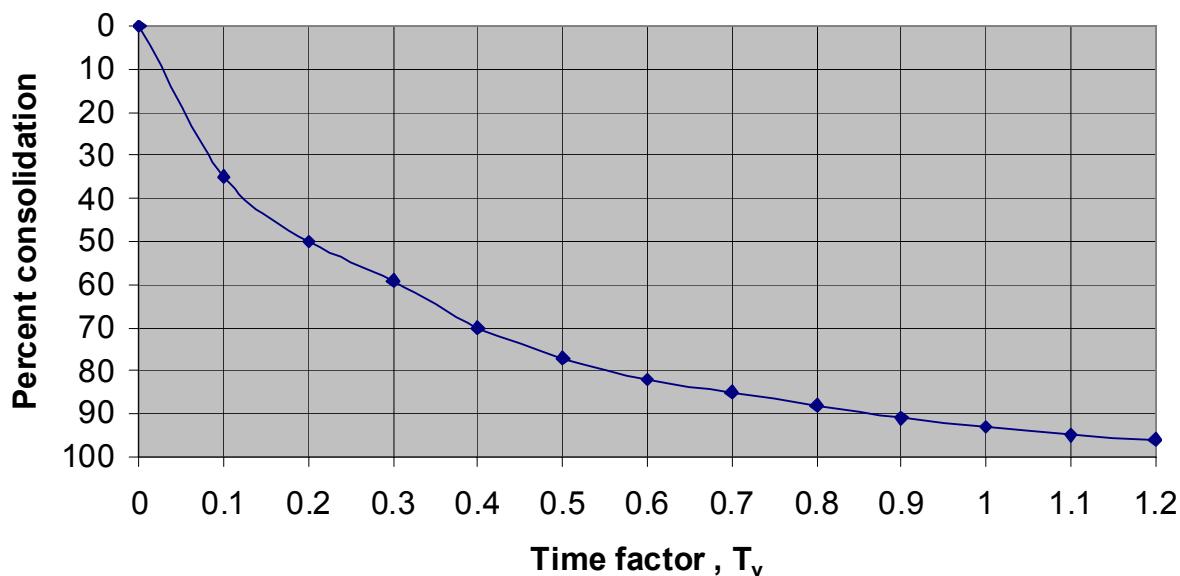
The coefficient of consolidation c_v for the marl can be found from the relation,

$$c_v = \frac{k(1+e_o)}{a_v \gamma_w} \quad \text{ft}^2/\text{day}$$

where the permeability $k = 1.0 \times 10^{-5} \, cms/s$, and $a_v = 2.9 \times 10^{-4} \, ft^2/lb$.

Find the time required (in days) for the marl to attain 50% consolidation.

Solution.

$$c_c = 0.75(e_o - 0.30) = 0.75(0.59 - 0.30) = 0.218$$

At midpoint of the marl, the in-situ stress σ_o is,

$$\sigma_o = \gamma h = \left(120 \frac{lb}{ft^3}\right)(20 ft) = 2,400 \, psf$$

The increase in stress due to the surcharge $\Delta\sigma$ is,

$$\Delta\sigma = \gamma \Delta h = \left(125 \frac{lb}{ft^3}\right)(20 ft) = 2,500 \, psf$$

Therefore, the consolidation settlement of the marl is,

$$\therefore \Delta = \frac{HC_c}{1+e_o} \log_{10}\left(\frac{\sigma_o + \Delta\sigma}{\sigma_o}\right) = \frac{(40 ft)(12 in/ft)(0.218)}{(1+0.59)} \log_{10}\left(\frac{2.4 + 2.5}{2.4}\right) = 20.4 \, inches$$

The time required for 50% of consolidation to take place is found through the coefficient of consolidation c_v,

$$c_v = k\left(\frac{1+e_o}{a_v \gamma_w}\right) = \frac{(1.0 \times 10^{-5} \, cm/\sec)(1+0.59)}{(2.9 \times 10^{-4} \, ft^2/lb)(62.4 lb/ft^3)} \left(\frac{1 in}{2.54 cm}\right)\left(\frac{1 ft}{12 in}\right)\left(\frac{86.4 \times 10^3 \, \sec}{day}\right) = 2.5 \, ft^2/day$$

The time factor for 50% consolidation is $T_v = 0.2$, therefore the time t required is,
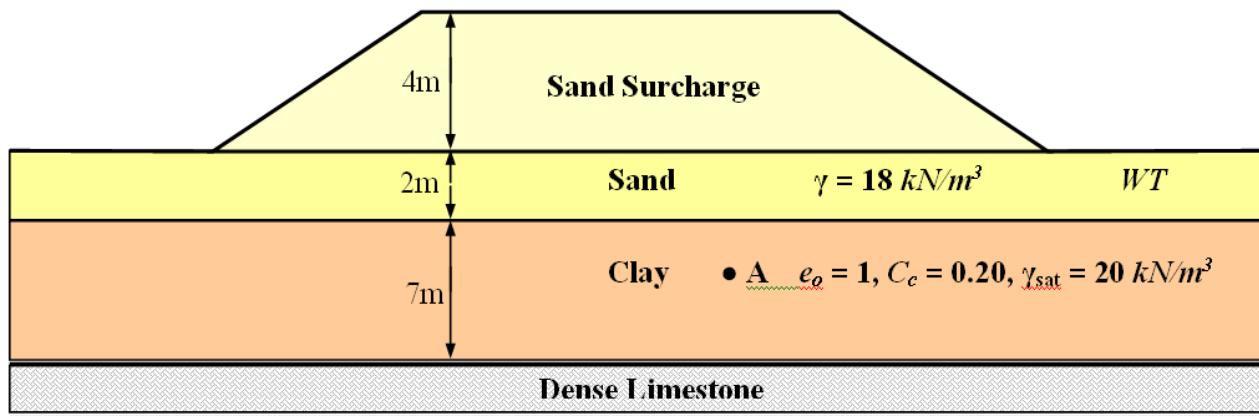
$$t = \frac{T_v H_{dr}^2}{c_v} = \frac{(0.2)(40 ft)^2}{2.5 \, ft^2/day} = 128 \, days$$

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**Plastic Settlement-19: *Differential settlements under a levee.*

(Revision: Aug-08)

An oedometer (consolidation) test was performed on a clay sample 3 cm high, drained on both sides, and taken from mid-stratum shown below. Seventy percent consolidation was attained in 6.67 minutes. Find:

(b) The time required to attain 70% consolidation of the clay stratum.

(c) The magnitude of that settlement in that time.

Solution.

(a) Since the soil is the same clay in the laboratory and the field, and both are 70% consolidation,

$$c_v = \frac{T_v H_{dr}^2}{t} = \left[\frac{T_v H_{dr}^2}{t}\right]_{laboratory} = \left[\frac{T_v H_{dr}^2}{t}\right]_{field} \qquad \therefore \frac{t_{field}}{t_{lab}} = \frac{H_{field}^2}{H_{lab}^2}$$

$$or \quad t_{field} = \frac{t_{lab} H_F^2}{H_L^2} = (6.67\,\text{min})\left(\frac{700cm}{1.5cm}\right)^2 = 1.45 \times 10^6 \,\text{min} = 2.76\,years$$

(b) The amount of settlement that takes place at 70% consolidation is,
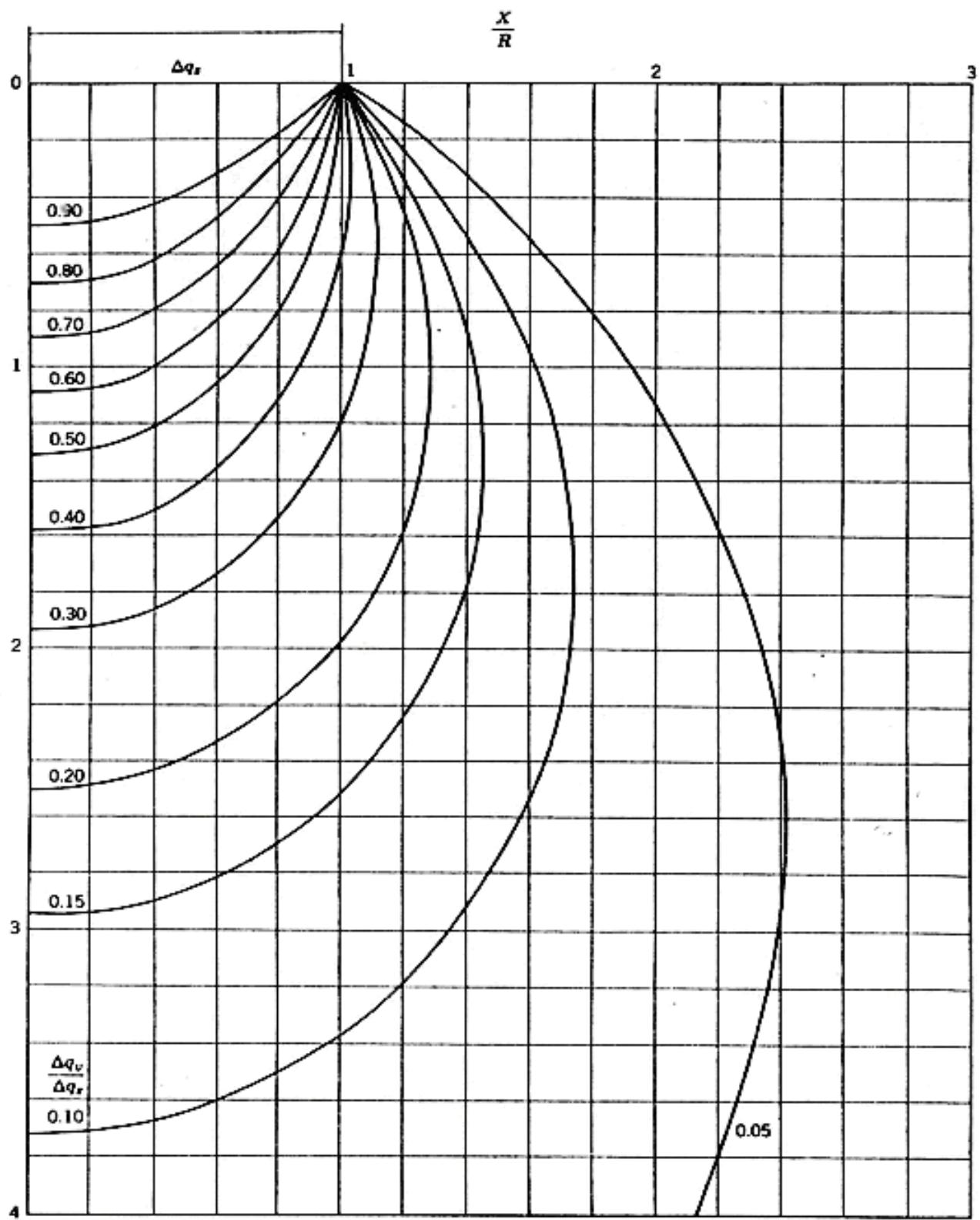
$$\Delta H_{70\%} = (0.70)\left\{\frac{C_c H}{1+e_o}\log_{10}\left(\frac{p_o' + \Delta p'}{p_o'}\right)\right\}$$

The in-situ stress at mid-clay stratum before the surcharge was applied was,

$$\therefore p_o' = \sum_i \gamma_i h_i = (18kN/m^3)(2m) + (20 - 9.81)kN/m^3(3.5m) = 71.7kN/m^2$$

and $\Delta p' = 72\ kN/m^2$

$$\therefore 70\%\Delta H = \frac{0.7 C_c H}{1+e_o}\log_{10}\left(\frac{p_o' + \Delta p'}{p_o'}\right) = \frac{0.7(0.20)(700cm)}{(1+1)}\log_{10}\left(\frac{71.7+72}{71.7}\right) = 14.8\ \text{cms}$$

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Vertical stresses induced by a uniform load on a circular area

***Plastic Settlement-20: Estimate of the coefficient of consolidation c_v.

(Revised Oct-09)

An oedometer test was performed on a peat soil sample from an FDOT project in the Homestead area. The results are shown below. The initial sample thickness was 20 mm with two way drainage through porous stones, simulating field conditions. The vertical stress increment is 10 kPa. Use an estimate of the coefficient of consolidation c_v as,
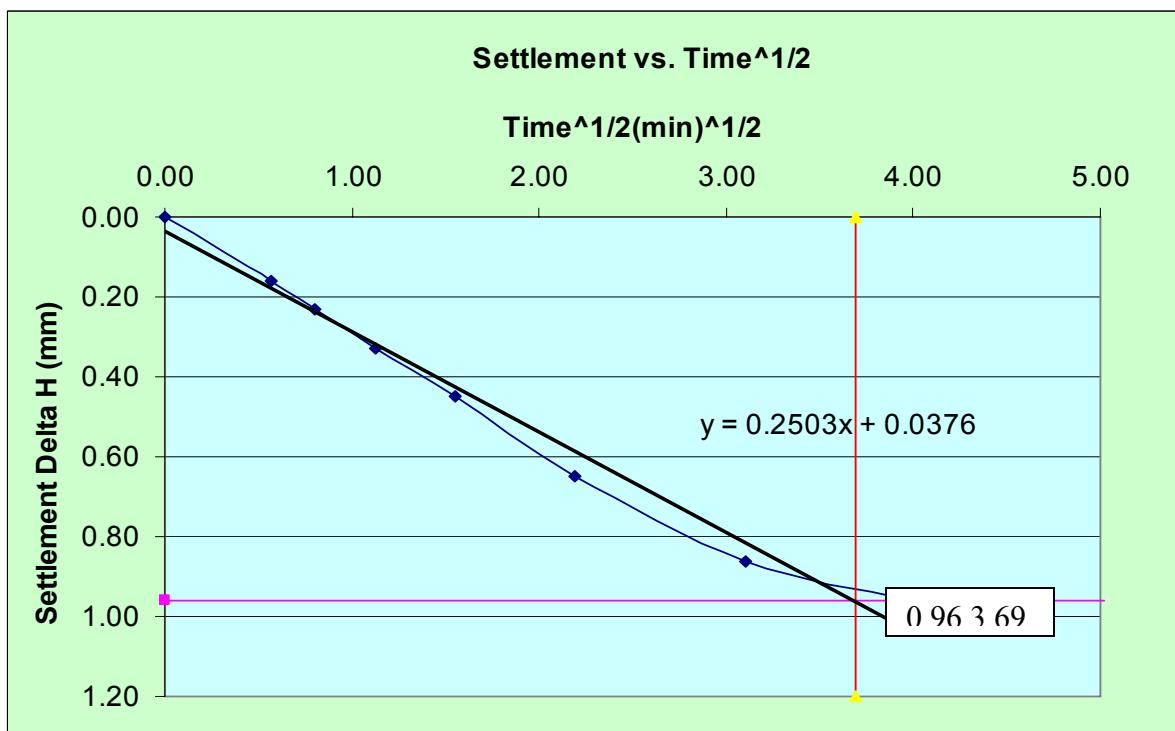
$$c_v = \frac{3H_{dr}^2}{4t}$$

Given Data								
Time(min)	0.00	0.32	0.64	1.28	2.40	4.80	9.60	16.00
Settlement (mm)	0.00	0.16	0.23	0.33	0.45	0.65	0.86	0.96
Time^1/2(min)^1/2	0.00	0.57	0.80	1.13	1.55	2.19	3.10	4.00

(a) Plot a graph of settlement against the square root of time.

(b) Determine the value of c_v ($\frac{m^2}{s}$).

(c) Estimate what could be a good estimate of the elastic modulus of this soil, E_o (in kPa).

(d) What sort of permeability k ($\frac{m}{s}$) could you estimate for this soil (from E_o)?

Solution:

(a)

Settlement vs. Time^1/2

y = 0.2503x + 0.0376

0 96 3 69

(b) $\sqrt{t_x} = 3.69\,\text{min}^{1/2} \rightarrow t_x = 13.62\,\text{min}$

$$c_v = \frac{3d^2}{4t_x} = \frac{3(10\,\text{min})^2\,(1\,\text{min})(1m)^2}{4(13.62\,\text{min})(60\,\text{sec})(1000mm)^2} = 9.178*10^{-8}\,\frac{m^2}{\text{sec}}$$

(c) The strain at the end of the loading is:

$$\varepsilon_v = \frac{\Delta}{L} = \frac{(1mm)}{(20mm)} = 0.05 \text{ or } 5\%$$

$$E_o = \frac{\sigma_v}{\varepsilon_v} = \frac{10*10^3 N}{0.05m^2} = 200kPa \text{ Vertical Modulus}$$

From Coduto (pages 390 and 391) $\varepsilon_v \approx \dfrac{C_c}{1+e_o}$ but Coduto (page 424)

$$c_v = \frac{2.30\sigma_z k}{\gamma_w}\left(\frac{1+e_0}{C_c}\right) \text{ or } c_v = \frac{\sigma_z}{\varepsilon_z}\left(\frac{k}{\gamma_w}\right) = E_o\,\frac{k}{\gamma_w}$$

*note: (2.30 is for ln; use 1.0 for log)

$$k = \frac{c_v \gamma_w}{E_o} = \left(\frac{\left(9.178*10^{-8}\,\dfrac{m^2}{\text{sec}}\right)\left(9.81\,\dfrac{kN}{m^3}\right)}{\left(200\,\dfrac{kN}{m^2}\right)}\right) = 4.50*10^{-9}\,\frac{m}{s}$$

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(Revised Oct-09)

Find the C_c and the "apparent" OCR for a superficial clay stratum that is 4m thick. The water is at the surface. A sample of the clay has a water content of 27.9%, and a specific gravity of 2.65. An oedometer test showed the following results:

Voids Ratio	Vertical Effective Stress
0.736	15
0.733	28
0.73	60
0.675	230
0.638	480
0.6	930

Solution:

$C_c = [C_a\text{-}C_b]/[\log(O'_{z\,b})\text{-}\log(O'_{z\,a})] = [0.675\text{-}0.600]/[\log(930)\text{-}\log(230)] = 0.123$

$Se = wG_s$ (Saturation means S = 1)

$e = wG_s = (0.279)(2.65) = 0.73935 = 0.739$

$\gamma = [(Gs + e)\,\gamma_w]/[1 + e] = [(2.65 + 0.739)(9.81)]/[1 + 0.739] = 33.25/1.739 = 19.12$

OCR = (98)/(19.12) = 5.13

(Revised Oct-09)

Two tall buildings sit next to each other in the downtown area of Boston. They are separated by a narrow 5 foot alley and are both 50 stories high (550 feet tall). They also have similar foundations, which consist of a simple mat foundation. Each mat is a thick reinforced concrete slab, 5 feet thick and 100 feet by 100 feet in plan view. The total load (dead + live + wind) of each building is 150,000 kips.

The mats are sitting upon a thick prepared stratum of carefully improved soil 40 feet thick, that has an allowable bearing capacity of 17 ksf. Below the compacted fill stratum lays a medium to highly plastic clay stratum 38 feet thick. Below the clay stratum is a thick layer of permeable sand. The water table coincides with the interface between the improved soil and the clay. The dry unit weight of improved soil is 110 pcf, whereas the in-situ unit weight of the clay is 121 pcf.

The clay has a specific gravity of 2.68. Also a moisture content of 32%, a PI of 52% and a PL of 12% before building "A" was built in 1975. The clay has a consolidation coefficient of 10^{-4} in^2/sec. The second building "B" was finished by early 1995. Assume that each building's mat rotates as a rigid plate.

 How much do you predict will building 'A" drift towards building "B" by early 1996, in inches?

(Note: The drift of a building is its horizontal movement at the edge of the roof level due to lateral loads such as wind or earthquakes, or the differential settlement due to unequal pressures.)

Given: P = 150,000 kips

q_a = 17 ksf;

For clay: G_s = 2.68,

w = 32%,

P.I = 52%,
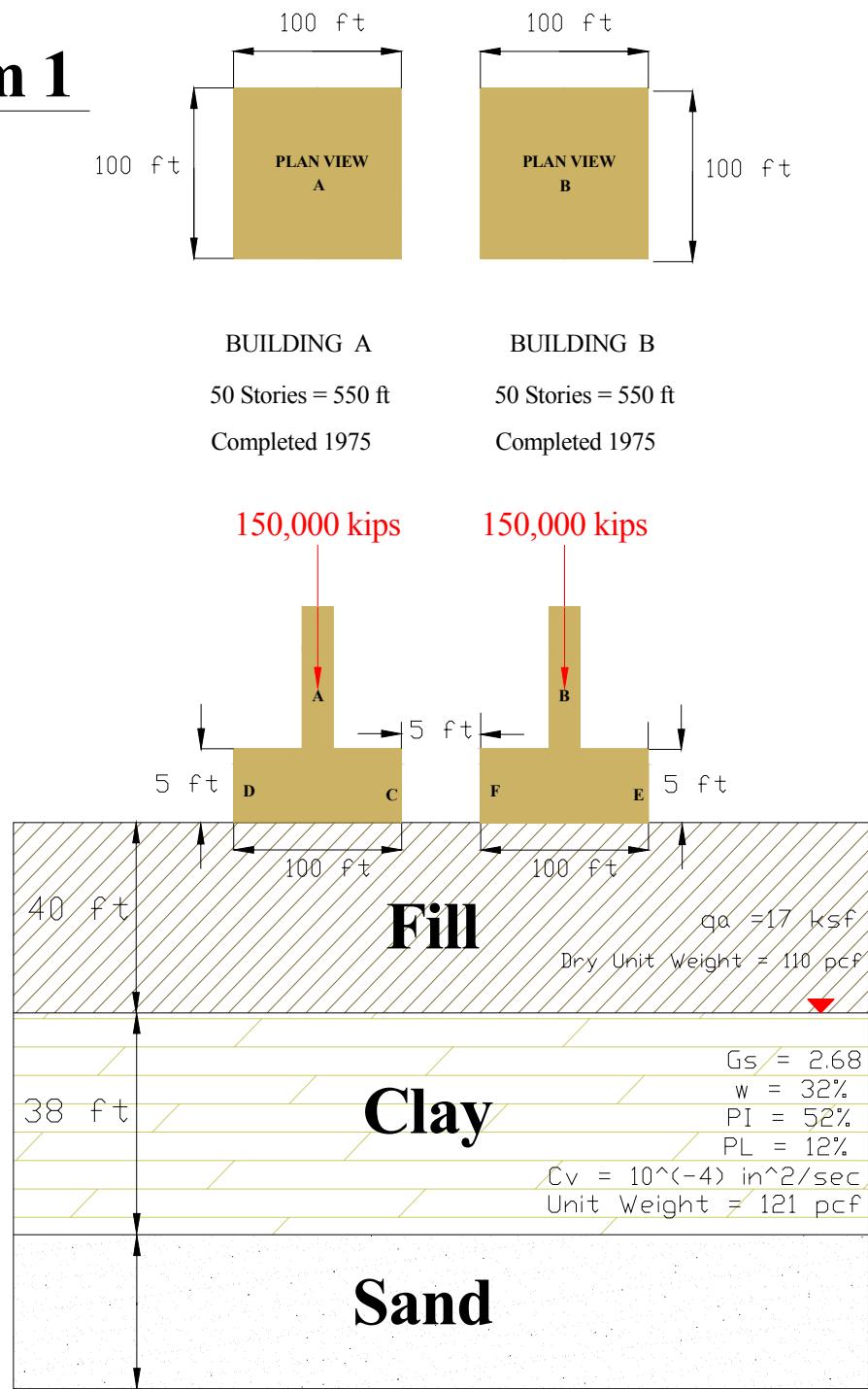
P.L = 12%,

γ =121 pcf,

$c_v = 10^{-4}$ in^2/sec

For the improved soil: γ_d =110 pcf

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Diagram 1

100 ft · 100 ft

100 ft · PLAN VIEW A · PLAN VIEW B · 100 ft

BUILDING A
50 Stories = 550 ft
Completed 1975

BUILDING B
50 Stories = 550 ft
Completed 1975

150,000 kips · 150,000 kips

A · B

5 ft

5 ft · D · C · F · E · 5 ft

100 ft · 100 ft

40 ft · **Fill** · $q_a = 17$ ksf
Dry Unit Weight = 110 pcf

38 ft · **Clay** · Gs = 2.68
w = 32%
PI = 52%
PL = 12%
Cv = 10^(-4) in^2/sec
Unit Weight = 121 pcf

Sand

<u>*Solution:*</u>

Initial Condition: Geostatic Stresses

$\sigma'_{zo} = \sum H\gamma - u = (40)(121) - (40)(62.4) = 2,344 \text{ lb/ ft}^2$

Induced Stresses:

To solve the induced stresses we will use the Boussinesq's Method:

$$(\sigma_z)_{induced} = \frac{q\left[2BLz_f\left(B^2 + L^2 + z_f^2\right)^{1/2}\right]}{4\Pi\left(z_f^2\left(B^2 + L^2 + z_f^2\right) + B^2L^2\right)} \times \frac{B^2 + L^2 + 2z_f^2}{B^2 + L^2 + z_f^2} + \Pi - \sin^{-1} - \frac{2BLz_f\left(B^2 + L^2 + 2z_f^2\right)^{1/2}}{z_f^2\left(B^2 + L^2 + z_f^2\right) + B^2L^2}$$

$q = \frac{p}{A} = \frac{150,000 \times 10^3}{(100)(100)} = 15,000 \text{ lbs}$

$z_f = 58$ ft (at mid-clay)

Replacing the values of q, B, L and z_f into the Boussinesq's equation yields:

$(\sigma_z)_{induced} = 8008.65 \text{ lb/ ft}^2$, 1059.79 lb/ ft^2

Exists differential stresses at the corners of the foundation.

Stresses due to fill:

$(\sigma_z)_{fill} = H_{fill} \; \gamma_{fill} = (110)(38) = 4,180 \text{ lb/ ft}^2$

• The total final stresses at corner "C"
$(\sigma'_{zf})_C = \sigma'_{zo} + (\sigma_z)_{induced} + (\sigma_z)_{fill}$

$= 2344 \text{ lb/ ft}^2 + 8008.65 \text{ lb/ ft}^2 + 4180 \text{ lb/ ft}^2 = 14,532.65 \text{ lb/ ft}^2 \approx 14,533 \text{ lb/ ft}^2$

• The total final stresses at corner "D"
$(\sigma'_{zf})_D = \sigma'_{zo} + (\sigma_z)_{induced} + (\sigma_z)_{fill}$

$= 2344 \text{ lb/ ft}^2 + 1059.79 \text{ lb/ ft}^2 + 4180 \text{ lb/ ft}^2 = 7,583.79 \text{ lb/ ft}^2 \approx 7,584 \text{ lb/ ft}^2$

(Note: Both buildings A and B will experience the same final stress due to fill, loadings and geostatic factors).

<u>*Time Factor*</u>

Determining the time factor for building A:

$Tv = \dfrac{4C_v t}{H^2_{dr}}$

For 21 years: t = 662,256,000 sec

$Tv_A = \dfrac{4(10^{-4} in^2 / \sec)(662,256,000 \, sec)}{(40)^2} = 165.56$

<u>*Degree of Consolidation*</u>

$U = [1-10^{-(0.085+Tv)/0.933}] \times 100\%$

The degree of consolidation at $Tv_A = 165.56$ is:
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$U = [1-10^{-(0.085+166)/0.933}] \times 100\%$ $=100\%$

By early 1996, 21 yrs after the load was applied, the soils are completely consolidated.

Therefore,

$$U = \frac{\delta_c}{(\delta_c)_{ult}}$$

$$\delta_c = (\delta_c)_{ult}$$

Assuming normally consolidated soils

$$\delta_c = \sum \frac{C_c}{1+e_0} H \log\left(\frac{\sigma_{z_f}}{\sigma_{z_0}}\right) \quad \textbf{(Eqn 1-1)}$$

        $1 + e_o$ σ'_{zo}

$Cc = 0.009 \, (LL - 10)$

$LL = PL + PL = 52 + 12 = 64 \, \%$

Therefore, $Cc = 0.009 \, (64 - 10) = 0.486$

$$e_0 = \frac{G_s \gamma_w}{\gamma_d} - 1$$

$$\gamma_d = \frac{\gamma}{1+w} = \frac{121}{1+0.32} = 91.67 \quad \therefore \quad e_0 = \frac{(2.68)(62.4)}{91.67} - 1 = 0.82$$

Recall that uniform stress does not exist beneath the corners of the mat foundation. Therefore, we will have differential settlement.

At corner C:

    $H = 40 \, ft$ $(\sigma'_{zf})_C = 14{,}533 \, lb/ft^2$ $\sigma'_{zo} = 2{,}344 \, lb/ft^2$

  Replacing values of H, $(\sigma'_{zf})_C$, σ'_{zo}, Cc and e_o into **(Eqn 1-1):**

$$\delta_c = \frac{0.486}{1+0.82}(40 \, ft)\log\left(\frac{14{,}533}{2{,}344}\right) = 8.46 \, ft = 101.55 \, in$$
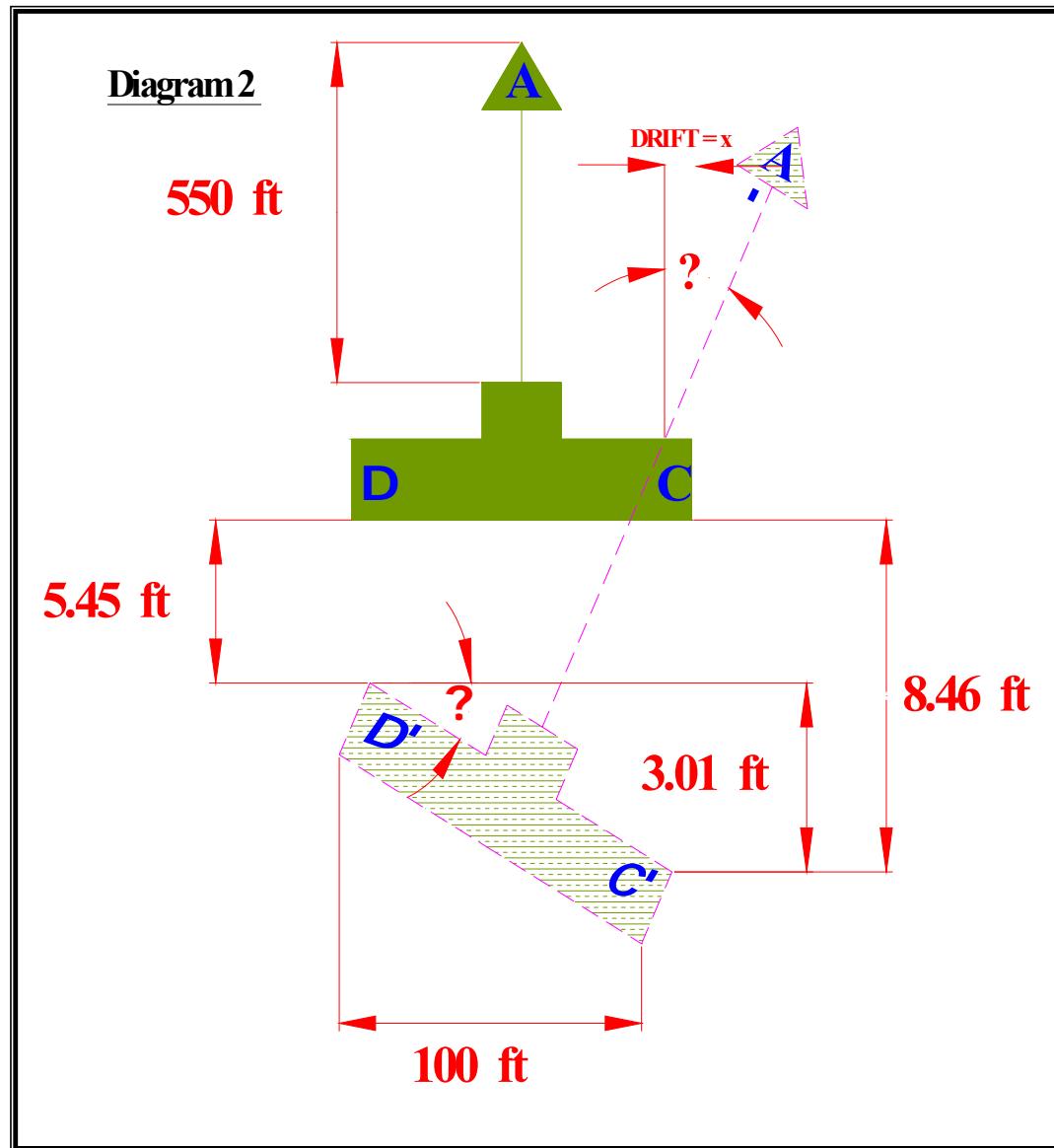
At corner D:

    $H = 40 \, ft$ $(\sigma'_{zf})_D = 7584 \, lb/ft^2$ $\sigma'_{zo} = 2{,}344 \, lb/ft^2$

Replacing values of H, $(\sigma'_{zf})_D$, σ'_{zo}, Cc and e_o into **(Eqn 1-1):**

$$\delta_c = \frac{0.486}{1+0.82}(40\ ft)\log\left(\frac{7,584}{2,344}\right) = 5,45\ ft = 65.35\ in$$

The $\Delta\delta c = 8.46 - 5.45 = 3.01$ ft

Diagram 2
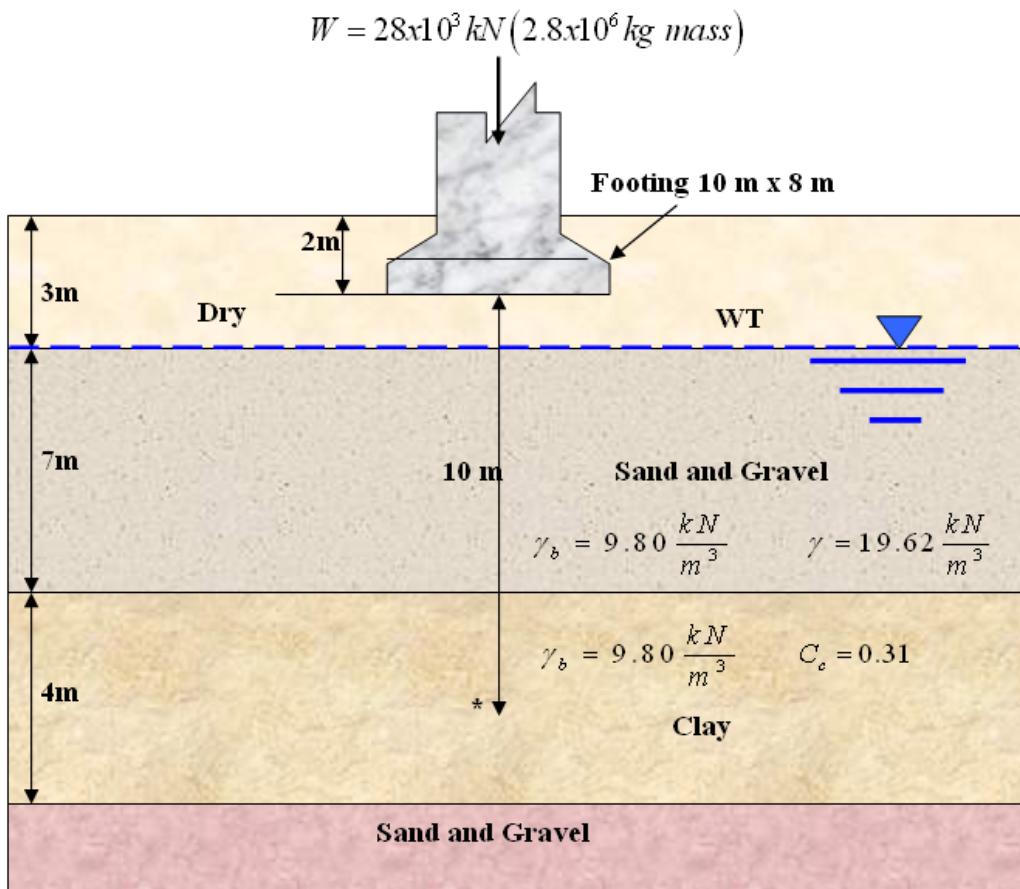
$\Delta = \tan^{-1}(3.01/100) = 1.72°$

Drift = x = 550 tan 1.72 = **16.52 ft = 200 inches.**

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(Revision: Aug-08)

Estimate the average settlement from primary consolidation of the clay stratum under the center of the bridge pier.

$$W = 28x10^3 \, kN \left(2.8x10^6 \, kg \; mass\right)$$

Footing 10 m x 8 m

2m

3m

Dry

WT

7m

10 m

Sand and Gravel

$$\gamma_b = 9.80 \, \frac{kN}{m^3} \qquad \gamma = 19.62 \, \frac{kN}{m^3}$$

$$\gamma_b = 9.80 \, \frac{kN}{m^3} \qquad C_c = 0.31$$

4m

*

Clay

Sand and Gravel

Solution:

Stress at mid-clay stratum:

$$3 \, m \; x \; 19.62 \, \frac{kN}{m^3} = 58.9 \, \frac{kN}{m^2}$$

$$7 \, m \; x \; 9.80 \, \frac{kN}{m^3} = 68.6 \, \frac{kN}{m^2} \qquad \text{for a total} \quad \rho_o = 146.7 \, \frac{kN}{m^2}$$
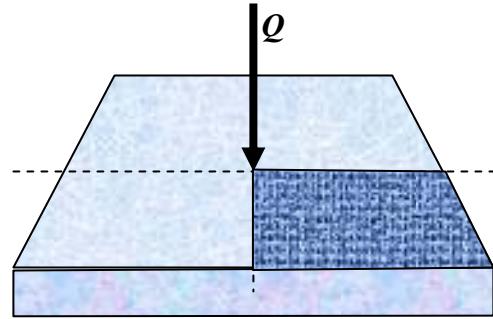
$$2 \, m \; x \; 9.60 \, \frac{kN}{m^3} = 19.2 \, \frac{kN}{m^2}$$

Determine e_o : From chart $p_c \sim 150 < p_o$ normally consolidated, $e_o = 0.81$.

Determine $\Delta\sigma$ or Δp:

$$m = \frac{4\,m}{10\,m} = 0.286 \quad and \quad n = \frac{5\,m}{10\,m} = 0.500 \quad \therefore \ I_4 = 0.071$$

$$\Delta\sigma = 4I_4 \frac{Q}{A} = 4\,(0.071)\left(\frac{28\,x10^3 \ kN}{8m \ x \ 10m}\right) = 99.5 \, \frac{kN}{m^2}$$

Determine the settlement ΔH :

$$\Delta H = H\left(\frac{C_c}{1+e_o}\right)\log_{10}\left(\frac{p_o+\Delta p}{p_o}\right) = 4m\left(\frac{0.31}{1+0.81}\right)\log_{10}\left(\frac{146.7+99.5}{146.7}\right) = 0.15 \ m$$

Chapter 12
Shear Strength of Soils

$c \rightarrow$ *The cohesion of a soil particle.*

$c_u \rightarrow$

$p \rightarrow$

$p' \rightarrow$

$q \rightarrow$

$q' \rightarrow$

$q_u \rightarrow$ *Ultimate shear strength of a soil.*

$u \rightarrow$ *Pore water pressure.*

$u_c \rightarrow$

$u_d \rightarrow$

$\sigma \rightarrow$ *The normal axial stress.*

$\sigma_1 \rightarrow$ *stress*

$\sigma_1' \rightarrow$ *stress*

$\sigma_3' \rightarrow$ *stress*

$\sigma_3 \rightarrow$ *Confirming pressure*

$\sigma_d \rightarrow$

$\tau \rightarrow$ *The shear stress.*

$\tau_f \rightarrow$ *The normal shear stress at a failure.*

$\phi \rightarrow$ *The angle of internal friction of the soil.*

$\varphi \rightarrow$ *The angle of inclination of the plane of failure caused by the failure shear stress.*

$\sigma' \rightarrow$ *Effective stress.*

(Revision: Oct-08)

A consolidated un-drained triaxial test was performed on a specimen of saturated clay with a chamber pressure $\sigma_3 = 2.0\dfrac{kg}{cm^2}$. At failure,

$\sigma_1 - \sigma_3 = 2.8\dfrac{kg}{cm^2}$, $u = 1.8\dfrac{kg}{cm^2}$ and the failure plane angle $\theta = 57°$.

Calculate (1) the normal stress σ and (2) shear stress τ on the failure surface and (3) the maximum shear stress on the specimen.

Solution:

$$\sigma_1 = 2.8 + 2.0 = 4.8 \ \frac{kg}{cm^2}$$

$$\sigma_3 = 2.0 \ \frac{kg}{cm^2}$$

Shear stress $\quad \tau = \dfrac{\sigma_1 - \sigma_3}{2}\sin 2\theta = \left(\dfrac{4.8 - 2}{2}\right)\sin 114° = 1.27 \ \dfrac{kg}{cm^2}$

Normal stress $\quad \sigma = \left(\dfrac{\sigma_1 + \sigma_3}{2}\right) + \left(\dfrac{\sigma_1 - \sigma_3}{2}\right)\cos 2\theta = \left(\dfrac{4.8 + 2}{2}\right) + \left(\dfrac{4.8 - 2}{2}\right)\cos 114° = 2.83 \ \dfrac{kg}{cm^2}$

Maximum shear $\quad \tau_{MAX} = \dfrac{\sigma_1 - \sigma_3}{2}\sin 2\theta = \left(\dfrac{4.8 - 2}{2}\right) = 1.4 \ \dfrac{kg}{cm^2} \quad$ at $\quad \theta = 45°$

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(Revised Oct-09)

Using the results of the previous Problem 01, and $\phi = 24°$, $c' = 0.80\dfrac{kg}{cm^2}$, show why the sample failed at 57 grades instead of the plane of maximum shear stress.

Solution:

On failure plane

$$\sigma' = (\sigma - u) = 2.83 - 1.8 = 1.03\frac{kg}{cm^2}$$

$$S_{57°} = c' + \sigma'\tan\phi = 0.8 + (1.03\tan 24°) = 1.27\frac{kg}{cm^2}$$

Compare that to $\tau = 1.27$ kg/cm from the previous problem, and note that they are equal, and so for both, $S_{57°} = \tau_{57°}$ failed. .

Now at the plane of maximum shear stress $\theta = 45°$

$$\sigma = \left(\frac{4.8 + 2}{2}\right) + \left(\frac{4.8 - 2}{2}\right)\cos 90° = 3.4\frac{kg}{cm^2}$$

$$\sigma' = 3.4 - 1.8 = 1.6\frac{kg}{cm^2}$$

$$S_{45°} = c' + \sigma'\tan\phi = 0.8 + (1.60\tan 24°) = 1.51\frac{kg}{cm^2}$$

The shear <u>strength</u> at 45º is much larger than at 57º, therefore failure does not occur.

Shear strength–03: Find the maximum principal stress σ_1.

(Revised Oct-09)

Continuing with the data from the two previous problems, the same soil specimen is now loaded slowly to failure in a drained test, that is $u = 0$, with $\sigma_3 = 2.0\dfrac{kg}{cm^2}$. What will be the major principal stress at failure?

Solution:
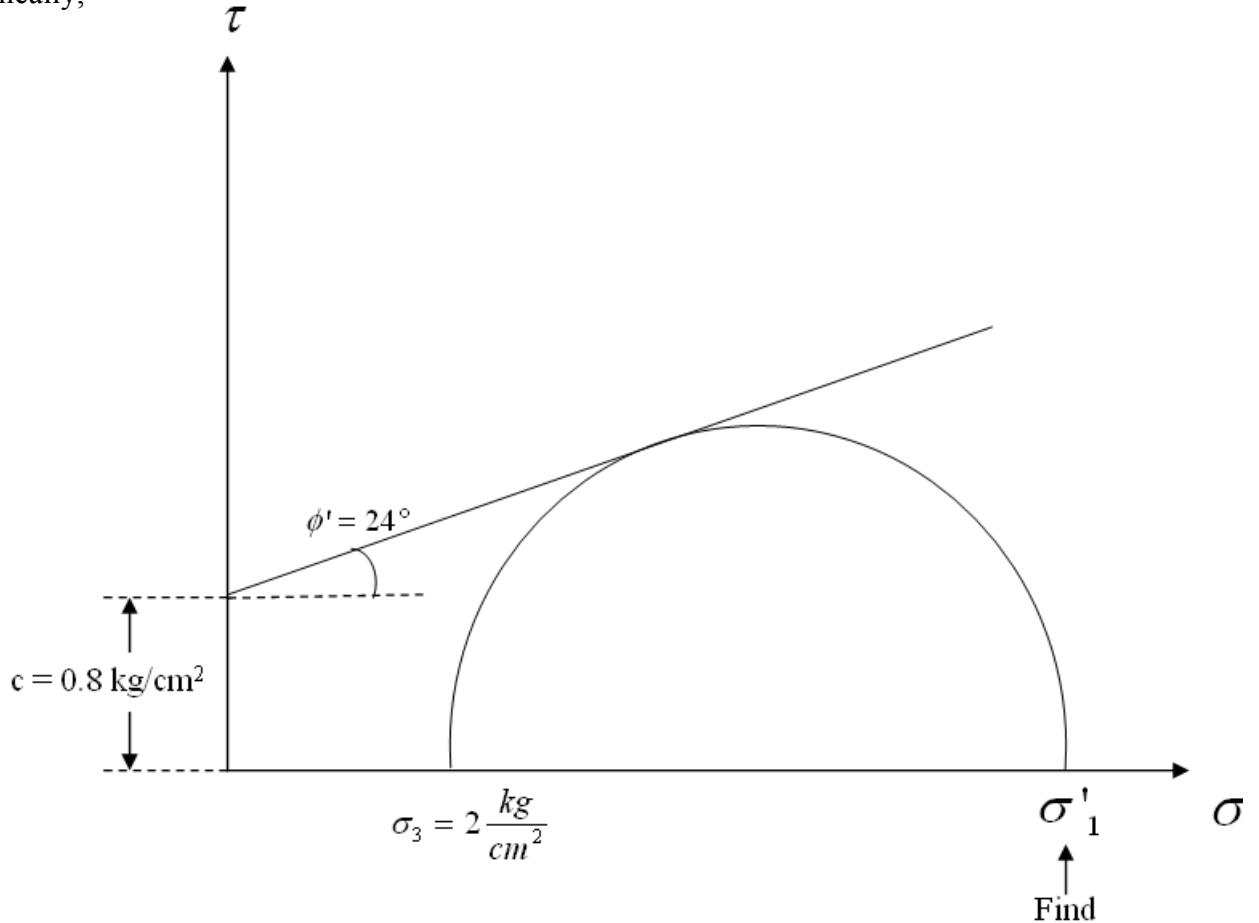
a) Analytically, in a drained test $u = 0$; at failure $\sigma'_3 = \sigma_3 = 2\dfrac{kg}{cm^2}$, on the failure plane $\theta = 57°$.

$$s = c' + \sigma' \tan\phi = 0.80 + \left[\left(\frac{\sigma'_1 + 2}{2} \right) + \left(\frac{\sigma'_1 - 2}{2} \right) \cos 114° \right] \tan 24° = (1.426) + (0.132)\sigma'_1$$

$$and \quad \tau = \left(\frac{\sigma'_1 - 2}{2} \right) \sin 114° = (0.457)\sigma'_1 - (0.914)$$

At failure $s = \tau$ $\therefore$ $\sigma'_1 = 7.31\dfrac{kg}{cm^2}$; $\sigma_\theta = 3.6\dfrac{kg}{cm^2}$ *and* $\tau_\theta = 2.38\dfrac{kg}{cm^2}$

b) Graphically,

$\phi' = 24°$

$c = 0.8$ kg/cm²

$\sigma_3 = 2\dfrac{kg}{cm^2}$

σ'_1

Find

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(Revised Oct-09)

A drained triaxial test on a normally consolidated clay showed that the failure plane makes an angle of 58° with the horizontal. If the sample was tested with a chamber confining pressure of 103.5 kN/m², what was the major principal stress at failure?

Solution:

$$\theta = 45° + \frac{\phi}{2} \qquad \therefore \qquad 58° = 45° + \frac{\phi}{2} \therefore \qquad \phi = 26°$$

Using the equation that relates the major principal stress σ_1 to the minor principal stress σ_3, and with c = 0 (the value of cohesion for a normally consolidated clay),

$$\sigma_1' = \sigma_3' \tan^2\left(45° + \frac{\phi_1}{2}\right) = (103.5)\left[\tan^2\left(45° + \frac{26}{2}\right)\right] = 265\frac{kN}{m^2}$$

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*Shear strength–05: *Using the p-q diagram.

Triaxial tests performed on samples from our Miami Pamlico formation aeolian sand, showed the peak stresses listed below. Plot these values on a *p-q* diagram to find the value of the internal angle of friction.

$\sigma_1 = $ 76 psi   for $\sigma_3 = $ 15 psi $p = $ 45.5, $q = $ 30.5 psi   (1)
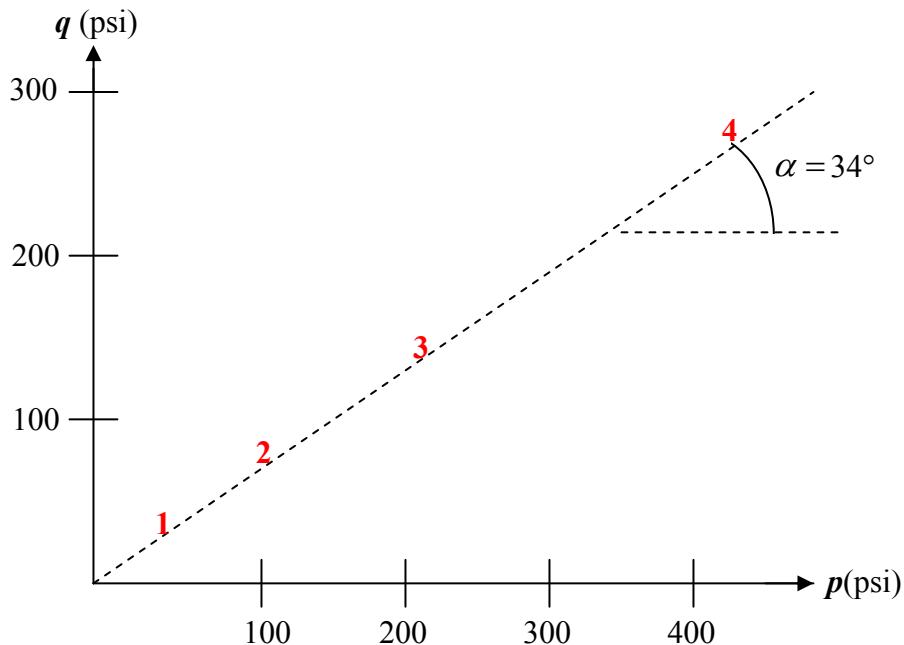
$\sigma_1 = $ 148 psi   for $\sigma_3 = $ 30 psi $p = $ 89.0, $q = $ 59.0 psi   (2)

$\sigma_1 = $ 312 psi   for $\sigma_3 = $ 60 psi $p = $ 186.0, $q = $ 126.0 psi   (3)

$\sigma_1 = $ 605 psi   for $\sigma_3 = $ 120 psi $p = $ 362.5, $q = $ 242.5 psi   (4)

Solution:

Remember that $p = (\sigma_1 + \sigma_3)/2$ and $q = (\sigma_1 - \sigma_3)/2$

From the *p-q* diagram:

$$\tan \alpha \approx \frac{q_4}{p_4} = \frac{242.5}{362.5} = 0.668$$

$$\sin \phi = \tan \alpha = 0.668 \quad \therefore \quad \phi = 42°$$

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(Revised Oct-09)

A consolidated-drained triaxial test was conducted on a normally consolidated clay. The results are as follows: $\qquad \sigma'_3 = 276 \dfrac{kN}{m^2} \qquad (\Delta\sigma_d)_f = 276 \dfrac{kN}{m^2}$

Determine:

(a) The angle of friction φ;

(b) The angle θ that the failure plane makes with the major principal plane, and

(c) The normal stress σ' and shear stress τ_f on the failure plane.

<u>Solution:</u>

For normally consolidated soil the failure envelope equation is:

$$\tau_f = \sigma' \tan\phi \quad \text{because } c = 0$$
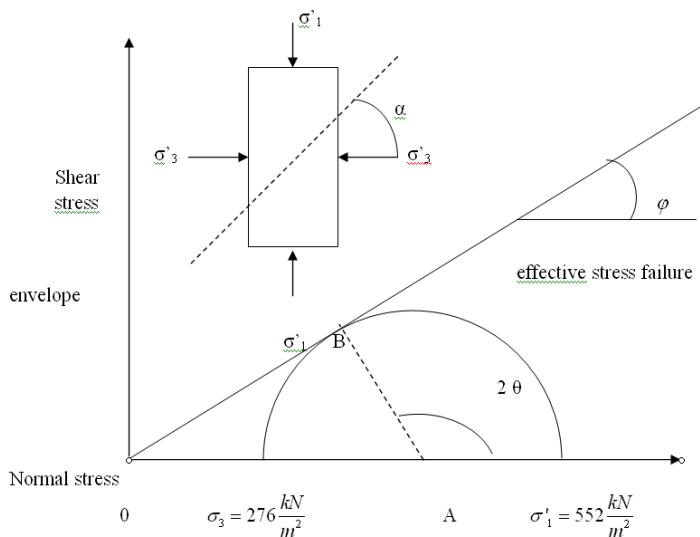
For the triaxial test, the effective major and minor principal stresses at failure are as follows:

$$\sigma'_1 = \sigma_1 = \sigma_3 + (\Delta\sigma_d)_f = 276 + 276 = 552 \frac{kN}{m^2} \quad \text{and} \quad \sigma'_3 = \sigma_3 = 276 \frac{kN}{m^2}$$

<u>Part A.</u>

The Mohr circle and the failure envelope are shown in the figure below, from which:

$$\sin\phi = \frac{AB}{OA} = \frac{\dfrac{\sigma'_1 - \sigma'_3}{2}}{\dfrac{\sigma'_1 + \sigma'_3}{2}} = \frac{\sigma'_1 - \sigma'_3}{\sigma'_1 + \sigma'_3} = \frac{552 - 276}{552 + 276} = 0.333 \quad \therefore \quad \phi = 19.45°$$

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Part B.

$$\theta = 45° + \frac{\phi}{2} = 45° + \frac{19.45°}{2} = 54.7°$$

Part C.

$$\sigma'_{(on\ the\ failure\ plane)} = \frac{\sigma'_1 + \sigma'_3}{2} + \frac{\sigma'_1 - \sigma'_3}{2}\cos 2\theta$$

and $\tau_f = \dfrac{\sigma'_1 - \sigma'_3}{2}\sin 2\theta$

Substituting the values of $\sigma'_1 = 552\dfrac{kN}{m^2}$, $\sigma'_3 = 276\dfrac{kN}{m^2}$ and $\theta = 54.7°$ in the preceding equations,
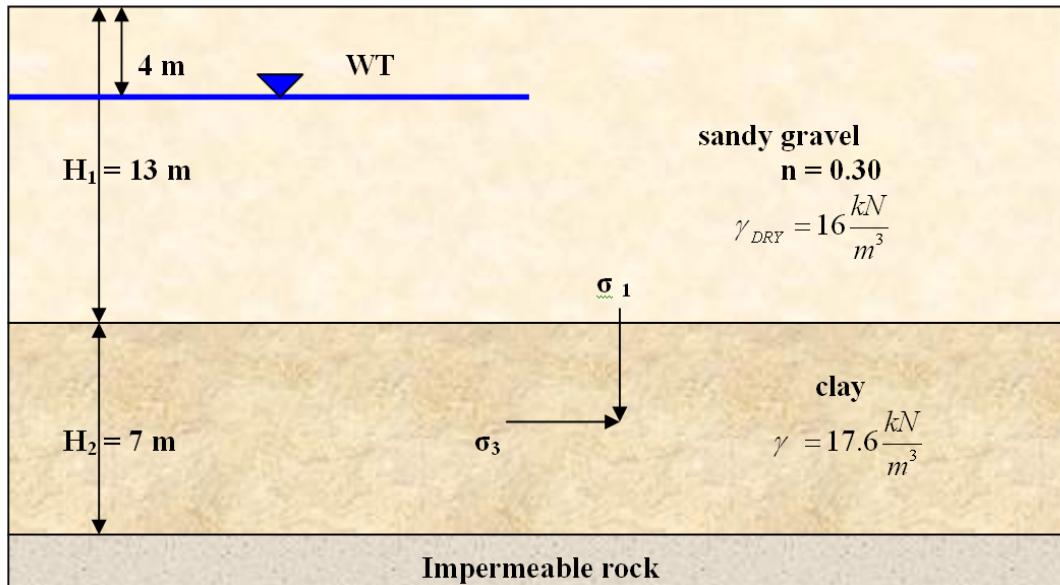
$$\sigma' = \frac{552 + 276}{2} + \frac{552 - 276}{2}\cos 2(54.7) = 368\frac{kN}{m^2}$$

$$and \quad \tau_f = \frac{552 - 276}{2}\sin 2(54.7) = 130\frac{kN}{m^2}$$

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Triaxial un-drained tests were performed on clay samples taken from the stratum shown below. The tests were taken with pore water pressure measurements, and $c' = 20 \dfrac{kN}{m^2}$, *and* $\phi = 24°$

(a) Find the clay shear strength at its mid-stratum, and

(b) Find the effective and total stresses at the same level acting on a vertical face of a soil element.

Solution:

(a) For the gravel:

$$\gamma_{sat} = \gamma_d + n\gamma_w = (16 + 0.3x10) = 19\frac{kN}{m^3}$$

$$\therefore For \quad \sigma_1 = 4m\left(16\frac{kN}{m^3}\right) + 9m\left(19\frac{kN}{m^3}\right) = 3.5\left(17.6\frac{kN}{m^3}\right) = 297\frac{kN}{m^2}$$
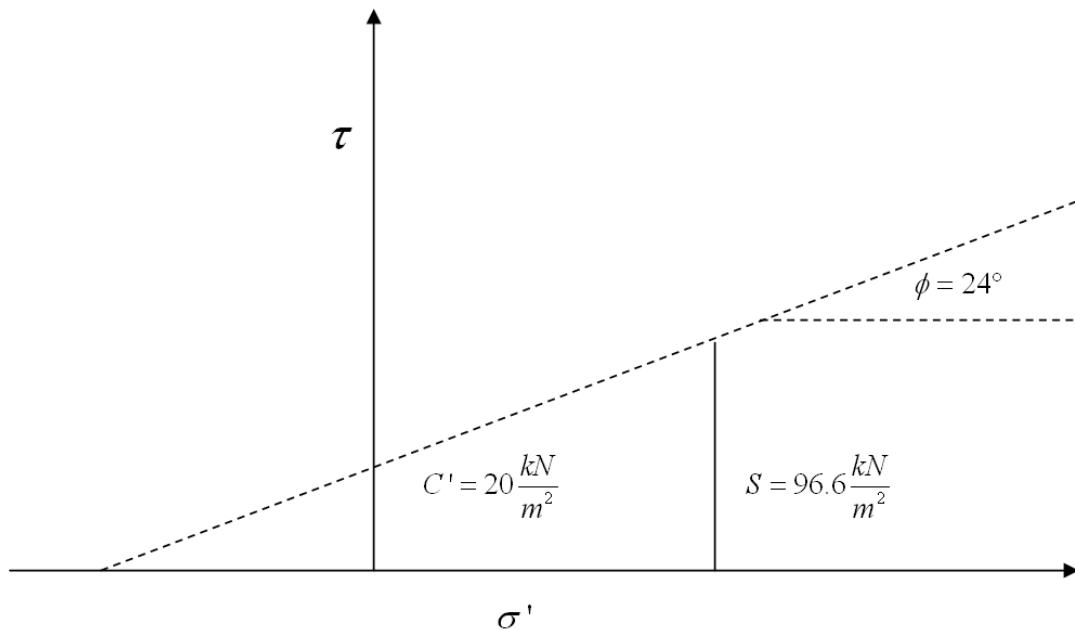
$$\therefore For \quad \sigma'_1 = \sigma_1 - u = \sigma_1 - \gamma_w(9 + 3.5) = 297 - \left(10\frac{kN}{m^3}\right)(12.5m) = 173\frac{kN}{m^2}$$

$$\therefore S = c' + \sigma'_1 \tan \phi' = 20\frac{kN}{m^2} + 173\frac{kN}{m^3}\tan 24° = 96.6\frac{kN}{m^2}$$

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(b)   Since the clay is saturated,

$$\sigma'_3 = K_o\sigma'_1 = 0.5\sigma'_1 = 0.5(173) = 86\frac{kN}{m^2}$$

$$and \quad \sigma_3 = \sigma'_3 + u = 86\frac{kN}{m^2} + (13m + 3.5m)10\frac{kN}{m^3} = 251\frac{kN}{m^2}$$

τ

$\phi = 24°$

$C' = 20\frac{kN}{m^2}$

$S = 96.6\frac{kN}{m^2}$

σ'

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Two similar clay soil samples were pre-consolidated in triaxial equipment with a chamber pressure of 600 kN/m². Consolidated-drained triaxial tests were conducted on these two specimens. The following are the results of the tests:

Specimen 1:
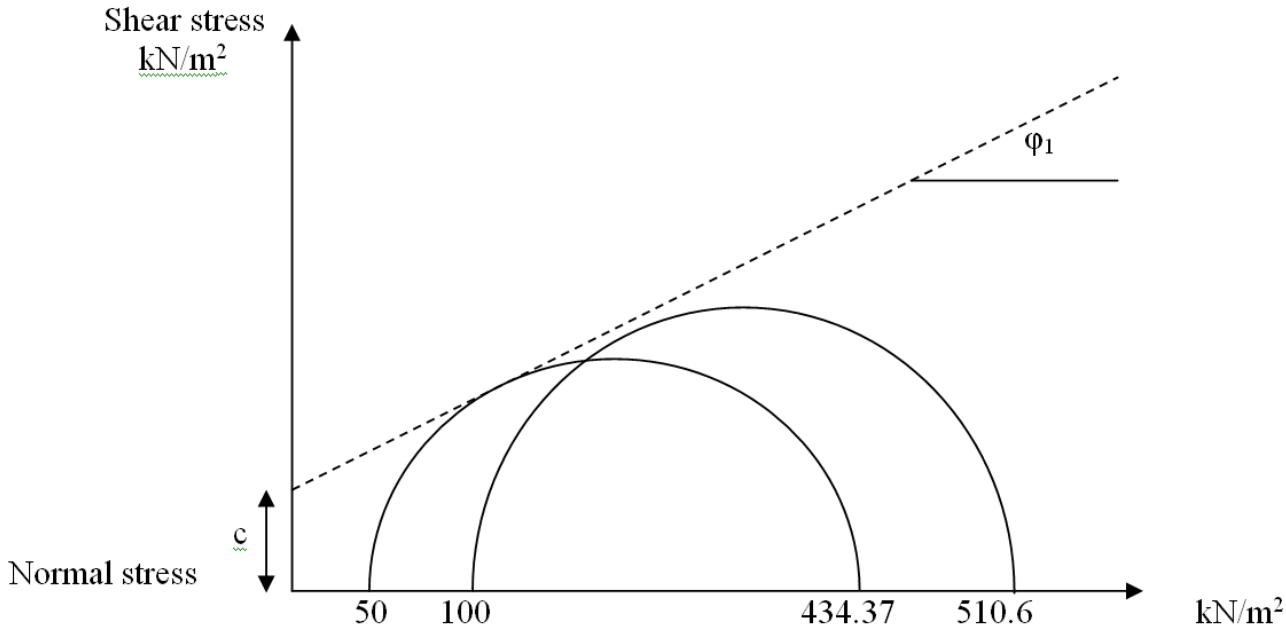
$$\sigma_3 = 100 \frac{kN}{m^2}$$

$$(\Delta \sigma_d)_f = 410.6 \frac{kN}{m^2}$$

Specimen 2:

$$\sigma_3 = 50 \frac{kN}{m^2}$$

$$(\Delta \sigma_d)_f = 384.37 \frac{kN}{m^2}$$

Find the values of the cohesion c and the angle of internal friction φ.

Solution:

For Specimen 1, the principal stresses at failure are,

$$\sigma_3' = \sigma_3 = 100 \frac{kN}{m^2} \quad and \quad \sigma_1' = \sigma_1 = \sigma_3 + (\Delta \sigma_d)_f = 100 + 410.6 = 510.6 \frac{kN}{m^2}$$

Similarly, the principal stresses at failure for specimen 2 are

$$\sigma_3' = \sigma_3 = 50 \frac{kN}{m^2} \quad and \quad \sigma_1' = \sigma_1 = \sigma_3 + (\Delta \sigma_d)_f = 50 + 384.4 = 434.4 \frac{kN}{m^2}$$

These two samples are over-consolidated. Using the relationship given by equation

$$\sigma_1' = \sigma_3' \tan^2\left(45° + \frac{\phi_1}{2}\right) + 2c\tan\left(45° + \frac{\phi_1}{2}\right)$$

Thus, for specimen 1

$$(510.6) = (100)\tan^2\left(45° + \frac{\phi_1}{2}\right) + 2c\tan\left(45° + \frac{\phi_1}{2}\right)$$

and for specimen 2

$$(434.4) = (50)\tan^2\left(45° + \frac{\phi_1}{2}\right) + 2c\tan\left(45° + \frac{\phi_1}{2}\right)$$

Subtracting both equations $(76.2) = (50)\tan^2\left(45° + \frac{\phi_1}{2}\right)$ $\quad \therefore \quad \phi_1 = 12°$

Substituting ϕ_1 into the equation,
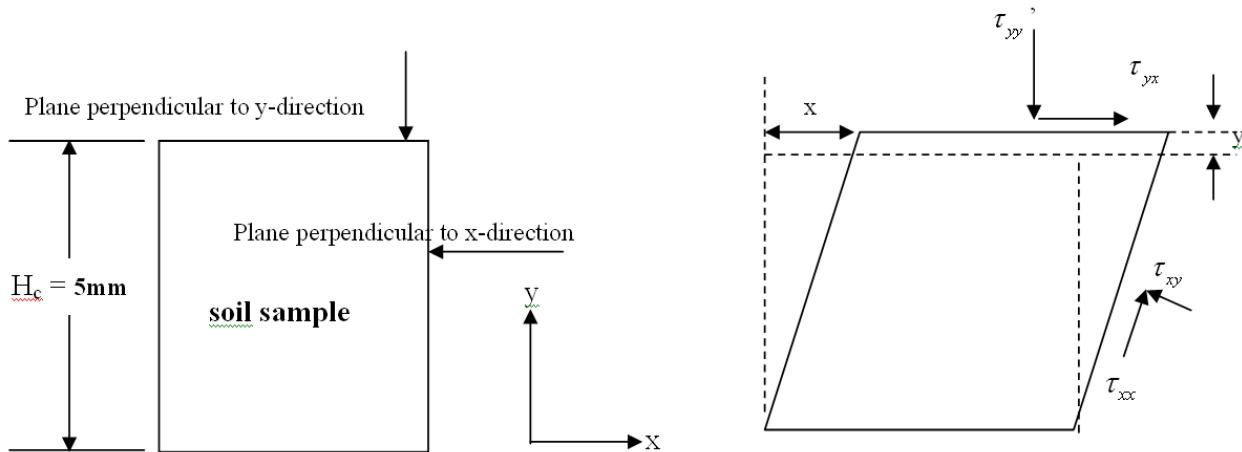
$$(510.6) = (100)\tan^2\left[45° + \left(\frac{12}{2}\right)\right] + 2c\tan\left[45° + \left(\frac{12}{2}\right)\right]$$

$$510.6 = 152.5 + 2.47c \quad \therefore \quad c = 145\frac{kN}{m^2}$$

***Shear strength-09:** Plots of the progressive failure in a shear-box.*

(Revised Oct-09)

A soil test is performed in the shear-box shown below. The test data lists the stresses and displacements. Assign positive normal stresses to compression and positive shear stresses are counter-clockwise. Plot the Mohr circles of stress for each stage.
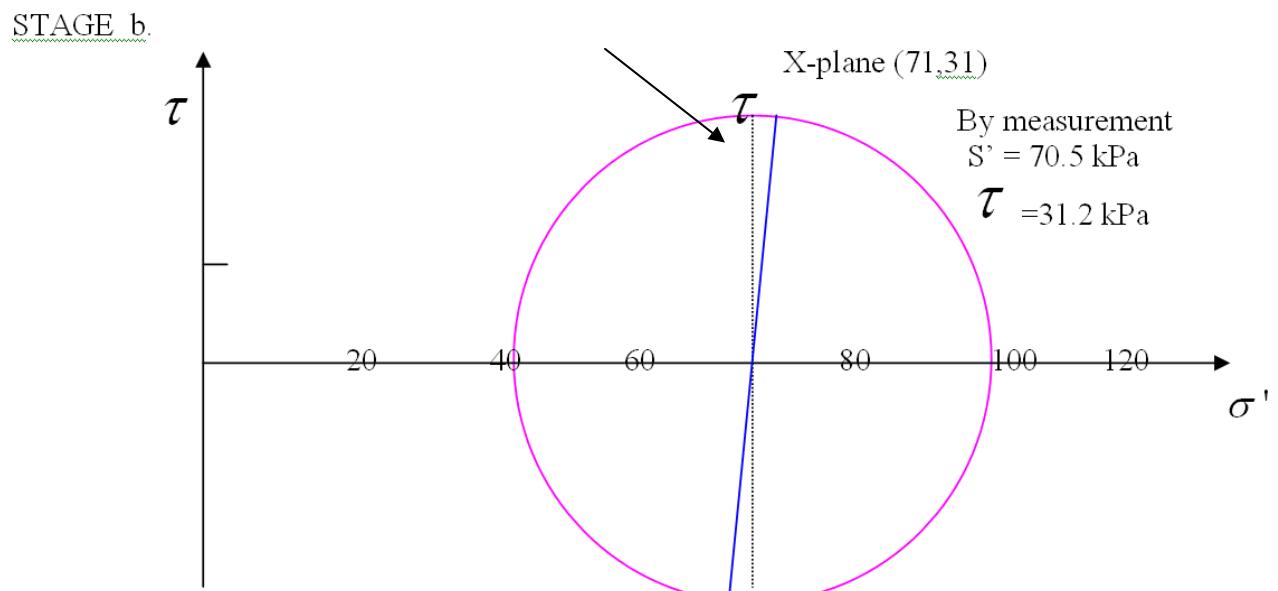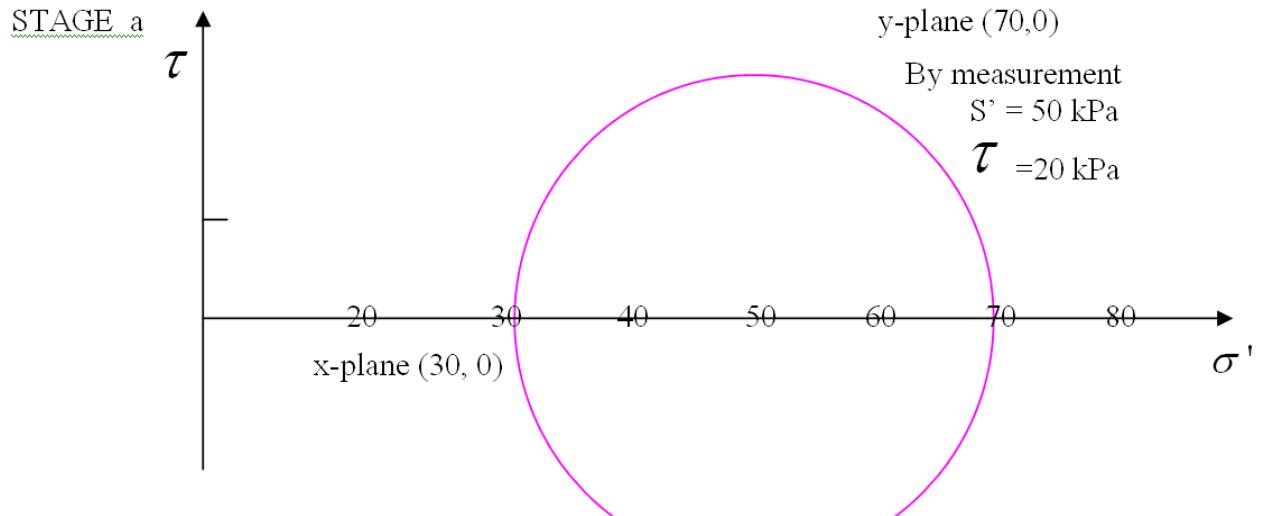
Stage	DISPLACEMENTS(mm)		σ'_{XX} (kpa)	σ'_{YY} (kpa)	τ_{xy} (kpa)	τ_{yx} (kpa)
	x	y				
a	0	0	30	70	0	0
b	0.30	-0.50	71	70	31.0	-31.3
c (peak)	2.50	-0.60	145	70	43.3	=49.0
d	3.00	-0.82	-	-	-	-
e	10.00	1.50	90.6	70	24.5	-32.0
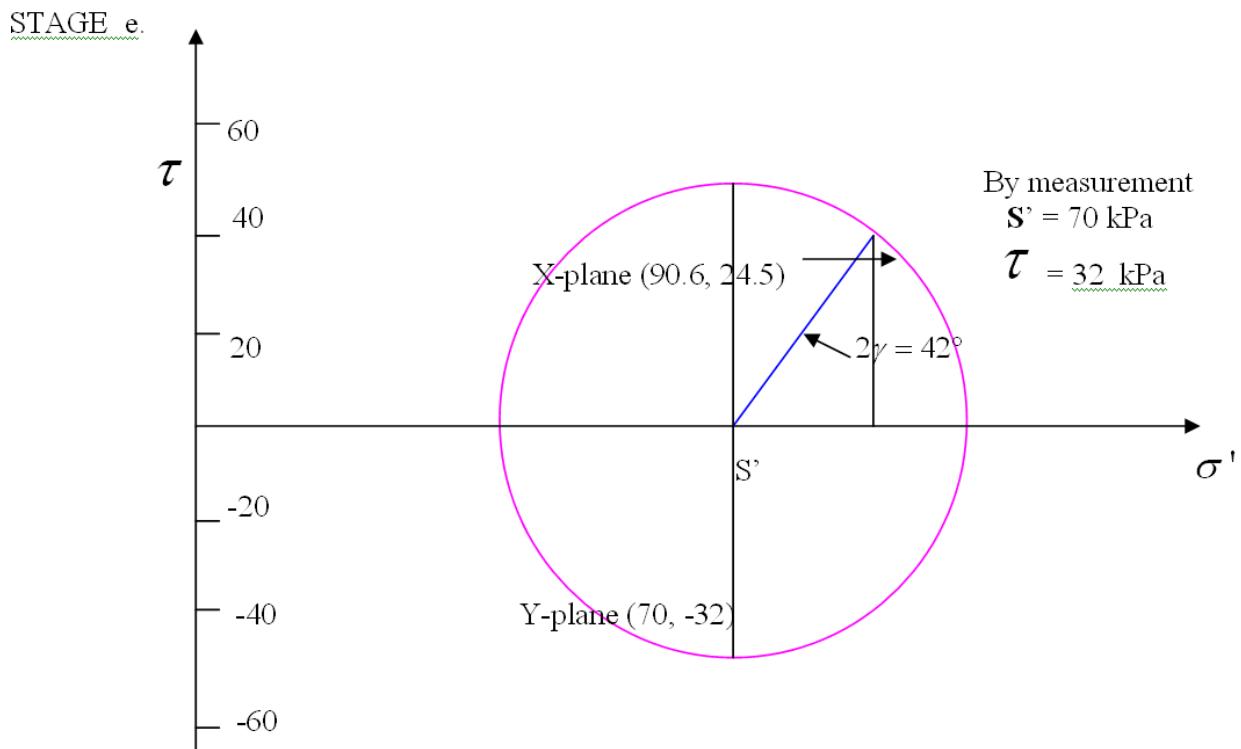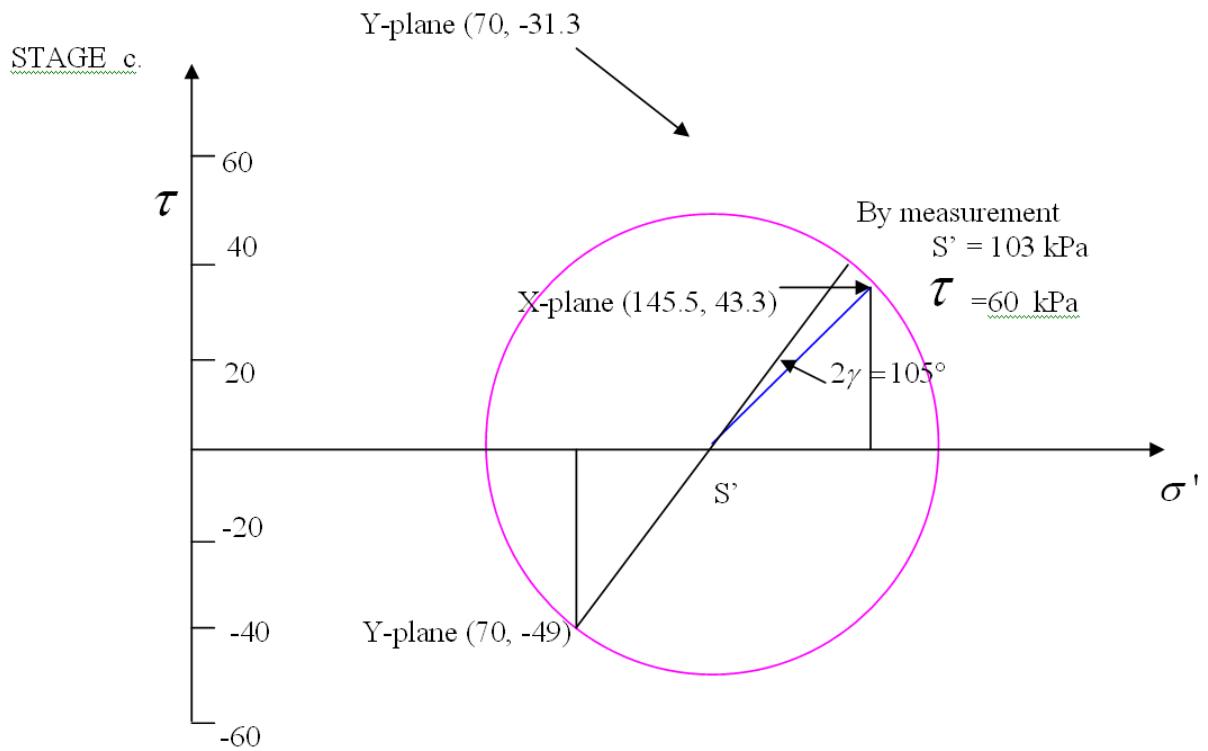
Solution:

For small displacements, the x and y planes remain perpendicular. Use a compass to locate by trial and error the center of the Mohr circle. The center of the circle must lie on the σ' axis, and it must be equidistant from the two stress points $(\sigma'_{xx}, \tau_{xy})$ and $(\sigma'_{yy}, \tau_{yx})$.

TABLE OF VALUES

STAGE	$S' = \dfrac{\sigma_1' + \sigma_3'}{2}$ kPa	$\tau = \dfrac{\sigma_1' - \sigma_3'}{2}$ kPa	$\dfrac{\tau}{S'}$	Change in angle between x + y plan (in degrees)
a	50	20	0.40	0°
b	70.5	31.2	0.44	0°
c (peak)	103	60	0.58	5.25°
e	70	32	0.46	21°

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STAGE a

τ

y-plane (70,0)

By measurement
S' = 50 kPa

τ =20 kPa

20   30   40   50   60   70   80 σ'

x-plane (30, 0)

STAGE b.

τ

X-plane (71,31)

By measurement
S' = 70.5 kPa

τ =31.2 kPa

τ

20   40   60   80   100   120 σ'

225

STAGE c.

Y-plane (70, -31.3

τ

60

40

By measurement
S' = 103 kPa
τ =60 kPa

X-plane (145.5, 43.3)

20

$2\gamma = 105°$

S'

σ'

-20

-40   Y-plane (70, -49)

-60

STAGE e.

τ

60

40

By measurement
S' = 70 kPa
τ = 32 kPa

X-plane (90.6, 24.5)

20

$2\gamma = 42°$

S'

σ'

-20

-40   Y-plane (70, -32)

-60

226
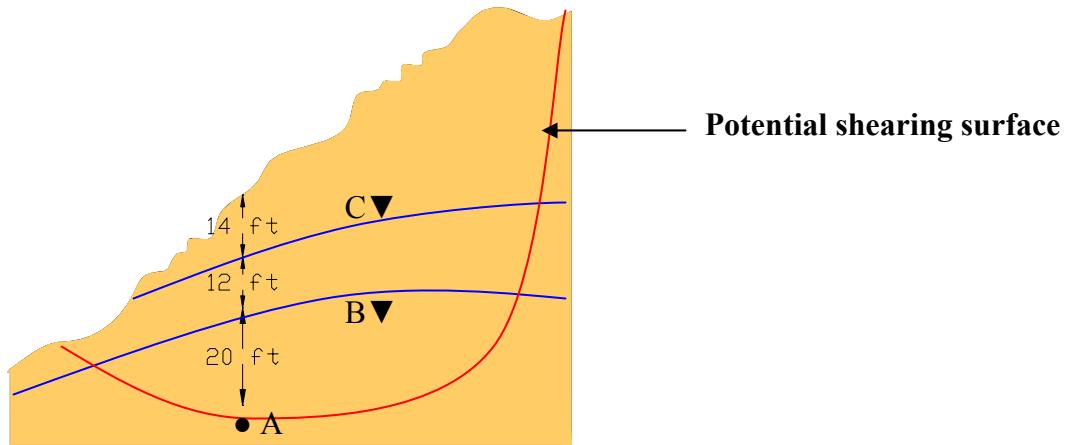
(Revision Oct-09)

An engineer is evaluating the stability of the slope in the figure below, and considers that the potential for a shear failure occurs along the shear surface shown. The soil has an angle $\phi' = 30°$ and no cohesive strength. Compute the shear strength at point A along this surface when the groundwater table is at level B, then compute the new shear strength if it rose to level C. The unit weight of the soil is 120 lb/ft^3 above the WT and 123 lb/ft^3 below.

Potential shearing surface

C▼

14 ft

12 ft

B▼

20 ft

• A

Solution:

When the groundwater table is at B:

$$u = \gamma_z z_w = \left(62.4 \frac{lb}{ft^3}\right)(20 \ ft) = 1248 \ \frac{lb}{ft^2}$$

$$\sigma'_z = \sum \gamma H - u = \left(120 \frac{lb}{ft^3}\right)(26 \ ft) + \left(123 \frac{lb}{ft^3}\right)(20 ft) - 1248 \frac{lb}{ft^2} = 4332 \ \frac{lb}{ft^2}$$

The potential shear surface is horizontal, so $\sigma' = \sigma'_z$

$$s = \sigma' \ \tan \phi' = \left(4332 \ \frac{lb}{ft^2}\right) \tan \ 30° = 2501 \ \frac{lb}{ft^2}$$

When the groundwater table is at C:

$$u = \gamma_z z_w = \left(62.4 \frac{lb}{ft^3}\right)(32 \ ft) = 1997 \ \frac{lb}{ft^2}$$
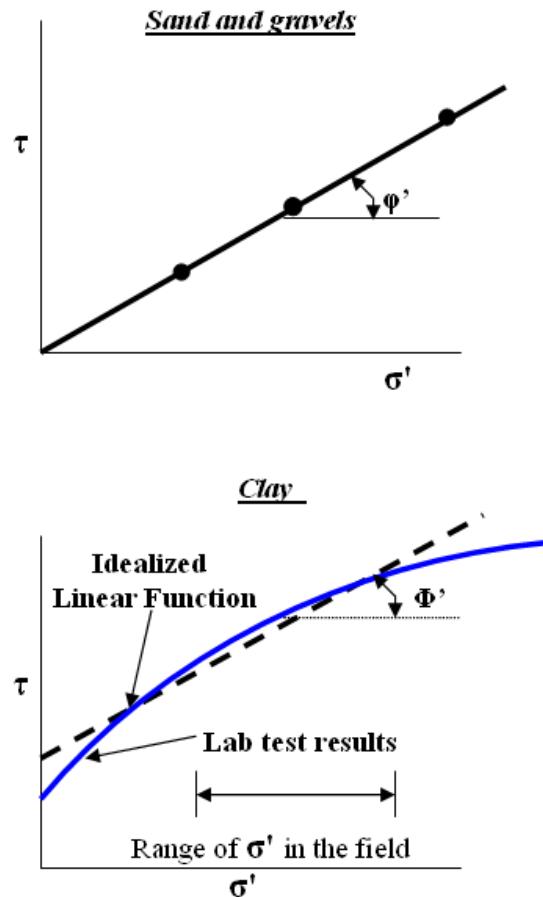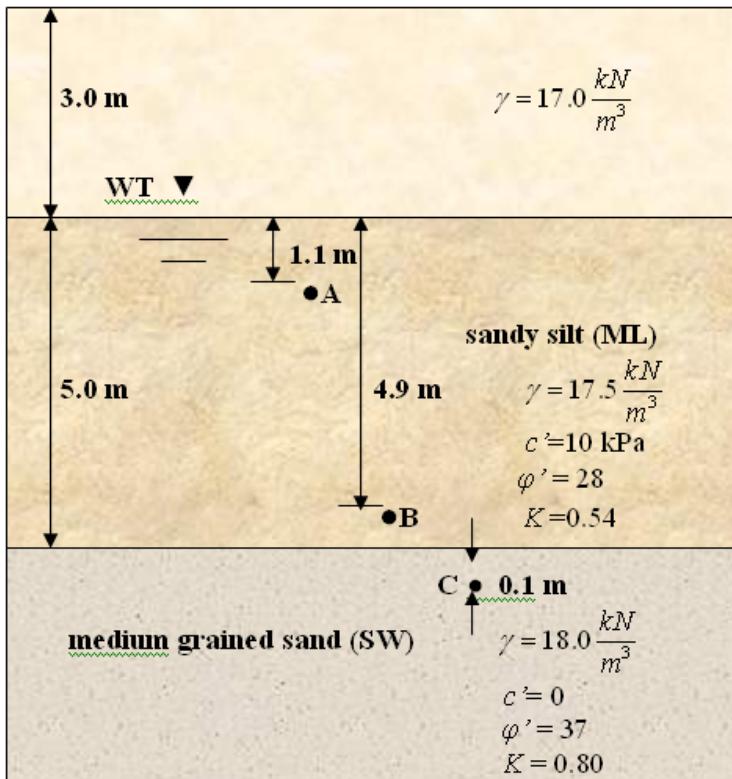
$$\sigma'_z = \sum \gamma H - u = \left(120 \frac{lb}{ft^3}\right)(14 \ ft) + \left(123 \frac{lb}{ft^3}\right)(32 ft) - 1997 \frac{lb}{ft^2} = 3619 \ \frac{lb}{ft^2}$$

$$s = \sigma' \ \tan \phi' = \left(3619 \ \frac{lb}{ft^2}\right) \tan \ 30° = 2089 \ \frac{lb}{ft^2}$$

***Shear strength-11: Use of the Mohr-Coulomb failure envelope.

(Revised Oct-09)

Samples have been obtained from both soil strata shown in the figure below. A series of shear strength tests were then performed on both samples and plotted in diagrams below. The *c'* and *φ'* values obtained from these diagrams are shown in the figure below. Using this data, compute the shear strength on the horizontal and vertical planes at points *A*, *B*, and *C*.

Sand and gravels

Clay

Solution:

Point A - Horizontal plane:

$$\sigma' = \sum \gamma H - u = (17.0 \frac{kN}{m^3})(3.0m) + (17.5 \frac{kN}{m^3})(1.1m) - (9.8 \frac{kN}{m^3})(1.1m)$$

$$\sigma'_z = 59.8 \ kPa$$

$$s = c' + \sigma' \tan \phi' = 10 kPa + (59.5 kPa) \tan 28° = 41.6 \ kPa$$

Point A - Vertical plane:

$$\sigma'_z = K\sigma'_z = (0.54)(59.5 \ kPa) = 32.1 \ kPa$$

$$s = c' + \sigma' \tan \phi' = 10 \ kPa + (32.1 \ kPa) \tan 28° = 27.1 \ kPa$$

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Using similar computations:

Point **B** vertical plane $s = 57.2$ kPa

Point **B** horizontal plane $s = 35.5$ kPa

Point **C** vertical plane $s = 68.1$ kPa

Point **C** horizontal plane $s = 54.4$ kPa

Commentary

At each point the shear strength on a vertical plane is less than that on a horizontal plane because

$K < 1$. In addition, the shear strength at point B is greater than that at point A, because the effective strength is greater. The strength at point C is even higher than at point B because it is in a new strata with different c', φ', and K values. Thus, the strength would increase gradually with depth within each stratum, but change suddenly at the boundary between the two strata.

Draw the shear strength envelope for the ML stratum and then plot the upper half of the Mohr circle for point A on this diagram. Assume the principal stresses act vertically and horizontally.

Failure envelope and Mohr's circle

(Revised Oct-09)

Samples have been obtained from both soil strata shown in the figure below. A series of shear strength tests were then performed on both samples and plotted in diagrams below. The *c'* and *φ'* values obtained from these diagrams are shown in the figure below. Using this data, compute the shear strength on the horizontal and vertical planes at points *A*, *B*, and *C*.

Solution:

Point A - Horizontal plane:

$$\sigma^{'} = \sum \gamma H - u = (17.0\frac{kN}{m^3})(3.0m) + (17.5\frac{kN}{m^3})(1.1m) - (9.8\frac{kN}{m^3})(1.1m)$$

$\sigma'_z = 59.8 \ kPa$

$s = c' + \sigma' \tan \phi' = 10kPa + (59.5kPa)\tan 28° = 41.6 \ kPa$

Point A - Vertical plane:

$\sigma'_z = K\sigma'_z = (0.54)(59.5 \ kPa) = 32.1 \ kPa$

$s = c' + \sigma' \tan \phi' = 10 \ kPa + (32.1 \ kPa)\tan 28° = 27.1 \ kPa$

Using similar computations:

        Point **B** vertical plane $s = 57.2$ kPa

        Point **B** horizontal plane $s = 35.5$ kPa

        Point **C** vertical plane $s = 68.1$ kPa

        Point **C** horizontal plane $s = 54.4$ kPa

Commentary

At each point the shear strength on a vertical plane is less than that on a horizontal plane because

K < 1. In addition, the shear strength at point B is greater than that at point A, because the effective strength is greater. The strength at point C is even higher than at point B because it is in a new strata with different ***c', φ'***, and **K** values. Thus, the strength would increase gradually with depth within each stratum, but change suddenly at the boundary between the two strata.

Draw the shear strength envelope for the ML stratum and then plot the upper half of the Mohr circle for point A on this diagram. Assume the principal stresses act vertically and horizontally.

Failure envelope and Mohr's circle

Shear strength-12: *Triaxial un-drained tests.*

(Revised Oct-09)

Triaxial un-drained tests were performed on clay samples taken from the stratum shown below. The test were taken with pure water pressure measurements and yield a c` = 20 $^{kN}/_m{}^3$, and φ= 24°.Find (1) the clay shear strength at mid-stratum, and (2) the effective and total stresses at that same level acting on a vertical face of a small element.

Impermeable rock stratum

Solution:

For the gravel: $\gamma_{sat} = \gamma_{sat} + n\gamma_w = [16 + (0.3)10] = \mathbf{19}\,^{kN}/_m{}^3$

For the clay: $\sigma_1 = (4m)(16\,^{kN}/_m{}^3) + (9m)(19\,^{kN}/_m{}^3) + (3.5m)(17.6\,^{kN}/_m{}^3) = \mathbf{297}\,^{kN}/_m{}^3$

$\sigma_1{}' = \sigma_1 - u = \sigma_1 - \gamma_w(9 + 3.5) = 297 - 10\,^{kN}/_m{}^3\,[12.5m] = \mathbf{172}\,^{kN}/_m{}^3$

$S = c' + \sigma_1{}' \tan\phi = (20\,^{KN}/_m{}^3 + 172\,^{kN}/_m{}^3 \tan 24°) = \mathbf{96.6}\,^{NM}/_m{}^3$

Since the clay is saturated, $\sigma_3{}' = k\,\sigma_1{}' = 0.5(172) = \mathbf{86}\,^{kN}/_m{}^3$

$\sigma_3 = \sigma_3{}' + u = 86\,^{kN}/_m{}^3 + (86m + 86m)\,10\,^{kN}/_m{}^3 = \mathbf{251}\,^{kN}/_m{}^3$

(Revised Oct-09)

Triaxial un-drained tests were performed on clay samples taken from the stratum shown below. The test were taken with pure water pressure measurements and yield a c` $= 20$ $^{kN}/_m{}^3$, and $\varphi= 24°$.Find (1) the clay shear strength at mid-stratum, and (2) the effective and total stresses at that same level acting on a vertical face of a small element.

___Solution:___

For the gravel: $\gamma_{sat} = \gamma_{sat} + n\gamma_w = [16 + (0.3)10] = \mathbf{19}\,^{kN}/_m{}^3$

For the clay: $\sigma_1 = (4m)(16\,^{kN}/_m{}^3) + (9m)(19\,^{kN}/_m{}^3) + (3.5m)(17.6\,^{kN}/_m{}^3) = \mathbf{297}\,^{kN}/_m{}^3$

$\sigma_1{}' = \sigma_1 - u = \sigma_1 - \gamma_w\,(9 + 3.5) = 297 - 10\,^{kN}/_m{}^3\,[12.5m] = \mathbf{172}\,^{kN}/_m{}^3$

$S = c' + \sigma_1{}'\,\tan\phi = (20\,^{KN}/_m{}^3 + 172\,^{kN}/_m{}^3\,\tan 24°) = \mathbf{96.6}\,^{NM}/_m{}^3$

Since the clay is saturated, $\sigma_3{}' = k\,\sigma_1{}' = 0.5(172) = \mathbf{86}\,^{kN}/_m{}^3$

$\sigma_3 = \sigma_3{}' + u = 86\,^{kN}/_m{}^3 + (86m + 86m)\,10\,^{kN}/_m{}^3 = \mathbf{251}\,^{kN}/_m{}^3$

(Revised Oct-09)

A clay layer, 20 feet thick is covered by a 40 foot sandy gravel stratum with a porosity of 30%, and a dry unit weight of 103 pcf. Tests on the un-drained samples of the clay gave c = 2.9 psi, γ_{SAT} = 112 psf and φ' = 24°. Find:

(a) the soil shear strength $s = c + \sigma' \tan\varphi'$ at the clay's midlevel (point A), and

(b) the effective and total stress acting on the **vertical** face of a soil element at the clay midlevel (point A).

Solution:

(a) In order to find s, it is required to know the γ_{sat} of the sand.

$$e = \frac{n}{1-n} = \frac{0.30}{1-0.30} = \frac{0.30}{0.70} = 0.429 \quad \text{and} \quad G_S = \frac{\gamma_d(1+e)}{\gamma_W} = \frac{103(1+0.429)}{62.4} = 2.36$$

$$\gamma_{sat} = \frac{(G_S+e)\gamma_W}{1+e} = \frac{(2.36+0.429)62.4}{1.429} = 122pcf$$

*Assume that the clay was **normally consolidated** to find σ' at midlevel in the clay (point A), that is $c = 0$.

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$$\sigma'_A = \gamma h_S + \gamma' h_S + \gamma' h_C = (0.103)(13) + \left[(0.122 - 0.0624)(27)\right] + \left[(0.122 - 0.0624)(10)\right] = 3.4ksf$$

and

$$s = c + \sigma'_A \tan \phi'$$

$$s = 2.9 \frac{lb}{in^2} \times \frac{kip}{10^3 lb} \times \frac{144 in^2}{ft^2} + 3.44 \frac{k}{ft^2} (\tan 24^o) = 0.42ksf + 1.53ksfs = 1.95ksf$$

The pore water pressure u is,

$$u = (10 + 27)(0.0624) = 2.3ksf$$

Therefore, the toatl stress is,

$$\sigma_A = \sigma'_A + u = 3.4 + 2.3 = 5.7ksf$$

(b) To find the stress on the vertical face of the soil element at A, we find θ through a graphical solution as follows,

$$\theta = 45^o + \frac{\phi}{2} = 45^o + \frac{24^o}{2} = 57^o \_ 2\theta = 114^o$$

$$x_1 = \frac{c}{\tan \phi'} = \frac{0.42}{\tan 24^o} = 0.939ksf$$

$$x_2 = \frac{s}{\tan \phi'} = \frac{1.95}{\tan 24^o} = 4.38ksf$$

$$x_3 = s(\tan 24^o) = 1.95(\tan 24^o) = 0.868ksf$$

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$$R = \frac{s}{\cos \phi'} = \frac{1.95}{\cos 24^o} = 2.13ksf$$

$\sigma N_3 = x_2 + x_3 - x_1 - R = 4.38 + 0.868 - 0.939 - 2.13 = \underline{\textbf{2.18 ksf}}$

$\sigma N_1 = 2.18 + 2R = 2.18 + 2(2.13) = \underline{\textbf{6.44 ksf}}$

$\sigma_3 = \sigma N_3 + u = 2.18 + 2.31 = \underline{\textbf{4.5 ksf}}$

(Revised Oct-09)

A clay layer, 20 feet thick is covered by a 40 foot sandy gravel stratum with a porosity of 30%, and a dry unit weight of 103 pcf. Tests on the un-drained samples of the clay gave $c = 2.9$ psi, $\gamma_{SAT} = 112$ psf and $\varphi' = 24^\circ$. Find (1) the soil shear strength $s = c + \sigma' \tan\varphi'$ at the clay's midlevel (point A), and (2) the effective and total stress acting on the **vertical** face of a soil element at the clay midlevel (point A).

D=13'

H=40'   Sand

$\gamma_{d}= 103$ pct

n= 30%

• A

Clay

$\gamma_{sat}= 103$ pct

H= 20'

c = 2.9 psi

$\varphi' = 24^\circ$

Rock

Solution:

In order to find s, it is required to know the γ_{sat} of the sand.

$$e = \frac{n}{1-n} = \frac{0.30}{1-0.30} = \frac{0.30}{0.70} = 0.429 \quad \text{and} \quad G_S = \frac{\gamma_d(1+e)}{\gamma_W} = \frac{103(1+0.429)}{62.4} = 2.36$$

$$\gamma_{sat} = \frac{(G_S + e)\gamma_W}{1+e} = \frac{(2.36 + 0.429)62.4}{1.429} = 122pcf$$

* Assume that the clay was <u>normally consolidated</u> to find σ' at midlevel in the clay (point A), that is $c = 0$.

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$$\sigma'_A = \gamma h_S + \gamma' h_S + \gamma' h_C = (0.103)(13) + \left[(0.122 - 0.0624)(27)\right] + \left[(0.122 - 0.0624)(10)\right] = 3.4 ksf$$

and

$$s = c + \sigma'_A \tan \phi'$$

$$s = 2.9 \frac{lb}{in^2} \times \frac{kip}{10^3 lb} \times \frac{144 in^2}{ft^2} + 3.44 \frac{k}{ft^2}(\tan 24^\circ) = 0.42 ksf + 1.53 ksfs = 1.95 ksf$$
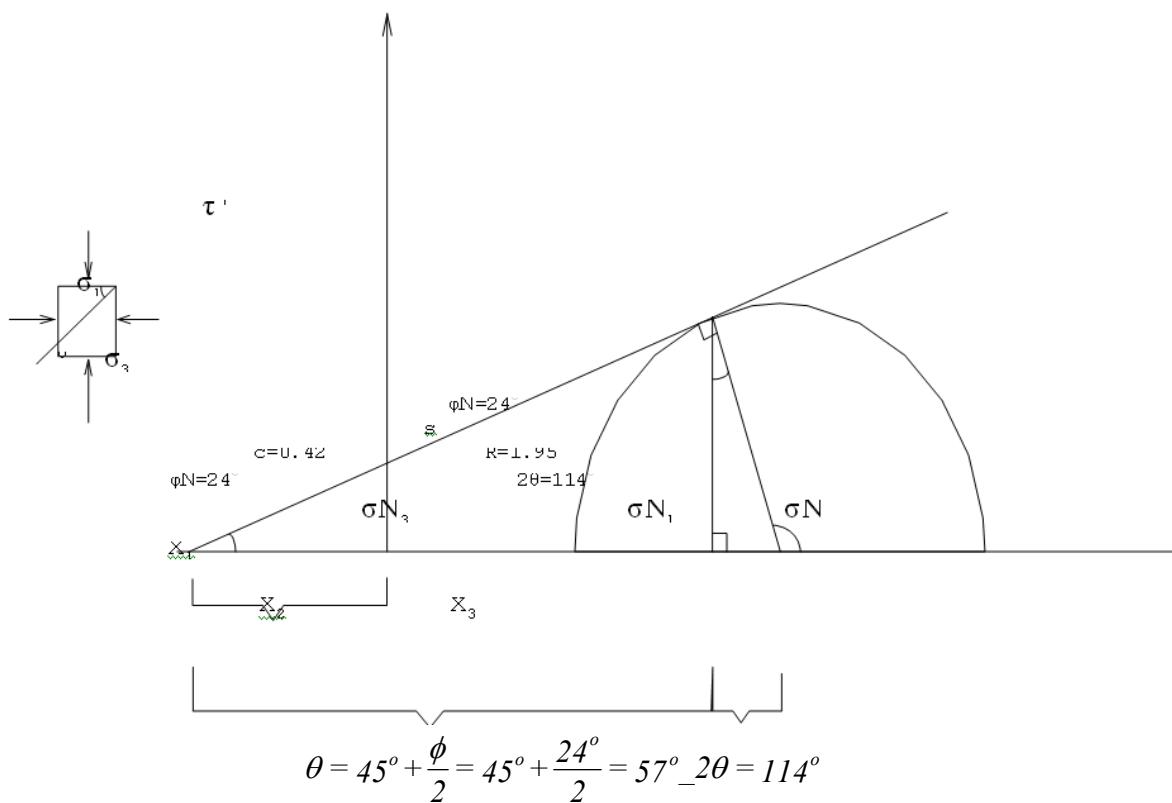
The pore water pressure u is,

$$u = (10 + 27)(0.0624) = 2.3 ksf$$

Therefore, the toatl stress is,

$$\sigma_A = \sigma'_A + u = 3.4 + 2.3 = \textcolor{red}{5.7 ksf}$$

To find the stress on the vertical face of the soil element at A, we find θ through a graphical solution as follows,

$$\theta = 45^\circ + \frac{\phi}{2} = 45^\circ + \frac{24^\circ}{2} = 57^\circ \_ 2\theta = 114^\circ$$

$$x_1 = \frac{c}{\tan \phi'} = \frac{0.42}{\tan 24^\circ} = 0.939 ksf$$

$$x_2 = \frac{s}{\tan \phi'} = \frac{1.95}{\tan 24^\circ} = 4.38 ksf$$

$$x_3 = s(\tan 24^\circ) = 1.95(\tan 24^\circ) = 0.868 ksf$$

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$$R = \frac{s}{\cos \phi'} = \frac{1.95}{\cos 24°} = 2.13 ksf$$

$\sigma N_3 = x_2 + x_3 - x_1 - R = 4.38 + 0.868 - 0.939 - 2.13 = \underline{\textbf{2.18 ksf}}$
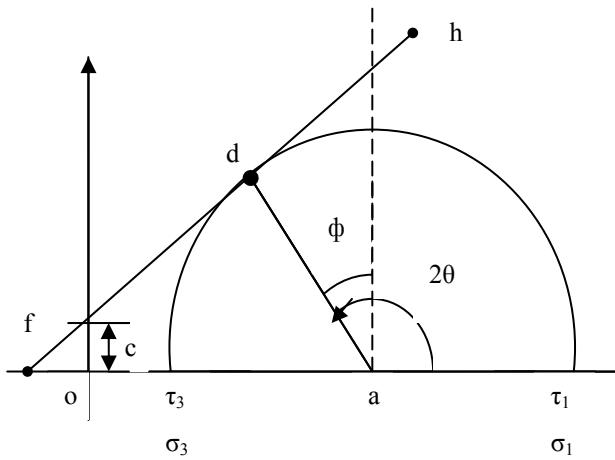
$\sigma N_1 = 2.18 + 2R = 2.18 + 2(2.13) = \underline{\textbf{6.44 ksf}}$

$\sigma_3 = \sigma N_3 + u = 2.18 + 2.31 = \underline{\textbf{4.5 ksf}}$

(Revised Oct-09)
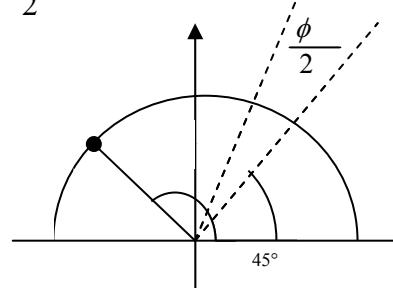
Derive the general formula that gives the value of the major principal stress σ1 as a function of the minor principal stress σ3, the cohesion c and the angle of internal friction φ.

Solution:

$$\theta = 45° + \frac{\phi}{2} \quad \longrightarrow \quad 2\,\theta = 90 + \varphi$$

$$\overline{ad} = \frac{\sigma_1 - \sigma_3}{2}$$

From the figure,

$$\sin \varphi = \frac{\overline{ad}}{\overline{fa}}$$

(1)

$$\overline{fa} = \overline{fo} + \overline{oa} = (c) \cot \varphi + \frac{\sigma_1 + \sigma_3}{2}$$

$$\tan \varphi = \frac{c}{fo} \quad \longrightarrow \quad \frac{fo}{c} = \cot \varphi \quad \longrightarrow \quad \overline{fo} = (c)(\cot \varphi)$$

Using the properties of the Mohr circle,

$$\overline{oa} = \sigma_3 + \frac{(\sigma_1 - \sigma_3)}{2} \quad \longrightarrow \quad \overline{oa} = \frac{(\sigma_1 + \sigma_3)}{2} \quad \longrightarrow \quad \overline{fa} = (c) \cot \varphi + \frac{(\sigma_1 + \sigma_3)}{2}$$

(2)

Introducing (1) into (2): $\sin \varphi = \dfrac{\dfrac{(\sigma_1 - \sigma_3)}{2}}{(c)\cot \phi + \dfrac{(\sigma_1 + \sigma_3)}{2}} \quad \longrightarrow \quad (\sin \varphi)\left[(c) \cot \varphi + \dfrac{\sigma_1 + \sigma_3}{2}\right] = \dfrac{\sigma_1 - \sigma_3}{2}$

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$\longrightarrow [(c) \sin \varphi \cot \varphi] + \left[\dfrac{\sigma_1 + \sigma_3}{2} \sin \varphi \right] = \dfrac{\sigma_1 - \sigma_3}{2} \longrightarrow (c) \sin \varphi \cot \varphi = \left[\dfrac{\sigma_1 - \sigma_3}{2} \right] - \left[\dfrac{\sigma_1 + \sigma_3}{2} \sin \varphi \right]$

$\longrightarrow (2) [(c) \sin \varphi \cot \varphi] = [\sigma_1 - (\sin \varphi)(\sigma_1)] - [\sigma_3 + (\sin \varphi)(\sigma_3)]$

$\longrightarrow (2) \left[(c) \sin \varphi \dfrac{\cos \phi}{\sin \phi} \right] = \sigma_1 [1 - (\sin \varphi)] - \sigma_3 [1 + (\sin \varphi)]$

$\dfrac{2(c) \cos \phi}{(1 - \sin \phi)} + (\sigma_3) \dfrac{(1 + \sin \phi)}{(1 - \sin \phi)} = \sigma_1$

Since $\dfrac{\cos \phi}{(1 - \sin \phi)} \approx \tan (45° + \dfrac{\phi}{2})$ **and** $\dfrac{(1 + \sin \phi)}{(1 - \sin \phi)} \approx \tan^2 (45° + \dfrac{\phi}{2})$

$$\sigma_1 = \sigma_3 \tan^2 \left(45° + \dfrac{\phi}{2} \right) + 2c \tan \left(45° + \dfrac{\phi}{2} \right)$$

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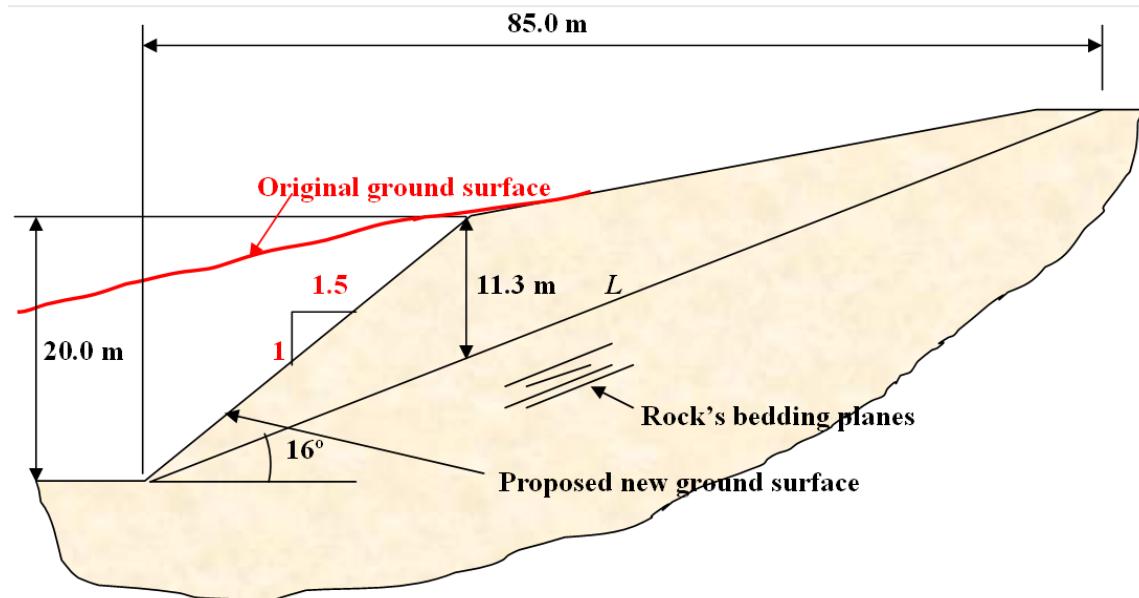
Chapter 13
Slope Stability

Symbols for Slope Stability

(Revision: Oct.-08)

A slope cut to 1.5H:1V will be made in a shale rock stratum that has bedding planes that have an apparent dip of 16° (see the figure below). If the acceptable factor of safety against failure is at least 2 along the lower-most bedding plane, is this slope stable? Use a unit weight of 20.1 kN/m^3, and bedding strength parameters of $c = 22$ kPa and $\varphi = 30°$.

Solution:

The traingule of rock above the potential slip plane has a weight W per unit width,

$$W = \frac{1}{2}(85.0m)(11.3m)\left(20.1\frac{kN}{m^3}\right) = 9,650\frac{kN}{m}$$

The length L of the slip plane is,

$$L = \frac{85.0\ m}{\cos 16°} = 88.4\ m$$

Therefore,
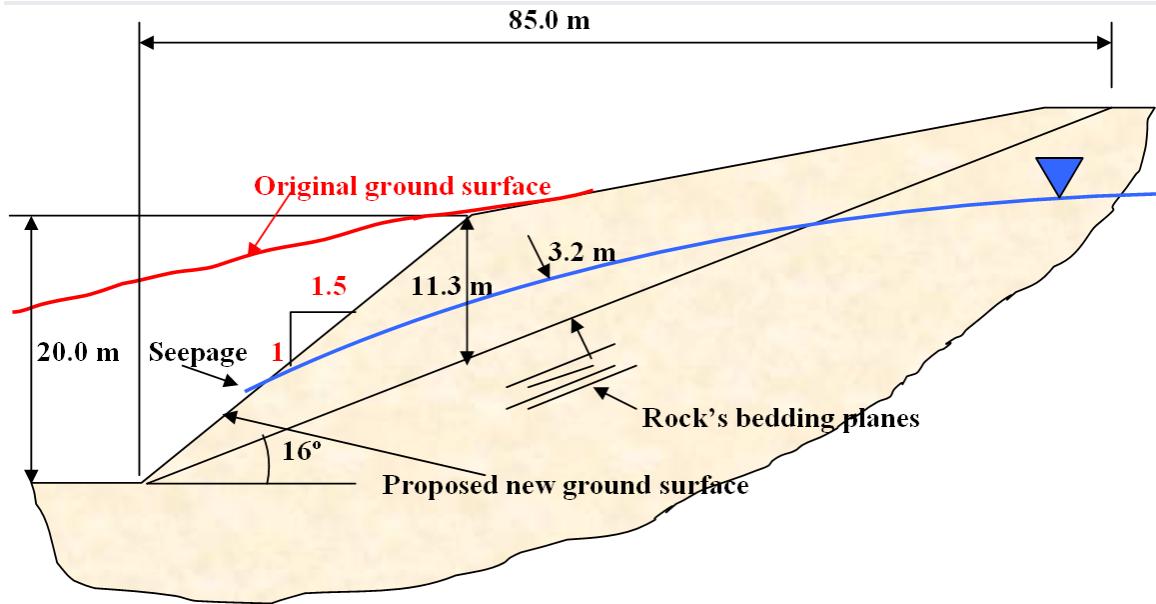
$$FS = \frac{\text{Resisting Forces}}{\text{Driving Forces}} = \frac{cL + \left[(W)\cos\alpha\right]\tan\phi}{(W)\sin\alpha} =$$

$$FS = \frac{\left(22\frac{kN}{m^2}\right)(88.4\ m) + \left[\left(9,650\frac{kN}{m}\right)\cos 16°\right]\tan 30°}{\left(9,650\frac{kN}{m}\right)\sin 16°} = 2.7 > 2 \quad OK$$

(Revision: Oct.-08)

In the previous problem the slope appeared to be stable with a factor of safety = 2.7. What happens to that factor of safety if the water table rises to the level shown below? Use a unit weight of 20.1 kN/m³, and bedding strength parameters are reduced by the effective parameters of *c'* = 15 kPa and *φ'* = 20°.

85.0 m

Original ground surface

3.2 m

1.5   11.3 m

20.0 m   Seepage   1

16°

Rock's bedding planes

Proposed new ground surface

Solution:

The weight W of the rock triangle per unit width is still $9,650 \frac{kN}{m}$

The length L of the slip plane is still $88.4\ m$.

The pore water pressure is based on an estimate of its value along the length L, at water depth z_w above the plane that range from 0 to $3.2\ m$; conservatively,

$$u = \gamma_w z_w = \left(9.81 \frac{kN}{m^3}\right)(3.2\ m) = 31.4\ kPa$$

$$FS = \frac{Resisting\ Forces}{Driving\ Forces} = \frac{c'L + \left[(W)\cos\alpha - u\right]\tan\phi}{(W)\sin\alpha} =$$
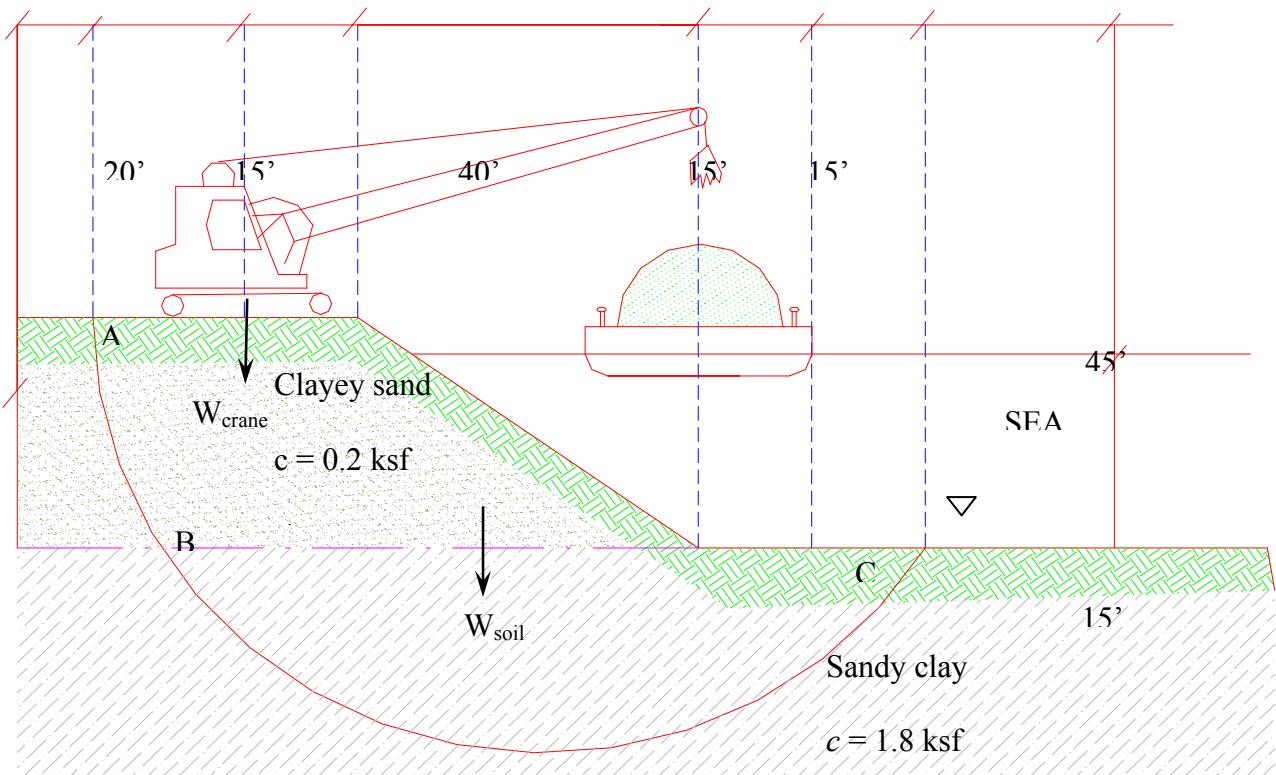
$$FS = \frac{\left(15 \frac{kN}{m^2}\right)(88.4\ m) + \left[\left(9,650 \frac{kN}{m}\right)\cos 16° - (31.4\ kPa)\right]\tan 20°}{\left(9,650 \frac{kN}{m}\right)\sin 16°} = 1.76 < 2 \quad NG$$

The computed factor of safety of 1.76 is less than the minimum acceptable value of 2, therefore this design is NOT acceptable. *Notice that a rising WT decreases the stability of the slope.*

*Slope-03: *Is a river embankment safe with a large crane?*

(Revision: Oct.-08)

Determine if the work site shown below is safe, provided you consider the minimum acceptable factor of safety for the man-made waterfront slope shown below to be 2. Assume the arc radius is 80 feet; the circular lengths are AB = 22 feet and BC = 102 feet. The total weight of the soil per unit width are W_{soil} = 205 kips and W_{crane} =70 kips. The site is located in a seismic zone with a seismic coefficient of 0.15.

Solution:

$Mr = R[S1(AB)+S2(BC)] = R[(C_1'+\sigma_1'\tan\phi_1)AB + (C_2'+\sigma_2'\tan\phi_2)BC]$

$= 80'[(0.2+0.125(8')(\tan40)22' + (1.8+(0.130 - 0.064)(21)(\tan15)(102)] = 80[23k + 221k] = \underline{\textbf{19,500 k-ft}}$

$M_0 = Wb_1 - W_{wH}(d_1) - W_{WV}(b_2) + Vb_3 + \dfrac{W_{ae}(d_2)}{g}$

$= 205(40)–(1/2)(0.064)(15)^2 - (0.064)[30(15)+(1/2)(40)(15)]15+70(55)+205(0.15)(50)$

Mo = 8,200 - 7.2 –720 +3850 +1540 = **12,900 k-ft**

Therefore: FS = Mr/Mo = 19,500/12,900 = 1.51   ➔**Not Good!**

Removing Crane ➔ Mo =9,050 k-ft

Therefore: FS = Mr/Mo = 19,500/9050 = 2.15   ➔ **GOOD!**

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(Revision: Oct.-08)

The stability of a slope was analyzed by the method of slices. One of the trial curved surfaces through the soil mass yielded the shearing and normal components of each slice as listed below. The curved length of the trial curved surface is 40 feet, the soil parameters are c = 225 lb/ft^2 and φ = 15°. Determine the factor of safety along this trial surface.

Solution:
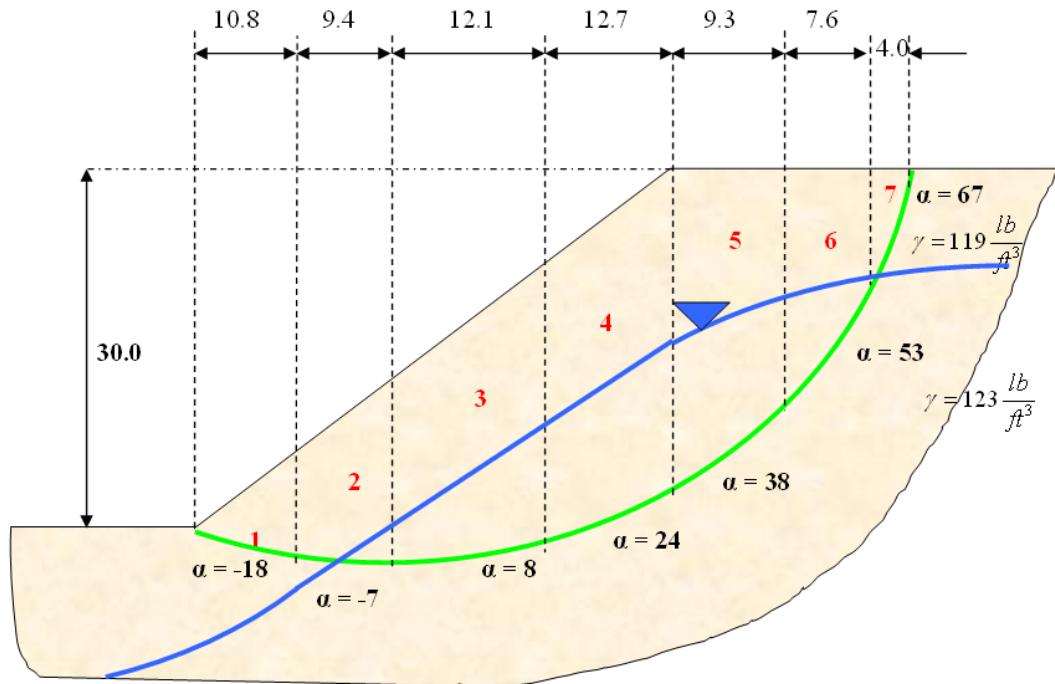
Slice Number	Shearing Component (W sin α) (lb/ft)	Normal Component (W cos α) (lb/ft)
1	-38	306
2	-74	1410
3	124	2380
4	429	3050
5	934	3480
6	1570	3540
7	2000	3210
8	2040	2190
9	766	600
	Σ = 7,751 lb/ft	Σ = 20,166 lb/ft

$$FS = \frac{cL + \sum (W \cos \alpha) \tan n\phi}{\sum W \sin \alpha} = \frac{(225\,psf)(40\,ft) + 20,166\,plf}{7,751\,plf} = 1.86 < 2 \quad NG$$

Slope-05: *Method of slices to find the factor of safety of a slope with a WT.*

A 30 ft tall, 1.5H:1V slope is to be built as shown below. The soil is homogeneous, with $c' = 400 \dfrac{lb}{ft^2}$ **and** $\phi' = 29°$. **The unit weight is 119 pcf above the groundwater table, and 123 pcf below. Using the ordinary method of slices, compute the factor of safety along the trial circle.**

Solution:

Weights:

$$\frac{W_1}{b} = 10.8 \left(\frac{10.3}{2} \right) 119 = 6,620 \frac{lb}{ft}$$

$$\frac{W_2}{b} = 9.4 \left(\frac{10.3 + 12.5}{2} \right) 119 + 9.4 \left(\frac{5.2}{2} \right) 123 = 15,800 \frac{lb}{ft}$$

$$\frac{W_3}{b} = 12.1 \left(\frac{12.5 + 14.6}{2} \right) 119 + 12.1 \left(\frac{5.2 + 10.0}{2} \right) 123 = 30,800 \frac{lb}{ft}$$

$$\frac{W_4}{b} = 2.9 \left(\frac{5.0}{2} \right) 17.0 + 7.1 \left(\frac{12.9 + 8.0}{2} \right) 17.8 = 1620 \frac{lb}{ft}$$

$$\frac{W_5}{b} = 9.3 \left(\frac{16.8 + 12.8}{2} \right) 119 + 9.3 \left(\frac{10.7 + 7.3}{2} \right) 123 = 39,900 \frac{lb}{ft}$$

$$\frac{W_6}{b} = 7.6 \left(\frac{12.8 + 9.9}{2} \right) 119 + 7.6 \left(\frac{7.3}{2} \right) 123 = 26,700 \frac{lb}{ft}$$

$$\frac{W_7}{b} = 4.0 \left(\frac{9.9}{2} \right) 119 = 2,400 \frac{lb}{ft}$$

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Average pore water pressure at base of each slice:

$u_1 = 0$

$u_2 = \left(\dfrac{5.2}{2}\right)62.4 = 160 \dfrac{lb}{ft^2}$

$u_3 = \left(\dfrac{5.2 + 10.0}{2}\right)62.4 = 470 \dfrac{lb}{ft^2}$

$u_4 = \left(\dfrac{10.0 + 10.7}{2}\right)62.4 = 650 \dfrac{lb}{ft^2}$

$u_5 = \left(\dfrac{10.7 + 7.3}{2}\right)62.4 = 560 \dfrac{lb}{ft^2}$

$u_6 = \left(\dfrac{7.3}{2}\right)62.4 = 230 \dfrac{lb}{ft^2}$

$u_7 = 0$

Slice	$\dfrac{W}{b}(lb)$	α (Deg)	$c'\left(\dfrac{lb}{ft^2}\right)$	ϕ (Deg)	$u\left(\dfrac{lb}{ft^2}\right)$	$l\ (ft)$	$c'l + \left(\dfrac{W}{b}\right)(\cos\alpha - ul)\tan\phi'$	$\dfrac{W}{b}(lb)$
1	6620	-18	400	29	0	11.4	8,000	-2,000
2	15,800	-7	400	29	160	9.5	11,700	-1,900
3	30,800	8	400	29	470	12.2	18,600	4,300
4	39,900	24	400	29	650	13.9	20,800	16,200
5	26,700	38	400	29	560	11.8	12,700	16,400
6	13,700	53	400	29	230	12.6	8,000	10,900
7	24,00	67	400	29	0	10.2	4,600	2,200
							$\Sigma = 84,400$	$\Sigma = 46,100$

Therefore, the factor of safety is,

$$FS = \dfrac{c'l + \left(\dfrac{W}{b}\right)(\cos\alpha - ul)\tan\phi'}{\left(\dfrac{W}{b}\right)} = \dfrac{84,400}{46,100} = 1.83 < 2 \quad NG$$

Note how slices # 1 and 2 have a negative α because they are inclined backwards.
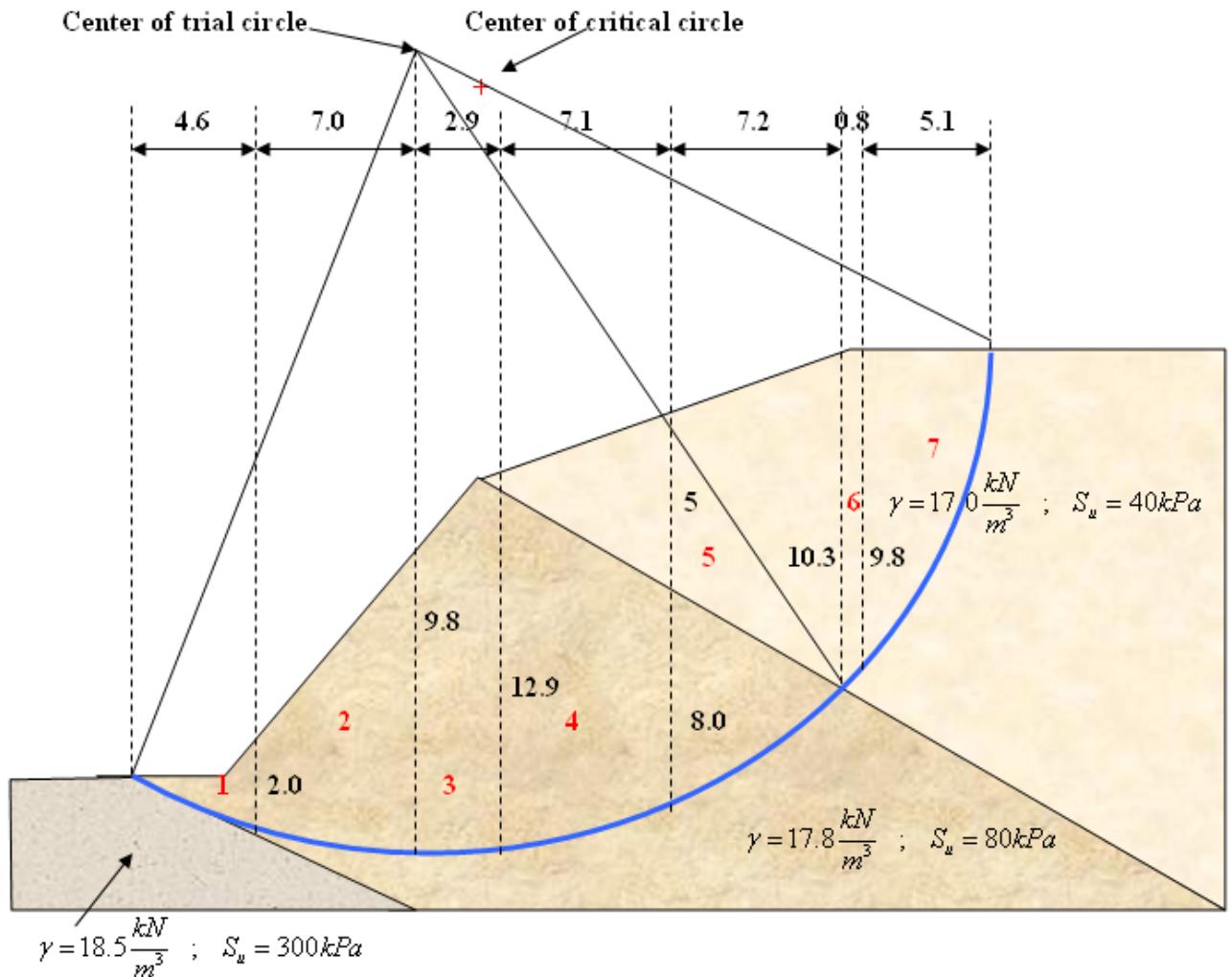
(Revision: Oct.-08)

Using the Swedish slip circle method, compute the factor of safety along the trial circle shown in the figure below.

Solution:

Divide the slide mass into vertical slices as shown. One of the slice borders should be directly below the center of the circle (in this case, the border between slices 2 and 3). For convenience of computations, also draw a slice border wherever the slip surface intersects a new soil stratum and whenever the ground surface has a break in slope. Then, compute the weight and moment arm for each slide using simplified computations as follows:

Solution:

Weights

$$\frac{W_1}{b} = 4.6\left(\frac{2.0}{2}\right)17.8 = 80 \frac{kN}{m}$$

$$\frac{W_2}{b} = 7.0\left(\frac{2.0 + 9.8}{2}\right)17.8 = 130 \frac{kN}{m}$$

$$\frac{W_3}{b} = 2.9\left(\frac{9.8 + 12.9}{2}\right)17.8 = 590 \frac{kN}{m}$$

$$\therefore \frac{W_4}{b} = 2.9\left(\frac{5.0}{2}\right)17.0 + 7.1\left(\frac{12.9 + 8.0}{2}\right)17.8 = 1620 \frac{kN}{m}$$

$$\frac{W_5}{b} = 7.2\left(\frac{5.0 + 10.3}{2}\right)17.0 + 7.2\left(\frac{8.0}{2}\right)17.8 = 1450 \frac{kN}{m}$$

$$\frac{W\,6}{b} = 0.8\left(\frac{10.3 + 9.8}{2}\right)17.0 = 140 \frac{kN}{m}$$

$$\frac{W_7}{b} = 5.1\left(\frac{9.8}{2}\right)17.0 = 420 \frac{kN}{m}$$

Moment arms:

$$d_1 = -7.0 - \frac{4.6}{3} = -8.5 \ m$$

$$d_2 = \frac{-7.0}{2} = -3.5 \ m$$

$$d_3 = \frac{2.9}{2} = 1.5 \ m$$

$$d_4 = 2.9 + \frac{7.1}{2} = 6.5 \ m$$

$$d_5 = 2.9 + 7.1 + \frac{7.1}{2} = 10.9 \ m$$

$$d_6 = 2.9 + 7.1 + 7.2 + \frac{0.8}{2} = 17.6 \ m$$

$$d_7 = 2.9 + 7.1 + 7.2 + 0.8 + \frac{5.1}{3} = 19.7 \ m$$

Slice	S_u (kPa)	θ (Deg)	$S_u\,\theta$	$\dfrac{W}{b}\left(\dfrac{kN}{m}\right)$	$d\,(m)$	$\left(\dfrac{W}{b}\right)d$
1				80	-8.5	-690
2				130	-3.5	-450
3	80	76	6080	390	1.5	890
4				1620	6.5	10,530
5				1450	10.9	15,800
6	40	30	1200	140	17.6	2,460
7				420	19.7	8,280
			$\Sigma = 7280$			$\Sigma = 36,830$

$$FS = \frac{\pi R^2}{180}\frac{\sum S_u\theta}{\sum\left(\dfrac{W}{b}\right)d} = \frac{\pi\left(23.6\right)^2}{180}\ \frac{7,280}{36,830} = 1.92 < 2 \quad \textit{Not Good}$$

Chapter 14
Statistical Analysis of Soils

Chapter 15
Lateral Pressures from Soils

Symbols for Lateral Pressures from Soils

$D_x \rightarrow$ *Diameter of the grains distributed (represent % finer by weight).*

$e \rightarrow$ *The voids ratio.*

$G_S \rightarrow$ *Specific gravity of the solids of a soil.*

$H \rightarrow$ *Maximum depth of excavation or thickness of a soil layer.*

$h_{soil} \rightarrow$ *depth of the soil.*

$i_{critical} \rightarrow$ *Critical hydraulic gradient.*

$k_H \rightarrow$ *Horizontal permeability.*

$k_V \rightarrow$ *Vertical permeability.*

$u \rightarrow$ *pore water pressure.*

$\sigma' \rightarrow$ *Effective stress.*

$\sigma_V' \rightarrow$ *Vertical Effective stress.*

$g' \rightarrow$ *Bouyant unit weight of a soil.*

$\gamma_{SAT} \rightarrow$ *Saturated unit weight of a soil*

$\gamma_W \rightarrow$ *Unit weight of water.*

$V_W \rightarrow$ *Volume of water.*

$w \rightarrow$ *water content.*

Formulas and Figures for Lateral Stresses.
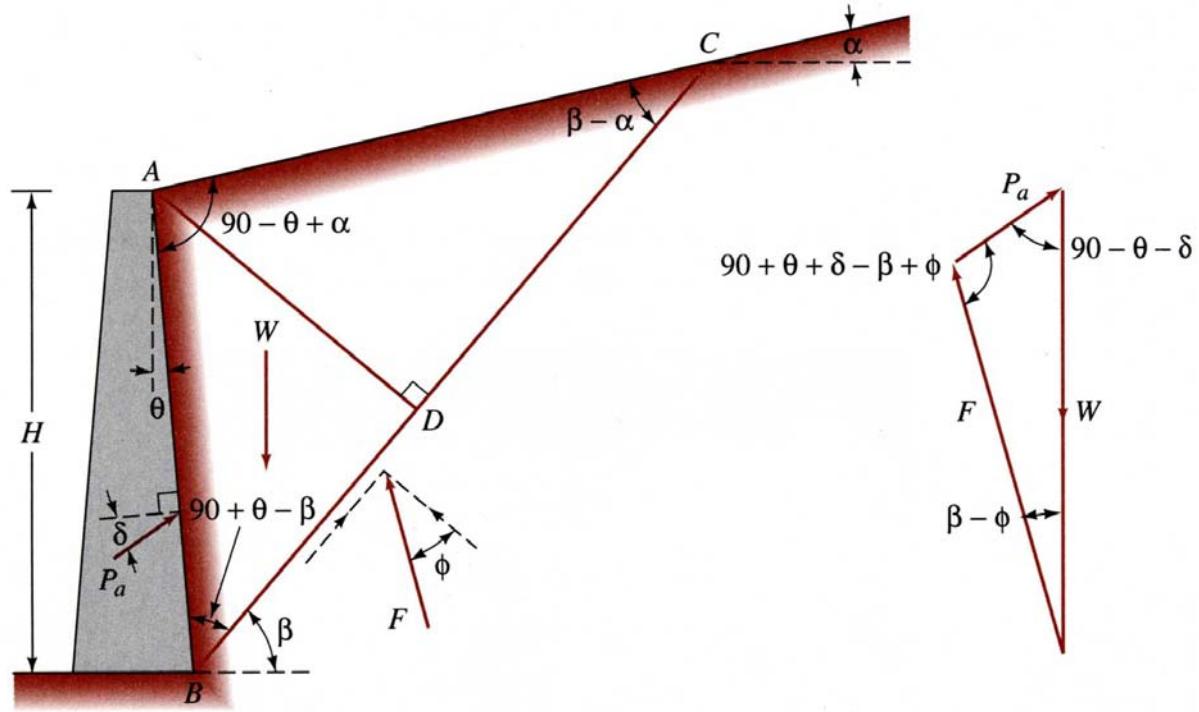
Figure for symbols used in the Coulomb earth pressures.

Coulomb's lateral pressure coefficients K_a and K_p.

$$K_a = \frac{\cos^2(\phi - \theta)}{\cos^2\theta \, \cos(\delta+\theta)\left[1+\sqrt{\dfrac{\sin(\delta+\phi)\sin(\phi-\alpha)}{\cos(\delta+\theta)\cos(\theta-\alpha)}}\right]^2}$$

$$K_p = \frac{\cos^2(\phi + \theta)}{\cos^2\theta \, \cos(\delta-\theta)\left[1-\sqrt{\dfrac{\sin(\phi-\delta)\sin(\phi+\alpha)}{\cos(\delta-\theta)\cos(\alpha-\theta)}}\right]^2}$$

K_a for the case of $\theta = 0^\circ$ and $\alpha = 0^\circ$.

↓ φ (deg)	δ (deg) → 0	5	10	15	20	25
28	0.3610	0.3448	0.3330	0.3251	0.3203	0.3186
30	0.3333	0.3189	0.3085	0.3014	0.2973	0.2956
32	0.3073	0.2945	0.2853	0.2791	0.2755	0.2745
34	0.2827	0.2714	0.2633	0.2579	0.2549	0.2542
36	0.2596	0.2497	0.2426	0.2379	0.2354	0.2350
38	0.2379	0.2292	0.2230	0.2190	0.2169	0.2167
40	0.2174	0.2089	0.2045	0.2011	0.1994	0.1995
42	0.1982	0.1916	0.1870	0.1841	0.1828	0.1831
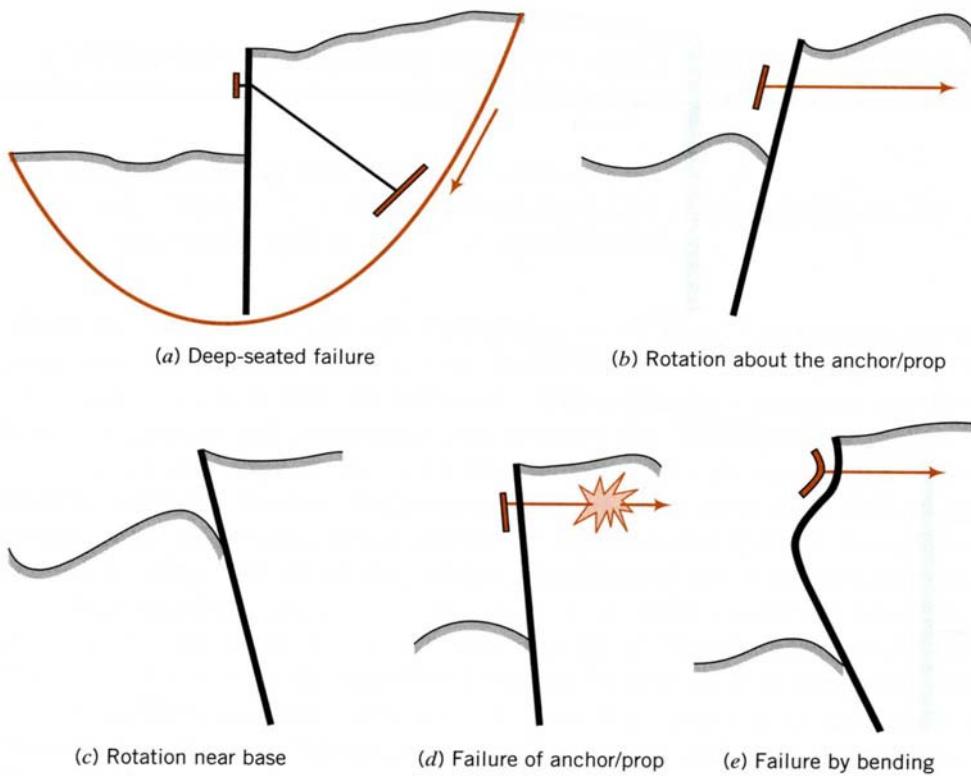
K_a for the case where $\delta = 2/3\,\varphi$.

α (deg)	φ (deg)	θ (deg) 0	5	10	15	20	25
0	28	0.3213	0.3588	0.4007	0.4481	0.5026	0.5662
	30	0.2973	0.3349	0.3769	0.4245	0.4794	0.5435
	32	0.2750	0.3125	0.3545	0.4023	0.4574	0.5220
	34	0.2543	0.2916	0.3335	0.3813	0.4367	0.5017
	36	0.2349	0.2719	0.3137	0.3615	0.4170	0.4825
	38	0.2168	0.2535	0.2950	0.3428	0.3984	0.4642
	40	0.1999	0.2361	0.2774	0.3250	0.3806	0.4468
	42	0.1840	0.2197	0.2607	0.3081	0.3638	0.4303
5	28	0.3431	0.3845	0.4311	0.4843	0.5461	0.6191
	30	0.3165	0.3578	0.4043	0.4575	0.5194	0.5926
	32	0.2919	0.3329	0.3793	0.4324	0.4943	0.5678
	34	0.2691	0.3097	0.3558	0.4088	0.4707	0.5443
	36	0.2479	0.2881	0.3338	0.3866	0.4484	0.5222
	38	0.2282	0.2679	0.3132	0.3656	0.4273	0.5012
	40	0.2098	0.2489	0.2937	0.3458	0.4074	0.4814
	42	0.1927	0.2311	0.2753	0.3271	0.3885	0.4626
10	28	0.3702	0.4164	0.4686	0.5287	0.5992	0.6834
	30	0.3400	0.3857	0.4376	0.4974	0.5676	0.6516
	32	0.3123	0.3575	0.4089	0.4683	0.5382	0.6220
	34	0.2868	0.3314	0.3822	0.4412	0.5107	0.5942
	36	0.2633	0.3072	0.3574	0.4158	0.4849	0.5682
	38	0.2415	0.2846	0.3342	0.3921	0.4607	0.5438
	40	0.2214	0.2637	0.3125	0.3697	0.4379	0.5208
	42	0.2027	0.2441	0.2921	0.3487	0.4164	0.4990
15	28	0.4065	0.4585	0.5179	0.5869	0.6685	0.7671
	30	0.3707	0.4219	0.4804	0.5484	0.6291	0.7266
	32	0.3384	0.3387	0.4462	0.5134	0.5930	0.6895
	34	0.3091	0.3584	0.4150	0.4811	0.5599	0.6554
	36	0.2823	0.3306	0.3862	0.4514	0.5295	0.6239
	38	0.2578	0.3050	0.3596	0.4238	0.5006	0.5949
	40	0.2353	0.2813	0.3349	0.3981	0.4740	0.5672
	42	0.2146	0.2595	0.3119	0.3740	0.4491	0.5416
20	28	0.4602	0.5205	0.5900	0.6715	0.7690	0.8810
	30	0.4142	0.4728	0.5403	0.6196	0.7144	0.8303
	32	0.3742	0.4311	0.4968	0.5741	0.6667	0.7800
	34	0.3388	0.3941	0.4581	0.5336	0.6241	0.7352
	36	0.3071	0.3609	0.4233	0.4970	0.5857	0.6948
	38	0.2787	0.3308	0.3916	0.4637	0.5587	0.6580
	40	0.2529	0.3035	0.3627	0.4331	0.5185	0.6243
	42	0.2294	0.2784	0.3360	0.4050	0.4889	0.5931

K_p **for** $\theta = 0^\circ$ **and** $\alpha = 0^\circ$.

↓ ϕ (deg)	δ (deg) →				
	0	5	10	15	20
15	1.698	1.900	2.130	2.405	2.735
20	2.040	2.313	2.636	3.030	3.525
25	2.464	2.830	3.286	3.855	4.597
30	3.000	3.506	4.143	4.977	6.105
35	3.690	4.390	5.310	6.854	8.324
40	4.600	5.590	6.946	8.870	11.772

Failure modes for flexible walls (sheet-piling).

(a) Deep-seated failure

(b) Rotation about the anchor/prop

(c) Rotation near base

(d) Failure of anchor/prop

(e) Failure by bending
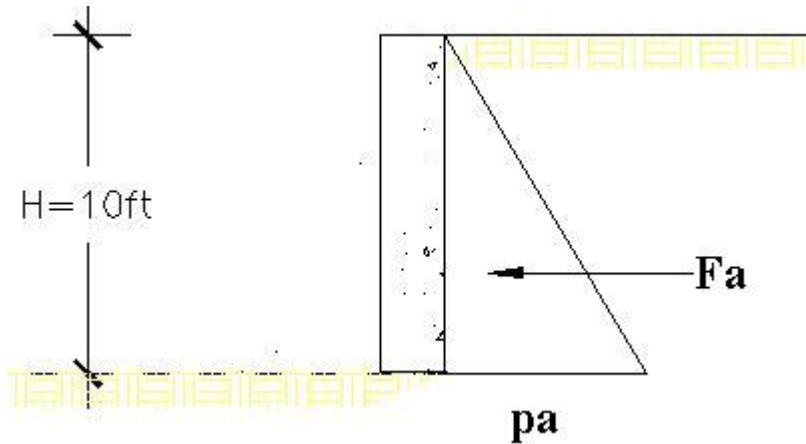
(Revision: Sept.-08)

Consider a small 10-foot tall and 3 feet thick concrete retaining wall. The backfill behind the wall will be from local sandy gravel with a dry unit weight of 115 pcf and an angle of internal friction of 30 degrees. The wall will not have to retain water.

Estimate, (a) the lateral force on the wall from the backfill in an active pressure condition, (b) its stability against overturning, and (c) its stability against sliding (use a Factor of Safety ≥ 2).

pa

Solution:

(a) The Rankine *active* earth pressure coefficient is, $K_a = \tan^2\left(45° - \dfrac{\phi}{2}\right) = \tan^2\left(45° - \dfrac{30°}{2}\right) = 0.33$

The lateral pressure at the bottom of the wall is $p_a = \gamma h K_a = (0.115\,kcf)(10\,ft)(0.33) = 0.38\,ksf$

The force against the wall is $F_a = \dfrac{1}{2}(p_a)h = (0.5)(0.38)(10) = 1.9\,kips\ per\ foot\ of\ wall$

(b) The stability of the wall against overturning is found by taking moments about the point "O" at the toe of the wall,

$Factor\ of\ Safety\,(FS) = \dfrac{resisting\ moment}{overturning\ moment} = \dfrac{(3')(10')(1')(0.150\,kcf)(1.5\,ft)}{(1.9\,kips)(10/3\,ft)} = 1.07 < 2 \quad NG$

(c) The stability of the wall against sliding towards the left is found by,

$Factor\ of\ Safety\,(FS) = \dfrac{resisting\ force}{driving\ force} = \dfrac{(3')(10')(1')(0.150\,kcf)(\tan 30°)}{(1.9\,kips)} = 1.37 < 2 \quad NG$

(Revision: Sept-2008)

(a) **Compare the Rankine and Coulomb lateral earth pressure coefficients for a wall that retains a granular backfill soil with $\phi = 35°$, $\delta = 12°$, $\theta = 0°$ and $\alpha = 20°$. (Note: δ is the angle of friction between the soil and the backside of the wall; α is the angle of the slope for the backfill behind the wall and θ is the back of the wall's angle with respect to the vertical).**

(b) **What is the passive earth force on the wall at failure if the wall is 10 m high, $\gamma = 18.1$ kN/m^3 and $c = 9$ kN/m^2?**

Solution:

(a) Rankine's active and passive earth pressure coefficients,

$$K_a = \tan^2\left(45° - \frac{\phi}{2}\right) = \tan^2\left(45° - \frac{35°}{2}\right) = 0.271$$

$$K_p = \tan^2\left(45° + \frac{\phi}{2}\right) = \tan^2\left(45° + \frac{35°}{2}\right) = 3.690 \quad \text{Note that} \quad K_a = \frac{1}{K_P}$$

Coulomb's active and passive earth pressure coefficients,

$$K_a = \frac{\cos^2(\phi - \theta)}{\cos^2\theta\,\cos(\delta+\theta)\left[1+\sqrt{\dfrac{\sin(\delta+\phi)\sin(\phi-\alpha)}{\cos(\delta+\theta)\cos(\theta-\alpha)}}\right]^2} = \frac{\cos^2(35 - 0)}{\cos^2 0\,\cos(12+0)\left[1+\sqrt{\dfrac{\sin(12+35)\sin(35-20)}{\cos(12+0)\cos(0-20)}}\right]^2} = 0.323$$

$$K_p = \frac{\cos^2(\phi + \theta)}{\cos^2\theta\,\cos(\delta-\theta)\left[1-\sqrt{\dfrac{\sin(\phi-\delta)\sin(\phi+\alpha)}{\cos(\delta-\theta)\cos(\alpha-\theta)}}\right]^2} = \frac{\cos^2(35 + 0)}{\cos^2 0\,\cos(12-0)\left[1-\sqrt{\dfrac{\sin(35-12)\sin(35+20)}{\cos(12-0)\cos(20-0)}}\right]^2} = 3.517$$

When $\alpha = 0°$, $\theta = 0°$ and $\delta = 0°$ the Coulomb formula becomes identical to Rankine's.

(b) Therefore, the Rankine coefficient is 3.690 versus 3.517 for Coulomb's. Using these values, the total passive force F_p on the wall per unit length is,
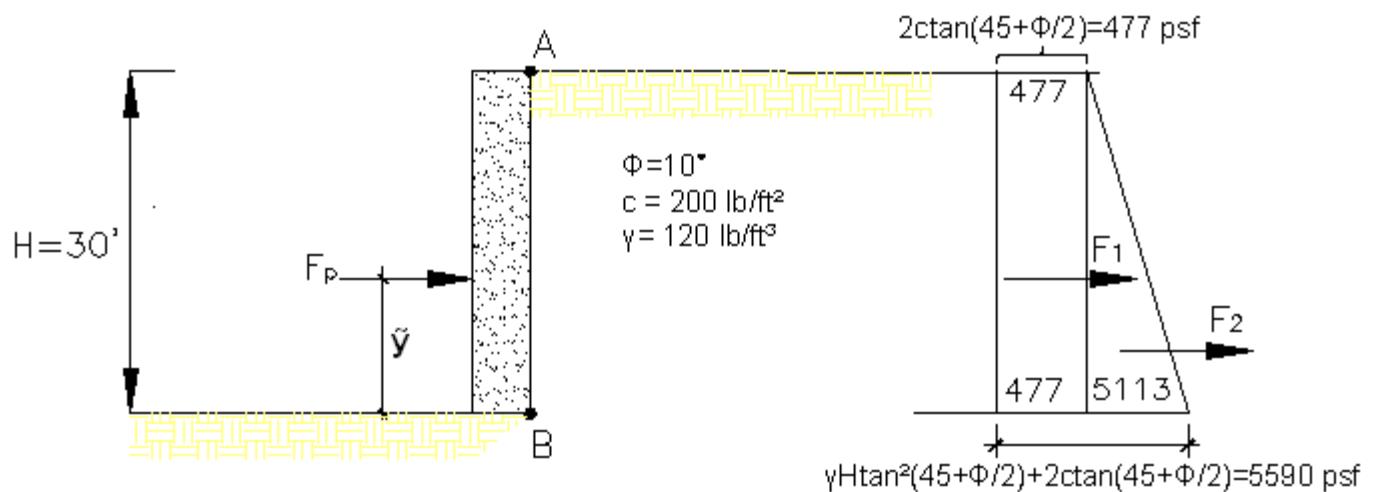
$Rankine's \quad F_p = 0.5\gamma h^2 K_p + 2ch\sqrt{K_p} = (0.5)(18.1)(10)^2(3.690) + 2(9)(10)\sqrt{3.690} = 3,685\ kN/m^2$

$Coulomb's \quad F_p = 0.5\gamma h^2 K_p + 2ch\sqrt{K_p} = (0.5)(18.1)(10)^2(3.517) + 2(9)(10)\sqrt{3.517} = 3,520\ kN/m^2$

(Revision: Sept-08)

Using the Rankine method, find the magnitude and location of the passive pressure force F_p with respect to the heel of the wall (point B), exerted upon a temporary retaining wall by a large jacking system (which is not shown in the figure).

$\Phi=10°$
$c = 200 \text{ lb/ft}^2$
$\gamma = 120 \text{ lb/ft}^3$

$2c\tan(45+\Phi/2)=477 \text{ psf}$

477

F_1

F_2

477 5113

$\gamma H\tan^2(45+\Phi/2)+2c\tan(45+\Phi/2)=5590 \text{ psf}$

H=30'

F_p

$\tilde{y}$

A

B

Solution:

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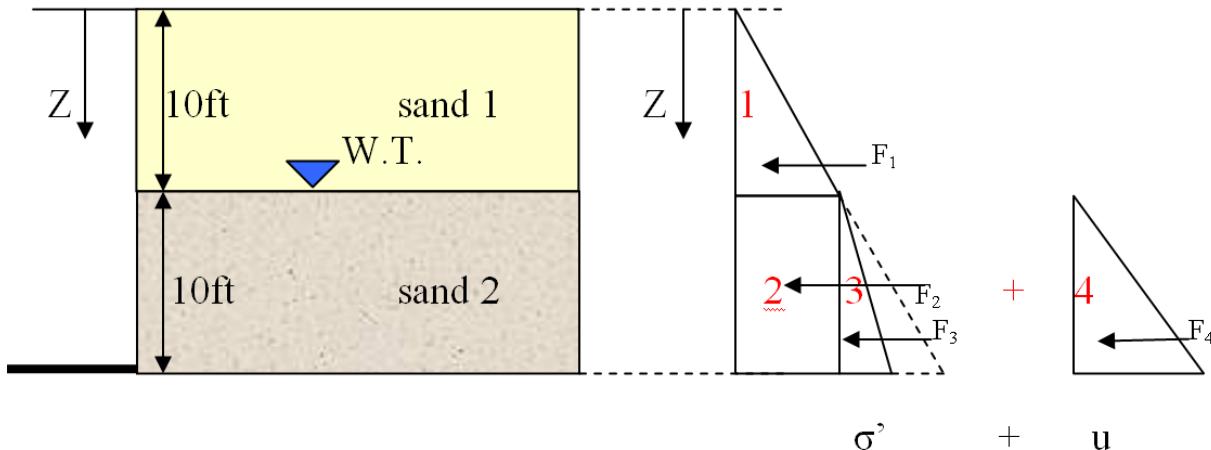
(Revision: Sept-08)

Find the lateral "at-rest" force F_o on the wall and its location with respect to the top of the wall. Given: Sand #1 has a unit weight of 105 pcf, $c = 0$ psf and $\varphi = 30°$; Sand #2 has a unit weight of 122 pcf, $c = 0$ psf and $\varphi = 30°$.

Solution:

From Jaky's empirical relation, $K_o = 1 - \sin \phi' = 1 - \sin 30° = 0.50$

at $z = 0$ feet $\sigma' = 0$ ksf, because there is no surcharge loading upon the surface of Sand #1.

at $z = 10$ feet $\sigma'_h = K_o \sigma'_v = (0.5)(0.105kcf)(10ft) = 0.525$ ksf

at $z = 20$ feet $\sigma'_h = (0.5)\left[(0.105)(10) + (0.122 - 0.0624)10\right] = 0.823$ ksf

$$\sigma_w = \gamma_w h = (0.0624 pcf)(10 ft) = 0.624 \ ksf$$

$$F_o = \sum_i f_i = F_1 + F_2 + F_3 + F_4 = \frac{1}{2}(0.525)(10) + (0.525)(10) + \frac{1}{2}(0.302)(10) + \frac{1}{2}(0.624)(10)$$

$$F_o = 2.63 + 5.25 + 1.49 + 3.12 = 12.5 \ \frac{kip}{ft}$$

$$z = \frac{(2.63)(6.67) + (5.25)(15) + (1.49)(16.67) + (3.12)(16.67)}{12.5 \ kip} = \frac{173.1 \ kip - ft}{12.5 \ kip}$$
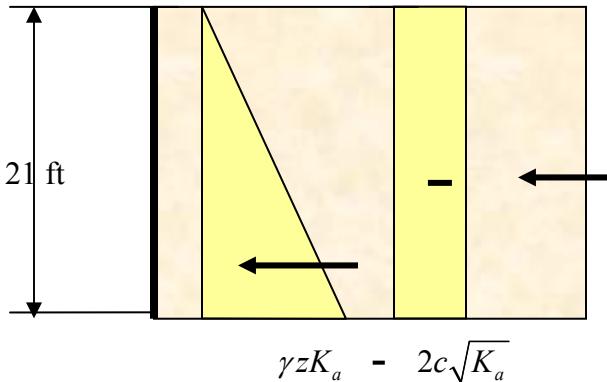
$z = 13.8 \ ft \ from \ the \ top \ of \ the \ wall.$

(Revision: Sept-08)

A 21 foot high retaining wall supports a purely cohesive soil ($\varphi = 0°$) with a cohesion of 630 psf and a unit weight of 113 pcf. Find:

(a) The Rankine active earth pressure on the wall.

(b) Estimate the depth of separation of the clay from the wall, and (c) find the lateral force upon the wall whilst considering the clay separation.

$$\gamma z K_a \quad - \quad 2c\sqrt{K_a}$$

Solution:

a) The coefficient of active earth pressure is,

$$K_a = \tan^2\left(45° - \frac{\phi}{2}\right) = \tan^2\left(45° - \frac{0°}{2}\right) = \tan^2 45° = 1$$

The net active earth pressure p_a on the wall is,

$$p_a = \sigma_3 = \gamma h K_a - 2c\sqrt{K_a}$$
$$= (0.113 \, kcf)(21ft)(1) - 2(0.630 ksf)\sqrt{1} = 2.37 - 1.26 = 1.11 \; ksf$$

b) The crack stops where the pressure is zero, $p_a = 0$,

$$p_a = \gamma h K_a - 2c\sqrt{K_a} \quad \therefore \quad \gamma h K_a = 2c\sqrt{K_a}$$

$$\therefore h_{crack} = \frac{2c\sqrt{K_a}}{\gamma K_a} = \frac{2c}{\gamma\sqrt{K_a}} = \frac{2(0.630 ksf)}{(0.113 kcf)(1)} = 11.2 \; feet$$

c) The total (Rankine) active earth force upn the wall F_a is,

$$F_a = \frac{1}{2}\gamma H^2 K_a - 2cH\sqrt{K_a}$$

but there is no contact on the wall where the tension crack exists, therefore

$$F_a = \frac{1}{2}\left(\gamma H K_a - 2c\sqrt{K_a}\right)\left(H - \frac{2c}{\gamma\sqrt{K_a}}\right) = \frac{1}{2}\gamma H^2 - 2cH\sqrt{K_a} + \frac{2c^2}{\gamma}$$
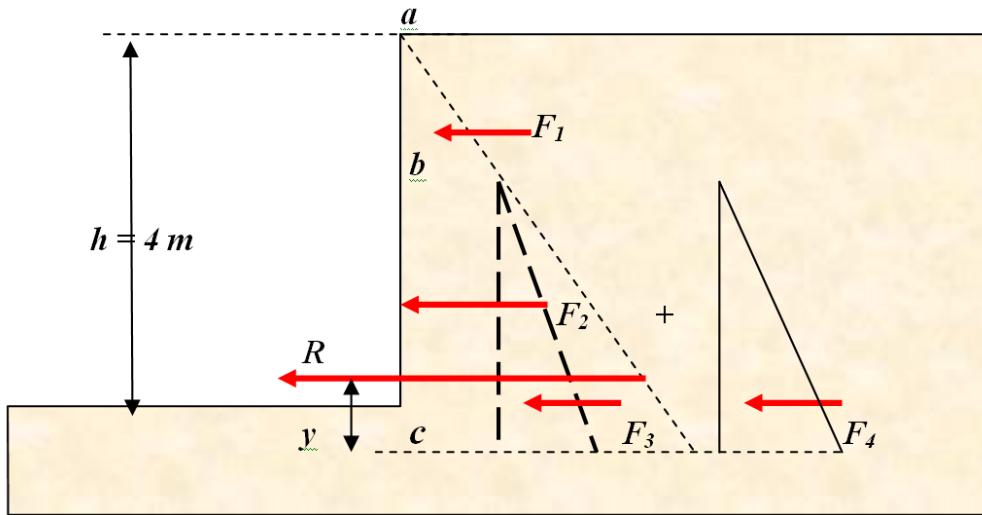
$$F_a = \frac{1}{2}(0.113 kcf)(21ft)^2(1) - 2(0.63 ksf)(21ft)(1) + \frac{2(0.63 ksf)^2}{(0.113 kcf)} = 5.48 \; k / ft \; of \; wall$$

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(Revision: Sept-08)

A 4 m wall retains a dry sand backfill with a unit weight of 18.3 kN/m³, an angle of internal friction of 36° and a porosity of 31%. The backfill is fully drained through weep holes.

1) What is the magnitude of the backfill force on a 1 m wide slice of wall if it is not allowed to deflect?

2) What is the magnitude of the backfill force on the same 1 m wide slice, if the wall does deflect enough to develop a Rankine active earth pressure condition?

3) What is the new force on the wall, and its location from its heel, if the wall's weep holes are clogged and the water table now rises to within 1 m of the ground surface behind the wall?

Solution:

1) No deflection of the wall means the soil is "at rest" and $K_0 = 1 - \sin\phi = 1 - \sin 36° = 0.41$

$The\ force\ F_o = \frac{1}{2}\gamma_d h^2 K_o = \frac{1}{2}\left(18.3\frac{kN}{m^3}\right)(4m)^2 (0.41) = 60\,kN\ per\ meter\ of\ wall$

2) When the wall deflects to the left sufficiently to develop an active pressure condition,

$K_a = \tan^2\left(45° - \frac{\phi}{2}\right) = \tan^2\left(45° - \frac{36°}{2}\right) = 0.26$

$The\ force\ F_a = \frac{1}{2}\gamma_d h^2 K_a = \frac{1}{2}\left(18.3\frac{kN}{m^3}\right)(4m)^2 (0.26) = 38\,kN\ per\ meter\ of\ wall$

3) The buoyant weight γ' of the flooded sand is,

$\gamma' = \gamma_{sat} - \gamma_w = \gamma_d - n\gamma_w - \gamma_w = 18.3 + (0.31)(9.81) - (9.81) = 11.5\frac{kN}{m^3}$

The stress at point "a" is $\sigma_a = 0$, and at "b" which is 1 meter below the surface,

$\sigma_b = \gamma_d h K_a = (18.3\frac{kN}{m^3})(1m)(0.26) = 4.8\ kN/m^2 \quad \therefore\ F_1 = \frac{1}{2}(4.8kN/m^2)(1m) = 2.4\ kN/m$

$$\sigma_{bc} = \gamma_d h K_a = (18.3\frac{kN}{m^3})(1m)(0.26) = 4.8 \; kN/m^2 \quad \therefore \; F_2 = (4.8kN/m^2)(3m) = 14.4 \; kN/m$$

$$\sigma_c = \gamma' h K_a = (11.5\frac{kN}{m^3})(3m)(0.26) = 9.0 \; kN/m^2 \quad \therefore \; F_3 = \frac{1}{2}(9.0kN/m^2)(3m) = 13.5 \; kN/m$$

The water pressure and force,

$$\sigma_w = \gamma_w h = (9.81\frac{kN}{m^3})(3m) = 29.4 \; kN/m^2 \qquad \qquad \therefore \; F_4 = \frac{1}{2}(29.4kN/m^2)(3m) = 44.1 \; kN/m$$

Therefore $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad R = \displaystyle\sum_{i=1}^{4} F = 74.4 \; kN/m$

The location of the resultant is $\overline{y}$,

$$\overline{y} = \frac{F_1 d_1 + F_2 d_2 + F_3 d_3 + F_4 d_4}{R} = \frac{(2.4)(3.33m)+(14.4)(1.5m)+(13.5)(1m)+(44.1)(1)}{74.4}$$

$\overline{y} = 1.17m$ from the bottom of the wall.
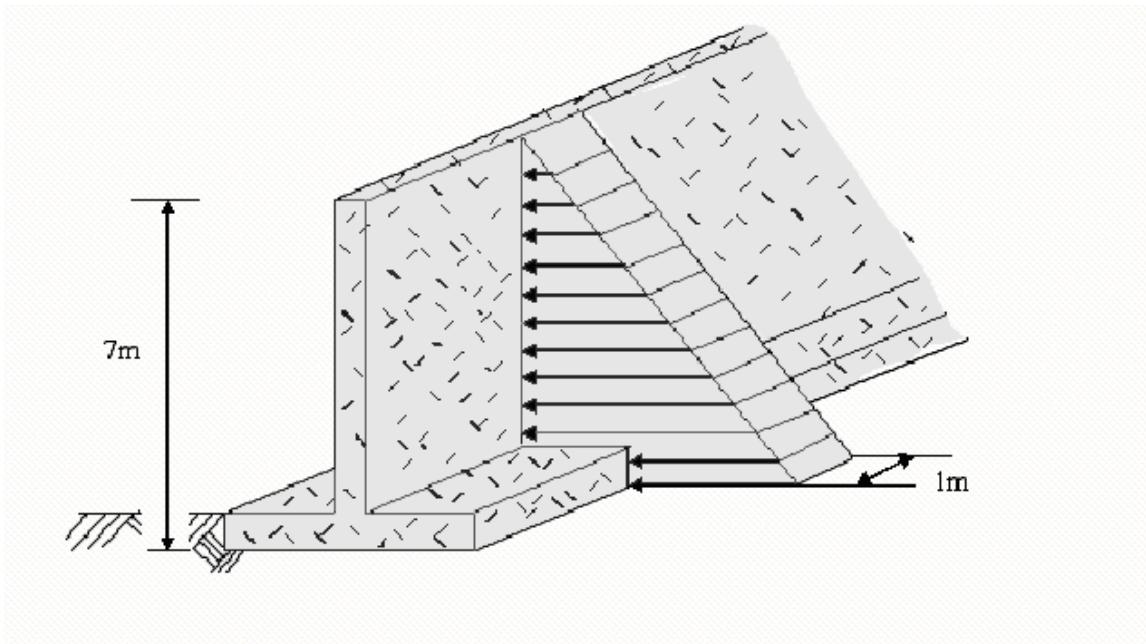
The percent increase in load upon the wall due to flooding is,

$$\Delta F = \frac{(74.4kN - 38kN)}{38kN} = 96\% \; increase.$$

(Revision: Sept-08)

A 7.0 m high retaining wall has a horizontal backfill of dry sand with a unit weight of 17.2 kN/m³ and an angle of internal friction φ = 32°. The wall is cast-in-place concrete, with a friction angle δ = 20°. Ignoring the effect of the passive pressure upon the toe of the footing, find the magnitude of the active earth force upon a length of wall equal to 3.5 m assuming Rankine conditions.

Solution:

The force applied to the wall first requires the coefficient of active earth pressure,

$$K_a = \tan^2(45° - \frac{\phi}{2}) = \tan^2(45° - \frac{32°}{2}) = 0.307$$

The horizontal force F_H per unit width of wall is,
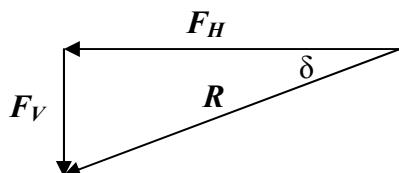
$$F_H = \frac{1}{2}\gamma h^2 K_a = \frac{1}{2}(17.2 kN/m^3)(7m)^2(0.307) = 129.5 \ kN/m$$

The F_H is related to the total force R on the wall as a function of the angle of wall friction δ,

$$F_H = R\cos\delta \quad \therefore \quad R = \frac{F_H}{\cos\delta} = \frac{129.5 kN/m}{\cos 20°} = 138 \ kN/m$$

We are asked what is the total force every 3.5 m,
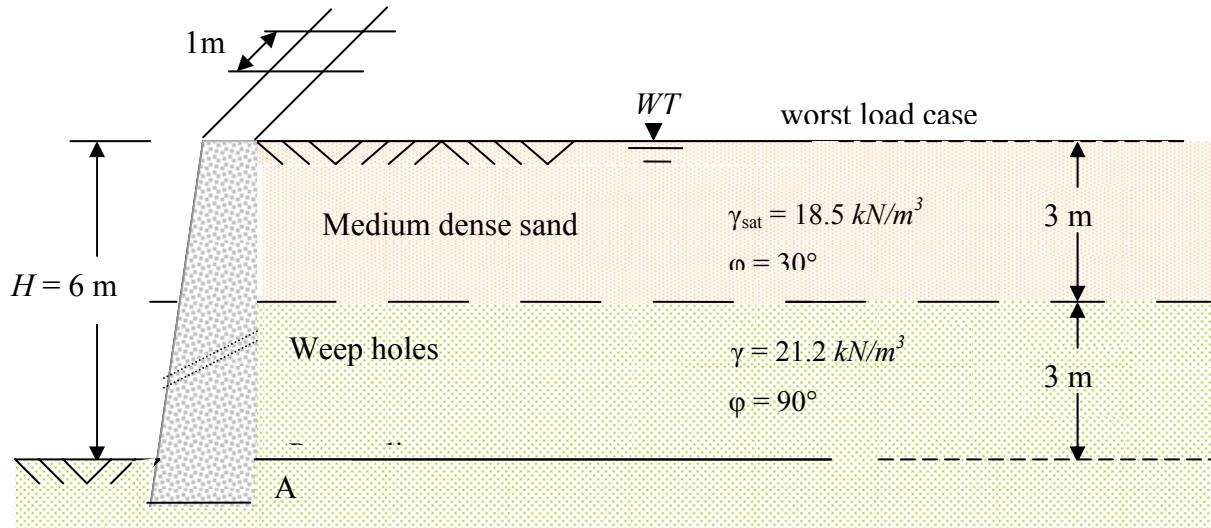
$$Total \ Active \ Force \ every \ 3.5 \ m = (138)(3.5m) = 482kN$$

(Revision: Sept-08)

Calculate the active force F_a and its location $\hat{y}$ with respect to the heel of the 6 m wall (point A), for the worst case (clogged weep holes).

Solution.

The worst active pressure load occurs when the water table raises to the top of the wall.

$$K_{a\ sand} = \tan^2(45° - \frac{\varphi}{2}) = \tan^2(45° - \frac{30°}{2}) = 0.333$$

$$K_{a\ limestone} = \tan^2(45° - \frac{\varphi}{2}) = \tan^2(45° - \frac{90°}{2}) = 0 \quad \therefore \quad \text{the limestone does not load the wall.}$$

The $\phi = 90°$ is really a combination of shear and cohesion ("cementation").

$$p_1 = \gamma'h_1 K_a = (\gamma_{SAT} - \gamma_w)h_1 K_a = (18.5 - 9.8)(3)(0.33) = \quad 8.7 \ kN/m^2$$
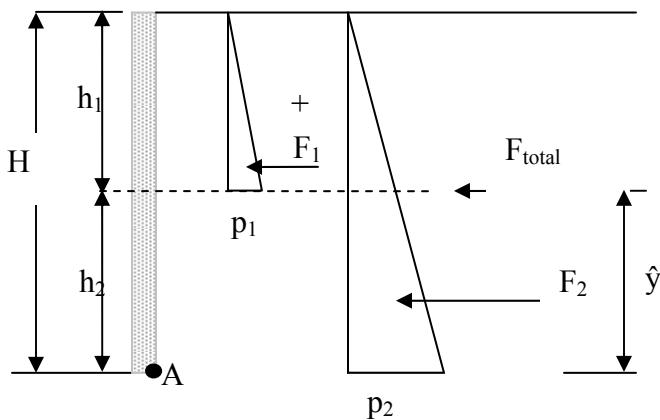
$$p_2 = \gamma_w H = (9.8)(6 m) \qquad\qquad\qquad = \quad 58.8 \ kN/m^2$$

$$F_1 = \frac{1}{2}\ p_1 h_1 = (0.5)(8.7)(3 m) \qquad\qquad = \quad 13.1 \ kN/m$$

$$F_2 = \frac{1}{2}\ p_2 H = (0.5)(58.8)(6 m) \qquad\qquad = \underline{176.4 \ kN/m}$$

$$F_{total} \qquad\qquad\qquad\qquad\qquad\qquad\qquad = 189.5 \ kN/m$$
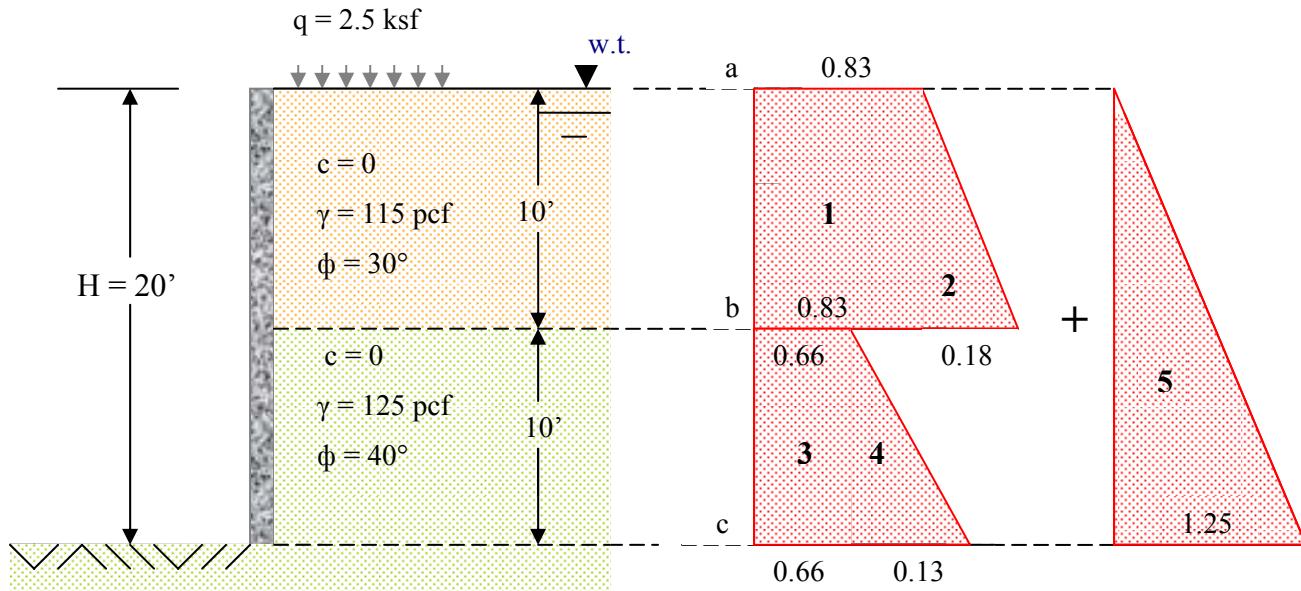
The location $\bar{y} = \dfrac{y_1 F_1 + y_2 F_2}{F_{total}} = \dfrac{(4\ m)(13.1) + (2\ m)(176.4)}{(189.5)} = 2.1m$ from A.

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(Revised Oct-09)

Draw the pressure diagram on the wall in an active pressure condition, and find the resultant F_{total} on the wall and its location with respect to the top of the wall.

Solution:

Step 1

Ka1 = tan2 (45°- 30°/2) = 0.333
Ka 2 = tan2 (45°- 40°/2) = 0.217

Step 2

The stress on the wall at point a is: pa = q Ka 1 = (2.5) (0.333) = 0.83 ksf
The stress at b (within the top stratum) is: pb'+ = (q + γ'h) Ka 1

= [2.5 + (0.115 - 0.0624) (10')] [0.333]   = 1.01 ksf

The stress at b (within bottom stratum) is: pb' - = (q + γ'h) Ka 2

= [2.5 + (0.115 – 0.0624) (10')] [0.217]   = 0.66 ksf

The stress at point c is: pc' = [q + (γ'h)1 + (γ'h)2] Ka 2

= [2.5 + (0.115 – 0.0624) (10') + (0.125 – 0.0624)(10')] [0.217]   = 0.79 ksf

The pressure of the water upon the wall is: pw = γwh = (0.0624) (20') = 1.25 ksf

Step 3

The forces from each area:

$F_1 = (10') (0.83) \quad = \quad 8.30 \ \text{kips/ft}$

$F_2 = \frac{1}{2} (10')(0.18) = \quad 0.90 \ \text{kips/ft}$

$F_3 = (10') (0.66) \quad = \quad 6.60 \ \text{kips/ft}$

$F_4 = \frac{1}{2} (10')(0.13) = \quad 0.65 \ \text{kips/ft}$

$F_5 = \frac{1}{2} (1.25) (20') = 12.5 \ \text{kips/ft}$

F_{total} = 29.0 kips/ft

Step 4

The location of forces $\hat{y}$ is at:

$$\hat{y} = \frac{5 \cdot 8.3 + \frac{20}{3} \cdot 0.9 + 15 \cdot 6.6 + \frac{50}{3} \cdot 0.65 + \frac{40}{3} \cdot 12.5}{29} = 0.66 \ ksf$$
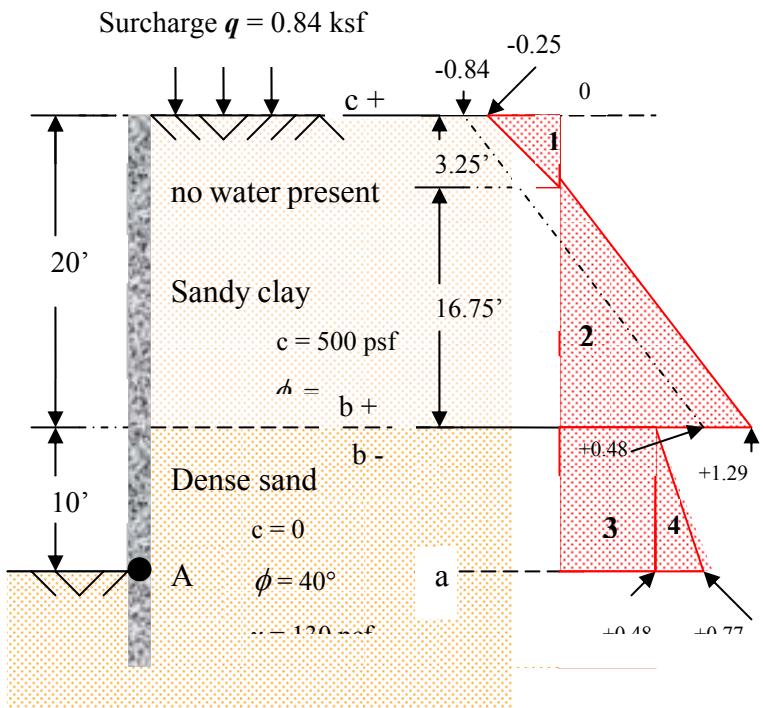
The stress at point c is: **$\hat{y}$ = 11.2 feet from top of wall**

The sheet pile wall shown below is flexible enough to permit the retained soil to develop an active earth pressure condition. Calculate the magnitude of the resultant F_{total} of the active force above the point "A" upon the wall. Assume Rankine conditions.

Solution:

Notice that the vertical pressure diagram will always increase in magnitude, but the horizontal pressures are governed by the K_a coefficient, which may increase or decrease the pressures on the wall.

Lateral load from the surcharge

$\sigma_{c+} = K_{a1} q = (0.70)(0.84 \ kcf) = 0.59$ ksf

$\sigma_{c-} = -2c \sqrt{K_{a1}} = -2(0.5) \sqrt{0.70} = -0.84$ ksf

$\therefore \sum \sigma_c = 0.59 - 0.84 = -0.25$ ksf

$\sigma_{b+} = K_{a1} \gamma h - 2c \sqrt{K_{a1}} + q K_a$

$= (0.7)(0.11)(20') - (2)(0.50)\sqrt{0.70}$

$= 1.29$ ksf

$\sigma_{b-} = K_{a\,2}\,\gamma\,h - 0 = (0.22)\,(0.11)\,(20') \quad = 0.48 \text{ ksf}$

$\sigma_a = 0.48 + K_{a\,2}\,\gamma\,h = 0.48 + (0.22)(0.13)(10') \quad = 0.48 + 0.29 = 0.77 \text{ ksf}$
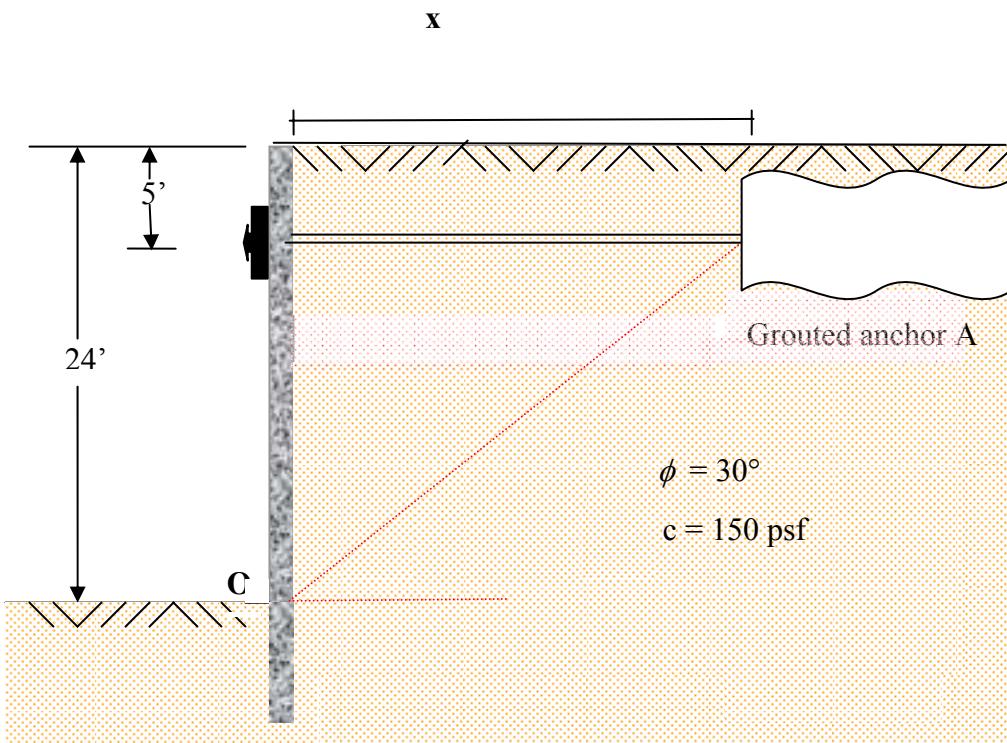
$K_{a\,1} = \tan^2(45° - \phi/2) \quad = \tan^2 40° = 0.70$ $F_1 = \frac{1}{2}(-0.25)(3.25') \quad = -0.41 \text{ k/ft (tension)}.$

$K_{a\,2} = \tan^2(45° - 40°/2) = \tan^2 25° = 0.22$ $F_2 = \frac{1}{2}(1.29)(16.75') \quad = +10.80 \text{ k/ft}$

$F_3 = (0.48)(10') \quad = +4.80 \text{ k/ft}$

$F_4 = \frac{1}{2}(0.29)(10') \quad \underline{= +1.45 \text{ k/ft}}$

$\boldsymbol{F_{total}} \qquad \textcolor{red}{\boldsymbol{= +16.6 \text{ kip/ft}}}$

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**Lateral-11: *Anchoring to help support a wall.*

(Revision: Sept.-08)

The wall shown below will be used to retain the sides of an excavation for the foundations of a large building. The engineer has decided to use earth anchors in lieu of braces or rakers to stabilize the wall.

(1) **What is the minimum distance *x* from the anchor to behind the wall?**

(2) **What is your recommended factor of safety for the anchor? What is an economical load for the anchor?**

x

Grouted anchor A

24'

$\phi = 30°$

$c = 150$ psf

C

Solution:

(1) The anchor must be beyond the passive slip plane, or (x) tan 30° = 19' or x = 33 feet.

(2) $K_a = \tan^2(45° - \varphi/2) = 0.33$ and $K_p = \tan^2(45° + \varphi/2) = 3.0$

The active force upon the wall per unit width F_a is:

$F_a = \frac{1}{2}\gamma H^2 K_a - 2cH\sqrt{K_a}$ = ½(0.105)(24')2(0.33) - 2(0.15)(24)$\sqrt{0.33}$ = 5.84 kip/ft   with the force located at $\hat{y} = \frac{1}{3}(19') = 6.33'$ above point O (note that the tensile portion does not load the wall).

The potential passive failure force (from the anchor) on the wall F_p is:

$F_p = \frac{1}{2}\gamma H^2 K_p + 2cH\sqrt{K_p}$ = ½ (0.105)(24)^2(3) + 2(0.15)(24)$\sqrt{3}$ = 103 kip/ft

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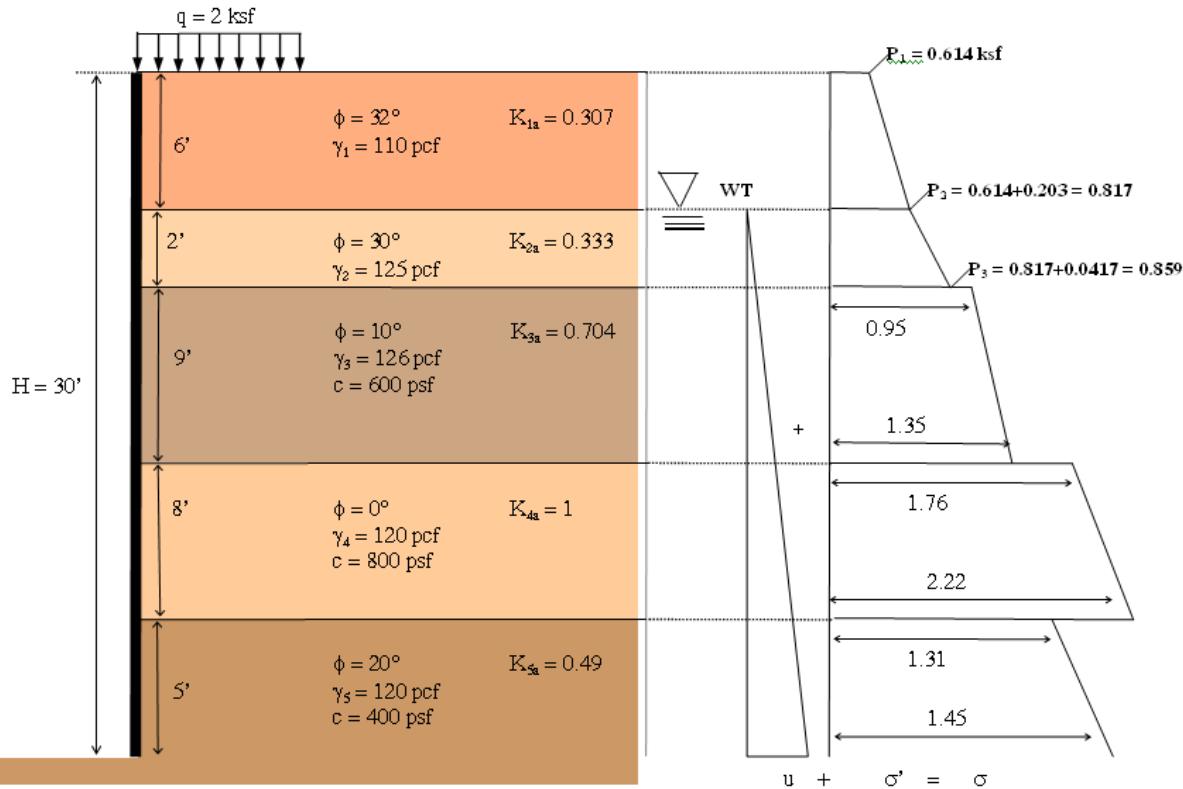
The factor of safety should be the same for an active failure as a passive failure. Therefore, a simple equation could be written as, $F_a(FS) = \dfrac{F_p}{(FS)}$ or $(FS)^2 = \dfrac{F_p}{F_a} = \dfrac{103\,kips}{5.84\,kips} = 17.6$ $\therefore$ $FS = 4.2$

Note that this corresponds to a load in the anchor of (5.84)(4.2) = <u>24.5 kips/ft</u> (which is the same as using the passive force = (103)/(4.2) = 24.5 kips/ft). The horizontal spacing of the anchors is not influenced by this analysis, and depends on cost factors. A common spacing would be 10 feet, which means *A = 245 kips*.

(Revision Oct-09)

Plot the pressure diagram and find the resultant force F and its location under an active pressure condition.

At h=0' $p_1 = q\,K_{1a} = (2)\,(0.307) = 0.614$ ksf

at h = -6' $\Delta p_2 = \gamma_1 h\,K_{1a} = (0.110)(6)\,(0.307) = 0.203$ ksf

at h = -8' $\Delta p_3 = (\gamma_2 - \gamma_w)h\,K_{2a} = (0.125 - 0.0624)(2)(0.333) = 0.417$ ksf

at h = -(8+dh)' $= [q + (\gamma_1)\,6' + (\gamma_2 - \gamma_w)\,2']\,K_{3a} - 2c\,\sqrt{(K_{3a}}$ from $p = \gamma h\,K_a - 2c\sqrt{K_a}$

$= [2 + (0.11)6' + (0.125 - 0.0624)2'](0.704) - 2(0.6)(0.84) = 0.95$ ksf
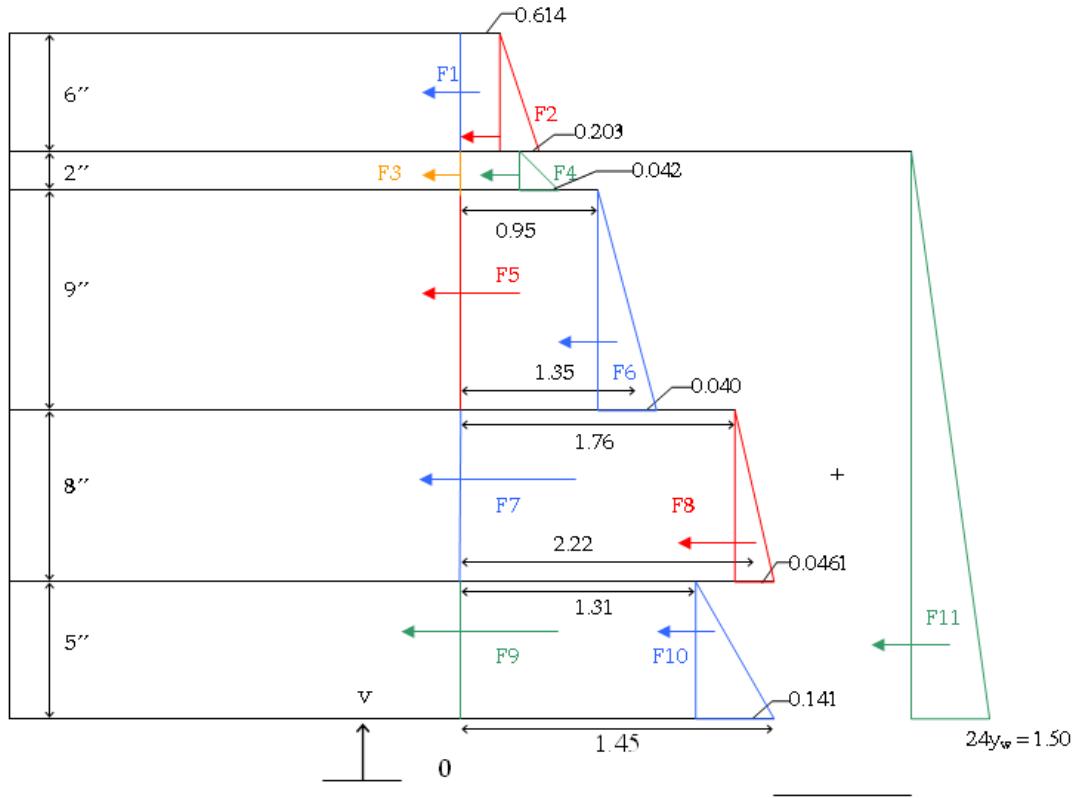
at h= -17' $\Delta p_4 = (\gamma_3 - \gamma_w)h\,K_{3a} = (0.126-0.0624)(9)(0.704) = 0.403$ ksf $\therefore 0.95+0.403 = 1.35$ ksf

at h = -(17 + dh)' $= [2 + 0.66 + 0.125 + (0.0626)\,(9)](1) -2(0.8)(1) = 1.76$ ksf

at h = -25' $\Delta p_5 = (\gamma_4 - \gamma_w)h\,K_{4a} = (0.120 - 0.0624)(8)(1) = 0.46$ ksf $\therefore 1.76 + 0.46 = 2.22$ ksf

at h = -(25 + dh)' $= [2 + 0.66 + 0.125 + 0.572 + 8(0.120 - 0.0624)](0.49) - 2(0.4)(0.7) = 1.13$ ksf

at h = -30' $\Delta p_6 = (\gamma_5 - \gamma_w)h\,K_{5a} = (0.120-0.0624)(5)\,(0.49) = 0.141$ ksf $\therefore 1.31+0.14 = 1.45$ ksf

F1 = (0.614)(6) = 3.68 kips   The resultant R is, **R = ∑ Fi = 57.1 kips**

F2 = 0.5(0.203)(6) = 0.61 kips

F3 = (0.817)(2) = 1.63 kips

F4 = 0.5(0.042)(2) = 0.04   The location of R is…….∑M$_0$ = 0 (about 0)

F5 = (0.95)(9) = 8.55 kips 57.09(y) = (3.68)(27) + (0.61)(26) = (1.63)(23)

F6 = 0.5(0.40(9) = 1.80 kip

F7 = (1.758)(8) = 14.1 kips

F8 = 0.5(0.461)(8) = 1.84 kips

F9 = (1.31)(5) = 6.55 kips

F10 = 0.5(0.141)(5) = 0.35 kips

F11 = 0.51(1.50)(24) = 18.0 kips   ∴ y = 611 / 57.1 = **10.7 feet above "0"**
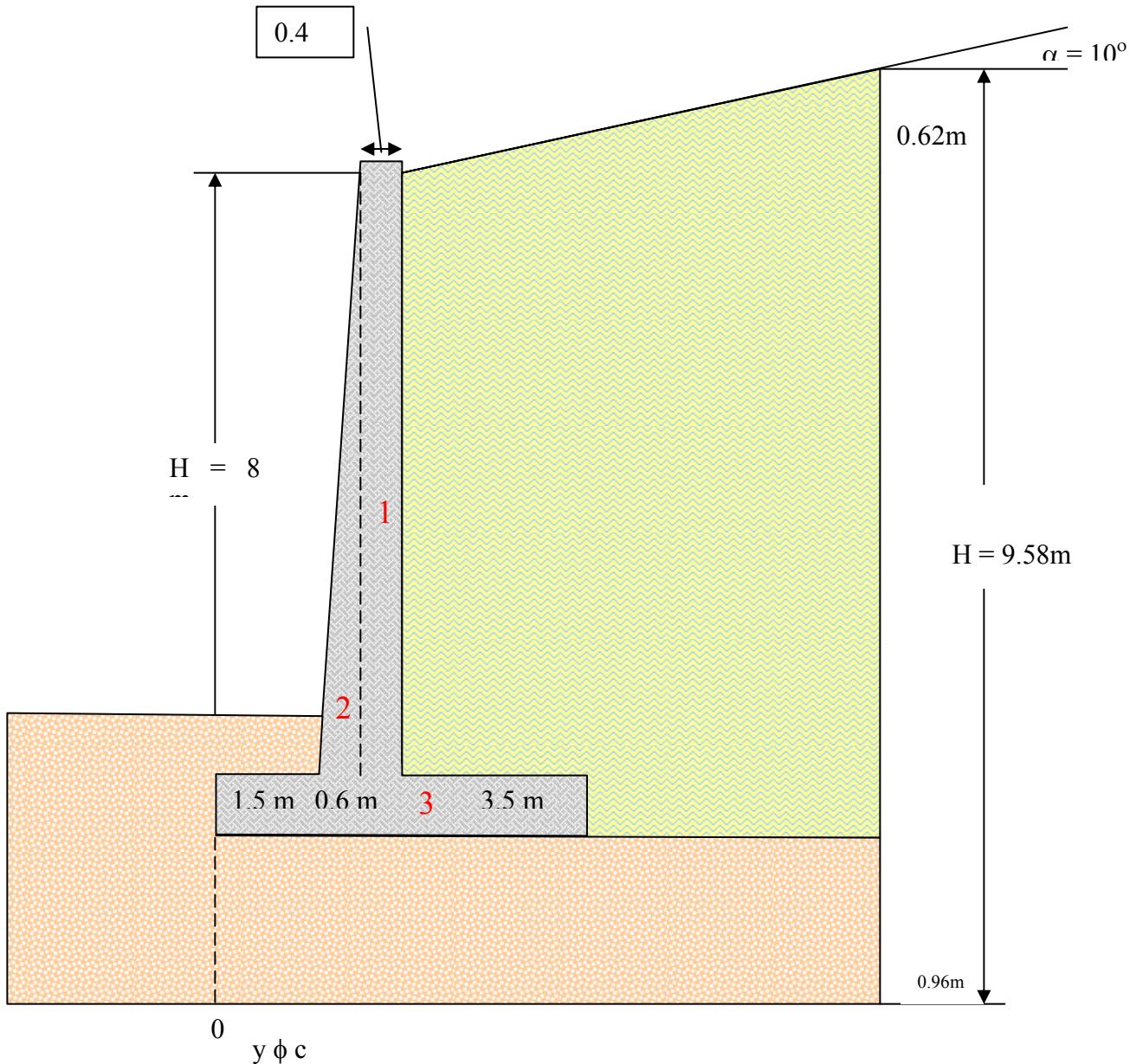
          57.1 kips

(Revised Oct-09)

Calculate the Factor of Safety against, (a) overturning, (b) sliding, and (c) bearing capacity failures.

$$\gamma_1 = 16.8 \frac{kN}{m^3}$$

$$\gamma_{conc} = 23.6 \frac{kN}{m^3}$$

$$\phi_1 = 32°$$

$$c_1 = 0$$

$$\gamma_2 = 17.6 \frac{kN}{m^3}$$

$$\phi_2 = 28°$$

$$c_2 = 30 \frac{kN}{m^2}$$

$$Ka = \cos\alpha \frac{\cos\beta - \sqrt{\cos^2\beta - \cos^2\phi'}}{\cos\beta + \sqrt{\cos^2\beta - \cos^2\phi'}} = \cos 10^o \frac{\cos 10^o - \sqrt{\cos^2 10^o - \cos^2 32^o}}{\cos 10^o + \sqrt{\cos^2 10^o - \cos^2 32^o}} = 0.322$$

Fa = (1/2) H^2 γ_1 Ka = (1/2)(9.58 m)^2(16.8 kN/m^3)(0.322) = 248 kN/m

F_v = F_a sin10° = (248 kN/m)(0.174) = 43.1 kN/m

F_h = F_a cos10° = (248 kN/m)(0.985) = 244 kN/m

a) The factor of safety against overturning is found by taking moments about point "O".

The resisting moment against overturning is M_R,

M_R = 23.6 kN/m^3[(0.4m)(8m)(1.90m) + (1/2)(0.2m)(8m)(1.63m) + (0.96m)(5.6m)(2.8m)] (1m)

    + 16.8 kN/m^3 [(3.5m)(8m)(3.85m) + (1/2)(0.617m)(3.5m)(4.43m)] (1m)

    + 43.1 kN/m (5.6m)(1m) = 2661 kN-m

and the overturning moment is M_O = Fh (1/3) H' = 244 kN/m (9.58m)(1/3) = 777 kN-m

FS_O = M_R / M_O = 3 .42

b) The factor of safety (FS_S) against sliding failure,

K_1 = K_2 = 2/3

Kp = tan^2(45° + 28°/2) = 2.77

Fp = (1/2) γ_2 H^2 Kp + 2 c_2 H $\sqrt{Kp}$

  = (0.5)(2.77)(17.6 kN/m^3)(1.75 m)^2 + (2)(30kN/m^2) $\sqrt{2.77}$ (1.75 m) = 249 kN/m

the driving force   = F_h = 244 kN/m

the resisting force = F_R = ΣV tan(2/3)(28) + (5.6)(2/3)(30) = 355 kN/m

FS_S = F_h / F_R = 1.46

c) the factor of safety (FS_{BC}) against a bearing capacity failure,

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$$e = \frac{B}{2} - \frac{\Sigma Mr - \Sigma Mo}{\Sigma V} = \quad e = 2.8 - \frac{2661 - 777}{749} = 0.31 \text{ m}$$

$$q_{toe} = \frac{\Sigma V}{B}\left[1 + \frac{6e}{B}\right] = \frac{749}{5.6}\left[1 + \frac{6(0.31)}{5.6}\right] = 178 \text{ kN/m}^2$$

B' = B – 2e = 5.6m – 2 (0.31m) = 4.98 m

$q_u = (1/2) \gamma_2 \, B' \, N_\gamma \, F_{\gamma d}{}^s \, F_{\gamma i}{}^d \; + \; C_2 \, N_c \, F_{cd}{}^s \, F_{ci}{}^d \; + \; q \, N_q \, F_{qd}{}^s \, F_{qi}{}^d$

$q = \gamma_2 \, D = (17.6)(1.75) = 30.8 \text{ kN/m}^2$

using $\phi_2 = 28° \; \rightarrow \; N_c = 25.8$

$$N_q = 14.7$$

$$N_\gamma = 16.7$$

$$F_{cd} = 1 + 0.4\frac{Df}{B'} = 1 + 0.4\frac{1.75}{4.98} = 1.14$$

$$\psi = \tan^{-1}\left(\frac{Ph}{\Sigma V}\right) = \tan^{-1}(244 / 749) = 18.04°$$

$$F_{qi} = F_{ci} = \left(1 - \frac{\psi}{90}\right)^2 = 0.96$$

$$F_{\gamma i} = \left(1 - \frac{\psi}{\phi}\right)^2 = \left(1 - \frac{18.04}{28}\right)^2 = 0.58$$

$F_{\gamma d} = 1$

$$F_{qd} = 1 + 2\tan\phi(1 - \sin\phi)^2\left(\frac{Df}{B}\right) = 1 + 2\tan 28°(1-\sin 28°)^2 \, (1.75/5.6)) =$$

$$= 1.08$$

$q_u = (1/2)(17.6)(4.98)(16.7)(1)(0.58) + (30)(25.8)(1.14)(0.96) + (30.8)(14.7)(1.08)(0.96) =$

$= 1740 \text{ kN/m}^2$

FS$_{BC}$ = q$_u$ / q$_{toe}$ = 1740 / 178.00 = 9.78
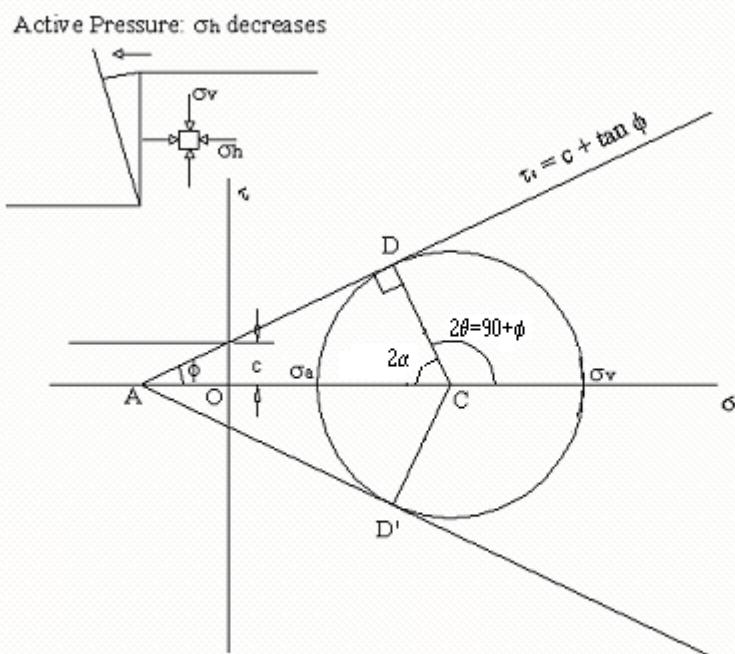
Derive a formula that provides K and σ_H as a function of σ_v.

(Revised Oct-09)

Using the Mohr-Coulomb failure criterion combined with Rankine's theory, find the coefficient of active earth pressure K_a as an investigation of the stress conditions in soil at a state of plastic equilibrium (in other words, when the soil mass is on the verge of failure).

Solution:

The definition of an "active pressure" condition is when σ_h decreases until it touches point D on the Mohr-Coulomb failure envelope.

Find σ_a:

From the figure,

$$\sin \phi = \frac{CD}{AC} = \frac{CD}{AO + OC}$$

$$CD = \text{radius of failure circle} = \left(\frac{\sigma_1 - \sigma_3}{2}\right) = \left(\frac{\sigma_v - \sigma_a}{2}\right)$$

$$AO = c \cot \phi$$

$$OC = \left(\frac{\sigma_1 + \sigma_3}{2}\right) = \left(\frac{\sigma_v - \sigma_a}{2}\right)$$

Substituting values into the equation for *CD, AO* and *OC* gives:

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$$\sin \phi = \frac{\left(\dfrac{\sigma_v - \sigma_a}{2}\right)}{\left[c \cot \phi + \left(\dfrac{\sigma_v + \sigma_a}{2}\right)\right]}$$

Rearrange the equation to make σ_a the subject:

This gives:

$$2c.\cos \phi + \left(\frac{\sigma_v + \sigma_a}{2}\right)\sin \phi = \frac{\sigma_v - \sigma_a}{2}$$

This then gives:

$$\sigma_a (\sin \phi + 1) = \sigma_v (1 - \sin \phi) - 2c.\cos \phi$$

$$\therefore \sigma_a = \sigma_v \left(\frac{1 - \sin \phi}{1 + \sin \phi}\right) - 2c\left(\frac{\cos \phi}{1 + \sin \phi}\right)$$

Solution of the trigonometric expressions:

$$2\alpha + (90° + \phi) = 180°$$

$$\alpha = \frac{90° + \phi}{2} = 45° - \frac{\phi}{2}$$
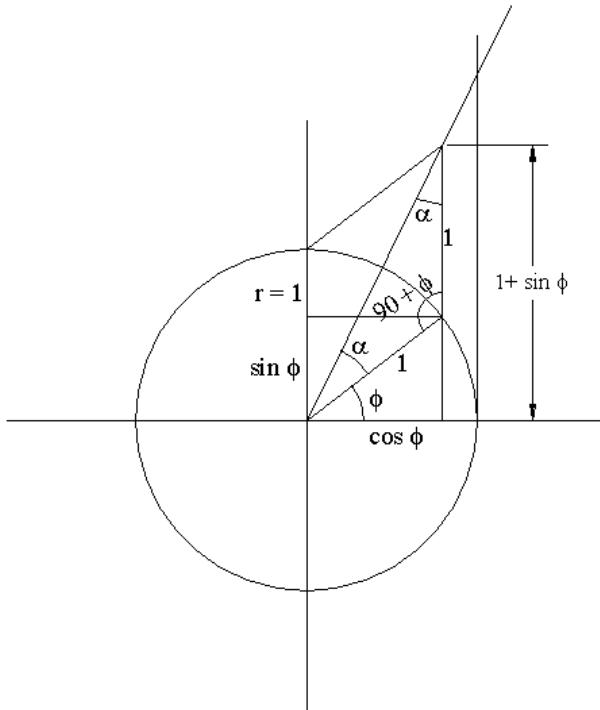
$$\frac{1 + \sin \phi}{\cos \phi} = \tan (\alpha + \phi)$$

$$= \tan \left(45° - \frac{\phi}{2} + \phi\right)$$

$$= \tan \left(45° + \frac{\phi}{2}\right)$$

$$= \frac{\sin \left(45° + \dfrac{\phi}{2}\right)}{\cos \left(45° + \dfrac{\phi}{2}\right)}$$

$$\therefore \frac{\cos \phi}{1 + \sin \phi} = \frac{\cos \left(45° + \dfrac{\phi}{2}\right)}{\sin \left(45° + \dfrac{\phi}{2}\right)}$$

But for any complementary angles β and $(90° - \beta)$, $\cos \beta = \sin (90° - \beta)$.

Thus, $\cos \left(45° \cdot \frac{\phi}{2}\right) \cdot \sin \left(45° - \frac{\phi}{2}\right)$ and $\sin \left(45° \cdot \frac{\phi}{2}\right) \cdot \cos \left(45° \cdot \frac{\phi}{2}\right)$

$$\therefore \frac{\cos\phi}{1+\sin\phi} = \frac{\sin\left(45° - \dfrac{\phi}{2}\right)}{\cos\left(45° - \dfrac{\phi}{2}\right)} = \tan\left(45° - \frac{\phi}{2}\right)$$
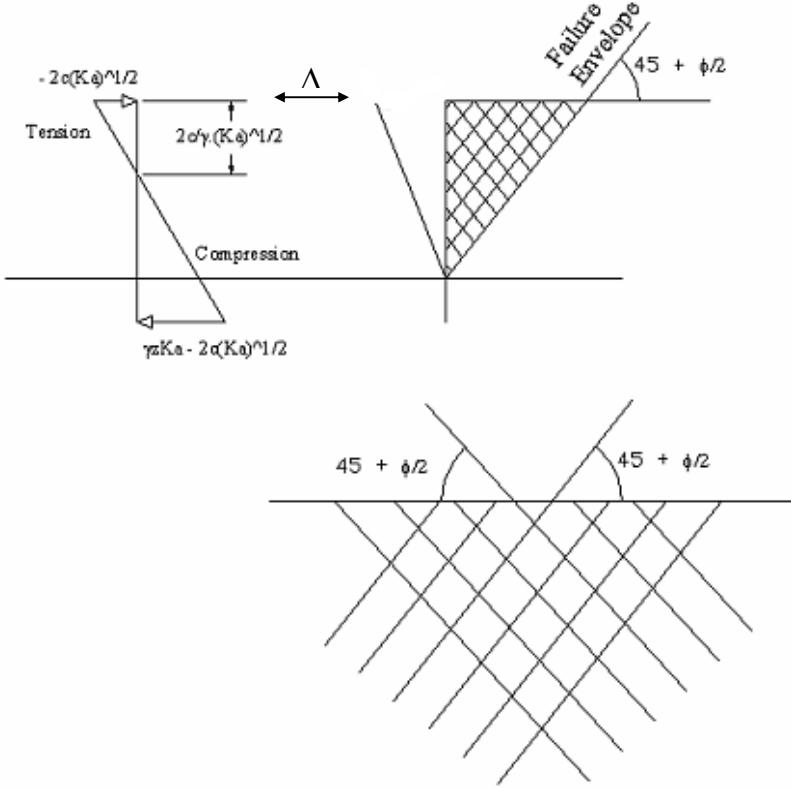
$$\therefore \frac{1-\sin\phi}{1+\sin\phi} = \left(\frac{1-\sin\phi}{1+\sin\phi}\right) \times \left(\frac{1+\sin\phi}{1+\sin\phi}\right) = \frac{1^2 - \sin^2\phi}{(1+\sin\phi)^2} = \frac{\cos^2\phi}{(1+\sin\phi)^2} = \tan^2\left(45° - \frac{\phi}{2}\right)$$

$$\sigma_a = \sigma_v \tan^2\left(45° - \frac{\phi}{2}\right) - 2c \tan\left(45° - \frac{\phi}{2}\right) \quad \longleftarrow \quad \boxed{\text{Rankine's expression gives the effective}}$$

$$\boxed{\therefore \sigma_a = (\gamma z)\, K_a - 2c\,(K_a)^{\frac{1}{2}} \ ; \text{ where } K_a = \tan^2\left(45° - \frac{\phi}{2}\right)}$$

Using this equation, the slip planes can be described by the grid of lines shown below:
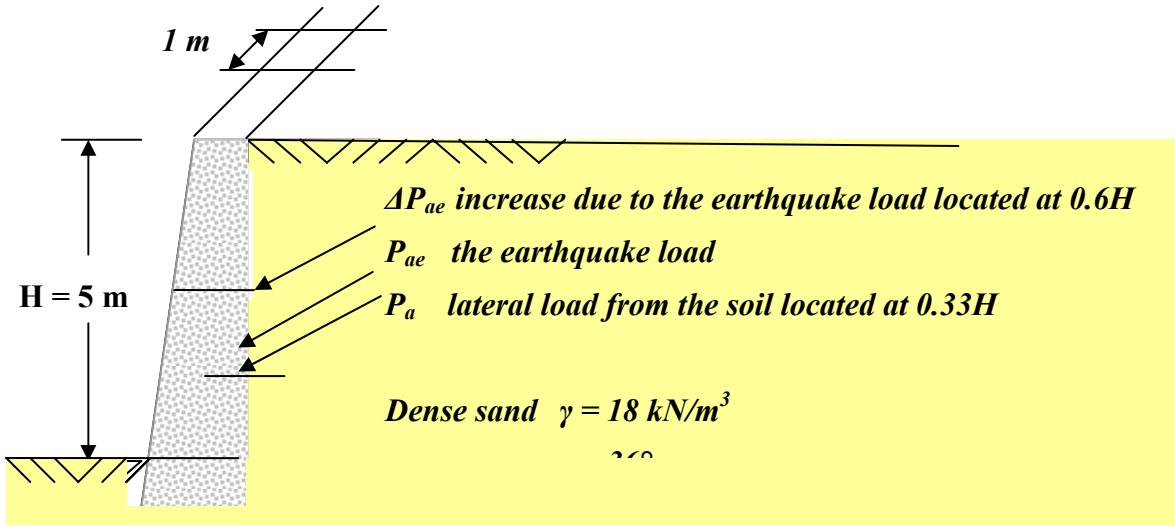
**Lateral-15: *The magnitude and location of a seismic load upon a retaining wall.*

(Revision: Sept-08)

The reinforced concrete retaining wall shown below will be subjected to a horizontal seismic load of 0.2 g without a vertical component. Determine,

(a) The magnitude of the active earth force P_a on the wall;

(b) The magnitude of the earthquake active earth force P_{ae} on the wall;

(c) The location of the resultant of both forces.

ΔP$_{ae}$ increase due to the earthquake load located at 0.6H
P$_{ae}$ the earthquake load
P$_a$ lateral load from the soil located at 0.33H
Dense sand γ = 18 kN/m³

___

<u>*Solution.*</u>

Calculate the coefficient of active earth pressure (Coulomb) K_a using,
$\phi = 36°, \alpha = 0°, \beta = 90°$ and $\delta = 2/3\phi = 24°$,

$$K_a = \frac{\sin^2(\beta+\phi')}{\sin^2\beta\sin(\beta-\delta)\left[1+\sqrt{\frac{\sin(\phi'+\delta)\sin(\phi'-\alpha)}{\sin(\beta-\delta)\sin(\alpha+\beta)}}\right]^2}$$

$$K_a = \frac{\sin^2(90°+36°)}{\sin^2(90°)\sin(90°-24°)\left[1+\sqrt{\frac{\sin(36°+24°)\sin(36°-0°)}{\sin(90°-24°)\sin(0°+90°)}}\right]^2}$$

$$K_a = \frac{\sin^2(126°)}{\sin(66°)\left[1+\sqrt{\frac{\sin(60°)\sin(36°)}{\sin(66°)}}\right]^2} = \frac{0.654}{(0.914)\left[1+\sqrt{\frac{(0.866)(0.588)}{(0.914)}}\right]^2} = 0.2346$$

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The active earth force P_a is,

$$P_a = \frac{1}{2}\gamma H^2 K_a = (0.5)(18kN/m^3)(5m)^2(0.2346) = 53\ kN/m$$

Calculate the earthquake coefficient of active earth pressure (Coulomb) K_{ae},

$$k_h = 0.2,\ k_V = 0,\ \delta = 2/3\phi = 2/3(36°) = 24°\ and\quad \theta' = \tan^{-1}\left[\frac{k_h}{1-k_V}\right] = \tan^{-1}(0.2) = 11.3°$$

$$\therefore K_{ae} = \frac{\sin^2(\phi+\beta-\theta')}{\cos\theta'\sin^2\beta\sin(\beta-\theta'-\delta)\left[1+\sqrt{\frac{\sin(\phi+\delta)\sin(\phi-\theta'-\alpha)}{\sin(\beta-\delta-\theta')\sin(\alpha+\beta)}}\right]^2}$$

$$K_{ae} = \frac{\sin^2(36°+90°-11.3°)}{\sin^2(90°)\sin(90°-24°)\left[1+\sqrt{\frac{\sin(36°+24°)\sin(36°-0°)}{\sin(90°-24°)\sin(0°+90°)}}\right]^2} = 0.372$$

The Mononobe-Okabe earthquake active earth force P_{ae} is,

$$P_{ae} = \frac{1}{2}\gamma H^2(1-k_V)K_{ae} = (0.5)(18kN/m^3)(5m)^2(1-0)(0.372) = 83.7\ kN/m$$

The earthquake force is $\Delta P_{ae} = P_{ae} - P_a = 83.7 - 53 = 30.7\ kN/m$

The location of the resultant earthquake force $\bar{z}$ is, found by locating the force P_{ae} at a height 0.6H above the base of the wall; the active earth force P_a is obviously $(0.33)H$ above the base.
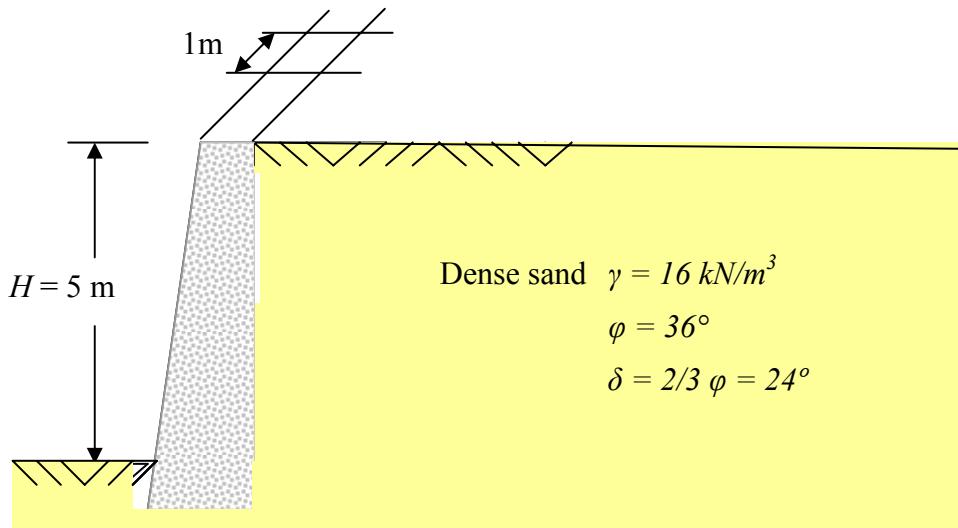
$$\bar{z} = \frac{(0.6H)(\Delta P_{ae})+(1/3)H(P_a)}{P_{ae}} = \frac{(0.6)(5m)(30.7kN/m^2)+(1/3)(5m)(53kN/m^2)}{(83.7kN/m^2)} = 2.1\ m$$

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**Lateral-16: Seismic loading upon a retaining wall.

(Revision: Aug-08)

The reinforced concrete retaining wall shown below will be designed to a horizontal seismic loading of 0.2 g. Assume no vertical seismic component (k_v=0). Determine,

(a) The weight of the wall W_w under static conditions;

(b) The weight of the wall under seismic conditions, for zero lateral displacement;

(c) The weight of the wall under seismic conditions, for a lateral displacement = 1.5 inches.

Solution.

Chapter 16
Braced Cuts for Excavations
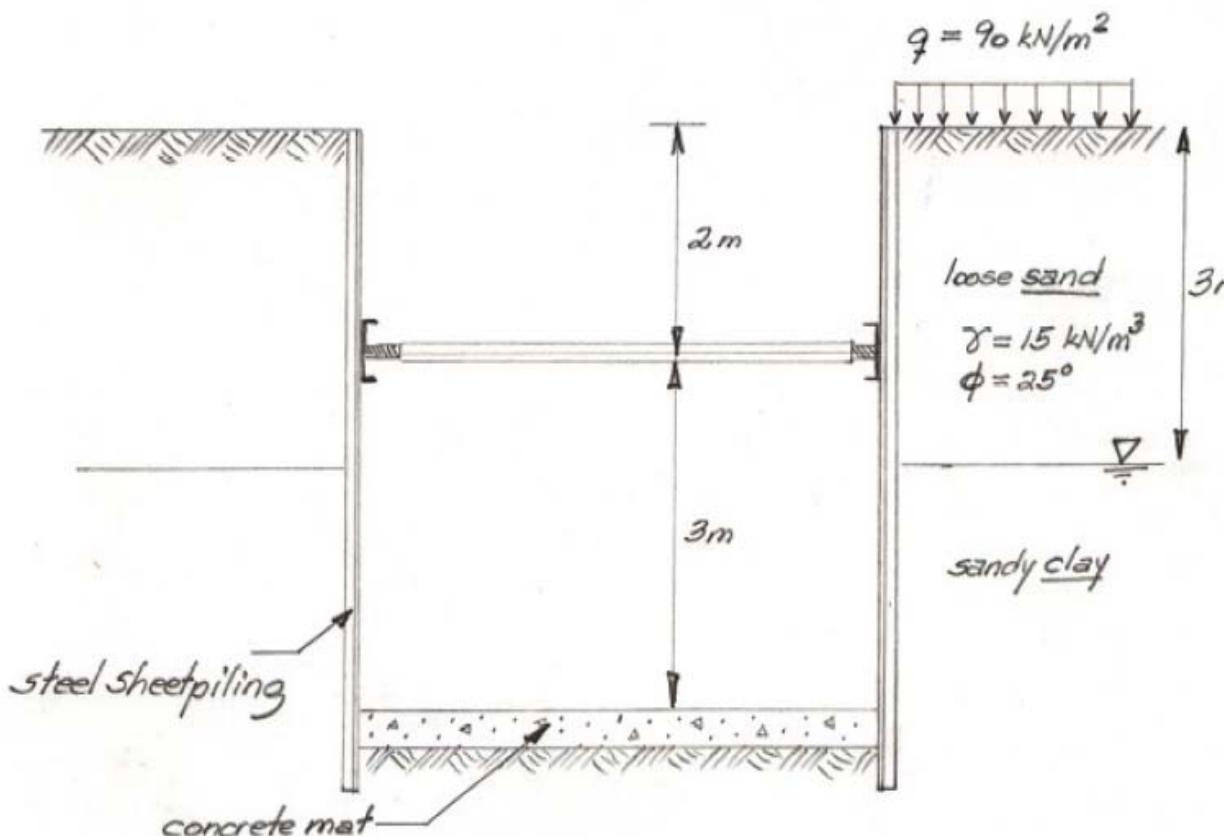
*Braced-cuts-01: *Forces and moments in the struts of a shored trench.*

(Revision: Sept-08)
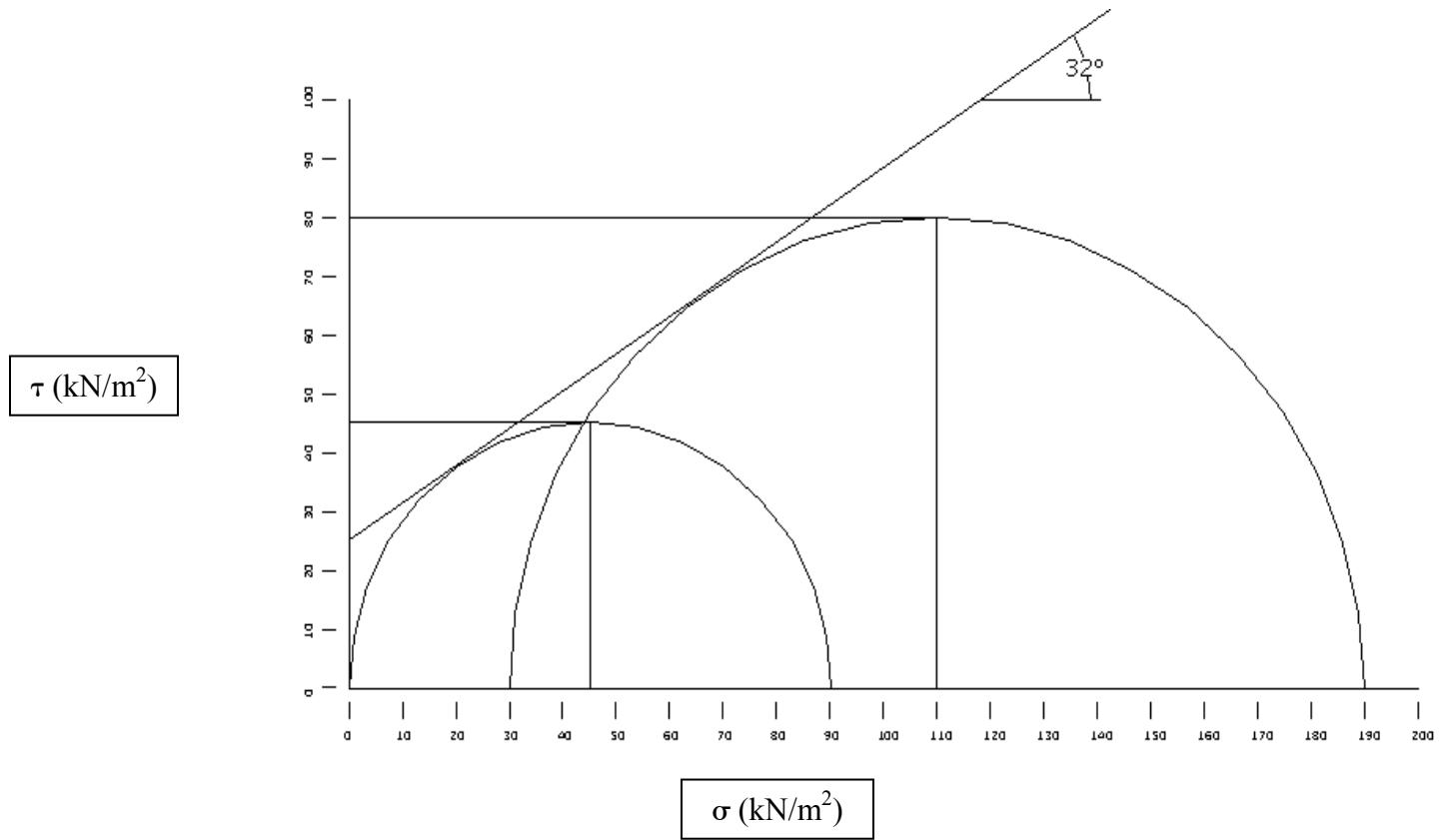
You have been asked by a contractor to design the internal supports (struts) of a temporary utility trench, as shown below. In order to design the steel horizontal strut shown, you must first *find the force and moment* on one of them, if they are spaced every 4 m horizontally.

Two triaxial laboratory tests were performed on samples of the clayey sand. The first sample had a confining pressure of 0 kN/m², and the sample reached failure with a deviator stress of 90 kN/m². (N.B.: the deviator stress is the additional vertical stress required to reach failure, i.e. s-1 to s-3). The second sample had its confining stress increased to 30 kN/m². The deviator stress needed to attain failure was 160 kN/m².

Further laboratory tests show that this clayey sand had an in-situ voids ratio of 0.46 at a moisture of 34% (assume $G_s = 2.65$). Show all your calculations.

$q = 90 \ kN/m^2$

2 m

loose sand   3r

$\gamma = 15 \ kN/m^3$
$\phi = 25°$

3m

sandy clay

steel sheetpiling

concrete mat

Effective Stress Mohr's Circle for failure Angle

τ (kN/m²)

σ (kN/m²)
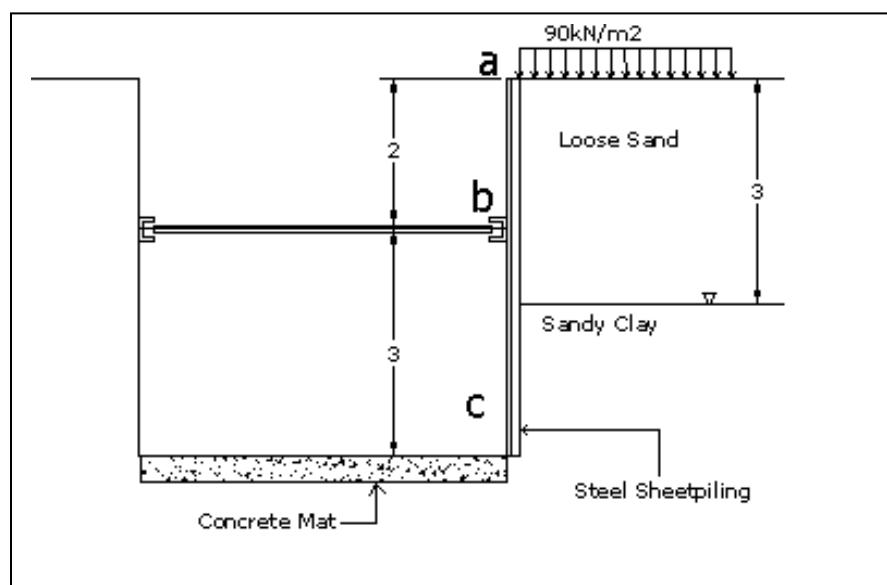
From the Mohr's Circle, we can get that $\phi_2 = \underline{\mathbf{\textit{32}^o}}$

$G_s = 2.65$; $\gamma_w = 9810$ N/m² $\rightarrow$ $\gamma_s = \dfrac{(G_s)(\gamma_W)}{1+e} = \dfrac{(2.65)(9810)}{1+0.46}$ $\rightarrow$ $\gamma_s = \gamma_2 = \underline{\mathbf{\textit{17.8 kN/m}^2}}$

$$\boxed{K_A = \tan^2(45^o - \frac{\phi}{2})}$$

$K_{A1} = \tan^2(45^o - \dfrac{25^o}{2}) = \mathbf{\textit{0.406}}$

$K_{A2} = \tan^2(45^o - \dfrac{32^o}{2}) = \mathbf{\textit{0.307}}$

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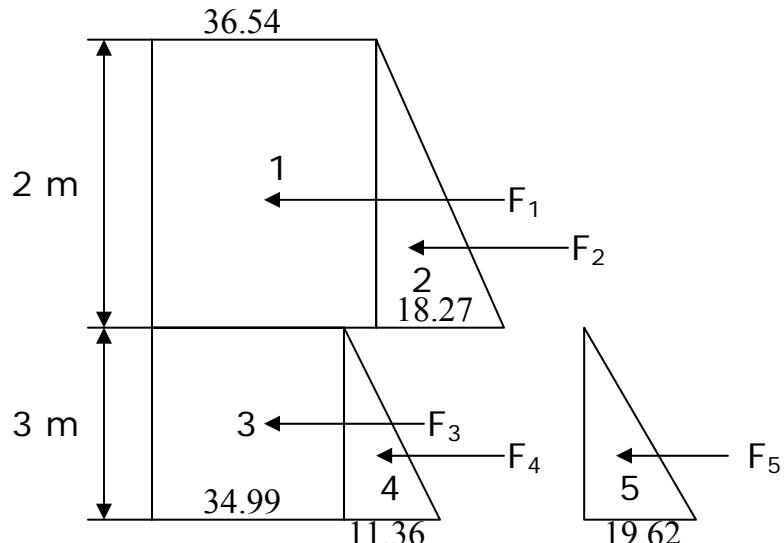
$$\mathbf{P_a} = (q)\,(K_{A1}) = (90\text{kN/m}^2)\,(.406) \rightarrow \boldsymbol{36.54\ kN/m^2}$$

$$\mathbf{P_{b'}+} = [K_{A1}(q + \gamma_1 h_1)] = [(.406)\,(15\text{kN/m2} \times 3\text{m})] \rightarrow \boldsymbol{54.81\ kN/m^2}$$

$$\mathbf{P_{b'-}} = [K_{A2}(q + (\gamma_2\text{-}\gamma_w)\,h] = [(.307)\,(90 + (17.8\text{-}9.81)\,(3)] \rightarrow \boldsymbol{34.99\ kN/m^2}$$

$$\mathbf{P_c} = [(q + \gamma_1 h_1 + (\gamma_2\text{-}\gamma_w)\,h_2]\,K_{A2} = .307\,[90 + (15)(3) + (17.8\text{-}9.81)(2)] \rightarrow \boldsymbol{46.35\ kN/m^2}$$

$$\mathbf{P_w} = \gamma_w\,h_w = (9.81)(2) \rightarrow \boldsymbol{19.62\ kN/m^2}$$

Location of the Forces (with respect to the top datum):

$\mathbf{F_1}$: 3m (1/2) = **1.5m**

$\mathbf{F_2}$: 3m (2/3) = **2.0m**

$\mathbf{F_3}$: 3m + 2m (1/2) = **4.0m**

$\mathbf{F_4}$: 3m + 2m (2/3) = **4.33m**

$\mathbf{F_5}$: 3m + 2m (2/3) = **4.33m**

Magnitude of the Forces:

$$F_1 = (P_a)(\,h_1) = (36.54\ \text{kN/m}^2)(3\text{m}) = \boldsymbol{109.6\ kN/m}$$

$$F_2 = (P_{b}+ \text{-}\ P_a)(\,h_1/2) = (54.81\text{-}36.54)(3/2) = \boldsymbol{27.4\ kN/m}$$

$$F_3 = (P_b\text{-})(\ h_2) = (34.99 \ kN/m^2) \ (2m) = \textbf{\textit{69.98 kN/m}}$$
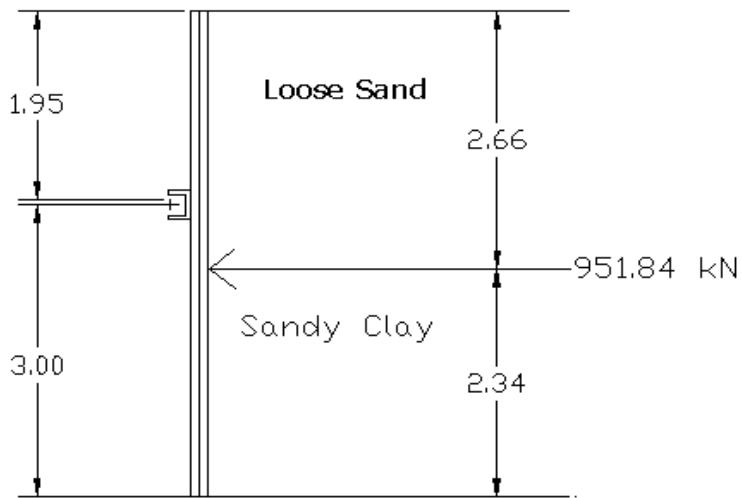
$$F_4 = (P_c - P_b\text{-})(\ h_2/2) = (46.35\text{-}34.99)(2/2) = \textbf{\textit{11.36 kN/m}}$$

$$F_5 = (P_w) \ (h_w/2) = (19.62)(2/2) = \textbf{\textit{19.62 kN/m}}$$

$$\sum F = F_1 + F_2 + F_3 + F_4 + F_5 = \textbf{\textit{237.96 kN/m}}$$

$$F_{tot} = \left(\sum F\right)(\text{space b/t struts}) = (237.96 kN/m)(4m) \rightarrow \underline{\textbf{\textit{951.84 kN}}}$$

Located at $y_f = \dfrac{109.6(1.5) + 27.4(2) + 69.98(4) + 11.36(4.33) + 19.62(4.33)}{237.96} = 2.66m$

$\boxed{\sum M_c = 0}$ Where C is located at the bottom of the trench along with R_A
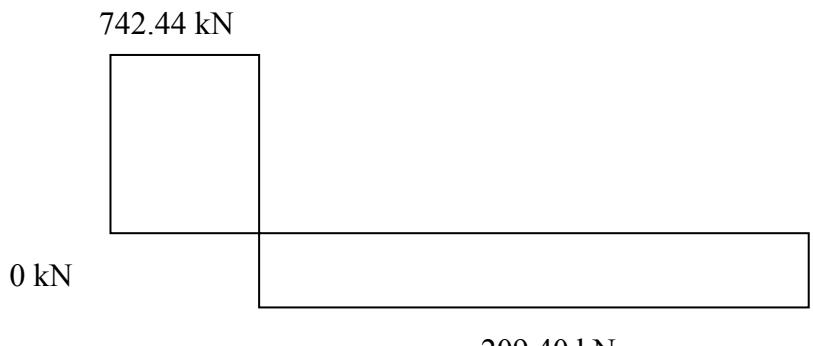
R_B is located at the end of the strut.

$\rightarrow R_B \ (3m) - 951.84 \ kN \ (2.34m) = 0$
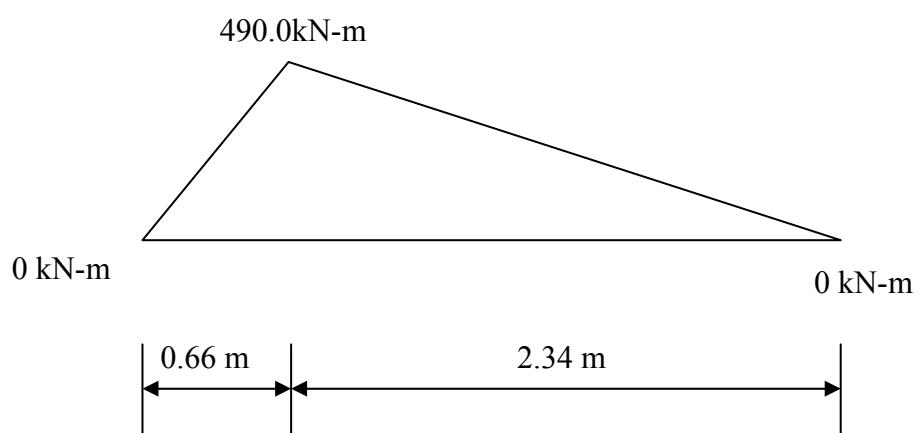
$\rightarrow \textbf{\textit{R}}_B = \textbf{\textit{742.44 kN}}$

$\rightarrow \textbf{\textit{R}}_A = \textbf{\textit{209.40 kN}}$

Shear Diagram
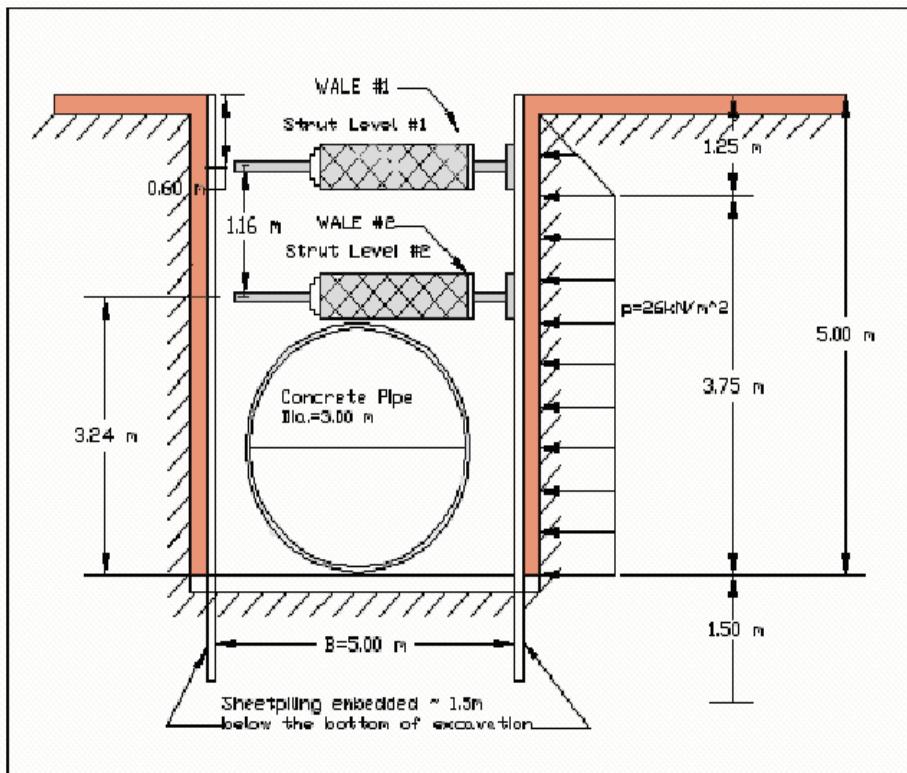
742.44 kN

0 kN

-209.40 kN

Moment Diagram

490.0kN-m

0 kN-m

0 kN-m

0.66 m

2.34 m

(Revision: Sept-08)

Design a braced excavation for a large sanitary sewer force-main, which is a reinforced concrete pipe with a diameter of 3 m. The trench should be 5 m deep and 5 m wide. The phreatic surface is below bottom of excavation. The SPT for the silty clay is $N_{avg} = 20$, and $\gamma = 17$ kN/m³. Assume $\varphi = 0$.

Solution:

Use Stroud's relation to estimate the un-drained cohesion of the soil (the previous problem provided the shear strength):

$c_u = KN = (3.5 \text{ kN/m}^2)(20) = 70$ kN/m².

Therefore,

$$if \quad \frac{\gamma H}{c_u} > 4 \quad the\,clay\,is\,soft\,to\,medium$$

$$if \quad \frac{\gamma H}{c_u} \le 4 \quad the\,clay\,is\,stiff$$

In this problem, $\dfrac{\gamma H}{c_u} = \dfrac{(17)(5)}{(70)} = 1.21 < 4$ ∴ this is a stiff clay

Also, since $\gamma H / c_u < 6$, the sheet-piling should extend at least 1.5 m below bottom.

Step 1. Establish the lateral earth pressure distribution.

Using Peck's (1967) apparent pressure envelope, we must choose the larger of,

$$(1)\ p_a = \gamma H \left[1 - \left(\frac{4c_u}{\gamma H} \right) \right]$$
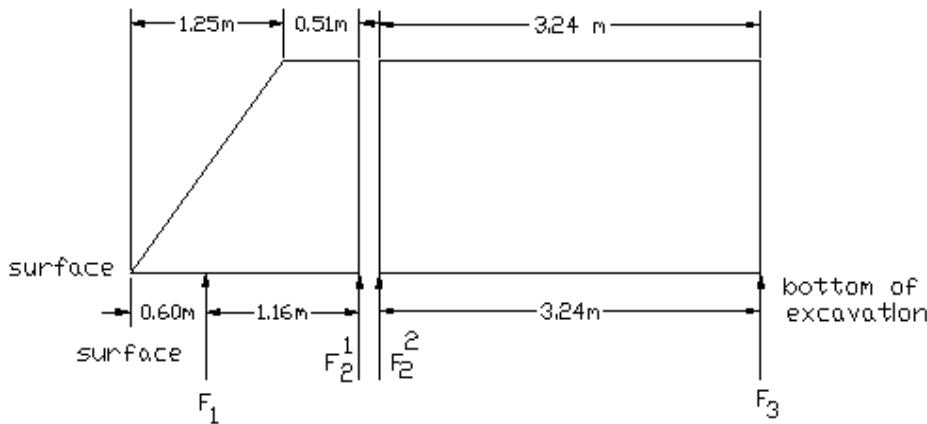
$$(2)\ p_a = 0.3\gamma H = 0.3(17)(5) = 25.5 \, kN/m^2$$

The location of the top strut should be less then the depth of the tensile crack z_c. Since $\varphi = 0$,

$$Ka \rightarrow \sqrt{Ka} = 1. \text{ therefore} \qquad \sigma_3 = \sigma_a = (\gamma)(z_c)K_a - 2c\sqrt{Ka}$$

therefore $z_c = 2c/\gamma = 2(70 \text{ kN/m}^2)/ 17 \text{ kN/m}^3 = 8.2 \text{ m} >> 0.6 \text{ m}$ **OK**

Step 2: Determine the lateral loads at strut locations and excavation bottom.

Isolation the left portion between the surface and strut #2.

$\sum M_{F'_2} = 0 \qquad = F_1(1.16m)-(0.5)(1.25m)(26)[0.51+1.25/3]-(0.51)(26)[0.51/2] = 0$

therefore, $F_1 = 15.9 \text{ kN/m}$

$\sum F_y = 0 \qquad = -15.9 + 1/2\ (1.25)(26)+(0.51)(26)- F'_2 = 0 \quad$ therefore, $F'_2 = 13.6 \text{ kN/m}$

Isolating the right portion between strut #2 and the trench bottom, by symmetry

$F_2^2 = F'_2 = 13.6 \text{ kN/m} \qquad \sum F_y = 0 \qquad = -F_2^2 + (3.75-0.51)(26)- F_3$

therefore, $F_3 = 70.6 \text{ kN/m}$

Step 3: Find the maximum moment M_{max} in the sheet-piling.

Finding moments at A, B, & C (that is, the areas under the shear diagram):

$M_A = \frac{1}{2}(0.60)(12.48)(0.60/3) = 0.75$ kN-m/m

$M_B = \frac{1}{2}(1.25)(26)(1.25/3)-15.9(0.65) = 3.56$ kN-m/m

$M_C = (2.71)(26)(2.71/2) = 96$ kN-m/m

Obviously $M_{max} = 96$ kN-m/m

Step 4: Select the steel-piling .

Assume $f_y = 50$ ksi $= 345$ MN/m², therefore $\sigma_{allow} = 50\%f_y = 172$ MN/m²

The required section modulus S

$S = M_{Max}/ \sigma_{all} = 96$ kN-m/ $172,000$ kN/m² $= 0.00056$m³ $= 56$ m³/ m-10^5

Choose a PDA-27 section, which provides 57.46 m³/ m-10^5.

Step 5: Select the horizontal waler at each strut level.

At strut level #1 the load F1 is 16 kN/m. Select the horizontal spacings to be 4 m.   (May use 3 m to reduce steel size, but increases the difficulty of placing the concrete pipes).

$M_{max} = F_1s^2/8 = (16)(4)^2/8 = 32$ kN-m (where s is the spacing)

therefore, $S_{wale\ at\ 1} = M_{max}/\sigma_{allow} = 32$ kN-m/ $172,000$ kN/m² $= 18.6$ m³/ m-10^5

 At strut level #2 the load is 27.2 kN/m; the spacing s is $= 4$ m.

$M_{max=} F_2s^2/8 = (27.2)(4)^2/8 = 54.4$ kN-m

Therefore, $S_{wale\ at\ 2} = M_{max}/\sigma_{allow\ =}$ 54.4 kN-m/ $172,000$ kN/m² $= 31.6$ m³/ m-10^5

Notes: 1. The bottom of the trench has the highest lateral load, with 70.6 kN per every meter. Propose to cast a concrete "mud" slab at the bottom of the trench. Design the thickness of the slab (diaphragm).

 2. Wales are commonly channels or WF beams. Design the steel pipe wales and the struts, calculated in Step 6 below.

Step 6: Select the struts.

Level # 1 strut $= F_1s = (16$ kN/m$)(4$m$) = 64$ kN

Level # 2 strut $= 2\ F_2s = (27.2$ kN/m$)(4$m$) = 109$ kN (Design the steel for the struts).

Step 7: Check for possible heave of the excavation bottom.

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Braced cuts in clay may become unstable if the bottom heaves upward and fails a section of wall.

$FS_{against\ heaving}$ = [cN_c(0.84 + 0.16 B/L)]/ γH = (70)(6.4)(0.84)/(17)(5) = 4.4 > 2 **O.K.**

Step 8: Expected lateral yielding of the sheet-piling and ground settlement behind the wall.

Expect δ_h from 5 to 10 cms.
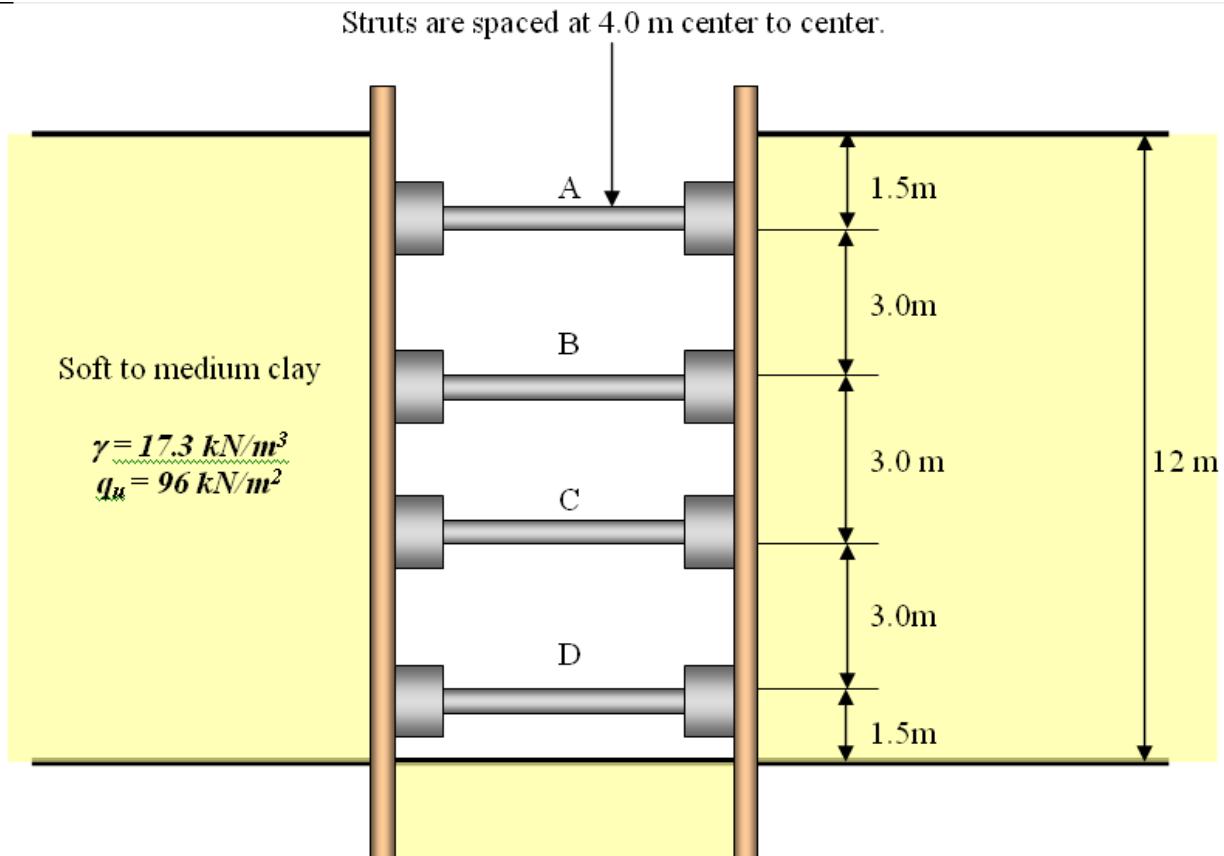
$\quad$ δ_γ from 1 to 5 cms.

(Revision: Sept-08)

A four-strut braced sheet pile installation is designed for an open cut in a clay stratum, as shown below. The struts are spaced longitudinally (in plan view) at 4.0 m center to center. Assume that the sheet piles are pinned or hinged at strut levels B and C.

Find: 1. The lateral earth pressure diagram for the braced sheet pile system.

2. The loads on struts A, B, C, and D.

Struts are spaced at 4.0 m center to center.

Soft to medium clay

$\gamma = 17.3\ kN/m^3$
$q_u = 96\ kN/m^2$

A — 1.5m

3.0m

B

3.0 m

C

3.0m

D

1.5m

12 m

Solution:

From Terzaghi and Peck (1967), a clay is soft, medium or stiff,

$$if \quad \frac{\gamma H}{c_u} > 4 \quad the\ clay\ is\ soft\ to\ medium \quad then \quad \sigma_a = \gamma H \left(1 - \frac{4c_u}{\gamma H}\right)$$

$$if \quad \frac{\gamma H}{c_u} \leq 4 \quad the\ clay\ is\ stiff \quad then \quad \sigma_a = 0.2\gamma H\ to\ 0.4\gamma H$$

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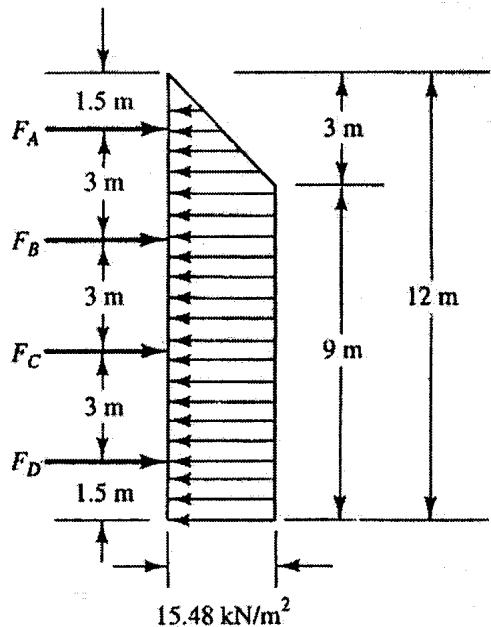
Determine the cohesion from Mohr's circle $c_u = \dfrac{q_u}{2} = \dfrac{96 \ kN/m^2}{2} = 48 \ kN/m^2$

$\therefore \dfrac{\gamma H}{c_u} = \dfrac{(17.3 \ kN/m^3)(12 \ m)}{(48 \ kN/m^2)} = 4.33 > 4 \quad \therefore this \ is \ a \ soft \ to \ medium \ clay$

Peck (1969) provided a criterion for soft to medium clays,

$p_a = \gamma H \left(1 - \dfrac{4 c_u}{\gamma H} \right) = (17.3 \ kN/m^3)(12 \ m) \left[1 - \dfrac{(4)(48 \ kN/m^2)}{(17.3 \ kN/m^3)(12 \ m)} \right] = 15.48 \ kN/m^2$

The lateral earth pressure diagram for the braced sheet pile system in soft clays is,

15.48 kN/m²

2. In the free body diagram, part (a), $\sum M_B = 0$

$\left(\tfrac{1}{2} \right)(15.48 \ kN/m^2)(3.0 \ m)(4.0 \ m)\left(1.5 \ m - \dfrac{3.0 \ m}{3} \right) + (1.5 \ m)(15.48 \ kN/m^2)(4.0 \ m)\left(\dfrac{1.5 \ m}{2} \right) - (F_A)(3.0 \ m) = 0$

$\therefore F_A = 100.6 \ kN$

From $\sum H = 0$

$F_{B1} = \left(\tfrac{1}{2} \right)(1.5 \ m + 4.5 \ m)(15.48 \ kN/m^2)(4.0 \ m) - 100.6 \ kN = 85.2 \ kN$

In the free body diagram, part (b)

$F_{B2} = F_{C1} = \left(\tfrac{1}{2} \right)(3.0 \ m)(15.48 \ kN/m^2)(4.0 \ m) = 92 \ kN$

In the free body diagram, part (c), $\sum M_C = 0$
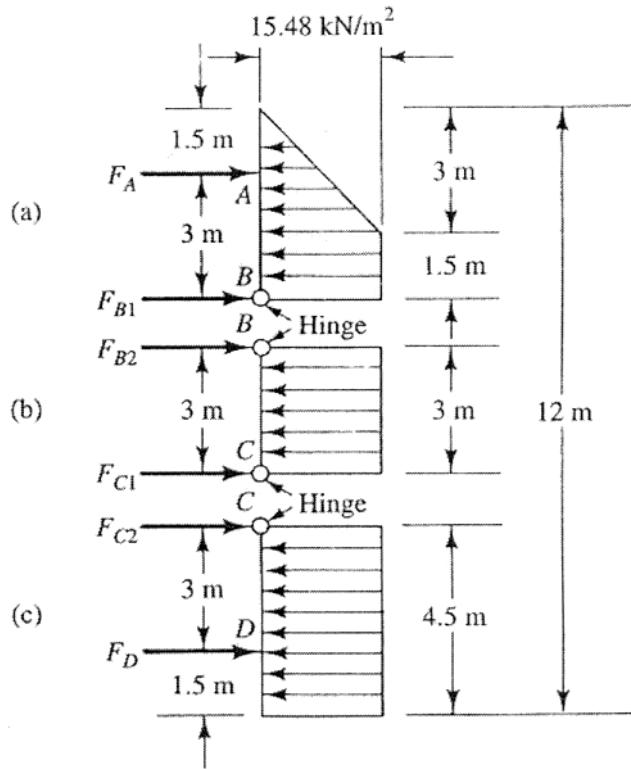
$(F_D)(3.0 \ m) - (4.5 \ m)(15.48 \ kN/m^2)(4.0 \ m)\left(\dfrac{4.5 \ m}{2} \right) = 0$

$\therefore F_D = 209.0 \ kN$

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From $\sum H = 0$

$$F_{C2} + F_D - (4.5\ m)(15.48\ kN/m^2)(4.0\ m) = 0$$

$$F_{C2} = (4.5\ m)(15.48\ kN/m^2)(4.0\ m) - 209.0\ kN = 69.6\ kN$$

Therefore,

$$F_A = 100.6\ kN$$

$$F_B = 85.2\ kN + 92.9\ kN = 178.1\ kN$$

$$F_C = 92.9\ kN + 69.6\ kN = 162.5\ kN$$

$$F_D = 209.0\ kN$$

Chapter 17
Bearing Capacity of Soils

Bearing Capacity Factors for General Shear

Angle φ (Degrees)	Angle φ (Radians)	Terzaghi				Meyerhof			Hansen		
		Kpγ	Nc	Nq	Nγ	Nc	Nq	Nγ	Nc	Nq	Nγ
0	0.0000	10.18	5.70	1.00	0.00	5.10	1.00	0.00	5.10	1.00	0.00
1	0.0175	10.61	6.00	1.10	0.08	5.38	1.09	0.00	5.38	1.09	0.00
2	0.0349	11.07	6.30	1.22	0.18	5.63	1.20	0.01	5.63	1.20	0.01
3	0.0524	11.56	6.62	1.35	0.28	5.90	1.31	0.02	5.90	1.31	0.02
4	0.0698	12.07	6.97	1.49	0.39	6.19	1.43	0.04	6.19	1.43	0.05
5	0.0873	12.61	7.34	1.64	0.51	6.49	1.57	0.07	6.49	1.57	0.07
6	0.1047	13.19	7.73	1.81	0.65	6.81	1.72	0.11	6.81	1.72	0.11
7	0.1222	13.80	8.15	2.00	0.80	7.16	1.88	0.15	7.16	1.88	0.16
8	0.1396	14.44	8.60	2.21	0.96	7.53	2.06	0.21	7.53	2.06	0.22
9	0.1571	15.13	9.09	2.44	1.15	7.92	2.25	0.28	7.92	2.25	0.30
10	0.1745	15.87	9.60	2.69	1.35	8.34	2.47	0.37	8.34	2.47	0.39
11	0.1920	16.65	10.16	2.98	1.58	8.80	2.71	0.47	8.80	2.71	0.50
12	0.2094	17.49	10.76	3.29	1.84	9.28	2.97	0.60	9.28	2.97	0.63
13	0.2269	18.38	11.41	3.63	2.12	9.81	3.26	0.74	9.81	3.26	0.78
14	0.2443	19.33	12.11	4.02	2.44	10.37	3.59	0.92	10.37	3.59	0.97
15	0.2618	20.36	12.86	4.45	2.79	10.98	3.94	1.13	10.98	3.94	1.18
16	0.2793	21.46	13.68	4.92	3.19	11.63	4.34	1.37	11.63	4.34	1.43
17	0.2967	22.65	14.56	5.45	3.63	12.34	4.77	1.66	12.34	4.77	1.73
18	0.3142	23.92	15.52	6.04	4.13	13.10	5.26	2.00	13.10	5.26	2.08
19	0.3316	25.30	16.56	6.70	4.70	13.93	5.80	2.40	13.93	5.80	2.48
20	0.3491	26.80	17.69	7.44	5.34	14.83	6.40	2.87	14.83	6.40	2.95
21	0.3665	28.42	18.92	8.26	6.07	15.81	7.07	3.42	15.81	7.07	3.50
22	0.3840	30.18	20.27	9.19	6.89	16.88	7.82	4.07	16.88	7.82	4.13
23	0.4014	32.10	21.75	10.23	7.83	18.05	8.66	4.82	18.05	8.66	4.88
24	0.4189	34.19	23.36	11.40	8.90	19.32	9.60	5.72	19.32	9.60	5.75
25	0.4363	36.49	25.13	12.72	10.12	20.72	10.66	6.77	20.72	10.66	6.76
26	0.4538	39.01	27.09	14.21	11.53	22.25	11.85	8.00	22.25	11.85	7.94
27	0.4712	41.78	29.24	15.90	13.15	23.94	13.20	9.46	23.94	13.20	9.32
28	0.4887	44.85	31.61	17.81	15.03	25.80	14.72	11.19	25.80	14.72	10.94
29	0.5061	48.26	34.24	19.98	17.21	27.86	16.44	13.24	27.86	16.44	12.84
30	0.5236	52.05	37.16	22.46	19.75	30.14	18.40	15.67	30.14	18.40	15.07
31	0.5411	56.29	40.41	25.28	22.71	32.67	20.63	18.56	32.67	20.63	17.69
32	0.5585	61.04	44.04	28.52	26.20	35.49	23.18	22.02	35.49	23.18	20.79
33	0.5760	66.40	48.09	32.23	30.33	38.64	26.09	26.17	38.64	26.09	24.44
34	0.5934	72.48	52.64	36.50	35.23	42.16	29.44	31.15	42.16	29.44	28.77
35	0.6109	79.40	57.75	41.44	41.08	46.12	33.30	37.15	46.12	33.30	33.92
36	0.6283	87.33	63.53	47.16	48.11	50.59	37.75	44.43	50.59	37.75	40.05
37	0.6458	96.49	70.07	53.80	56.62	55.63	42.92	53.27	55.63	42.92	47.38
38	0.6632	107.13	77.50	61.55	67.00	61.35	48.93	64.07	61.35	48.93	56.17
39	0.6807	119.59	85.97	70.61	79.77	67.87	55.96	77.33	67.87	55.96	66.76
40	0.6981	134.31	95.66	81.27	95.61	75.31	64.20	93.69	75.31	64.20	79.54
41	0.7156	151.89	106.81	93.85	115.47	83.86	73.90	113.99	83.86	73.90	95.05
42	0.7330	173.09	119.67	108.75	140.65	93.71	85.37	139.32	93.71	85.37	113.96
43	0.7505	198.99	134.58	126.50	173.00	105.11	99.01	171.14	105.11	99.01	137.10
44	0.7679	231.10	151.95	147.74	215.16	118.37	115.31	211.41	118.37	115.31	165.58
45	0.7854	271.57	172.29	173.29	271.07	133.87	134.87	262.74	133.87	134.87	200.81
46	0.8029	323.57	196.22	204.19	346.66	152.10	158.50	328.73	152.10	158.50	244.65
47	0.8203	391.94	224.55	241.80	451.28	173.64	187.21	414.33	173.64	187.21	299.52
48	0.8378	484.34	258.29	287.85	600.15	199.26	222.30	526.45	199.26	222.30	368.67
49	0.8552	613.53	298.72	344.64	819.31	229.92	265.50	674.92	229.92	265.50	456.40
50	0.8727	801.95	347.51	415.15	1155.97	266.88	319.06	873.86	266.88	319.06	568.57

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The bearing capacity of a soil is its ability to carry loads without failing in shear. There are four major methods to predict failure. The fist method was developed by Karl Terzaghi in 1943. Field tests in Canada by Meyerhof (1963) lead to modification factors. Finally, Brinch Hansen in Denmark (1970) and Vesic in the USA modified these factor to a greater refinement.

These bearing capacity factors are based on these three authors:

Terzaghi (1943):

For square footings, $\qquad\qquad q_{ult} = 1.3c'N_c + \overline{q}N_q + 0.4\gamma BN_\gamma$

For continuous or wall footings $\qquad q_{ult} = c'N_c + \overline{q}N_q + 0.5\gamma BN_\gamma$

where, $\quad \overline{q} = \gamma D_f \quad$ *and the factors are,*

$$N_q = \frac{a^2}{a\cos^2\left(45° - \varphi/2\right)} \quad \text{where } a = e^{(0.75\pi - \phi/2)\tan\phi}$$

$$N_c = \left(N_q - 1\right)\cot\phi$$

$$N_\gamma = \frac{\tan\phi}{2}\left(\frac{K_{p\gamma}}{\cos^2\phi} - 1\right)$$

Meyerhof (1963):

For vertical loads, $\qquad q_{ult} = cN_cF_{sc}F_{dc} + \overline{q}N_qF_{sq}F_{dq} + 0.4\gamma BN_\gamma F_{s\gamma}F_{d\gamma}$

and for inclined loads, $\quad q_{ult} = cN_cF_{ic}F_{dc} + \overline{q}N_qF_{iq}F_{dq} + 0.4\gamma BN_\gamma F_{i\gamma}F_{d\gamma}$

and the factors are,

$$N_q = e^{\pi\tan\phi}\tan^2\left(45° - \phi/2\right)$$

$$N_c = \left(N_q - 1\right)\cot\phi$$

$$N_\gamma = \left(N_q - 1\right)\tan\left(1.4\phi\right)$$

Brinch Hansen (1970):

The general equation, $\qquad q_{ult} = cN_cF_{sc}F_{dc}F_{ic} + \overline{q}N_qF_{sq}F_{dq}F_{iq} + 0.4\gamma BN_\gamma F_{s\gamma}F_{d\gamma}F_{i\gamma}$

and the factors are,

$$N_q = e^{\pi\tan\phi}\tan^2\left(45° - \phi/2\right)$$

$$N_c = \left(N_q - 1\right)\cot\phi$$

$$N_\gamma = 1.5\left(N_q - 1\right)\tan\phi$$

(Revision: Sept-08)
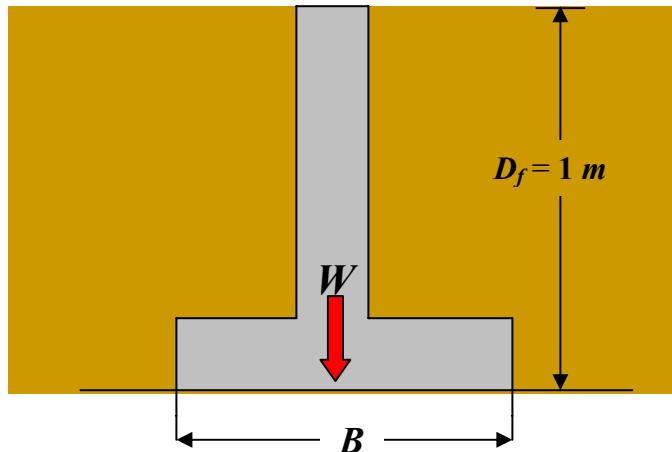
The square footing shown below must be designed to carry a 294 *kN* load. Use *Terzaghi's* bearing capacity formula to determine *B* of the square footing with a Factor of Safety =3.

$W = 294\ kN$

$D_f = 1\ m$

$\gamma\ = 18.15\ kN/m^3$

$\phi\ = 35°$

$c\ = 0$

W

B

Solution:

Terzaghi's formula for the ultimate bearing capacity q_{ult} of a square footing is,

$$q_{ult}\ =\ 1.3c'N_c\ +\ \bar{q}N_q\ +\ 0.4\gamma BN_\gamma \quad where \quad \bar{q} = D_f\gamma$$

The allowable bearing capacity q_{all} with the factor of safety of 3 is,

$$q_{all} = \frac{q_{ult}}{3} = \frac{1}{3}\left(1.3c'N_c + \bar{q}N_q + 0.4\gamma BN_\gamma\right) \quad and \quad q_{all} = \frac{W}{B^2} = \frac{294\ kN}{B^2}$$

$$or \quad \frac{294}{B^2} = \frac{1}{3}\left(1.3c'N_c + \bar{q}N_q + 0.4\gamma BN_\gamma\right)$$

For $\phi=35°$, $N_c=57.8$, $N_q=41.4$, and $N_\gamma=41.1$.

Substituting these values into Terzaghi's equation, we get

$$\frac{294}{B^2} = \frac{1}{3}\left[(0)+(18.15)(1)(41.4)+(0.4)(18.15)B(41.1)\right]$$
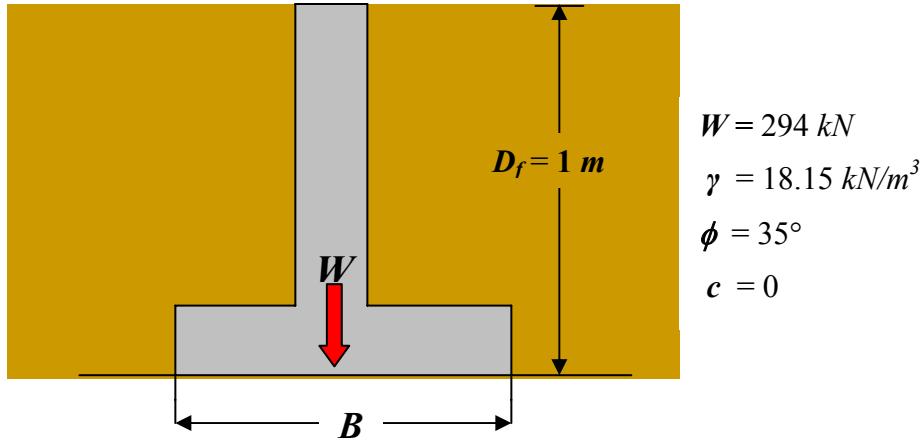
$$\frac{294}{B^2} = 250.5 + 99.5B$$

$$B^3 + 2.52B^2 - 2.96 = 0 \quad \therefore \quad B = \ 0.90\ m$$

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(Revision: Sept-08)

The square footing shown below must be designed to carry a 294 *kN* load. Use Meyerhof's bearing capacity formula to determine *B* with a factor of safety =3.

$W = 294\ kN$

$\gamma = 18.15\ kN/m^3$

$\phi = 35°$

$c = 0$

Solution:

Meyerhof's formula for the ultimate bearing capacity q_{ult} of a square footing is,

$$q_{ult} = c'N_cF_{sc}F_{dc}F_{ic} + \bar{q}N_qF_{sq}F_{dq}F_{iq} + 0.4\gamma BN_\gamma F_{s\gamma}F_{d\gamma}F_{i\gamma} \quad where \quad \bar{q} = D_f\gamma$$

Since the load is vertical, all three inclination factors $F_{ic}=F_{iq}=F_{i\gamma}=1$.

$$F_{sq} = 1 + \left(\frac{B}{L}\right)\tan\phi = 1 + \left(\frac{1}{1}\right)\tan 35° = 1.70 \quad and \quad F_{s\gamma} = 1 - 0.4\left(\frac{B}{L}\right) = 1 - 0.4(1) = 0.6$$

$$F_{dq} = 1 + 2\tan\phi(1-\sin\phi)^2\left(\frac{D_f}{B}\right) = 1 + 2(\tan 35°)(1-\sin 35°)^2\left(\frac{1}{B}\right) \approx 1.25 \quad and \quad F_{d\gamma} = 1$$

The allowable bearing capacity q_{all} with the factor of safety of 3 is,

$$q_{all} = \frac{q_{ult}}{3} = \frac{1}{3}\left(c'N_cF_{sc}F_{dc} + \bar{q}N_qF_{sq}F_{dq} + 0.4\gamma BN_\gamma F_{s\gamma}F_{d\gamma}\right) \quad and \quad q_{all} = \frac{W}{B^2} = \frac{294\ kN}{B^2}$$

$$or \quad \frac{294}{B^2} = \frac{1}{3}\left(c'N_cF_{sc}F_{dc} + \bar{q}N_qF_{sq}F_{dq} + 0.4\gamma BN_\gamma F_{s\gamma}F_{d\gamma}\right)$$

For $\phi = 35°$, $N_c = 46.12$, $N_q = 33.30$, and $N_\gamma = 37.15$.

Substituting these values into Meyerhof's equation, we get
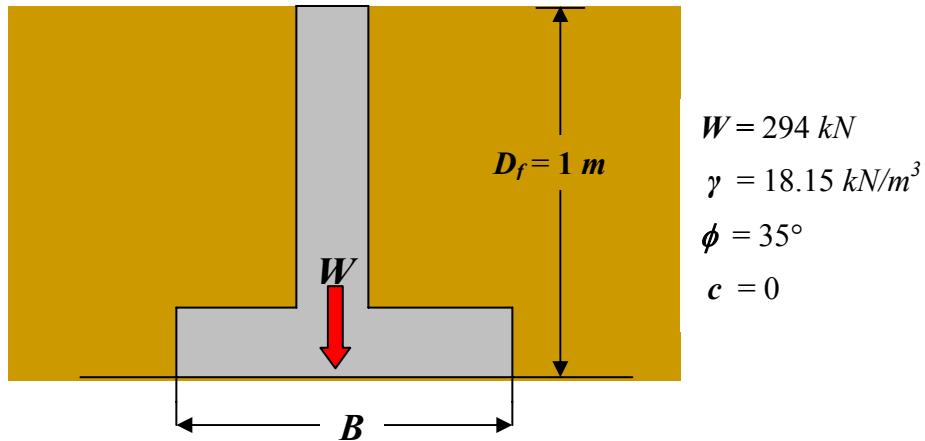
$$\frac{294}{B^2} = \frac{1}{3}\left[(0) + (18.15)(1)(33.3)(1.7)(1.25) + (0.4)(18.15)B(37.15)(0.6)(1)\right]$$

$$\frac{294}{B^2} = 428.1 + 53.94B \quad or \quad B^3 + 7.94B - 5.45 = 0 \quad \therefore B = 0.65\ m$$

(Revision: Sept-08)

The square footing shown below must be designed to carry a 294 *kN* load. Use Brinch Hansen's bearing capacity formula to determine *B* with a factor of safety =3.

$D_f = 1\ m$

$W = 294\ kN$
$\gamma = 18.15\ kN/m^3$
$\phi = 35°$
$c = 0$

W

B

Solution:

Hansen's formula for the ultimate bearing capacity q_{ult} of a square footing is,

$$q_{ult} = c' N_c F_{sc} F_{dc} F_{ic} + \bar{q} N_q F_{sq} F_{dq} F_{iq} + 0.4 \gamma B N_\gamma F_{s\gamma} F_{d\gamma} F_{i\gamma} \quad where \quad \bar{q} = D_f \gamma$$

Since the load is vertical, all three inclination factors $F_{ic} = F_{iq} = F_{i\gamma} = 1$.

$$F_{sq} = 1 + \left(\frac{B}{L}\right) \tan\phi = 1 + \left(\frac{1}{1}\right) \tan 35° = 1.7 \quad and \quad F_{s\gamma} = 1 - 0.4\left(\frac{B}{L}\right) = 1 - 0.4(1) = 0.6$$

$$F_{dq} = 1 + 2\tan\phi(1 - \sin\phi)^2 \left(\frac{D_f}{B}\right) = 1 + 2(\tan 35°)(1 - \sin 35°)^2 \left(\frac{1}{B}\right) \approx 1.255 \quad and \quad F_{d\gamma} = 1$$

The allowable bearing capacity q_{all} with the factor of safety of 3 is,

$$q_{all} = \frac{q_{ult}}{3} = \frac{1}{3}\left(c' N_c F_{sc} F_{dc} + \bar{q} N_q F_{sq} F_{dq} + 0.4 \gamma B N_\gamma F_{s\gamma} F_{d\gamma}\right) \quad and \quad q_{all} = \frac{W}{B^2} = \frac{294\ kN}{B^2}$$

$$or \quad \frac{294}{B^2} = \frac{1}{3}\left(c' N_c F_{sc} F_{dc} + \bar{q} N_q F_{sq} F_{dq} + 0.4 \gamma B N_\gamma F_{s\gamma} F_{d\gamma}\right)$$

For $\phi = 35°$, $N_c = 46.12$, $N_q = 33.30$, and $N_\gamma = 33.92$.

Substituting these values into Hansen's equation, we get
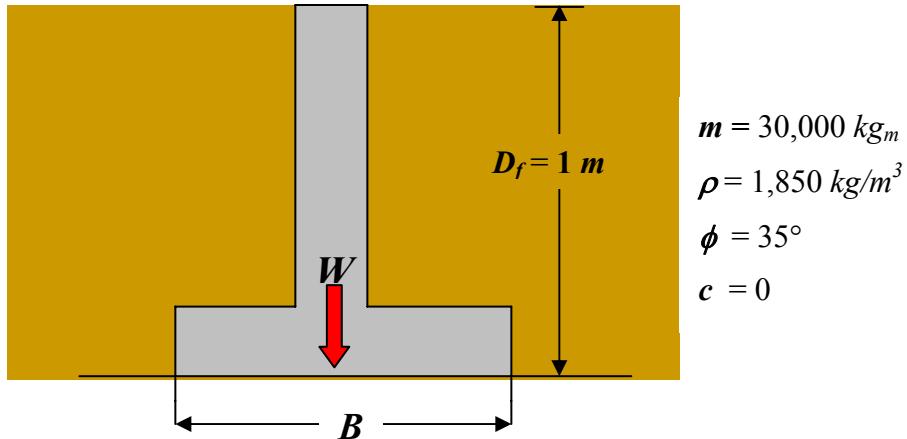
$$\frac{294}{B^2} = \frac{1}{3}\left[(0) + (18.15)(1)(33.3)(1.7)(1.255) + (0.4)(18.15)B(33.92)(0.6)(1)\right]$$

$$\frac{294}{B^2} = 429.8 + 49.25B \quad or \quad B^3 + 8.73B - 5.97 = 0 \quad \therefore B = 0.70\ m$$

(Revision: Sept-08)

The square footing shown below must be designed to a load of 30,000 kg_m. Using a factor of safety of 3 and using *Terzaghi's* method, determine the size B of the square footing.

$D_f = 1\ m$

$m = 30,000\ kg_m$

$\rho = 1,850\ kg/m^3$

$\phi = 35°$

$c = 0$

W

B

<u>Solution:</u>

The soil density $\rho = 1,850\ kgm/m^3$ converts to a unit weight via $\gamma = \rho g$ $(like\ F = ma)$,

$$\gamma = \rho g = \frac{\left(1,850\dfrac{kg_m}{m^3}\right)\left(9.81\dfrac{m}{s^2}\right)}{(1,000\ N/kN)} = 18.15\ kN/m^3 \text{ and the load to be supported by the footing is,}$$

$$W = ma = \frac{(30,000\ kg_m)\left(9.81\dfrac{m}{s^2}\right)}{(1,000\ N/kN)} = 294\ kN$$

Terzaghi's ultimate bearing capacity of a square footing is given by,

$$q_{ult} = 1.3c'N_c + \overline{q}N_q + 0.4\gamma BN_\gamma$$

$$\therefore q_{all} = \frac{q_{ult}}{3} = \frac{1}{3}\left(1.3c'N_c + \overline{q}N_q + 0.4\gamma BN_\gamma\right) \quad and \quad q_{all} = \frac{P}{B^2} = \frac{294}{B^2}$$

$$or \quad \frac{294}{B^2} = \frac{1}{3}\left(1.3c'N_c + \overline{q}N_q + 0.4\gamma BN_\gamma\right)$$

For $\phi = 35°$, $N_c = 57.8$, $N_q = 41.4$, and $N_\gamma = 41.1$,
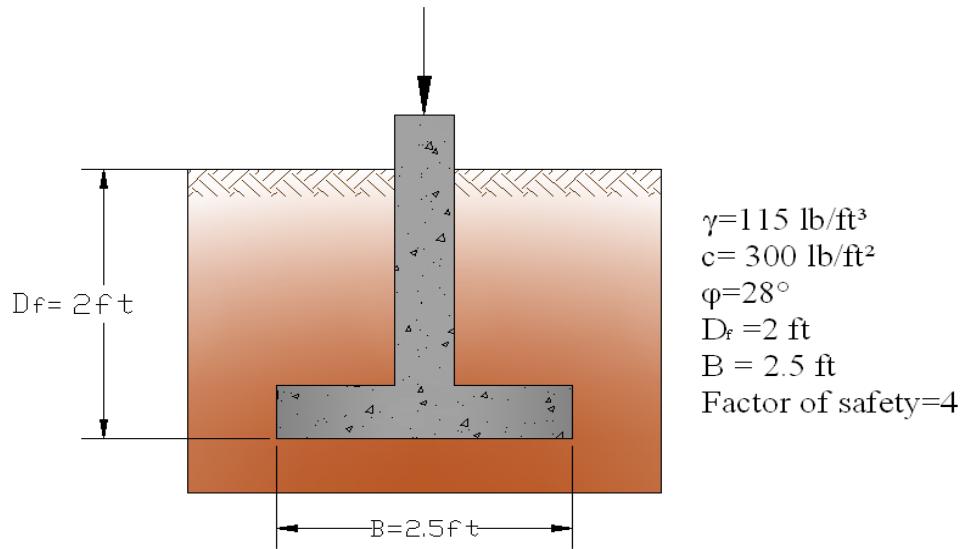
$$\frac{294}{B^2} = \frac{1}{3}\left[(0) + (18.15)(1)(41.4) + (0.4)(18.15)B(41.1)\right] \therefore B^3 + 2.52B^2 - 2.96 = 0$$

$B = 0.90\ m$

(Revision: Sept-08)

Using Terzaghi's method, distinguish between the value of the local shear failure versus the general shear failure.

γ=115 lb/ft³
c= 300 lb/ft²
φ=28°
D_f =2 ft
B = 2.5 ft
Factor of safety=4

Df= 2ft

B=2.5ft

Solution:

Terzahi's general bearing capacity failure of a square footing is,

$$q_{ult} = 1.3c'N_C + \overline{q}N_q + 0.4\gamma BN_\gamma$$

For $\phi = 28°$ $N_c = 31.6$, $N_q = 17.8$, $N_\gamma = 15.0$ and $\overline{q} = \gamma D_f = (0.115)(2) = 0.23 \, ksf$

Therefore,

$$q_{ult} = 1.3(0.30)(31.6) + (0.23)(17.8) + 0.4(0.115)(2.5)(15.0) = 18.1 \, ksf$$

To find the value of the bearing capacity of a local shear failure, the cohesion and angle of internal friction are reduced by two-thirds,

$$q'_{ult-local} = 1.3c'N'_c + \overline{q}N'_q + 0.4\gamma BN'_\gamma \quad where \; c' = \frac{2}{3}c = \frac{2}{3}(0.30) = 0.2 \, ksf$$

$$and \; \phi' = \frac{2}{3}(\phi) = \frac{2}{3}(28°) = 18.7° \; which \; give \; N'_c = 16.2, \; N'_q = 6.5 \; and \; N'_\gamma = 4.52$$
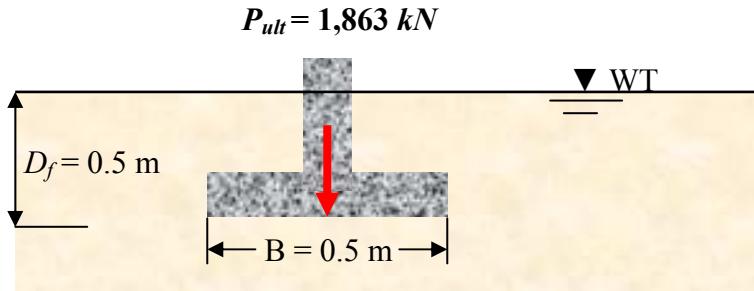
$$\therefore q'_{ult-local} = (1.3)(0.2)(16.2) + (0.23)(6.5) + (0.4)(0.115)(2.5)(4.52) = 6.2 \, ksf$$

$q_{ult-general \; failure} = 18.1 \, ksf \quad versus \quad q_{ult-local \; failure} = 6.2 \, ksf \quad (Almost \; a \; three - to - one)$

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(Revision: Sept-08)

Compare the results of the *Hansen* and the *Meyerhof* bearing capacity formulas to the results of a field test that took a rectangular footing to failure when the load reached 1,863 kN. Given B = 0.5 m, L = 2.0 m, c = 0, $\varphi_{triaxial}$ = 42° and γ' = 9.31 kN/m³ (the WT is at the surface).

$$P_{ult} = 1,863 \ kN$$

$D_f = 0.5$ m

▼ WT

B = 0.5 m

Solution:

$$q_{ult} = \frac{P_{ult}}{BL} = \frac{1,863 \ kN}{(0.5 \ m)(2.0 \ m)} = 1,863 \ kPa \quad \text{was the field measured failure load.}$$

(1) The Hansen formula predicts an ultimate bearing capacity of,

$$q_{ult} = 0 + \overline{q} N_q F_{qs} F_{qd} + 0.5\gamma BN_\gamma F_{\gamma s} F_{\gamma d}$$

Lee's adjustment formula is $\phi_{ps} = 1.5\phi_{triaxial} - 17° = 1.5(42°) - 17° = 46°$

For $\varphi = 46°$, $N_q = 158.5$ *and* $N_\gamma = 244.65$

$$F_{qs} = 1 + \left(\frac{B}{L}\right)\tan\varphi = 1 + \left(\frac{0.5}{2}\right)\tan 46° = 1.26$$

$$F_{\gamma s} = 1 - 0.4\left(\frac{B}{L}\right) = 1 - 0.4\left(\frac{0.5}{2}\right) = 0.9$$

$$F_{qd} = 1 + 2\tan\varphi(1 - \sin\varphi)^2\left(\frac{D_f}{B}\right) = 1 + 2\tan 46°(1 - \sin 46°)^2\left(\frac{0.5}{0.5}\right) = 1.16$$

$$F_{\gamma d} = 1.0$$

∴ $q_{ult} = 0 + (9.31)(0.5)(159)(1.27)(1.16) + (0.5)(9.31)(0.5)(245)(0.9)(1.0)$

∴ $q_{ult} = 1,485 \ kPa$ *versus* 1,863 *kPa measured (Hansen underestimates by* 20%)

(2) The Meyerhof formula with $\phi = 46°$, $N_q = 158.5$ and $N_\gamma = 328.73$,

$$q_{ult} = 0 + \overline{q} N_q F_{qs} F_{qd} + 0.5\gamma BN_\gamma F_{\gamma s} F_{\gamma d}$$

∴ $q_{ult} = 0 + (9.31)(0.5)(158.5)(1.27)(1.16) + (0.5)(9.31)(0.5)(328.73)(0.9)(1.0)$

∴ $q_{ult} = 1,782 \ kPa$ *versus* 1,863 *kPa (Meyerhof underestimates by* 4 %).
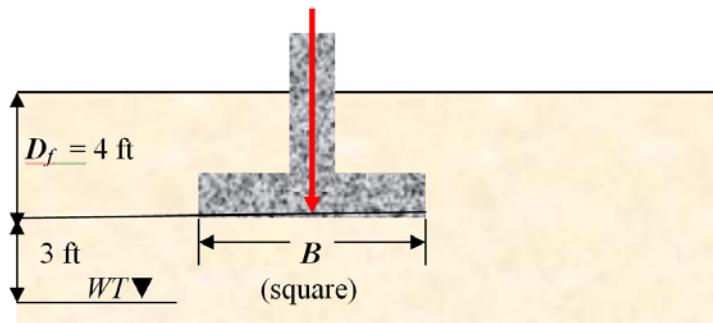
(Revision: Sept-08)

Use Meyerhof's bearing capacity formula (with a factor of safety = 3) to select a footing's width B if, (a) the water table is as shown below, and (b) if the water table rises to the ground surface?

The soil has a unit weight of 112 pcf, a moisture of 10%, $\varphi = 25°$, a cohesion $c_u = 240$ *psf* and a specific gravity of solids of $G_s = 2.68$.

$$Q = 200 \ kips$$

$D_f = 4$ ft

3 ft

B

$WT▼$

(square)

Solution:

(a) Find γ_{SAT} to determine γ',

$$\gamma_{dry} = \frac{\gamma}{1 + w_N} = \frac{112}{1.10} = 101.8 \ pcf \ and \ V_s = \frac{W_s}{G_s \gamma_w} = \frac{\gamma_{dry}}{G_s \gamma_w}$$

$$set \ V = 1 \ ft^3 \ \therefore \ V_s = \frac{101.8}{2.68(62.4)} = 0.61 \ ft^3 \ \therefore \ V_v = V - V_s = 1 - 0.61 = 0.39 \ ft^3$$

$$but \ \gamma_{sat} = \gamma_{dry} + n\gamma_w = \gamma_{dry} + \left(\frac{V_v}{V}\right)\gamma_w \ \therefore \ \gamma_{sat} = 101.8 + (0.39)(62.4) = 126.2 \ pcf$$

$and \ \gamma' = \gamma_{sat} - \gamma_w = 126.2 - 62.4 = 63.8 \ pcf$

Try $B = 5.7$ feet with Meyerhof's equation,

$$q_{ult} = c' N_c (F_{cs} F_{cd} F_{ci}) + \overline{q} N_q (F_{qs} F_{qd} F_{qi}) + 0.5 \gamma BN_\gamma (F_{\gamma s} F_{\gamma d} F_{\gamma i})$$

where the load inclination factors F_{ci}, F_{qi} *and* $F_{\gamma i} = 1$

$$For \ \varphi \geq 10° \quad K_p = \tan^2\left(45° + \frac{\varphi}{2}\right) = \tan^2\left(45° + \frac{25°}{2}\right) = 2.46, \ therefore$$

$$F_{cs} = 1 + (0.2)\left(\frac{B}{L}\right) K_p = 1 + (0.2)\left(\frac{5.7}{5.7}\right)(2.46) = 1.49$$

$$F_{cd} = 1 + (0.2)\left(\frac{D_f}{B}\right)\sqrt{K_p} = 1 + (0.2)\left(\frac{4}{5.7}\right)\sqrt{2.46} = 1.22$$

$$F_{qd} = F_{\gamma d} = 1 + (0.1)\left(\frac{D_f}{B}\right)\sqrt{K_p} = 1 + (0.1)\left(\frac{4}{5.7}\right)\sqrt{2.46} = 1.11$$

$$F_{qs} = F_{\gamma s} = 1 + (0.1)\left(\frac{B}{L}\right) K_p = 1 + (0.1)(\frac{5.7}{5.7})(2.46) = 1.25$$

The Meyerhof bearing capacity factors for $\phi = 25°$ are

$N_c = 20.7$, $N_q = 10.7$, *and* $N_\gamma = 6.77$

$q_{ult} = c' N_c (F_{cs} F_{cd} F_{ci}) + \bar{q} N_q (F_{qs} F_{qd} F_{qi}) + 0.5 \gamma B N_\gamma (F_{\gamma s} F_{\gamma d} F_{\gamma i})$

$q_{ult} = (0.24)(20.7)(1.49)(1.22)(1) + (0.112)(4)(10.7)(1.25)(1.11)(1) + (0.5)(0.112)(5.7)(6.67)(1.25)(1.11)(1)$

$q_{ult} = 18.6 \ ksf$

$q_{all} = \dfrac{q_{ult}}{FS} = \dfrac{18.6}{3} = 6.2 \ ksf$ *therefore* $B^2 = \dfrac{Q}{q_{all}} = \dfrac{200}{6.2} = 32.25 \ ft^2 \therefore B = 5.7 \ ft$

Therefore the choice of $B = 5.7$ ft was a good choice.

(b) When the water table rises to the ground surface, need a larger footing; try $B = 7.0$ feet.

$F_{cd} = 1 + 0.2\left(\dfrac{B}{L}\right) K_p = 1 + 0.2\left(\dfrac{7}{7}\right)(2.46) = 1.49$

$F_{cs} = 1.49$ *same as above*

$F_{qd} = F_{\gamma d} = 1 + 0.1\left(\dfrac{D}{B}\right)\sqrt{K_p} = 1 + 0.1\left(\dfrac{4}{7}\right)\sqrt{2.46} = 1.09$

$F_{qs} = F_{\gamma s} = 1.25$ *same as above*

$q_{ult} = (0.24)(20.7)(1.49)(1.18) + (0.062)(4)(10.7)(1.25)(1.09) + (0.5)(0.062)(7)(6.67)(1.09)(1.25)$

$q_{ult} = 16.62 \ ksf$
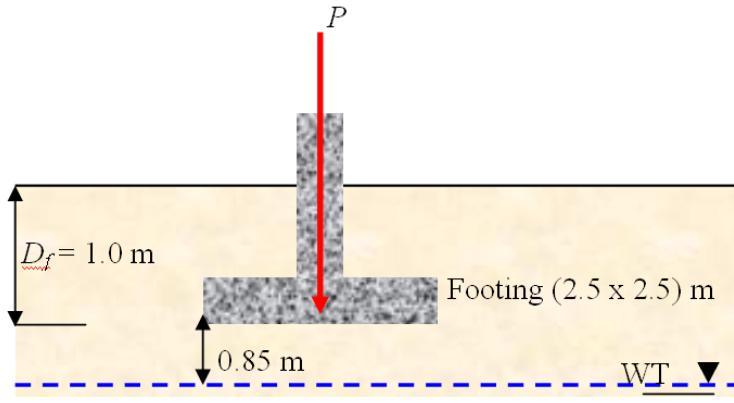
$q_{all} = \dfrac{q_{ult}}{FS} = \dfrac{16.62}{3} = 5.54 \ ksf$ *and* $B^2 = \dfrac{Q}{q_{all}} = \dfrac{200}{5.54} = 36.1 \ ft^2 \therefore B = 6.01 \ ft$

Iterate once more, and find $B = 7.5$ feet.

Bearing–08: *The effect of the WT upon the bearing capacity.*

(Revision: Sept-08)

Using the Hansen method, what are the ultimate and allowable bearing capacities for the footing shown below if you require a factor of safety of at least 2?

$c = 0$

$\phi_t = 35°$

$\gamma = 18.10 \dfrac{kN}{m^3}$

$w_N = 10\%$, and $G_s = 2.68$

D_f= 1.0 m

Footing (2.5 x 2.5) m

0.85 m

WT ▼

Solution:

Always use the effective unit weight of water in the bearing capacity formulas. The average effective weight γ_e of the soil can also be given by the formula:

$$\gamma_e = (2H - d_w)\frac{d_w}{H^2}\gamma_{wet} + \frac{\gamma'}{H^2}(H - d_w)^2$$

$$where \quad H = (0.5)B\tan\left(45° + \frac{\varphi}{2}\right) = (0.5)(2.5)\tan\left(45° + \frac{35°}{2}\right) = 2.40\ m$$

and d_w = depth to the WT below the footing invert = 0.85 m

Set the total volume $V = 1\ m^3$

$$\gamma_{dry} = \frac{\gamma_{wet}}{1+w} = \frac{18.10}{1+0.10} = 16.5\ \frac{kN}{m^3} \quad and \quad V_s = \frac{\gamma_{dry}}{G_s\gamma_{wet}} = \frac{16.5}{(2.68)(9.8)} = 0.63\ m^3$$

$$V_v = 1.0 - V_s = 1 - 0.63 = 0.37\ m^3 \quad and \quad \gamma_{sat} = \gamma_{dry} + n\gamma_{wet} = 16.5 + (0.37)(9.8) = 20.1\ \frac{kN}{m^3}$$

$$\therefore\ \gamma_e = ((2)(2.40 - 0.85))\left[\frac{0.85(18.10)}{(2.4)^2}\right] + \left[\frac{20.1 - 9.8}{(2.4)^2}\right](2.40 - 0.85)^2 = 12.6\ \frac{kN}{m^3}$$

Using Hansen's method with $\varphi = 35°$, the bearing capacity factors are $N_q = 33.3$ and $N_\gamma = 33.92$.

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$$F_{qs} = 1 + \frac{B}{L} \tan \phi = 1 + \left(\frac{2.5}{2.5} \right) \tan 35° = 1.70$$

$$F_{qd} = 1 + 2 \tan \phi \left(1 - \sin \phi \right)^2 \left(\frac{D_f}{B} \right) = 1 + 2 \tan 35° \left(1 - \sin 35° \right)^2 \left(\frac{1}{2.5} \right) = 1.10$$

$$F_{\gamma s} = 1 - 0.4 \frac{B}{L} = 1 - 0.4 (\frac{2.5}{2.5}) = 0.6$$

$$F_{\gamma d} = 1.0$$

Therefore, the ultimate and allowable bearing capacities are,

$$q_{ult} = 0 + \overline{q} N_q (F_{qs} F_{qd}) + 0.5 \gamma_e B N_\gamma (F_{\gamma s} F_{\gamma d})$$

$$q_{ult} = (18.1)(1.0)(33)(1.70)(1.10) + (0.5)(12.6)(2.5)(34)(0.6)(1)$$
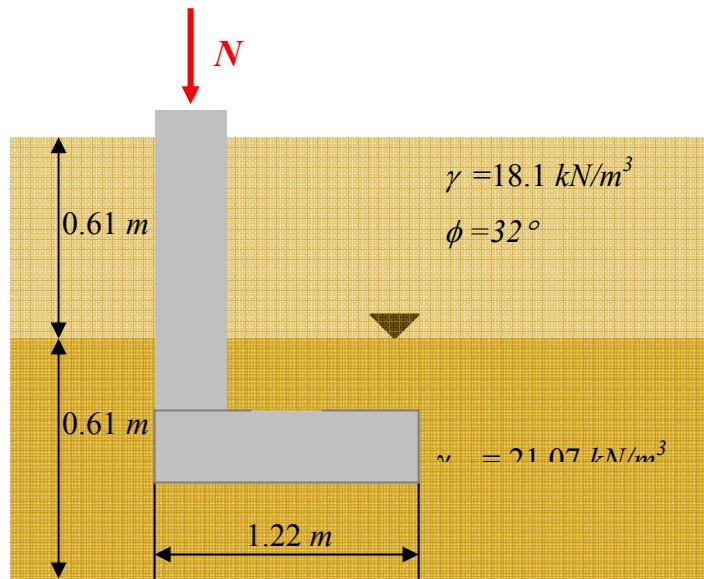
$$q_{ult} = 1,497 \ kPa$$

$$q_{all} = \frac{1,497}{2} = 749 \ kPa$$

(Revision: Sept-08)

Use the Hansen formula to determine the gross normal load N on the column shown below using a factor of safety of 3.

$\gamma =18.1 \ kN/m^3$

$\phi =32°$

0.61 m

0.61 m

$\gamma \quad = 21.07 \ kN/m^3$

1.22 m

**Solution:**

The Hansen formula for a footing is,

$$q_{ult} = cN_c \ F_{cs}F_{cd} + qN_q \ F_{qs}F_{qd} + 0.5\gamma BN_y \ F_{ys}F_{yd}$$

The inclination factors F_{ci}, F_{qi}, and F_{yi} are all equal to 1 because the load is vertical.

For $\phi = 32°$, $N_c = 35.49$, $N_q = 23.18$ and $N_y = 20.79$ and $B / L = 1$

$$F_{cs} =1+\left(\frac{N_q}{N_c}\right)=1+(23.20/35.50)=1.65$$

$$F_{qs} =1+ \tan \phi =1+0.62 =1.62$$

$$F_{ys} =1-0.4\left(\frac{B}{L}\right)=1-0.4 = 0.60$$

$$F_{qd} =1+2 \tan \phi \left(1- \sin \phi\right)^2\left(\frac{D_f}{B}\right)=1+ (2)(0.62)(0.22)(1) = 1.273 \text{ for } D_f / B \leq 1$$

$$F_{yd} =1$$

$$F_{cd} = F_{qd} -\left[\frac{\left(1-F_{qd}\right)}{N_q \ \tan \phi}\right]=1.273 -\left[\left(\frac{1-1.273}{23.20\times 0.62}\right)\right]=1.292$$

The WT is located above the footing, therefore,

$$\bar{q} = (0.61m)(18.1\,kN/m^3) + (0.61m)(21.07 - 9.81) = 17.9\,kN/m^2$$

$$\therefore q_{ult} = (17.9)(1.62)(1.273)(23.20) + (0.5)(0.6)(21.07 - 9.81)(1.22)(20.8)(1) = 981\,kN/m^2$$

Therefore,

$$q_{all} = \left(\frac{q_{ult}}{3}\right) = \left(\frac{981kN/m^2}{3}\right) = 327\,kN/m^2$$
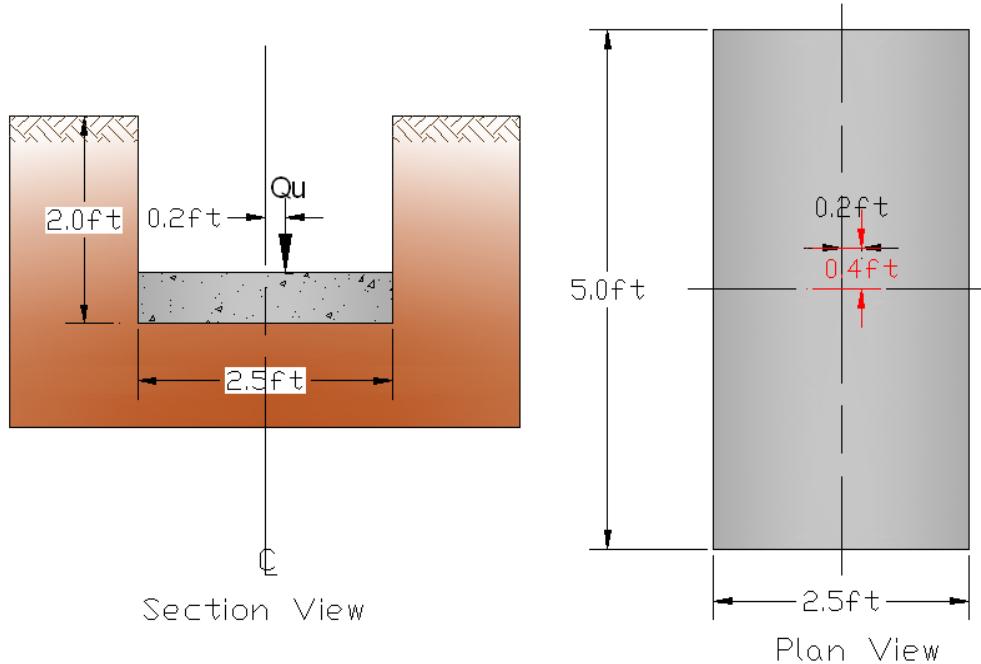
Hence, the total gross load N is,

$$N = q_{all}B^2 = (327kN/m^2)(1.22m)^2 = 487\,kN$$

(Revision: Sept-08)

A rectangular footing measures 5 feet by 2.5 feet. Determine the gross ultimate load Q_{ult} applied eccentrically upon the footing, and the ultimate bearing capacity of the soil q_{ult}, given that $\gamma = 115$ pcf, $c = 0$ and $\phi = 30°$.

Qu

2.0ft 0.2ft

5.0ft

0.2ft

0.4ft

2.5ft

Section View

2.5ft

Plan View

Solution:

The effective width footing width $B' = B - 2e_x = (2.5) - 2(0.2) = 2.1$ ft
and the effective length $L' = L - 2e_y = (5) - 2(0.4) = 4.2$ ft.
Meyerhof's ultimate bearing capacity formula with $c = 0$ is,

$$q_{ult} = 0 + \bar{q} N_q F_{qs} F_{qd} + 0.5\gamma B' N_\gamma F_{ys} F_{\gamma d}$$

$$For\ \phi = 30°, N_q = 18.4\ and\ N_\gamma = 15.67$$

$$F_{qs} = 1 + \left(\frac{B'}{L'}\right)\tan\phi = 1 + \left(\frac{2.1}{4.2}\right)(0.58) = 1.29$$

$$F_{qd} = 1 + 2(\tan 30°)(1 - \sin 30°)^2 \left(\frac{2}{2.1}\right) = 1.275$$

$$F_{ys} = 1 - 0.4\left(\frac{B'}{L'}\right) = 1 - 0.4\left(\frac{2.1}{4.2}\right) = 0.8$$

$$F_{\gamma d} = 1$$

$$q_{ult} = (2)(0.115)(18.4)(1.29)(1.275) + (0.5)(0.115)(2.1)(15.67)(0.8)(1) = 8.47\ ksf$$
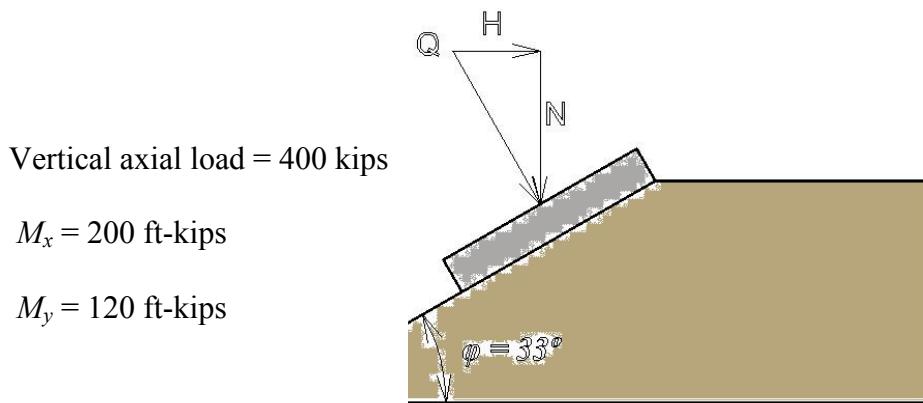
$$Hence,\quad Q_{ult} = q_{ult}(B'L') = (8.47)(2.1)(4.2) = 74.73\ kips$$

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(Revision: Sept-08)

A square 8' x 8' footing is loaded with an axial load of 400 kips and M_x = 200 ft-kips, M_y = 120 ft-kips. Un-drained triaxial tests (the soil is not saturated) gave φ = 33° and c = 200 psf. The footing depth D_f = 6.0 feet, the soil unit weight is 115 pcf, and the *WT* was not found.

Use the *Hansen* equation with the *Meyerhof* reduction factors and a *FS = 3* to find the

Vertical axial load = 400 kips

M_x = 200 ft-kips

M_y = 120 ft-kips

Solution:

Eccentricities $\quad e_x = \dfrac{M_y}{Q} = \dfrac{120\,ft-k}{400} = 0.3\ feet \quad$ and $\quad e_y = \dfrac{M_x}{Q} = \dfrac{200\,ft-k}{400} = 0.5\ feet$

$\therefore\ \ B_r = B - 2e_y = 8'-1' = 7\ feet \quad$ and $\quad L_r = L - 2e_x = 8' - 0.6' = 7.4\ feet \quad$ (ie. $L_r > Br$)

Adjusting the ϕ from triaxial (ϕ_{tr}) to a plane-strain value (ϕ_{ps}) via Lee's formulation,

$\phi_{ps} \cong 1.1\phi_{tr} = 1.1(32.7°) = 36°$

$N_q = e^{\pi \tan 36°} \tan^2\left(45° + \dfrac{36°}{2}\right) = 37.8$

$N_c = (N_q - 1)\ \cot\phi = (36.8)\cot 36° = 50.6$

$N_\gamma = (N_q - 1)\tan(1.4\phi) = (36.8)\tan 50.4° = 44.4$

$N_\gamma = 1.5(N_q - 1)\tan\phi = 1.5(36.8)\tan 36° = 40.1$

$\therefore\quad S_c = 1 + 0.2K_p\left(\dfrac{B_r}{L_r}\right) = 1 + 0.2(3.85)\dfrac{7}{7.4} = 1.73$

and $\quad d_c = 1 + 0.2\sqrt{K_p}\left(\dfrac{D}{B_r}\right) = 1 + 0.2(\sqrt{3.85})\left(\dfrac{6}{7}\right) = 1.34$

Since $\phi > 10°$, $S_q = S_\gamma \cong 1.0 \quad$ and $\quad d_q = d_\gamma = 1.0$.

Hansen's $\quad q_{ult} = 0.5\gamma BN_\gamma S_\gamma d_\gamma i_\gamma g_\gamma b_\gamma + cN_c S_c d_c i_c g_c b_c + q_q N_q S_q d_q i_q g_q b_q$

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Also $i = g = b = 1.0$ for this problem, since $\alpha = 0 = i$ (inclination factor f / load Q with t vertical) $\eta = 4$

g (ground factor with t inclined ground on side of footing)

b (base factor with t inclined ground under the footing)

$$q_{ult} = 0.5(0.115)(7)(40.1)(1) + (0.200)(50.6)(1.73)(1.34) + (0.115)(6)(37.8)(1) =$$
$$q_{ult} = 16.1 + 23.5 + 26.1 = 65.7 \ ksf$$

$$q_{all} = \frac{q_{ult}}{FS} = \frac{65.7}{3} = 21.9 \ ksf$$

$$R_{ex} = 1 - \left(\frac{e_x}{B}\right)^{\frac{1}{2}} = 1 - \left(\frac{0.3}{8}\right)^{\frac{1}{2}} = 0.81$$

$$R_{ey} = 1 - \left(\frac{e_y}{B}\right)^{\frac{1}{2}} = 1 - \left(\frac{0.5}{8}\right)^{\frac{1}{2}} = 0.75$$

$$Q_{all} = q_{all}\left(B^2\right)\left(R_{ex}\right)\left(R_{ey}\right) = 21.9(8x8)(0.81)(0.75) = 851 \ kips$$

$$q_{all} = \frac{Q_{all}}{B^2} = \left(\frac{851}{64}\right) = 13.3 \ ksf$$
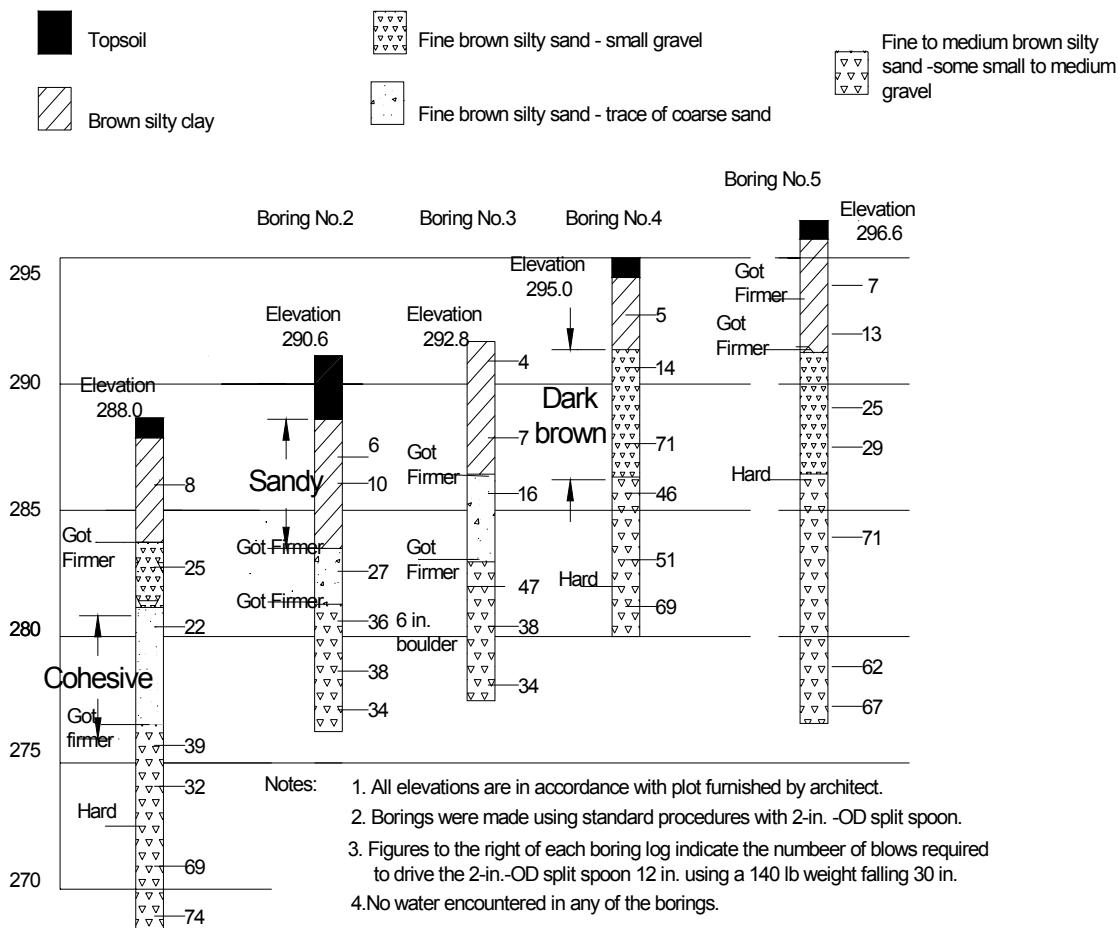
(The contact load $q_o = 13\left(\frac{400}{851}\right) = 6.1 \ ksf$)

(Revision: Sept-08)

Use the boring logs show below to recommend an allowable soil pressure q_{all} for the footings located in the vicinity of elevation 284, boring No. 2?

The building is a four-story (five on the low side) office building with column loads around 160 kips. State your reasons.

Solution:

It is presumed that all the building's footings will be placed at roughly elevation 284 or thereabouts. This is fine for the building area covered by borings # 3, 4 and 5 because they have good SPT values.

Meyerhof has proposed formulas for the allowable bearing capacity adjusted so that the settlement is limited to 1-inch. These formulas are:

$$q_{all} = \frac{N}{4}(K_D) \text{ for B} \le 4 \text{ ft}$$

$$q_{all} = \frac{N}{6}\left(\frac{B+1}{B}\right)^2 (K_D) \text{ for B> 4ft}$$

where $K_D = 1 + 0.33\left(\frac{D_f}{B}\right) \leq 1.33$

For the silty sand use $N = \frac{47+51+71}{3} = 56.33 \square 56$ (#3, 4, and 5)

Let's assume B=4.5 ft and $D_f = 0$

$q_{all} = \frac{56}{6}\left(\frac{4.5+1}{45}\right)^2 (1) = 13.9 \; ksf$ This suggests that a B = 4.5 feet is excessive since

$q_0 = \frac{Q}{B^2} = \frac{160 kips}{20.25 sf} = 7.9 \square \; q_{all} = 13.9 \text{ ksf}$

Assume B < 4 ft, say B~ 3.5 ft , and use formula

$q_{all} = \frac{N}{4}(K_D)$ $k_d = 1 + 0.33 D_f/B$ $\frac{56}{4}\left(1+\frac{0.33 D_f}{B}\right)$ and $D_f = 0$ $q_{all} = 14$ ksf

$\therefore q_0 = \frac{Q}{B^2} = \frac{160 \; kips}{(3.5)^2} = 13.06 \cong 13$ ksf ≤ 14 ksf   OK

For footings in area of borings # 1 and #2, they will be deeper by 1-story (ie. for 5-story building). That places the shallow foundation at elevation 274 ft. This area will have bearing in the same strata. N= 32 and using B= 3.50' and D_f= 4.5'

$q_{all} = \frac{N}{4} k_d$ $k_d = 1 + 0.33 D_f/B \leq 1.33$ $K_d = \frac{32}{4}\left(1+\frac{0.33 x 4.5}{3.50}\right) = 1.33$

$q_{all} = 10.64 ksf < 13$ ksf   NOT GOOD

Let's use B= 3.90 feet $q_{all} = 10.64$ ksf $q_0 = \left(\frac{Q}{B^2}\right) = 10.51$ ksf ≤ 10.64 ksf

Use __*B = 3.90 feet.*__

Chapter 18
Shallow Foundations

Properties of Reinforcing Steel (British and SI units).

ASTM STANDARD REINFORCING BARS

BAR SIZE DESIGNATION	WEIGHT POUNDS PER FOOT	NOMINAL DIMENSIONS — ROUND SECTIONS		
		DIAMETER INCHES	CROSS SECTIONAL AREA - SQ. INCHES	PERIMETER INCHES
#3	.376	.375	.11	1.178
#4	.668	.500	.20	1.571
#5	1.043	.625	.31	1.963
#6	1.502	.750	.44	2.356
#7	2.044	.875	.60	2.749
#8	2.670	1.000	.79	3.142
#9	3.400	1.128	1.00	3.544
#10	4.303	1.270	1.27	3.990
#11	5.313	1.410	1.56	4.430
#14	7.65	1.693	2.25	5.32
#18	13.60	2.257	4.00	7.09

METRIC REINFORCING BARS

BAR DESIG-NATION	MASS WEIGHT kg/m	NOMINAL DIMENSIONS		
		DIAMETER mm	CROSS SECTIONAL AREA mm²	PERIMETER mm
10M	0.785	11.3	100	35.5
15M	1.570	16.0	200	50.1
20M	2.355	19.5	300	61.3
25M	3.925	25.2	500	79.2
30M	5.495	29.9	700	93.9
35M	7.850	35.7	1000	112.2
45M	11.775	43.7	1500	137.3
55M	19.625	56.4	2500	177.2

The nominal dimensions of a deformed bar are equivalent to those of a plain round bar having the same mass per metre as the deformed bar.

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(Revision: Sept-08)

Design a square reinforced concrete footing for a column 15"x15" with 4 # 8 rebars, and,

$$D_L = 100 \ kips \qquad f_c' = 3,000 \ psi$$

$$L_L = 120 \ kips \qquad f_y = 60,000 \ psi$$

$$q_{allowable} = 4 \ ksf \quad from \quad q_{ult} = 10 \ ksf \ and \ FS = 2.5$$

Solution:

1) Footing size for service loads:

$$B = \sqrt{\frac{Q}{q_a}} = \sqrt{\frac{220 \ kips}{4 \ ksf}} = 7.42 \ feet \quad therefore \ use \ B = 7.5 \ feet.$$

2) Check ultimate parameters: that is the actual soil pressure q_o under Q_{ult},

$$Q_{ULT} = 1.2D_L + 1.6L_L = 1.2(100) + 1.6(120) = 120 + 192 = 312 \ kips$$
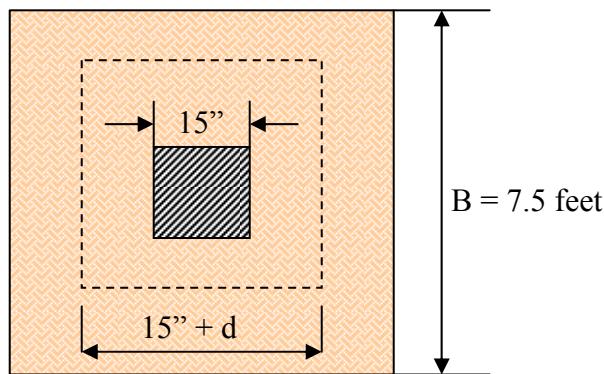
$$\therefore \quad q_O = \frac{Q_U}{B^2} = \frac{312}{(7.5 \ ft)^2} = 5.5 \ ksf \ < \ 10 \ ksf \ for \ q_{ult} \ ...GOOD$$

3) Compute the allowable concrete shear strength $v_c \ (allowable)$

$$v_{all} = 4\phi\sqrt{f'_C} \quad where \ \varphi = 0.75 \ for \ shear \ and \ torsion ,$$

$$v_C = v_{all} = 4(0.75)\sqrt{3,000 \ psi} = 164 \ psi \approx 23.7 \ ksf$$

4) Find *d, the effective depth,* (in this case two-way shear governs) in feet.

$$d^2\left(v_c+\frac{q_o}{4}\right)+d\left(v_c+\frac{q_o}{4}\right)w-\left(B^2-w^2\right)\left(\frac{q_o}{4}\right)=0$$

$$d^2\left(23.7+\frac{5.5}{4}\right)+d\left(23.7+\frac{5.5}{4}\right)\left(\frac{15}{12}\right)-\left[7.5^2-\left(\frac{15}{12}\right)^2\right]\left(\frac{5.5}{4}\right)=0$$

$$d^2+1.25d-3.0=0$$

Which yield to two solutions for $d=\begin{cases}+1.22\ ft\\-2.47\ ft\end{cases}$ and using the modified formula equation 2a:

When the column has a rectangular area bxc, the formula is,
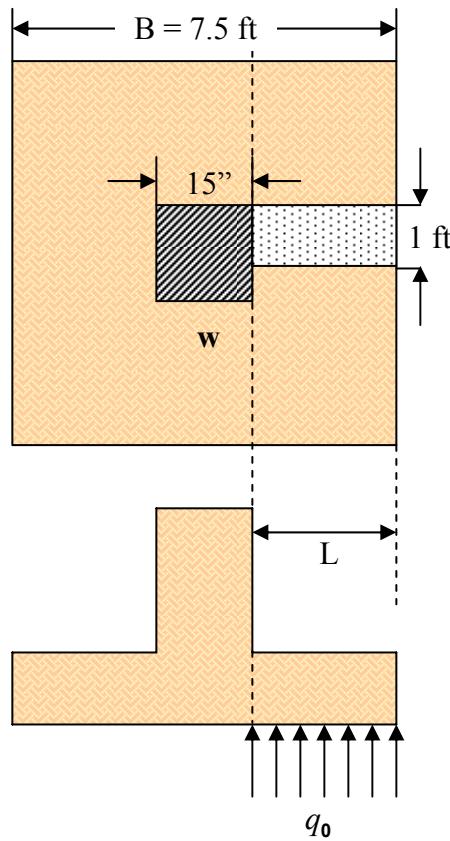
$$4d^2+2(b+c)d-\left(\frac{BLq_o}{v_c}\right)=4d^2+2\left(\frac{15}{12}+\frac{15}{12}\right)d-\left[\frac{(7.5)^2(5.5)}{23.7}\right]=0$$

$$d^2+1.25d-3.26=0 \quad \textit{which yields } d=+0.39\ ft$$

Use the largest $d=1.22\ feet=14.6$ inches; round-out to $d=15$ inches. It is not necessary to check for wide-beam shear on a square footing.

5) Compute the area of steel A_S for flexure.

Unit strip of the cantilever arm $=\left(\frac{B-w}{2}\right)=L \quad \therefore \quad L=\dfrac{\left(7.5-\left(\frac{15}{12}\right)\right)}{2}=3.13\ ft$

The cantilever moment:

$$M_U = \left(\frac{q_o L^2}{2}\right) = \frac{\left[\left(5.5\frac{k}{ft^2}\right)(3.13\ ft)^2\,(12\ in)\right]}{2} = 359\ in-kips$$

$$M_U = \phi A_S f_y \left(\frac{d-a}{2}\right) \qquad where \qquad \phi = 0.9\ for\ tension$$

$$a = \frac{A_S f_y}{0.85 f'_C\, b} = \frac{A_S(60)}{0.85(3)(12\ in)} = 1.96\ A_S$$

Substituting a into the M_U equation above,

$$M_U = (0.9)A_S f_y \left(\frac{d-a}{2}\right) \quad \therefore \quad \frac{M_U}{(0.9)f_y} = A_S\left(\frac{d-a}{2}\right)$$

$$\frac{359\ in-kip}{(0.9)(60\ ksi)} = A_S\left(15\ in - 1.96\frac{A_S}{2}\right)$$

$$15 A_S - 0.98 A_S^2 - 5.99 = 0$$

$$A_s^2 - 15.3 A_s + 6.11 = 0$$

$$A_S = 0.41\ in^2\ per\ foot\ of\ footing$$

The total steel required across the footing is $A_S = 7.5$ ft (0.41 in²/ft) = 3.08 in^2 (Check ACI 10.5.1) for minimum steel and ACI 7.12 for temperature and shrinkage,

$$\rho = \frac{A_S}{bd} = \frac{0.41}{12(15)} = 0.0023 > 0.0018$$

$$\rho_{min} = \frac{3\sqrt{f'_c}}{f_y} = 0.0018bh\ or$$

$$\rho_{min} = \frac{200}{f_y} = 0.0018$$
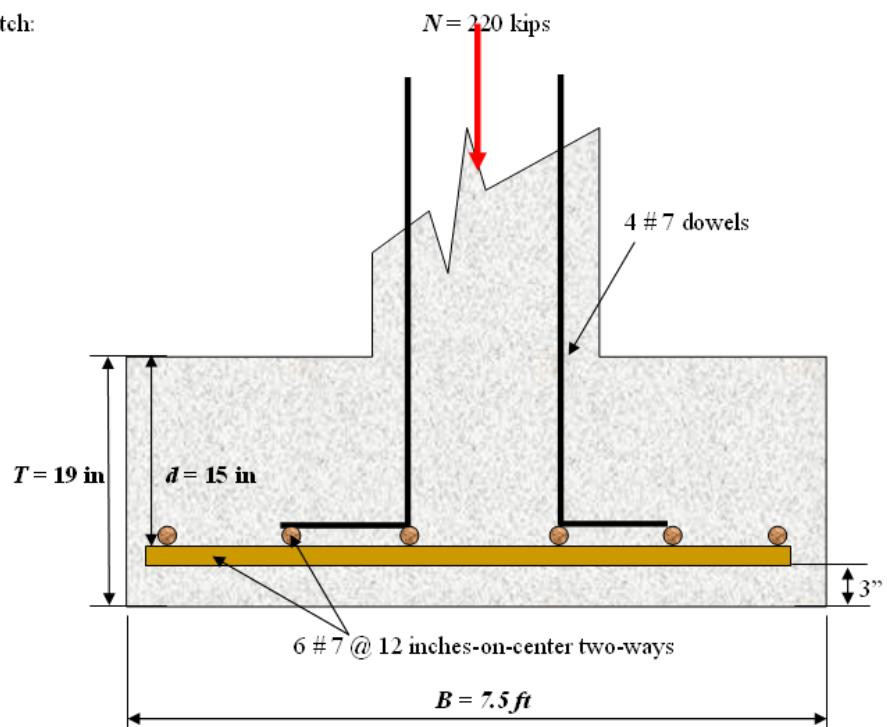
Therefore, $A_S = 7.5\ (0.0023)(12)(15) = 3.1\ in^2$ or 0.59 in² per foot of footing

For **B** = 90" (7.5') use 6 # 7 bars ($A_S = 3.18\ in^2$) @ 12 inches on center

or 5 # 8 bars ($A_S = 3.95\ in^2$) @ 12 inches on center

Check for Development length L_d (ACI-318-08.12.3), and the embedment length of the dowels.

N = 220 kips

4 # 7 dowels

T = 19 in

d = 15 in

3"

6 # 7 @ 12 inches-on-center two-ways

B = 7.5 ft
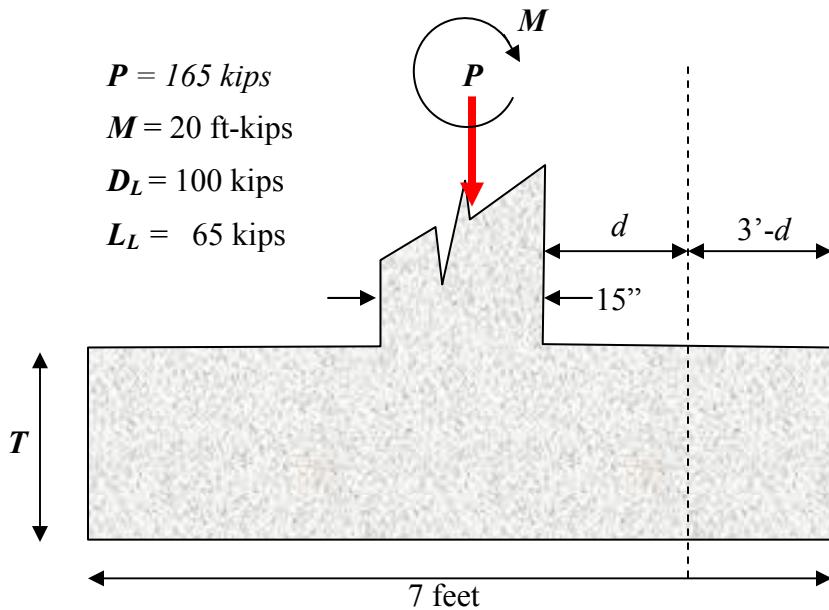
(Revision: Sept-08)

The footing shown below is a square footing with the dimensions and loads as shown.

a) **Compute the load's eccentricity *e*.**

b) **Check the bearing pressure at ultimate load.**

c) **Calculate the wide beam shear.**

$P = 165\ kips$

$M = 20\ \text{ft-kips}$

$D_L = 100\ \text{kips}$

$L_L = 65\ \text{kips}$

M

P

d   *3'-d*

15"

T

7 feet

d) **Determine the flexural moment for a strip of footing 1 foot wide.**

$q_{all} = 4\ ksf\ (with\ a\ FS = 1.5)\ ,\quad f'_C = 3\ ksi\ ,\quad f_y = 60\ ksi\ .\quad P = D_L + L_L$

Solution:

a) Compute the load eccentricity: $e = \dfrac{M}{P} = \dfrac{20}{165} = 0.121\ ft$

$$q = \left(\dfrac{P}{A}\right)\left(1\ \pm\ \dfrac{6e}{B}\right) = \left(\dfrac{165}{7'x7'}\right)\left(1\ \pm\ \dfrac{6(0.121')}{7'}\right) =$$

$$q_{max} = 3.72\ ksf\quad and\quad q_{min} = 3.02\ ksf$$

b) Check bearing pressure at ultimate load: $Q_u = 1.2(100) + 1.6(65) = 224\ kips$

$$q_{all} = q_{max}\left(\dfrac{Q_u}{P}\right) = 3.72\left(\dfrac{224}{165}\right) = 5.05\ ksf\quad and\quad q_{all} = q_{min}\left(\dfrac{Q_u}{P}\right) = 3.02\left(\dfrac{224}{165}\right) = 4.1\ ksf$$
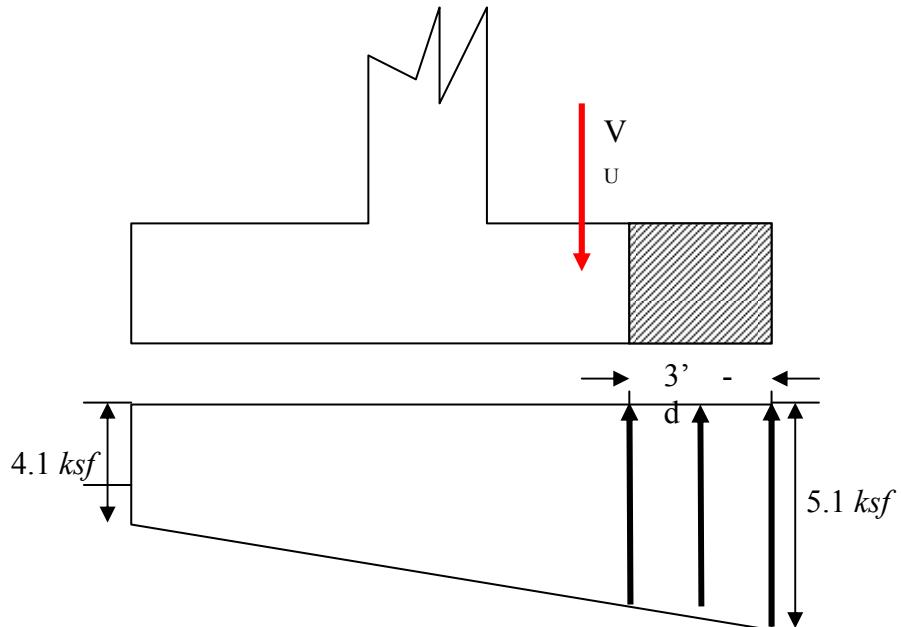
Then consider $q_{all} = 5.1\ ksf\ <\ q_{all}\ (FS) = 4\,(1.5)\ =\ 6\ ksf,$ GOOD.

c) Calculate the allowable one-way (wide beam) shear; assume a $d = 1$ ft; $\bar{q} = \dfrac{5.1+4.1}{2} = 4.6$

$$V_U = \bar{q}(\dfrac{B}{2} - T - \dfrac{c}{2}) = 4.6(\dfrac{7}{2} - 1 - \dfrac{1}{2}) = 9.2 \; kips \qquad \textbf{Equation (1)}$$

The allowable shear $= 2\phi\sqrt{f_c'} = 110 \; psi$

The required $d = \dfrac{V_u}{2\varphi\sqrt{f_c'}b} = \dfrac{(9.2)(1000)}{(0.75)(110)(12)} = 9.3 \; inches \approx 12 \; in \; assumed$

Equation (2)

d) Determine the flexural moment for a strip 1 foot wide:

$$M = \int_0^x V \, dx = \int_0^3 \left(\dfrac{5.9x - 0.157x^2}{2} \right) dx = 25.9 \; \dfrac{ft - kip}{ft}$$
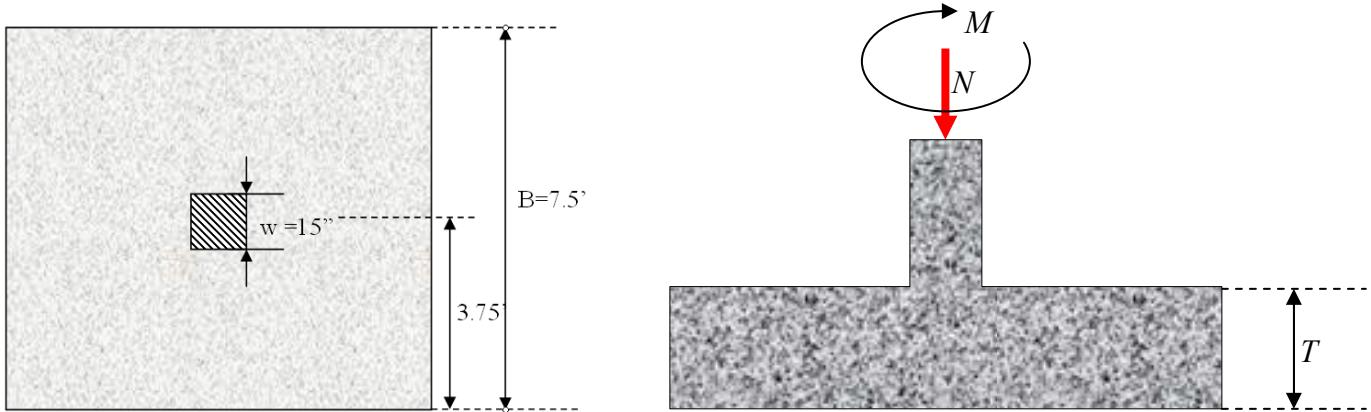
*Footings–03: **Find the thickness T and the A$_s$ of the previous problem.**

(Review:

Find, 1) The soil pressure under the footing for the given loads,

   2) The footing thickness *T*, and
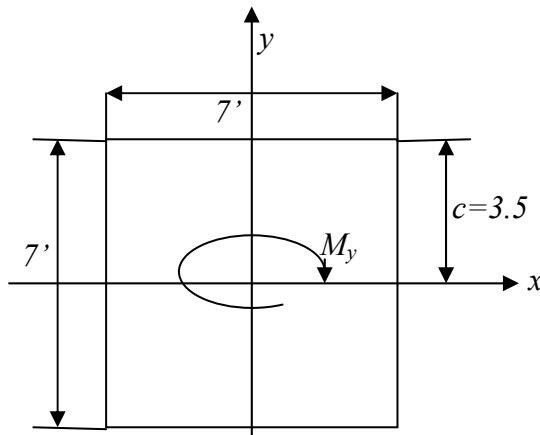
   3) The flexural steel reinforcement A$_s$.

Given: $f_y = 60 \ ksi$, $f'_c = 3 \ ksi$, $q_a = 4 \ ksf$, $D_L = 65 \ kips$, $L_L = 100 \ kips$, $M = 20 \ ft-kips$

$$N = D_L + L_L$$
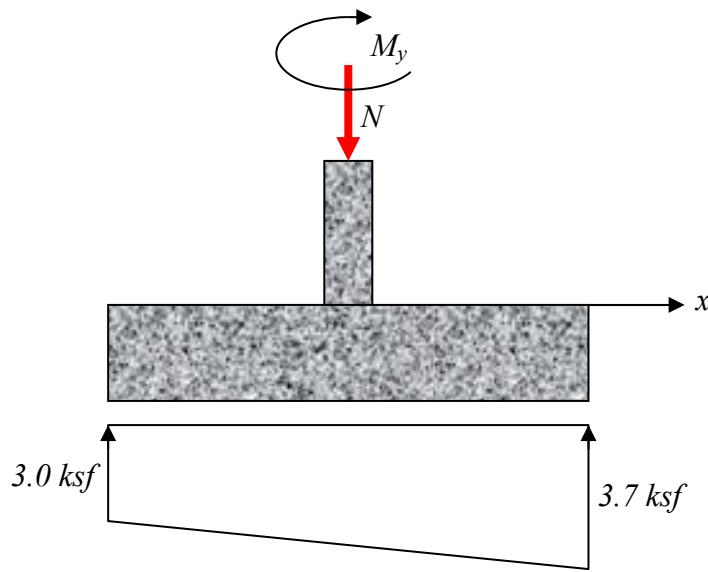
The actual soil pressure q_o (versus the allowable soil bearing capacity):

$$q_o = \frac{N}{A} \pm \frac{Mc}{I} \qquad where \qquad I = \frac{bd^3}{12} = \frac{(7')(7')^3}{12} = 200 \ ft^4$$

$$q_o = \frac{165k}{(7'x7')} \pm \frac{(20k.ft)(3.5ft)}{200 \ ft^4} =$$

$q_{o \ max} = 3.7 \ ksf \qquad and \qquad q_{o \ min} = 3.0 \ ksf < 4 \ ksf \ allowable \qquad GOOD$

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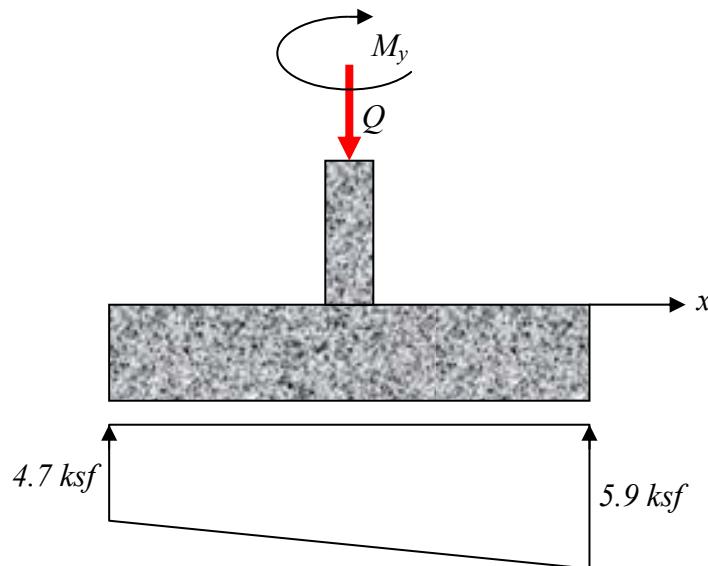
b) The ultimate load on the soil:

$$Q_u = 1.2D_L + 1.6L_L = 1.2(65 \ kips) + 1.6(100 \ kips) = 238 \ kips$$
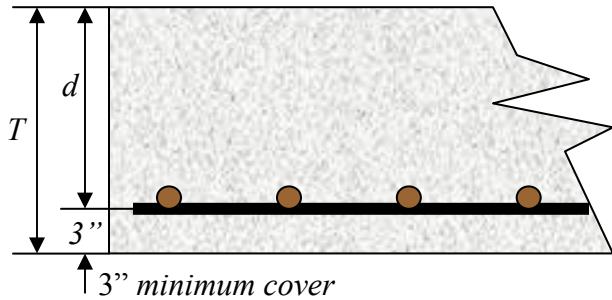
$$M_u = 1.6M_{LL} = 1.6(20 \ ft.kips) = 32 \ ft \cdot kips$$

$$q_u = \frac{Q_u}{A} \pm \frac{M_u c}{I} = \frac{238 kips}{(7')(7')} \pm \frac{(32 \ ft.kips)(3.5 \ ft)}{200 \ ft^4} =$$
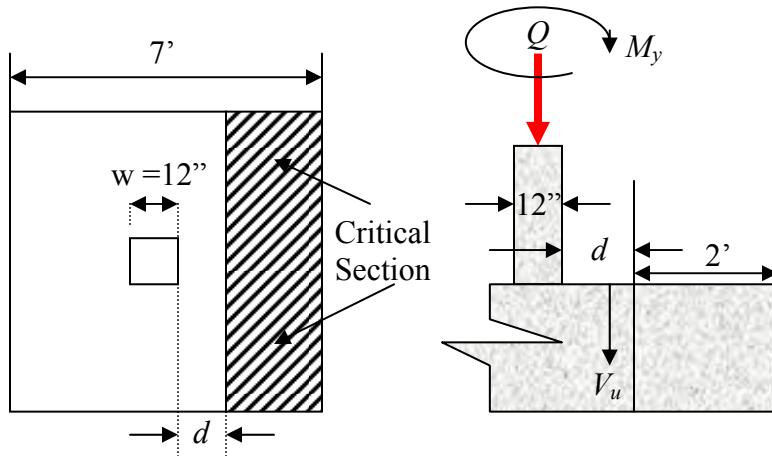
$$q_{u \ max} = 5.9 \ ksf \qquad and \qquad q_{u \ min} = 4.7 \ ksf$$

a) Determine the thickness of the footing **T**.

3" *minimum cover*

Check wide-beam shear: Assume d =12", (controls for rectangular footing).

From $\sum f_y = 0$, the ultimate shear is
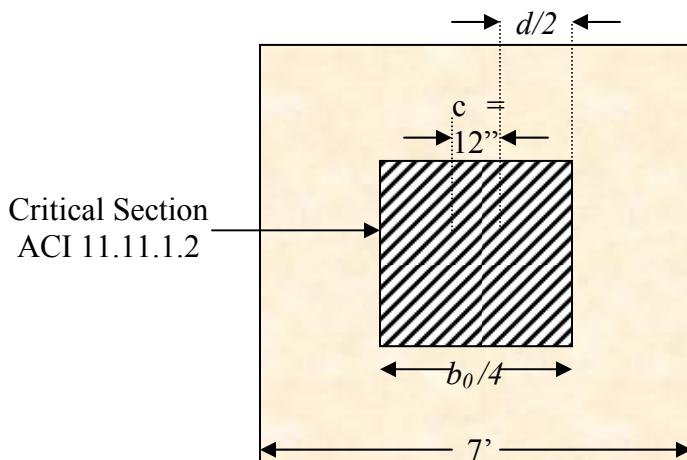
$V_u = 82,600 \ lbs$, but $V_u = \phi 2 \sqrt{f'_c} \ bd$,

$$ d = \frac{V_u}{\phi 2 \sqrt{f'_c} \ b} = \frac{82,600 \ lbs}{(0.85)(2)\sqrt{3000}(84 \ in)} = 10.6 \ in \ < 12 \ in \text{ assumed.} $$

Check for punching shear (controls for square footings, such as this one).

$V_u = \phi 4\sqrt{f'_c}\, b_o d$, where b_o is the perimeter of the critical section; $b_o = 4\left(c + 2\left(\dfrac{d}{2}\right)\right)$

$\therefore \quad d = \dfrac{V_u}{\phi 4\sqrt{f'_c}\, b} = \dfrac{\left(5.9\ ksf\ (7\,ft\ x\ 7\,ft - 2\,ft\ x\ 2\,ft)1000\right)}{(0.85)(4)\left(\sqrt{3000}\right)(4)(24\ in)} = 14.8\ in$ \quad assumed;

must increase d.

Recalculate by trying $d = 14.0$ in.
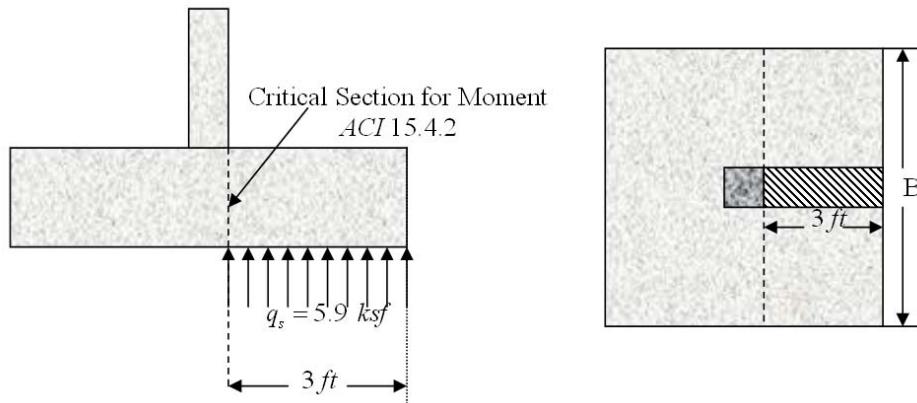
$\therefore \quad d = \dfrac{V_u}{\phi 4\sqrt{f'_c}\, b} = \dfrac{\left(5.9\ ksf\ (7\,ft\ x\ 7\,ft - 2.17\,ft\ x\ 2.17\,ft)1000\right)}{(0.85)(4)\left(\sqrt{3000}\right)(4)(26\ in)} = 13.5\ in$

$d = 13.5\ in\ <\ 14.0\ in$, therefore is Good!!

The footing thickness $T = 14\ in + 1$ bar diameter (1") + 3 in (cover) $= 18\ in$.

Footing dimensions: 7.5 feet x 7.5 feet x 18 in.

Finding the flexural reinforcement A_s:

$M_u = \dfrac{q_s B l^2}{2} = \dfrac{\left(5.9\ ksf\right)\left(7\,ft\right)\left(3\,ft\right)^2}{2} = 186\ kip.ft$

$a = \dfrac{A_s f_y}{0.85 f'_c b}$ \quad (where $b = B$) \quad then \quad $A_s = \dfrac{0.85 f'_c Ba}{f_y} = \dfrac{0.85(3000)(7\,ft)(a)}{60,000} = 0.3a$

The ultimate moment M_u is given by: $M_u = \phi A_s f_y \left(d - \dfrac{a}{2}\right)$ with $M_u = 186\ kip\text{-}ft$, and $\phi = 0.9$.

$\therefore \quad 186\ kip.ft = 0.9\left(0.3a\ ft^2\right)\left(60\ \dfrac{kip}{in^2}\right)\left(144\ \dfrac{in^2}{ft^2}\right)\left(1.16 - \dfrac{a}{2}\right)ft =$

$a = 0.07\ ft$ \quad and \quad $\therefore A_s = 0.3a = 0.021$

Percentage of steel \quad $p = \dfrac{A_s}{bd} = \dfrac{A_s}{Bd} = \dfrac{0.021\,ft^2}{(7\,ft)(1.167\,ft)} = 0.00257\ >\ p_{min} = 0.0018$

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$A_{s\ min} = 0.021\,ft^2 = 3.024\ in^2$ Therefore, use 6 # 7 bars at 12 *inches*.

e) Check development length, L_d (ACI 12.2.2) $L_d = 0.04 A_b \left(\dfrac{f_y}{\sqrt{f'_c}} \right)$ > $0.0004 d_b f_y$

$$L_d = 0.04 A_b \left(\frac{0.44\,(60,000)}{\sqrt{3000}} \right) \quad > \quad 0.0004\,(0.75)(60,000)$$

$L_d = 19.3\ in \qquad > \qquad 18\ in$

Actual $L_{d\ (provided)} = B - \dfrac{c}{2} - (cover) = \left(7\,ft\ \dfrac{12in}{1\,ft} \right) - 6\ in - 3\ in = 75\ in$

f) Check bearing strength on concrete (ACI 318-02 10.15)

Bearing strength = $0.85 \phi f'_c\, A_g \sqrt{\left(\dfrac{A_2}{A_g} \right)}$ where $\sqrt{\left(\dfrac{A_2}{A_g} \right)} \le 2$

A_g is the area of the column   and A_2 is the area of the footing.

Then $0.85 \phi f'_c\, A_g \sqrt{\left(\dfrac{A_2}{A_g} \right)} = 0.85\,(0.70)\left(\dfrac{3\ kips}{in^2} \right)(144\ in^2)(2) = 514\ kips$

But, the factored column load $U = 261\ kips < 514\ kips$. Good !
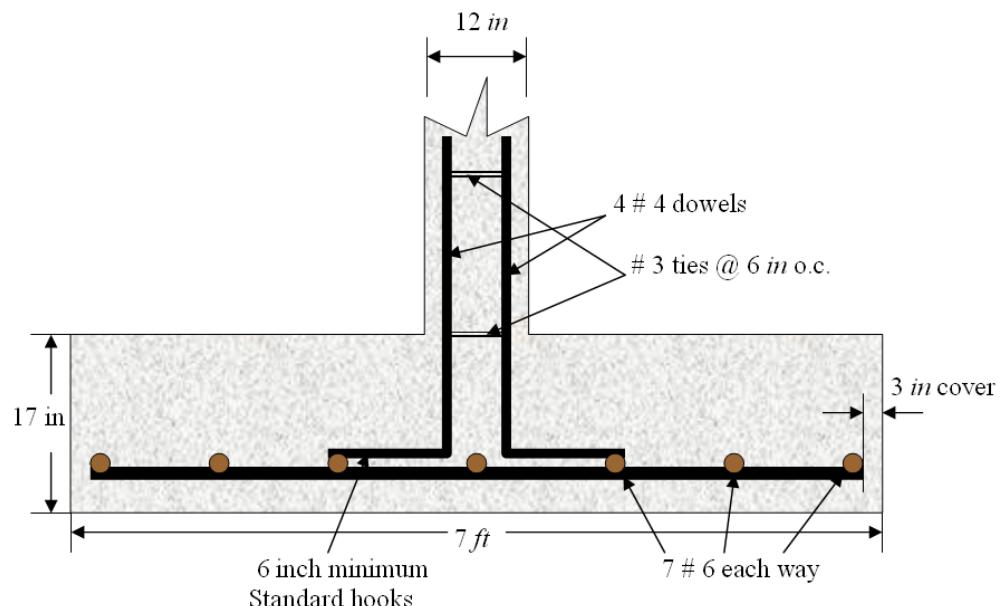
g) Dowels to column:

Since bearing strength is adequate, a minimum area of dowels should be provided across the interference of the column and footing (ACI 318-02 15.8.2.1)

Minimum area of steel = 0.005 (area of column) = 0.005 (144 in²) = 0.72 in².

Use 4 # 4 bars $A_S = 0.20\ in^2\,(4) = 0.80\ in^2$

h) Final design

12 in

4 # 4 dowels

3 ties @ 6 in o.c.

3 in cover

17 in

6 inch minimum
Standard hooks

7 ft

7 # 6 each way
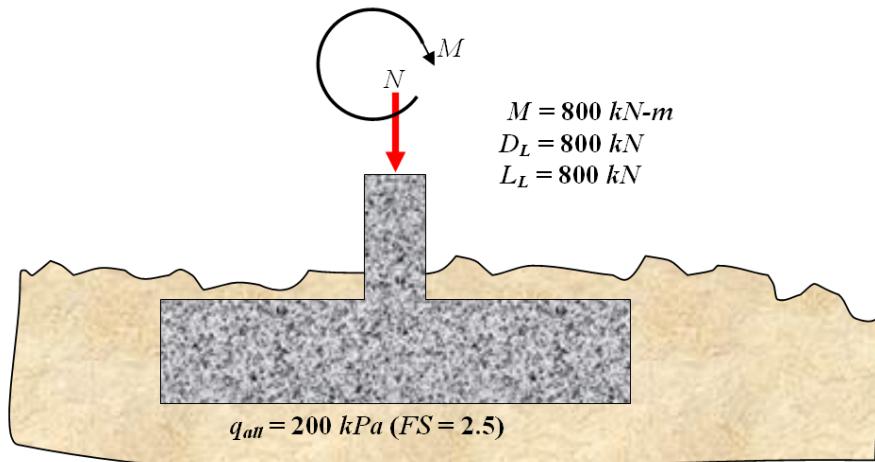
*Footings–04: Find the dimensions B x L of a rectangular footing.

Find the footing dimensions B x L to carry a moment induced by winds of 800 kN-m.

$M = 800\ kN$-m
$D_L = 800\ kN$
$L_L = 800\ kN$

$q_{all} = 200\ kPa\ (FS = 2.5)$

Solution:

Select a test value for B x L. Set B x $L = B^2$ and check the increase in soil pressure due to wind

$$B^2 = \frac{N}{q_a} = \frac{(800\ kN + 800\ kN)}{\left(200\ \frac{kN}{m^2}\right)} = 8\ m^2 \qquad \therefore \qquad B = 2.82\ m$$

$$e = \frac{M}{N} = \frac{800}{1600} = 0.5\ m \qquad\qquad \therefore \qquad L \geq 6(0.5) = 3\ m$$

If $L = 3$ m, try for $q_{max} \cong 2q_{avg}$ iterate by trying footing: 2.5 m x 4 m $= B$ x L.

$$q_{avg} = \frac{N}{A} = \frac{1600\ kN}{10\ m^2} = 160\ kPa$$

$$q_{max} = \frac{N}{BL\left(1 + \frac{6e}{L}\right)} = \frac{1600}{10}\left[1 + \frac{6(0.5)}{4}\right] = 280\ kPa$$

Note that q_{max} exceeds q_{avg} by 33 %, therefore increase the area.

Try using 2.75 m x 4.5 m dimensions;

$$q_{avg} = \frac{N}{A} = \frac{1600 \ kN}{2.75(4.5) \ m^2} = 130 \ kPa$$

$$q_{max} = \frac{N}{BL\left(1 + \dfrac{6e}{L}\right)} = 130\left[1 + \frac{6(0.5)}{4.5}\right] = 217 \ kPa$$

Iterate again, with $B = 3.0$ m and $L = 5.0$ m

$$q_{avg} = \frac{N}{A} = \frac{1600 \ kN}{3 \ m \ (5 \ m)} = 107 \ kPa$$
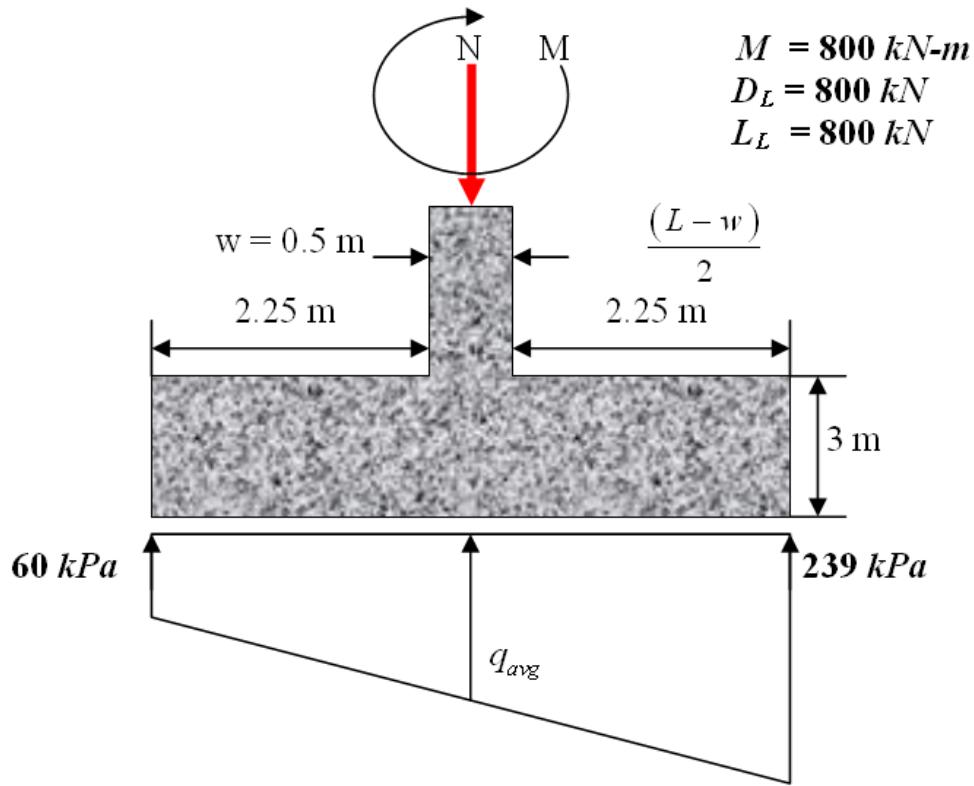
$$q_{max} = \frac{N}{BL\left(1 + \dfrac{6e}{L}\right)} = \frac{1600}{15}\left[1 + \frac{6(0.5)}{5}\right] = 171 \ kPa \quad GOOD$$

The footing dimensions are B = 3 m by L = 5 m.

(Revision: Sept-08)

Design the previous footing using $f'_c = 21 \ MPa$ *and* $f_y = 415 \ MPa$.

$N \quad M$

$M = 800 \ kN\text{-}m$
$D_L = 800 \ kN$
$L_L = 800 \ kN$

w = 0.5 m

$\dfrac{(L - w)}{2}$

2.25 m   2.25 m

3 m

60 kPa   239 kPa

q_{avg}

Solution.

1) Check the ultimate pressures:

$$N_u = 1.2(D_L) + 1.6(L_L) = 1.2(800) + 1.6(800) = 2240 \ kN$$

$$e = \frac{M}{N} = \frac{800}{1600} = 0.5 \ m$$

$$q_{max} = \frac{N_u}{A\left(1 + \dfrac{6e}{L}\right)} = \left(\frac{2240}{15}\right)\left[1 + \frac{6(0.5)}{5}\right] = 239 \ kPa$$

$$q_{min} = \frac{N}{A\left(1 - \dfrac{6e}{L}\right)} = \left(\frac{2240}{15}\right)\left[1 - \frac{6(0.5)}{5}\right] = 60 \ kPa$$

$q_{max} = 239 \ kPa$ and $q_{min} = 60 \ kPa$

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$$q_{avg} = \frac{2240\ kN}{15\ m^2} = 149\ kPa \quad < \quad q_{all} = 200\ kPa \ \textbf{Good}$$

2a) Calculate footing depth T based on punching shear, for a $f'_c = 21\ MPa$, and

$V_c = 1.29\ MPa$.
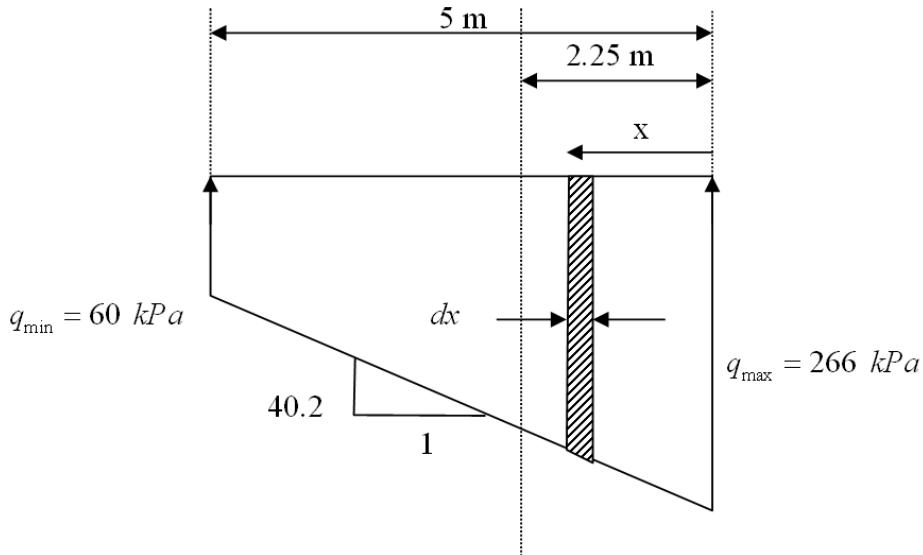
Using the simplified equation for a square footing:

$$4T^2 + 2(0.5\ m + 0.5\ m)T - \frac{(15\ x\ 149)}{2240} = 0$$

$$T^2 + 0.5T - 0.25 = 0 \quad \therefore \quad T \cong 0.50\ m$$

2b) Calculate footing depth T based on wide beam shear:

The shear is calculated from the outer edge of footing ($x = 0$) inwards towards a distance d from the column ($x = 2.225 - d$):

$$dv = qdx$$

$$V = \int_0^x qdx = \int_0^x (266 - 40.2x)dx = \left\{ 266x - \left(\frac{40.2x^2}{2} \right) \right\}_0^{2.25-T}$$

$$V = 598 - 266T - 20.1(2.25 - T)^2 =$$

$$V = \frac{V_c}{2T} = \frac{1290}{2T} \quad \therefore \quad T^2 + 40.8T - 24.7 = 0 \quad T = 0.60\ m$$

We will use the highest of (2a) or (2b), therefore $T = 0.60\ m$.

3) a) Find A_S (Longitudinal): $a = \dfrac{A_s f_y}{0.85 f'_c b} = \dfrac{415 \, A_s}{0.85(21)(1)} = 23.3 \, A_s$

$$M_u = \int_0^{2.25} V dx = \int_0^{2.25} \left(266x - \frac{40.2 x^2}{2} \right) dx = \left[\frac{266 x^2}{2} - \frac{40.2 x^3}{6} \right]_0^{2.25} = 597 \; kN.m$$

$$M_u = \phi A_s f_y \left(d - \frac{a}{2} \right) \qquad \therefore \qquad A_s \left(d - \frac{a}{2} \right) = \frac{M_u}{\phi f_y}$$

$$A_s \left(0.6 - \frac{23.3}{2} \right) = \frac{597}{0.9(415)} = A_s^2 - 0.0515 A_s + 1.37 x 10^{-2} = 0$$

$$A_s = 28.2 \, \frac{cm^2}{m}$$

Check $p = \dfrac{A_s}{bd} = \dfrac{0.00282}{1(0.6)} 0.0047 > 0.002$ GOOD

3) b) Find A_S (transverse):   Use the high average.

$$q = \sqrt{(q_{ave} + q_{max})} = \sqrt{(149 + 266)} = 20.37 \; kPa$$

$$M_u = \frac{w l^2}{2} = \frac{q \left[\dfrac{(B-0.5)}{2} \right]^2}{2} = \frac{\left(215 \left(\dfrac{(3-0.5)}{2} \right)^2 \right)}{2} = 168 \; kN.m$$

and $A_s \left(d - \dfrac{a}{2} \right) = \dfrac{M_u}{\phi f_y}$ $\therefore$ $A_s^2 - 0.0515 A_s + 0.000386 = 0 \therefore$ $A_s = 7.61 \, \dfrac{cm^2}{m}$

Check for $\rho = \dfrac{A_s}{bT} = \dfrac{7.61 \; x \; 10^{-4} \; m^2}{1(0.6)} = 0.00126 \; < \; 0.002$
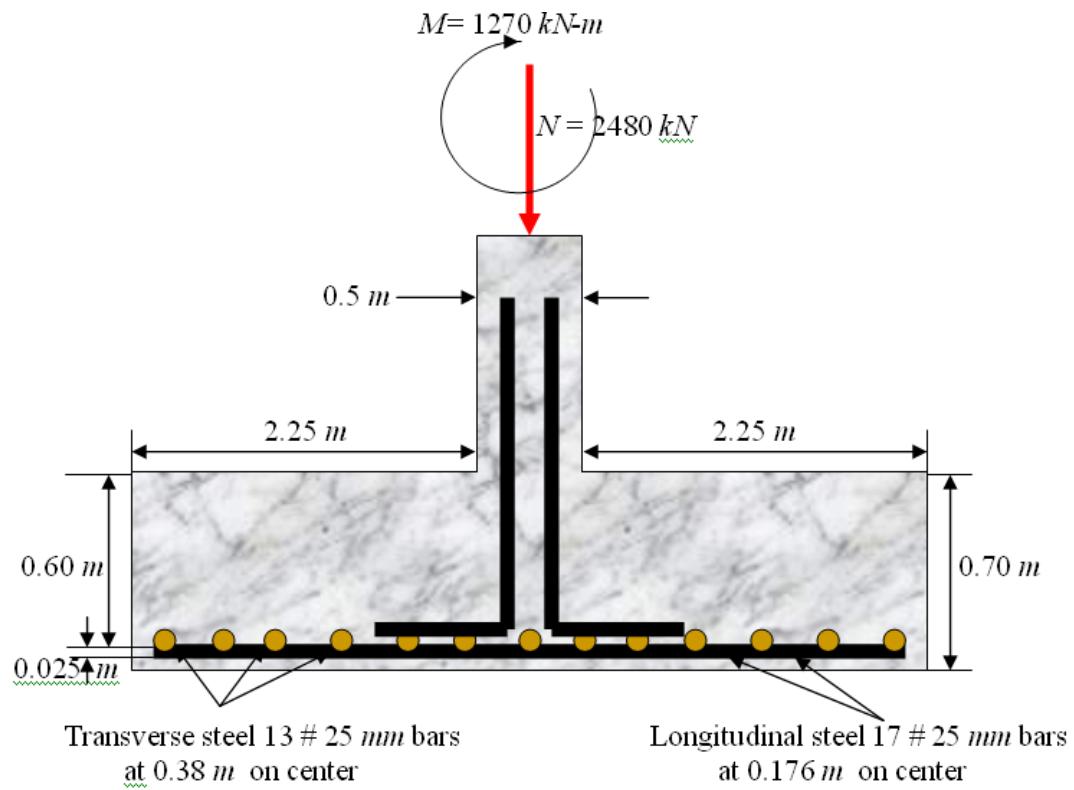
$\therefore$ Use minimum $A_s = 0.002(b)(T) = 0.002(1)(0.6) = 0.0012 \, \dfrac{m^2}{m} = 12 \dfrac{cm^2}{m}$

4) Check footing:

Longitudinal steel: $28.2 \; \dfrac{cm^2}{m} \; (3 \; m) = 84.6 \; cm^2 \; \dfrac{1 m^2}{10,000 cm^2} = 8.46 \; x 10^{-3} \; m$

Therefore use 17 # 25 mm bars at 17.6 $cms \; o - c$

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Sketch:

$M = 1270\ kN\text{-}m$

$N = 2480\ kN$

0.5 m

2.25 m

2.25 m

0.60 m

0.70 m

0.025 m

Transverse steel 13 # 25 mm bars
at 0.38 m on center

Longitudinal steel 17 # 25 mm bars
at 0.176 m on center
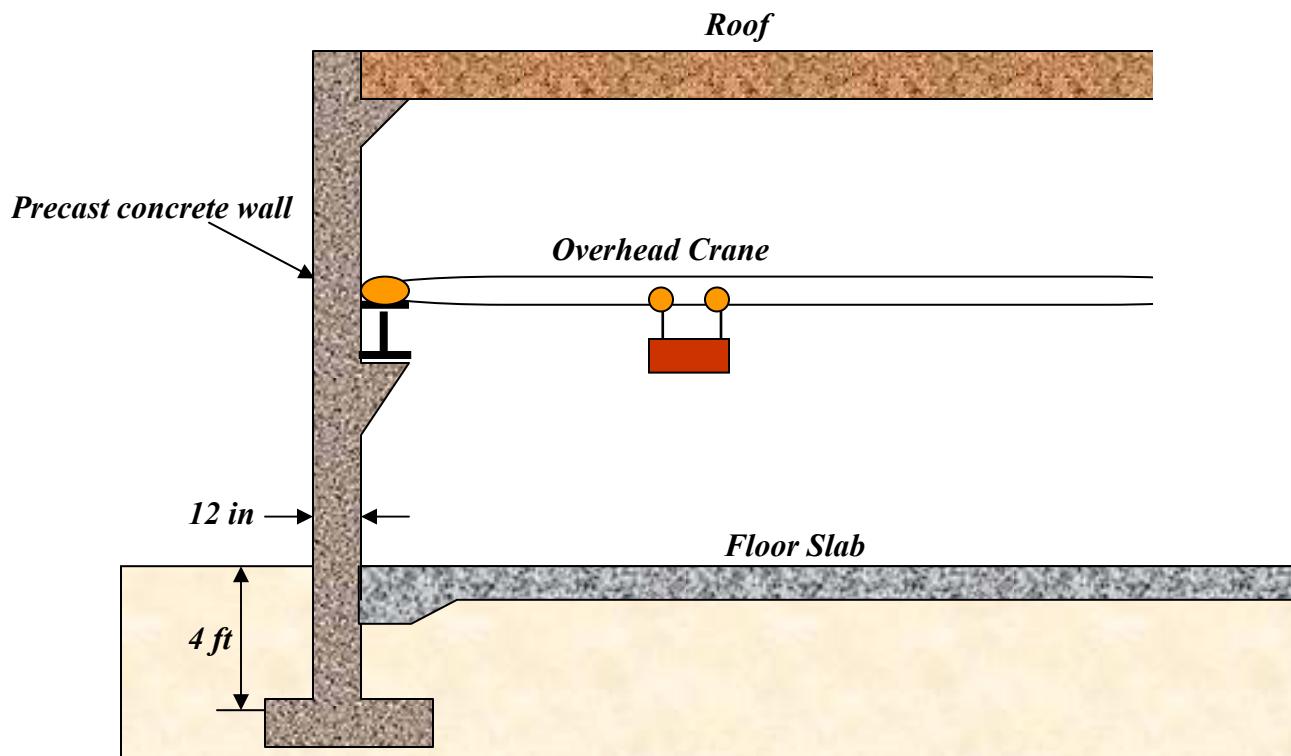
(Revision: Sept-08)

Design a continuous footing for the warehouse wall with the loads shown below:
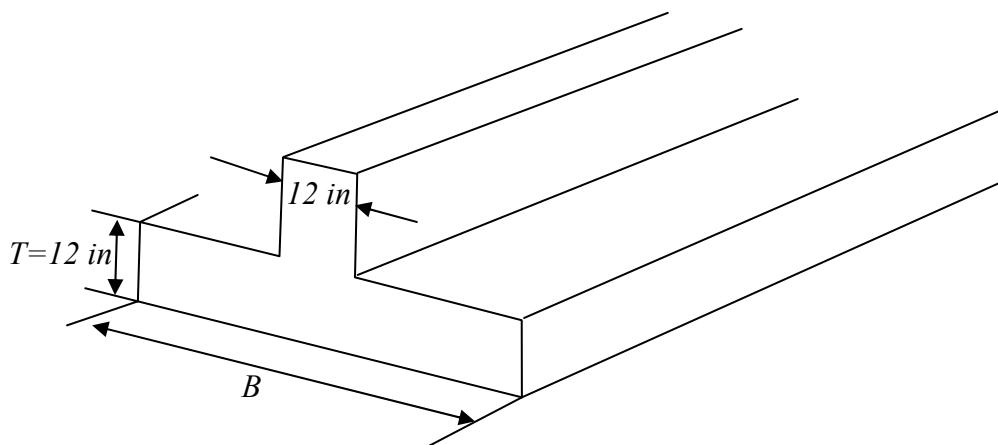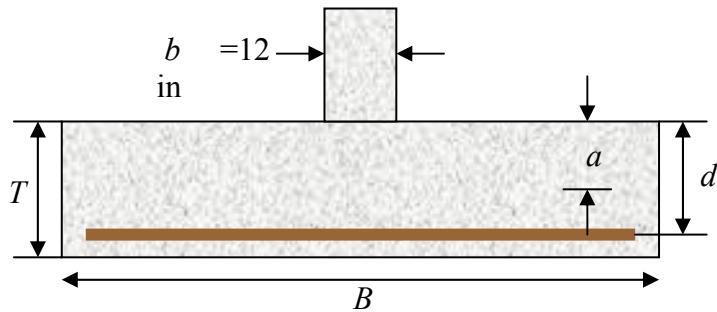
Solution:

1) <u>Assume an initial footing thickness $T = 12$":</u>
   From ACI 7.7.2, the minimum cover is 3" from the steel to the footing invert.   Assume # 4 bars.
   Therefore, d = 12" - 3" - 0.5" / 2 = 8.75"

Find the ultimate soil pressure q_{ult}.

The actual soil pressure from the structure is,

$q_o = q_{all} - q_{concrete} - q_{soil\text{-}above\,footing}$

$q_o = 2 \text{ ksf} - (1 \text{ ft })(0.15 \text{ k/ ft}^3) - (3 \text{ ft })(0.110 \text{ k/ ft}^3) = 2 - 0.15 - 0.33 = 1.52 \text{ ksf}$

Estimate $B = Q/ q_o = 4.2 \text{ k/ } 1.52 \text{ ksf} = 2.76$ ft. per unit length. Therefore, assume $B = 3$ ft.

Ultimate load, $U = 1.2 \text{ DL} + 1.6 \text{ LL} = 1.2 (3) + 1.6 (1.2) = 4.0 + 2.0 = 6.0$ kips

Therefore, the soil pressure at ultimate loads q_u is:

$q_u = U / (B)(1) = 6.0 \text{ kips/ } (3 \text{ ft })(1 \text{ ft }) = 2.0 \text{ ksf} < q_{ult} = 4 \text{ ksf} \rightarrow$ GOOD
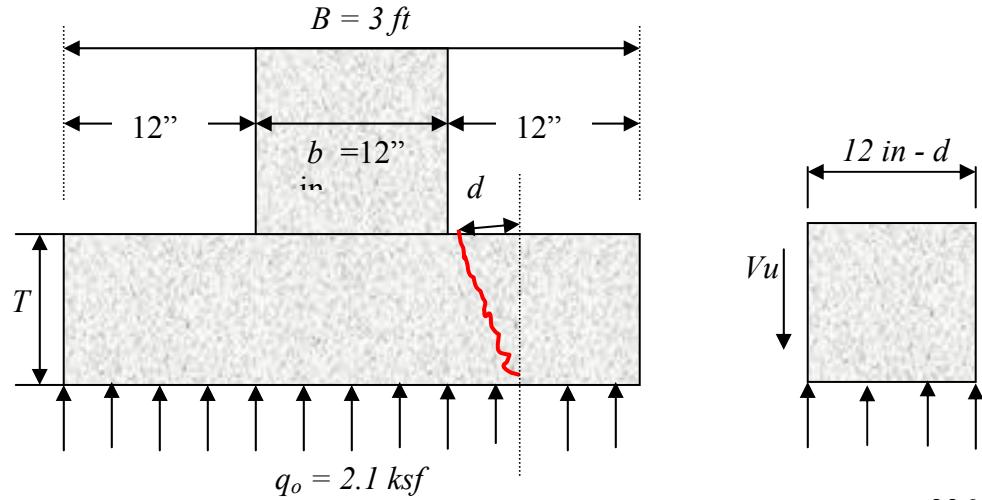
2) Check the shear strength of the footing:

The critical section for shear section occurs at a distance $\underline{d}$ from the face of the wall.

(ACI 15.5 & ACI 11.11.1).

$q_o = 2.1 \ ksf$

$V_u = (12 - d) q_o (1)$

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The ultimate shear $V_u = (12" - 8.75")(1 \text{ ft} / 12 \text{ in}) \times 2.1 \text{ k/ ft}^2 = 0.57 \text{ kips} / \text{ft of wall}$

The concrete shear strength must be: $V_u \le \phi V_c = \phi 2\sqrt{f'_c}\, bd$

$V_u = 0.85 \cdot 2 \cdot \sqrt{3000} \cdot 12in \cdot 8.75in = 9.8 \text{ kips} / \text{ ft of wall} >> 0.57 \text{ kips} / \text{ ft}$

Since $V_u << \phi V_c$, we can reduce the thickness of the footing $\underline{T}$ from 1.0 ft to say 0.85 ft (~10").
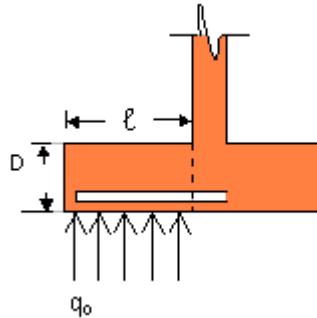
Therefore, $d = 10" - 3" - 0.5 / 2 = 6.5" > 6" = d_{min}$ from `ACI 15.4`

Rechecking, $\qquad V_u = 0.85 \cdot 2 \cdot \sqrt{3000} \cdot 12in \cdot 6.5in = 7.3 \text{ kips} / \text{ ft of wall} \rightarrow GOOD$

Therefore, the total thickness $T = d + \text{bar diameter} + 3" = 10" + 1" + 3" = \underline{14"}$

3) <u>Design the flexural reinforcement, A_s (`ACI 15.4`):</u>

$M_u = q_o\, \ell^2 / 2 \qquad\qquad$ where $\ell = 12"$

$M_u = (2.1 \text{ ksf} \times 1 \text{ ft}^2) / 2 = 1.05 \text{ kip-ft} / \text{ft of wall}$

But, $a = A_s f_y / [\, 0.85\, f'_c\, (b)\,] = [\,(60 \text{ ksi})\, A_s\,] / [\, 0.85\, (3 \text{ ksi})(12 \text{ in})\,] = 1.96\, A_s$ (inches)

and, $M_n = A_s f_y\, (d - a/2) = A_s\, (60 \text{ k} / \text{in}^2)\, (d - a/2) = 60\, A_s\, (6.5 \text{ in} - 9.8\, A_s)$

But, $M_u = \phi\, M_n = 0.9\, M_n$

Therefore, $\qquad 1.05 \text{ kips-in} / \text{in} = 0.9\, (60 \text{ k} / \text{in}^2)\, A_s\, (6.5 \text{ in} - 9.8\, A_s)$

$53\, A_s^2 - 351 A_s - 1.05 = 0$

Two possible answers: $\qquad A_s\, (1) = 6.6 \text{ in}^2$ per ft. of wall

$\qquad\qquad\qquad\qquad\qquad A_s\, (2) = 0.003 \text{ in}^2$ per ft. of wall

The percentages of steel with $A_s\, (1)$ & $A_s\, (2) \qquad$ (`ACI 7.12.2.1`)

337

$\rho_1 = A_{s(1)} / bd = 6.6 \text{ in}^2 / (12 \text{ in})(6.5 \text{ in}) = 0.085$

$\rho_2 = A_{s(2)} / bd = 0.003 \text{ in}^2 / (12 \text{ in})(6.5 \text{ in}) = 0.0004 < 0.0018$ minimum steel

The maximum steel percentages allowed $\rho_{max} = 0.75 \, \rho_b$, where

$\rho_b = (0.85 \, f'_c / f_y) \, \beta \, (87{,}000 / (87{,}000 + f_y))$

$= (0.85 \, (3) / 60) \, 0.85 \, (87{,}000 / (87{,}000 + 60{,}000)) = 0.021$
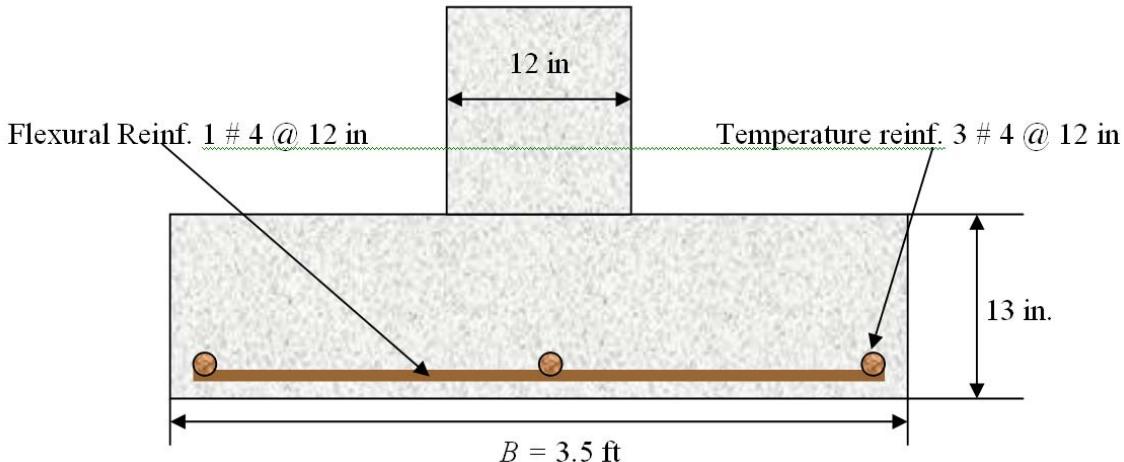
therefore,

$\rho_{max} = 0.75 \, \rho_b = 0.75 \, (0.021) = 0.016$

Note that $\rho_1 = 0.085 > \rho_{max} = 0.016$ therefore, <u>use $\rho_{min} = 0.0018$</u>

Therefore,

$A_s = \rho_{min} \, b \, d = (0.0018)(12 \text{ in})(6.5 \text{ in})$

$= \underline{0.14 \text{ in}^2 \text{ per ft. of wall}} \rightarrow \boxed{\text{use } 1 \ \#4 \text{ every ft. of wall } (A_s = 0.20 \text{ in}^2)}$

4) <u>Check the development length, L_d (`ACI 12.2`):</u>

12 in

Flexural Reinf. 1 # 4 @ 12 in   Temperature reinf. 3 # 4 @ 12 in

13 in.

B = 3.5 ft

$L_d = 0.04 \, A_b \, \dfrac{f_y}{\sqrt{f'_c}}$ *(But not less than $0.0004 \, d_b f_y$)* $= 0.04 \cdot 0.20 \cdot \dfrac{60{,}000}{\sqrt{3000}} = 8.8"$

$L_d = 8.8"$ or $12" \rightarrow$ Clearly, 12" controls.

Presently we have 12" – 3" cover = 9" < 12". Therefore, we are missing 3" on each side.

Increase the footing width <u>B</u> by 6" to ***B = 3.5 ft.***

(Note that increasing B, reduces q_o, and the design could be further optimized.)

Therefore,

B = 3.5 ft

T = 13"

$A_s = 1$ #4 @ 12" along the wall

Use minimum steel in longitudinal direction, to offset shrinkage and temperature effects (`ACI 7.12`):

$A_s = (0.0018)(b)(d) = 0.0018 (42 \text{ in})(6.5 \text{ in}) = 0.49 \text{ in}^2$
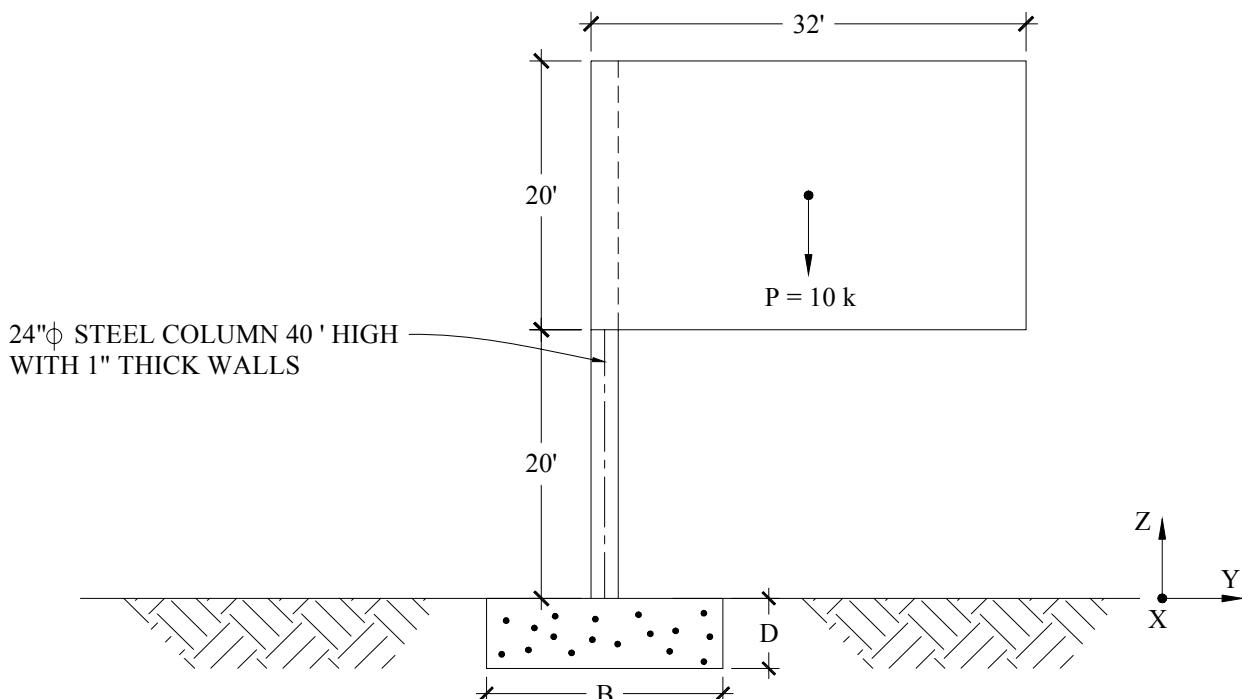
Provide 3 #4 bars at 12" o.c. ($A_s = 0.60 \text{ in}^2$)

**Footings–07: *Design the footings of a large billboard sign.*

(Revised Oct-09)

Design a spread footing for the billboard sign shown below using FBC-2004 and ASCE 7- 02. Ignore the torsion and the wind load on the column, and the water table.

Given: γ = 150 pcf c = 150 pcf ϕ = 20° V = 146 mph

solution:

STEP #1: Find the wind load as per ASCE 7-02, assuming an Exposure C, Category I.

$q_z = 0.00256\, K_z\, (IV)^2$

$K_z = 0.98$

$I = 1.05$ $F = q_z\, G_h\, C_f\, A_f$ $\therefore\ q_z = 52$ psf

$V = 146$ mph

$G_h = 1.26$

$C_f = 1.2$

The sign shape factor is $\dfrac{M}{N} = \dfrac{32}{20} = 1.6$

Therefore: $F = \dfrac{(34\text{ psf})\,(1.26)\,(1.2)\,(32\text{ ft x }20\text{ ft})}{1000} = 32.4$ kips

Step #2: Calculate loads on footing

Weight of steel column $= \gamma_s$ L A $= 0.49$

$M_x = 10$ kips x 15' $= 150$ k-ft

$M_y = 32.4$ k x 30' $= 972$ k-ft

$M_z = 32.4$ k x 15' $= 486$ k-ft

Total (normal) load N $= 10$ k $+ 5$ k $= 15$ kips

Step #3: Calculate the footing's bearing capacity using Hansen's formula.

c (cohesion) $= 0.150$ ksf

$q = \gamma D_f =$ (embedment pressure) $= (0.130$ ksf$)(3$ ft$) = 0.39$ ksf

$B =$ (footing width – initial assumptions) $= 5$ ft

$L =$ (footing length – initial assumptions) $= 15$ ft

N_q (factor for embedment at $\phi = 20°$) $= e^{\pi \tan \phi} \tan^2(45+\phi/2) = 6.40$

N_c (factor for cohesion at $\phi = 20°$) $= (N_q - 1)\cot \phi = 14.83$

N_γ (factor for width at $\phi = 20°$) $= 1.5 (N_q - 1)\tan \phi = 2.95$

$F_{sq} =$ (shape factor for embedment) $= 1.0 + (B/L) \sin \phi = 1.11$

$F_{sc} =$ (shape factor for cohesion) $= 1.0 + (N_q / N_c) (B/L) = 1.14$

$F_{s\gamma} =$ (shape factor for width) $= 1.0 - 0.4 (B/L) = 0.867$

$F_{dq} =$ (depth factor for embedment) $= 1 + 2 \tan \phi(1 - \sin \phi)^2 (D_f/B) = 1.19$

$F_{dc} =$ (depth factor for cohesion) $= 1.0 + 0.4 (D_f / B) = 1.24$

$F_{d\gamma} =$ (depth factor for width) $= 1.0$

$F_{ic} =$ (inclination factor) $= 0.5 - \sqrt{\left(\dfrac{1 - H}{(Af\,Ca)} \right)}$

where $c_a = (0.6$ to $1.0)$ c

$F_{iq} = [\ 1 - (0.5\ H) / (V + A_f\ C_a \cot \phi)]^d$ where $2 \le d \le 5$

$F_{i\gamma} = [1 - (0.7\ H) / (V + A_f\ C_a \cot \phi)]^\alpha$

$\boldsymbol{q_{ult}} = c'\ N_c\ F_{sc}\ F_{dc}\ F_{ic} + \bar{q}\ N_q\ F_{sq}\ F_{dq}\ F_{iq} + 0.5\ \gamma B'\ N_\gamma\ F_{s\gamma}\ F_{d\gamma}\ F_{i\gamma}$

Step #4: Assume:

$B = 10'$

$L = 50'$

$D = 3'$

B/L = 0.2

FS = 3.0

SC = 1.0 + (0.431 x 0.2) = 1.09

DC = 1.0 + (0.4)(3/10) = 1.12

Q = (130)x3 = 390

SQ = 1.0 + 0.2 sin 20 = 1.07

SJ = 1.0 – 0.4(0.2) = 0.92

DQ = 1 + (0.315)(3/10) = 1.09

$$q_{max,min} = \frac{P}{BL} + \left(\frac{-6P\,e_y}{B^2L}\right) + \left(\frac{-6P\,e_x}{BL^2}\right) \text{ where P} = 15 \text{ kips} + \frac{((3x10x50x0.150))}{FTG\,WT} = 240 \text{ kips}$$

$$e_x = \frac{M_y}{P} = \frac{972}{240} = 4.05'$$

$$e_y = \frac{M_x}{P} = \frac{150}{240} = 0.625'$$

$$q_{max,min} = \left(\frac{240 kips}{500}\right) + \left(\frac{-6(240)(0.625)}{102\,x\,50}\right) + \left(\frac{-6(240)(4.05)}{10\,x\,502}\right) = 0.89\ ksf < 2.5\ ksf \text{ GOOD}$$

Step #5: Check out (long direction),

MOT = 32.4 Kips (30+3) = 1069 kip-ft

MR = 5 Kips x 25' + 225 Kips x 25' = 5750 kip-ft

F.S. = 5750 / 1069 = 5.4 >> 1.5   GOOD

Check Sliding RS = Σ V tan f + CB = 240 tan f + 150 (10) = 1587 kips >> 32.4 kips

#3

LOAD COMBINATION = 0.75 (1.2D + 1.6L + 1.7W)

FACTORED LOADS: **_Pu_**   = 1.05 x 240 Kips = 252 kips

$\qquad\qquad\qquad\qquad\qquad$ **_Mu-x_** = 1.05 x 150 kip-ft = 158 kip-ft

$\qquad\qquad\qquad\qquad\qquad$ **_Mu-y_** = 1.275 x 972 kip-ft = 1239 kip-ft

$$\therefore\ Q_{MAX,MIN} = \left(\frac{336}{10\,x\,50}\right) \pm \left(\frac{6\,x\,336\,x\,0.625}{10^2\,x\,50}\right) \pm \left(\frac{6\,x\,336\,x\,4.9}{10\,x\,50^2}\right) = 0.989 \text{ ksf, } 0.019 \text{ ksf}$$

Check beam shear: D = 36" – 4" = 32"

V_U = 0.019x(50 – 21.33) + 0.15(50-21.33)(0.989 – 0.019) = 14.4 kip per feet

Punching shear will not govern by observation.

#3 Design for flexure in long direction

$f'_C = 3000$ PSI   and $f_Y = 60000$ PSI

$A = A_S \times F_Y / 0.85 \, F'C \, B$ $A_S = MU \times 12 / F \, FY \, (D - A/2)$

$A = 1.64$ in. $A_S = 0.83$ in^2

$$R = \frac{AS}{BD} = \left(\frac{0.83}{12 \times 32}\right) = 0.0022 > 0.0018 \text{ OK}$$

USE # 7 @ 8" O/C, $A_s = 0.90$ in^2 OK

Therefore; $M_U = (0.9)(60)\dfrac{\left(132 - \dfrac{1.64}{2}\right)}{12} = 126$ kip-ft > 116 kip-ft   GOOD

Use a footing 10 feet x 50 feet x 3 feet thick with # 7 bars @ 8" on-center top and bottom, each way.

NOTE: **In lieu of such a large and expensive footing, a short drilled shaft would be an efficient and inexpensive foundation. That alternative will be covered in the drilled shaft section later in this course.**

Chapter 19
Combined Footings

Symbols for Combined Footings

Chapter 20
Mat Foundations

(Revision: Sept-08)

Determine the ultimate bearing capacity of a mat foundation measuring 45 feet long by 30 feet wide placed 6.5 feet below the surface and resting upon a saturated clay stratum with c_u = 1,950 lb/ft² and $\varphi = 0^o$.

Solution:

Mat foundations in purely cohesive soils have the following ultimate bearing capacity:

$$q_{ult(net)} = 5.14\, c_u (1 + \frac{0.195B}{L})(1 + \frac{0.4D_f}{B})$$

$$q_{ult(net)} = 5.14(1.95\ ksf)\ [1 + \frac{0.195(30\,ft)}{(45\,ft)}]\ [1 + \frac{0.40(6.5\,ft)}{(30\,ft)}] = 12\ ksf$$

Chapter 21
Deep Foundations - Single Piles

(Revision: Oct.-08)

A concrete pile 20 m long with a cross section of 381 mm x 381 mm is fully embedded in a saturated clay stratum. The clay has $\gamma_{sat} = 18.5$ kN/m³, $\varphi = 0°$ and $c_u = 70$ kN/m². The water table lies below the tip of the pile. Determine the allowable capacity of the pile for a FS = 3 using the α-method.

Solution:

The ultimate capacity of the pile Q_{ult} is given by the simple formula,
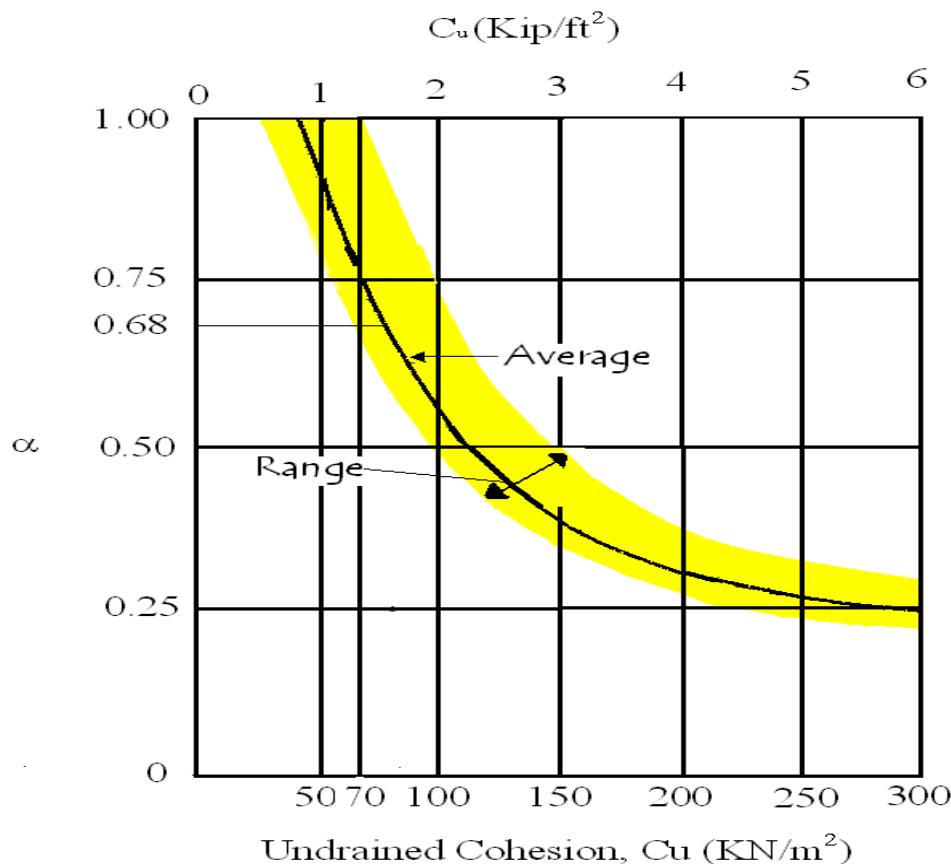
$$Q_{ult} = Q_{point} + Q_{shaft} = A_p \, q_p + (c_u)(perimeter)L = A_p(c_u)N_c + \alpha(c_u)(perimeter)L$$

Notice that the value of the cohesion is reduced by the "α" factor found in the graph below,

$$\therefore \quad Q_{ult} = (0.38m)^2 (70 \, kN/m^2)(10.97) + (0.75)(70 \, kN/m^2)4(0.38m)(20m) = 1,890 \, kN$$

The allowable capacity is,

$$Q_{all} = \frac{Q_{ult}}{3} = \frac{1,890 kN}{3} = 630 \, kN$$

$C_u (\text{Kip/ft}^2)$

Average

Range

α

Undrained Cohesion, Cu (KN/m²)

Chapter 22
Deep Foundations - Pile Groups and Caps

(Revision: Oct-08)

Design a pile cap footing to support an 18" square column subjected to a live load reaction of 180 kips and a dead load reaction of 160 kips at service loads. The testing laboratory recommends an ultimate pile load of 70 kips per pile, and a service pile load of 42 kips per pile. The vertical steel in the column consists of 12 No.7 bars. Use $f'_c = 3000$ psi, $f_y = 40,000$ psi, and 12" diameter piles.

Solution.

Since the footing weight will be about 3 kips/pile, the net service load per pile is 42.0-3.0 = 39.0 kips/pile. The number of piles required in $N=W/P = 340/39 = 8.7$, or 9 piles. Use a pile pattern as shown in Fig. 1. The net ultimate load is used to design the footing; thus $W_u = (1.4) (160) + (1.7) (180) = 530$ kips, and the load per pile is $P_u = 530/9 = 58.9$, say 59.0 kips/pile, which is less than the maximum ultimate load, 70 kips/ pile. Punching shear around a single pile often governs the footing depth determination, except in cases in which the loads are small. In this case, it will be shown that beam shear governs. Referring to Fig. 2, we calculate the punching shear stress.   After several trials, assume $d = 19.5"$. The shear perimeter is $b_o = \pi(12 + d) = 99.0"$. The permissible shear force around the pile will be,

$$V_c = 4\sqrt{f'_c}\, b_o d = 4\sqrt{3000}\ (99)\ (19.5) / 1000 = 423 \text{ kips}$$

Since the actual shear force is the nominal pile reaction, $P_n = P_u/\phi = 59.0/0.85 = 69.4$ kips < 423 kips, the pile will not punch through the pile cap (footing).
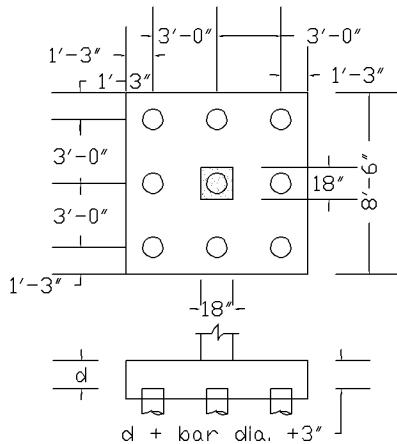

Figure 1
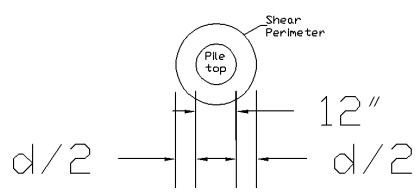
Figure 2
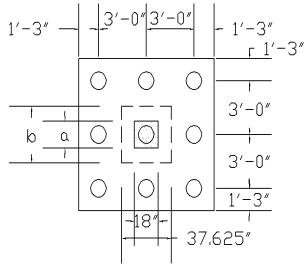
Figure 3
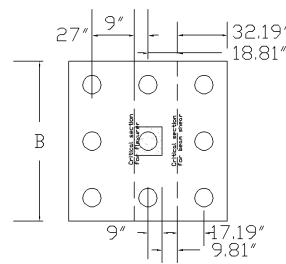
Figure 4

Perimeter shear (punching shear) must now be checked around the column in a similar manner. In this case, all of the nominal pile reactions outside of the critical section plus any partial reactions outside of the critical section will contribute to punching shear for the column. Refer to Figure 3. Assuming No. 6 bars will be used, clearance above the pile butts will be 3″ and embedment of the piles will be 6″. The total dept required will be 28.75″. For practical reasons use 29″; this furnishes an effective depth $d = 19.625″$. Thus $c = a + b = 18.0 + 19.625 = 37.625″$ and $b_o = 4(37.625) = 150.0″$. Hence, $V_{ou} = 472$ kips on 8 piles outside of the critical section as shown on Fig. 3. The permissible punching shear force ($\beta_c = 18/18 < 2$) is given by (6.12) as

$$V_c = 4 \sqrt{3000} \ (150) \ (19.625) \ / \ 1000 = 644.9 \text{ kips}$$

The force to be resisted is $V_n = V_{ou}/\phi = 472/0.85 = 555.3$ kips; therefore the pile cap (footing) is satisfactory for punching shear. Beam shear must now be checked. Refer to Fig. 4. Three piles exist beyond the critical section, so $V_u = (3) \ (59.0) = 177.0$ kips. Since $b = B = 8′\text{-}6″ = 102″$, the permissible beam shear (one-way shear) force on the critical section is

$$V_c = 2\sqrt{f'_c} \ bd = 2 \ \sqrt{3000} \ (102) \ (19.625) \ / \ 1000 = 219.3 \text{ kips}$$

The force to be resisted is the nominal shear force, $V_n = V_u \ / \ \phi = 177/ \ 0.85 = 208.2$ kips. Hence the footing is satisfactory for beam shear. The bending moment about the face of the column must now be investigated. Refer to Fig. 4

$$M_u = (177) \ (27/12) = 398.3 \text{ ft-kips}$$

$$\phi R_{u =} M_u \ / \ bd^2 = 398.3 \text{ x } 12,000 \ / \ (102)(19.625)^2 = 121.67 \text{ psi}$$

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Table 5.2 for $f'_c = 3000$ psi and $f_y = 40,000$ psi, discloses the fact that the steel ration required is less than the minimum steel ration, $\rho_{min} = 200 / f_y = 0.005$. Further, if the steel ration required is increased by 1/3, it will still be less than ρ_{min}. It would appear that 4/3 times the required steel ration would satisfy the 1983 ACI Code. However, the Code does not permit un-reinforced (plain concrete) pile caps. Since any section having less than minimum reinforcement is usually considered to be un-reinforced, the minimum are of steel will be provided. Thus,
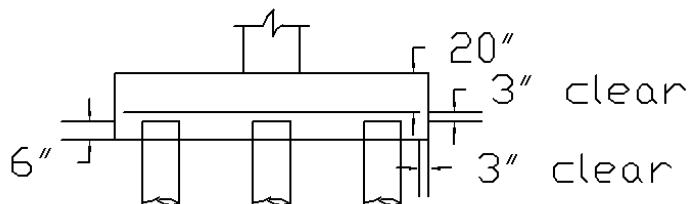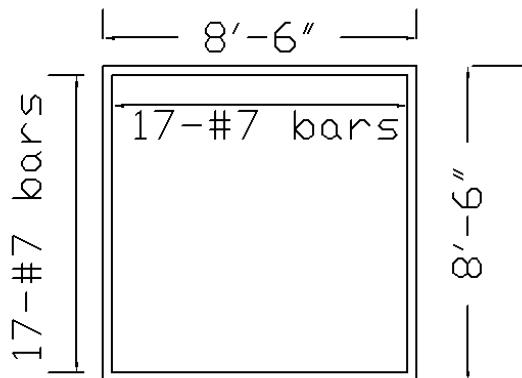
$$A_s = (200/f_y) \, bd = (200/40,000) \, (102) \, (19.925) = 10.0 \text{in.}^2$$

Use seventeen No. 7 bars ($A_s = 10.2$ in.^{2}).

The 1983 ACI Code is not explicit concerning minimum steel for footings. Hence, some structural engineers use 0.002bh for minimum steel area if f_y 40,000 psi and 0.0018bh if $f_y = 60,000$ psi. This corresponds to temperature and shrinkage reinforcement requirements. The assumed footing weight must finally be checked. The total weight is

$$W_F = (8.5) \, (8.5) \, (29) \, (12.5) / 1000 = 26.2 \text{ kips}$$

And the weight per pile is 26.2/ 9 = 2.91 kips / pile. The assumed weight of 3.0 kips / pile is most satisfactory. The final details are shown below
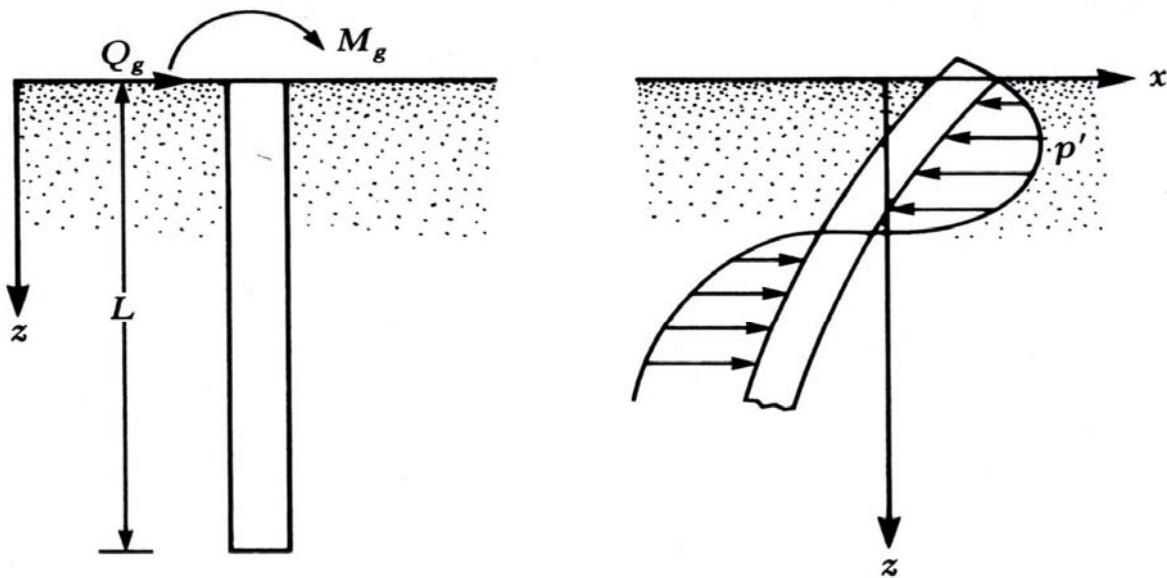
Chapter 23
Deep Foundations: Lateral Loads

(Revision: Oct-08)

Determine the lateral load capacity Q_g of a steel H-pile (HP 250 x 0.834) fully embedded to a depth of 25 m in very dense submerged sand. The top end of the pile is allowed to deflect laterally 8 mm. For simplicity assume that there is no moment applied to the top of the pile (that is, $M_g = 0$).

Solution:

The subgrade modulus k_s is a description of the reaction of the soil mass to vertical loads. The modulus of horizontal subgrade reaction n_h is a function of k_s at any depth z,

$$k_z = n_h z$$

Type of soil	n_h modulus of horizontal subgrade reaction	
	lb/in^3	kN/m^3
Dry or moist sand		
- loose	6.5 to 8.0	1,800 to 2,200
- medium	20 to 25	5,500 to 7,000
- dense	55 to 65	15,000 to 18,000
Submerged sand		
- loose	3.5 to 5.0	1,000 to 1,400
- medium	12 to 18	3,500 to 4,500
- dense	32 to 45	9,000 to 12,000
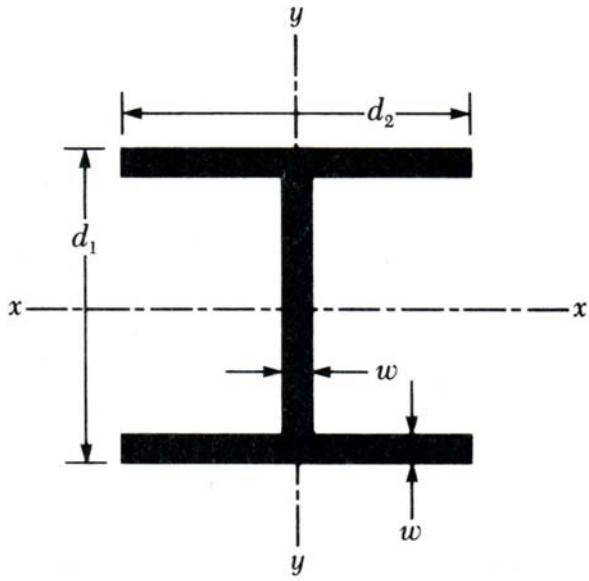
From this table and the soil conditions noted above, choose $n_h = 12$ MN/m³ for the modulus. Now choose the parameters for the steel H-pile,

| Designation, size (mm) × weight (kN/m)	Depth d_1 (mm)	Section area (m² × 10⁻³)	Flange and web thickness w (mm)	Flange width d_2 (mm)	Moment of inertia (m⁴ × 10⁻⁶) I_{xx}	I_{yy}
HP 200 × 0.52	204	6.84	11.3	207	49.4	16.8
HP 250 × 0.834	254	10.8	14.4	260	123	42
× 0.608	246	8.0	10.6	256	87.5	24
HP 310 × 1.226	312	15.9	17.5	312	271	89
× 1.079	308	14.1	15.49	310	237	77.5
× 0.912	303	11.9	13.1	308	197	63.7
× 0.775	299	10.0	11.05	306	164	62.9
HP 330 × 1.462	334	19.0	19.45	335	370	123
× 1.264	329	16.5	16.9	333	314	104
× 1.069	324	13.9	14.5	330	263	86
× 0.873	319	11.3	11.7	328	210	69
HP 360 × 1.707	361	22.2	20.45	378	508	184
× 1.491	356	19.4	17.91	376	437	158
× 1.295	351	16.8	15.62	373	374	136
× 1.060	346	13.8	12.82	371	303	109

For future reference, this is the same table in British units,

Designation size (in.) × weight (lb/ft)	Depth d_1 (in.)	Section area (in²)	Flange and web thickness w (in.)	Flange width d_2 (in.)	Moment of inertia (in⁴) I_{xx}	I_{yy}
HP 8 × 36	8.02	10.6	0.445	8.155	119	40.3
HP 10 × 57	9.99	16.8	0.565	10.225	294	101
× 42	9.70	12.4	0.420	10.075	210	71.7
HP 12 × 84	12.28	24.6	0.685	12.295	650	213
× 74	12.13	21.8	0.610	12.215	570	186
× 63	11.94	18.4	0.515	12.125	472	153
× 53	11.78	15.5	0.435	12.045	394	127
HP 13 × 100	13.15	29.4	0.766	13.21	886	294
× 87	12.95	25.5	0.665	13.11	755	250
× 73	12.74	21.6	0.565	13.01	630	207
× 60	12.54	17.5	0.460	12.90	503	165
HP 14 × 117	14.21	34.4	0.805	14.89	1220	443
× 102	14.01	30.0	0.705	14.78	1050	380
× 89	13.84	26.1	0.615	14.70	904	326
× 73	13.61	21.4	0.505	14.59	729	262

For the steel HP 250 x 0.834 pile, the moment of inertia about the strong axis is $I_p = 123$ x 10^{-6} m⁴, its modulus of elasticity is $E_p = 207$ x 10^6 kN/m², the steel's yield strength is $F_y = 248$ MN/m² and the pile depth $d_1 = 0.254$ m.

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The characteristic length *T* of a pile-soil system is given by,

$$T = \sqrt[5]{\frac{E_p I_p}{n_h}} = \sqrt[5]{\frac{(207 x 10^6)(123 x 10^{-6})}{(12,000)}} = 1.16 \ m$$

Therefore, the ratio *L* / *T* = 25 m / 1.16 m = 21.6 > 5, so this is a *long pile*.

The formula for the pile's top end lateral deflection *Δ* at any depth *z* is given by,

$$\Delta = A_z \frac{Q_g T^3}{E_p I_p} + B_z \frac{M_g T^2}{E_p I_p} \quad but \ M_g = 0$$

In this problem we are given this value of *Δ* = *8 mm*, and we want to find the allowable lateral load *Q_g*, at a depth *z* = 0, where the coefficient *A_z* is taken from a table of coefficients *k_z* = *n_h z* for long piles.

Z	A_x	A_θ	A_m	A_v	A'_p	B_x	B_θ	B_m	B_v	B'_p
0.0	2.435	−1.623	0.000	1.000	0.000	1.623	−1.750	1.000	0.000	0.000
0.1	2.273	−1.618	0.100	0.989	−0.227	1.453	−1.650	1.000	−0.007	−0.145
0.2	2.112	−1.603	0.198	0.956	−0.422	1.293	−1.550	0.999	−0.028	−0.259
0.3	1.952	−1.578	0.291	0.906	−0.586	1.143	−1.450	0.994	−0.058	−0.343
0.4	1.796	−1.545	0.379	0.840	−0.718	1.003	−1.351	0.987	−0.095	−0.401
0.5	1.644	−1.503	0.459	0.764	−0.822	0.873	−1.253	0.976	−0.137	−0.436
0.6	1.496	−1.454	0.532	0.677	−0.897	0.752	−1.156	0.960	−0.181	−0.451
0.7	1.353	−1.397	0.595	0.585	−0.947	0.642	−1.061	0.939	−0.226	−0.449
0.8	1.216	−1.335	0.649	0.489	−0.973	0.540	−0.968	0.914	−0.270	−0.432
0.9	1.086	−1.268	0.693	0.392	−0.977	0.448	−0.878	0.885	−0.312	−0.403
1.0	0.962	−1.197	0.727	0.295	−0.962	0.364	−0.792	0.852	−0.350	−0.364
1.2	0.738	−1.047	0.767	0.109	−0.885	0.223	−0.629	0.775	−0.414	−0.268
1.4	0.544	−0.893	0.772	−0.056	−0.761	0.112	−0.482	0.688	−0.456	−0.157
1.6	0.381	−0.741	0.746	−0.193	−0.609	0.029	−0.354	0.594	−0.477	−0.047
1.8	0.247	−0.596	0.696	−0.298	−0.445	−0.030	−0.245	0.498	−0.476	0.054
2.0	0.142	−0.464	0.628	−0.371	−0.283	−0.070	−0.155	0.404	−0.456	0.140
3.0	−0.075	−0.040	0.225	−0.349	0.226	−0.089	0.057	0.059	−0.213	0.268
4.0	−0.050	0.052	0.000	−0.106	0.201	−0.028	0.049	−0.042	0.017	0.112
5.0	−0.009	0.025	−0.033	0.015	0.046	0.000	−0.011	−0.026	0.029	−0.002

From *Drilled Pier Foundations*, by R. J. Woodwood, W. S. Gardner, and D. M. Greer. Copyright 1972 by McGraw-Hill. Used with the permission of McGraw-Hill Book Company.

The magnitude of the lateral load Q_g limited by the displacement condition only is,

$$\therefore \quad Q_g = \frac{\Delta(E_p I_p)}{A_z T^3} = \frac{(0.008 \text{ m})(207 \times 10^6 \text{ kN / m}^2)(123 \times 10^{-6} \text{ m}^4)}{(2.435)(1.16)^3} = 54 \text{ kN}$$

Since the value of the allowable lateral load Q_g found above is based on the limiting displacement conditions only, and ignores that the pile has a moment capacity, that moment capacity at any depth z is found through,

$$M_z = A_m Q_g T$$

The table above shows that the maximum value of A_m at any depth is 0.772. The maximum allowable moment that the pile can carry is,

$$M_{max} = F_y \frac{I_p}{d_1 / 2} = (248 \text{ MN / m}^2) \frac{(123 \times 10^{-6} \text{ m}^4)}{(0.254 \text{ m} / 2)} = 240 \text{ kN} - m$$

$$\therefore \quad Q_g = \frac{M_{max}}{A_m T} = \frac{(240 \text{ kN} - m)}{(0.772)(1.16 \text{ m})} = 268 \text{ kN}$$

This last value of Q_g emanating from the moment capacity is much larger than the value of $Q_g = 54$ kN found for the deflection criterion.

Therefore use, $Q_g = 54$ kN.

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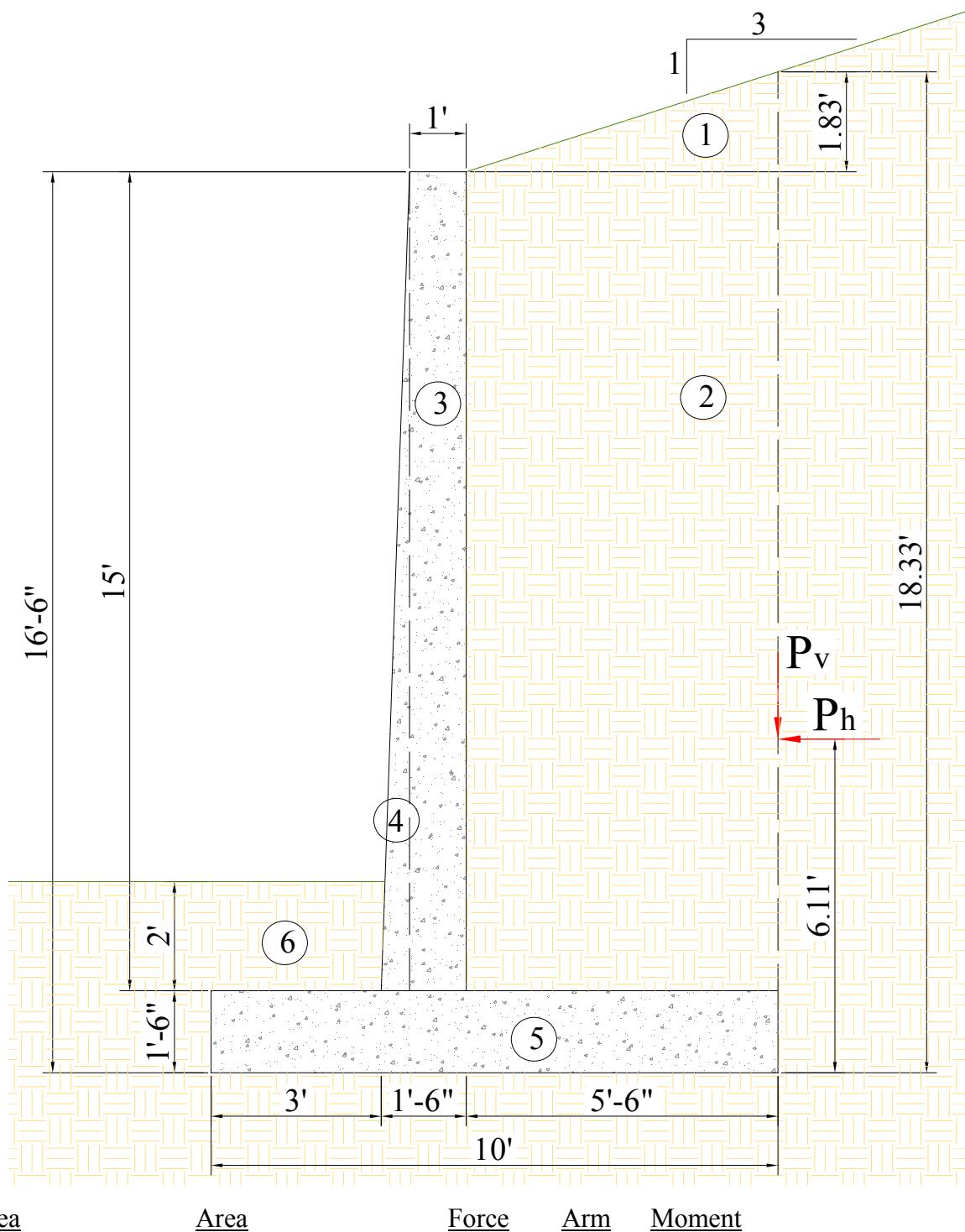
Chapter 24
# Reinforced Concrete Retaining Walls and Bridge Abutments

Symbols for Reinforced Concrete Retaining Walls

(Revision: Oct-08)

Design a reinforced concrete wall with a backfill γ = 125 pcf, an allowable soil bearing capacity of
q_{all} = 3 ksf, and a friction at the base of φ = 30°. Design the wall and check for it's stability under
working loads. (Note: All loads, shears and moments are per linear ft. of retaining wall).

Solution:

Step 1: Find

The active la

$$K_a = \tan^2 \left(4 \right.$$

The presure

$$p_b = \gamma H K_a =$$

The forces o

$$P_v = ½(12)(1$$

$$P_h = ½(39)(1$$

Step 2: Stabi

Moment
About A

Area   Area   Force   Arm   Moment

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			(kip)	(ft)	(kip-ft)
1	½ x 5.5x 1.83	= 5.03 x 0.125 =	0.63	1.83	1.2
and2	5.5 x 15.0	= 82.5 x 0.125 =	10.31	2.75	28.4
3	1.0 x 15.0	= 15.0 x 0.150 =	2.25	6.00	13.5
4	½ x 0.5 x 15.0	= 3.75 x 0.150 =	0.56	6.67	3.7
5	10.0 x 1.5	= 15.0 x 0.150 =	2.25	5.00	11.3
6	3.0 x 2.0	= 6.0 x 0.125 =	0.75	8.50	6.4
P_v			2.00		
P_h		$\Sigma H = 6.60$		6.11	40.4
			$\Sigma V = 18.75$		$\Sigma M = 104.9$

Location of Resultant

From point A, $\dfrac{104.9}{18.75} = 5.6$ ft

then e = $5.6 - \dfrac{10}{2} = 0.6$ ft   o.k. < $\dfrac{B}{6}$

Soil Pressure at Toe of Base

$q_{max} = \dfrac{18.75}{10} (1 + \dfrac{6 \times 0.6}{10}) = 1.875 (1 + 0.36) = 2.55$ ksf   OK < 3 ksf

Check F against Sliding

Shear available along base = 18.75 kips x 0.58 = 10.9 kips

Passive force at toe

Use S = 2/3 (30º) = 20º , $P_p = \dfrac{(\cos \delta)}{½ \gamma H^2}$ = 5.8 kips

$P_p = \dfrac{5.8 (0.125) (3.5)^2}{2 (0.940)}$ = 4.7 kips

Min. F = $\dfrac{10.9}{6.6}$ = 1.7 kips , Max F = $\dfrac{10.9 + 4.7}{6.6} = \dfrac{15.6}{6.6}$ = 2.4 kips   OK without Key

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Step 2: Design parameters.

<u>Load Factors</u>

Stem – Use 1.7 P_h

Base (toe and heel) – distribute ΣV uniformly over front B/3

<u>Concrete and Steel Data</u>

Capacity reduction factors: 0.90(flexure); 0.85(shear)
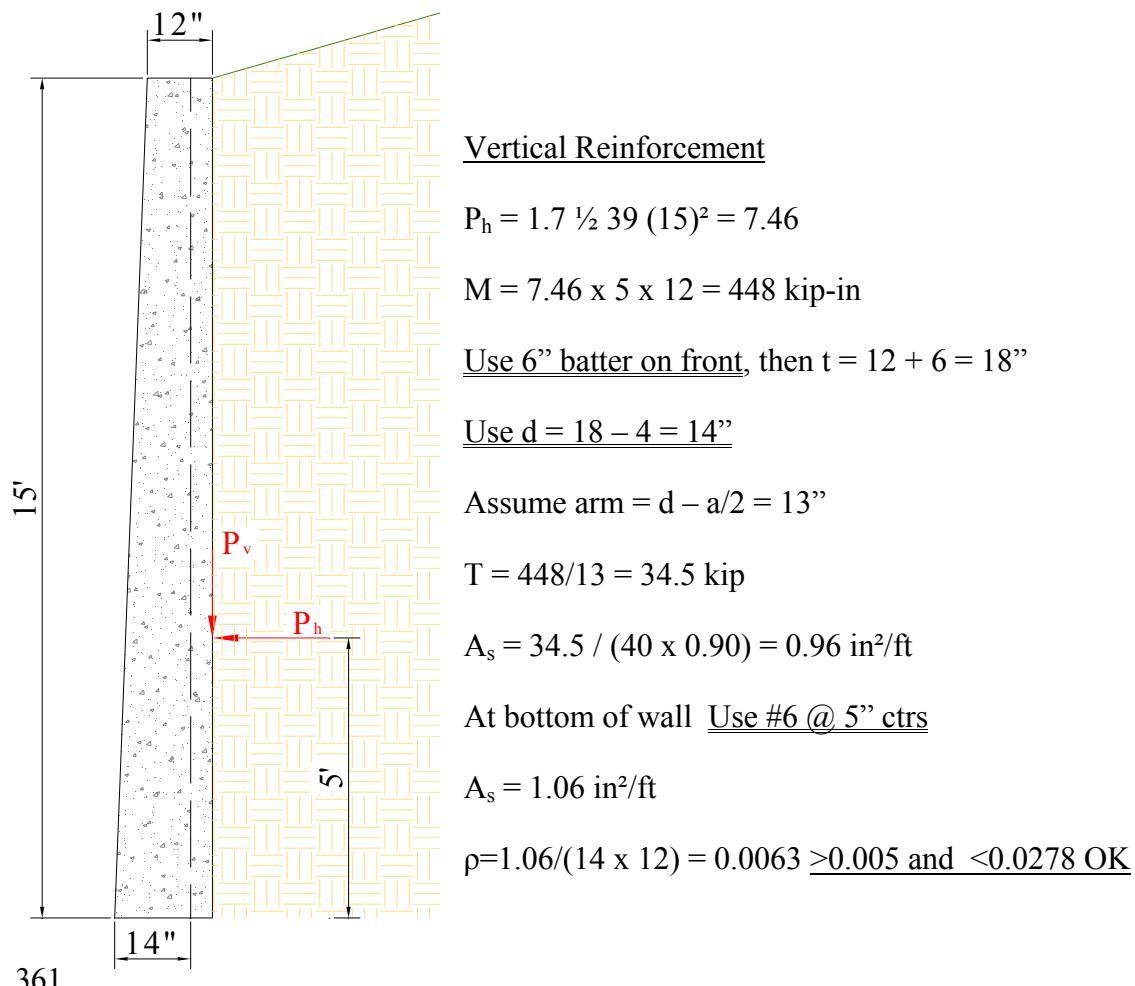
F'_c = 3,000psi x 0.85 = 2,550 psi (for stress block)

V_c = 2 $\sqrt{3,000}$ = 110 psi

f_y = 40,000 psi ; ρ_{min} = 0.005 ; ρ_{max} = 0.0278 ; $\rho_{shrinkage}$ = 0.002

l_d = $\dfrac{0.04\ A_b\ (40,000)}{\sqrt{3,000}}$ = 29.2 A_b (bottom bars) x 1.4 = 40.9 (top bars)

Step 3: Design the stem of the wall.

<u>Vertical Reinforcement</u>

P_h = 1.7 ½ 39 (15)² = 7.46

M = 7.46 x 5 x 12 = 448 kip-in

<u>Use 6" batter on front</u>, then t = 12 + 6 = 18"

<u>Use d = 18 – 4 = 14"</u>

Assume arm = d – a/2 = 13"

T = 448/13 = 34.5 kip

A_s = 34.5 / (40 x 0.90) = 0.96 in²/ft

At bottom of wall <u>Use #6 @ 5" ctrs</u>

A_s = 1.06 in²/ft

ρ=1.06/(14 x 12) = 0.0063 >0.005 and <0.0278 OK

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Check compressive stress block:

C = T = 34.5 / (2.55 x 12 x 0.90) = α_{min} = 1,25 in OK < 2.0

Check Shear:

7.46 / (14 x 12 x 0.85) = 52psi OK < 110 psi

Moment computations for steel cutoff:

At bottom : M = 448 kips

At 10' level : M = 448 (10/15)³ = 133 kip-in
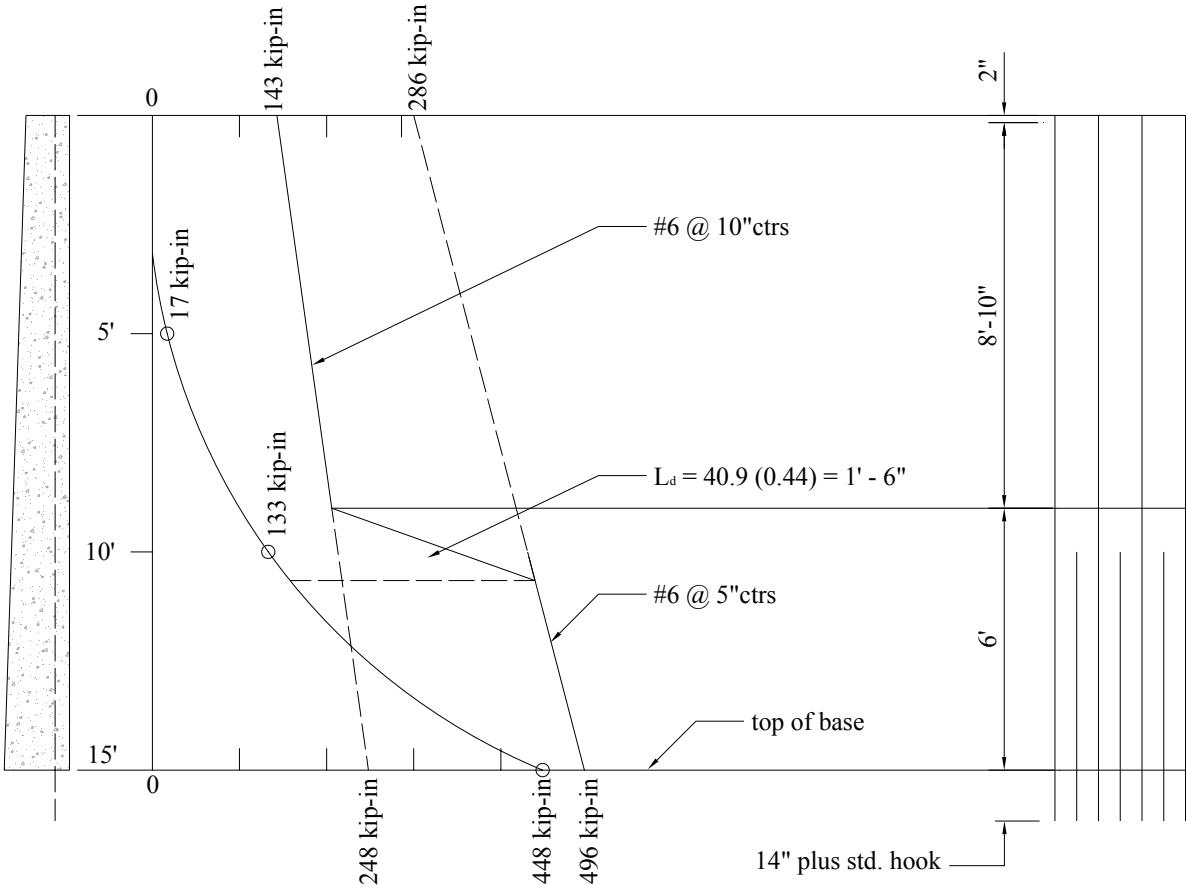
At 5' level : M = 448 (5/15)³ = 17 kip-in

Resisting moment of steel :

#6 @ 5" ctrs. ; A_s = 1.06 in/ft

At bottom : M_r = 1.06 (40) (0.90) (13) = 496 kip-in

At top : M_r = 106 (40) (0.90) (7.5) = 286 kip-in

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<u>Compute level of cutoff for #6 @ 10" ctrs.</u> :

As = 0.53 in²/ft ; ρ = 0.53 / ($\pm$11 x 12) = 0.004 > 0.002 < 0.005

Therefore, cutoff and develop bars above level where:

M = $\dfrac{0.53\ (40)\ (\pm 11)\ (0.90)}{1.33}$ = 158 kip-in

M = 158kip-ft @ $\pm$ 10'-6" level

Developmental length = 1'-6"

<u>Level of Cutoff = 9'-0"</u>

Step 4: Design the toe of the base of the wall.

Distribute ΣV over front B/3. Assume t = 18" , d =14"
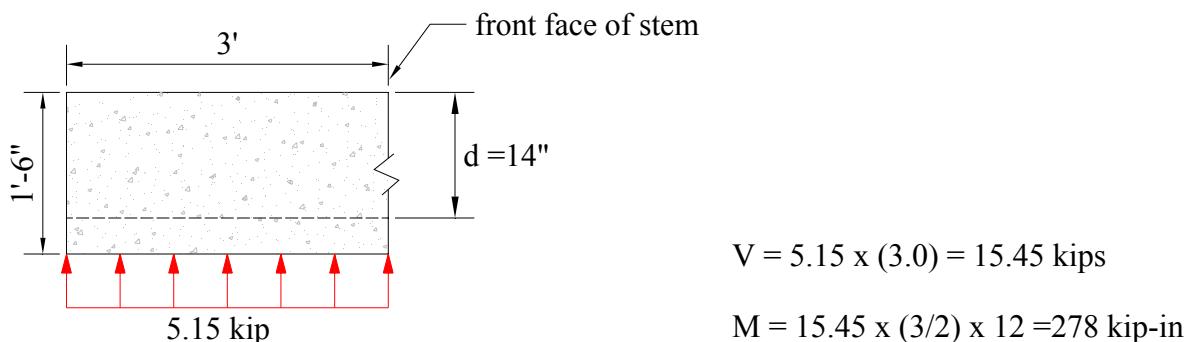
ΣV = $\dfrac{18.75\ kips}{(10\ /\ 3)}$ = q_{max} = 5.63 ksf

wt. of soil over toe = 2.0 (0.125) = 0.25 ksf

wt of concrete base = 1.5 (0.150) = 0.23 ksf

<u>Net toe pressure for design</u> = qmax - wt. of soil over toe - wt of concrete base

= 5.63ksf - 0.25ksf - 0.23 ksf = <u>5.15 ksf</u>

V = 5.15 x (3.0) = 15.45 kips

M = 15.45 x (3/2) x 12 =278 kip-in

Assume arm = 13 in

T = 278/13 = <u>21.4 kips</u> = 0.59 in²/ft

(40 x 0.9)

Check ρ = $\dfrac{0.59}{}$ = 0.0035 > 002, shrinkage OK

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14 x 12

P< 0.005, so increase A_s by 1/3 , then A_s = 0.59 x 1.33 = 0.80 in²/ft

use #6 @ 6" ctrs. A_s = 0.88 in²/ft

Compressive stress block and shear OK by inspection after stem computations

Development length : Extend full base width, therefore l_d OK

Step 5: Design the heel of the base of the wall.

At Stem:

Weight of soil above heel at back face of stem = 15.0 (0.125) = 1.88 ksf

Weight of concrete base = 1.5 (0.150) = 0.23 ksf
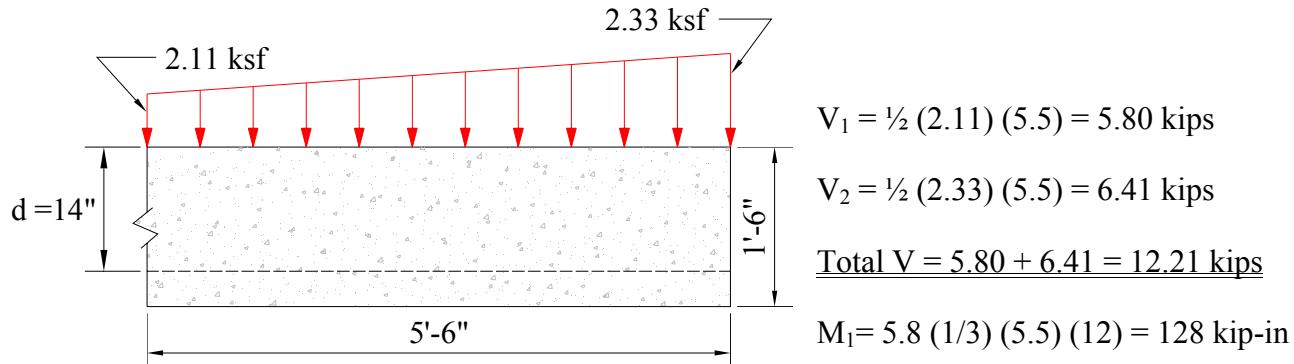
Net pressure for design = 1.88 + 0.23 = 2.11 ksf

At Back:

Weight of soil above hell at back = 16.83 (0.125) = 2.10 ksf

Weight of concrete base = 0.23 ksf

Net pressure for design = 2.33 ksf

V_1 = ½ (2.11) (5.5) = 5.80 kips

V_2 = ½ (2.33) (5.5) = 6.41 kips

Total V = 5.80 + 6.41 = 12.21 kips

M_1 = 5.8 (1/3) (5.5) (12) = 128 kip-in

M_2 = 6.41 (2/3) (5.5) (12)=282 kip-in

Total M = 128 + 282 = 410 kip-in

Assume arm = 13"

T= 410/13 = 31.5 kip

$A_s = \dfrac{31.5kip}{(40 \times 0.9)}$ = 0.88 in²/ft

Use #6 @ 6" ctrs. A_s = 0.88 in²/ft ρ > 0.002 and >0.005 and < 0.0278 OK

Compressive stress block and shear OK

Development length: Extend full base width, therefore l_d OK

Horizontal shrinkage steel in stem:

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Required: 0.002 (±15) (12) = 0.36 in²/ft of height
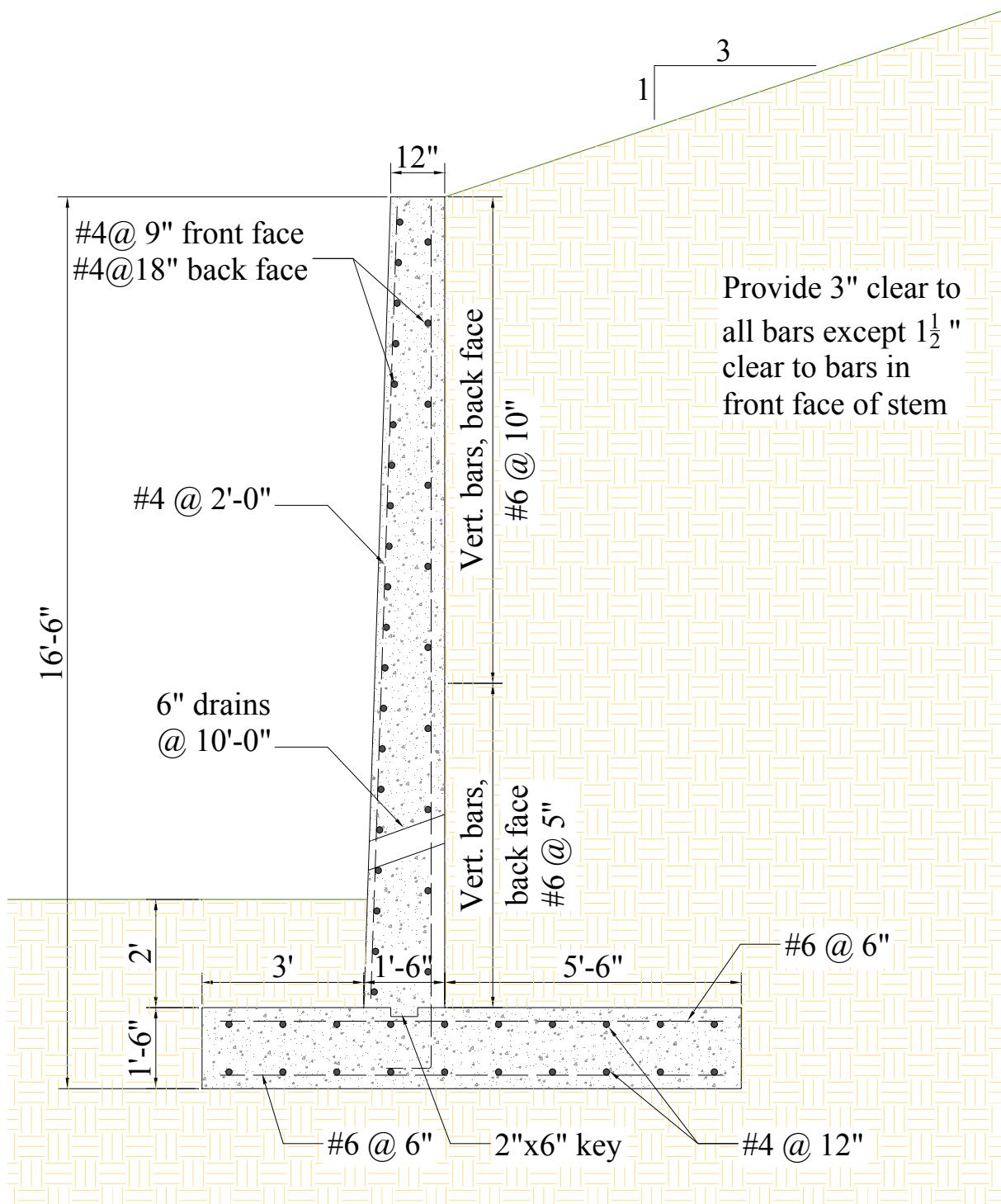
<u>Use #4 @ 9" ctrs. front</u> A_s = 0.27 in²/ft

<u>Use #4 @ 18" ctrs back</u> A_s = 0.13 in²/ft

<u>Total A_s = 0.27 + 0.13 = 0.40 in²/ft</u>

Horizontal shrinkage steel in base: <u>Use #4 @ 12 ctrs. top and bottom</u> A_s = 0.40 in²/ft

Step 6: Finished sketch of the wall.
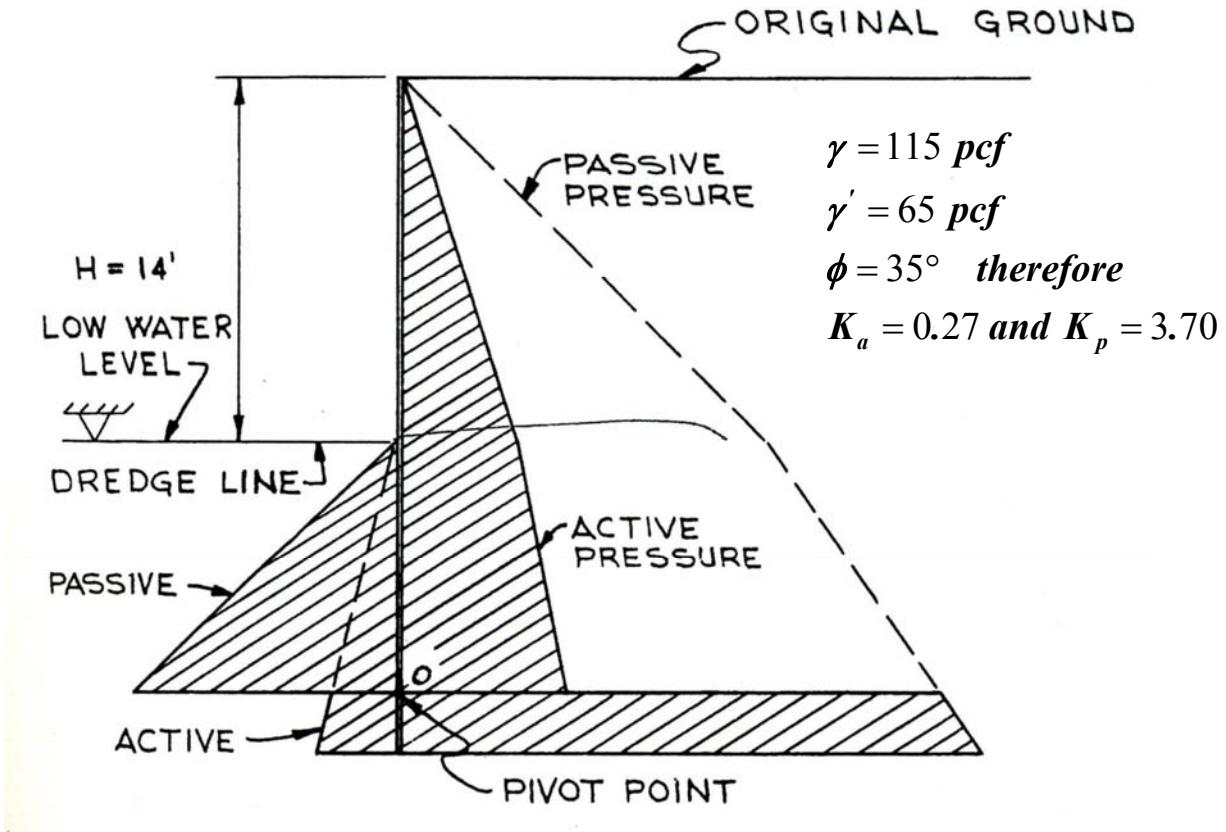
Chapter 25
Steel Sheet Pile Retaining Walls

Symbols for Steel Sheet Pile Retaining Walls
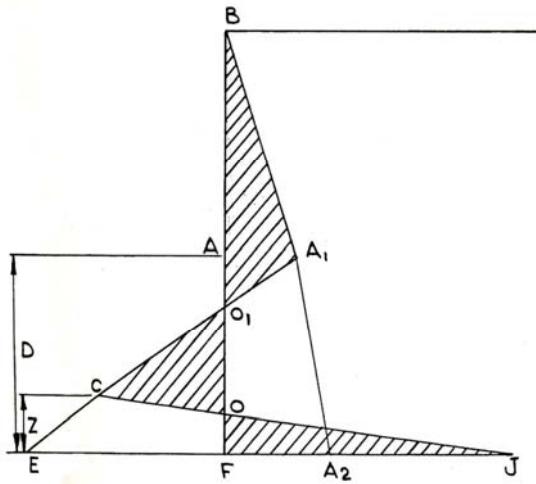
Select an appropriate steel sheet-pile section and its total length L to retain a medium sand backfill for the conditions shown below without an anchoring system.

$\gamma = 115 \ pcf$

$\gamma' = 65 \ pcf$

$\phi = 35° \quad therefore$

$K_a = 0.27 \ and \ K_p = 3.70$

Solution:

Step 1. Determine the pressures on the cantilevered wall.

Step 2. Determine the depth of embedment D.

The procedure will follow the following steps:

    a) From statics, find the dimension z shown in the figure of previous page,

    b) Assume a value for D,

    c) Calculate z,

    d) Use z to check if $\Sigma M = 0$; adjust D for convergence.

$$p_{A1} = \gamma H K_a = (115)(14)(0.27) = 435 \ psf$$

$$p_{A2} = p_{A1} + \gamma D K_a$$
$$= (435) + (65)(0.27)D = 435 + 17.6D$$

$$p_E = \gamma D\left(K_p - K_a\right) - p_{A1} = 65D(6.29) - 435$$
$$= 409D - 435$$

$$p_J = \gamma D\left(K_p - K_a\right) + \gamma H K_p$$
$$= 65D(6.29) + (115)(14)(6.56)$$
$$= 409D + 10,560$$

$Try \ D = 10.5 \ feet,$

$p_{A1} = 435 \ psf,$

$p_{A2} = 620 \ psf,$

$p_J = 14,860 \ psf$

$p_E = 3,860 \ psf$

$$(3,860 - 435)(10.5) - (14)(435)$$

(1) $\Sigma F_H = 0$

$Area(BAA_1) + Area(AA_1A_2F) + Area(ECJ) - Area(EA_1A_2) = 0$

$\dfrac{1}{2}Hp_{A1} + (p_{A1} + p_{A2})\left(\dfrac{D}{2}\right) + (p_E + p_J)\left(\dfrac{z}{2}\right) - (p_E + p_{A2})\left(\dfrac{D}{2}\right) = 0$
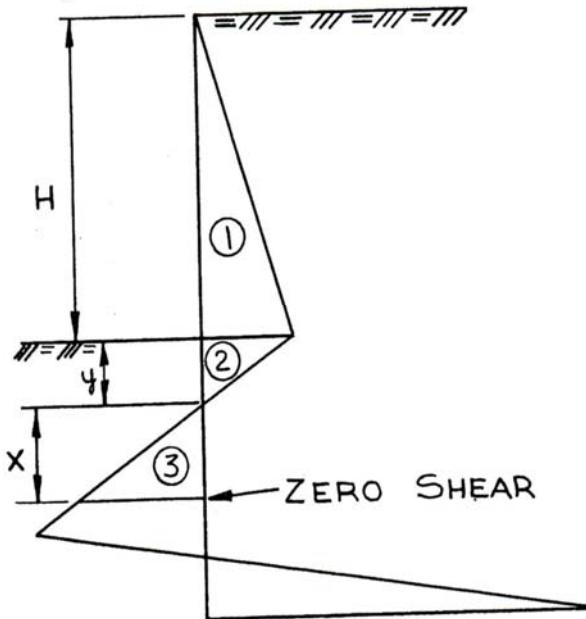
Solving for z,

$z = \dfrac{(p_E - p_{A1})D - Hp_1}{p_E + p_J}$

(2) $\Sigma M = 0$ *about any point, say F,*

$\dfrac{1}{2}(H)p_{A1}\left(D + \dfrac{H}{3}\right) + (p_{A1})\left(\dfrac{D^2}{2}\right) + (p_E + p_J)\left(\dfrac{z^2}{6}\right) - (p_E + p_{A2})\left(\dfrac{D^2}{6}\right) + (p_{A2} - p_{A1})\left(\dfrac{D^2}{6}\right) = 0$

<u>Step 3. Determine the maximum moment *Mmax* (point of zero shear).</u>

$y = \dfrac{p_{A1}}{\gamma'(K_p - K_a)} = \dfrac{435}{(65)(6.29)} = 1.06 \ feet$

$P_1 = \dfrac{1}{2}p_{A1}H = \dfrac{1}{2}(435)(14) = 3{,}040 \ lb$

$P_2 = \dfrac{1}{2}p_{A1}y = \dfrac{1}{2}(435)(1.0) = 218 \ lb$

also,

$\dfrac{1}{2}\gamma'(K_p - K_a)x^2 = P_1 + P_2$

$\therefore \quad x^2 = \dfrac{2(P_1 + P_2)}{\gamma'(K_p - K_a)} = \dfrac{2(3{,}040 + 218)}{(65)(6.29)} = 16 \quad \therefore x = 4.0 \ feet$

$P_3 = \dfrac{1}{2}\gamma'(K_p - K_a)(4)^2 = 3{,}280 \ lb$

$M_{max} = P_1 l_1 + P_2 l_2 - P_3 l_3$

where,
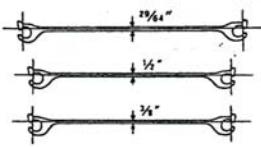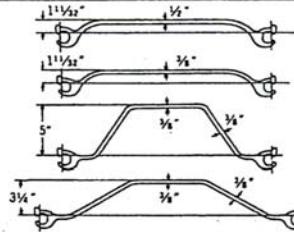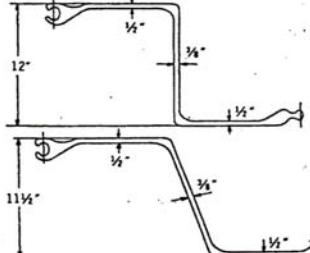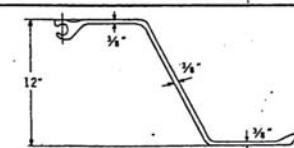
$l_1 = \dfrac{H}{3} + y + x$

$l_2 = \dfrac{2y}{3} + x$

$l_3 = \dfrac{x}{3}$

$M_{max} = P_1 l_1 + P_2 l_2 - P_3 l_3 = (3{,}040)\left(\dfrac{14}{3} + 1.0 + 4.0\right) + (218)\left(\dfrac{2(1.0)}{3} + 4.0\right) - (3{,}280)\left(\dfrac{4.0}{3}\right)$
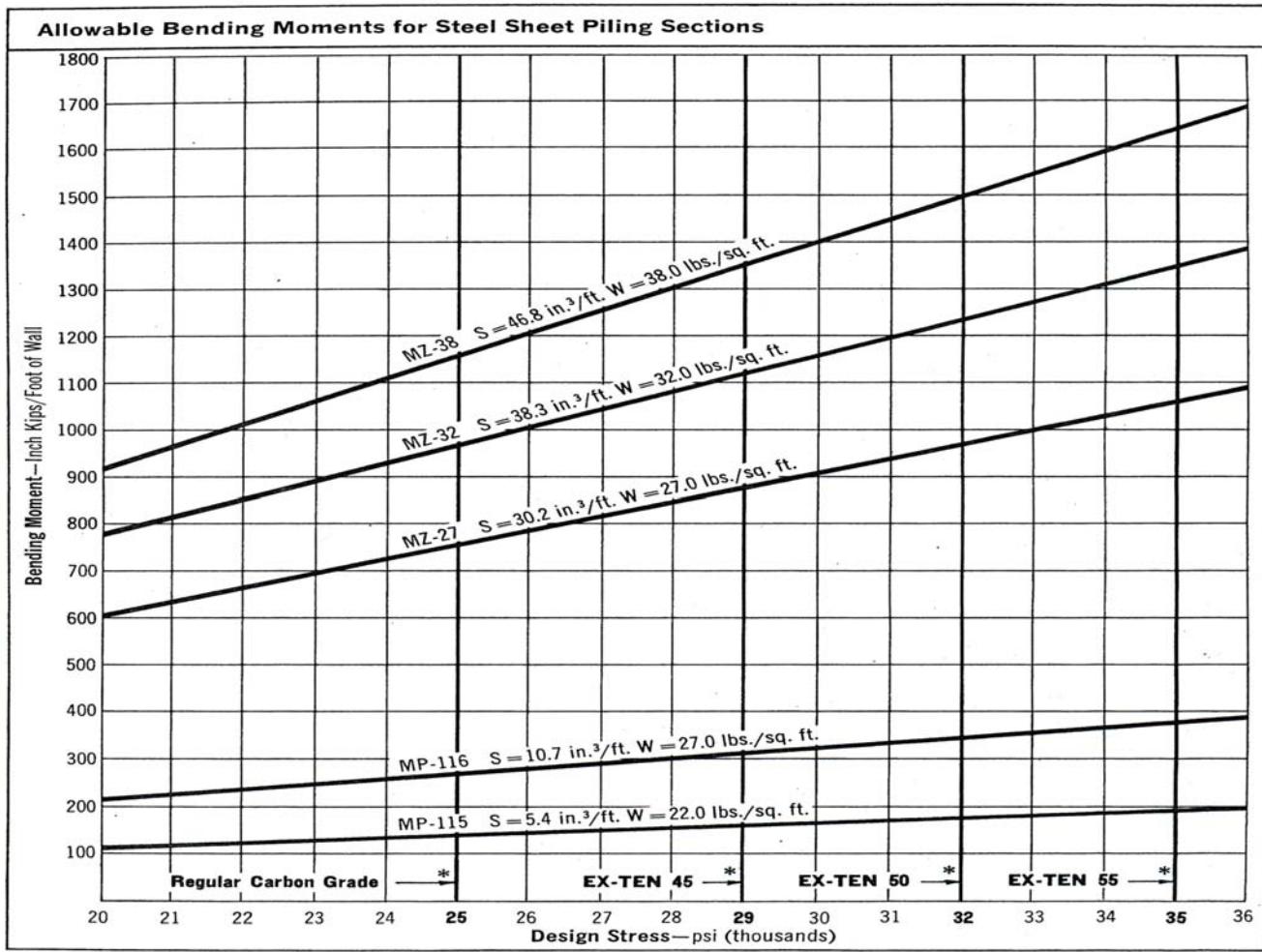
$M_{max} = 29{,}300 + 1{,}030 - 4{,}360 = 26{,}000 \ ft-lb = 312 \ in-kips$

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Steel Sheet Piling Sections

Profile		Section Index	District Rolled	Driving Distance per Pile (In.)	Weight Per Foot (Lbs.)	Weight Per Square Foot of Wall (Lbs.)	Web Thickness (In.)	Section Modulus Per Pile (In.³)	Section Modulus Per Foot of Wall (In.³)	Area Per Pile (In.²)	Moment of Inertia Per Pile (In.⁴)	Moment of Inertia Per Foot of Wall (In.⁴)
	Interlock with Each Other	MP 103	P.	16½	44.0	32.0	29/64	3.3	2.4	12.94	5.1	3.7
		MP 102*	P.C.	15	40.0	32.0	½	2.4	1.9	11.77	3.6	2.9
		MP 101	P.C.	15	35.0	28.0	3/8	2.4	1.9	10.30	3.5	2.8
	Interlock with Each Other	MP 113*	P.	16	37.3	28.0	½	3.3	2.5	10.98	6.0	4.5
		MP 112	P.C.	16	30.7	23.0	3/8	3.2	2.4	8.99	5.5	4.1
		MP 116	P.C.	16	36.0	27.0	3/8	14.3	10.7	10.59	53.0	39.8
		MP 115	P.C.	19⅝	36.0	22.0	3/8	8.8	5.4	10.59	22.4	13.7
	Interlock with Each Other and with MP 112 or MP 113	MZ 38	P.	18	57.0	38.0	3/8	70.2	46.8	16.77	421.2	280.8
		MZ 32	P.	21	56.0	32.0	3/8	67.0	38.3	16.47	385.7	220.4
	Interlocks with Itself and with MP 112 or MP 113	MZ 27	P.	18	40.5	27.0	3/8	45.3	30.2	11.91	276.3	184.2

Suggested Allowable Design Stresses—Sheet Piling

Steel Brand or Grade	Minimum Yield Point, psi	Allowable Design Stress, psi*
USS EX-TEN 55	55,000	35,000
USS EX-TEN 50	50,000	32,000
USS EX-TEN 45	45,000	29,000
Regular Carbon Grade	38,500	25,000

*Based on 65% of minimum yield point. Some increase for temporary overstresses generally permissible.

Allowable Bending Moments for Steel Sheet Piling Sections

If instead, an Ex − Ten − 45 steel section is used, $f_s = 29$ ksi

The required section modulus $S = \dfrac{M_{max}}{f_s} = \dfrac{(26,000)(12)}{(29,000)} = 10.8 \; in^3$

∴ Use a MP − 116 that provides a $S = 10.7 \; in^3$ at a lower cost.
This is shown plotted with the red lines.

Step 5. Determine the sheet-pile length L.

The total sheet-pile length $L = H + D = 14.0 + 13.5 = 27.5$ feet

Chapter 26
MSE (Mechanically Stabilized Earth) Walls
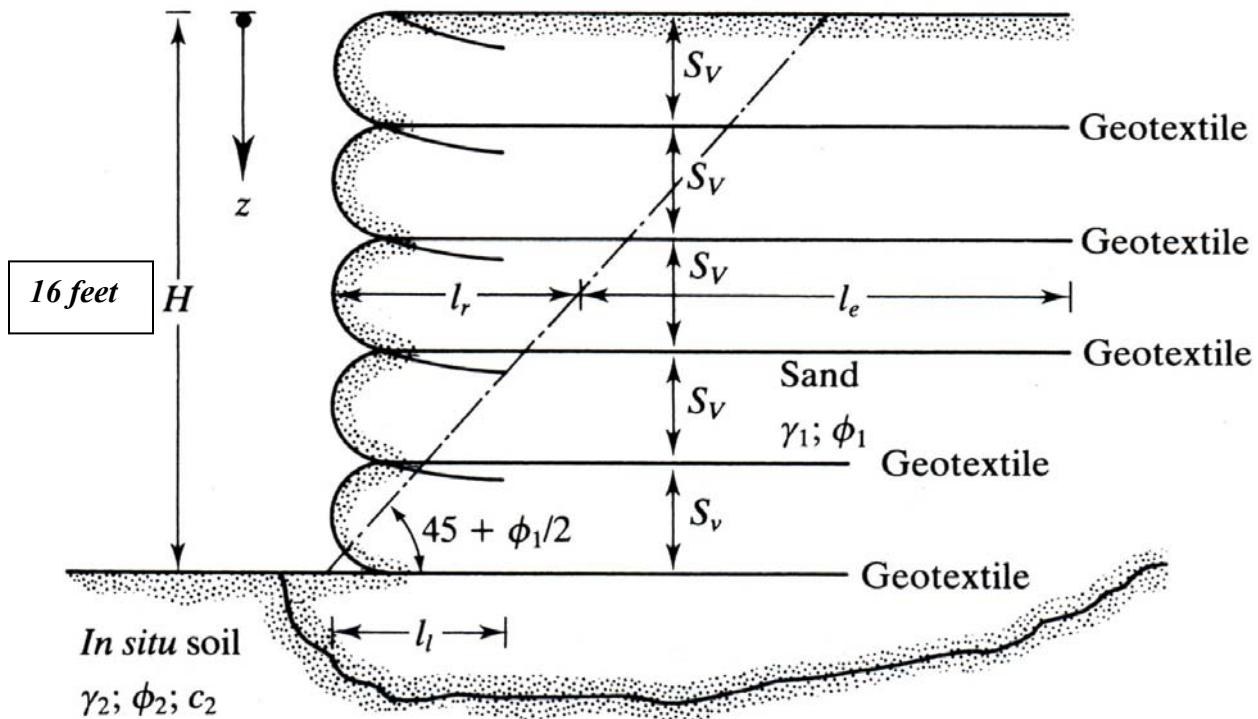
Symbols for Mechanically Stabilized Earth Walls

MSE Walls-01: *Design the length L of geotextiles for a 16 ft wall.*

(Revision: Oct-08)

Determine the length L of a geotextile-reinforcing for the 16 foot high temporary retaining wall shown below. Also determine the required vertical spacing of the reinforcing layers S_v and the required lap length l_l. The geotextile chosen has an allowable strength of σ_G of 80 lb/inch, and the available granular backfill has a unit weight of $\gamma = 110$ lb/ft³ and the angle of internal friction is $\varphi = 36°$.

Comment: These geotextile walls are usually used for temporary civil works, such as detour roadways, temporary abutments or excavation walls, etc. If the wall must become permanent, then the face is stabilized and the plastic geotextile is protected from ultra-violet light through a layer of shotcrete.

Solution:

The active pressure on the wall is,

$$\sigma_a = K_a\sigma_v = K_a(\gamma z) \quad where \; K_a = tan^2\left(45° - \varphi/2\right) = tan^2(45° - 36°/2) = 0.26$$

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Step 1: *Find the vertical spacing S_V of each layer* of geotextile, where the *Factor of Safety (FS)* is generally chosen between 1.3 to 1.5 for temporary walls. Permanent walls should use at least $FS \geq 2$. For this problem choose $FS = 1.5$.

$$S_V = \frac{\sigma_G}{\sigma_a FS} = \frac{\sigma_G}{(\gamma z K_a) FS}$$

At a depth of $z = 8$ feet from the top,
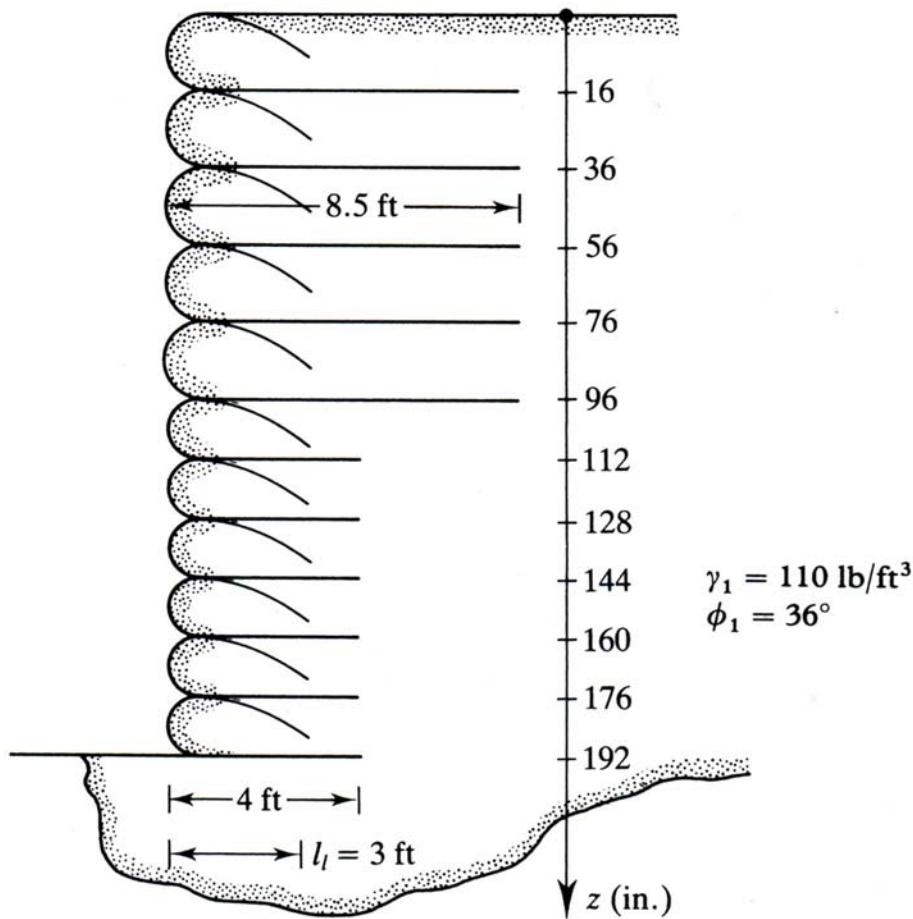
$$S_V = \frac{\sigma_G}{(\gamma z K_a) FS} = \frac{(80 x 12 \; lb / ft)}{(110)(8)(0.26)(1.5)} = 2.8 \; feet \approx 34 \; inches$$

At a depth of $z = 12$ feet from the top,

$$S_V = \frac{\sigma_G}{(\gamma z K_a) FS} = \frac{(80 x 12 \; lb / ft)}{(110)(12)(0.26)(1.5)} = 1.9 \; feet \approx 22 \; inches$$

At a depth of $z = 16$ feet from the top,

$$S_V = \frac{\sigma_G}{(\gamma z K_a) FS} = \frac{(80 x 12 \; lb / ft)}{(110)(16)(0.26)(1.5)} = 1.4 \; feet \approx 17 \; inches$$

Notice how the spacing becomes denser the deeper we go below the surface; choose $S_V = 20$ inches from $z = 0$ to $z = 8$ feet; below $z = 8$ feet use $S_V = 16$ inches throughout, as shown in the figure.

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Step 2: *Find the length of each layer of geotextile L*, which is composed of two parts, l_R which is the length of the geotextile within the Rankine failure zone, and l_e which is the effective length of the geotextile beyond the failure zone (see the first figure, page 408). Again use $FS = 1.5$, and the angle φ_F is the soil-to-geotextile angle of friction, which is usually assumed to be *2/3 φ* of the soil. Other values can be used, and a few are shown in this table,

Type	ϕ_F/ϕ_1
Woven — monofilament/concrete sand	0.87
Woven — silt film/concrete sand	0.8
Woven — silt film/rounded sand	0.86
Woven — silt film/silty sand	0.92
Nonwoven — melt-bonded/concrete sand	0.87
Nonwoven — needle-punched/concrete sand	1.0
Nonwoven — needle-punched/rounded sand	0.93
Nonwoven — needle-punched/silty sand	0.91

Therefore, the length L of the geotextile layer is,

$$L = l_R + l_e = \frac{(H-z)}{\tan(45° - \phi/2)} + \frac{S_V K_a (FS)}{2 \tan \varphi_F} = \frac{(H-z)}{\tan(45° - 36°/2)} + \frac{S_V (0.26)(1.5)}{2(0.445)}$$

$$L = (0.51)(H-z) + 0.438 S_V$$

From this equation, we can now prepare a table with the required lengths.

z		S_V	(0.51)(H-z)	(0.438S_V)	L
inches	feet	feet	feet	feet	feet
16	1.33	1.67	7.48	0.731	8.21
56	4.67	1.67	5.78	0.731	6.51
76	6.34	1.67	4.93	0.731	5.66
96	8.00	1.67	4.08	0.731	4.81
112	9.34	1.33	3.40	0.582	3.99
144	12.00	1.33	2.04	0.582	2.66
176	14.67	1.33	0.68	0.582	1.26

Based on this table, use *L = 8.5 feet for z ≤ 8 feet and L = 4.0 feet for z > 8 feet.*

Step 3: *Find lap length l_l for the geotextile, but never smaller than 3 feet,*

$$l_i = \frac{S_V \sigma_a (FS)}{4\sigma_v \tan \varphi_F} = \frac{S_V (0.26)(1.5)}{4 \tan\left[\left(\frac{2}{3}\right)(36°)\right]} = 0.219 S_V$$

For a depth of z=16 inches,

$$l_i = 0.219 S_V = (0.219)\left(\frac{20 \text{ inches}}{12 \text{ in / ft}}\right) = 0.365 \text{ feet} < 3 \text{ feet}$$

Therefore, use $l_i = 3$ feet

Comment:

These MSE problems commonly use Rankine's active pressure coefficient. However, the actual value of **K** must depend on the degree of restraint of the type of reinforcement, as shown in this figure:

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